

Title: Explorations in Particle Theory-5

Date: Apr 10, 2015 09:00 AM

URL: <http://pirsa.org/15040010>

Abstract:

Tutorial today
 on estimating rates
 & abundances
 (2-3:30 pm)
 Problem Set # 1 due
 next Friday

Boltzmann eq:

$$X \bar{X} \rightarrow f \bar{f} \quad \Gamma = n \langle \sigma v \rangle$$

$$\dot{N}_X + 3H n_X = - \langle \sigma v \rangle \left[n_X^2 - (n_X^e)^2 \right]$$

$$\frac{dY_X}{dx} = \frac{-X S(x)}{H(x=1)} \langle \sigma v \rangle \left[Y_X^2 - (Y_X^e)^2 \right]$$

where $Y_X = \frac{n_X}{S}$, $x = \frac{M_X}{T}$

Using $\langle \sigma v \rangle = \sigma_n X^{-n}$

$$\lambda \equiv \left[\frac{S \langle \sigma v \rangle}{H(x)} \right]_{x=1} \approx \left(\frac{\Gamma}{H} \right)_{x=1}$$

we find

$$Y_X^{\infty} = \frac{n+1}{\lambda} X_{f0}^{n+1}$$

$$X_{f0} = \log \left[\lambda \sqrt{\frac{10}{8\pi^3}} \frac{g_X}{\sqrt{S}} M_X M_p \sigma_n \right]$$

$\approx 20-30$

⑤ WIMP DM Properties

Today

- ① WIMP Miracle
- ② WIMP Properties + Models
- ③ Thermal Freeze-out Variants

$$\begin{aligned} \rho_{DM} &= M_{DM} \cdot n_{DM} \\ &= M_{DM} \left(\frac{n_{DM}}{S_{today}} \right) S_{today} = M_{DM} Y_{DM} S_{today} \end{aligned}$$

① WIMP Miracle

Match Planck measurement

$$\Omega_{DM} = 0.27$$

$$= \frac{\rho_{DM}}{\rho_{crit}}$$

If X is DM, $\Omega_{DM} = \Omega_X$

$$Y_x^\infty = \frac{n+1}{\lambda} X_B^{n+1}$$

$$\lambda = \frac{x S(x) \langle \sigma v \rangle}{H(x)} \Big|_{x=1}$$

$$S(T) = \frac{2\pi^2}{45} g_{\text{eff}} T^3$$

$$\xrightarrow{x=1} \frac{2\pi^2}{45} g_{\text{eff}} M_X^3$$

$$H(T) = 1.66 \sqrt{g_{\text{eff}}} \frac{T^2}{M_{\text{Pl}}}$$

$$\rightarrow 1.66 \sqrt{g_{\text{eff}}} \frac{M_X^2}{M_{\text{Pl}}}$$

$$\lambda = \text{coeff} \cdot \frac{M_X^3}{M_X^2 / M_{\text{Pl}}} \sigma_n$$

$$= \text{coeff} \cdot M_{\text{Pl}} M_X \sigma_n$$

$$Y_x^\infty \sim \frac{(n+1) X_B^{n+1}}{M_X M_{\text{Pl}} \sigma_n}$$

$$\rho_X = M_X Y_x^\infty S_0 \leftarrow 2000/\text{cm}^3$$

$$\rho_X = \frac{X_B^{n+1} S_0}{\sigma_n M_{\text{Pl}}}$$

$$\rho_X \sim \frac{1}{\langle \sigma v \rangle}$$

direct link b/w particle physics & cosmology.

$$\Omega_{\text{DM}} = \frac{\rho_X + \rho_{\bar{X}}}{\rho_{\text{crit}}} = \frac{2\rho_X}{\rho_{\text{crit}}}$$

- s-wave annihilation
($n=0$).

$$\langle \sigma v \rangle = a + b v^2 + \dots$$

$$g_2 \approx 100$$

$$X_H \approx 30$$

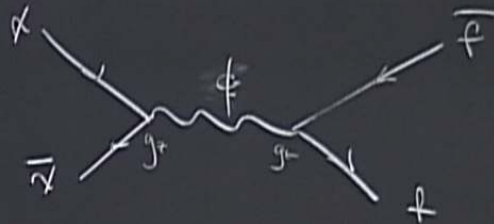
$$\Omega_{DM} = \frac{1.09 \times 10^{-9} \text{ GeV}^{-2}}{\langle \sigma v \rangle}$$

Planck \rightarrow

$$\langle \sigma v \rangle \approx 4.4 \times 10^{-9} \text{ GeV}^{-2}$$
$$\approx 6 \times 10^{-26} \text{ cm}^3/\text{s}$$

\rightarrow "Dirac" DM $\chi \neq \bar{\chi}$

\rightarrow "Majorana", $\chi = \bar{\chi} \rightarrow \langle \sigma v \rangle = 3 \times 10^{-26} \text{ cm}^3/\text{s}$



$$m_z, m_f \ll T \ll M_\chi$$

$$\langle \sigma v \rangle \approx \frac{1}{8\pi} \frac{g^4}{M_\chi^2} = (4.4 \times 10^{-9} \text{ (eV}^{-2}\text{)}) \left(\frac{g_\phi}{0.6} \right)^4 \left(\frac{700 \text{ GeV}}{M_\chi} \right)^2$$

WIMP Miracle \rightarrow "weakly interacting massive particle"


(perturbative)
Unitarity bound

$\uparrow m_\chi, \uparrow g(m_\chi)$

$$g \lesssim 4\pi$$

$$M_\chi \lesssim O(100) \text{ TeV}$$

 $\rightarrow \Delta m_h^2 \sim m_\chi^2$

 $\rightarrow \Delta m_h^2 \sim -m_\chi^2$
 $m \sim \text{weak scale.}$

• WIMPs have "large" $\langle \sigma v \rangle$ at $E \sim \text{TeV}$
 \rightarrow look for WIMPs in experiments.

• very predictive.

$$h \text{---} \textcircled{h} \text{---} h \rightarrow \Delta m_h^2 \sim m_h^2$$

$$\text{---} \textcircled{??} \text{---} \rightarrow \Delta m_h^2 \sim -m_h^2$$

$m \sim \text{weak scale.}$

= WIMPs have "large" $\langle \sigma v \rangle$ at $E \sim \text{TeV}$
 \rightarrow look for WIMPs in experiments.

= very predictive.

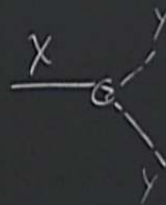
② WIMP Properties + Models

1) $\Omega_X = \Omega_{DM}$.

$$\langle \sigma v \rangle = 6 \times 10^{-26} \text{ cm}^3/\text{s}$$

2) Forbidden X decays.

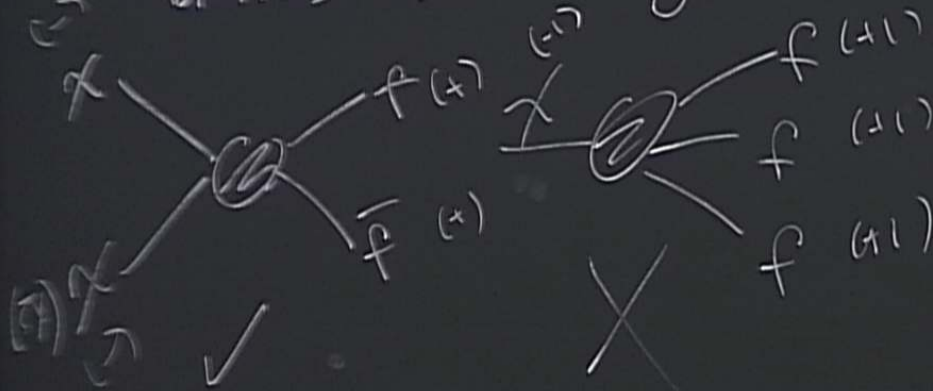
$$\tau_{\text{univ}} \sim 14 \text{ Gyr.}, \quad \Gamma_X \lesssim 10^{-40} \text{ GeV}^{-1}$$



$$\Gamma_X = \frac{g_{\text{decay}}^2}{8\pi} M_X \cdot \mathcal{I}_{\text{dec}} \lesssim 10^{-21}$$

Adding a symmetry that

forbids X decay



any interaction requires $2X$

Z_2 symmetry (parity)

X, \bar{X} charge -1

SM charge $+1$

nucleon = udd

$$\underbrace{\epsilon_{ijk} u^i d^j d^k}_{\text{colour-invariant}}$$

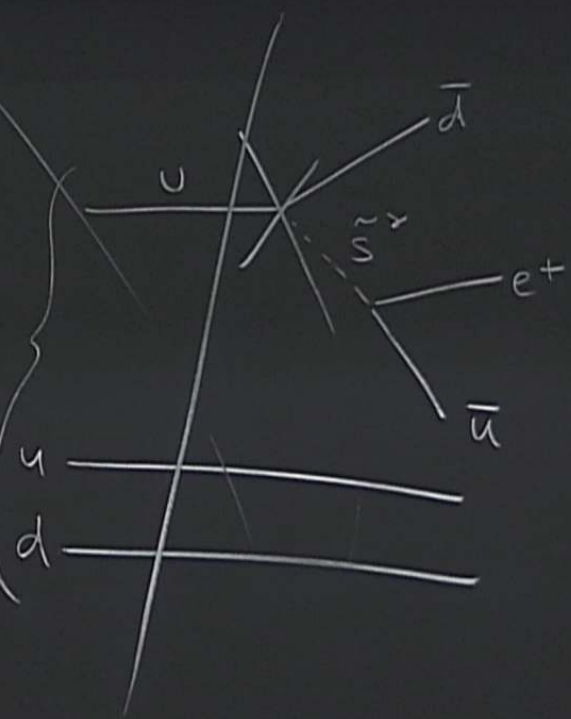
forbidden
in SM

$$\mathcal{L} = \epsilon_{ijk} u^i d^j \tilde{d}^k$$

Introduce \mathbb{Z}_2 called R-parity

$$\begin{aligned} \psi_{SM} & (+1) \\ \tilde{\psi} & (-1) \end{aligned}$$

udd
(proton)



$$\overline{u} \overline{d} \overline{u} d e^+$$
$$\pi^+ \pi^-$$

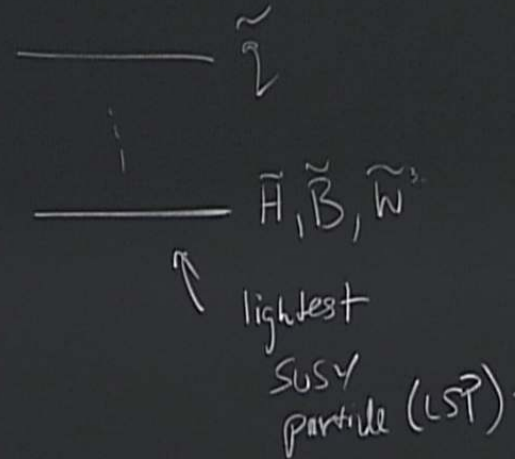
$$m_{\pi} \sim 130 \text{ MeV}$$
$$m_e \sim 0.5 \text{ MeV}$$

$$p \rightarrow \pi^+ \pi^- e^+$$
$$\tau \gtrsim 10^{34} \text{ s}$$

\mathbb{Z}_2 -even



\mathbb{Z}_2 -odd



$\{ \tilde{H}_0, \tilde{H}_d, \tilde{W}, \tilde{B} \}$
mix

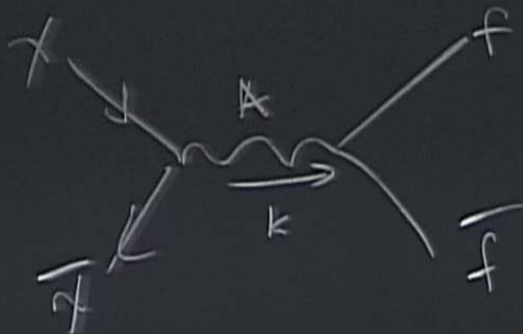
MSSM ~ 105 parameters.

③ Variants on Freeze-out

$\rho_{\text{kin}}^{\text{initial}}$ Lots of new particles in the spectrum

\rightarrow change the predictions of freeze-out.

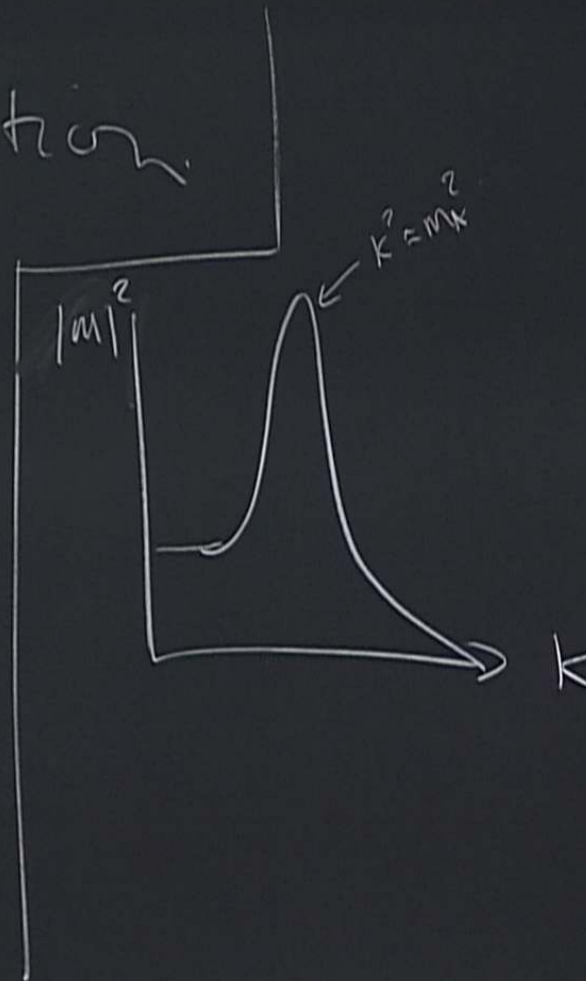
a) Resonant DM annihilation



$$m_A \gg m_X$$

$$M \sim \frac{1}{k^2 - m_A^2} \sim \frac{1}{m_X^2}$$

$$\sigma \sim |M|^2 \leftarrow \text{pole in } \sigma \text{ as well}$$



real part

$\text{Im}(\dots)$

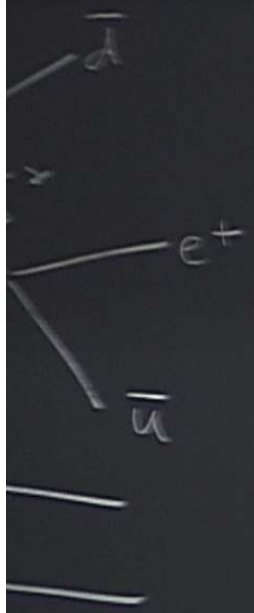
real part

$$\frac{i}{k^2 - m_A^2 + i m_A \Gamma_A}$$

$k^2 = m_A^2$

$$\text{Im} \left(\text{---} \bigcirc \text{---} + \text{---} \bigcirc \text{---} \bigcirc \text{---} + \text{---} \bigcirc \text{---} \bigcirc \text{---} \bigcirc \text{---} \right)$$

$$\sigma = \frac{1}{8\pi} \frac{g^4}{m_A^2} \rightarrow \frac{1}{8\pi} \frac{g^4}{m_A \Gamma_A}$$



$$\overline{u} \overline{d} \overline{d} e^+$$

$$\pi^+ \pi^-$$

$$m_\pi \sim 130 \text{ MeV}$$

$$m_e \sim 0.5 \text{ MeV}$$

$$p \rightarrow \pi^+ \pi^- e^+$$

$$\tau \gtrsim 10^{34} \text{ s}$$

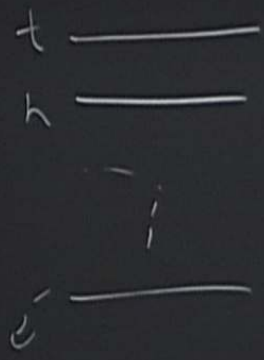
$$\Gamma_A \sim \frac{g^2}{8\pi} M_A$$

σ is enhanced
by $\frac{m_X^2}{M_A^2 g^2}$

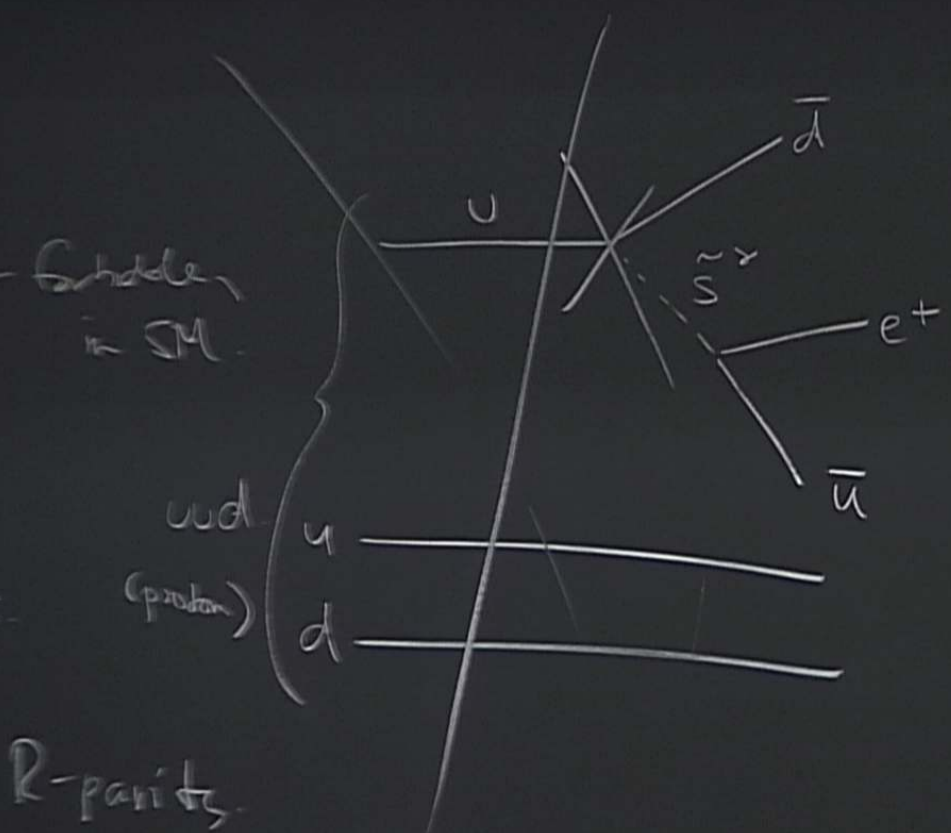
If g is too small
to give the
abundance,

OK if $m_X^2 = \frac{1}{4} M_A^2$

Z_2 -even



H_u



$$\bar{u} \bar{d} \bar{u} d e^+$$

$$\pi^+ \pi^-$$

$$m_{\pi} \sim 130 \text{ MeV}$$

$$m_e \sim 0.5 \text{ MeV}$$

$$p \rightarrow \pi^+ \pi^- e^+$$

$$\tau \gtrsim 10^{34} \text{ s}$$

$\Gamma_A \sim \frac{g^2}{8\pi} M_A$

σ is enhanced by $\frac{m_X^2}{M_A^2 g^2}$

If g is too small to ~~be~~ ^{give the} ~~relative~~ abundance,

OK if $m_X^2 = \frac{1}{4} M_A^2$