Title: Explorations in Cosmology-3

Date: Apr 08, 2015 10:15 AM

URL: http://pirsa.org/15040027

Abstract:

$$H(t) = \frac{1}{2}p^2 + \frac{1}{2}\omega(t)^2 x^2 + H_1(t)$$

$\omega(t) \to \omega_0$

$|4\rangle \to |0\rangle$

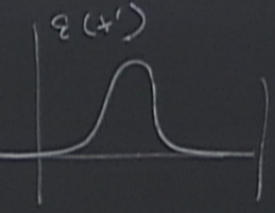
$$\hat{X}_I(t) = v(t)\hat{a}_s + v(t)^* \hat{a}_s^\dagger$$

$$\hat{P}_I(t) = \dot{v}(t)\hat{a}_s + \dot{v}(t)^* \hat{a}_s^\dagger$$

$$\ddot{v}(t) = -\omega(t)^2 v(t) \qquad v(t) \to \frac{e^{-i\omega_0 t}}{(2\omega_0)^{1/2}} \quad \text{AT EARLY TIMES}$$

$$\langle 4|O|4\rangle = \langle 0|O_I(t)|0\rangle - i\int_{-\infty}^{t} dt' \langle 0|[O_I(t), H_I(t')]|0\rangle + \cdots$$

$$H = \frac{1}{2}p^2 + \frac{1}{2}\omega(t)^2 x^2 + \varepsilon(t)\, p^3$$



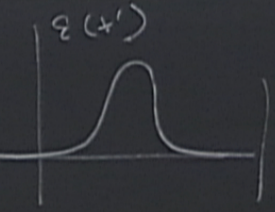
COMPUTE $\langle 4|x|4\rangle_\varepsilon$ TO FIRST ORDER $\mathcal{O}(\varepsilon)$

$$\langle 4|x|4\rangle = \langle 0|\, x_I(t)\,|0\rangle - i\int_{-\infty}^{t} dt' \left[x_I(t),\ \varepsilon(t')\, p_I(t')^3\right]|0\rangle$$

$$= 0 - i\int_{-\infty}^{t} dt'\, \varepsilon(t') \langle 0|\, x_I(t)\, p_I(t')^3\,|0\rangle + \text{c.c.}$$

$$= -i\int_{-\infty}^{t} dt'\, \varepsilon(t') \langle 0|\, \overbrace{x_I(t)\, p_I(t')}\, \overbrace{p_I(t')\, p_I(t')}\,|0\rangle + \text{c.c.}$$

$$H = \frac{1}{2}p^2 + \frac{1}{2}\omega(t)^2 x^2 + \varepsilon(t)\, p^3$$



COMPUTE $\langle 4|x|4\rangle_t$ TO FIRST ORDER $\mathcal{O}(\varepsilon)$

$$\langle 4|x|4\rangle = \langle 0|\, x_I(t)\,|0\rangle - i\int_{-\infty}^{t} dt' \left[x_I(t),\ \varepsilon(t')\, p_I(t')^3\right]|0\rangle$$

$$= 0 - i\int_{-\infty}^{t} dt'\, \varepsilon(t')\langle 0|\, x_I(t)\, p_I(t')^3\,|0\rangle + \text{c.c.}$$

$$= -3i\int_{-\infty}^{t} dt'\, \varepsilon(t')\langle 0|\, \overbrace{x_I(t)\, p_I(t')}\, \overbrace{p_I(t')\, p_I(t')}\,|0\rangle + \text{c.c.}$$

$$\langle 0|\tilde{x}\tilde{y}\tilde{z}\tilde{w}|0\rangle = \langle 0|xy|0\rangle\langle 0|zw|0\rangle$$

$$= -3i\int_{-\infty}^{0} dt'\, \varepsilon(t')\, v(t)\, \dot{v}(t')^{*}\, \dot{v}(t')\, \dot{v}(t')^{*} + \text{c.c.}$$

$$= -3i\int_{-\infty}^{0} dt'\, \varepsilon(t')\, |\dot{v}(t')|^{2} \left(v(t)\, i(t')^{*} - v(t)^{*}\, i(t') \right)$$

+ c.c.

QFT EXAMPLE:

- MASSLESS SCALAR FIELD ON NONDYNAMICAL DE SITTER METRIC

$$S[\phi] = \int d^4x \sqrt{-g}\left(-\frac{1}{2} g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi\right)$$

ϕ — NO g_{ij}!

DE SITTER: FRW WITH CONSTANT H $[w=-1]$

$$\Rightarrow a = e^{Ht} \qquad \tau = \int \frac{dt}{a} = -\frac{1}{H} e^{-Ht} \qquad [-\infty < \tau < 0]$$

QFT EXAMPLE

- MASSLESS SCALAR FIELD ON NONDYNAMICAL DE SITTER METRIC

$$S[\phi] = \int d^4x \sqrt{-g}\left(-\frac{1}{2} g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi\right)$$

$\mathcal{L}$ NO g_{ij}!

DE SITTER: FRW WITH CONSTANT H $[w=-1]$

$$\Rightarrow a = e^{Ht} \qquad \tau = \int \frac{dt}{a} = -\frac{1}{H} e^{-Ht} \qquad [-\infty < \tau < 0]$$

$$= \frac{-1}{H\tau}$$

$$ds^2 = a^2\left[-d\tau^2 + dx^2\right]$$

$$= \frac{1}{(H\tau)^2}\left(-d\tau^2 + dx^2\right)$$

$$S[\phi] = \frac{1}{2}\int d\tau\, d^3x\, \frac{1}{(H\tau)^2}\left((\partial_\tau\phi)^2 - (\partial_i\phi)^2\right)$$

CANONICALLY NORMALIZE ϕ: DEFINE $\sigma = \frac{\phi}{H\tau}$

$$S[\sigma] = \frac{1}{2}\int d\tau\, d^3x\left[(\partial_\tau\sigma)^2 + \frac{2}{\tau}\sigma(\partial_\tau\sigma) + \frac{\sigma^2}{\tau^2} - (\partial_i\sigma)^2\right]$$

$$= \frac{1}{2}\int d\tau\, d^3x\left[(\partial_\tau\sigma)^2 + 2\frac{\sigma^2}{\tau^2} - (\partial_i\sigma)^2\right]$$

$$\frac{2}{\tau}\sigma\dot{\sigma} = \frac{1}{\tau}\frac{d}{d\tau}(\sigma^2)$$

$$= +\frac{1}{\tau^2}\sigma^2$$

HAMILTONIAN: INTRODUCE $\hat{\pi}$ MOMENTUM [CONJUGATE TO σ]

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\sigma}} = \dot{\sigma}$$

$$H = \int d^3x \left(\pi \dot{\sigma} - \mathcal{L} \right)$$

$$= \frac{1}{2} \int d^3x \left[\frac{1}{2} \pi^2 + 2 \frac{\sigma^2}{c^2} + (\partial_i \sigma)^2 \right]$$

$$= \int \frac{d^3k}{(2\pi)^3} \left(\frac{1}{2} \pi_k^* \pi_k + \frac{1}{2} \omega(k,\tau)^2 \sigma_k^* \sigma_k \right)$$

WHERE

$$\omega(k,\tau) = k^2 - \frac{2}{\tau^2}$$

HAMILTONIAN: INTRODUCE $\hat{\pi}$ MOMENTUM [CONJUGATE TO σ]

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\sigma}} = \dot{\sigma}$$

$$H = \int d^3x \left(\pi \dot{\sigma} - \mathcal{L} \right)$$

$$= \frac{1}{2} \int d^3x \left[\pi^2 + 2 \frac{\sigma^2}{c^2} + (\partial_i \sigma)^2 \right]$$

$$= \int \frac{d^3k}{(2\pi)^3} \left(\frac{1}{2} \pi_k^* \pi_k + \frac{1}{2} \omega(k,\tau)^2 \sigma_k^* \sigma_k \right)$$

WHERE

$$\omega(k,\tau) = k^2 - \frac{2}{\tau^2}$$

FREE QFT HAS THE SOLUTION

$$\hat{\sigma}^H_k(\tau) = V_k(\tau)\, a_k + V_k^*(\tau)\, a^\dagger_{-k}$$

$$\hat{\pi}^H_k(\tau) = \dot{V}_k(\tau)\, a_k + \dot{V}_k^*(\tau)\, a^\dagger_{-k} \qquad \text{WHERE } [a_k, a^\dagger_{k'}] = (2\pi)^3 \delta^3(k-k')$$

WHERE $\ddot{V}_k(\tau) = -\left(k^2 - \frac{2}{\tau^2}\right) V_k(\tau)$ AND $V_k(\tau) \to \frac{e^{-ik\tau}}{(2k)^{1/2}} \quad \tau \to -\infty$

$$\Rightarrow V_k(\tau) = \frac{1}{(2k)^{1/2}} \left(1 - \frac{i}{k\tau}\right) e^{-ik\tau}$$

IN ORIGINAL ϕ VARIABLE

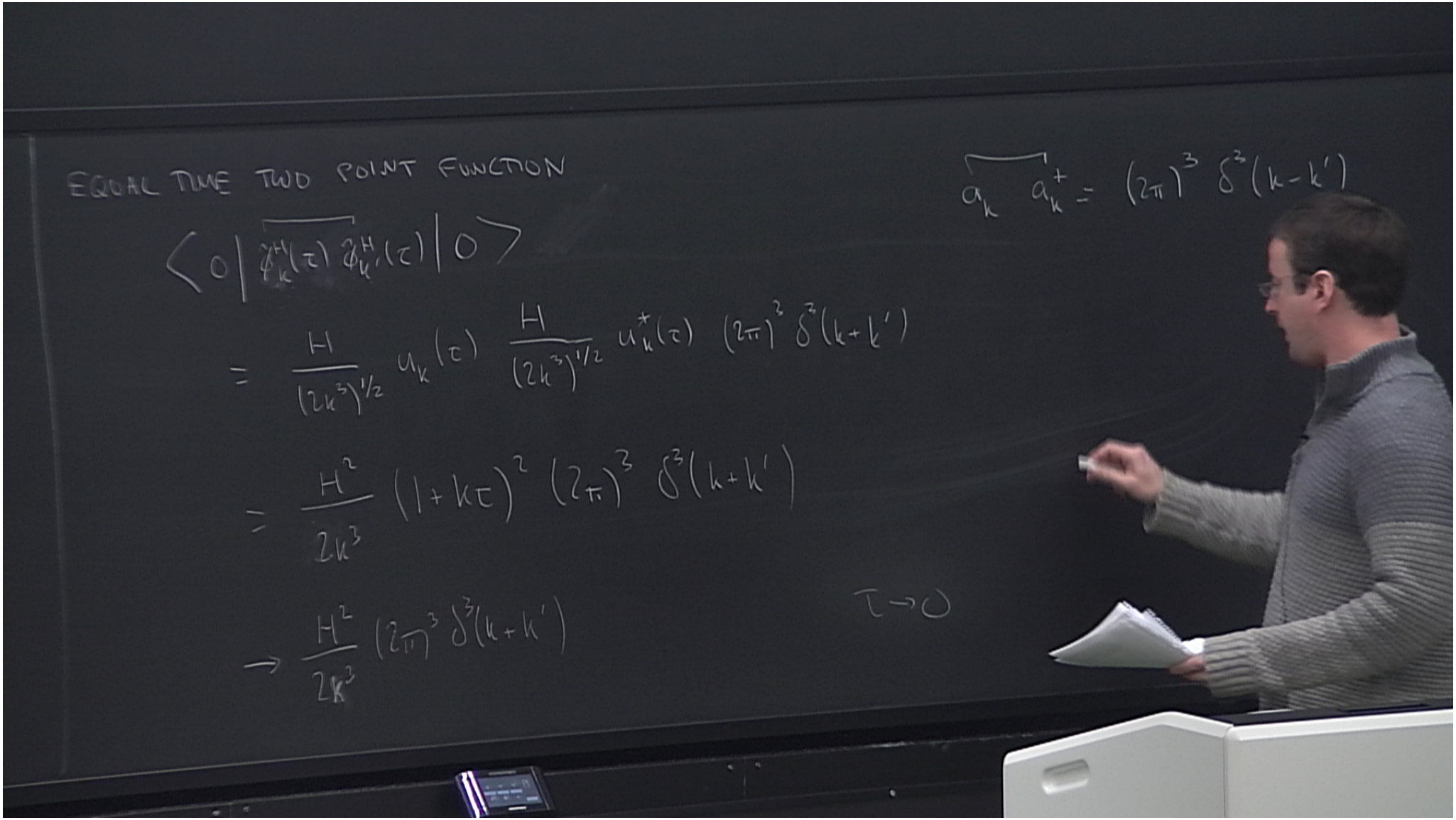
$$\hat{\phi}_k^H(\tau) = H\tau\, \hat{\sigma}_k^H(\tau)$$

$$= \frac{H}{(2k^3)^{1/2}} \left(u_k(\tau)\, a_k + u_k^*(\tau)\, a_{-k}^\dagger \right)$$

$$u_k(\tau) = -i(1 + ik\tau) e^{-ik\tau}$$

$$\dot{u}_k(\tau) = -ik^2 \tau e^{-ik\tau}$$

EQUA


EQUAL TIME TWO POINT FUNCTION
$\overline{a_k \; a_k^+} = (2\pi)^3 \delta^3(k-k')$
$\langle 0 | \overline{\phi_k^{H\dagger}(\tau) \phi_{k'}^H(\tau)} | 0 \rangle$
$= \frac{H}{(2k^3)^{1/2}} u_k(\tau) \frac{H}{(2k^3)^{1/2}} u_k^*(\tau) (2\pi)^3 \delta^3(k+k')$
$= \frac{H^2}{2k^3} (1+k\tau)^2 (2\pi)^3 \delta^3(k+k')$
$\tau \to 0$
$\to \frac{H^2}{2k^3} (2\pi)^3 \delta^3(k+k')$

EQUAL TIME TWO POINT FUNCTION

$$\overbrace{a_k \; a_k^\dagger} = (2\pi)^3 \delta^3(k-k')$$

$$\langle 0 | \overbrace{\hat{\phi}_k^H(\tau) \hat{\phi}_{k'}^H(\tau)} | 0 \rangle$$

$$= \frac{H}{(2k^3)^{1/2}} u_k(\tau) \frac{H}{(2k^3)^{1/2}} u_k^*(\tau) (2\pi)^3 \delta^3(k+k')$$

$$= \frac{H^2}{2k^3} (1+k\tau)^2 (2\pi)^3 \delta^3(k+k') \qquad \tau \gg -1/k$$

$$\to \frac{H^2}{2k^3} (2\pi)^3 \delta^3(k+k') \qquad \tau \to 0$$

EQUAL TIME TWO POINT FUNCTION

$$\overbrace{a_k \; a_k^\dagger} = (2\pi)^3 \delta^3(k-k')$$

$$\langle 0 | \overbrace{\hat{\phi}_k^{H\dagger}(\tau) \hat{\phi}_{k'}^{H}(\tau)} | 0 \rangle$$

$$= \frac{H}{(2k^3)^{1/2}} u_k(\tau) \frac{H}{(2k^3)^{1/2}} u_k^*(\tau) (2\pi)^3 \delta^3(k+k')$$

$$= \frac{H^2}{2k^3} (1+k\tau)^2 (2\pi)^3 \delta^3(k+k')$$

$\tau \gg -1/k$

$$\rightarrow \frac{H^2}{2k^3} (2\pi)^3 \delta^3(k+k') \qquad P_\phi(k) \approx \frac{1}{k^3}$$

"SCALE INVARIANT"

$$\int_{-\infty}^{t} dt' \, \varepsilon(t') \, \langle 0 | \, \hat{\chi}_I(t) \, \hat{P}_I(t') \, \hat{P}_I(t') \, \hat{P}_I(t') \, | 0 \rangle + \text{c.c.}$$

$$\hat{H}_1(\tau) = \frac{\varepsilon}{H^3} \int d^3x \, \frac{(\partial_\tau \phi)^3}{\tau}$$

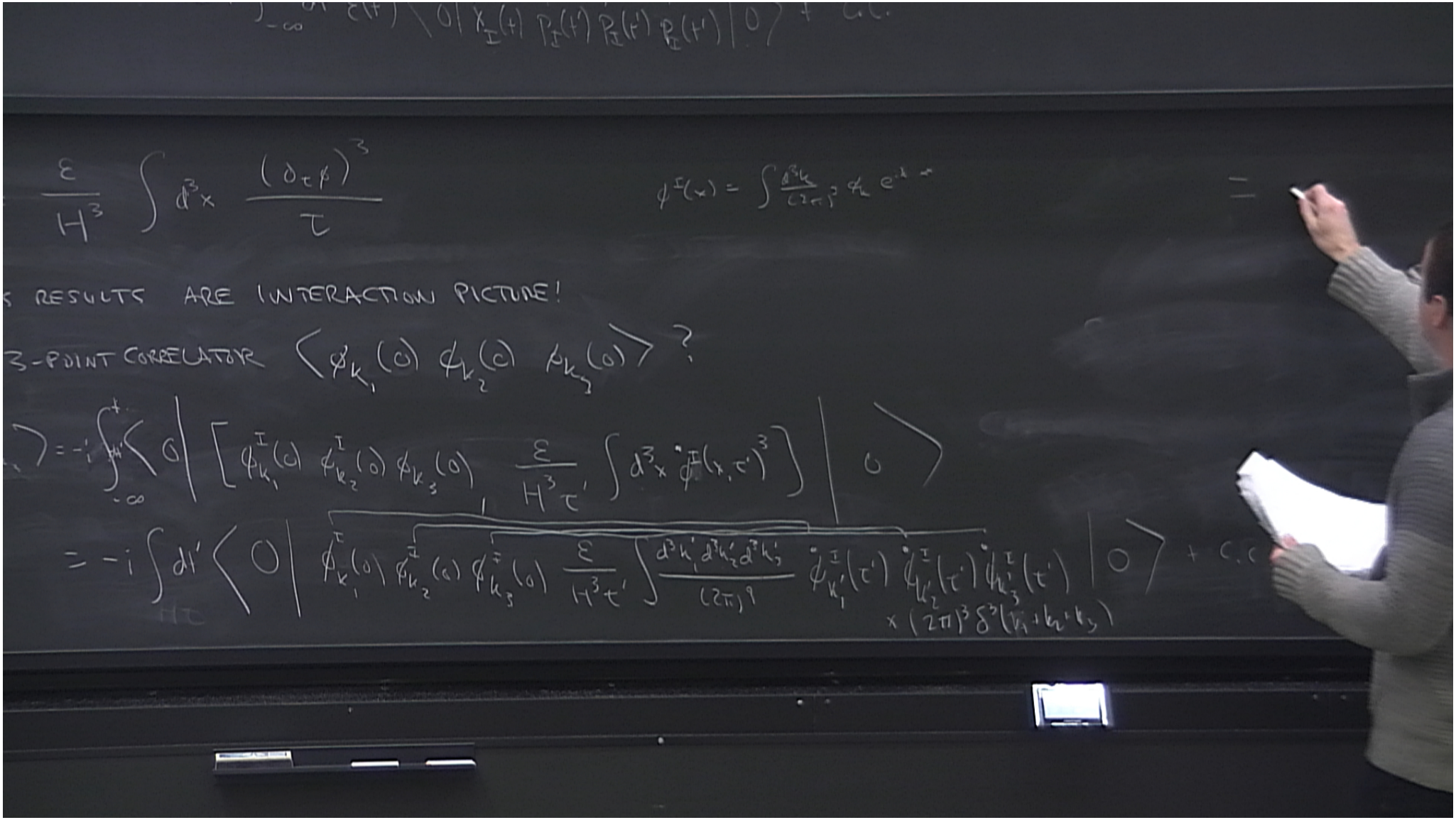
$$\phi^I(x) = \int \frac{d^3k}{(2\pi)^3} \, \phi_k \, e^{-ik \cdot x}$$

ALL PREVIOUS RESULTS ARE INTERACTION PICTURE!

WHAT IS 3-POINT CORRELATOR $\langle \phi_{k_1}(0) \, \phi_{k_2}(0) \, \phi_{k_3}(0) \rangle$?

$$\langle \phi_{k_1} \phi_{k_2} \phi_{k_3} \rangle = -i \int_{-\infty}^{t} \langle 0 | \left[\phi^I_{k_1}(0) \, \phi^I_{k_2}(0) \, \phi_{k_3}(0), \frac{\varepsilon}{H^3 \tau'} \int d^3x \, \dot{\phi}^I(x, \tau')^3 \right] | 0 \rangle$$

$$= -i \int d\tau' \langle 0 | \, \phi^I_{k_1}(0) \, \phi^I_{k_2}(0) \, \phi^I_{k_3}(0) \, \frac{\varepsilon}{H^3 \tau'} \int \frac{d^3k_1 \, d^3k_2 \, d^3k_3}{(2\pi)^9} \, \dot{\phi}^I_{k_1}(\tau') \, \dot{\phi}^I_{k_2}(\tau') \, \dot{\phi}^I_{k_3}(\tau') \, | 0 \rangle$$


$\frac{\varepsilon}{H^3}\int d^3x\,\frac{(\partial_\tau\phi)^3}{\tau}$
$\phi^I(x)=\int\frac{d^3k}{(2\pi)^3}\phi_k\,e^{ik\cdot x}$
RESULTS ARE IN INTERACTION PICTURE!
3-POINT CORRELATOR $\langle\phi_{k_1}(0)\,\phi_{k_2}(0)\,\phi_{k_3}(0)\rangle$?
$\left[\phi^I_{k_1}(0)\,\phi^I_{k_2}(0)\,\phi_{k_3}(0),\ \frac{\varepsilon}{H^3\tau'}\int d^3x\,\dot\phi^I(x,\tau')^3\right]$
$\frac{d^3k_1'\,d^3k_2'\,d^3k_3'}{(2\pi)^9}\,\dot\phi^I_{k_1'}(\tau')\,\dot\phi^I_{k_2'}(\tau')\,\dot\phi^I_{k_3'}(\tau')$
$\times(2\pi)^3\delta^3(k_1'+k_2'+k_3')$

$$H_1(\tau) = \frac{\varepsilon}{H^3} \int d^3x \, \frac{(\partial_\tau \phi)^3}{\tau}$$

$$\phi^I(x) = \int \frac{d^3k}{(2\pi)^3} \phi_k \, e^{i k \cdot x}$$

ALL PREVIOUS RESULTS ARE IN INTERACTION PICTURE!

WHAT IS 3-POINT CORRELATOR $\langle \phi_{k_1}(0) \, \phi_{k_2}(0) \, \phi_{k_3}(0) \rangle$? $\quad k_i \neq 0$

$$\langle \phi_{k_1} \phi_{k_2} \phi_{k_3} \rangle = -i \int_{-\infty}^{t} dt' \langle 0 | \left[\phi^I_{k_1}(0) \, \phi^I_{k_2}(0) \, \phi_{k_3}(0), \frac{\varepsilon}{H^3 \tau'} \int d^3x \, \dot{\phi}^I(x, \tau')^3 \right] | 0 \rangle$$

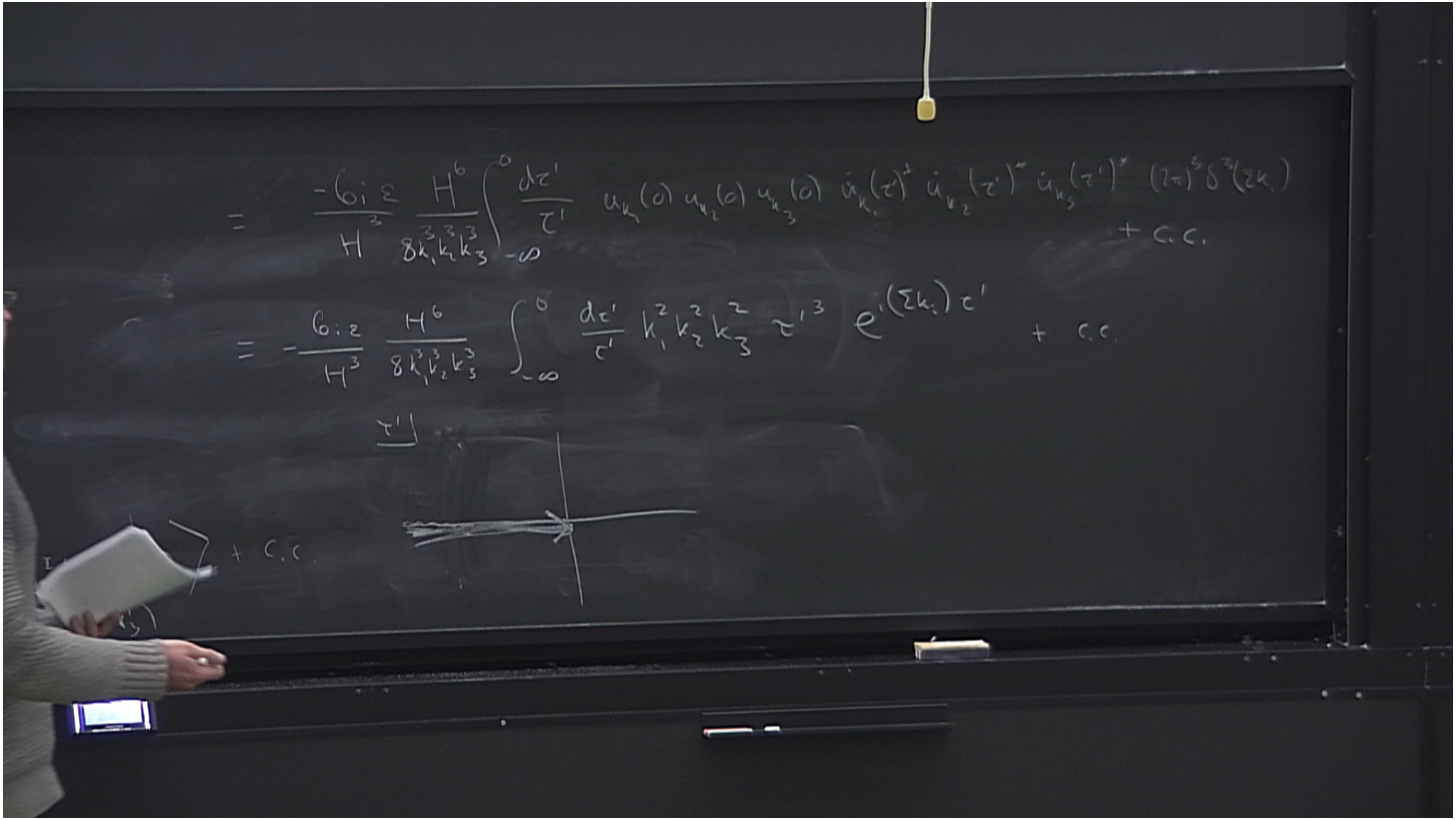
$$= -i \int d\tau' \langle 0 | \phi^I_{k_1}(0) \, \phi^I_{k_2}(0) \, \phi^I_{k_3}(0) \, \frac{\varepsilon}{H^3 \tau'} \int \frac{d^3k_1' \, d^3k_2' \, d^3k_3'}{(2\pi)^9} \dot{\phi}^I_{k_1'}(\tau') \, \dot{\phi}^I_{k_2'}(\tau') \, \dot{\phi}^I_{k_3'}(\tau') \times (2\pi)^3 \delta(k_1' + k_2' + k_3')$$

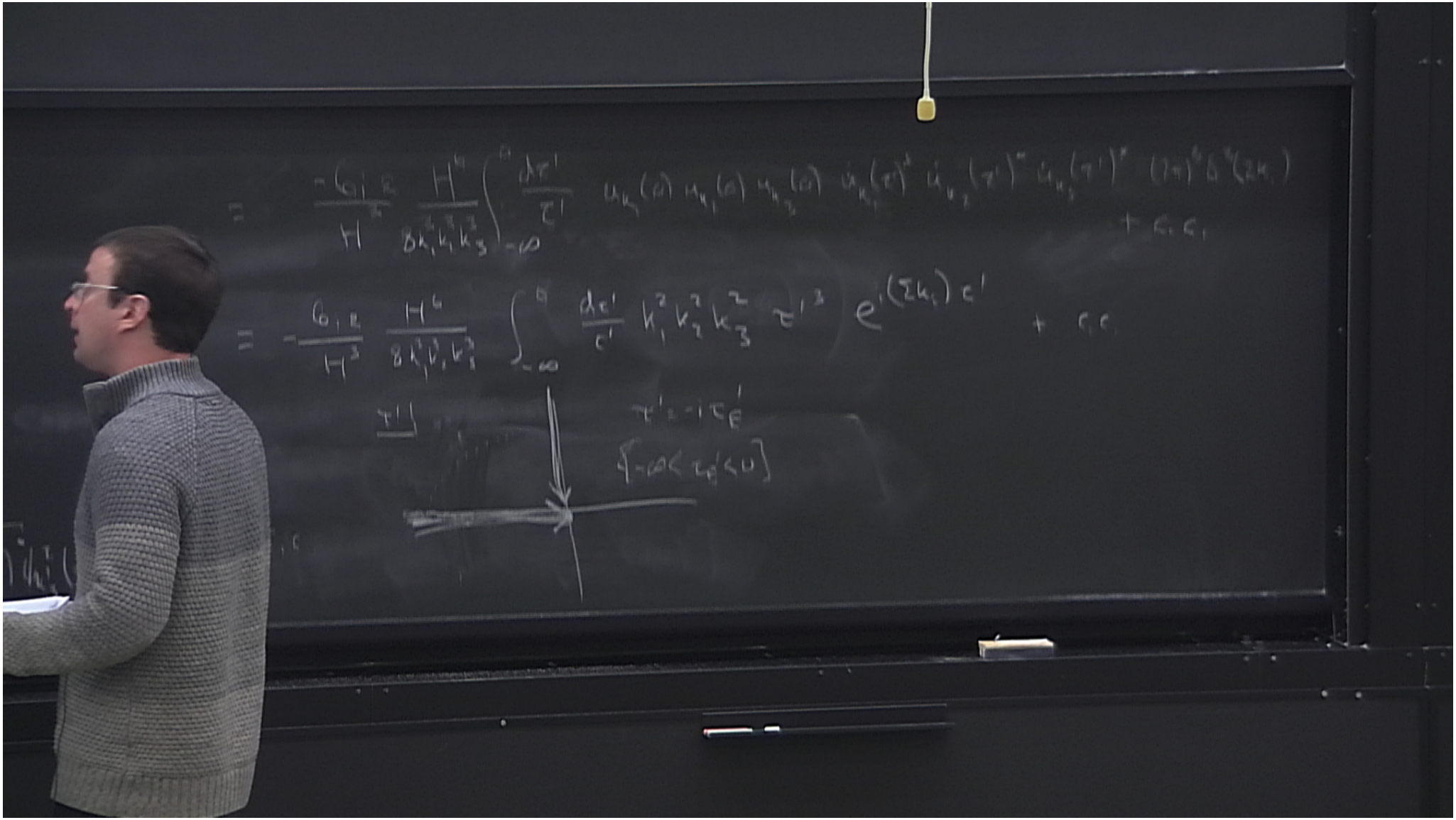
$$= \frac{-6i\varepsilon}{H^3} \frac{H^6}{8k_1^3 k_2^3 k_3^3} \int_{-\infty}^{0} \frac{d\tau'}{\tau'} u_{k_1}(0)\, u_{k_2}(0)\, u_{k_3}(0)\, \dot{u}_{k_1}(\tau')^*\, \dot{u}_{k_2}(\tau')^*\, \dot{u}_{k_3}(\tau')^*\, (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) + \text{c.c.}$$

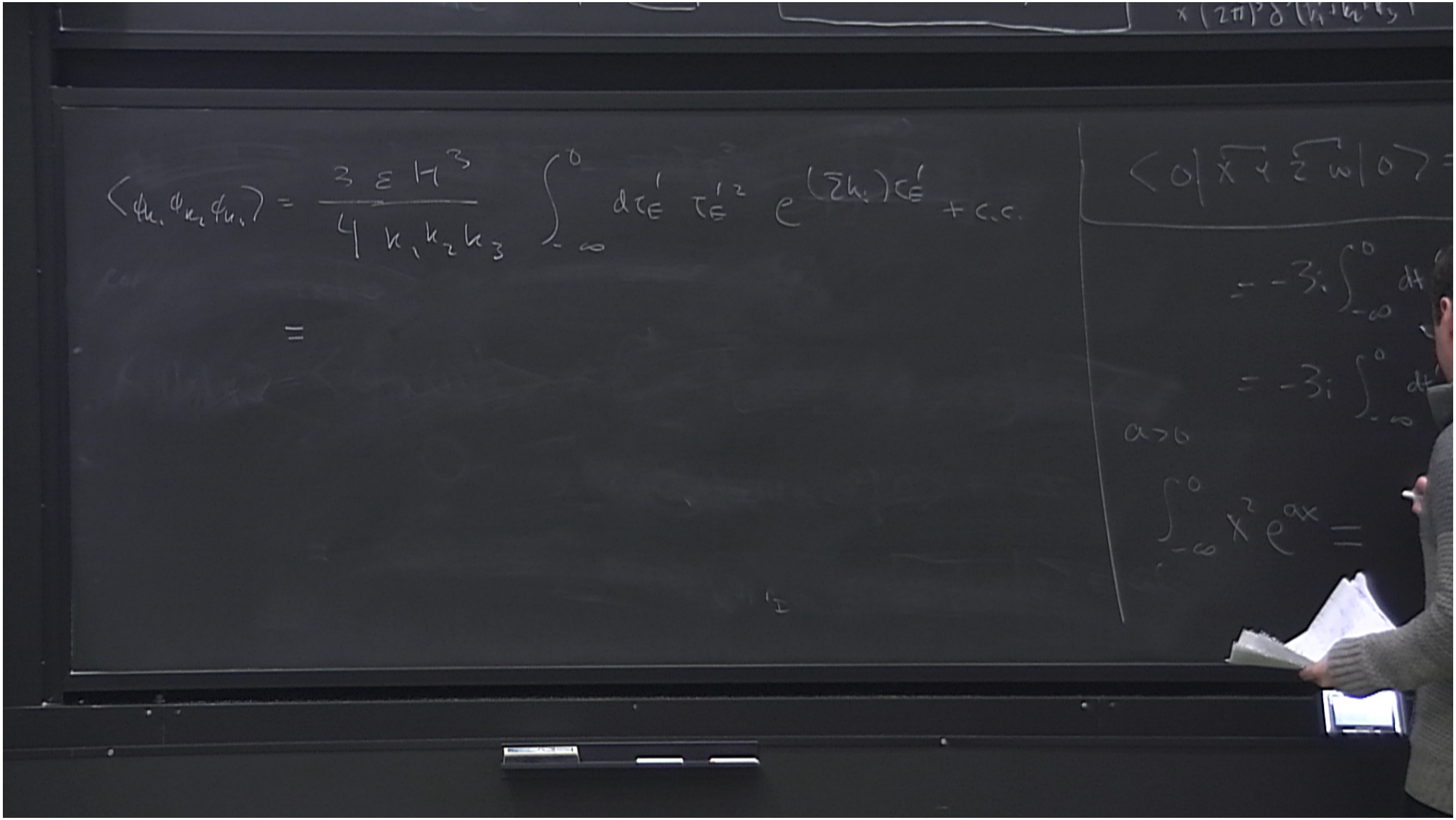
$$= -\frac{6i\varepsilon}{H^3} \frac{H^6}{8k_1^3 k_2^3 k_3^3} \int_{-\infty}^{0} \frac{d\tau'}{\tau'} k_1$$

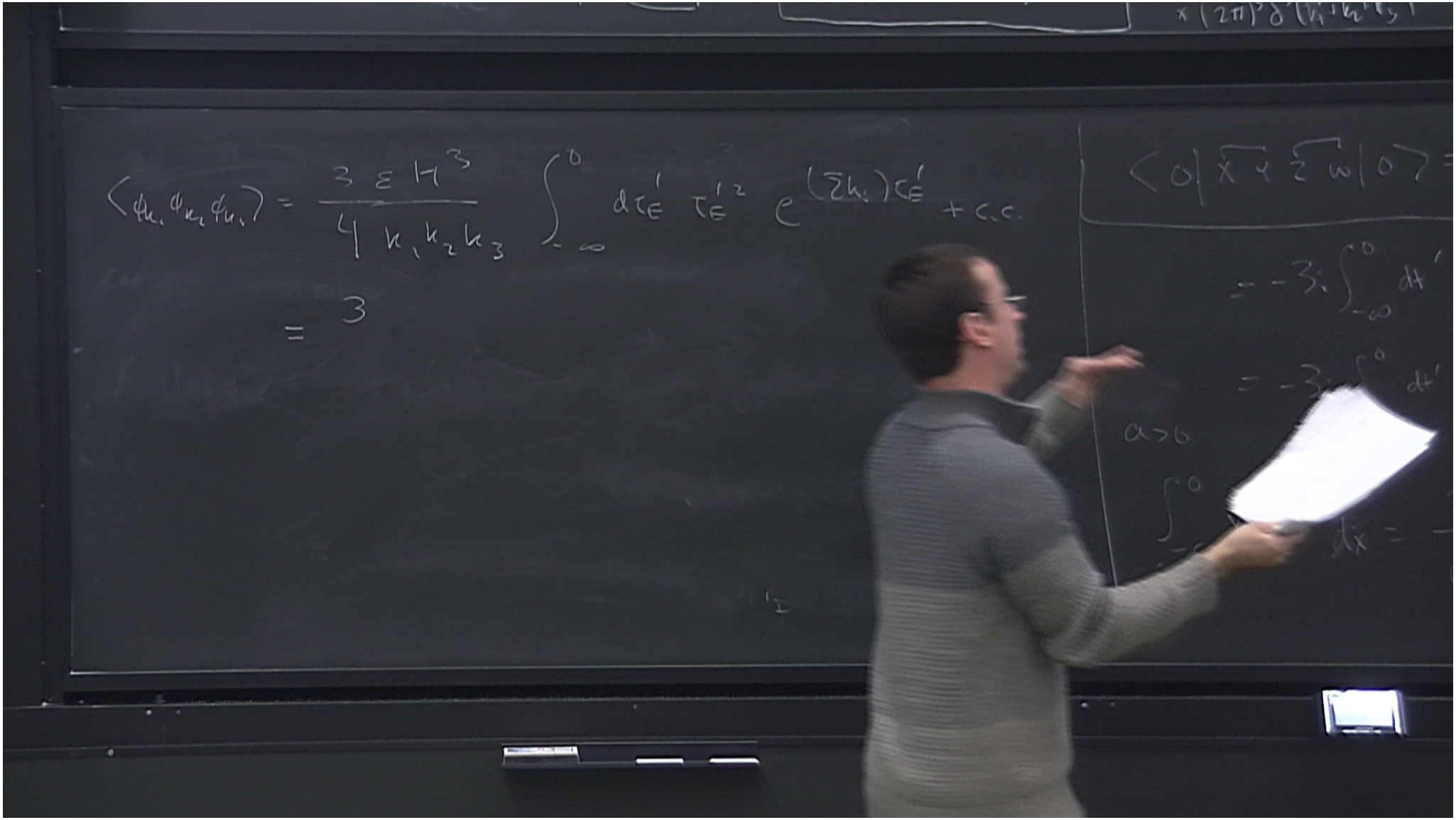
$\dot{u}_{k_3}^{I}(t') \,|0\rangle + \text{c.c.}$

$\delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$


$= \frac{-6i\varepsilon}{H^3} \frac{H^6}{8k_1^3k_2^3k_3^3} \int_{-\infty}^{0} \frac{d\tau'}{\tau'} u_{k_1}(0)\, u_{k_2}(0)\, u_{k_3}(0)\, \dot{u}_{k_1}(\tau')^* \dot{u}_{k_2}(\tau')^* \dot{u}_{k_3}(\tau')^* (2\pi)^3 \delta^3(\Sigma k_i)$ + c.c.
$= -\frac{6i\varepsilon}{H^3} \frac{H^6}{8k_1^3k_2^3k_3^3} \int_{-\infty}^{0} \frac{d\tau'}{\tau'} k_1^2 k_2^2 k_3^2 \tau'^3 e^{i(\Sigma k_i)\tau'}$ + c.c.
τ'
+ c.c.




$\langle \phi_{k_1} \phi_{k_2} \phi_{k_3} \rangle = \frac{3\varepsilon H^3}{4 k_1 k_2 k_3} \int_{-\infty}^{0} d\tau_E' \, \tau_E'^2 \, e^{(\Sigma k_i)\tau_E'} + c.c.$
=
$\times (2\pi)^3 \delta(k_1 + k_2 + k_3)$
$\langle 0 | \hat{x} \hat{y} \hat{z} \hat{w} | 0 \rangle =$
$= -3i \int_{-\infty}^{0} dt$
$= -3i \int_{-\infty}^{0} dt$
$a > 0$
$\int_{-\infty}^{0} x^2 e^{ax} =$


$\langle \phi_{k_1} \phi_{k_2} \phi_{k_3} \rangle = \frac{3 \varepsilon H^3}{4 k_1 k_2 k_3} \int_{-\infty}^{0} d\tau_\varepsilon' \, \tau_\varepsilon'^2 \, e^{(\Sigma k_i)\tau_\varepsilon'} + c.c.$
$= 3$
$\langle 0 | \hat{x} \hat{y} \hat{z} \hat{w} | 0 \rangle =$
$= -3i \int_{-\infty}^{0} dt'$
$= -3i \int_{-\infty}^{0} dt'$
$a > 0$
$\int_{-\infty}^{0}$
$dx =$

$$\langle \phi_{k_1} \phi_{k_2} \phi_{k_3} \rangle = \frac{3\varepsilon H^3}{4 k_1 k_2 k_3} \int_{-\infty}^{0} d\tau_E' \, \tau_E'^2 \, e^{(\Sigma k_i)\tau_E'} + \text{c.c.}$$

$$= \frac{3H^3}{4 k_1 k_2 k_3} \cdot \frac{2}{(\Sigma k_i)^3} + \text{c.c.}$$

$$= \frac{3H^3}{k_1 k_2 k_3 (k_1+k_2+k_3)^3} (2\pi)^3 \delta^3(k_1+k_2+k_3)$$

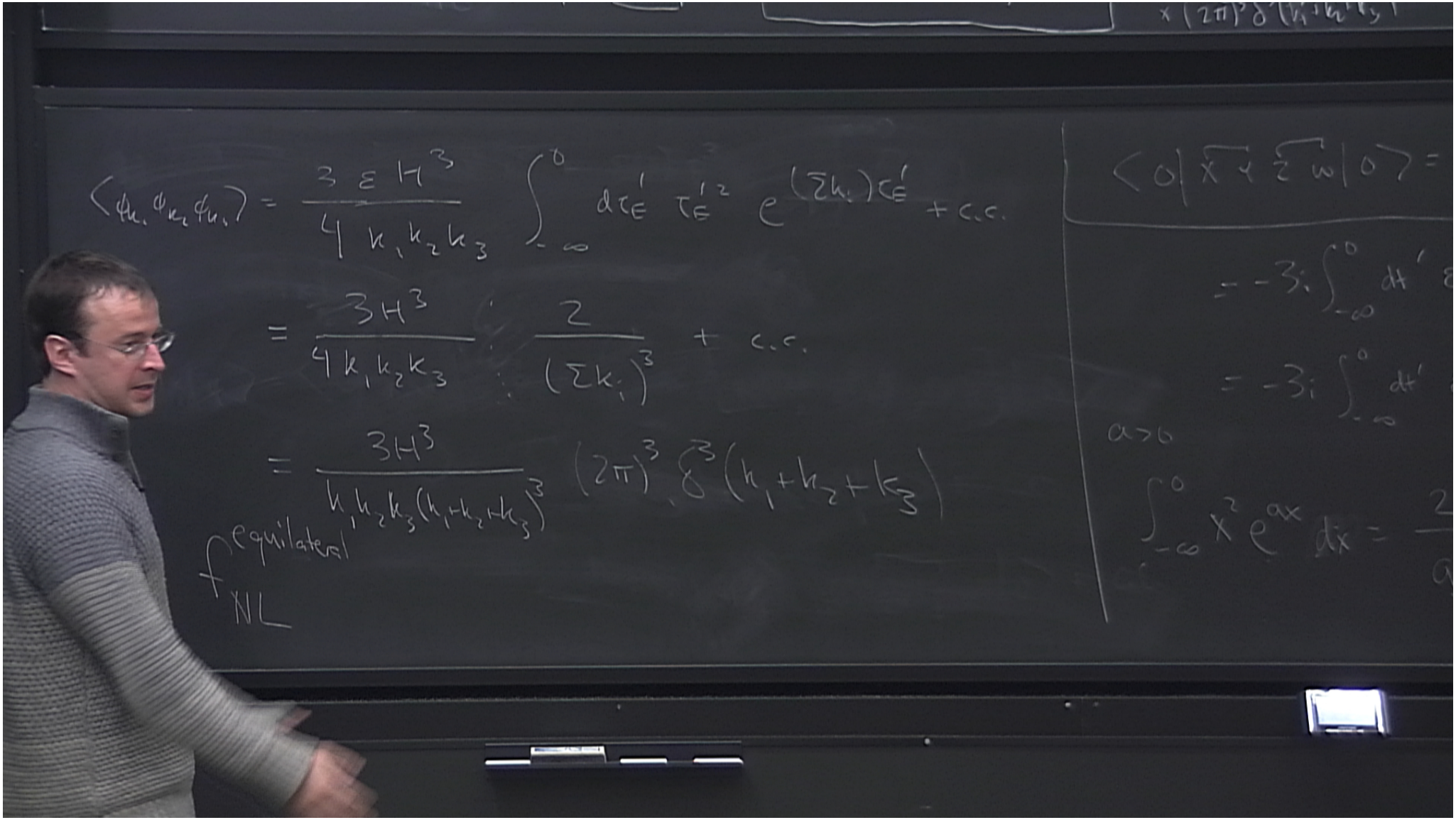
$$\langle 0 | \hat{x} \hat{y} \hat{z} \hat{w} | 0 \rangle =$$

$$= -3i \int_{-\infty}^{0} dt'$$

$$= -3i \int_{-\infty}^{0} dt'$$

$a > 0$

$$\int_{-\infty}^{0} x^2 e^{ax} dx = \frac{2}{a}$$


$\langle \phi_{k_1} \phi_{k_2} \phi_{k_3} \rangle = \frac{3\varepsilon H^3}{4 k_1 k_2 k_3} \int_{-\infty}^{0} d\tau_E' \, \tau_E'^2 \, e^{(\Sigma k_i)\tau_E'} + c.c.$
$= \frac{3H^3}{4 k_1 k_2 k_3} \cdot \frac{2}{(\Sigma k_i)^3} + c.c.$
$= \frac{3H^3}{k_1 k_2 k_3 (k_1+k_2+k_3)^3} (2\pi)^3 \delta^3(k_1+k_2+k_3)$
$f_{NL}^{equilateral}$
$\langle 0 | \hat{x} \hat{y} \hat{z} \hat{w} | 0 \rangle =$
$= -3i \int_{-\infty}^{0} dt'$
$= -3i \int_{-\infty}^{0} dt'$
$a > 0$
$\int_{-\infty}^{0} x^2 e^{ax} dx =$

$$\langle \phi_{k_1} \phi_{k_2} \phi_{k_3} \rangle = \frac{3\varepsilon H^3}{4 k_1 k_2 k_3} \int_{-\infty}^{0} d\tau_E' \, \tau_E'^2 \, e^{(\Sigma k_i)\tau_E'} + \text{c.c.}$$

$$= \frac{3H^3}{4 k_1 k_2 k_3} \cdot \frac{2}{(\Sigma k_i)^3} + \text{c.c.}$$

$$= \frac{3H^3}{k_1 k_2 k_3 (k_1+k_2+k_3)^3} (2\pi)^3 \delta^3(k_1+k_2+k_3)$$

$$f_{NL}^{\text{equilateral}}$$

$$\langle 0 | \hat{x} \hat{y} \hat{z} \hat{w} | 0 \rangle =$$

$$= -3i \int_{-\infty}^{0} dt'$$

$$= -3i \int_{-\infty}^{0} dt'$$

$a > 0$

$$\int_{-\infty}^{0} x^2 e^{ax} dx = \frac{2}{a}$$