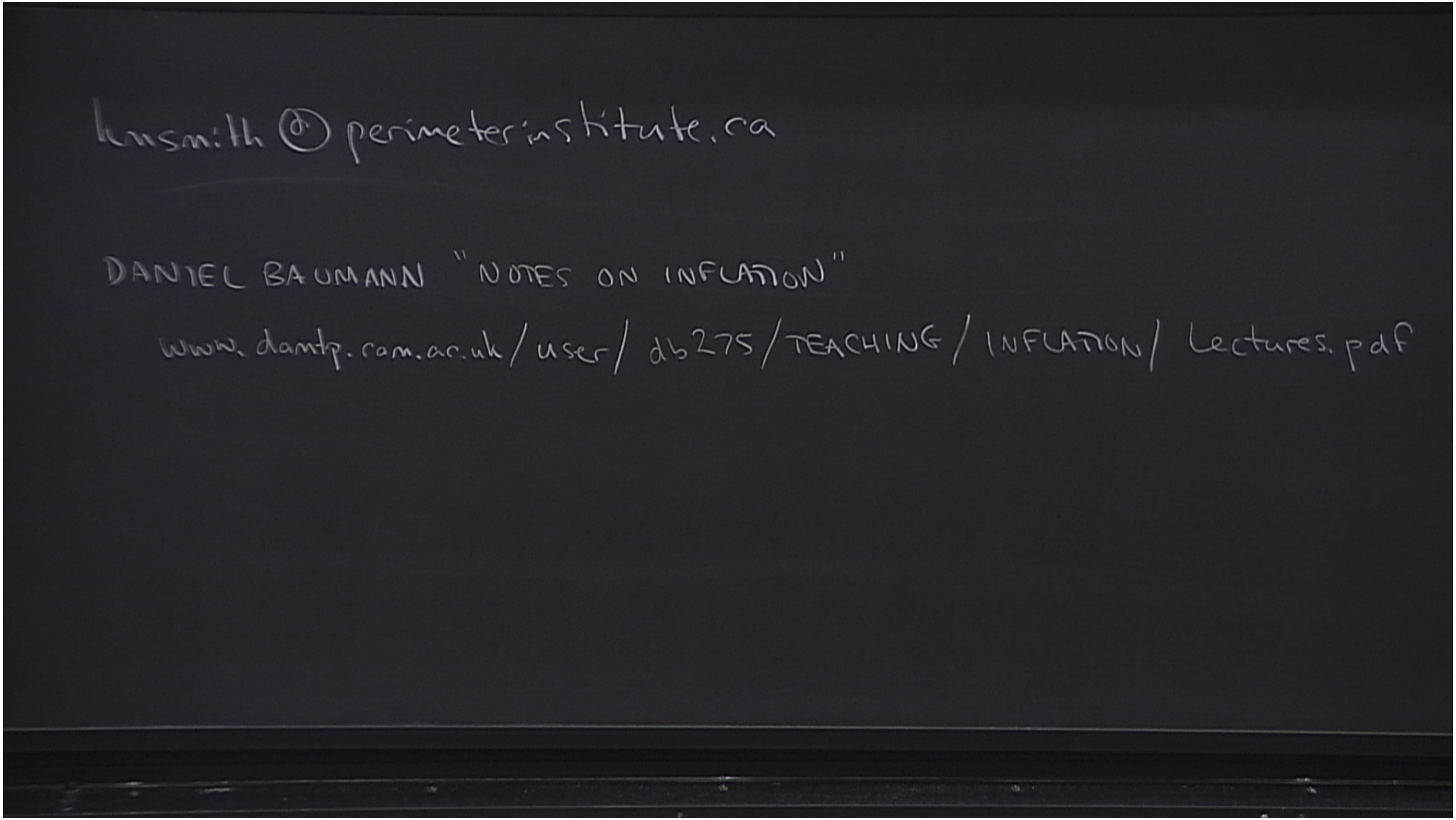
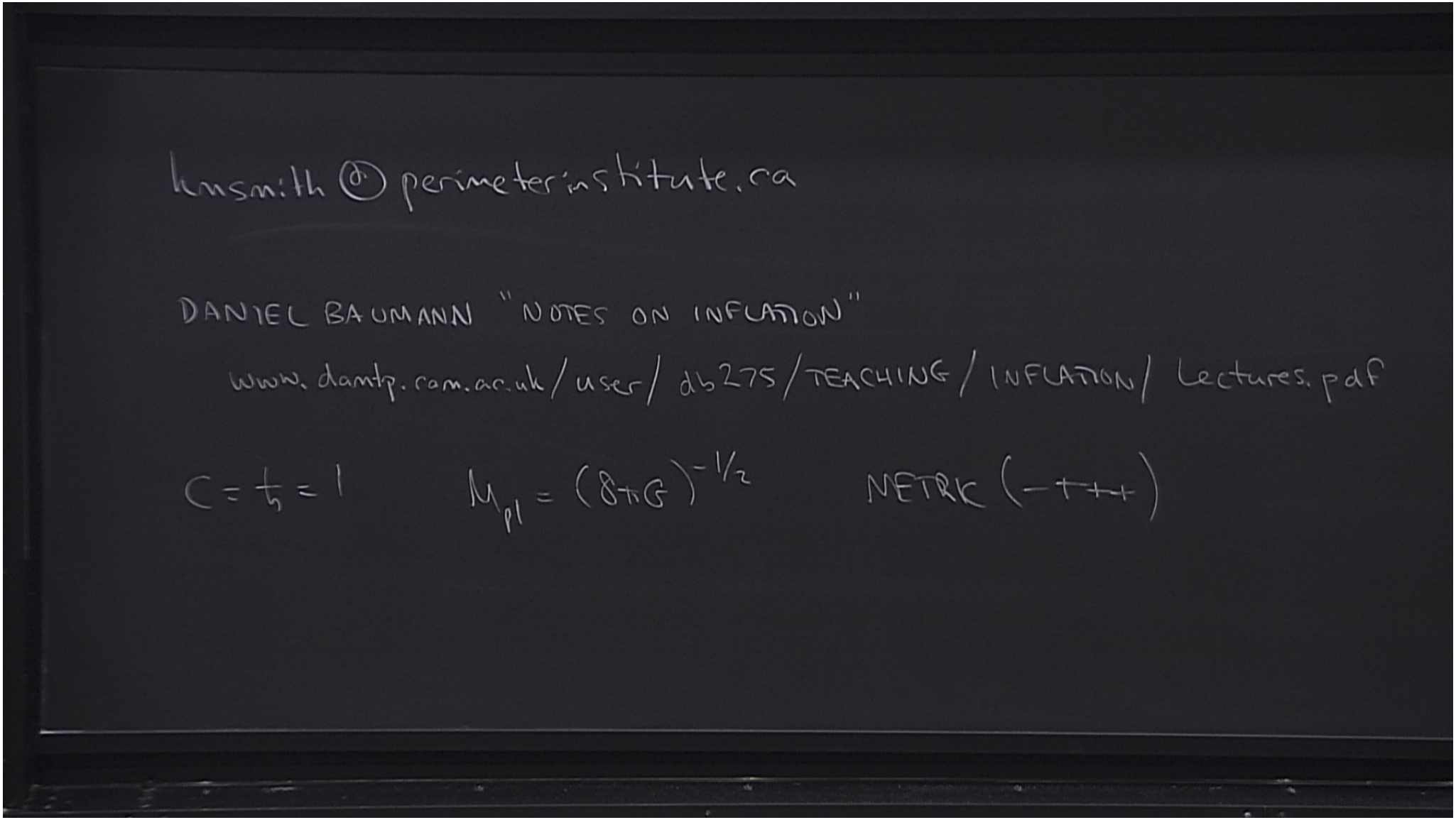
Title: Explorations in Cosmology-1

Date: Apr 06, 2015 10:15 AM

URL: http://pirsa.org/15040025

Abstract:


knsmith @ perimeterinstitute.ca
DANIEL BAUMANN "NOTES ON INFLATION"
www.damtp.cam.ac.uk/user/db275/TEACHING/INFLATION/Lectures.pdf


lmsmith @ perimeterinstitute.ca
DANIEL BAUMANN "NOTES ON INFLATION"
www.damtp.cam.ac.uk/user/db275/TEACHING/INFLATION/Lectures.pdf
$c = \hbar = 1$
$M_{Pl} = (8\pi G)^{-1/2}$
METRIC $(-+++)$

FRW METRIC



OBSERVED: ANGULAR

IMPLIED: ROTATIONS + TRANSLATION

FRW: $ds^2 = -dt^2 + a(t)^2 \gamma_{ij} dx^i dx^j$

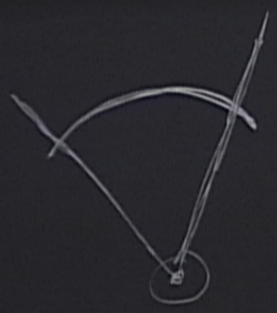
WHERE $\gamma_{ij} = \delta_{ij}$ IN FLAT UNIVERSE, GENERALIZATION TO CURVED SHORTLY

$a(t)$ = "SCALE FACTOR" MONOTONE INCREASING FROM $a=0$ [BIG BANG] TO $a=1$ [TODAY]

FRW METRIC

OBSERVED: ANGULAR

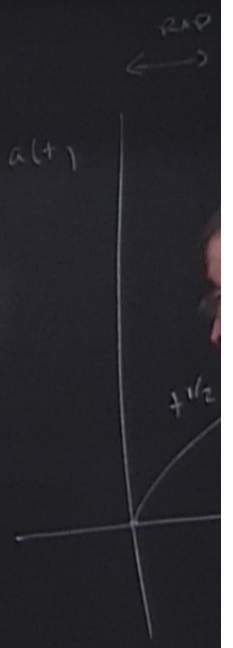
IMPLIED: ROTATIONS + TRANSLATION



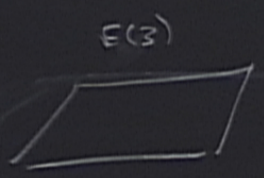
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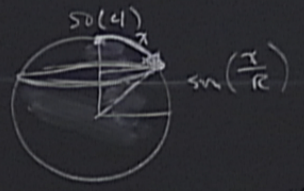
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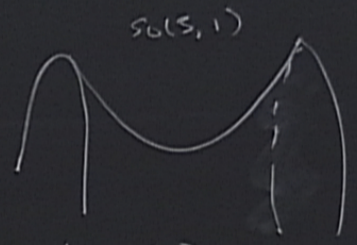
SPATIAL CURVATURE $K = ((\text{LENGTH})^{-2}$



$K = 0$



$K > 0$ [RADIUS $K^{-1/2}$]



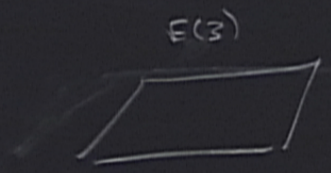
$K < 0$

$$\gamma_{ij}\,dx^i\,dx^j = dx^2 + S(x)^2\,d\Omega_2^2$$

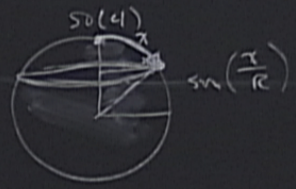
$$= \frac{dr^2}{1-Kr^2} + r^2\,d\Omega_2^2$$

$$S(x) = \begin{cases} x & K = 0 \\ \dfrac{\sin(K^{1/2}x)}{K^{1/2}} & K > 0 \\ \dfrac{\sinh[(-K)^{1/2}x]}{(-K)^{1/2}} & K < 0 \end{cases}$$

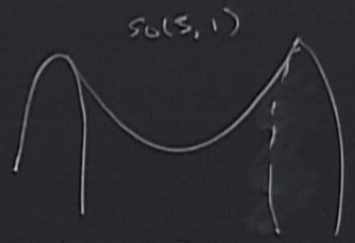
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UNIVERSE HAS A REST FRAME



$x(t) = \text{const.}$

$\frac{dx^\mu}{dt} = (1, 0, 0, 0)$

$(\Delta x)_{PHYS} = a(t)\,(\Delta x)$

$H(t) = \frac{1}{a}\frac{da}{dt}$

$H^{-1} \sim 4000\ \text{Mpc}$ COSMOLOGICAL SCALE

$\sim 14\ \text{Gyr}$

STRESS ENERGY PARAMETERIZED BY TWO FUNCTIONS OF TIME

$\rho(t) = \frac{\text{ENERGY}}{\text{PHYSICAL VOLUME}}$

$p(t) = \frac{\text{FORCE}}{\text{PHYSICAL AREA}}$

RUNNING THE CP MACHINE

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho}\left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu}\right)$$

$$\Gamma^{0}_{ij} = a\dot{a}\gamma_{ij}$$

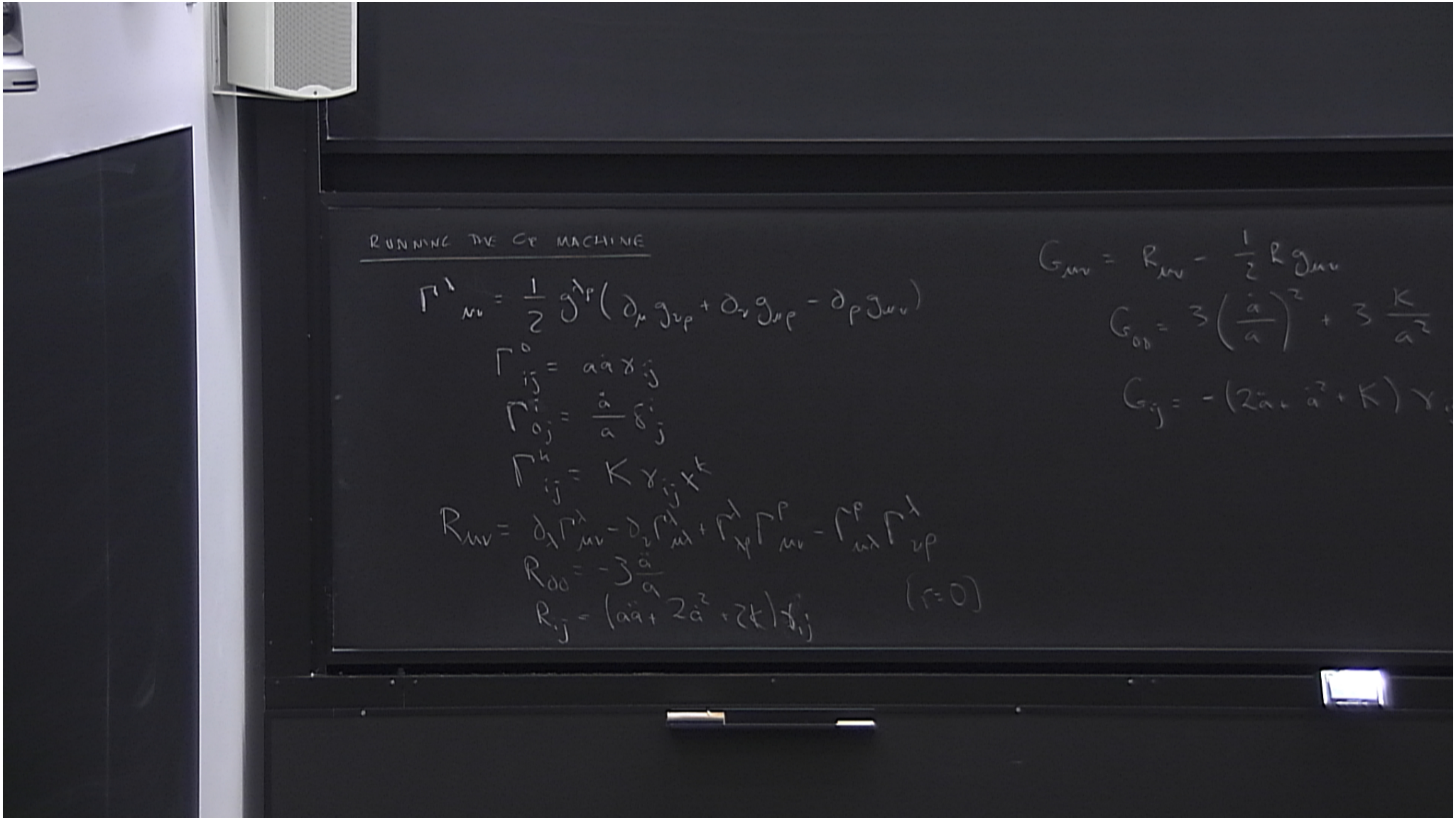
$$\Gamma^{i}_{0j} = \frac{\dot{a}}{a}\delta^{i}_{j}$$

$$\Gamma^{k}_{ij} = K\gamma_{ij}t^{k}$$

$$R_{\mu\nu} = \partial_\lambda \Gamma^{\lambda}_{\mu\nu} - \partial_\nu \Gamma^{\lambda}_{\mu\lambda} + \Gamma^{\lambda}_{\lambda\rho}\Gamma^{\rho}_{\mu\nu} - \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\rho}$$

$$R_{00} = -3\frac{\ddot{a}}{a}$$

$$R_{ij} = \left(a\ddot{a} + 2\dot{a}^2 + 2k\right)\gamma_{ij}$$


RUNNING THE GR MACHINE
$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho}(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu})$
$\Gamma^{0}_{ij} = a\dot{a}\gamma_{ij}$
$\Gamma^{i}_{0j} = \frac{\dot{a}}{a}\delta^{i}_{j}$
$\Gamma^{k}_{ij} = K\gamma_{ij}x^k$
$R_{\mu\nu} = \partial_\lambda\Gamma^{\lambda}_{\mu\nu} - \partial_\nu\Gamma^{\lambda}_{\mu\lambda} + \Gamma^{\lambda}_{\lambda\rho}\Gamma^{\rho}_{\mu\nu} - \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\rho}$
$R_{00} = -3\frac{\ddot{a}}{a}$
$R_{ij} = (a\ddot{a} + 2\dot{a}^2 + 2K)\gamma_{ij}$ $(r=0)$
$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu}$
$G_{00} = 3\left(\frac{\dot{a}}{a}\right)^2 + 3\frac{K}{a^2}$
$G_{ij} = -(2\ddot{a} + \dot{a}^2 + K)\gamma_{ij}$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

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$$G_{ij} = -\left(2\ddot{a} + \dot{a}^2 + K\right) \gamma_{ij}$$

EINSTEIN'S EQ^S

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$$

FRIEDMANN EQUATION

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

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COMBININ

CONTIN

COMBINING THE TWO GIVES

$$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a} \qquad \left[w = \frac{p}{\rho} \right]$$

CONTINUITY EQUATION $\nabla_\mu T^{\mu\nu} = 0$

FRIEDMANN EQUATION

COMBINING THE TWO GIVES

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CONTINUITY EQUATION $\nabla_\mu T^{\mu\nu} = 0$

...ANN EQUATION.

$R_{ij} = (a\ddot{a} + 2\dot{a}^2 + 2k) g_{ij}$ $(r=0)$

EXAMPLE SINGLE COMPONENT FLAT UNIVERSE W/ CONSTANT w $p = w\rho$

CONTINUITY EQUATION DETERMINES EVOLUTION OF ρ

$$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a} \Rightarrow \rho(t) \propto a(t)^{-3(1+w)}$$

NOW INTEGRATE FRIEDMANN'S EQ

$$a(t) \propto \begin{cases} t^{\frac{2}{3(1+w)}} & w \neq -1 \\ e^{Ht} & w = -1 \end{cases} \quad [\text{WHERE } H = \text{CONST.}]$$

= WP | ΛCDM

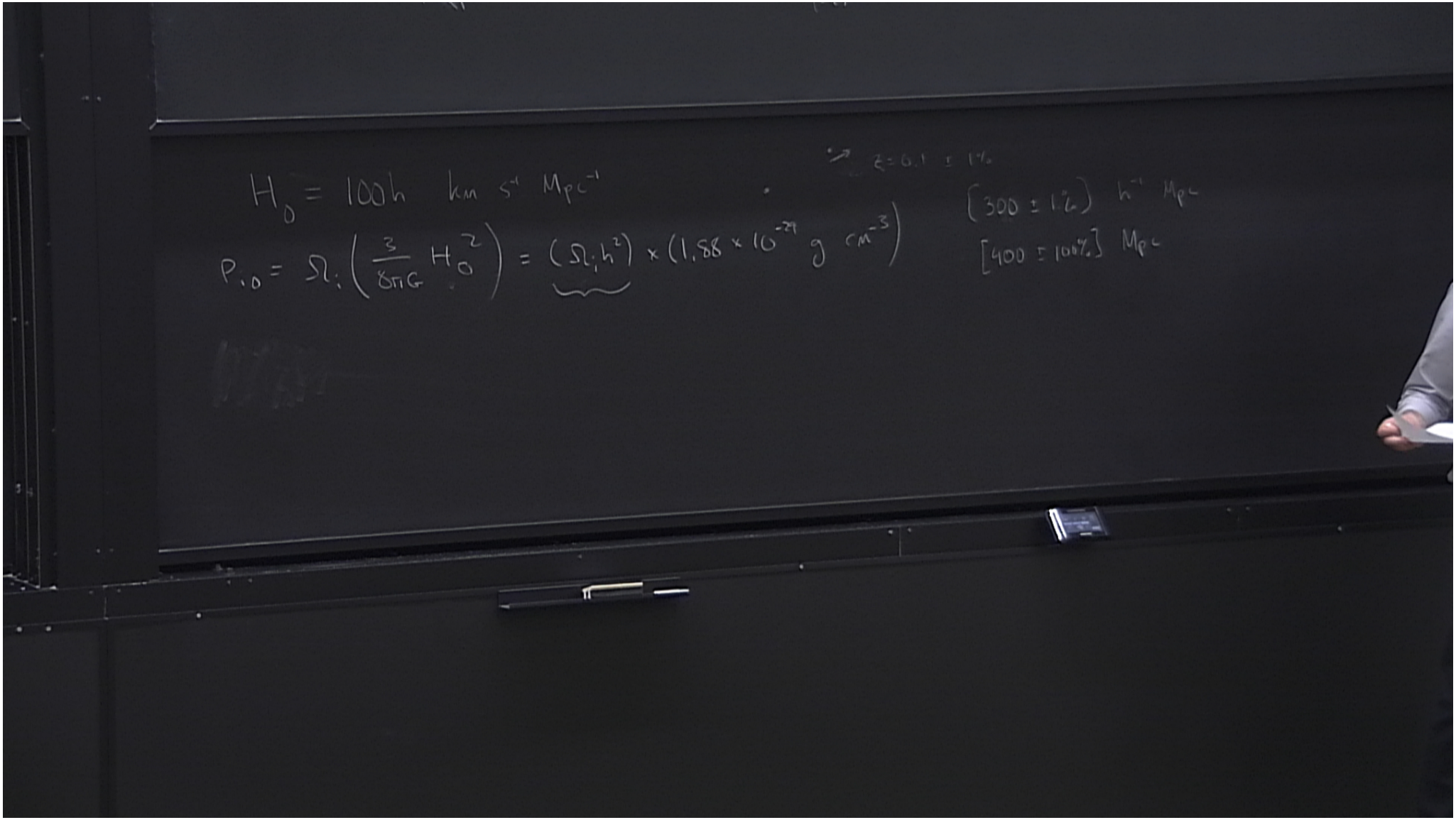
$$\rho = \rho_{b0} a^{-3} + \rho_{c0} a^{-3} + \rho_{r0} a^{-4} + \rho_{\Lambda 0}$$

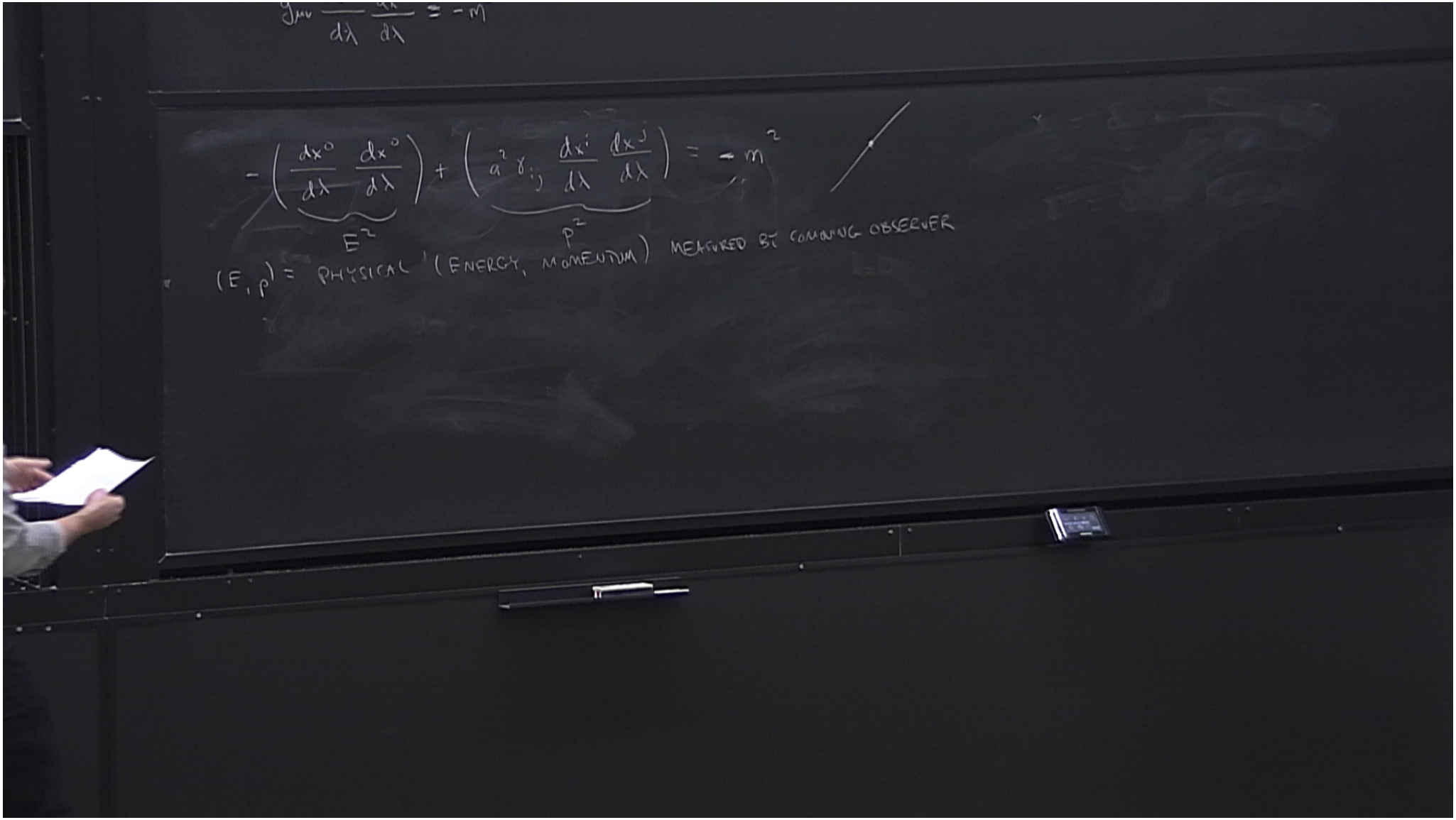
$$p = \frac{1}{3}\rho_{r0} a^{-3} - \rho_{\Lambda 0}$$

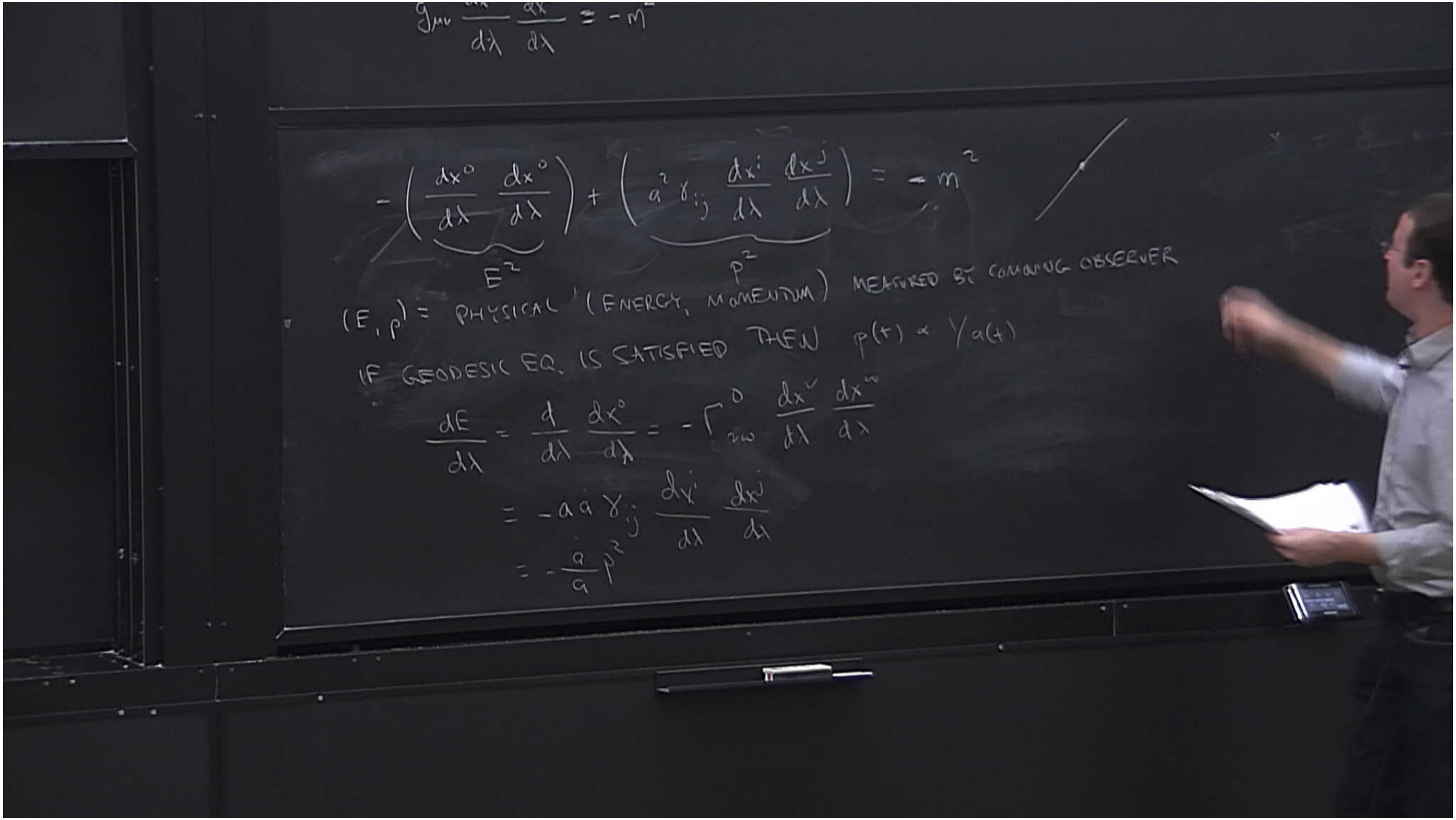
"PHYSICIST'S PARAMETRIZATION" $\{\rho_{b0}, \rho_{c0}, \rho_{r0}, \rho_{\Lambda 0}\}$

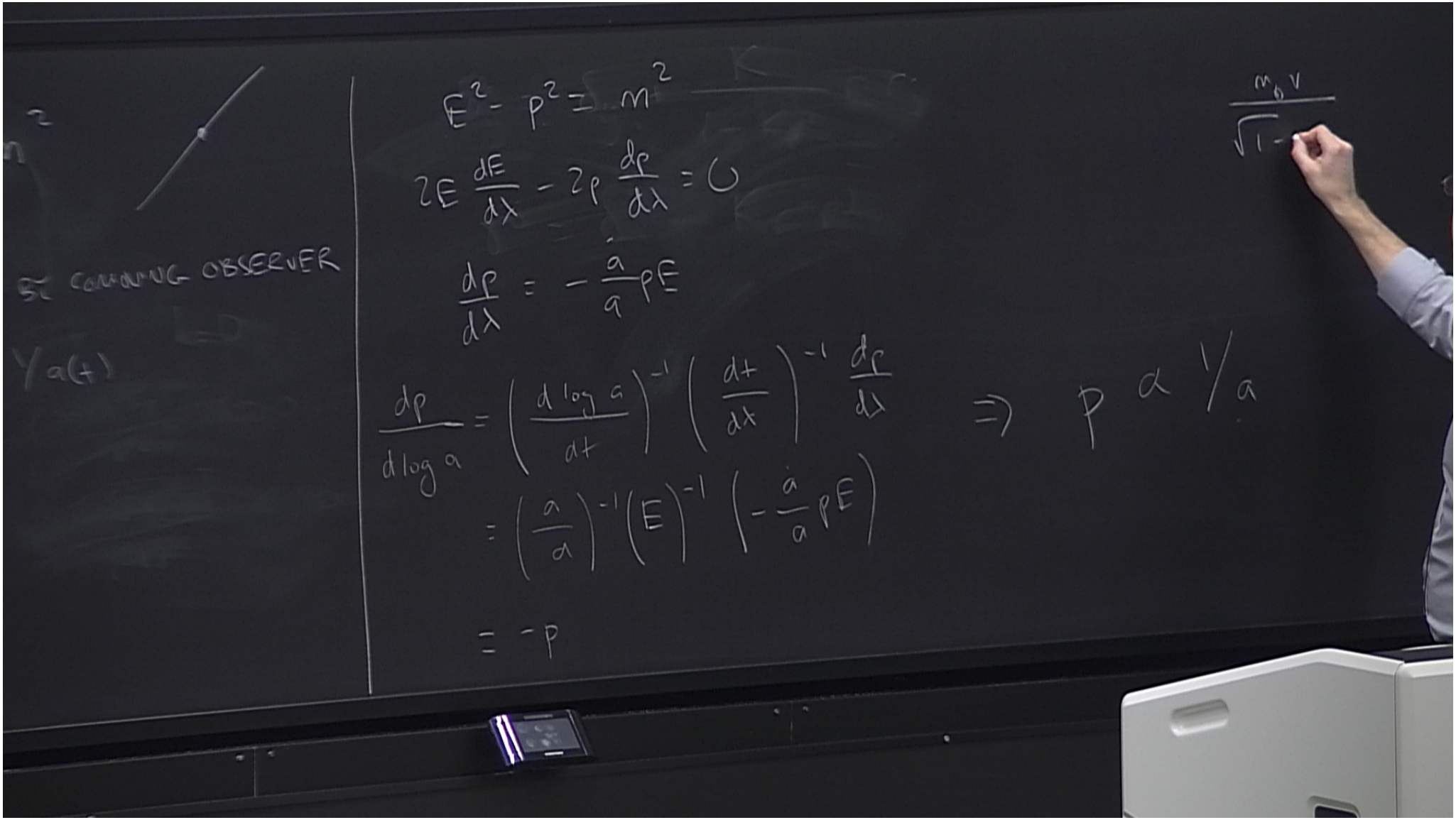
"ASTRO PARAMETRIZATION" $\{\Omega_{b0}, \Omega_{c0}, \Omega_{r0}, \Omega_{\Lambda 0}, h\}$

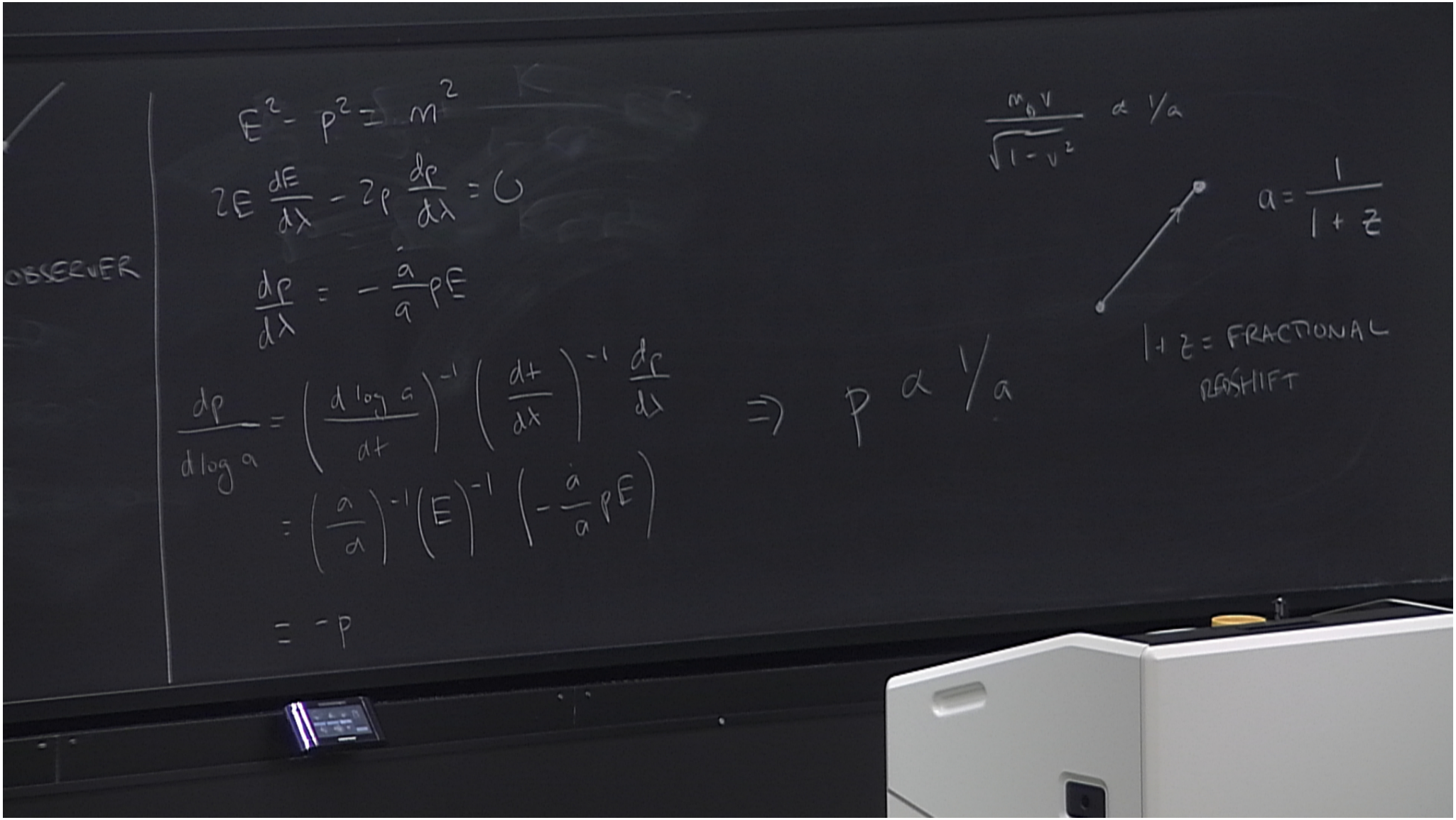
$$\Omega_{i0} = \frac{\rho_{i0}}{\Sigma \rho_{j0}} \quad [\Sigma \Omega_i = 1]$$


$H_0 = 100h \ \text{km s}^{-1} \ \text{Mpc}^{-1}$
$\rho_{\cdot 0} = \Omega_{\cdot} \left(\frac{3}{8\pi G} H_0^2 \right) = (\Omega_{\cdot} h^2) \times (1.88 \times 10^{-29} \ \text{g cm}^{-3})$
$\Rightarrow \xi = 0.1 \pm 1\%$
$(300 \pm 1\%) \ h^{-1} \ \text{Mpc}$
$[400 \pm 100\%] \ \text{Mpc}$


$g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = -m^2$
$-\left(\frac{dx^0}{d\lambda}\frac{dx^0}{d\lambda}\right) + \left(a^2\gamma_{ij}\frac{dx^i}{d\lambda}\frac{dx^j}{d\lambda}\right) = -m^2$
E^2
p^2
(E, p) = PHYSICAL (ENERGY, MOMENTUM) MEASURED BY COMOVING OBSERVER

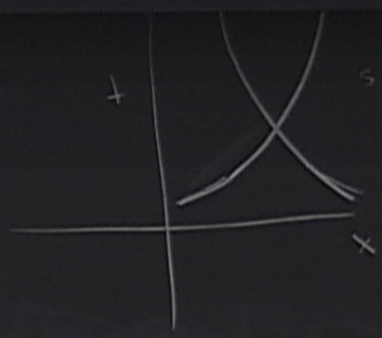

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E^2
p^2
(E, p) = PHYSICAL (ENERGY, MOMENTUM) MEASURED BY COMOVING OBSERVER
IF GEODESIC EQ. IS SATISFIED THEN $p(t) \propto 1/a(t)$
$\frac{dE}{d\lambda} = \frac{d}{d\lambda}\frac{dx^0}{d\lambda} = -\Gamma^0_{\nu\omega}\frac{dx^\nu}{d\lambda}\frac{dx^\omega}{d\lambda}$
$= -a\dot{a}\gamma_{ij}\frac{dx^i}{d\lambda}\frac{dx^j}{d\lambda}$
$= -\frac{\dot{a}}{a}p^2$


COMOVING OBSERVER
a(t)
$E^2 - p^2 = m^2$
$2E\frac{dE}{d\lambda} - 2p\frac{dp}{d\lambda} = 0$
$\frac{dp}{d\lambda} = -\frac{\dot{a}}{a}pE$
$\frac{dp}{d\log a} = \left(\frac{d\log a}{dt}\right)^{-1}\left(\frac{dt}{d\lambda}\right)^{-1}\frac{dp}{d\lambda}$
$= \left(\frac{\dot{a}}{a}\right)^{-1}(E)^{-1}\left(-\frac{\dot{a}}{a}pE\right)$
$= -p$
$\Rightarrow p \propto 1/a$
$\frac{m_0 v}{\sqrt{1-}}$


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$\Rightarrow p \propto 1/a$
OBSERVER
$\frac{m_0 v}{\sqrt{1-v^2}} \propto 1/a$
$a = \frac{1}{1+z}$
$1+z$ = FRACTIONAL REDSHIFT

$R_{ij} = (a\ddot{a} + 2\dot{a}^2 + 2k)\delta_{ij}$

TRAJECTORY OF LIGHTLIKE GEODESIC



$$\frac{dx_{phys}}{dt} = a(t)\frac{dx}{dt} = 1$$

$$X = \int \frac{dt}{a(t)}$$

ARE THE PAST/FUTURE LIGHTCONES BOUNDED OR UNBOUNDED?

[IN COMOVING COORDINATES]

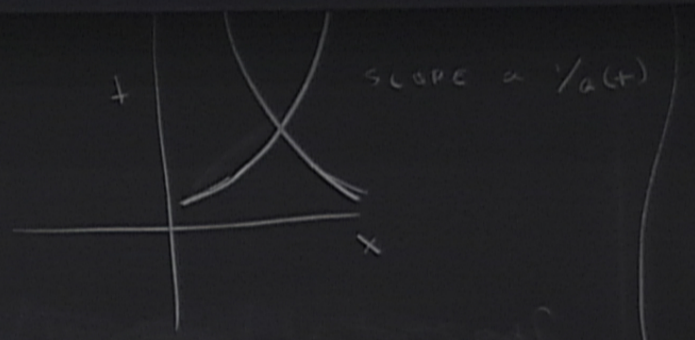
$$\Delta x = \int \frac{dt}{a} = \int \frac{da}{a^2 H} \propto \int \frac{da}{a^2 a^{-3(1+w)/2}} = \int a^{\frac{-1+3w}{2}} da$$

w >

w <

$R_{ij} = [a\ddot{a} + 2\dot{a}^2 + 2k]\delta_{ij}$

TRAJECTORY OF LIGHTLIKE GEODESIC



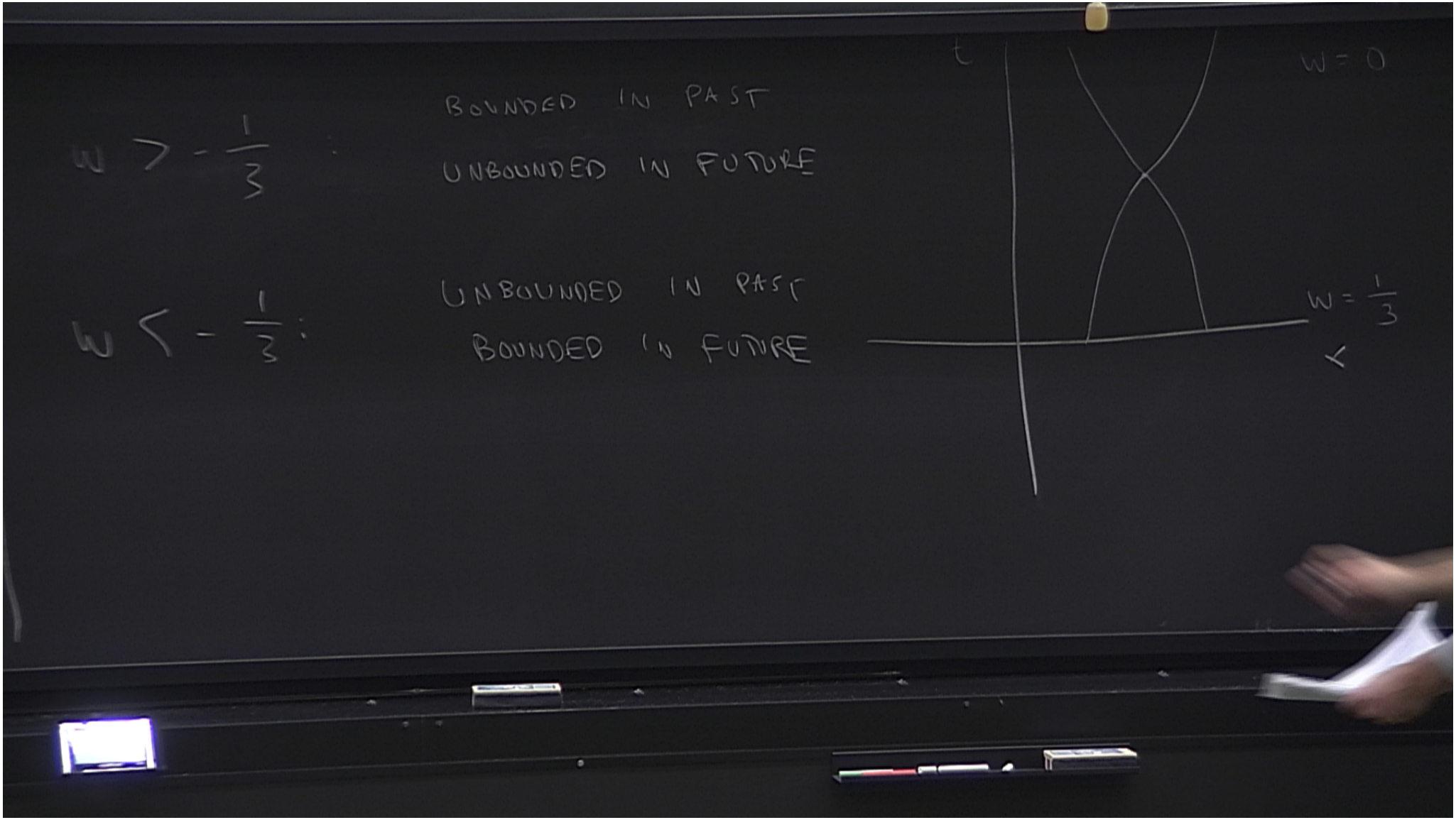
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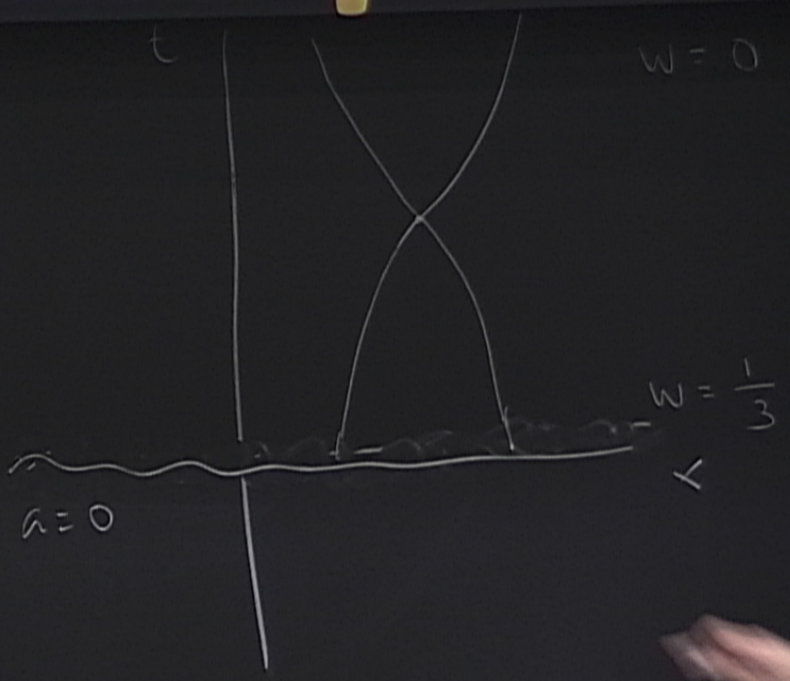
[IN COMOVING COORDINATES]

$$\Delta x = \int \frac{dt}{a} = \int \frac{da}{a^2 H} \propto \int \frac{da}{a^2 a^{-3(1+w)/2}} = \int a^{\frac{-1+3w}{2}} da$$


$w > -\frac{1}{3}$:
BOUNDED IN PAST
UNBOUNDED IN FUTURE
$w < -\frac{1}{3}$:
UNBOUNDED IN PAST
BOUNDED IN FUTURE
t
w = 0
$w = \frac{1}{3}$
r

$w > -\frac{1}{3}$: BOUNDED IN PAST
UNBOUNDED IN FUTURE

$w < -\frac{1}{3}$: UNBOUNDED IN PAST
BOUNDED IN FUTURE



$-\frac{1}{3}(\rho + 3p)$

$w > -\frac{1}{3}$: BOUNDED IN PAST
UNBOUNDED IN FUTURE

$w < -\frac{1}{3}$: UNBOUNDED IN PAST
BOUNDED IN FUTURE

