Title: Explorations in Particle Theory-3

Date: Apr 08, 2015 09:00 AM

URL: http://pirsa.org/15040008

Abstract:

First tutorial today

3:45 – 5:15

Ref for today/tomorrow

Kolb & Turner Ch. 5.1–5.2

Results from yesterday:

$$f_i(E) = \frac{1}{e^{(E-\mu_i)/T} \mp 1}$$

$$n_i(T, \mu=0) = \begin{Bmatrix} 1 \\ 3/4 \end{Bmatrix} g_i \frac{\zeta(3)}{\pi^2} T_i^3 , \quad T_i \gg m_i$$

$$= g_i \left(\frac{m_i T_i}{2\pi}\right)^{3/2} e^{-m_i/T_i} , \quad m_i \gg T_i$$

$T_i = T_j$ if i, j are in equilibrium

$S = s a^3$ is conserved

In radiation dominated universe:

$$\rho_{rad} = \frac{\pi^2}{30} g_* T^4$$

$$H = 1.66 \sqrt{g_*} \frac{T^2}{M_{Pl}}$$

$$s = \frac{2\pi^2}{45} g_{*s} T^3$$

radiation dominated

[u]niverse:

$$\rho_{rad} = \frac{\pi^2}{30} g_* T^4$$

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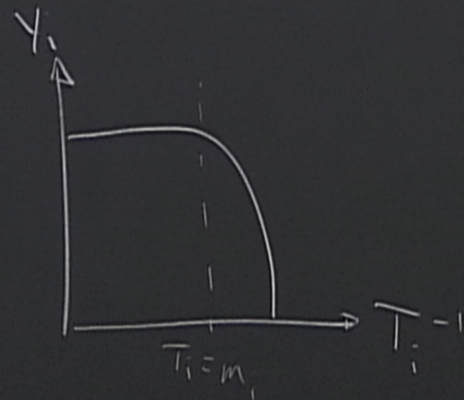
$$Y_i \equiv \frac{n_i}{s}$$



III Departure from equilibrium & thermal relics

DM interacts more weakly than SM

$\rightarrow$ is it in equilibrium?

"cold" $\rightarrow$ $m \gtrsim$ keV, $T_{CMB} \sim$ eV

n_{DM} @ CMB, $e^{-\text{keV}/\text{eV}} \sim e^{-1000}$

Today

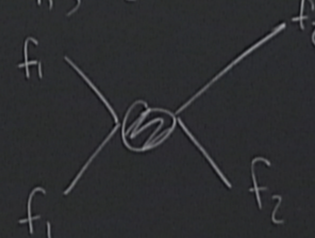
1. Departures from equilibrium
2. Intuitive overview of eq. departure
   → thermal freeze-out
3. Deriving the Boltzmann equations

① Departures from Equilibrium

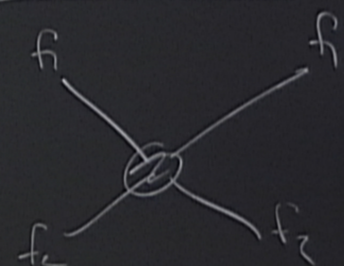
in-equilibrium: $\tau_{reaction} \lesssim \frac{1}{H}$

$$\boxed{\Gamma_{reaction} \gtrsim H}$$

f_1, f_2



INELASTIC



ELASTIC

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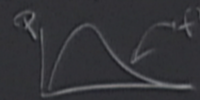
$E = \sqrt{p^2 + m^2}$



$$n_i(T, \mu=0) = \begin{Bmatrix} 1 \\ 3/4 \end{Bmatrix} g_i \frac{\zeta(3)}{\pi^2} T_i^3, \quad T_i \gg m_i$$

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In radiation do[...]

universe:

$\rho_{rad} = \frac{\pi^2}{30} g_*$ [...]

$H = 1.66 \sqrt{g_*}$ [...]

$s = \frac{2\pi^2}{45} g_*$ [...]

INELASTIC → chemical equilibrium

n_{f_1}, f_2 be in equilibrium

ELASTIC → kinetic equilibrium

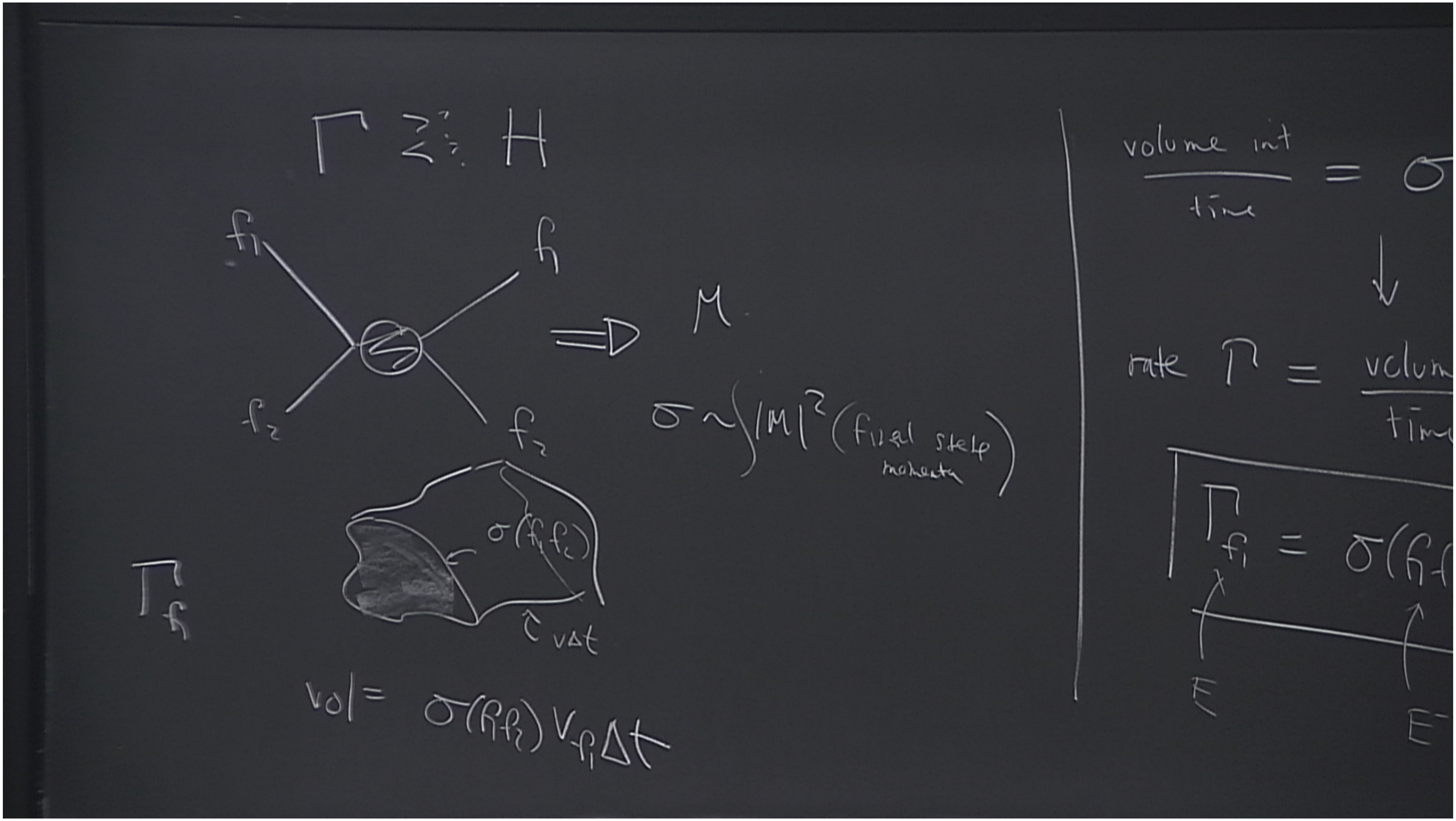
tends to make $\vec{p}_{f_1}, \vec{p}_{f_2}$ similar

Full equilibrium = kinetic + chemical equilibrium

↳ use the eq. formulae

Kinetic but NOT chemical

$$\boxed{f_i(|\vec{p}|) = \beta f_i^{eq}(|\vec{p}|)}$$



Γ ≷ H
f1
f1
M
f2
f2
σ ~ ∫|M|² (final state momenta)
σ(f1 f2)
τ vΔt
Γf1
vol = σ(f1f2) vf1 Δt
volume int
time
= σ
rate Γ = volum
time
Γf1 = σ(f1 f
E
E

$\Longrightarrow \mathcal{M}.$

$$\sigma \sim \int |\mathcal{M}|^2 \left(\text{final state momenta}\right)$$

$$\sigma(f_1 f_2)\, v_{f_1} \Delta t$$

$$\frac{\text{volume int}}{\text{time}} = \sigma(f_1 f_2)\, v_{f_1}$$

$$\downarrow$$

$$\text{rate } \Gamma = \frac{\text{volume}}{\text{time}} \times \frac{\text{number } f_2}{\text{volume.}}$$

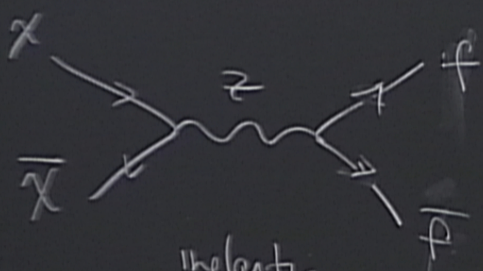
$$\boxed{\Gamma_{f_1} = \sigma(f_1 f_2)\, v_{f_1}\, n_{f_2}}$$

$E \qquad E^{-2} \qquad E^{3}.$

② Thermal Freeze-out

Introduce a new particle χ

$\longrightarrow$ couples to the Z boson



inelastic

elastic

Assume

$m_\chi \gg m_Z, m_f$

Assume that χ "starts" in equilibrium at high T

$$\Gamma_{\text{inelastic}} = \sigma(\chi\bar{\chi} \to f\bar{f})\, v_\chi\, n_{\bar{\chi}}$$

$T \gg m_\chi$:

$$v_\chi \sim c = 1$$

$$n_{\bar{\chi}} \sim \frac{3}{4} g_\chi \frac{\zeta(3)}{\pi^2} T^3$$

$$\sim \frac{6}{4} \frac{3}{10} T^3$$

$$\sim 0.5 T^3$$

... χ

... boson

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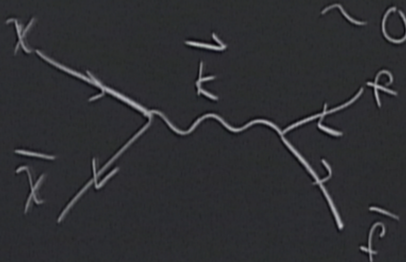
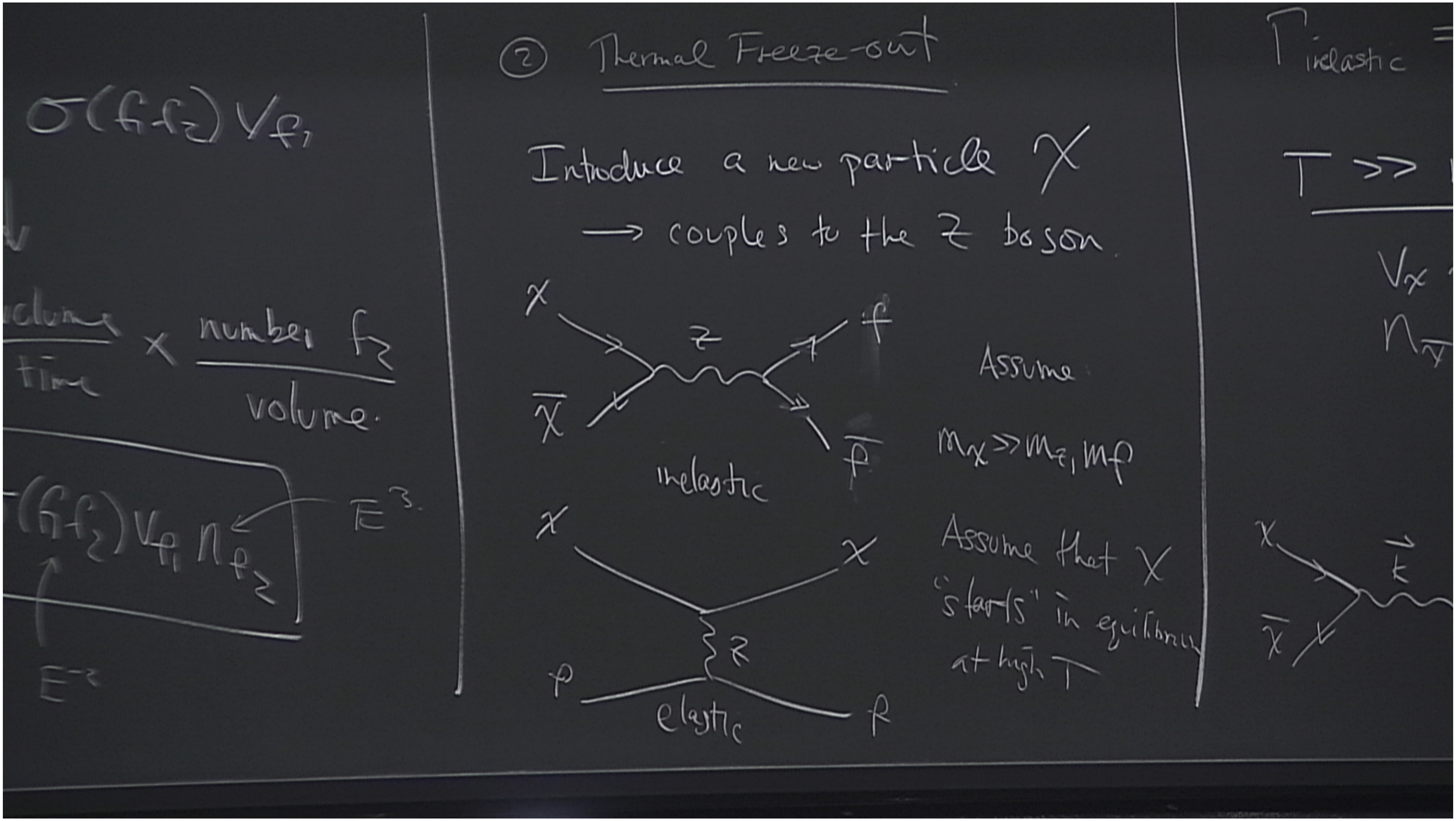


$$iM = \bar{v}_{\bar{\chi}} \left(g_{\chi V} \gamma^\mu + g_{\chi A} \gamma^\mu \gamma^5 \right) u_\chi \left[\frac{i g_{\mu\nu} - \frac{k_\mu k_\nu}{M_Z^2}}{k^2 - M_Z^2} \right] \bar{\psi}_f (\cdots) v_{\bar{f}} \cdots$$


② Thermal Freeze-out
Introduce a new particle χ
→ couples to the Z boson
χ
χ̄
Z
f
f̄
inelastic
χ
χ
Z
f
f
elastic
Assume
mχ >> mZ, mf
Assume that χ "starts" in equilibrium at high T
σ(f1f2) vf1
volume / time × number f2 / volume
E³
E⁻²
Γinelastic =
T >>
vχ
nχ̄
χ
χ̄
Z

...le χ

... boson

Assume

$m_\chi \gg m_Z, m_f$

assume that χ "starts" in equilibrium at high T

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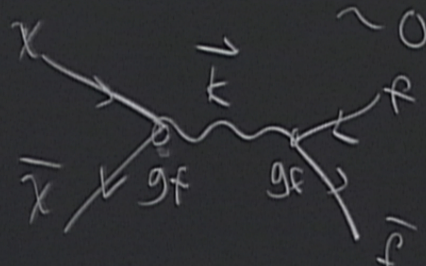

$$iM = \bar{v}_{\bar{\chi}}\left(g_{\chi V}\gamma^\mu + g_{\chi A}\gamma^\mu\gamma^5\right)u_\chi$$

$$\left[\frac{i g_{\mu\nu} - \frac{k_\mu k_\nu}{M_Z^2}}{k^2 - M_Z^2}\right]$$

$$\bar{u}_f(\cdots)v_{\bar{f}} \ldots$$

$$M \sim \frac{g_\chi g_f}{k^2 - M_Z^2} \left(\begin{matrix} \text{dim.} \\ \# \end{matrix} \right)$$

$$\sigma \propto |M|^2$$

$$\sigma \sim \frac{g_\chi^2 g_f^2}{(\underbrace{k^2}_{\sim T^2} - \cancel{M_Z^2})^2} \left(\quad \right)$$

$$\sim \left(\frac{1}{8\pi}\right) \frac{g_\chi^2 g_f^2}{T^4} (T^2)$$

$$\sigma_{\chi\bar{\chi}} = \frac{1}{8\pi} \frac{g_\chi^2 g_f^2}{T^2}$$

$$\Gamma_{\chi\,\text{inel}} = \left(\frac{1}{2} T^3\right)(1)\left(\frac{1}{10} \frac{g_\chi^2 g_f^2}{T^2}\right)$$

$$\boxed{\Gamma = \frac{1}{20} g_\chi^2 g_f^2 T}$$

$$\frac{\Gamma}{H} = \frac{1}{20} g_\chi^2 g_f^2 T \cdot \frac{M_{Pl}}{1.66 \sqrt{g_*}\, T^2} \approx \boxed{g_\chi^2 g_f^2 \frac{M_{Pl}}{T} = \frac{\Gamma}{H}}$$

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$10^{?}$ GeV

$\hookrightarrow$ "more" in eq. as $T \to 0$

② Thermal Freeze-out

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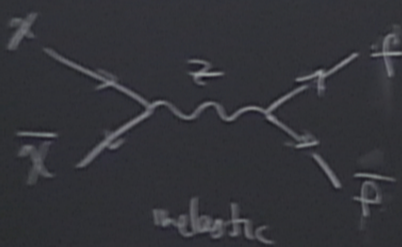

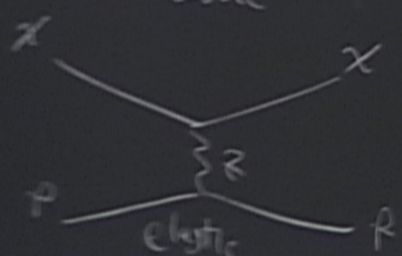

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Assume that χ "starts" in equilibrium at high T

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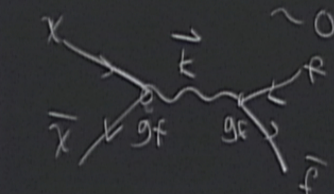
$(m_\chi, \vec{0}) + (m_\chi, \vec{0})$



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$\underline{T \lesssim m_\chi}$:

$v_\chi \sim O(1) - 0.1$

$$n_\chi = 2\left(\frac{m_\chi T}{2\pi}\right)^{3/2} e^{-m_\chi/T}$$

$$\sigma_\chi \overset{T \to m_\chi}{=} \frac{1}{8\pi} \frac{g_\chi^2 g_f^2}{m_\chi^2}$$

$$\boxed{\frac{\Gamma}{H} \sim \frac{g_\chi^2 g_f^2}{\sqrt{m_\chi T}} M_{pl} e^{-m_\chi/T}}$$

For $T \ll m_\chi$

$\left(\frac{m_\chi}{T} \sim 20\right)$.

$$\underline{\Gamma < H}$$

Inelastic → changes χ number

$\Longrightarrow$ χ number <u>stops changing</u>

at $T_{fo} \approx \frac{m_\chi}{20}$

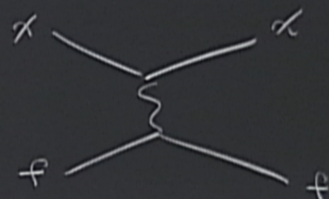
$\Longrightarrow$ thermal freeze-out

modulo expansion

$$Y_\chi(\text{today}) = Y_\chi(T_{\text{freezeout}})$$

↳ thermal relic

Γ_{elastic}



$$\Gamma_{\chi,el} = \sigma(\chi f \to \chi f)\, v_\chi \overline{n_f}$$

$$n_f = \# T^3$$

$$n_\chi = \# e^{-m/T}$$

$$\frac{\Gamma_{\chi,el}}{\Gamma_{\chi,inel}} = e^{m_\chi/T} \gg 1$$

if $\Gamma_{inel} \lesssim H$

$\underbrace{\Gamma_{el} \gg H}$

f_χ is valid

$$f_\chi = \beta f_\chi^{eq}$$

$$n_\chi \neq n_\chi^{eq}$$

③ Boltzmann Equation

continuity eq $\frac{\partial \rho}{\partial t} + \nabla \vec{J} = \Gamma_{\text{source}}$

Start with source-free equation (non-interacting particle)

↳ $f(x^\mu, p^\mu)$

$$\frac{df}{dt} = 0 \longrightarrow \frac{df}{dt} = \frac{\partial f}{\partial x^\mu}\frac{dx^\mu}{dt} + \frac{\partial f}{\partial p^\mu}\frac{dp^\mu}{dt} = 0$$

Example of non-relativistic particle

$$p^\mu = m\frac{dx^\mu}{dt}$$

Christoffel connection

$$\frac{dp^\mu}{dt} = m\frac{d^2x^\mu}{dt^2} = -m\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt}$$

Plug in FRW, homogeneous + isotropic

$$f(p^\mu, x^\mu) = f(|\vec{p}|)$$

$$E\frac{\partial f(|\vec{p}|,t)}{\partial t} - H|\vec{p}|^2\frac{\partial f}{\partial E} = 0$$

$\downarrow$ integrate over d^3p.

$$n = \int \frac{d^3p}{(2\pi)^3} f$$

$$\boxed{\frac{dn(t)}{dt} + 3Hn(t) = 0}$$

INELASTIC ELASTIC

-ticle

Christoffel connection

$$\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt}$$

isotropic

$f(|\vec p|)$

$$E\frac{\partial f(|\vec p|,t)}{\partial t} - H|\vec p|^2\frac{\partial f}{\partial E} = 0$$

$\downarrow$ integrate over d^3p

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$$\boxed{\frac{dn(t)}{dt} + 3Hn(t) = 0}$$

$$H = \dot a / a$$

$$n(t) = n(t_0)\left(\frac{a(t)}{a(t_0)}\right)^{-3}$$

↳ simple dilution of # density

$$Y = \frac{n}{s}, \quad s \propto a^{-3}$$

$$\boxed{\frac{dY}{dt} = 0} \rightarrow Y = \text{constant}.$$