Title: Explorations in Cosmology-4

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Abstract:

CLARIFY WICK'S THEOREM

$$\hat{X}_1 = \alpha_1 \hat{a} + \beta_1 \hat{a}^\dagger$$
$$\hat{X}_2 = \alpha_2 \hat{a} + \beta_2 \hat{a}^\dagger$$
$$\vdots$$

$$\langle 0|\hat{X}_1 \hat{X}_2|0\rangle = \langle 0|(\alpha_1 \hat{a})(\beta_2 \hat{a}^\dagger)|0\rangle = \alpha_1 \beta_2$$

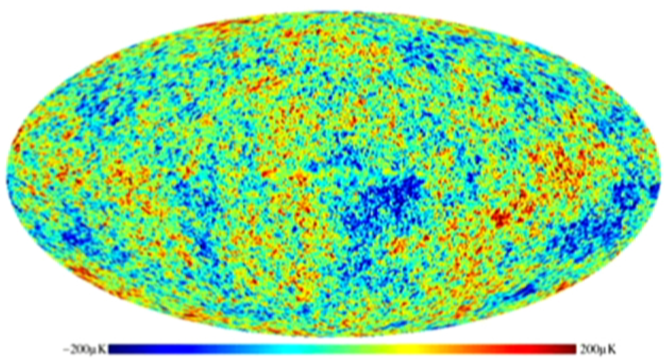
ODD CORRELATOR $\langle 0|\hat{X}_1 \cdots \hat{X}_{2N-1}|0\rangle = 0$

EVEN CORRELATOR

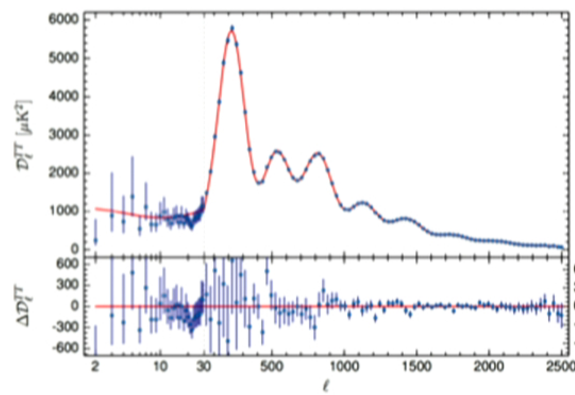
$$\langle 0|\hat{X}_1 \hat{X}_2 \cdots \hat{X}_{2N}|0\rangle = \langle 0|\hat{X}_1 X_2 X_3 X_4 \cdots|0\rangle + \langle 0|X_1 X_2 X_3 X_4 \cdots|0\rangle + \cdots$$
$$= \left(\langle 0|\hat{X}_1 \hat{X}_2|0\rangle\langle 0|\hat{X}_3 \hat{X}_4|0\rangle \cdots\right) + \left(\langle 0|\hat{X}_1 \hat{X}_3|0\rangle\langle 0|\hat{X}_2 \hat{X}_4|0\rangle \cdots\right) + \cdots = (\alpha_1 \beta_2$$

CMB temperature

Temperature map



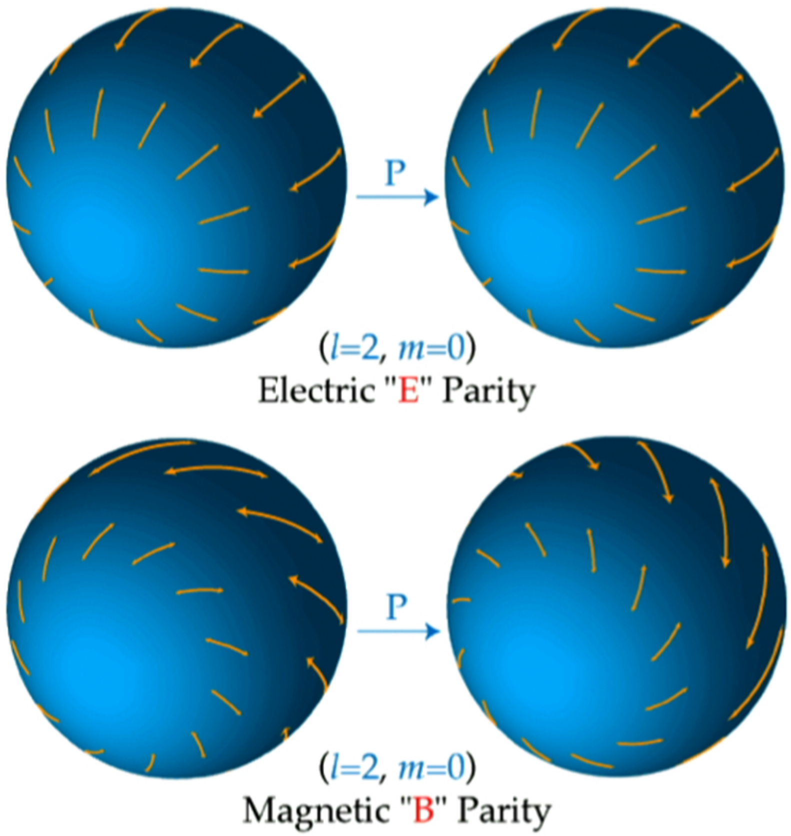
TT power spectrum



Fluctuations are ~100 microK

The CMB is very Gaussian! (~0.1%)

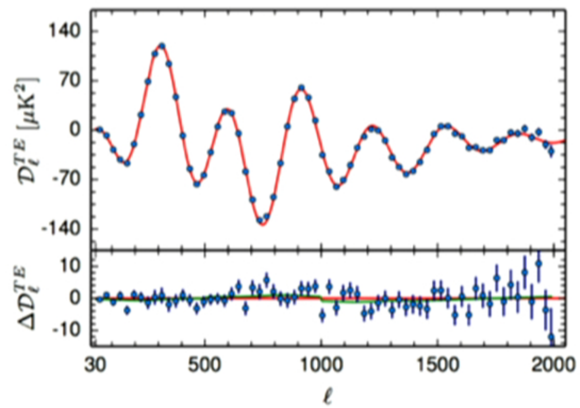
CMB polarization has an E-B decomposition



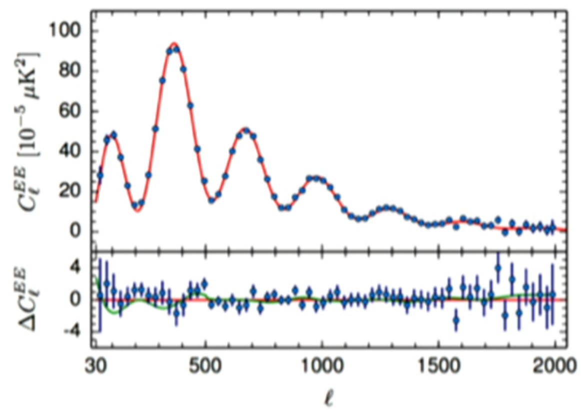
Wayne Hu

E-modes are qualitatively similar to T

TE power spectrum



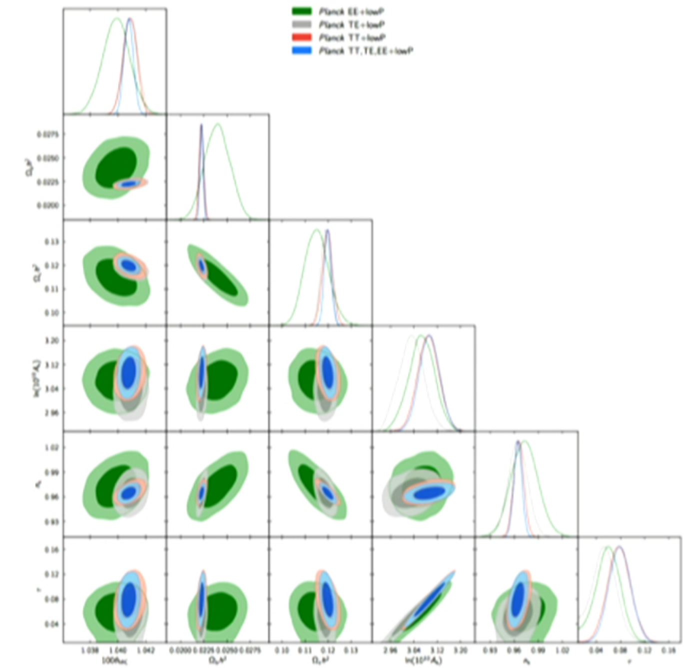
EE power spectrum



E-mode polarization is a few microK

T+E constrain cosmological parameters

Planck Collaboration: Cosmological parameters



Parameter	TT+lowP 68 % limits	TT+lowP+lensing 68 % limits	TT+lowP+lensing+ext 68 % limits	TT,TE,EE+lowP 68 % limits	TT,TE,EE+lowP+lensing 68 % limits	TT,TE,EE+lowP+lensing+ext 68 % limits
$\Omega_b h^2$	0.02222 ± 0.00023	0.02226 ± 0.00023	0.02227 ± 0.00020	0.02225 ± 0.00016	0.02226 ± 0.00016	0.02230 ± 0.00014
$\Omega_c h^2$	0.1197 ± 0.0022	0.1186 ± 0.0020	0.1184 ± 0.0012	0.1198 ± 0.0015	0.1193 ± 0.0014	0.1188 ± 0.0010
$100\theta_{MC}$	1.04085 ± 0.00047	1.04103 ± 0.00046	1.04106 ± 0.00041	1.04077 ± 0.00032	1.04087 ± 0.00032	1.04093 ± 0.00030
τ	0.078 ± 0.019	0.066 ± 0.016	0.067 ± 0.013	0.079 ± 0.017	0.063 ± 0.014	0.066 ± 0.012
$\ln(10^{10} A_s)$	3.089 ± 0.036	3.062 ± 0.029	3.064 ± 0.024	3.094 ± 0.034	3.059 ± 0.025	3.064 ± 0.023
n_s	0.9655 ± 0.0062	0.9677 ± 0.0060	0.9681 ± 0.0044	0.9645 ± 0.0049	0.9653 ± 0.0048	0.9667 ± 0.0040

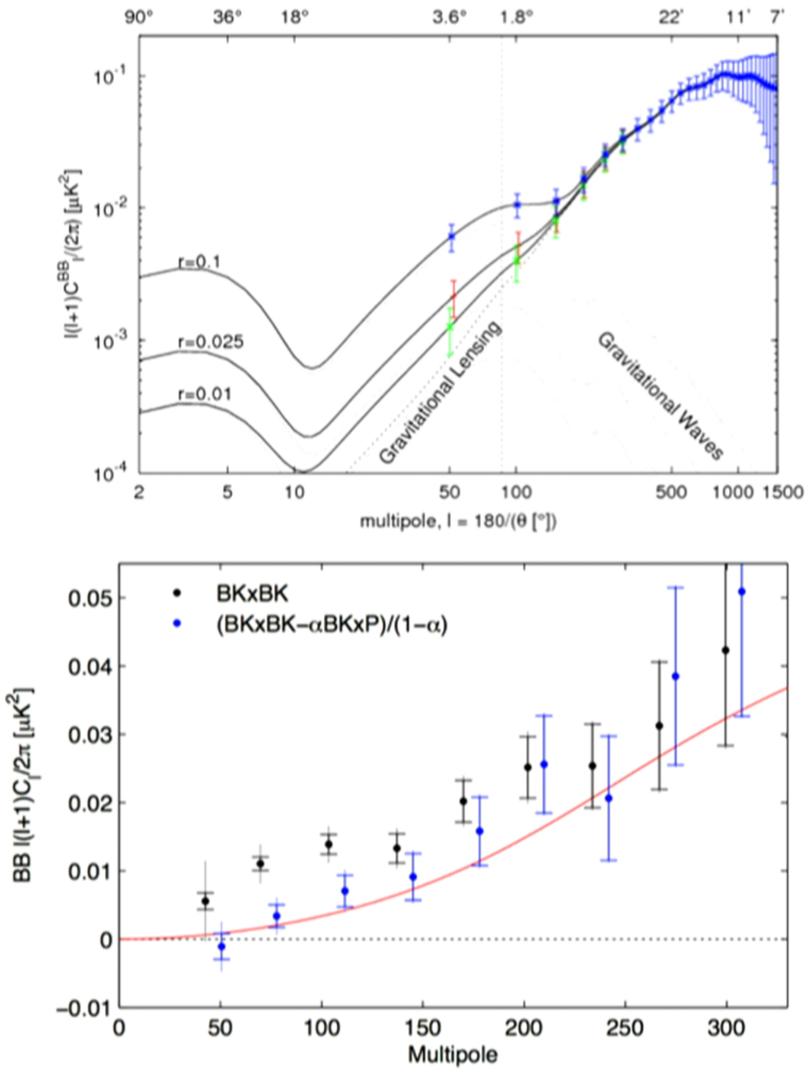
B-modes

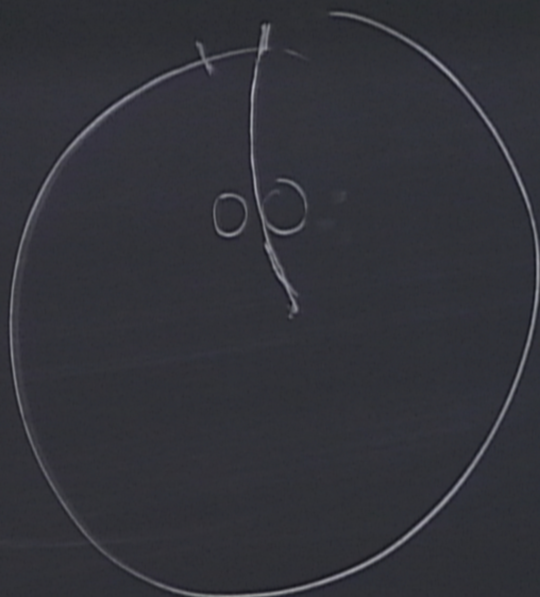
Theorem: scalar sources (e.g. inflaton) do not generate B-mode polarization in linear perturbation theory

Theorem: parity invariance => TB=EB=0
i.e. BB is the only B-mode related power spectrum

In slow-roll field inflation, the leading sources of B-modes are
- gravity waves from inflation!! (maybe)
- gravitational lensing (leading second-order effect; foreground for measuring gravity waves)

Because gravitational lensing is small (second order), B-mode polarization is small (~100 nano-K) and challenging to measure, but potentially a very powerful probe of gravity waves


90°
36°
18°
3.6°
1.8°
22'
11'
7'
r=0.1
r=0.025
r=0.01
Gravitational Lensing
Gravitational Waves
multipole, l = 180/(θ [°])
BKxBK
(BKxBK−αBKxP)/(1−α)
Multipole



$$|T_\ell|^2 \sim C_\ell^{TT}$$

$$|E_\ell|^2 \sim C_\ell^{EE}$$

$$|T_\ell E_\ell|^2 \sim C_\ell^{TE}$$

6-PARAMETER MODEL: $\{\Omega_b, \Omega_c, \Omega_\Lambda, \Delta_s^2, n_s, \tau\}$

- EXPANSION HISTORY: $\{\Omega_b, \Omega_c, \Omega_\Lambda\}$
- INITIAL FLUCTUATIONS $\{\Delta_s^2, n_s\}$

"ADIABATIC SCALAR GAUSSIAN POWER-LAW"

"ADIABATIC SCALAR": $ds^2 = -dt^2 + a(t)^2 e^{2\zeta(x)} dx^2$

"ADIABATIC CURVATURE"
FUNCTION OF x BUT NOT t

$\delta\rho_c^{(i)} = (\#)\,\zeta(x)$

"GAUSSIAN" $\langle \zeta_{\vec{k}} \zeta_{\vec{k}'}^{*} \rangle = P_\zeta(k)\,(2\pi)^3\,\delta^3(\vec{k}-\vec{k}')$

$$\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\zeta_{k_4}\rangle = \langle \zeta_{k_1}\zeta_{k_2}\rangle\langle\zeta_{k_3}\zeta_{k_4}\rangle + [2\ \text{PERM.}]$$

"POWER LAW"

$$\frac{k^3}{2\pi^2}P_\zeta(k) = \Delta_\zeta^2\left(\frac{k}{k_0}\right)^{n_s-1}$$

$n_s = 1 \Rightarrow$ SCALE INVARIANT

$$\langle \zeta^2(x)\rangle = \int \frac{d^3k}{(2\pi)^3}P_\zeta(k)$$

$$= \int_0^\infty d\log k\ \underbrace{\frac{k^3}{2\pi^2}P_\zeta(k)}$$

"GAUSSIAN" $\langle \zeta_{\vec{k}} \zeta_{\vec{k}'} \rangle = P_\zeta(k)\,(2\pi)^3\,\delta^3(\vec{k}-\vec{k}')$

$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle = \langle \zeta_{k_1} \zeta_{k_2} \rangle \langle \zeta_{k_3} \zeta_{k_4} \rangle + [2 \text{ PERM.}]$

$\frac{\ell^2}{2\pi} C_\ell$

"POWER LAW"

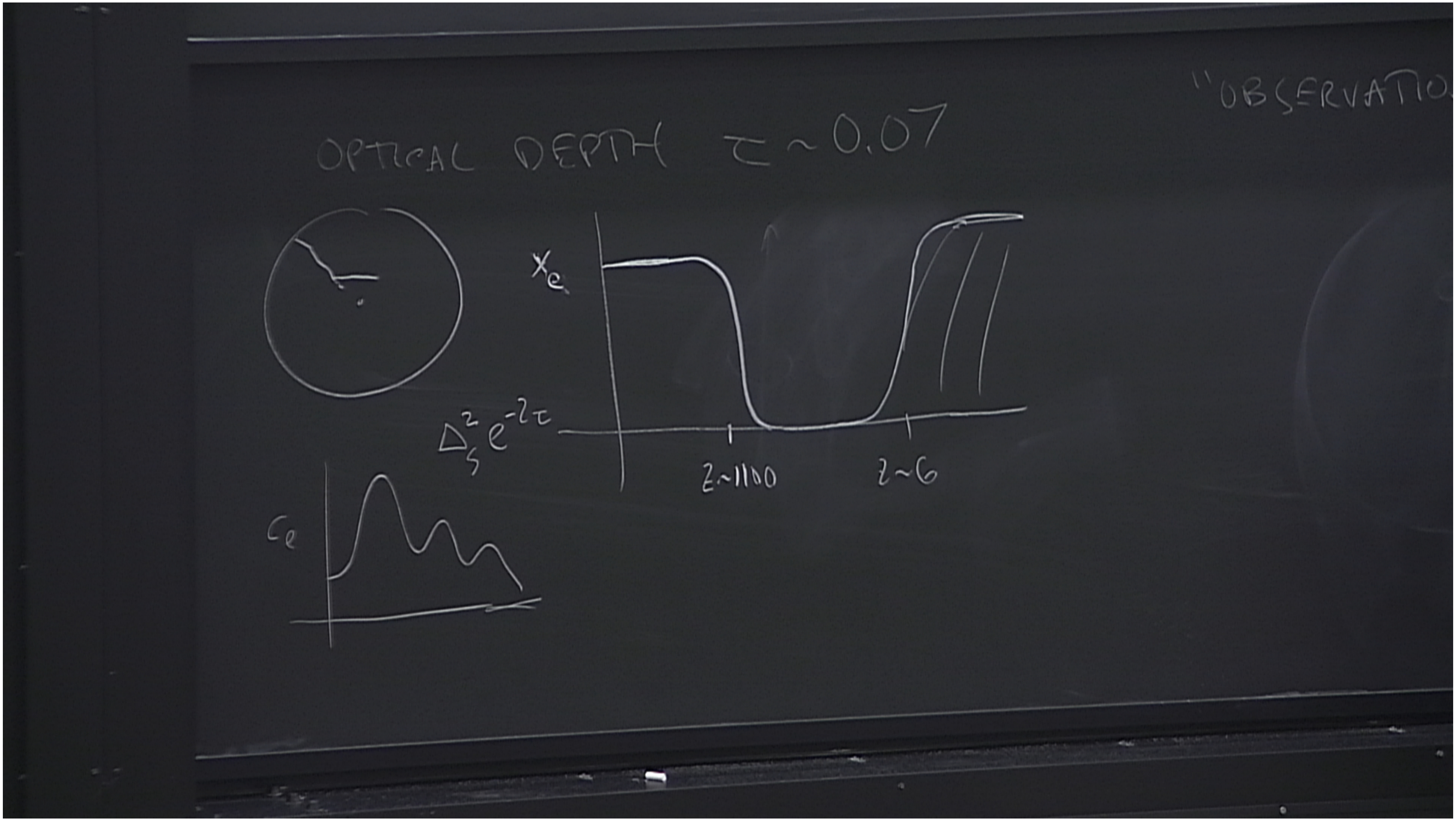
$\frac{k^3}{2\pi^2} P_\zeta(k) = \Delta_\zeta^2 \left(\frac{k}{k_0}\right)^{n_s - 1}$

$n_s = 1 \Rightarrow$ SCALE INVARIANT

$n_s = 0.9667 \pm 0.0040$

$\langle \zeta^2_{(x)} \rangle = \int \frac{d^3k}{(2\pi)^3} P_\zeta(k)$

$= \int_0^\infty d\log k \; \underbrace{\frac{k^3}{2\pi^2} P_\zeta(k)}$


"OBSERVATIO
OPTICAL DEPTH τ ~ 0.07
x_e
$\Delta_s^2 e^{-2\tau}$
z~1100
z~6
C_ℓ

"OBSERVATIONS" = CMB

CMB = COMPLICATED BUT <u>LINEAR</u> BLACK-BOX

$$\zeta_{k} \to \left(T + \text{POLARIZATION ON SKY} \right)$$

SKY IS A SPHERE η_{ab} = EUCLIDEAN 2-METRIC ON SPHERE

$$I_{ab}(\hat{n}) = \left\langle E_a(\hat{n}) \, E_b(\hat{n}) \right\rangle \quad \text{INTENSITY TENSOR}$$

TEMPERATURE $T(\hat{n}) = \underbrace{f(\omega)}_{\text{BLACKBODY}} \times \eta^{ab} I_{ab}(\hat{n})$

LINEAR POLARIZATION $P_{ab} = f(\omega) \times \left[I_{ab} - \frac{1}{2}\eta_{ab}\,\eta^{cd} I_{ab} \right]$ TRACELE

TEMPERATURE IS ANALYZED IN HARMONIC SPACE

IF SKY WERE FLAT:

$$T(\vec{x}) = \int \frac{d^2\ell}{(2\pi)^2} T_{\vec{\ell}}\, e^{i\vec{\ell}\cdot\vec{x}} \qquad \langle T_\ell T^*_{\ell'} \rangle = C_\ell\,(2\pi)^2\,\delta^2(\ell - \ell')$$

ON A SPHERICAL SKY

$$T(\hat{n}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a^T_{\ell m} Y_{\ell m}(\hat{n}) \qquad \langle a^T_{\ell m}\, a^{T*}_{\ell' m'} \rangle = C^{TT}_\ell\, \delta_{\ell\ell'}\,\delta_{mm'}$$

LINEAR POLARIZATION $P_{ab} = f(\omega) \times \left[I_{ab} - \frac{1}{2}\eta_{ab}\,\eta^{cd} I_{ab} \right]$ TRACELE

TEMPERATURE IS ANALYZED IN HARMONIC SPACE

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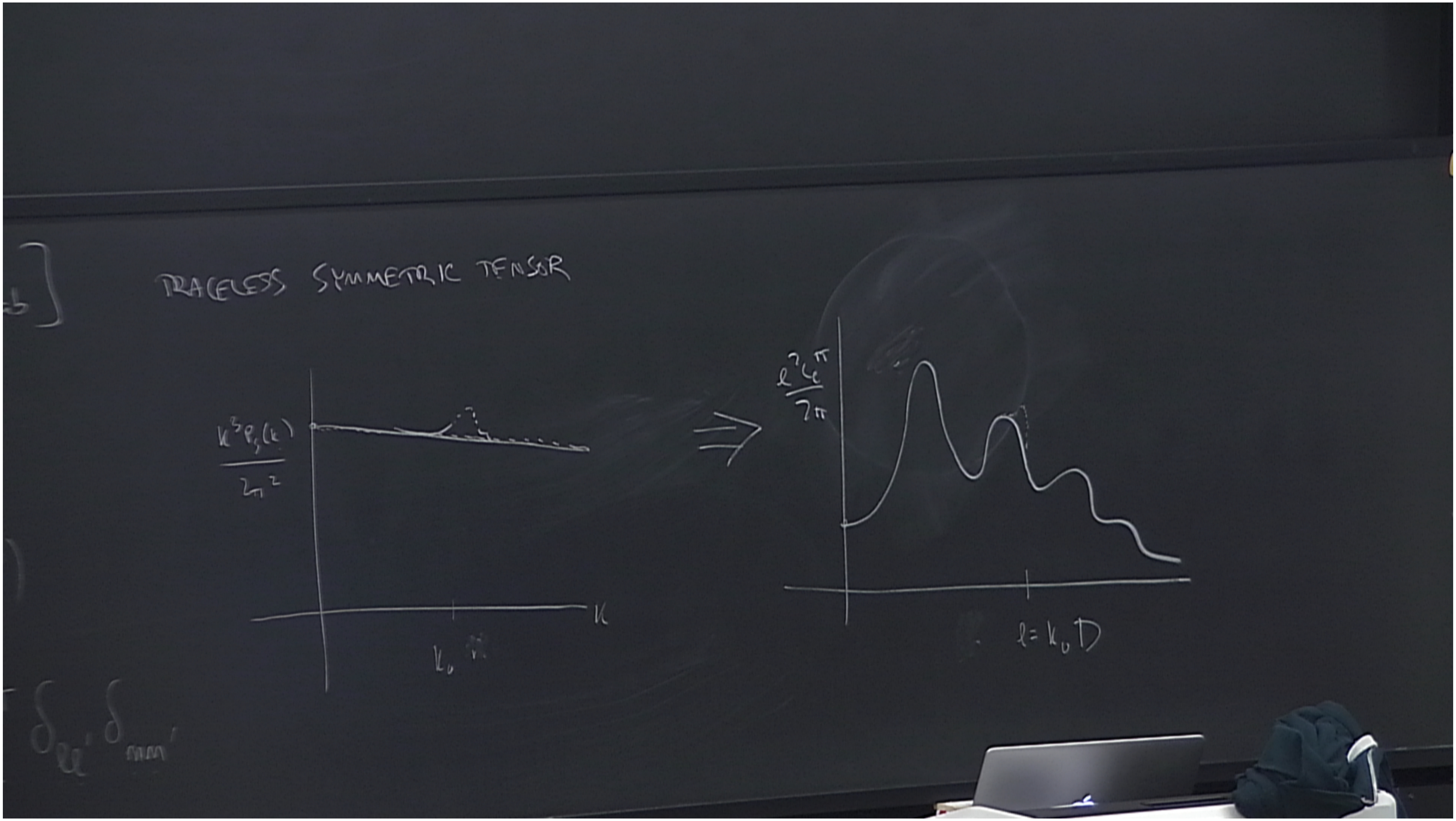
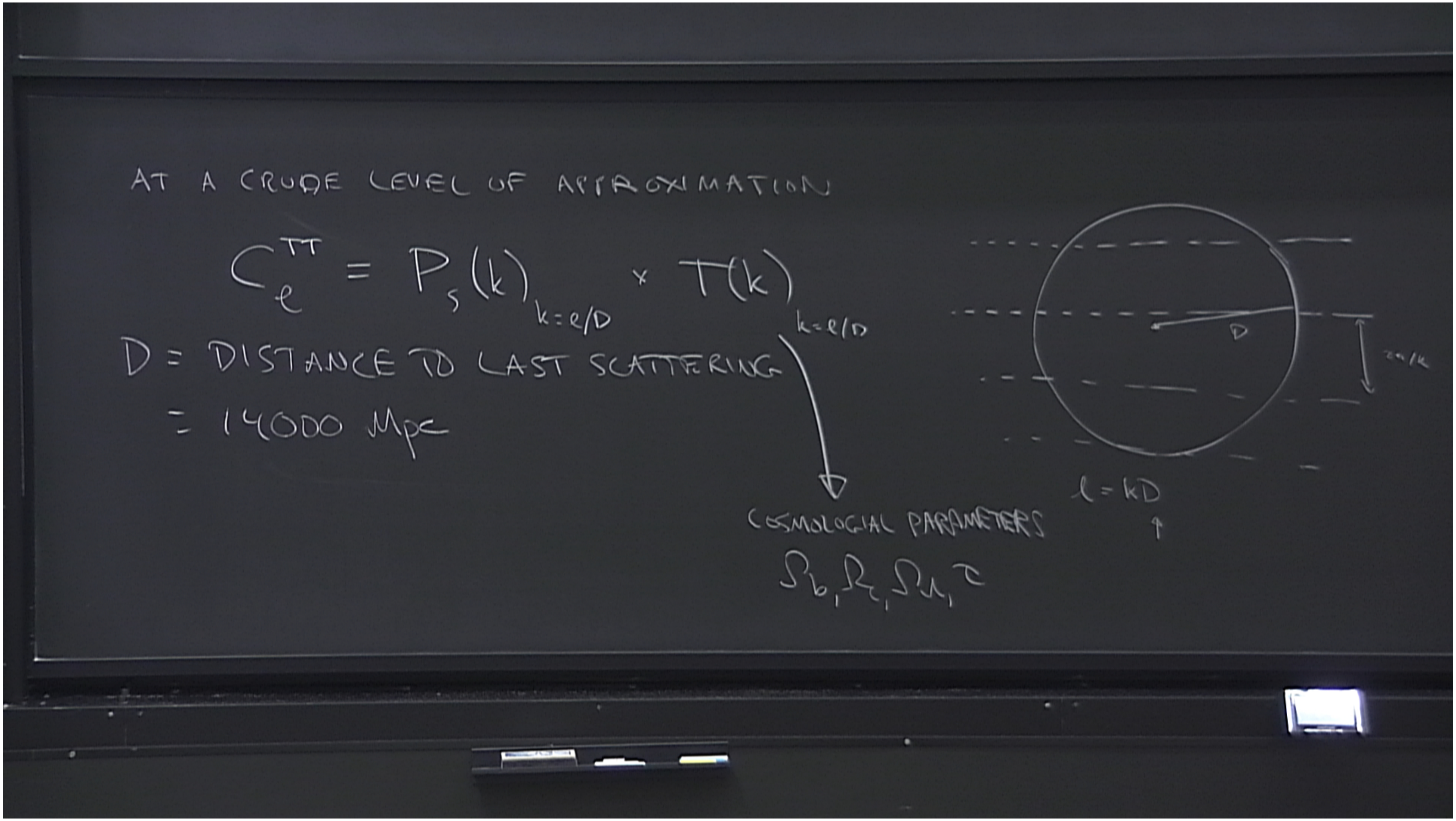
ON A SPHERICAL SKY

$$T(\hat{n}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a^T_{\ell m} Y_{\ell m}(\hat{n}) \qquad \langle a^T_{\ell m} a^{T*}_{\ell' m'} \rangle = \underline{\underline{C^{TT}_\ell}}\, \delta_{\ell\ell'} \delta_{mm'}$$


TRACELESS SYMMETRIC TENSOR
$\frac{k^3 P_s(k)}{2\pi^2}$
k
k_0
$\frac{\ell^2 C_\ell^{\pi}}{2\pi}$
$\ell = k_0 D$



AT A CRUDE LEVEL OF APPROXIMATION
$C_\ell^{TT} = P_s(k)\big|_{k=\ell/D} \times T(k)\big|_{k=\ell/D}$
D = DISTANCE TO LAST SCATTERING
= 14000 Mpc
COSMOLOGICAL PARAMETERS
$\Omega_b, \Omega_c, \Omega_\Lambda, \tau$
D
$2\pi/k$
$\ell = kD$

$$\sum_{\ell=0}\sum_{m=-\ell} a_{\ell m} Y_{\ell m} \quad =$$

WHAT IS THE ANALOG OF THE SPHERICAL TRANSFORM FOR A TRACELESS SYMMETRIC TENSOR P_{ab}?

FLAT SKY FIRST:

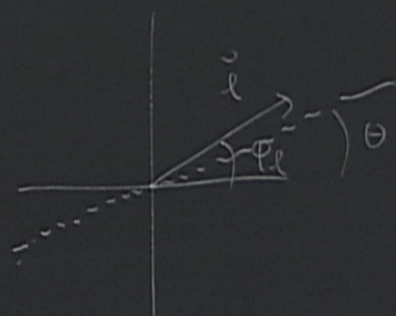
$$P_{ab}(\vec{x}) = Q(x)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + U(x)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

"STOKES PARAMETERS"

TWO NUMBERS FOR EACH $\vec{\ell}$: $Q_{\vec{\ell}}$, $U_{\vec{\ell}}$

MORE INTERESTING DECOMPOSITION:

$$\begin{pmatrix} E_{\vec{\ell}} \\ B_{\vec{\ell}} \end{pmatrix} = \begin{pmatrix} \cos 2\varphi_\ell & \sin 2\varphi_\ell \\ -\sin 2\varphi_\ell & \cos 2\varphi_\ell \end{pmatrix}\begin{pmatrix} Q_{\vec{\ell}} \\ U_{\vec{\ell}} \end{pmatrix}$$



ROTATION INVARIANT

$$\varphi' \to \varphi - \theta$$

$$\begin{pmatrix} Q' \\ U' \end{pmatrix} \to \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix}$$

$C_\ell \, \delta_{\ell\ell'} \, \delta_{mm'}$

$=$

...ELESS SYMMETRIC TENSOR P_{ab}?

"STOKES PARAMETERS"

PARITY

$E \to E$

$B \to (-B)$

ROTATION INVARIANT

$\varphi' \to \varphi - \Theta$

$$\begin{pmatrix} Q' \\ U' \end{pmatrix} \to \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$\begin{pmatrix} E' \\ B' \end{pmatrix} = \begin{pmatrix} E \\ B \end{pmatrix}$$