Title: Explorations in Particle Theory-13

Date: Apr 22, 2015 09:00 AM

URL: http://pirsa.org/15040022

Abstract:

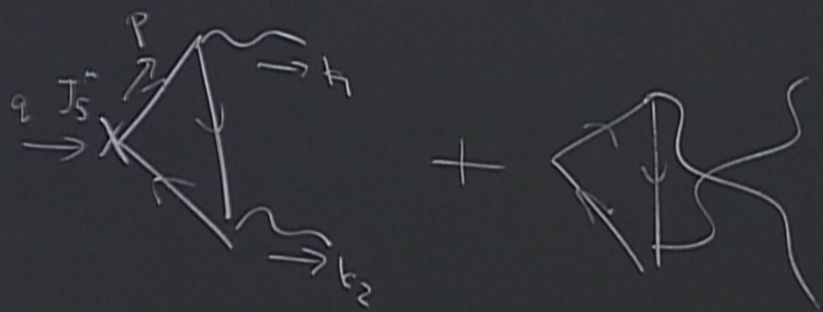
Anomalies & Strong CP Problem

$\psi \to e^{i\alpha\gamma^5}\psi$

We want $\partial_\mu \langle J_5^\mu \ldots \rangle = ?$ $\quad J_5^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi.$

any gauge coupled to ψ → gauge charge.

$$q_\lambda \langle A^\mu(k_1) A^\nu(k_2) \,|\, J_5^\lambda(q) \,|\, 0 \rangle \equiv g^2 q_\lambda \underbrace{\Delta^{\lambda\mu\nu}(k_1, k_2)}_{\text{3-point function}}$$



$\Delta^{\lambda\mu\nu}(k_1, k_2$

$$\Delta^{\lambda\mu\nu}(k_1,k_2) = (-1)\, i^3 \int \frac{d^4p}{(2\pi)^4} \Big[\mathrm{Tr}\Big(\gamma^\lambda \gamma^5 \frac{1}{\not p - \not q}\, \gamma^\nu \frac{1}{\not p - \not k_1}\, \gamma^\mu \frac{1}{\not p}\Big) + \mathrm{Tr}\Big(\gamma \ldots$$

↖ linear divergent

Q: Is $\Delta^{\lambda\mu\nu}(k_1 k_2)$ well-defined? Should not depend on choice of …

$p \to p + a$, is Δ the same?

$$\int_{-\infty}^{\infty} dp\, [f(p+a) - f(p)] = \int_{-\infty}^{\infty} dp\, a \frac{df}{dp} = a\,[f(\infty) - f(-\infty)]$$

← co…

←

$$\frac{1}{\not p - \not k_1}\gamma^\mu \frac{1}{\not p} \Big) + \mathrm{Tr}\left(\gamma^\lambda \gamma^5 \frac{1}{\not p - \not k_1}\gamma^\mu \frac{1}{\not p - \not k_2}\gamma^\nu \frac{1}{\not p}\right)\Big]$$

↖ linear divergent.

depend on choice of p.

$f(\infty) - f(-\infty)\big]$ ← convergent OR log divergent, $f \to 0$ for $p \to \pm\infty$ ✓

← linear + divergent, ✗

$$\Delta^{\lambda\mu\nu}(k_1,k_2) = (-i)\, i^3 \int \frac{d^4p}{(2\pi)^4} \Big[\mathrm{Tr}\Big(\gamma^\lambda \gamma^5 \frac{1}{\not p - \not q} \gamma^\nu \frac{1}{\not p - \not k_1} \gamma^\mu \frac{1}{\not p}\Big) + \mathrm{Tr}\Big(\gamma^\lambda \gamma^5 \frac{1}{\not p - \not q}$$

← linear divergent

Q: Is $\Delta^{\lambda\mu\nu}(k_1, k_2)$ well-defined? Should not depend on choice of p.

$p \to p + a$, is Δ the same?

$$\int_{-\infty}^{\infty} dp\,[f(p+a) - f(p)] = \int_{-\infty}^{\infty} dp\, a \frac{df}{dp} = a\,[f(\infty) - f(-\infty)]$$

← convergent OR

← linear + dive

Write $\Delta^{\lambda\mu\nu}(a, k_1, k_2)$

4d Minkowski:

$$\int d^4p\,(f(a+p) - f(p)) = \lim_{P\to\infty} i a^\mu \frac{P_\mu}{P} f(P)(2\pi^2 P^3)$$

← surface area of lower dim integral

$$\mathrm{Tr}\left(\gamma^\lambda \gamma^5 \frac{1}{\not{p} - \not{q}} \gamma^\nu \frac{1}{\not{p} - \not{k}_1} \gamma^\mu \frac{1}{\not{p}}\right) \qquad \frac{1}{\not{x}} = \frac{\not{x}}{x^2}$$

$$\Rightarrow \frac{\mathrm{Tr}\left(\gamma^\lambda \gamma^5 \not{p} \gamma^\nu \not{p} \gamma^\mu \not{p}\right)}{p^6}$$

$$= \frac{4i}{p^4} p_\sigma \varepsilon^{\sigma\nu\mu\lambda}$$

$$\Delta^{\lambda\mu\nu}(a, k_1, k_2) - \Delta^{\lambda\mu\nu}(k_1, k_2) = -\frac{4i}{8\pi^2} \lim_{p \to \infty} a^\omega \frac{p_\omega p_\sigma}{p^2} \varepsilon^{\sigma\nu\mu\lambda} + \begin{pmatrix} \mu \leftrightarrow \nu \\ k_1 \leftrightarrow k_2 \end{pmatrix}$$

$$\mathrm{Tr}\left(\gamma^\lambda\gamma^5 \frac{1}{\not{k}-\not{q}}\gamma^\nu \frac{1}{\not{p}-\not{k}_1}\gamma^\mu \frac{1}{\not{p}}\right) \qquad \frac{1}{\not{x}} = \frac{\not{x}}{x^2}$$

$$\Rightarrow \frac{\mathrm{Tr}\left(\gamma^\lambda\gamma^5 \not{p}\gamma^\nu \not{p}\gamma^\mu \not{p}\right)}{p^6} \qquad \int d^4p\, P_\alpha P_\beta = \int \frac{1}{4} P^2 g_{\alpha\beta}\, d^4p.$$

$$= \frac{4i}{p^4} P_\sigma \varepsilon^{\sigma\nu\mu\lambda}$$

$$\Delta^{\lambda\mu\nu}(a, k_1 k_2) - \Delta^{\lambda\mu\nu}(k_1, k_2) = -\frac{4i}{8\pi^2} \lim_{P\to\infty} a^\omega \frac{P_\omega P_\sigma}{P^2} \varepsilon^{\sigma\nu\mu\lambda} + \begin{pmatrix} \mu \leftrightarrow \nu \\ k_1 \leftrightarrow k_2 \end{pmatrix}$$

$$= -\frac{i}{8\pi^2} \varepsilon^{\sigma\nu\mu\lambda} a_\sigma + (\leftrightarrow)$$

Choose the "right" β.

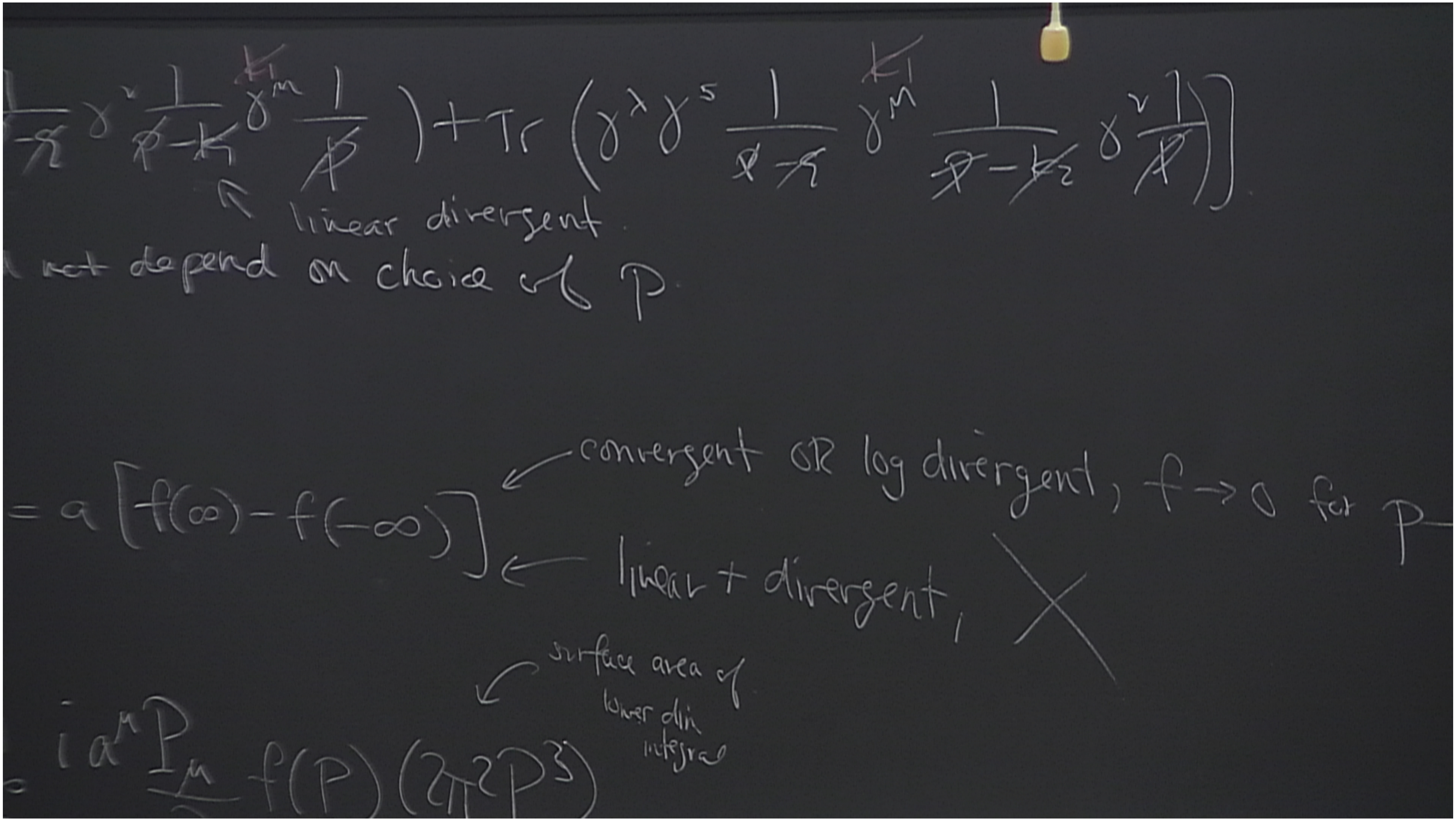
Impose vector current conservation.

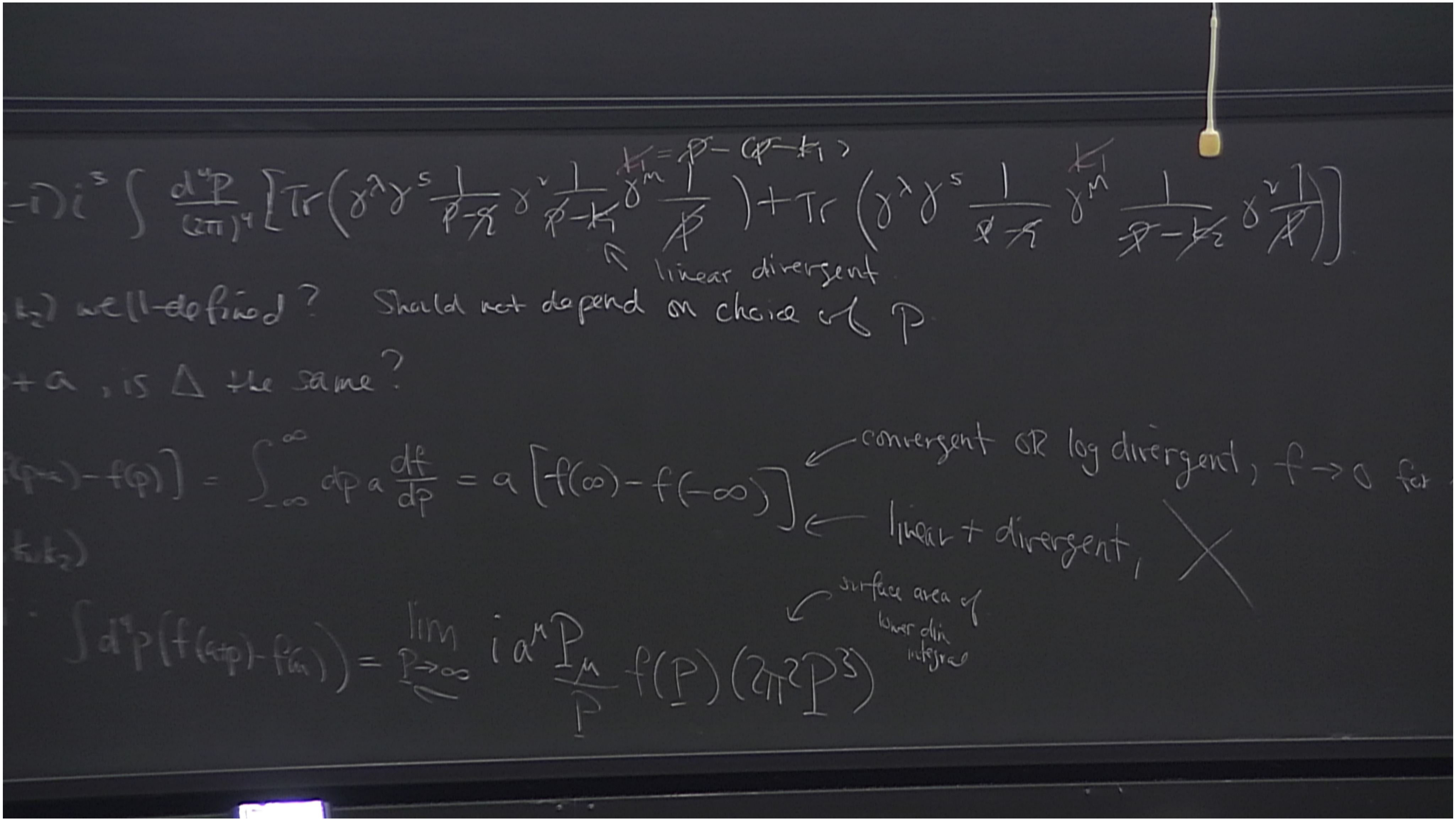
$$\partial_\mu J^\mu = 0$$

$\longrightarrow$ fermion number conservation

$\longrightarrow$ EM exact vector gauge symmetry.

$$k_{1\mu} \Delta^{\lambda\mu\nu} = k_{2\nu} \Delta^{\lambda\mu\nu} = 0$$


+ Tr
linear divergent
not depend on choice of P
convergent OR log divergent, f → 0 for P
= a [f(∞) − f(−∞)]
linear + divergent,
surface area of lower dim integral
i a^μ P_μ f(P) (2π²P³)


$k_1 = p - (p - k_1)$
$(-1) i^3 \int \frac{d^4 p}{(2\pi)^4} \left[Tr\left(\gamma^\lambda \gamma^5 \frac{1}{\not p - \not q} \gamma^\nu \frac{1}{\not p - \not k_1} \gamma^\mu \frac{1}{\not p}\right) + Tr\left(\gamma^\lambda \gamma^5 \frac{1}{\not p - \not q} \gamma^\mu \frac{1}{\not p - \not k_2} \gamma^\nu \frac{1}{\not p}\right)\right]$
linear divergent
well-defined? Should not depend on choice of p
+a, is Δ the same?
$\left[f(p+a) - f(p)\right] = \int_{-\infty}^{\infty} dp \, a \frac{df}{dp} = a\left[f(\infty) - f(-\infty)\right]$
convergent OR log divergent, f → 0 for
linear + divergent,
$\int d^4p \left(f(a+p) - f(p)\right) = \lim_{P \to \infty} i a^\mu \frac{P_\mu}{P} f(P) (2\pi^2 P^3)$
surface area of lower dim integral

$$k_{1\mu}\Delta^{\lambda\mu\nu} = i\int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\left[\gamma^\lambda\gamma^5\frac{1}{\not p - \not k}\gamma^\nu\frac{1}{\not p - \not k_1} - \gamma^\lambda\gamma^5\frac{1}{\not p - \not k_2}\gamma^\nu\frac{1}{\not p}\right]$$

$p \to p - k_1$

$$= 0 + (\text{shift from } p \to p - k_1)$$

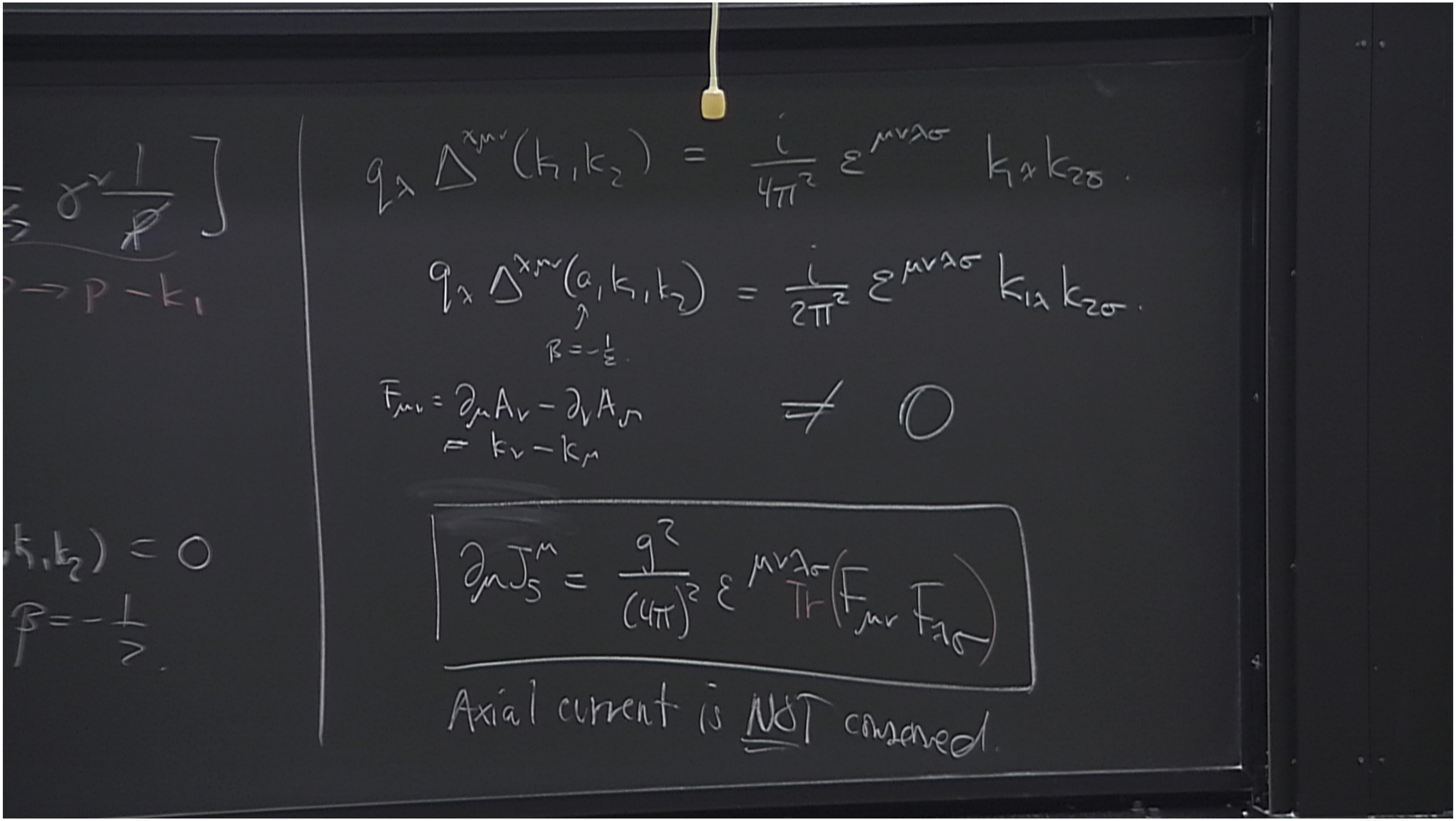
$$\boxed{k_{1\mu}\Delta^{\lambda\mu\nu}(k_1,k_2) = \frac{i}{8\pi^2}\varepsilon^{\lambda\nu\tau\sigma}k_{1\tau}k_{2\sigma}}$$

$$k_{1\mu}\Delta(a,k_1,k_2) = 0$$

when $\beta = -\frac{1}{2}$.

$$\boxed{\beta = -\frac{1}{2}}$$

so that properly defined)


$p \to p - k_1$
$(k_1, k_2) = 0$
$\beta = -\frac{1}{2}$.
$q_\lambda \Delta^{\lambda\mu\nu}(k_1, k_2) = \frac{i}{4\pi^2} \varepsilon^{\mu\nu\lambda\sigma} k_{1\lambda} k_{2\sigma}$.
$q_\lambda \Delta^{\lambda\mu\nu}(a, k_1, k_2) = \frac{i}{2\pi^2} \varepsilon^{\mu\nu\lambda\sigma} k_{1\lambda} k_{2\sigma}$.
$\beta = -\frac{1}{2}$.
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$= k_\nu - k_\mu$
$\neq 0$
$\partial_\mu J_5^\mu = \frac{g^2}{(4\pi)^2} \varepsilon^{\mu\nu\lambda\sigma} \mathrm{Tr}(F_{\mu\nu} F_{\lambda\sigma})$
Axial current is NOT conserved.

Choose the "right" β.

Impose vector current conservation.

$$\partial_\mu J^\mu = 0 \longleftarrow \text{holds in any correlation fn.}$$

$\longrightarrow$ fermion number conservation

$\longrightarrow$ EM exact vector gauge symmetry

$$k_{1\mu} \Delta^{\lambda\mu\nu} = k_{2\nu} \Delta^{\lambda\mu\nu} = 0$$

$$k_{1\mu} \Delta^{\lambda\mu\nu} = i \int$$

$$= 0$$

$$k_{1\mu} \Delta^{\lambda\mu\nu} ($$

β

$$\partial_\mu \left(\bar{\psi} \gamma^\mu \frac{1}{2}(1 \pm \gamma^5) \psi \right) = \pm \frac{1}{2} \frac{g^2}{(4\pi)^2} \varepsilon^{\mu\nu\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma}$$

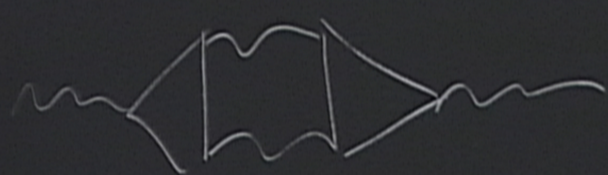
↖ chiral anomaly.

Implications:

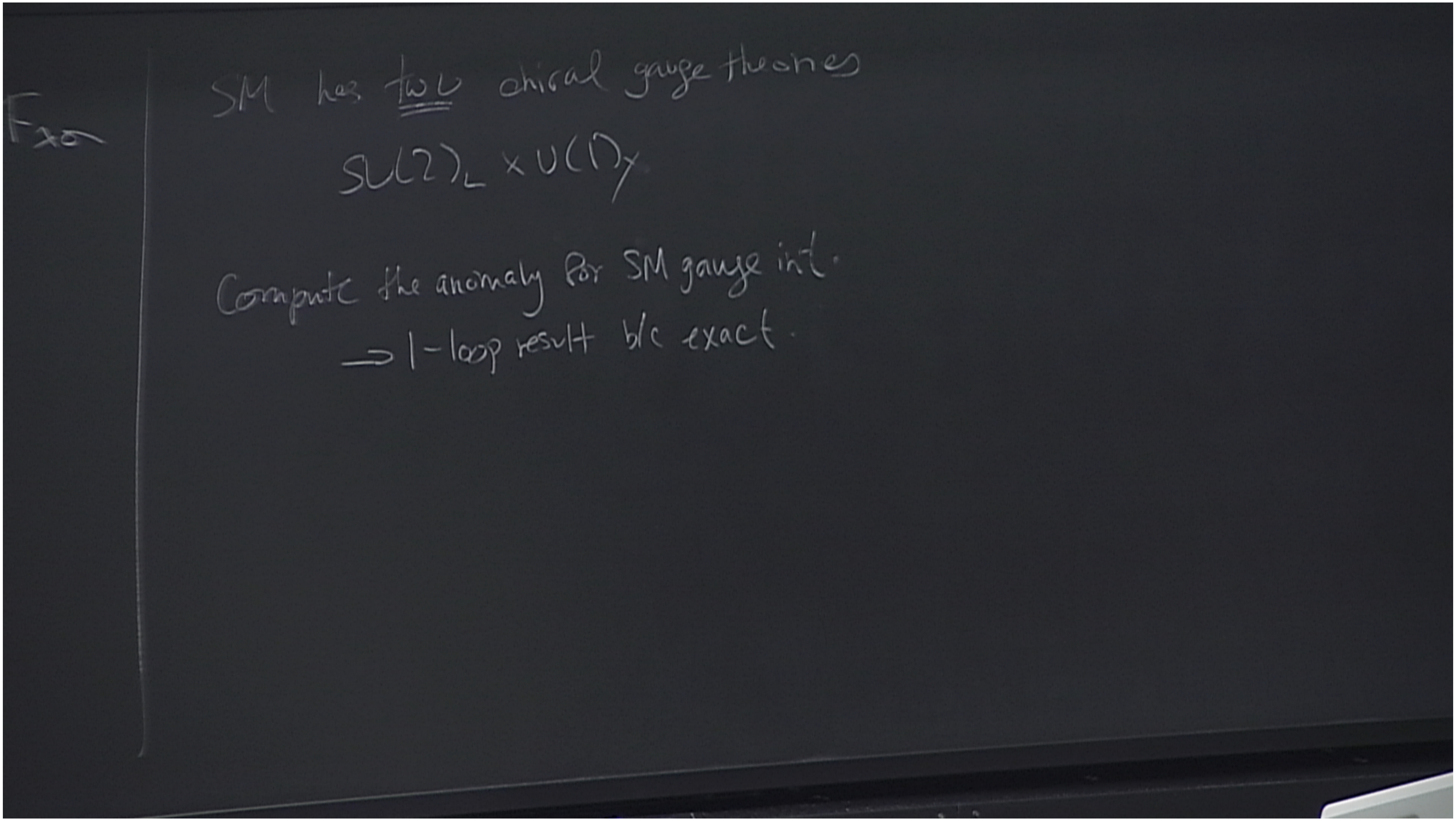
$U(1)_A$ is global: annoying

$U(1)_A$ is gauge: catastrophe.

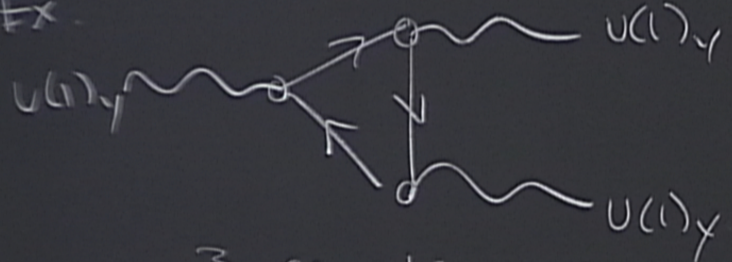
↳ inconsistent

A_5^μ  A_5^ν ~ mass for gauge.

SM


SM has two chiral gauge theories
SU(2)_L × U(1)_Y
Compute the anomaly for SM gauge int.
→ 1-loop result b/c exact.

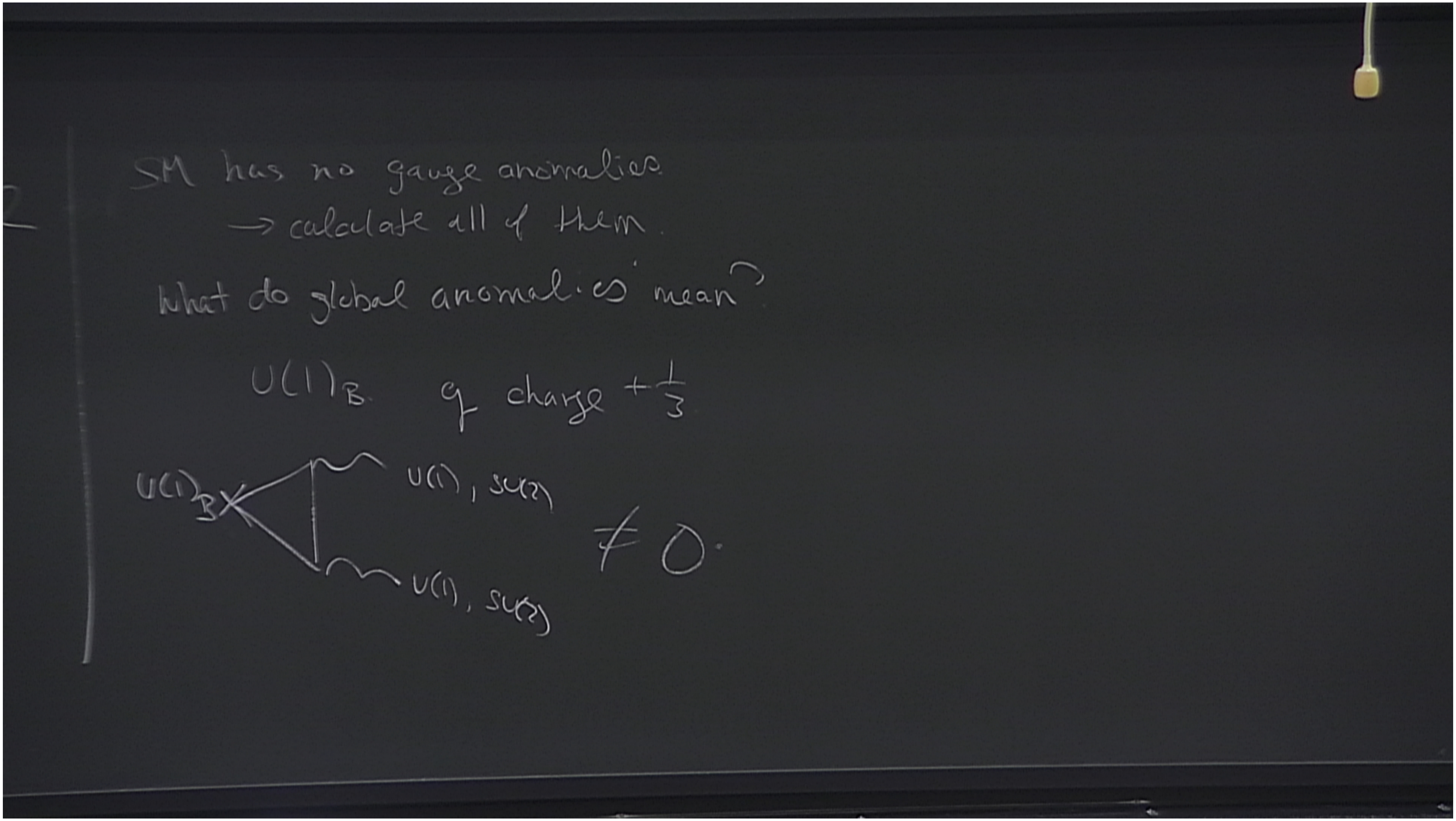
Ex.



SM: 3 generations of fermions

	$SU(3) \times SU(2) \times U(1)$
Q	$(3, 2, \frac{1}{6})$
u^c	$(\bar{3}, 1, -\frac{2}{3})$
d^c	$(\bar{3}, 1, \frac{1}{3})$
L	$(1, 2, -\frac{1}{2})$
E^c	$(1, 1, 1)$

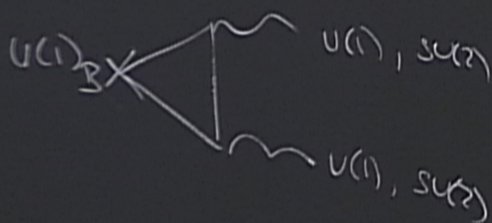
$$\text{anomaly} = F_{\mu\nu}F^{\mu\nu}\Big[\left(\tfrac{1}{6}\right)^3 \times 3 \times 2 + \left(-\tfrac{2}{3}\right)^3 \times 3 \times 1 + \left(\tfrac{1}{3}\right)^3 \times 3 \times 1 + \left(-\tfrac{1}{2}\right)^3 \times 1 \times 2 + (1)^3 \times 1 \times 1 \Big] = \emptyset$$


SM has no gauge anomalies
→ calculate all of them.
What do global anomalies mean?
U(1)_B q charge +1/3
U(1)_B
U(1), SU(2)
U(1), SU(2)
≠ 0.

SM has no gauge anomalies
→ calculate all of them.

What do global anomalies mean?

$U(1)_B$ q charge $+\frac{1}{3}$

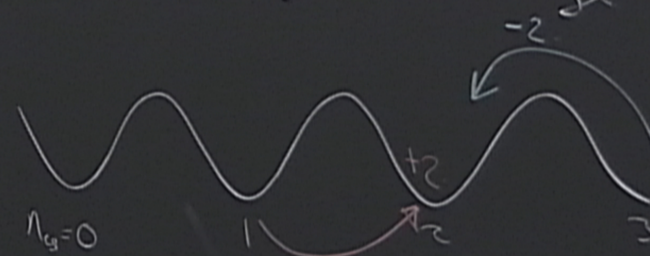


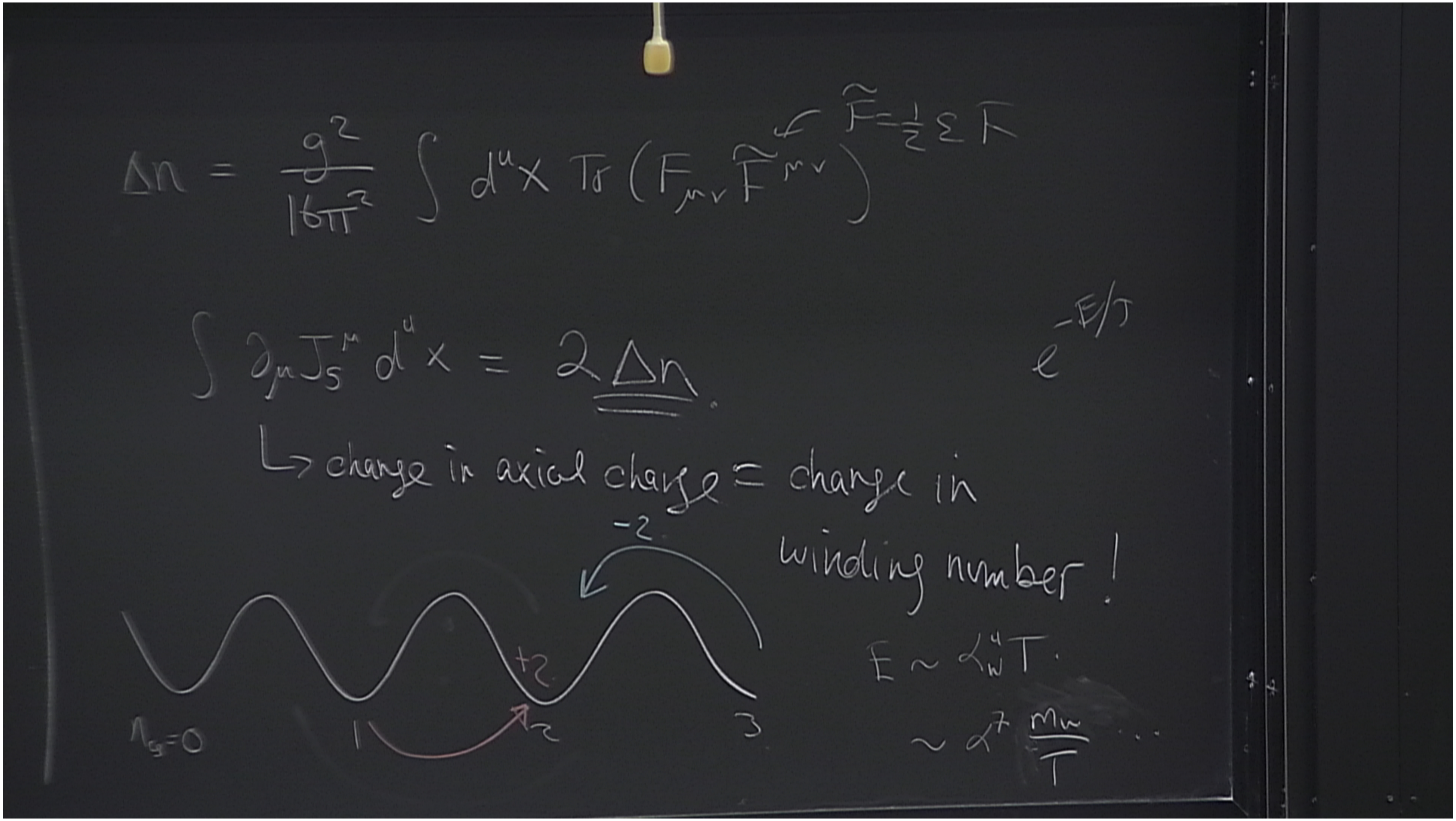
$\neq 0$.

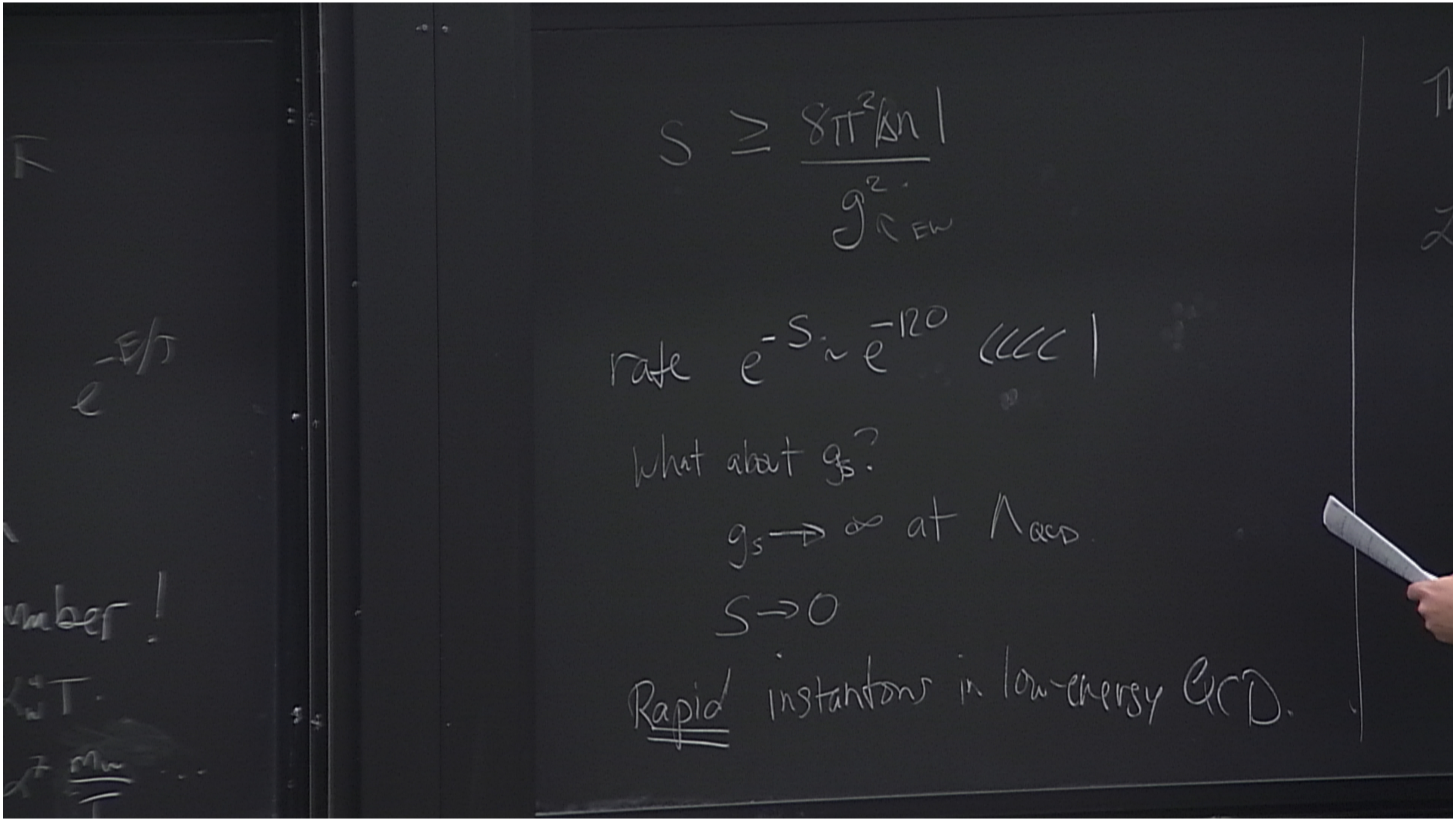
$$\Delta n = \frac{g^2}{16\pi^2} \int d^4x \, \mathrm{Tr}\left(F_{\mu\nu} \tilde{F}^{\mu\nu}\right) \qquad \tilde{F} = \frac{1}{2} \varepsilon F$$

$$\int \partial_\mu J_5^\mu \, d^4x = 2 \Delta n .$$

↳ change in axial charge = change in winding number!




$\Delta n = \frac{g^2}{16\pi^2} \int d^4x \, \mathrm{Tr}(F_{\mu\nu} \tilde{F}^{\mu\nu})$
$\tilde{F} = \frac{1}{2} \epsilon F$
$\int \partial_\mu J_5^\mu \, d^4x = 2 \Delta n$
$e^{-E/T}$
↳ change in axial charge = change in winding number!
-2
$+2$
$n_{cs} = 0$
1
2
3
$E \sim \alpha_W^4 T$
$\sim \alpha^7 \frac{m_W}{T} \ldots$


S ≥ 8π²|Δn| / g²_EW
rate e^-S ~ e^-120 <<<< 1
What about g_s?
g_s → ∞ at Λ_QCD
S → 0
Rapid instantons in low-energy QCD.

The Strong CP Problem

$$\mathcal{L} = q^\dagger \bar{\sigma}^\mu D_\mu q - M_{ij}\, q_{iL}\, q^c_{jR} + \text{h.c.}$$

$$- \frac{1}{2} \mathrm{Tr}\left(G^{\mu\nu} G_{\mu\nu}\right)$$

$$- \frac{g^2 \theta}{16\pi^2} \mathrm{Tr}\left(G_{\mu\nu} \tilde{G}^{\mu\nu}\right)$$

↑ θ-term.

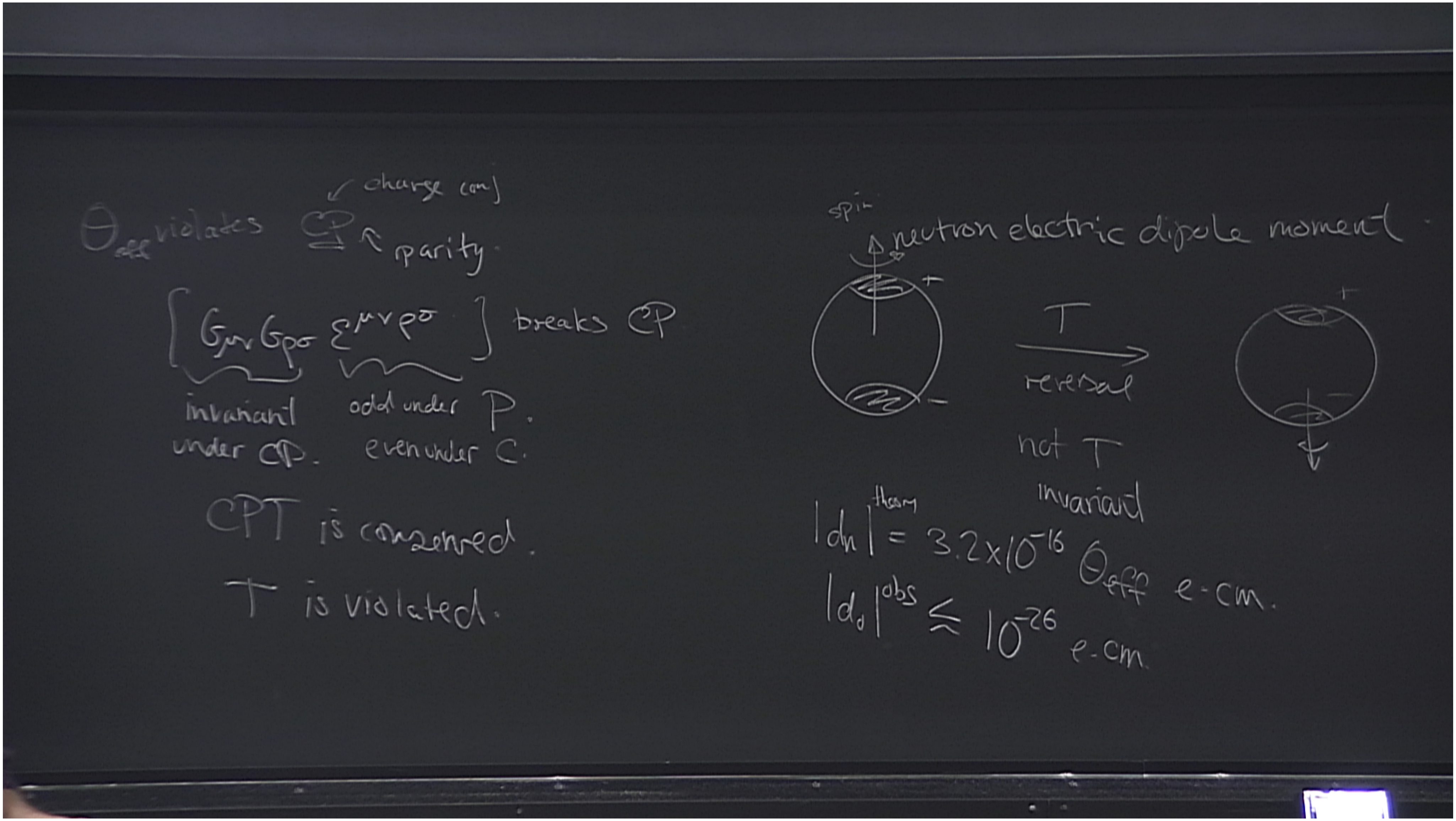
Generally, M is complex

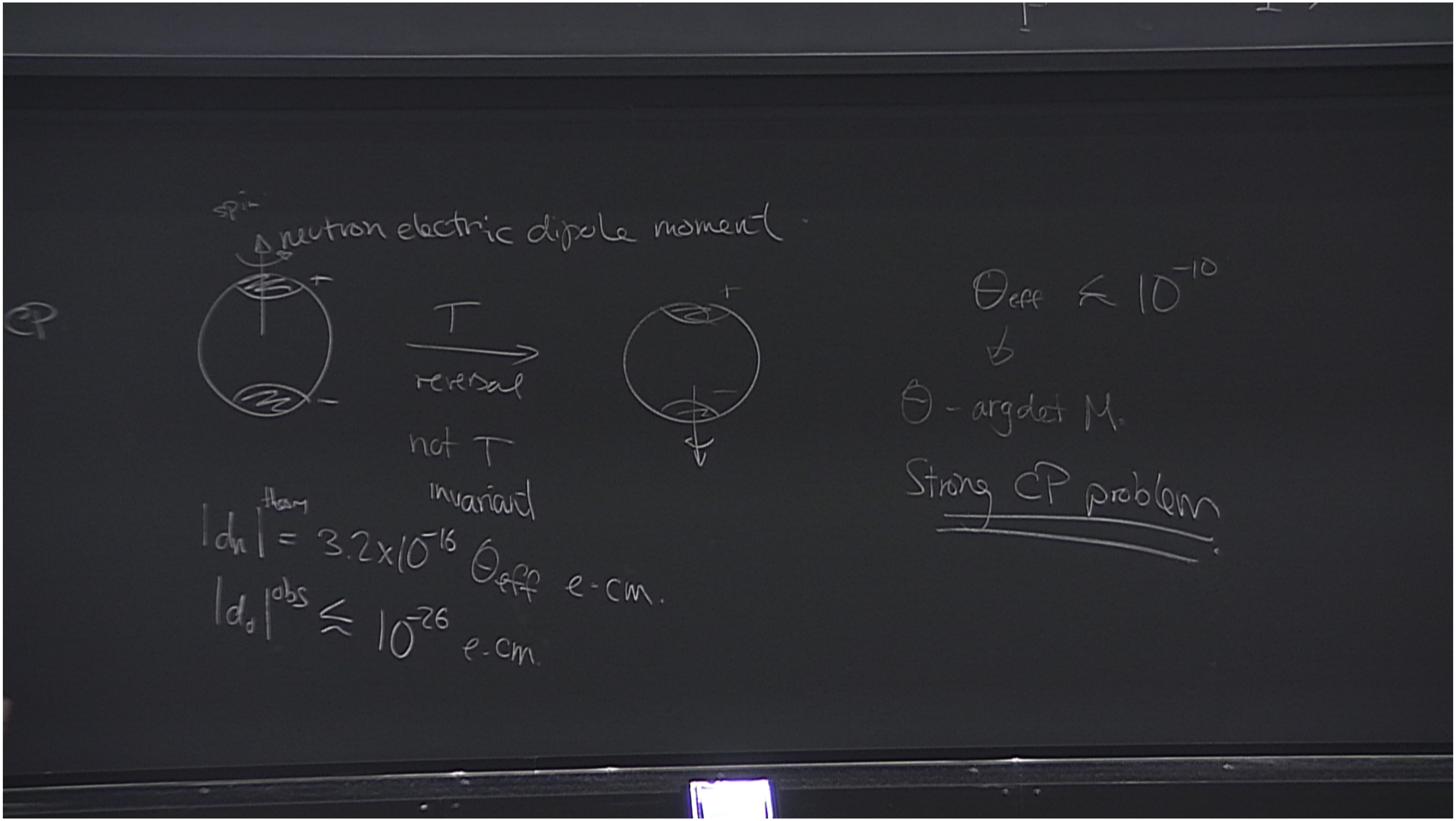
$$|M| e^{i\phi} q_L q_R^c$$

$$q_L \to e^{-i\phi} q_L$$

$$\theta \to \theta - \phi$$

$$\boxed{\theta_{\text{eff}} = \theta - \arg\det M}$$


charge conj
θeff violates CP
parity.
[Gμν Gρσ εμνρσ] breaks CP
invariant under CP.
odd under P.
even under C.
CPT is conserved.
T is violated.
spin
neutron electric dipole moment.
+
−
T
reversal
not T invariant
|dn|theory = 3.2×10−16 θeff e-cm.
|dn|obs ≲ 10−26 e-cm


spin
neutron electric dipole moment
CP
T
reversal
not T
invariant
$|d_n|^{theory} = 3.2 \times 10^{-16}\, \theta_{eff}$ e-cm.
$|d_n|^{obs} \lesssim 10^{-26}$ e-cm.
$\theta_{eff} \lesssim 10^{-10}$
θ - arg det M.
Strong CP problem