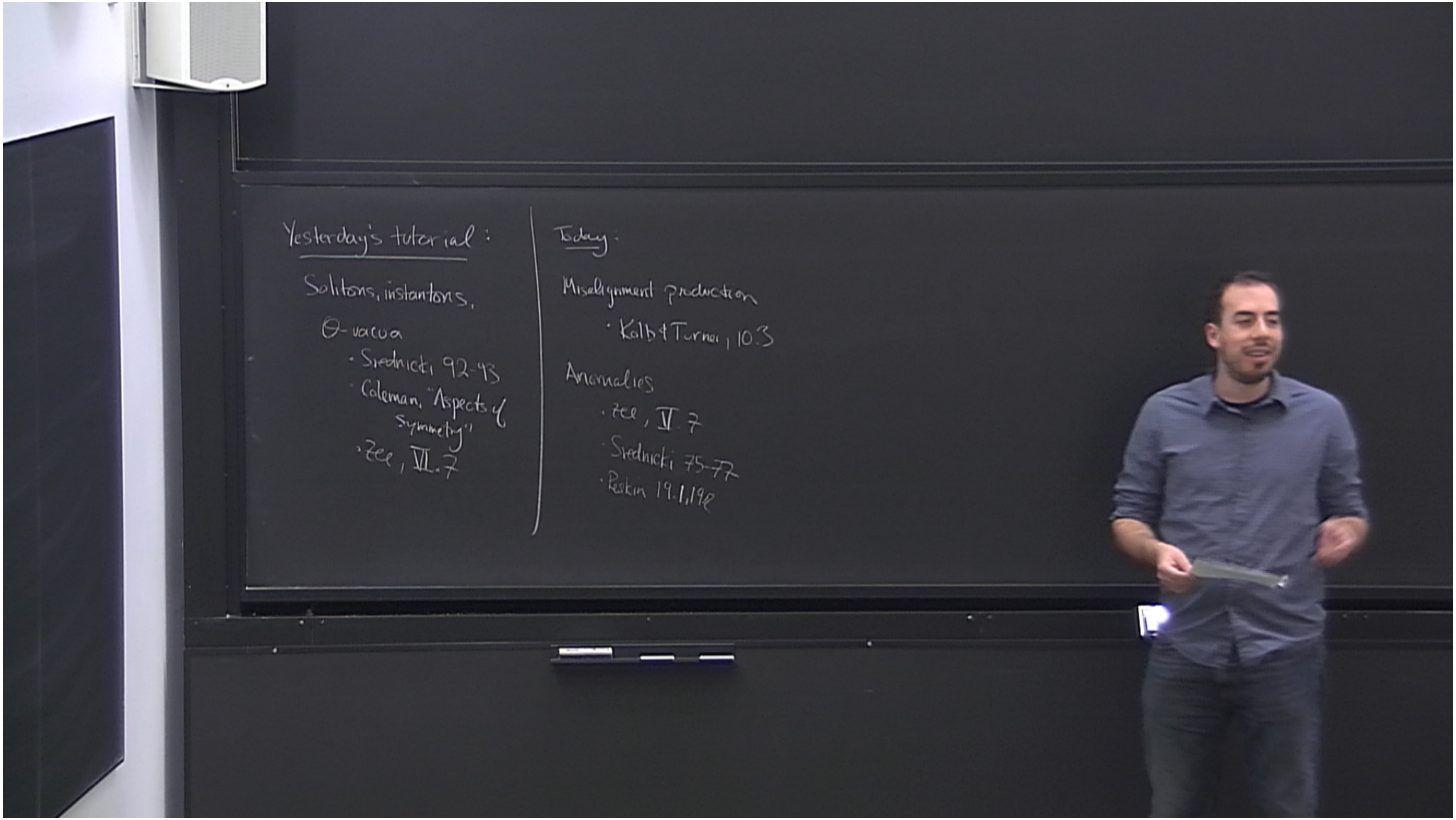
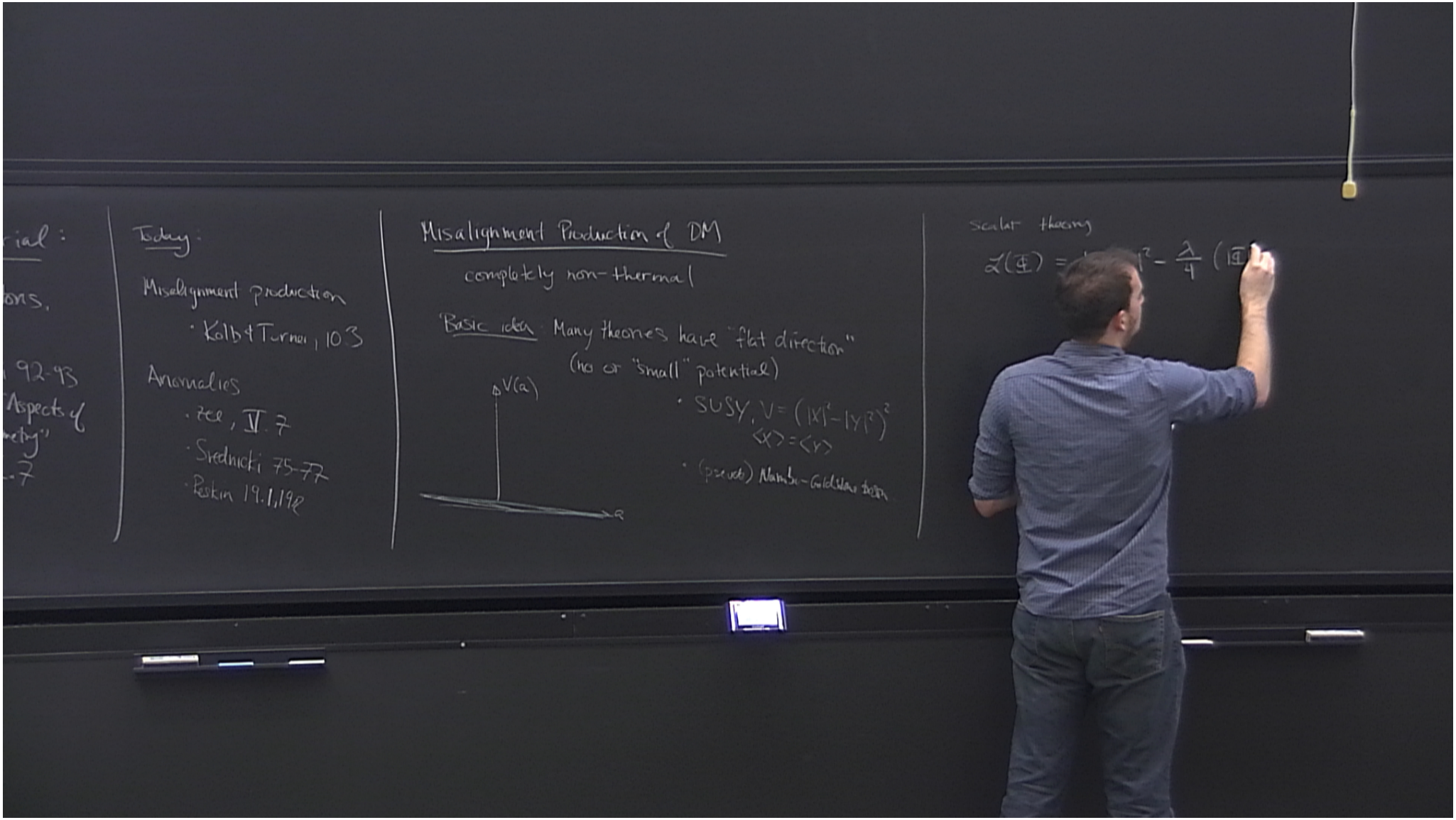
Title: Explorations in Particle Theory-12

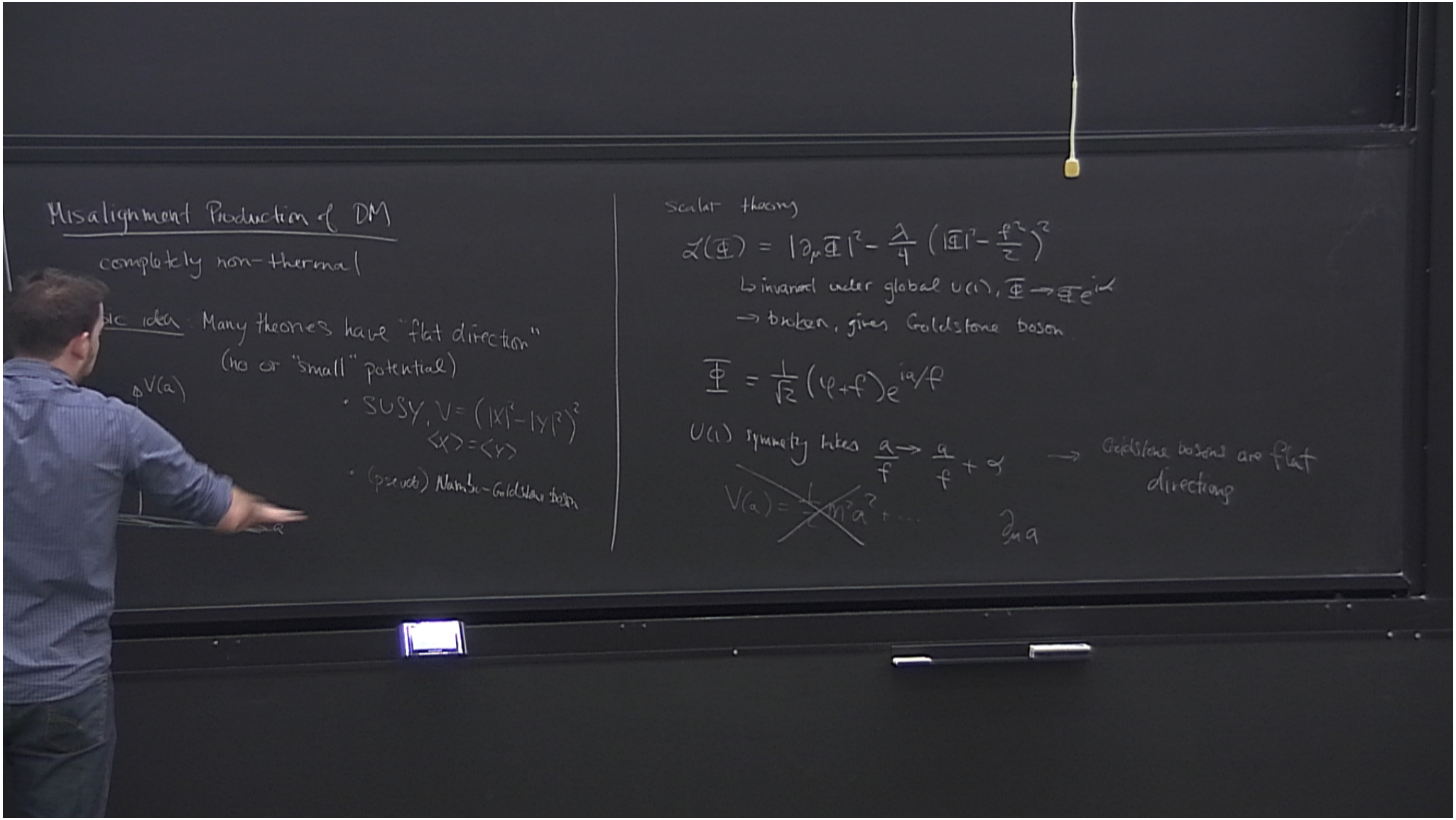
Date: Apr 21, 2015 09:00 AM

URL: http://pirsa.org/15040021

Abstract:


Yesterday's tutorial:
Solitons, instantons,
θ-vacua
· Srednicki 92-93
· Coleman, "Aspects of Symmetry"
· Zee, VI.7
Today:
Misalignment production
· Kolb & Turner, 10.3
Anomalies
· Zee, V.7
· Srednicki 75-77
· Peskin 19.1.1982

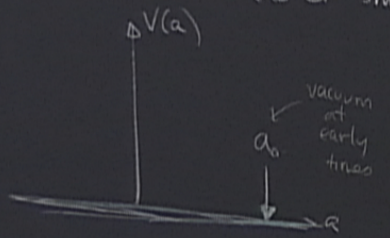

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92-93
Aspects of
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-7
Today:
Misalignment production
· Kolb & Turner, 103
Anomalies
· Zee, V.7
· Srednicki 75-77
· Peskin 19.1, 19.2
Misalignment Production of DM
completely non-thermal
Basic idea: Many theories have "flat direction"
(no or "small" potential)
V(a)
a
· SUSY, V = (|X|² - |Y|²)²
⟨X⟩ = ⟨Y⟩
· (pseudo) Nambu-Goldstone boson
Scalar theory
ℒ(Φ) =
λ/4 (|Φ|


Misalignment Production of DM
completely non-thermal
basic idea: Many theories have "flat direction"
(no or "small" potential)
V(a)
• SUSY, $V = (|X|^2 - |Y|^2)^2$
$\langle X \rangle = \langle Y \rangle$
• (pseudo) Nambu-Goldstone boson
scalar theory
$\mathcal{L}(\Phi) = |\partial_\mu \Phi|^2 - \frac{\lambda}{4}\left(|\Phi|^2 - \frac{f^2}{2}\right)^2$
↳ invariant under global U(1), $\Phi \to \Phi e^{i\alpha}$
→ broken, gives Goldstone boson
$\Phi = \frac{1}{\sqrt{2}}(\rho + f)e^{ia/f}$
U(1) symmetry takes $\frac{a}{f} \to \frac{a}{f} + \alpha$ → Goldstone bosons are flat directions
$V(a) = \frac{1}{2}m^2 a^2 + \ldots$
$\partial_\mu a$

Misalignment Production of DM

completely non-thermal!

Basic idea: Many theories have "flat direction" (no or "small" potential)



- SUSY, $V = (|X|^2 - |Y|^2)^2$, $\langle X \rangle = \langle Y \rangle$
- (pseudo) Nambu-Goldstone boson

scalar theory

$$\mathcal{L}(\Phi) = |\partial_\mu \Phi|^2 - \frac{\lambda}{4}\left(|\Phi|^2 - \frac{f^2}{2}\right)^2$$

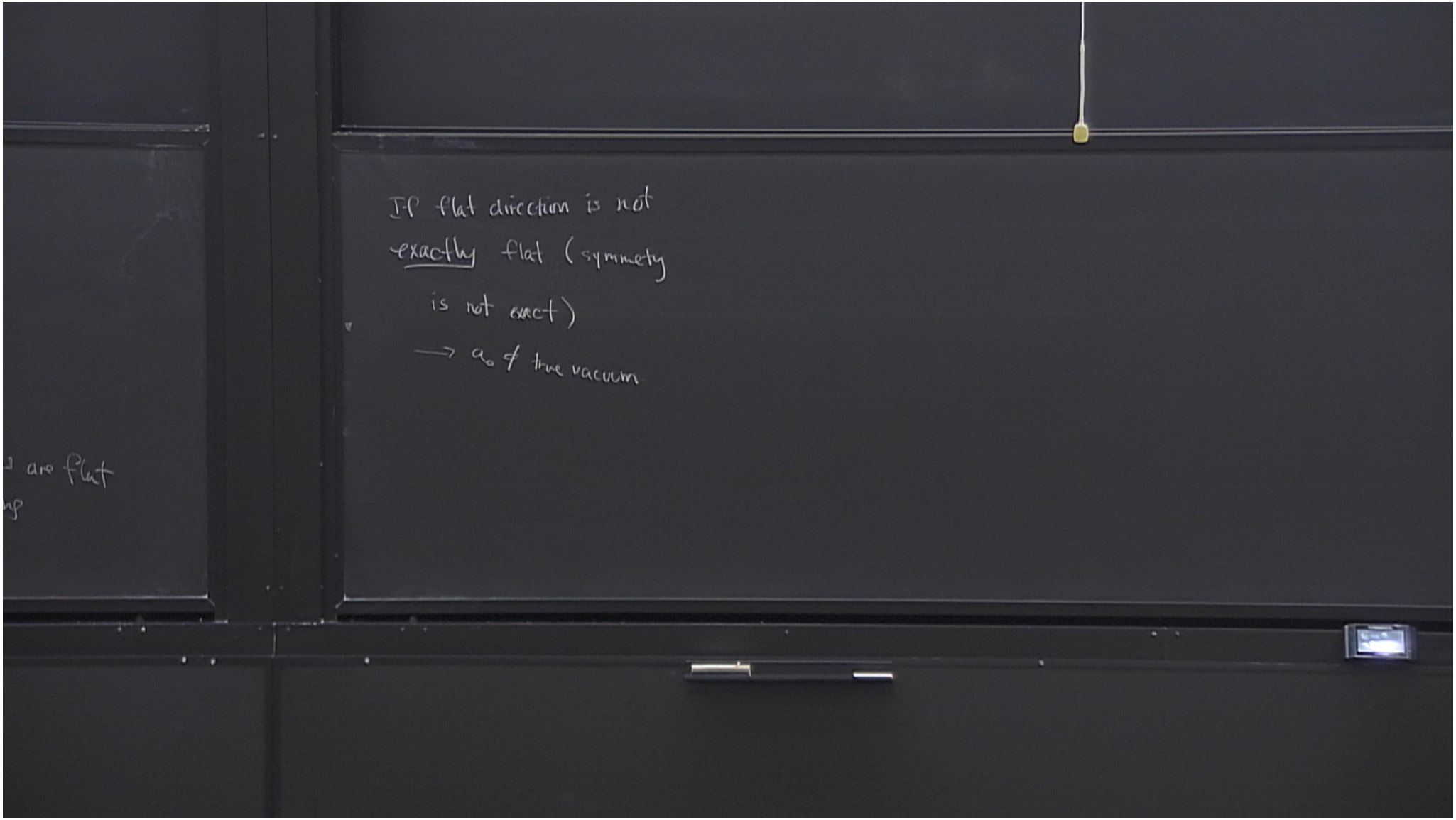
↳ invariant under global U(1), $\Phi \to \Phi e^{i\alpha}$

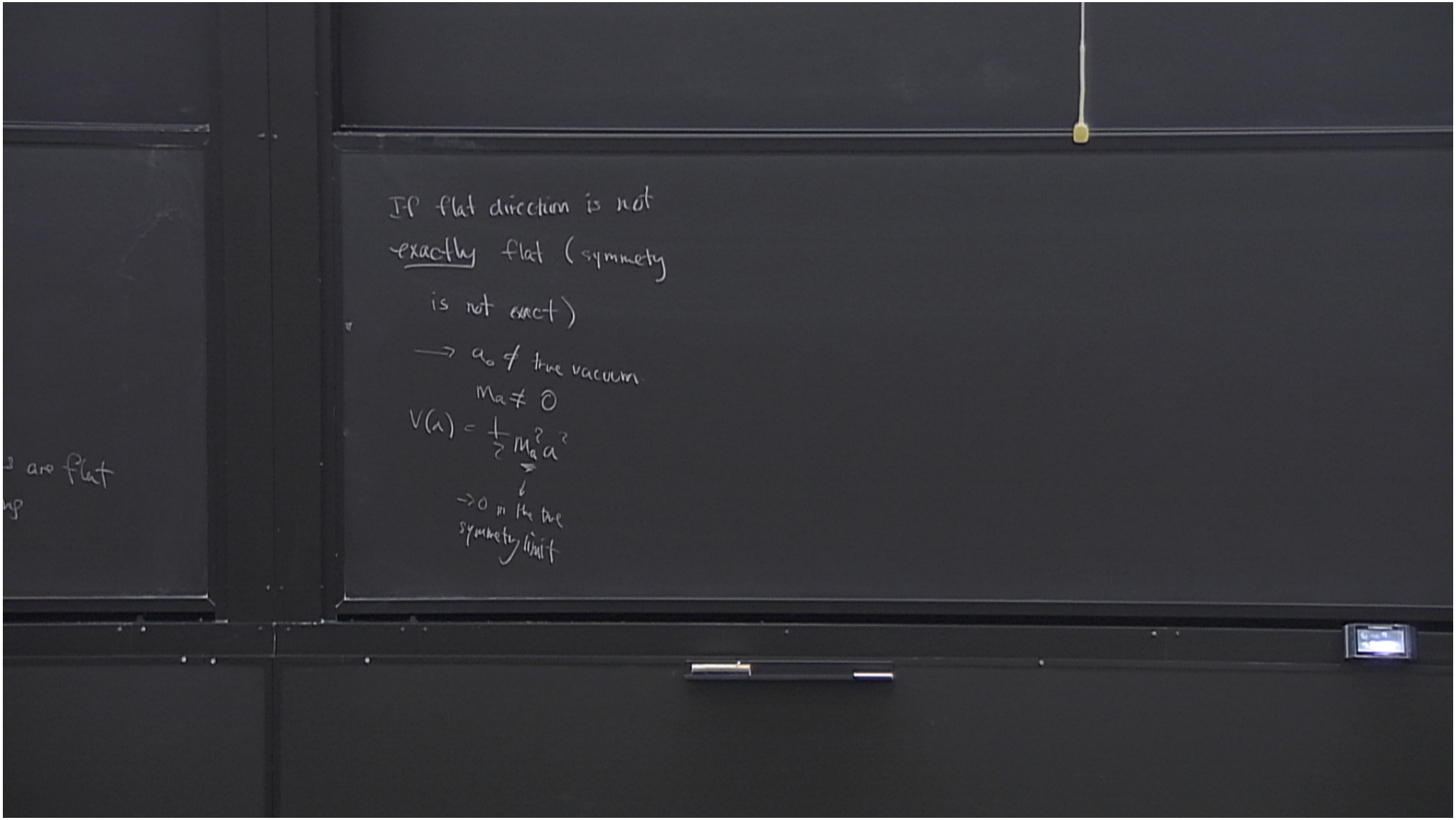
→ broken, gives Goldstone boson

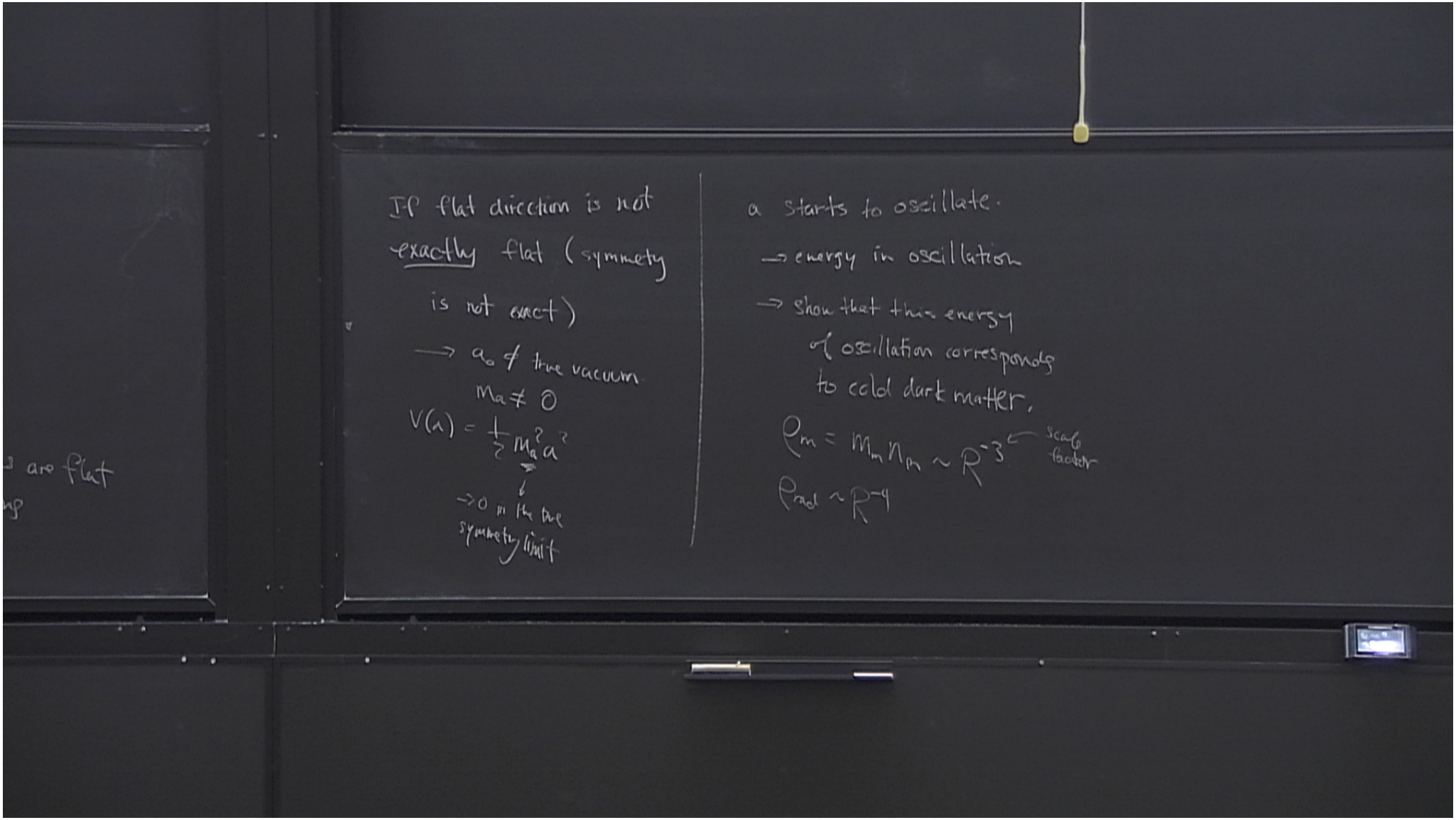
$$\Phi = \frac{1}{\sqrt{2}}(\rho + f)e^{ia/f}$$

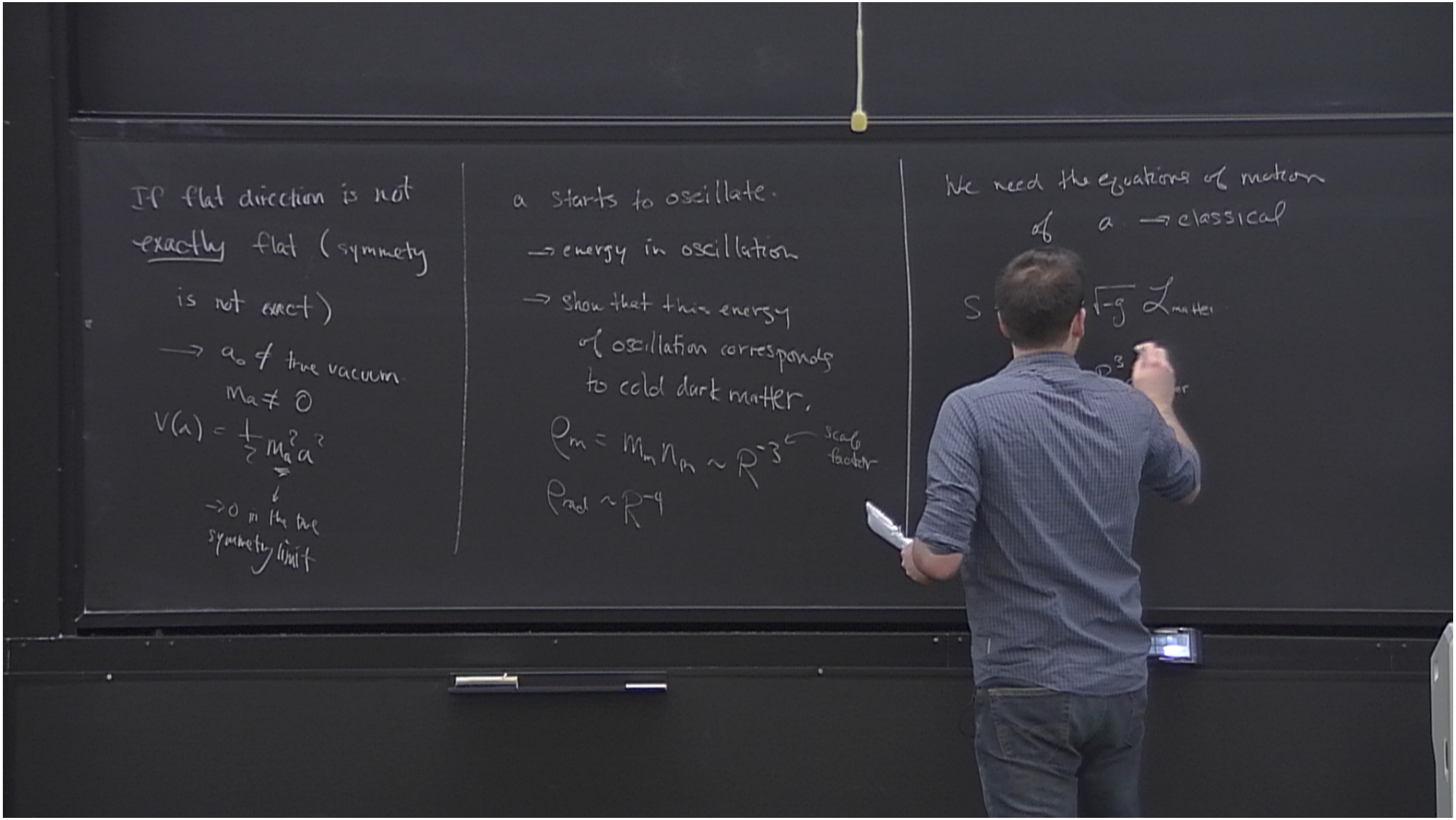
U(1) symmetry takes $\frac{a}{f} \to \frac{a}{f} + \alpha$ → Goldstone bosons are flat directions

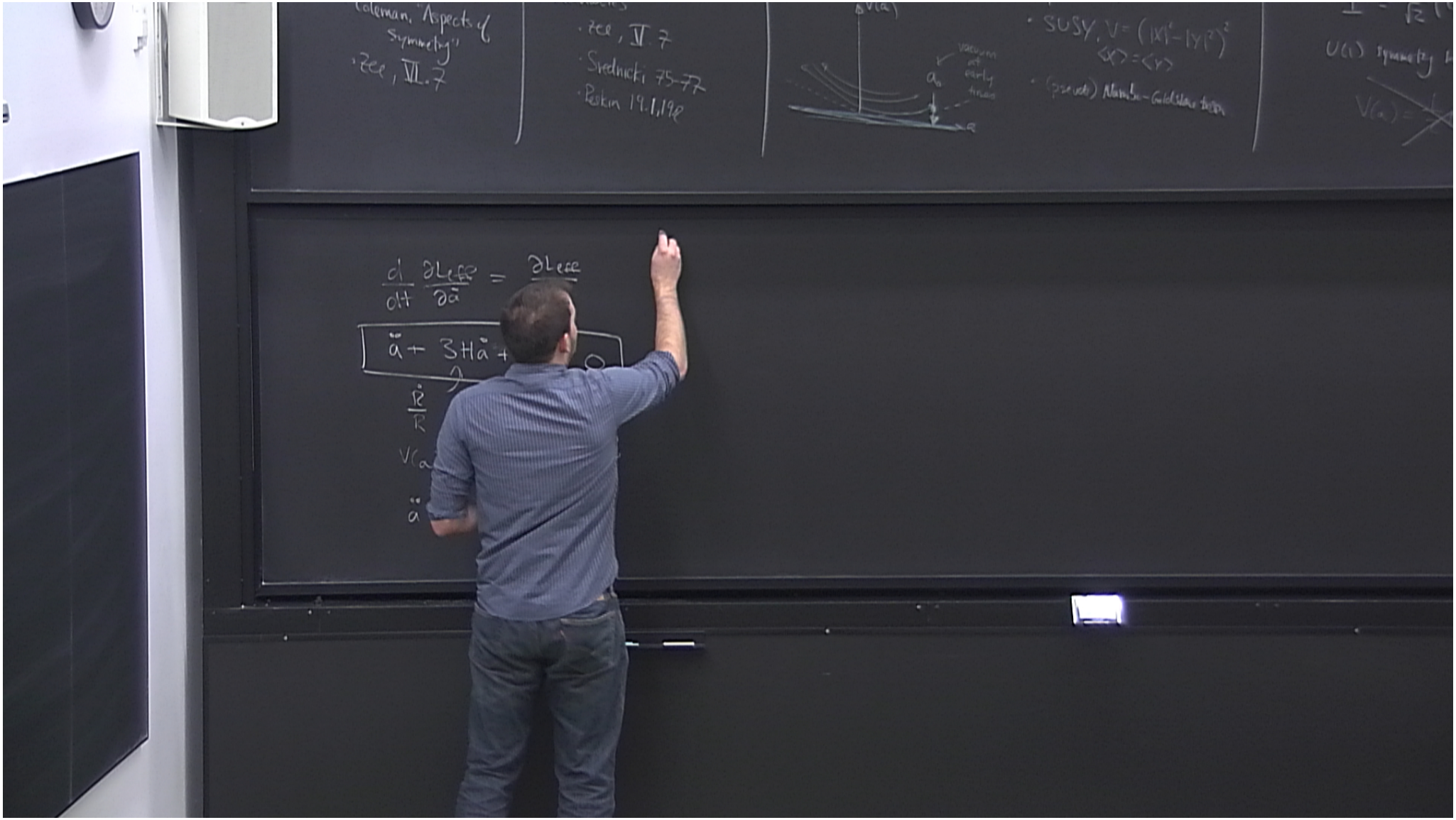
$V(a) = \frac{1}{2}m^2 a^2 + \dots$ ~~$\partial_\mu a$~~

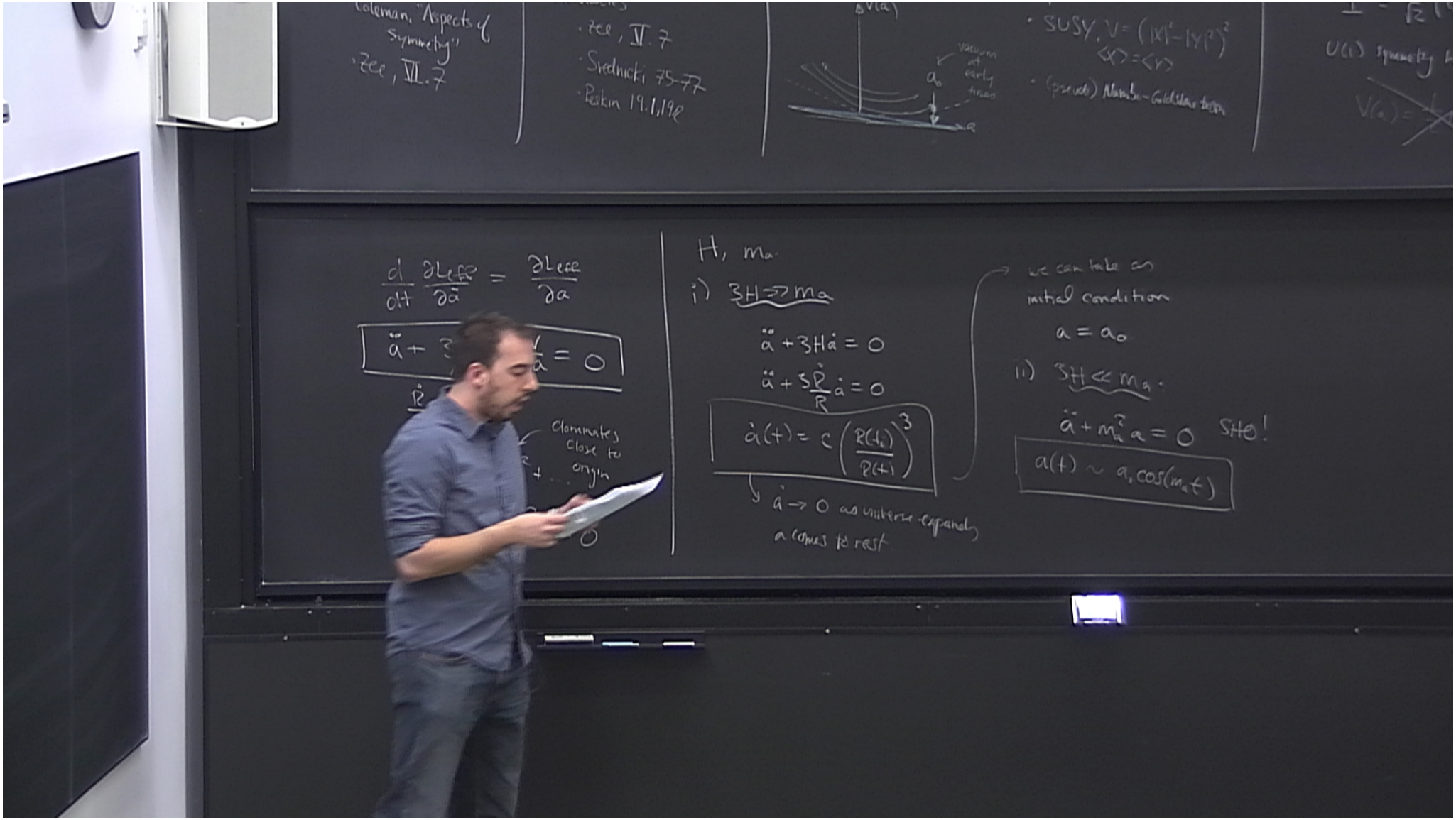

If flat direction is not
exactly flat (symmetry
is not exact)
⟶ a_0 of true vacuum
are flat

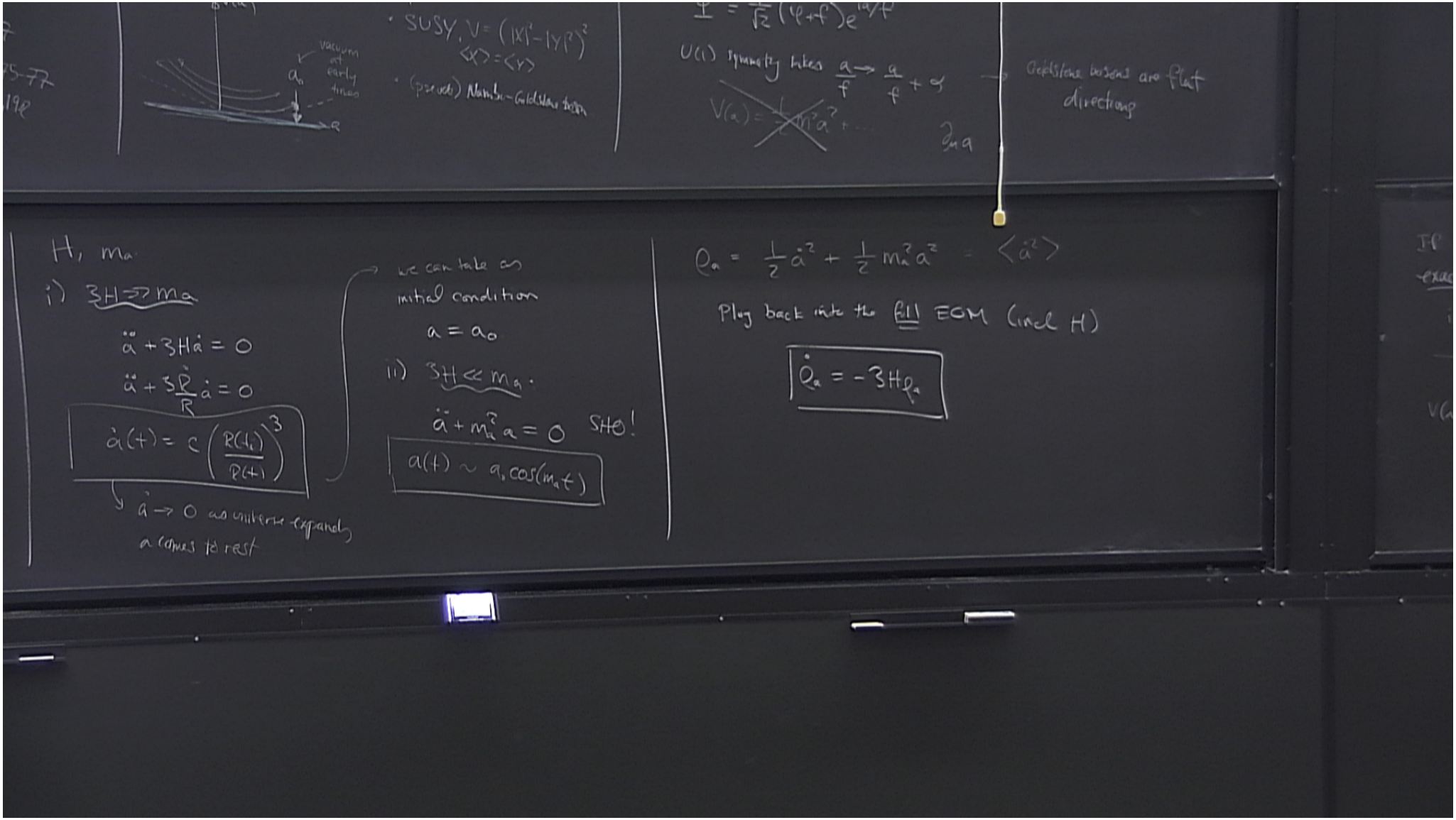

are flat
If flat direction is not
exactly flat (symmetry
is not exact)
$\longrightarrow$ a_0 of true vacuum
$m_a \neq 0$
$V(a) = \frac{1}{2} m_a^2 a^2$
$\to 0$ in the true
symmetry limit

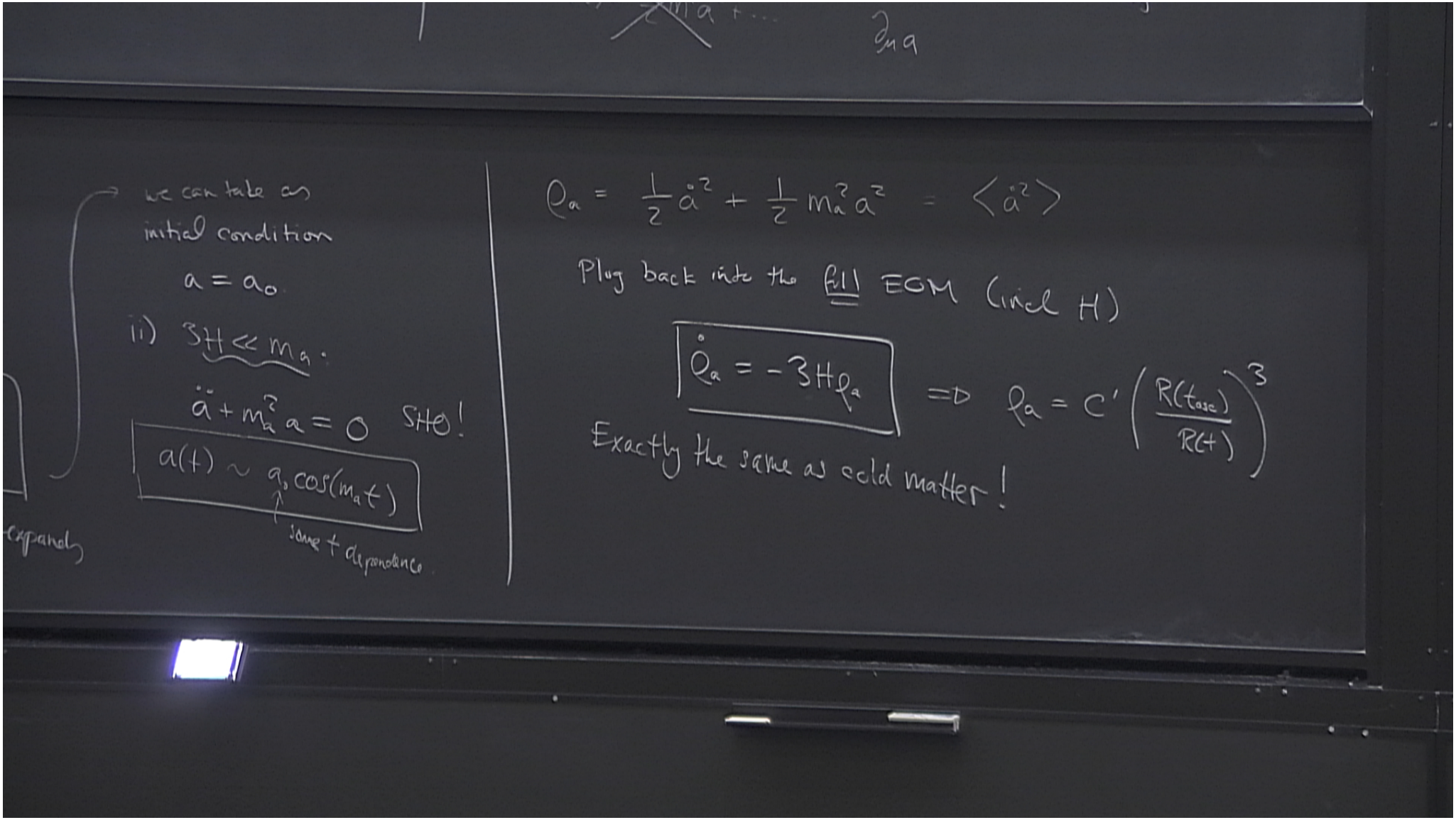

are flat
If flat direction is not
exactly flat (symmetry
is not exact)
⟶ a_0 ≠ true vacuum
$m_a \neq 0$
$V(a) = \frac{1}{2} m_a^2 a^2$
⇒
↓
→0 in the true
symmetry limit
a starts to oscillate.
→ energy in oscillation
→ show that this energy
of oscillation corresponds
to cold dark matter.
$\rho_m = m_m n_m \sim R^{-3}$ ← scale factor
$\rho_{rad} \sim R^{-4}$

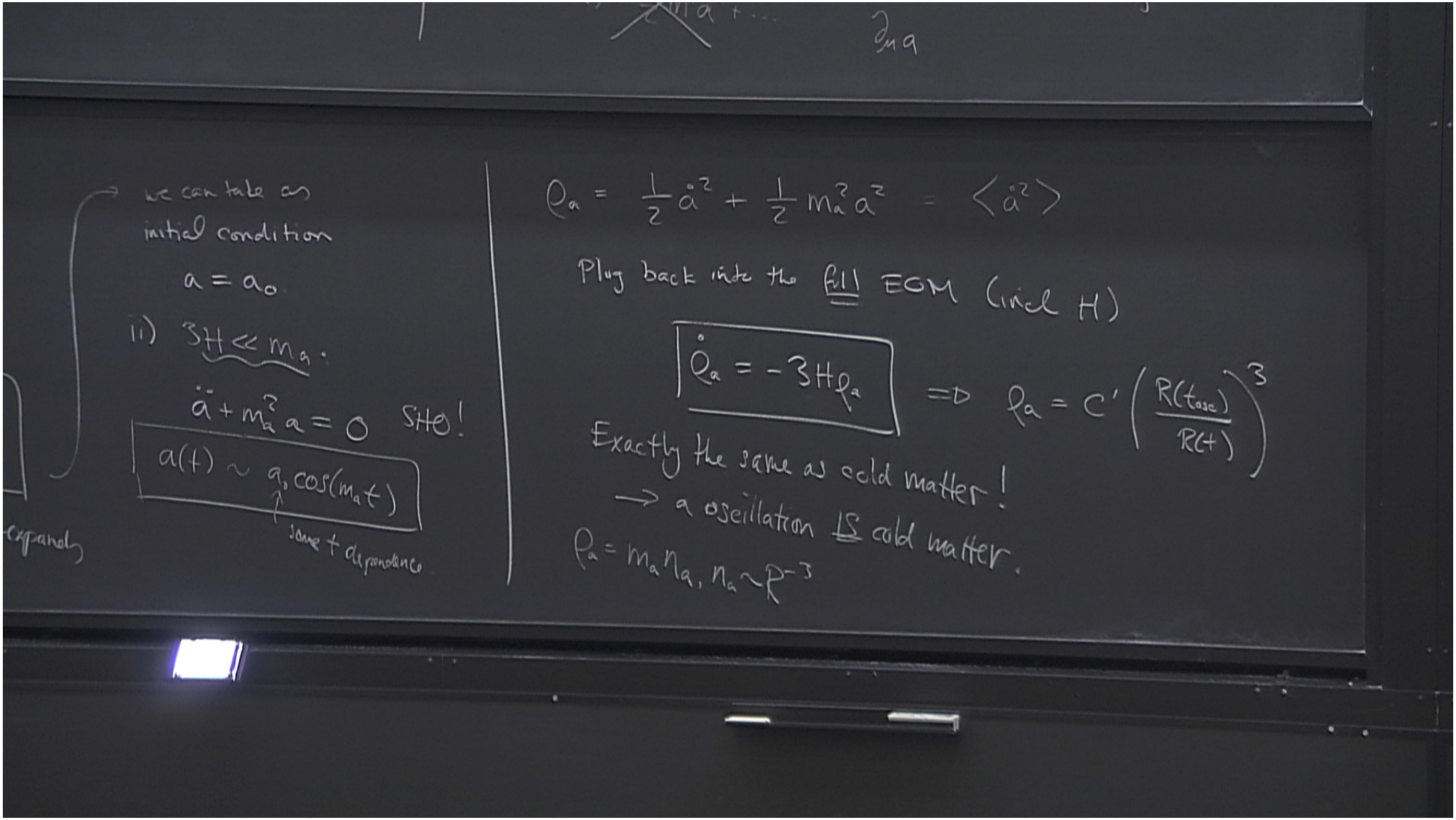

If flat direction is not exactly flat (symmetry is not exact)
→ a_0 of true vacuum
$m_a \neq 0$
$V(a) = \frac{1}{2} m_a^2 a^2$
→ 0 in the true symmetry limit
a starts to oscillate.
→ energy in oscillation
→ Show that this energy of oscillation corresponds to cold dark matter.
$\rho_m = m_m n_m \sim R^{-3}$ ← scale factor
$\rho_{rad} \sim R^{-4}$
We need the equations of motion of a → classical
$S \sim \int \sqrt{-g}\, \mathcal{L}_{matter}$

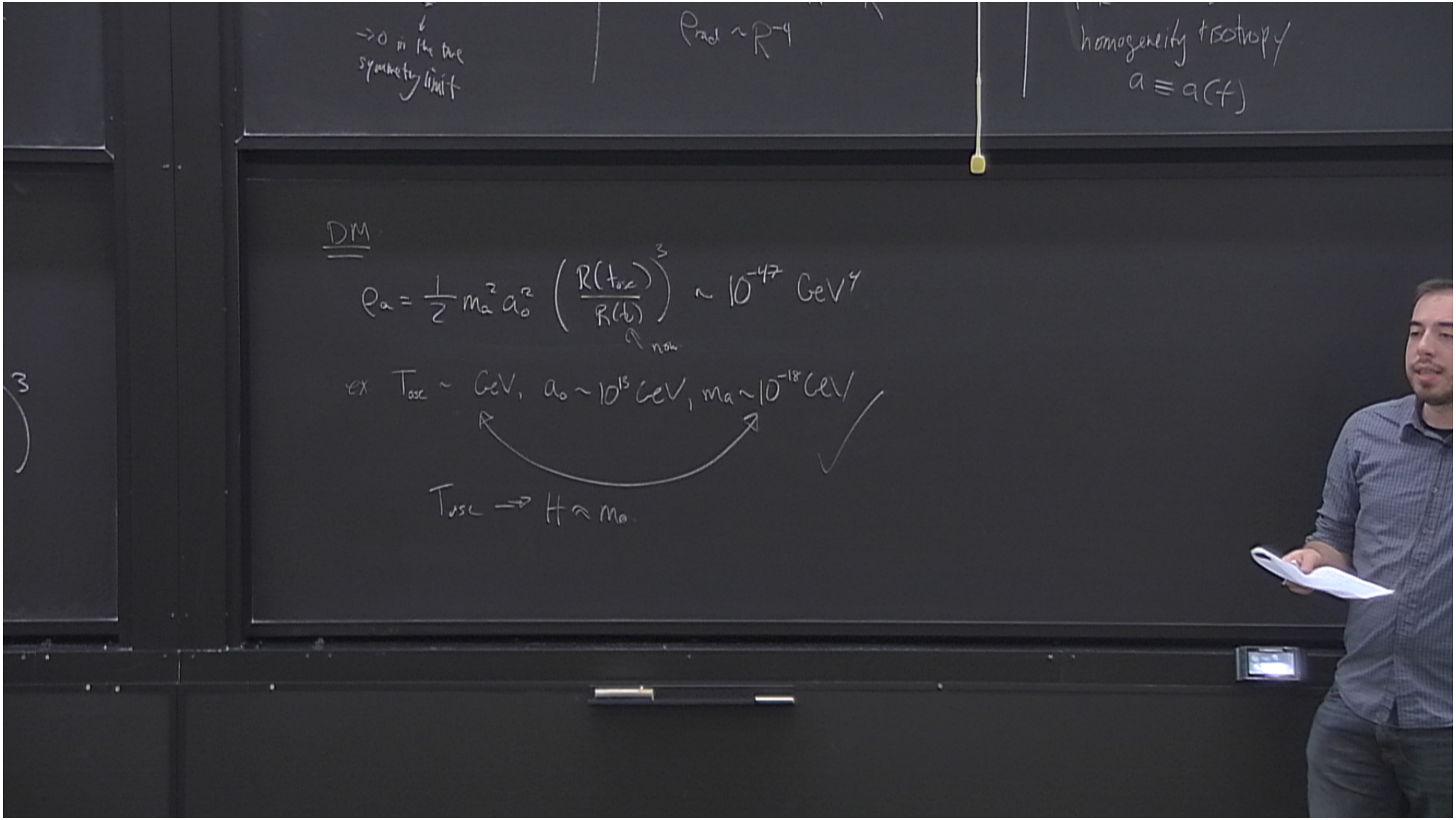

Coleman, "Aspects of Symmetry"
· Zee, VI. 7
· Zee, V. 7
· Srednicki 75-77
· Peskin 19.1, 19.2
· SUSY, V = (|X|² − |Y|²)²
⟨X⟩ = ⟨Y⟩
· (pseudo) Nambu–Goldstone
U(1) symmetry
d/dt ∂L_eff/∂ȧ = ∂L_eff/∂a
ä + 3Hȧ +
Ṙ/R
V(a
ä


Coleman, "Aspects of Symmetry"
Zee, VI.7
Zee, V.7
Srednicki 75-77
Peskin 19.1, 19.2
SUSY
vacuum at early times
$\frac{d}{dt}\frac{\partial L_{eff}}{\partial \dot{a}} = \frac{\partial L_{eff}}{\partial a}$
$\ddot{a} + \frac{\dot{R}}{R}\dot{a} = 0$
dominates close to origin
H, m_a
i) $3H \Rightarrow m_a$
$\ddot{a} + 3H\dot{a} = 0$
$\ddot{a} + 3\frac{\dot{R}}{R}\dot{a} = 0$
$\dot{a}(t) = c\left(\frac{R(t_0)}{R(t)}\right)^3$
$\dot{a} \to 0$ as universe expands
a comes to rest
we can take as initial condition
$a = a_0$
ii) $3H \ll m_a$
$\ddot{a} + m_a^2 a = 0$ SHO!
$a(t) \sim a_0 \cos(m_a t)$


vacuum at early times
a_0
a
SUSY, $V = (|X|^2 - |Y|^2)^2$
$\langle X \rangle = \langle Y \rangle$
(proved) Nambu–Goldstone boson
$U(1)$ symmetry takes $a \to a + \alpha f$
$V(a) = \frac{1}{2} m^2 a^2 + \dots$
$\partial_\mu a$
Goldstone bosons are flat directions
H, m_a
i) $3H \gg m_a$
$\ddot{a} + 3H\dot{a} = 0$
$\ddot{a} + 3\frac{\dot{R}}{R}\dot{a} = 0$
$\dot{a}(t) = c\left(\frac{R(t_i)}{R(t)}\right)^3$
$\dot{a} \to 0$ as universe expands, a comes to rest
we can take as initial condition
$a = a_0$
ii) $3H \ll m_a$
$\ddot{a} + m_a^2 a = 0$ SHO!
$a(t) \sim a_0 \cos(m_a t)$
$\rho_a = \frac{1}{2}\dot{a}^2 + \frac{1}{2} m_a^2 a^2 = \langle \dot{a}^2 \rangle$
Plug back into the full EOM (incl H)
$\dot{\rho}_a = -3H\rho_a$


we can take as
initial condition
$a = a_0$
ii) $3H \ll m_a$.
$\ddot{a} + m_a^2 a = 0$ SHO!
$a(t) \sim a_0 \cos(m_a t)$
same t dependence
$\rho_a = \frac{1}{2}\dot{a}^2 + \frac{1}{2} m_a^2 a^2 = \langle \dot{a}^2 \rangle$
Plug back into the full EOM (incl H)
$\dot{\rho}_a = -3H\rho_a$
$\Rightarrow \rho_a = C' \left(\frac{R(t_{osc})}{R(t)} \right)^3$
Exactly the same as cold matter!


we can take as
initial condition
$a = a_0$
ii) $3H \ll m_a$.
$\ddot{a} + m_a^2 a = 0$ SHO!
$a(t) \sim a_0 \cos(m_a t)$
same t dependence
$\rho_a = \frac{1}{2}\dot{a}^2 + \frac{1}{2} m_a^2 a^2 = \langle \dot{a}^2 \rangle$
Plug back into the full EOM (incl H)
$\dot{\rho}_a = -3H\rho_a \Rightarrow \rho_a = C' \left(\frac{R(t_{osc})}{R(t)} \right)^3$
Exactly the same as cold matter!
→ a oscillation IS cold matter.
$\rho_a = m_a n_a$, $n_a \sim R^{-3}$


→0 in the true symmetry limit
$\rho_{rad} \sim R^{-4}$
homogeneity + isotropy
$a \equiv a(t)$
DM
$\rho_a = \frac{1}{2} m_a^2 a_0^2 \left(\frac{R(t_{osc})}{R(t_0)}\right)^3 \sim 10^{-47}\ GeV^4$
now
ex $T_{osc} \sim$ GeV, $a_0 \sim 10^{15}$ GeV, $m_a \sim 10^{-18}$ GeV
$T_{osc} \Rightarrow H \approx m_a$

→0 in the the symmetry limit

$\rho_{rad} \sim R^{-4}$

homogeneity + isotropy

$a \equiv a(t)$

DM

$$\rho_a = \frac{1}{2} m_a^2 a_0^2 \left(\frac{R(t_{osc})}{R(t_0)}\right)^3 \sim 10^{-47}\ \text{GeV}^4$$

now ~ $f \sim 10^{15}$ GeV

ex. $T_{osc} \sim$ GeV, $a_0 \sim 10^{15}$ GeV, $m_a \sim 10^{-18}$ GeV ✓

$T_{osc} \rightarrow H \approx m_a$

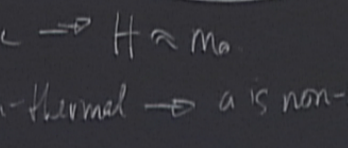
Non-thermal → a is non-rel even though $m_a \sim 10^{-18}$ GeV

There are thermal interactions of a

If there are other fields charged under U(1) global symmetry..

$i\psi^\dagger \bar{\sigma}^m \partial_m \psi$ $\quad \psi \rightarrow \psi e^{ia/f}$

$\frac{1}{f} \partial_m a \psi^\dagger \bar{\sigma}^m \psi$ f very large



$\sim \frac{1}{f^2}$ = rates very suppressed at low energies

Questions

1) Why is the ma

2) Are there mode

Peskin 19.1, 19.2

Origin of mass

m comes from explicit breaking of U(1)

i) explicit breaking in $\mathcal{L}$

ii) anomaly

i) Explicit breaking

$$\mathcal{L} = |\partial_\mu \Phi|^2 - \frac{\lambda}{4}\left(|\Phi|^2 - f^2/2\right)^2 + \mu^2 \Phi^2 + h.c.$$

$\hookrightarrow \sim \mu$

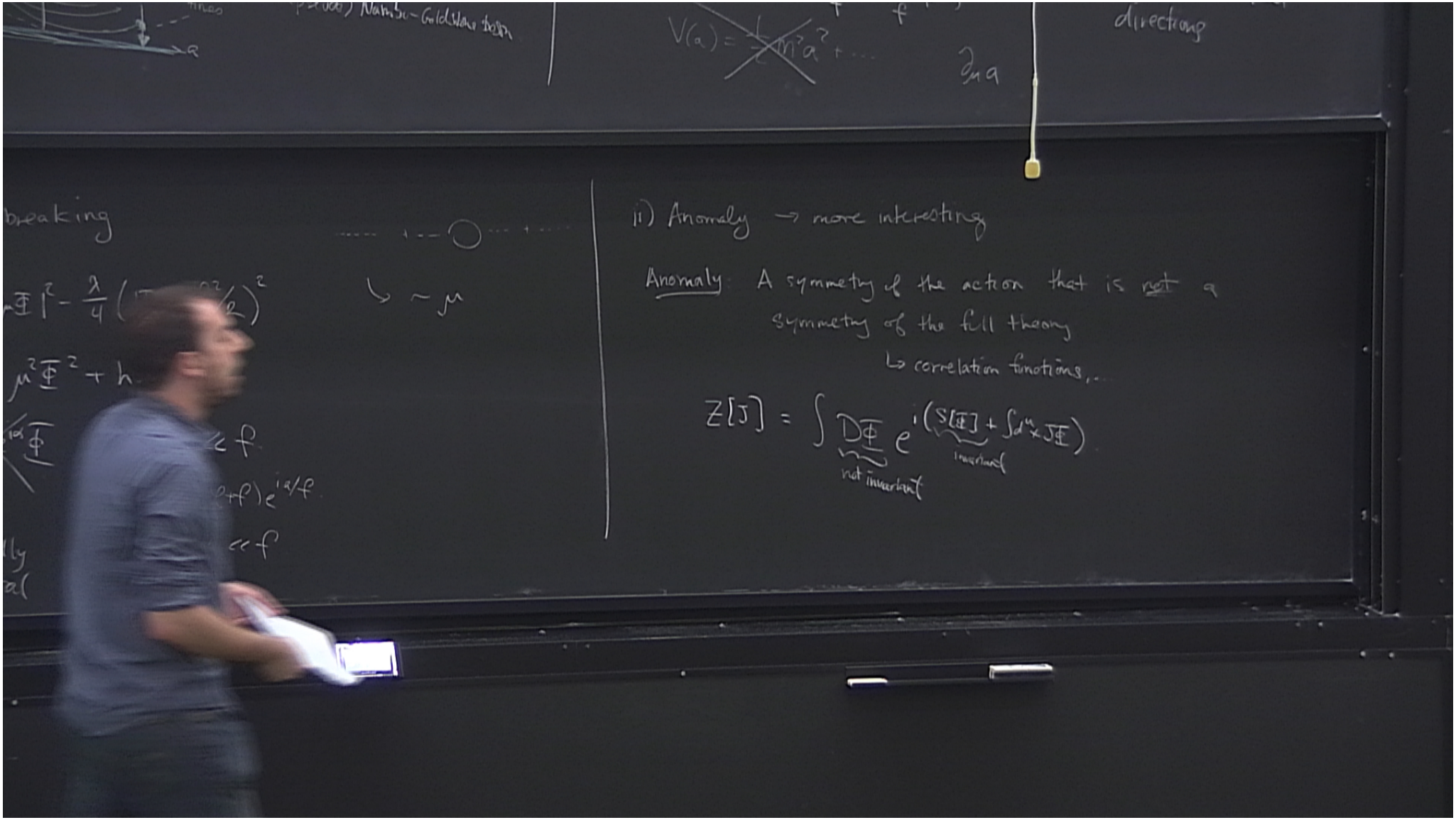
$\Phi \to e^{i\alpha}\Phi$

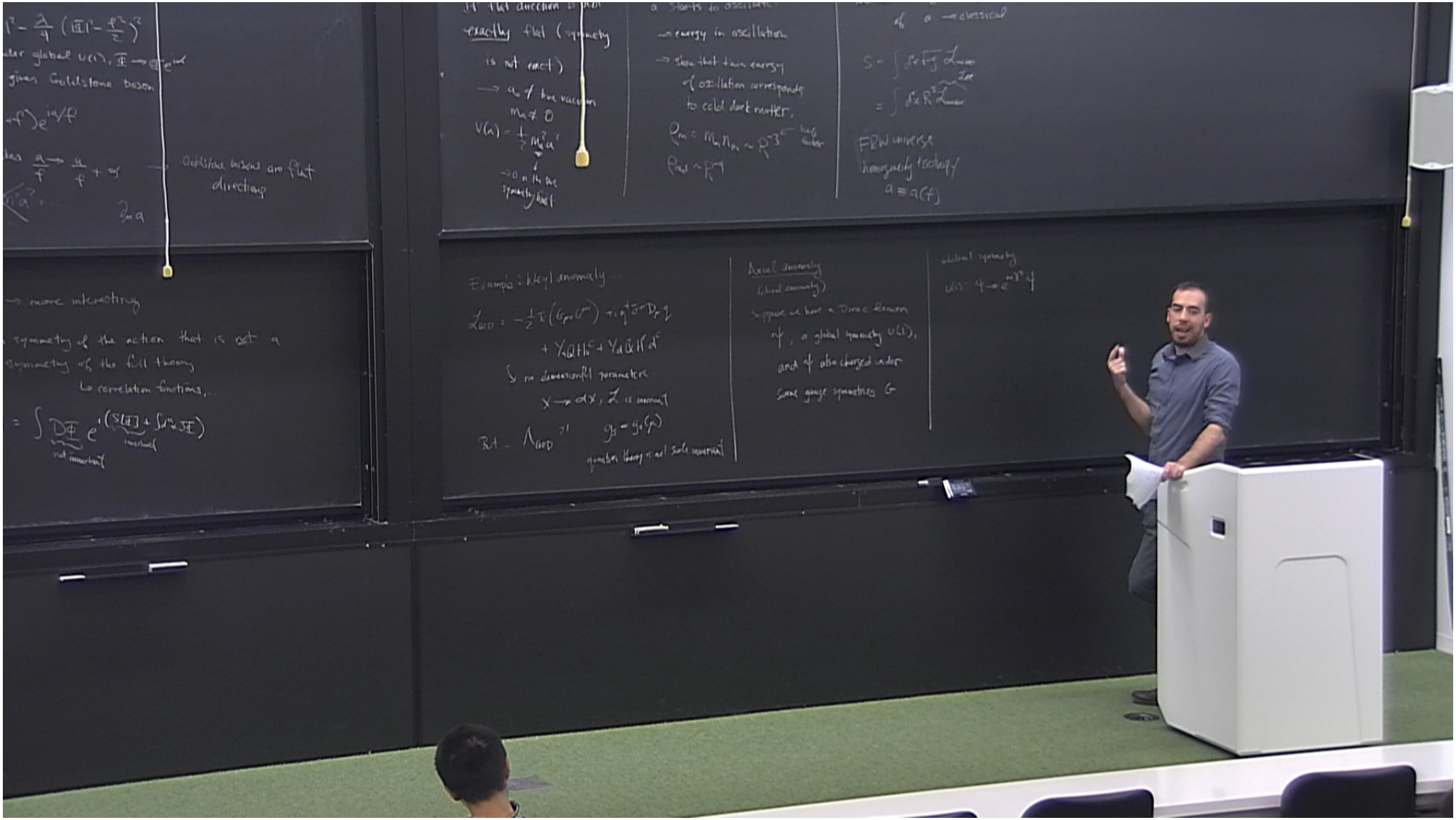
But $\mu \ll f$

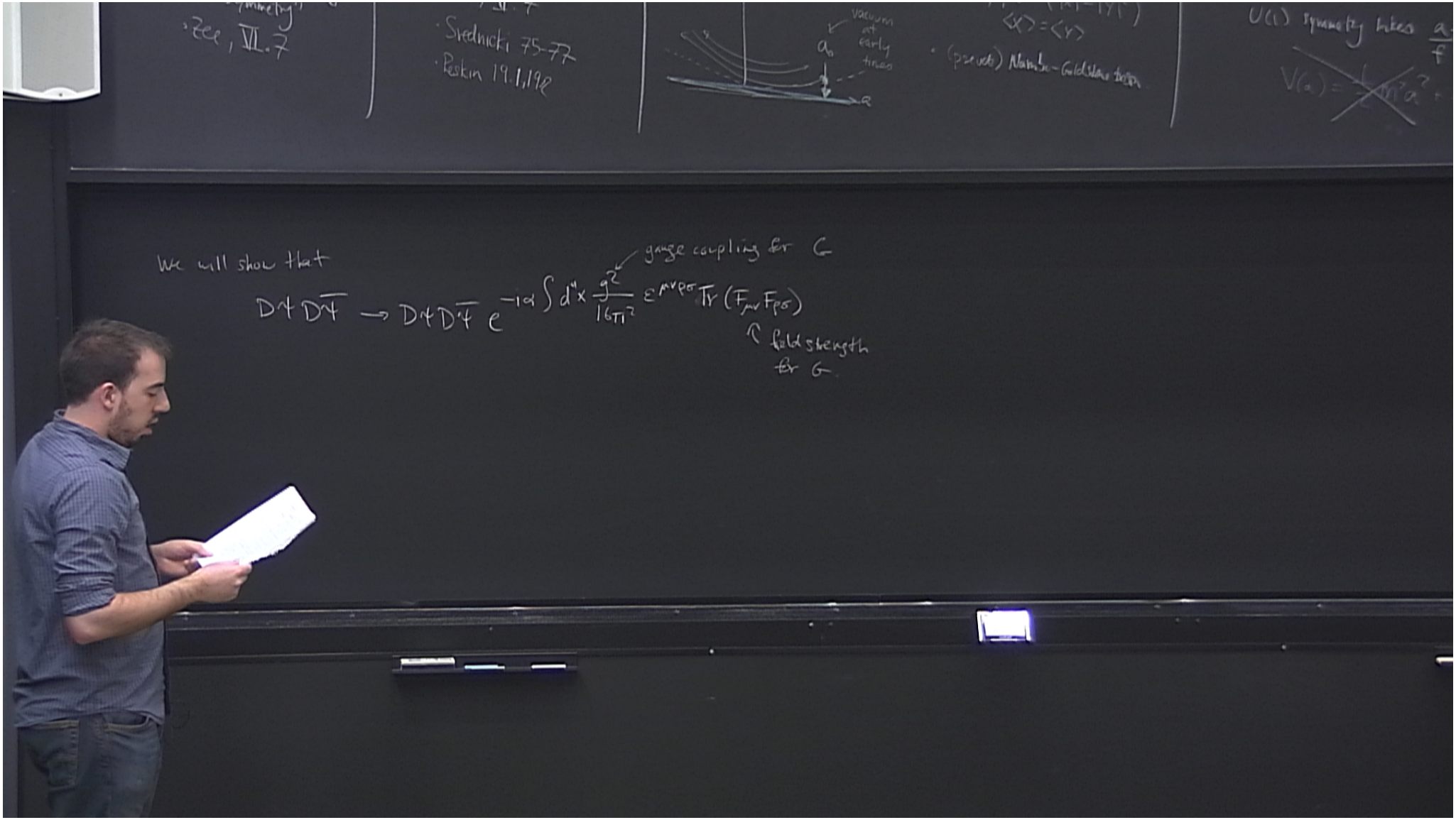
$\Phi = \frac{1}{\sqrt{2}}(\varrho + f)e^{i\pi/f}$

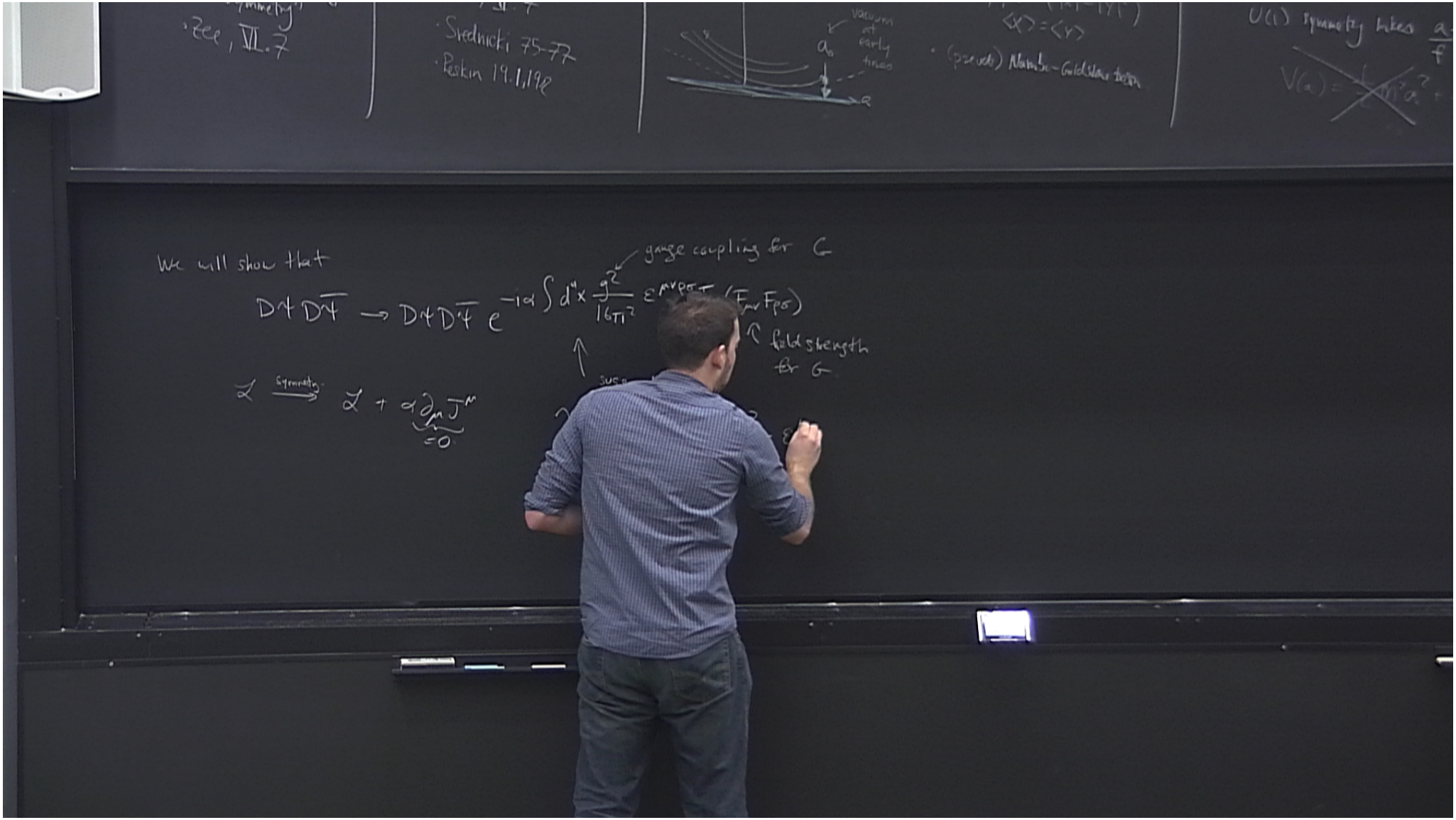
technically natural $\rightarrow$ $m_\pi \sim \mu \ll f$

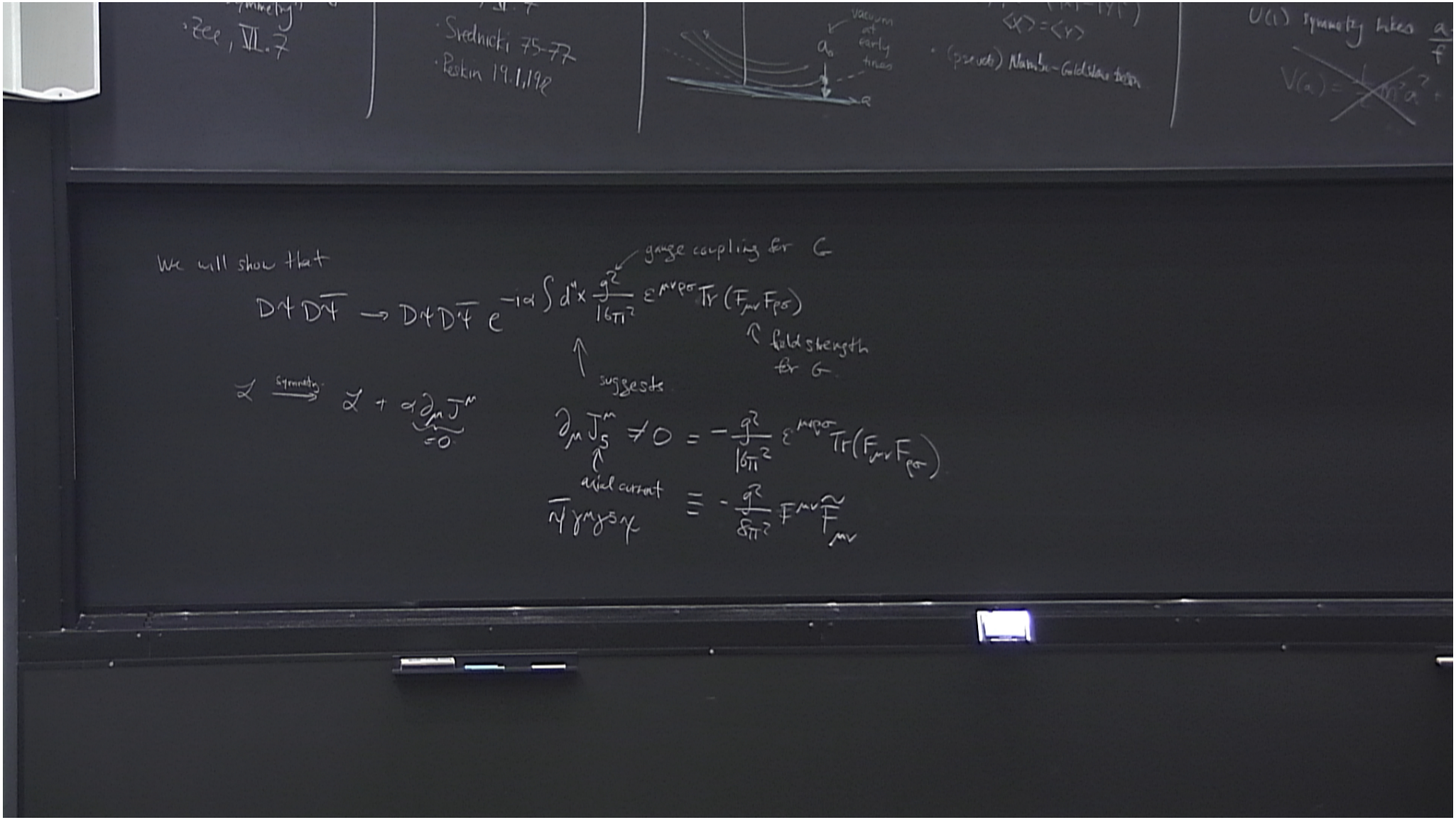
ii) Anomaly

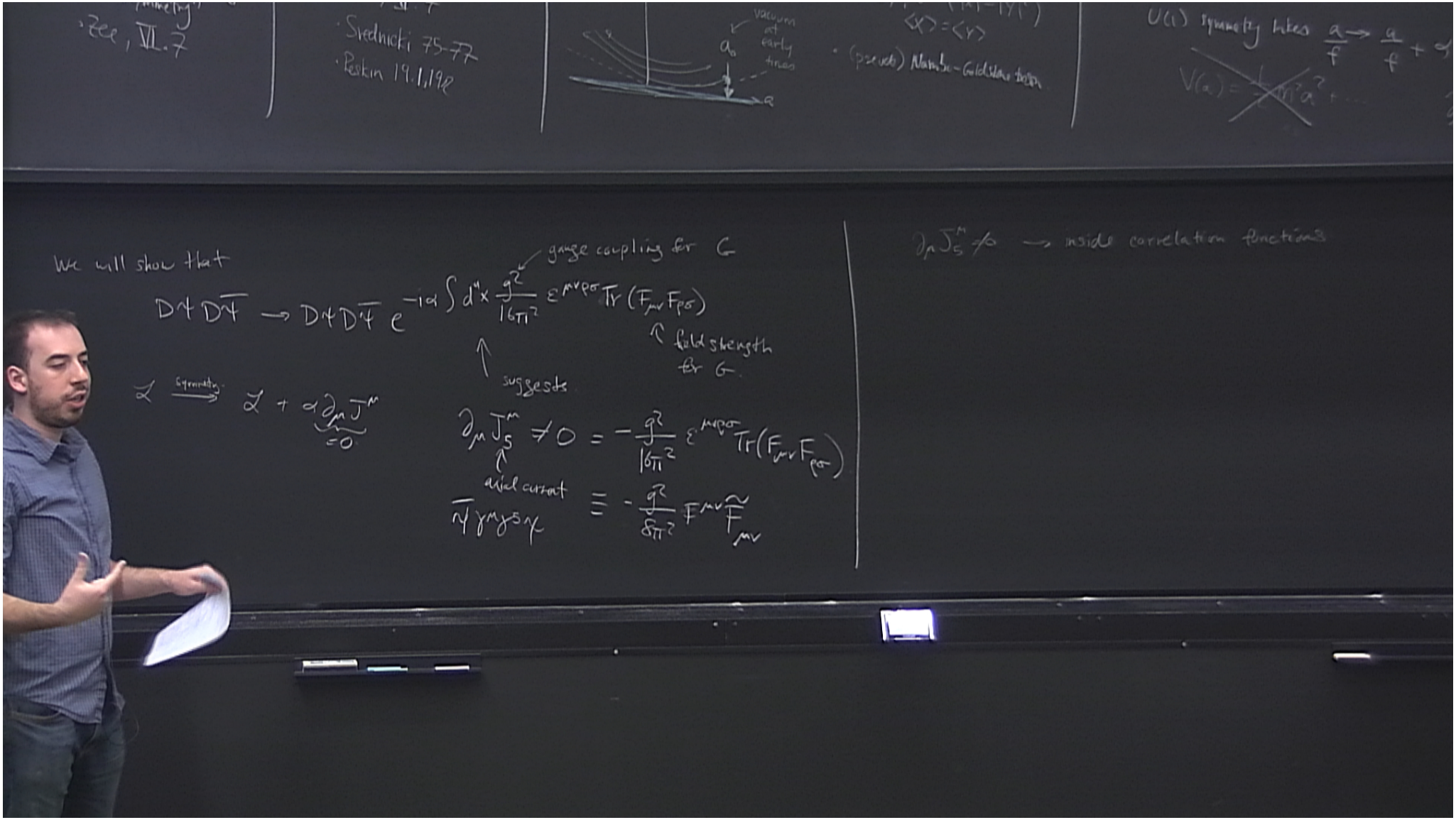

directions
breaking
ii) Anomaly → more interesting
Anomaly: A symmetry of the action that is not a symmetry of the full theory
↳ correlation functions, ...
$Z[J] = \int \mathcal{D}\Phi\, e^{i\left(S[\Phi] + \int d^4x\, J\Phi\right)}$
not invariant
invariant


exactly flat (symmetry
is not exact)
→ energy in oscillation
Example: Peccei-Quinn
Axial symmetry
global symmetry
FRW universe
homogeneity
isotropy

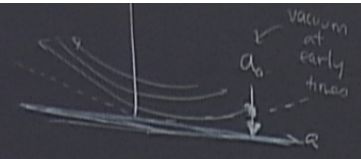

Zee, VI.7
Srednicki 75-77
Peskin 19.1, 19.2
Vacuum at early times
a_0
$\langle X \rangle = \langle Y \rangle$
(pseudo) Nambu-Goldstone boson
U(1) symmetry likes $\frac{a}{f}$
$V(a) = -\frac{1}{2}m^2a^2 +$
We will show that
gauge coupling for G
$DA\,D\bar{A} \rightarrow DA\,D\bar{A}\; e^{-iq\int d^4x \frac{g^2}{16\pi^2}\varepsilon^{\mu\nu\rho\sigma}\mathrm{Tr}(F_{\mu\nu}F_{\rho\sigma})}$
field strength for G


Zee, VI.7
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(pseudo) Nambu-Goldstone boson
U(1) symmetry takes
We will show that
gauge coupling for G
field strength for G
Symmetry


Zee, VI.7
Srednicki 75-77
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Vacuum at early times
$\langle X \rangle = \langle Y \rangle$
(pseudo) Nambu-Goldstone boson
U(1) symmetry likes
We will show that
gauge coupling for G
$\mathcal{D}\psi\mathcal{D}\bar{\psi} \to \mathcal{D}\psi\mathcal{D}\bar{\psi}\, e^{-i\alpha \int d^4x \frac{g^2}{16\pi^2} \varepsilon^{\mu\nu\rho\sigma} \mathrm{Tr}(F_{\mu\nu}F_{\rho\sigma})}$
field strength for G
suggests
$\mathcal{L} \xrightarrow{\text{symmetry}} \mathcal{L} + \alpha \underbrace{\partial_\mu J^\mu}_{=0}$
$\partial_\mu J^\mu_5 \neq 0 = -\frac{g^2}{16\pi^2} \varepsilon^{\mu\nu\rho\sigma} \mathrm{Tr}(F_{\mu\nu}F_{\rho\sigma})$
axial current
$\bar{\psi}\gamma^\mu\gamma^5\psi \equiv -\frac{g^2}{8\pi^2} F^{\mu\nu}\tilde{F}_{\mu\nu}$


Zee, VI.7
Srednicki 75-77
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vacuum at early times
a_0
$\langle \chi \rangle = \langle \chi \rangle$
• (pseudo) Nambu-Goldstone boson
U(1) symmetry takes $\frac{a}{f} \to \frac{a}{f} + \alpha$
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We will show that
$\mathcal{D}\psi\mathcal{D}\bar{\psi} \to \mathcal{D}\psi\mathcal{D}\bar{\psi}\, e^{-i\alpha \int d^4x \frac{g^2}{16\pi^2} \varepsilon^{\mu\nu\rho\sigma} \mathrm{Tr}(F_{\mu\nu}F_{\rho\sigma})}$
gauge coupling for G
field strength for G
suggests
$\mathcal{L} \xrightarrow{\text{symmetry}} \mathcal{L} + \alpha \underbrace{\partial_\mu J^\mu}_{=0}$
$\partial_\mu J_5^\mu \neq 0 = -\frac{g^2}{16\pi^2} \varepsilon^{\mu\nu\rho\sigma} \mathrm{Tr}(F_{\mu\nu}F_{\rho\sigma})$
axial current
$\bar{\psi}\gamma^\mu\gamma^5\psi \equiv -\frac{g^2}{8\pi^2} F^{\mu\nu}\tilde{F}_{\mu\nu}$
$\partial_\mu J_5^\mu \neq 0$ → inside correlation functions

75-77
1,19



$\langle \chi \rangle = \langle \chi \rangle$

• (pseudo) Nambu–Goldstone boson

$U(1)$ symmetry takes $\frac{a}{f} \to \frac{a}{f} + \alpha$

$V(a) = \cancel{-m^2 a^2 + \dots}$

$\partial_\mu a$

→ Goldstone bosons are flat directions

gauge coupling for G

$$-i\alpha \int d^4x \frac{g^2}{16\pi^2} \varepsilon^{\mu\nu\rho\sigma} \mathrm{Tr}\left(F_{\mu\nu} F_{\rho\sigma}\right)$$

↑ field strength for G

↑ suggests

$$\partial_\mu J_5^\mu \neq 0 = -\frac{g^2}{16\pi^2} \varepsilon^{\mu\nu\rho\sigma} \mathrm{Tr}\left(F_{\mu\nu} F_{\rho\sigma}\right)$$

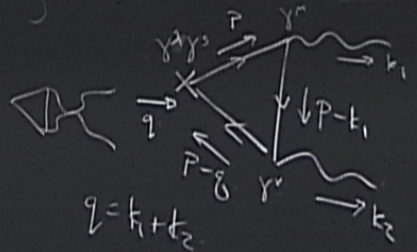
↑ axial current

$$\bar{\psi} \gamma_\mu \gamma_5 \psi \equiv -\frac{g^2}{8\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

$\partial_\mu J_5^\mu \neq 0$ → inside correlation functions

Consider a particular correlation fcn

$$\int d^4x\, \partial_\lambda \langle A^\mu(k_1) A^\nu(k_2) | J_5^\lambda(q) | 0 \rangle e^{ix} = q_\lambda \langle A^\mu(k_1) A^\nu(k_2) | J_5^\lambda(q) | 0 \rangle \stackrel{?}{=} 0$$



$q = k_1 + k_2$

fermion loop

$$\Delta^{\lambda\mu\nu}(k_1, k_2) = (-1)\, i^3 \int \frac{d^4p}{(2\pi)^4} \left[\mathrm{Tr}\left(\gamma^\lambda \gamma^5 \frac{1}{\not{p} - \not{q}} \gamma^\nu \frac{1}{\not{p} - \not{k}_1} \gamma^\mu \frac{1}{\not{p}} \right) + \mathrm{Tr}\left(\gamma^\lambda \gamma^5 \frac{1}{\not{p} - \not{q}} \gamma^\mu \frac{1}{\not{p} - \not{k}_2} \gamma^\nu \frac{1}{\not{p}} \right) \right]$$