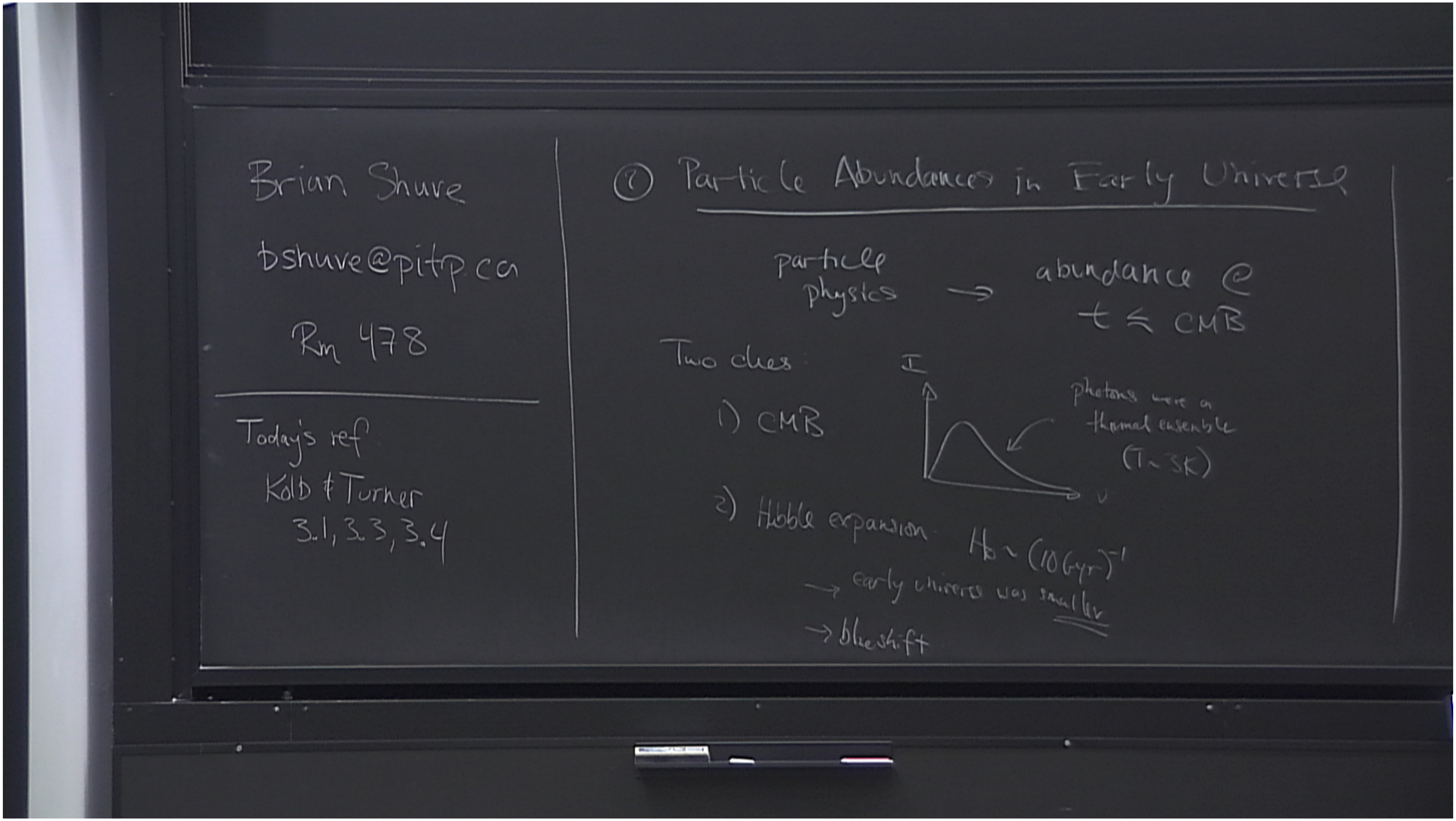
Title: Explorations in Particle Theory-2

Date: Apr 07, 2015 09:00 AM

URL: http://pirsa.org/15040007

Abstract:


Brian Shuve
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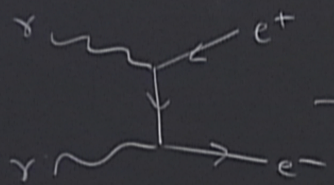
Rm 478
Today's ref
Kolb & Turner
3.1, 3.3, 3.4
② Particle Abundances in Early Universe
particle physics → abundance @ $t \lesssim$ CMB
Two clues:
1) CMB
I
ν
photons were a thermal ensemble
(T ~ 3K)
2) Hubble expansion
$H_0 \sim (10\,Gyr)^{-1}$
→ early universe was smaller
→ blueshift

Earlier times $\rightarrow$ higher T_γ.



$\rightarrow T_\gamma \gtrsim m_e$

↳ in thermal bath

- $T_\gamma \gtrsim m_t \rightarrow$ all SM particles in thermal bath

Today:

① Friedmann–Robertson–Walker universe

② Equilibrium thermodynamics

③ Thermodynamics in expanding bkd

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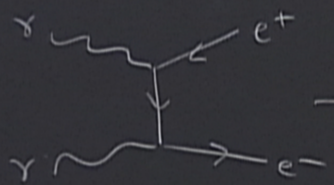

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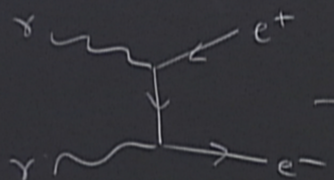
① FRW universe

Homogeneity & isotropy

Earlier times → higher T_γ.



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① Friedmann–Robertson–Walker universe

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③ Thermodynamics in expanding bkd

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Homogeneity & isotropy

$$ds^2 = dt^2 - a(t)^2 d^3x$$

↳ ruler size

$$d^3x = \frac{dr^2}{1-kr^2} + r^2 d\Omega^2$$

↑ curvature

$a(t) \equiv$ scale factor

Change in scale factor (expansion rate) is called Hubble scale

$$\boxed{H(t) \equiv \frac{\dot{a}(t)}{a(t)}}$$

Einstein's equations give $a(t)$ as a function of energy density

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$G \to 1/M_{Pl}^2$

$G_{\mu\nu} \to$ derivatives of $a(t)$

$$T_{\mu\nu} \underset{\text{isotropy}}{\overset{\text{homogeneity}}{=\!=}} \begin{pmatrix} \rho & & & 0 \\ & -p & & \\ & & -p & \\ 0 & & & -p \end{pmatrix}$$

Relationship b/w ρ, p determined by what kind of "stuff" you have.

"Equations of state"

non-relativistic matter

↳ $p = 0$

relativistic matter

$p = \rho/3$

$a(t) \equiv$ scale factor

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$\hookrightarrow P = 0$.

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$P = \rho/3$.

vacuum energy

$P = -\rho$

Take ds^2 & plug into E.E.

① $\dot{\rho} + 3H(\rho + P) = 0$

$\hookrightarrow$ continuity eq

② $H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho$

$\hookrightarrow k = 0$

$$\boxed{\rho_{crit} \equiv \frac{3H^2}{8\pi G}}$$

$$\sum_i \rho_i = \rho_{crit}.$$

$$\boxed{\Omega_i \equiv \frac{\rho_i}{\rho_{crit}}} \qquad \left(\sum_i \Omega_i = 1\right)$$

Solve Friedmann eq.

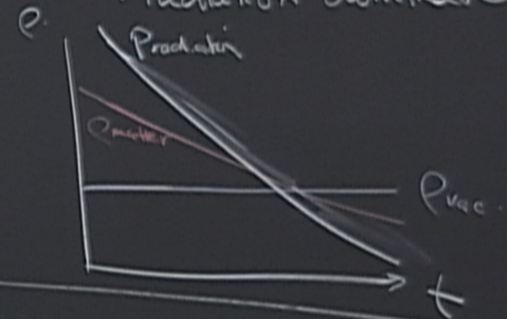
$$\rho_{rad.} \propto a^{-4}$$

$$\rho_{matter} \propto a^{-3}$$

$$\rho_{vacuum} \propto \text{constant}$$

Early universe:

→ radiation dominated



$$\boxed{H = \frac{1}{2t} \propto \frac{1}{a^2}, \quad a(t) = c\sqrt{t}}$$

↑ radiation dominated

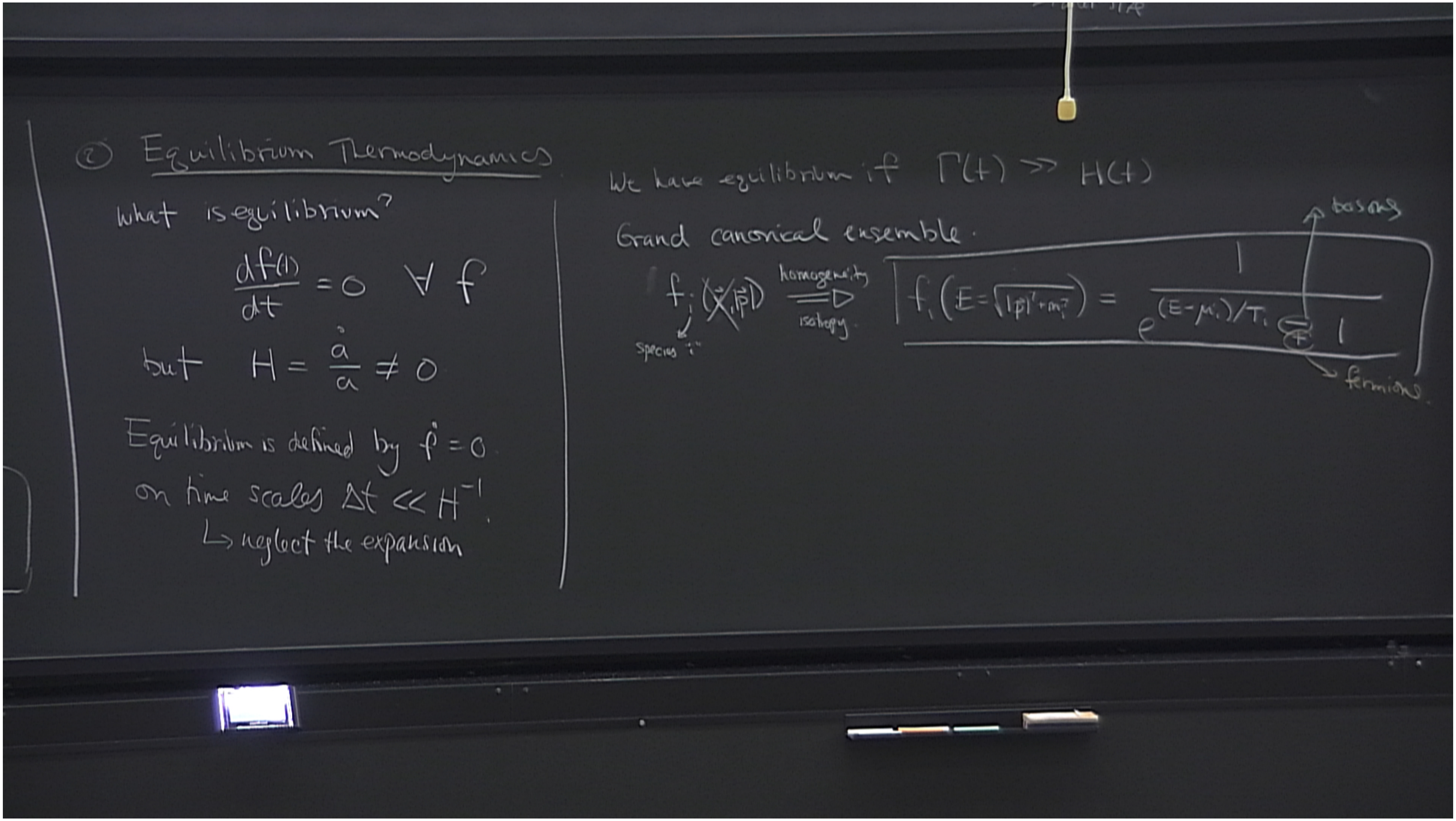
② Equ

what

but

Equili

on h


② Equilibrium Thermodynamics
what is equilibrium?
$\frac{df(t)}{dt} = 0 \quad \forall f$
but $H = \frac{\dot{a}}{a} \neq 0$
Equilibrium is defined by $\dot{f} = 0$
on time scales $\Delta t \ll H^{-1}$
↳ neglect the expansion
We have equilibrium if $\Gamma(t) \gg H(t)$
Grand canonical ensemble.
$f_i(\vec{x}, |\vec{p}|) \xRightarrow[\text{isotropy}]{\text{homogeneity}}$
species "i"
$f_i\left(E = \sqrt{|\vec{p}|^2 + m_i^2}\right) = \frac{1}{e^{(E-\mu_i)/T_i} \mp 1}$
bosons
fermions

② Equilibrium Thermodynamics

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→ T_i (temp), mean kinetic energy of species.

→ μ_i, characterizes "particle number"

② Equilibrium Thermodynamics

what is equilibrium?

$$\frac{df(1)}{dt} = 0 \quad \forall f$$

but $H = \frac{\dot{a}}{a} \neq 0$

Equilibrium is defined by $\dot{f} = 0$ on time scales $\Delta t \ll H^{-1}$.

↳ neglect the expansion

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Grand canonical ensemble.

$$f_i(\cancel{\vec{x}}, |\vec{p}|) \underset{\text{isotropy}}{\overset{\text{homogeneity}}{\Longrightarrow}} \boxed{f_i\left(E = \sqrt{|\vec{p}|^2 + m_i^2}\right) = \frac{1}{e^{(E-\mu_i)/T_i} \mp 1}}$$

species "i"

$-$ bosons

$+$ fermions

→ T_i (temp), mean kinetic energy of species

→ μ_i, characterizes "particle number"

↳ $n_i - n_{\bar{i}} \sim \mu$

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \sim \frac{\mu_B}{T} \sim 10^{-9}$$

② Equilibrium Thermodynamics

What is equilibrium?

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Grand canonical ensemble.

$$f_i(\cancel{\vec{x}}, |\vec{p}|) \xrightarrow[\text{isotropy}]{\text{homogeneity}} \boxed{f_i\left(E = \sqrt{|\vec{p}|^2 + m_i^2}\right) = \frac{1}{e^{(E-\mu_i)/T_i} \mp 1}}$$

species "i"

bosons ($-$)

fermions ($+$)

→ T_i (temp), mean kinetic energy of species.

→ μ_i, characterizes "particle number"

↳ $n_i - n_{\bar{i}} \sim \mu$

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \sim \frac{\mu_B}{T} \sim 10^{-9} \Longrightarrow \boxed{\mu \simeq 0}$$

Compute # particles &
energy in each species.

$$\boxed{\begin{aligned} n_i(T) &= g_i \int \frac{d^3p}{(2\pi)^3} f_i(|\vec{p}|) \\[2ex] \rho_i(T) &= g_i \int \frac{d^3p}{(2\pi)^3} E(|\vec{p}|) f_i(|\vec{p}|) \end{aligned}}$$

Relativistic limit

"radiation"

$$n_i = \begin{Bmatrix} 1 \\ 3/4 \end{Bmatrix} g_i \frac{\zeta(3)}{\pi^2} T_i^3$$

$$\rho_i = \begin{Bmatrix} 1 \\ 7/8 \end{Bmatrix} g_i \frac{\pi^2}{30} T_i^4$$

$$T_i \gg m_i$$

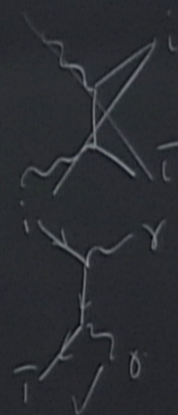
Non-relativistic limit

$$n_i = g_i \left(\frac{m_i T_i}{2\pi}\right)^{3/2} e^{-m_i/T_i}$$

↑ Boltzmann factor

$$\rho_i = g_i m_i \left(\frac{m_i T_i}{2\pi}\right)^{3/2} e^{-m_i/T}.$$

$$T_i \ll m_i$$



→ blueshift

B/c the universe is rad-dominated

$$\boxed{\rho_{rad} \equiv \frac{\pi^2}{30} g_* T_\gamma^4}$$

→ T_γ standard ruler.

$$g_* \equiv \sum_{bosons} g_i \left(\frac{T_i}{T_\gamma}\right)^4 \Theta(T_i - m_i)$$

$$+ \sum_{fermions} g_i \left(\frac{T_i}{T_\gamma}\right)^4 \frac{7}{8} \Theta(T_i - m_i)$$

g_* = effective # of relativistic species

③ Thermo in Expanding Bkd

$T \sim a^{-n}$?

Use entropy to determine this.

Second law of thermo:

$$dU = TdS - pdV$$

$$V = a^3$$

$$dU = d(\rho V) = TdS - pdV$$

$$S(T,V)$$

Integrability condition:

$$\frac{\partial^2 S}{\partial T_V \partial V_T} = \frac{\partial^2 S}{\partial V_T \partial T_V}$$

$$dS = d\Big[\underbrace{\left(\frac{\rho+p}{T}\right)V + \text{const}}_{S}\Big]$$

$$\Big\downarrow \text{continuity eqn}$$

$$\boxed{dS = 0}$$

$$S = \frac{(p+\rho)a^3}{T} \text{ is conserved}$$

density, you have.

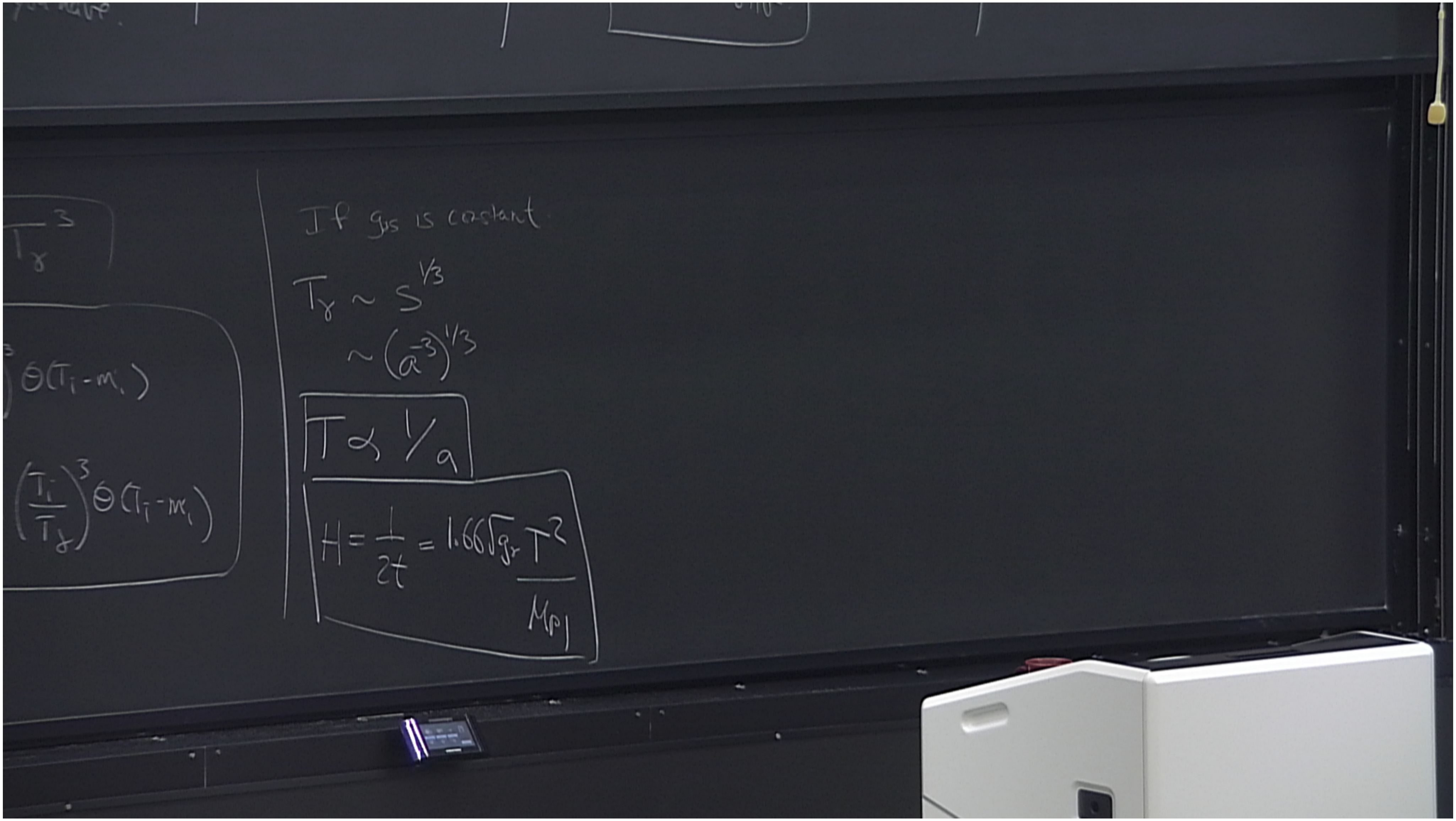
Define entropy density:

$$s \equiv \frac{S}{V} = \frac{S}{a^3} = \frac{p+\rho}{T}$$

$$\boxed{s = \frac{S}{a^3}}$$

$$\boxed{s = \frac{2\pi^2}{45} g_{*S} T_\gamma^3}$$

$$\boxed{g_{*S} = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T_\gamma}\right)^3 \Theta(T_i - m_i) + \sum_{\text{fermion}} \frac{7}{8} g_i \left(\frac{T_i}{T_\gamma}\right)^3 \Theta(T_i - m_i)}$$

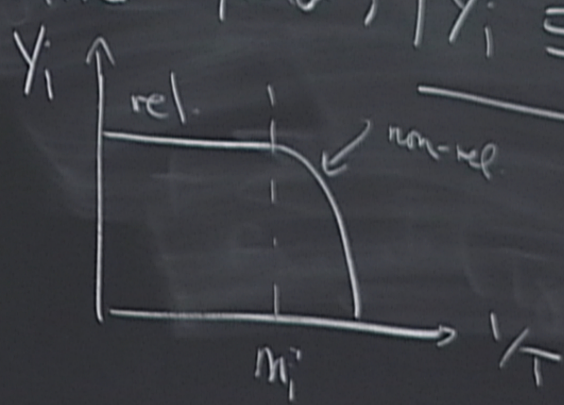

If g_{*s} is constant
$T_\gamma \sim s^{1/3}$
$\sim (a^{-3})^{1/3}$
$T \propto 1/a$
$H = \frac{1}{2t} = 1.66\sqrt{g_*}\,\frac{T^2}{M_{Pl}}$
$\left(\frac{T_i}{T_\gamma}\right)^3 \Theta(T_i - m_i)$

s has well-defined scaling, a^{-3}.

n_i changes from

- dilution (expansion)
- $e^{-m_i/T}$, ...

Define "yield", $\boxed{Y_i \equiv \frac{n_i}{s}}$



$\boxed{s_\gamma(t_0) = 2\times 10^3\ \mathrm{cm}^{-3}}$