Title: Explorations in Cosmology-2

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URL: http://pirsa.org/15040026

Abstract:

CONFORMAL TIME: TIME COORDINATE DEFINED BY $d\tau = dt/a$

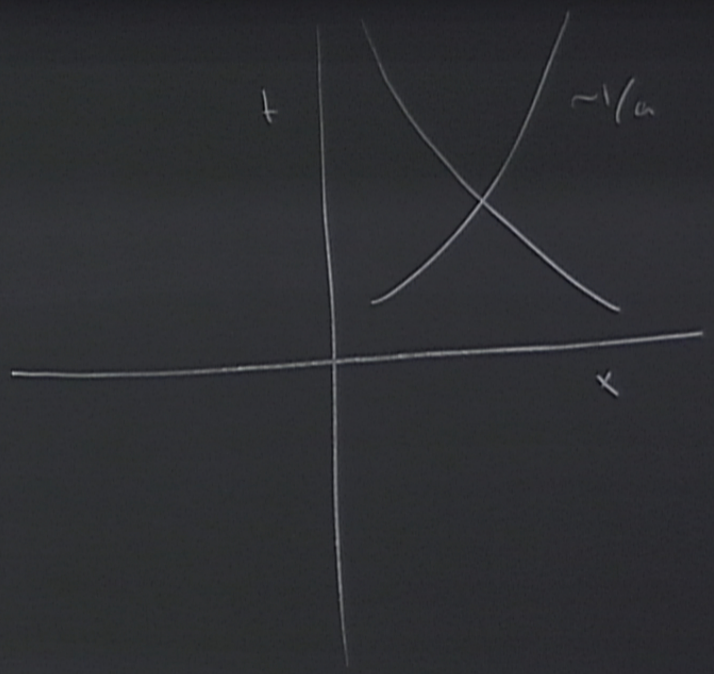
MAKES METRIC CONFORMALLY STATIC

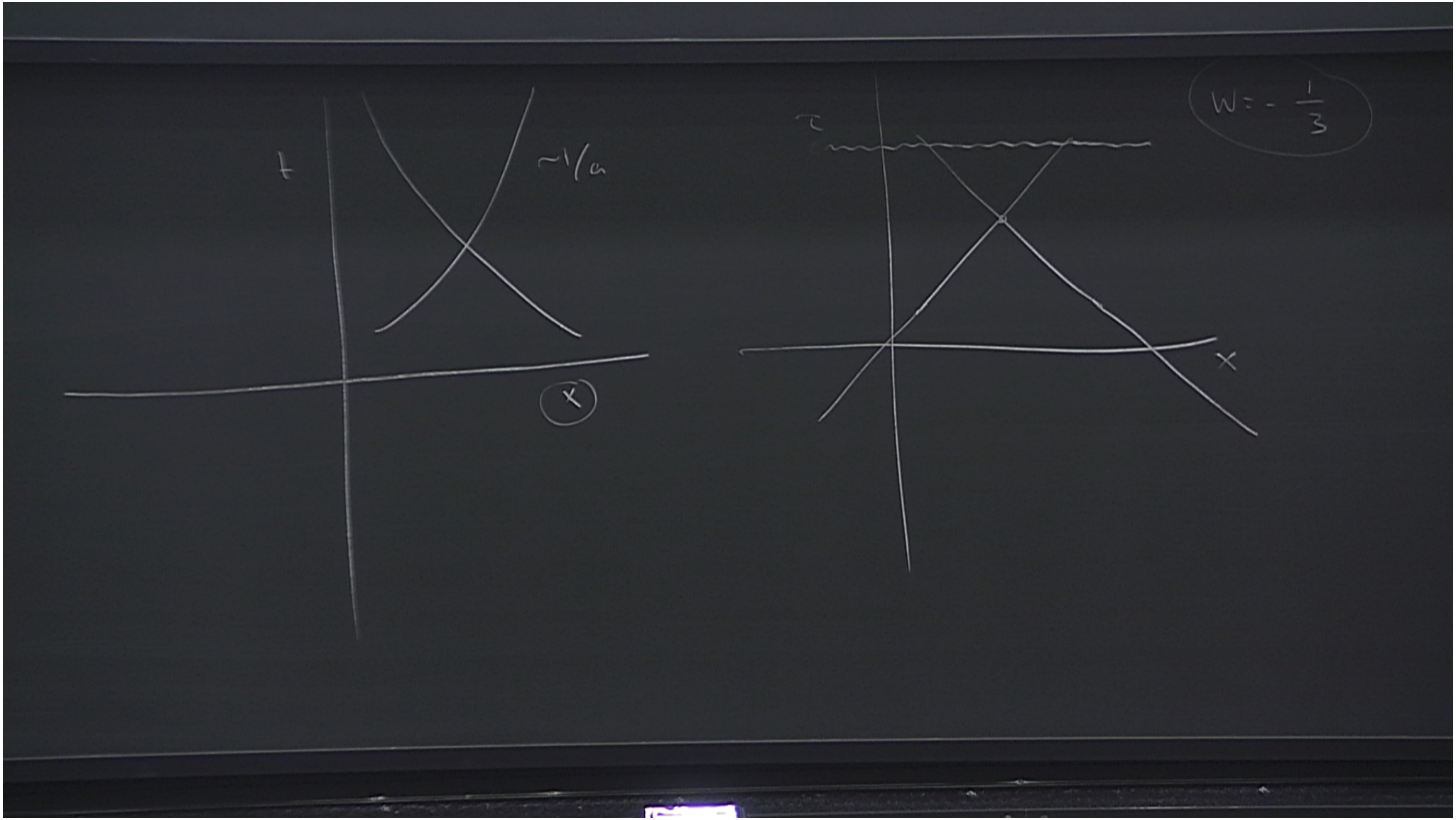
$$ds^2 = -dt^2 + a(t)^2\, d\Sigma_3^2 \quad [= dx^2 \text{ IF FLAT}]$$

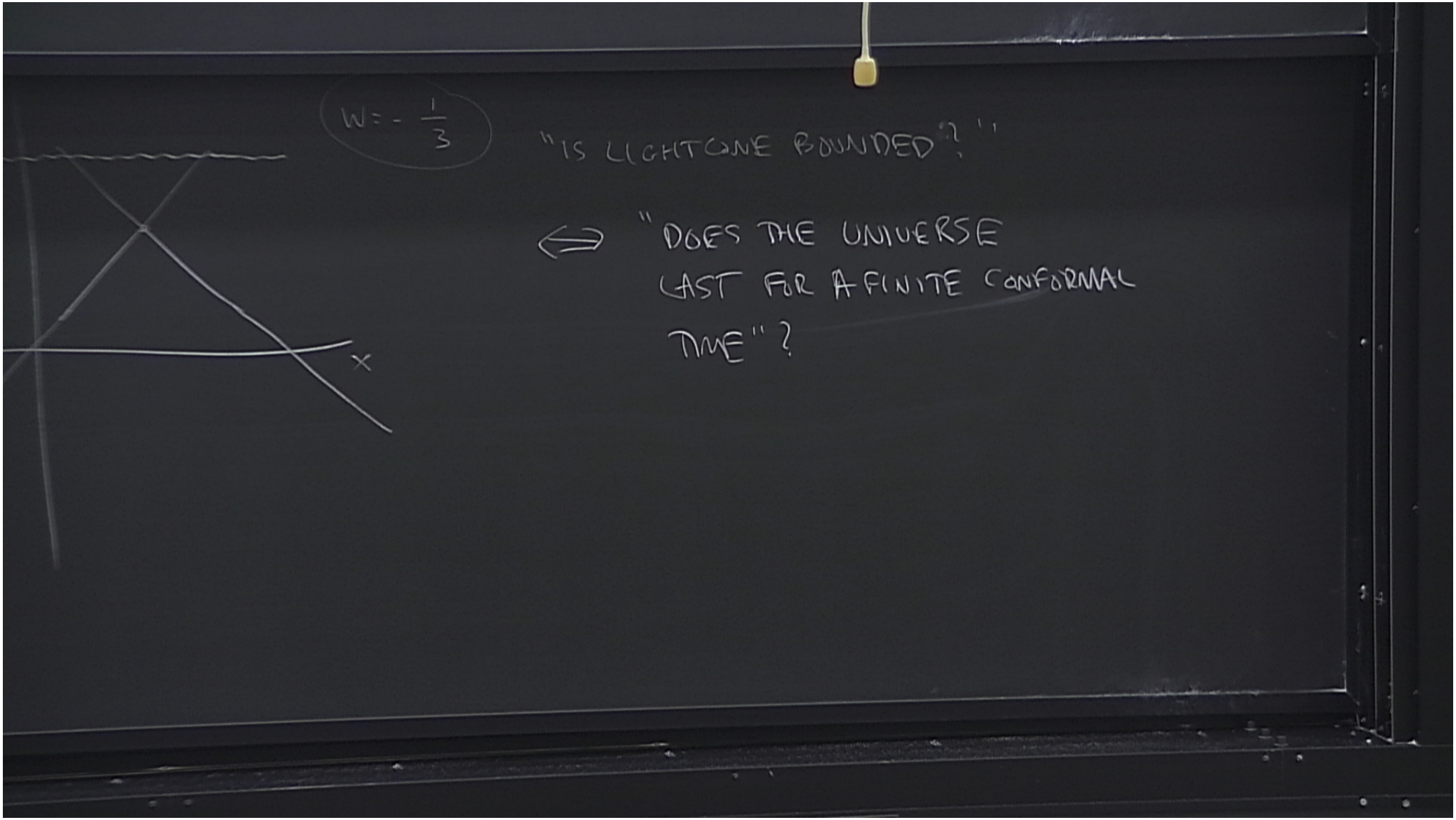
$$= a(t)^2\left[-d\tau^2 + d\Sigma_3^2\right]$$

LIGHTLIKE GEODESICS SATISFY $ds^2 = 0$

$$\frac{dx}{d\tau} = 1$$




t
~1/a
x
τ
x
W = - 1/3


$w = -\frac{1}{3}$
"IS LIGHTCONE BOUNDED?"
$\Leftrightarrow$ "DOES THE UNIVERSE LAST FOR A FINITE CONFORMAL TIME"?
x

EOM OF A MINIMALLY COUPLED TEST SCALAR FIELD

$$S = \frac{1}{2}\int d^4x \sqrt{-g}\, g^{\mu\nu}\, \partial_\mu \phi\, \partial_\nu \phi$$

$$\frac{\partial^2 \phi}{\partial t^2} + 3H\frac{\partial \phi}{\partial t} - a^{-2}\frac{\partial^2 \phi}{\partial x^2} = 0$$

$$\frac{\partial^2 \phi}{\partial \tau^2} + 2H\frac{\partial \phi}{\partial \tau} - \frac{\partial^2 \phi}{\partial x^2} = 0$$

$$\int dt = \int \frac{da}{aH} = \int \frac{da}{aH_0\left[\Omega_\Lambda + \Omega_m a^{-3} + \Omega_r a^{-4}\right]^{1/2}}$$

$$H(a) = H_0\left[\Omega_\Lambda + \Omega_m a^{-3} + \Omega_r a^{-4}\right]^{1/2}$$

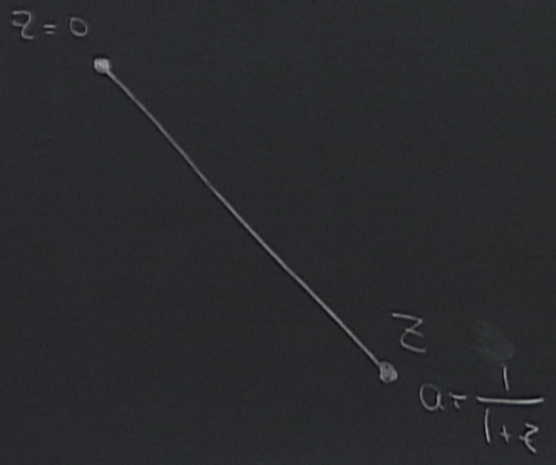
DISTANCES IN FRW

$$ds^2 = -dt^2 + a(t)^2 \, d\Sigma_3$$

$$d\Sigma_3 = dx^2 + r(x)^2 \, d\Omega_2$$

$$= \frac{dr^2}{1 - Kr^2} + r^2 \, d\Omega_2$$

$$r = \begin{cases} x & K = 0 \\ \dfrac{\sin K^{1/2} x}{K^{1/2}} & K > 0 \\ \dfrac{\sinh (-K)^{1/2} x}{(-K)^{1/2}} & K < 0 \end{cases}$$



- METRIC DISTANCE χ
- ANGULAR DIAMETER DISTANCE $d_A = \frac{R}{\Delta\theta}$

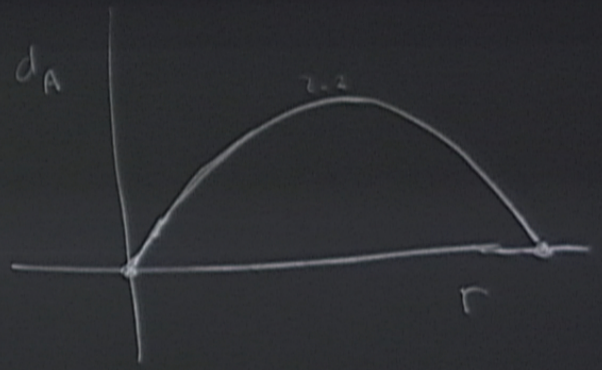
R = PHYSICAL LENGTH OF "STANDARD RULER"

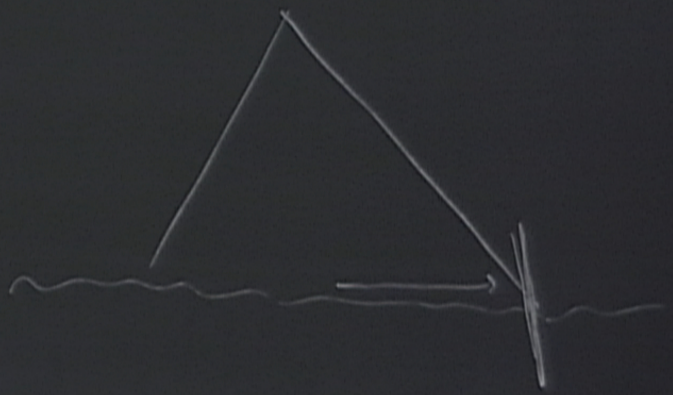
$\Delta\theta$ = ANGLE SUBTENDED ON SKY

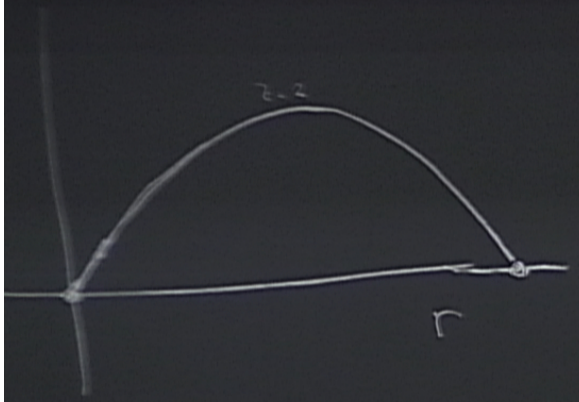


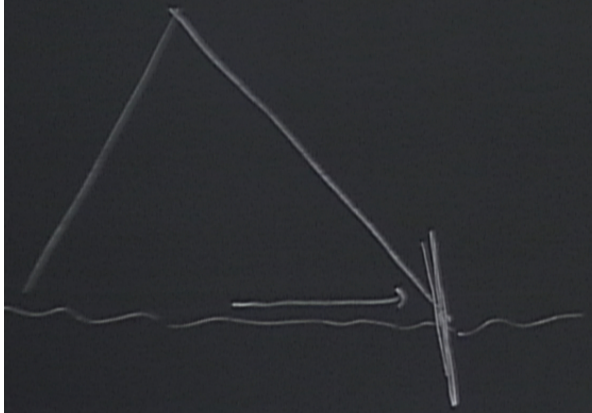
$R = a r \Delta\theta$

$\Rightarrow d_A = a r$









- LUMINOSITY DISTANCE d_L

DEFINED BY $F = \frac{L}{4\pi d_L^2}$

L = LUMINOSITY OF "STANDARD CANDLE" $\left[\frac{\text{ENERGY}}{\text{TIME}}\right]$

F = OBSERVED FLUX $\left[\frac{\text{ENERGY}}{\text{TIME} \cdot \text{AREA}}\right]$

$$\frac{\text{PHOTONS SENT}}{\text{TIME}} = \frac{L}{\omega}$$

$$\frac{\text{PHOTONS RECEIVED}}{\text{TIME}}$$

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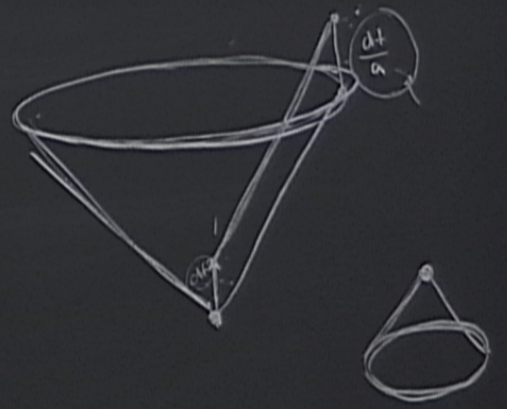
$\frac{\text{PHOTONS SENT}}{\text{TIME}} = \frac{L}{\omega}$

$\frac{\text{PHOTONS RECEIVED}}{\text{TIME}} = \frac{aL}{\omega}$

$\frac{\text{ENERGY RECVD}}{\text{TIME}} = a^2 L$

$F = \frac{a^2 L}{4\pi r^2}$

$\Rightarrow \boxed{d_L = \frac{r}{a}}$



QUANTUM MECHANICS

CONSIDER A 1D TIME DEPENDENT HARMONIC OSCILLATOR

$$H(t) = \frac{1}{2}p^2 + \frac{1}{2}\omega(t)^2 x^2$$

SUPPOSE THAT AT EARLY TIMES

1) $\omega(t) \to \omega_0$

2) OSCILLATOR IS IN ITS GROUND STATE $|0\rangle$

IC OSCILLATOR

KEY FACT: HEISENBERG-PICTURE EOM ARE LINEAR

$$\frac{d}{dt}\hat{X}_H = \hat{P}_H \qquad \frac{d}{dt}\hat{P}_H = -\omega(t)^2 \hat{X}_H$$

$\Rightarrow$ SOLUTION IN HEISENBERG PICTURE IS

$$\hat{X}_H(t) = v(t)\,\hat{a}_S + v(t)^*\,\hat{a}_S^\dagger$$

$$\hat{P}_H = \dot{v}(t)\,\hat{a}_S + \dot{v}(t)^*\,\hat{a}_S^\dagger$$

WHERE v, v^* ARE INDEPENDENT SOLUTIONS OF THE CLASSICAL EOM $\ddot{v}(t) = -\omega(t)^2 v(t)$

TO DETERMINE $\hat{a}_s$ WE MATCH TO EARLY TIMES

$$\hat{x}_H(t) = \cos(\omega_0 t)\,\hat{x}_s + \frac{1}{\omega_0}\sin(\omega_0 t)\,\hat{p}_s$$

$$= \frac{e^{-i\omega_0 t}}{(2\omega_0)^{1/2}}\,\hat{a}_s + \frac{e^{i\omega t}}{(2\omega_0)^{1/2}}\,\hat{a}_s^\dagger$$

CHOOSE v S.T. $v(t) \to \dfrac{e^{-i\omega_0 t}}{(2\omega_0)^{1/2}}$ AS $t \to -\infty$

THEN $\hat{a}_s = \hat{a}_s$

E.G. $\langle \psi(t) | \, x^2 \, | \psi(t) \rangle$

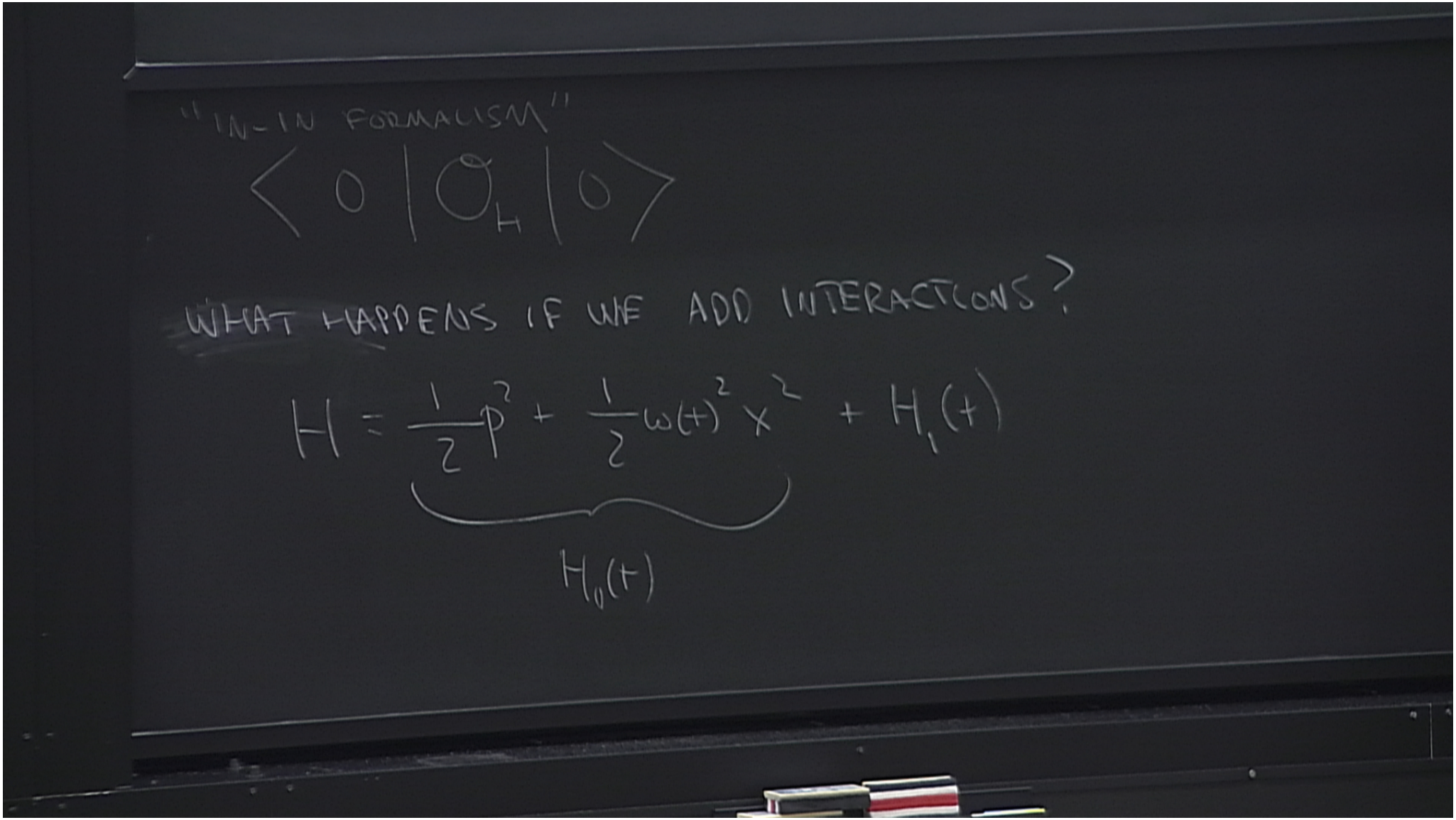
$= \langle 0 | \, x_H(t)^2 \, | 0 \rangle$

$= v(t)\, v(t)^* \langle 0 | \, a a^\dagger \, | 0 \rangle$

$= |v(t)|^2$

FREQUENCY-ARISING SPECIAL CASE $\omega(t)^2 = \omega_0^2 - \frac{2}{t^2}$ WITH $-\infty < t < 0$

$\Rightarrow v(t) = \frac{1}{(2\omega_0)^{1/2}} \left(1 - \frac{i}{\omega_0 t}\right) e^{-i\omega_0 t}$


"IN-IN FORMALISM"
$\langle 0 | \mathcal{O}_H | 0 \rangle$
WHAT HAPPENS IF WE ADD INTERACTIONS?
$H = \frac{1}{2}p^2 + \frac{1}{2}\omega(t)^2 x^2 + H_1(t)$
$H_0(t)$

SCHRODINGER PICTURE

$$|\psi_S(t)\rangle = U(t,t_0)\,|0\rangle$$

$$\hat{\mathcal{O}}_S = \text{CONST.}$$

HEISENBERG PICTURE

$$|\psi_H(t)\rangle = |0\rangle$$

$$\mathcal{O}_H(t) = U(t,t_0)^\dagger\,\hat{\mathcal{O}}_S\,U(t,t_0)$$

INTERACTION PICTURE

$$|\psi_I(t)\rangle = U_0(t,t_0)^\dagger\,U(t,t_0)\,|0\rangle$$

INTERACTION PICTURE

$$|\psi_I(t)\rangle = U_0(t,t_0)^\dagger\, U(t,t_0)\,|0\rangle$$

$$\mathcal{O}_I(t) = U_0(t,t_0)^\dagger\, \mathcal{O}_S\, U_0(t,t_0)$$

EQUIVALENCE

$$\langle \psi_S(t)|\,\mathcal{O}_S\,|\psi_S(t)\rangle = \langle 0|\,\mathcal{O}_H(t)\,|0\rangle$$

$$= \langle \psi_I(t)|\,\mathcal{O}_I(t)\,|\psi_I(t)\rangle$$

$$= \langle \psi|\mathcal{O}|\psi\rangle$$

FIELD

$$\int dt = \int \frac{da}{aH} = \int \frac{da}{aH_0\left[\Omega_\Lambda + \Omega_m a^{-3} + \Omega_r a^{-4}\right]^{1/2}}$$

DISTANCES (N

$$ds^2 = -dt^2 +$$

INTERACTION

$$\frac{d}{dt}\mathcal{O}_I(t) = i\left[H_0(t),\ \mathcal{O}_I(t)\right]$$

H_I = "INTERAC
IN THE

$$\frac{d}{dt}|\psi_I(t)\rangle = U_0(t,t_0)^\dagger\,(iH_0(t))\,U(t,t_0)|0\rangle + U_0(t,t_0)^\dagger\,(-i$$

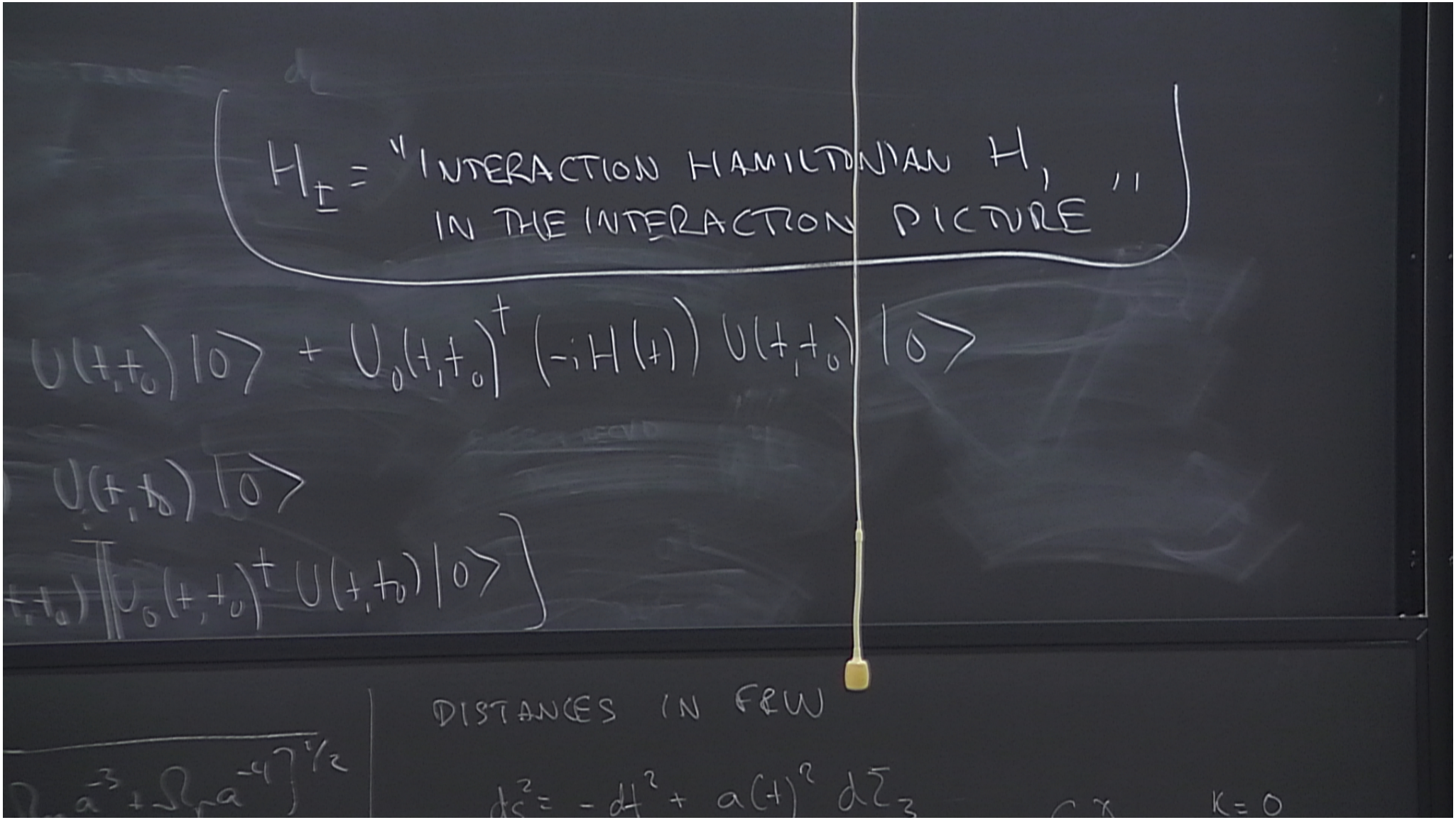
$$= -i\,U_0(t,t_0)^\dagger\,H_1(t)\,U(t,t_0)|0\rangle$$

$$= -i\left[U_0(t,t_0)^\dagger H_1 U_0(t,t_0)\right]\left[U_0(t,t_0)^\dagger\,U(t,t_0)|0\rangle\right]$$

$$\int dt = \int \frac{da}{aH} = \int \frac{da}{aH_0\left[\Omega_\Lambda + \Omega_m a^{-3} + \Omega_r a^{-4}\right]^{1/2}}$$

DISTANCE

$d_c^2 =$


H_I = "INTERACTION HAMILTONIAN H_1 IN THE INTERACTION PICTURE"
$U(t,t_0)|0\rangle + U_0(t,t_0)^\dagger (-iH(t)) U(t,t_0)|0\rangle$
$U(t,t_0)|0\rangle$
$U_0(t,t_0)^\dagger U(t,t_0)|0\rangle$
DISTANCES IN FRW
$ds^2 = -dt^2 + a(t)^2 d\Sigma_3$
$K=0$

$$\frac{d}{dt} U(t, t_0) = -iH(t)\ U(t, t_0)$$

INTERACTION

SCHRODINGER $\frac{d}{dt}|\psi_S(t)\rangle = -iH|\psi_S(t)\rangle$

HEISENBERG $\frac{d}{dt}\hat{O}_H(t) = i\left[H(t), O_H(t)\right]$

$\frac{d}{dt} O_I(t) =$

$\frac{d}{dt}|\psi_I($

$$= -i H_I(t) |\psi_I(t)\rangle$$

$$|\psi_I(t)\rangle = T \exp\left(-i \int_{t_0}^{t} dt' H_I(t')\right) |0\rangle$$

$$= \left(1 - i \int_{t_0}^{t} dt' H_I(t) - \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' H_I(t') H_I(t'')\right) |0\rangle$$

$$\langle \psi | \mathcal{O} | \psi \rangle = \langle 0 | \overline{T} \exp\left(i\right.$$

$$= -i H_I(t) |\psi_I(t)\rangle$$

$$|\psi_I(t)\rangle = T \exp\left(-i \int_{t_0}^{t} dt' H_I(t')\right) |0\rangle$$

$$= \left(1 - i \int_{t_0}^{t} dt' H_I(t) - \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' H_I(t') H_I(t'')\right) |0\rangle$$

$$\langle\psi|\mathcal{O}|\psi\rangle = \langle \psi_I(t)| \mathcal{O}_I(t) |\psi_I(t)\rangle$$

$$= \langle 0| \bar{T} \exp\left(i \int_{t_0}^{t} dt' H_I(t')\right) \mathcal{O}_I(t) T \exp\left(-i \int_{t_0}^{t} dt' H_I(t')\right) |0\rangle$$

$$= \langle 0|\mathcal{O}_I(t)|0\rangle - i \int_{t_0}^{t} dt' \langle 0|[\mathcal{O}_I(t), H_I(t')]|0\rangle + \cdots$$