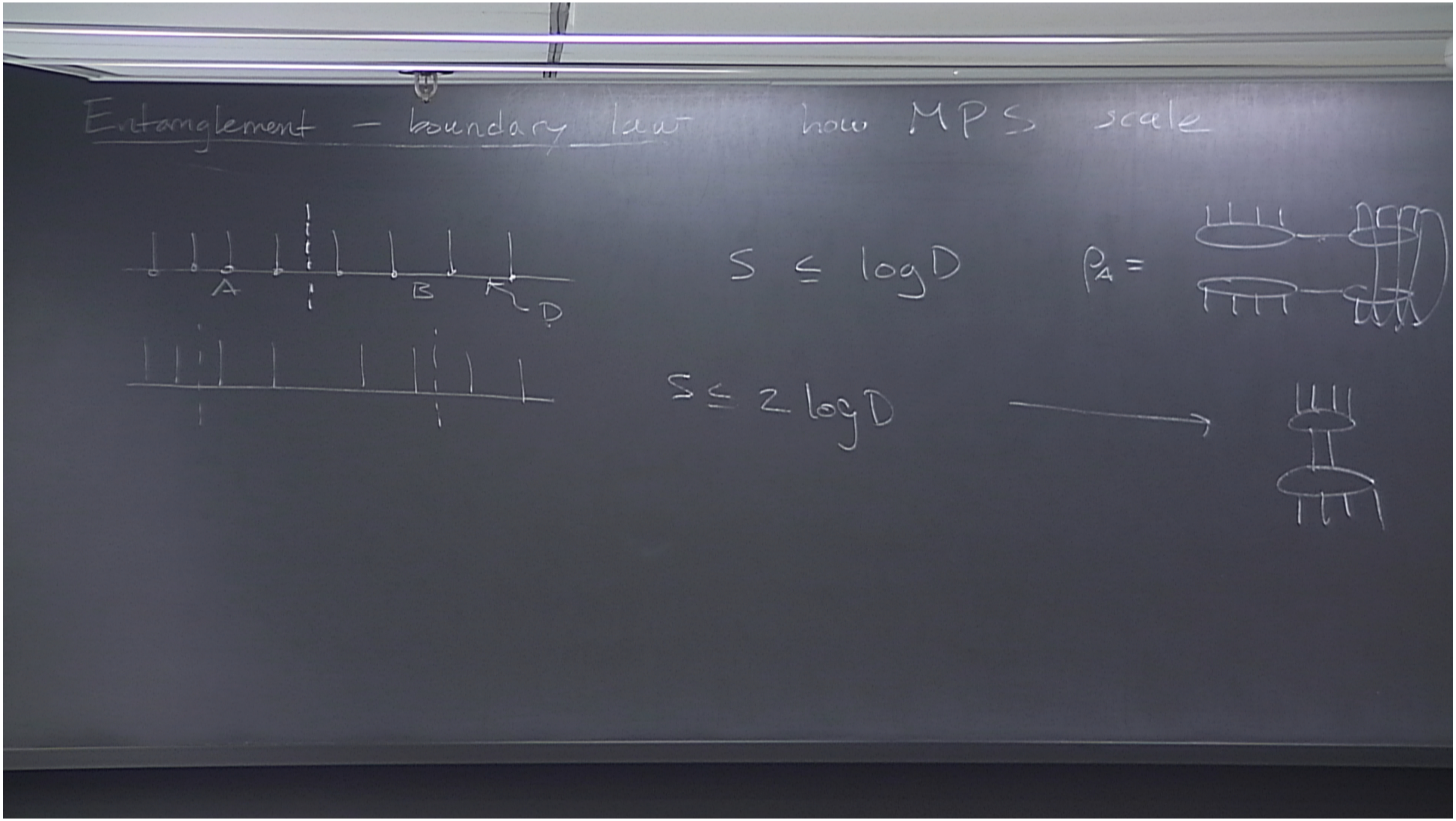
Title: Explorations in Condensed Matter-14

Date: Apr 02, 2015 10:15 AM

URL: http://pirsa.org/15040003

Abstract:

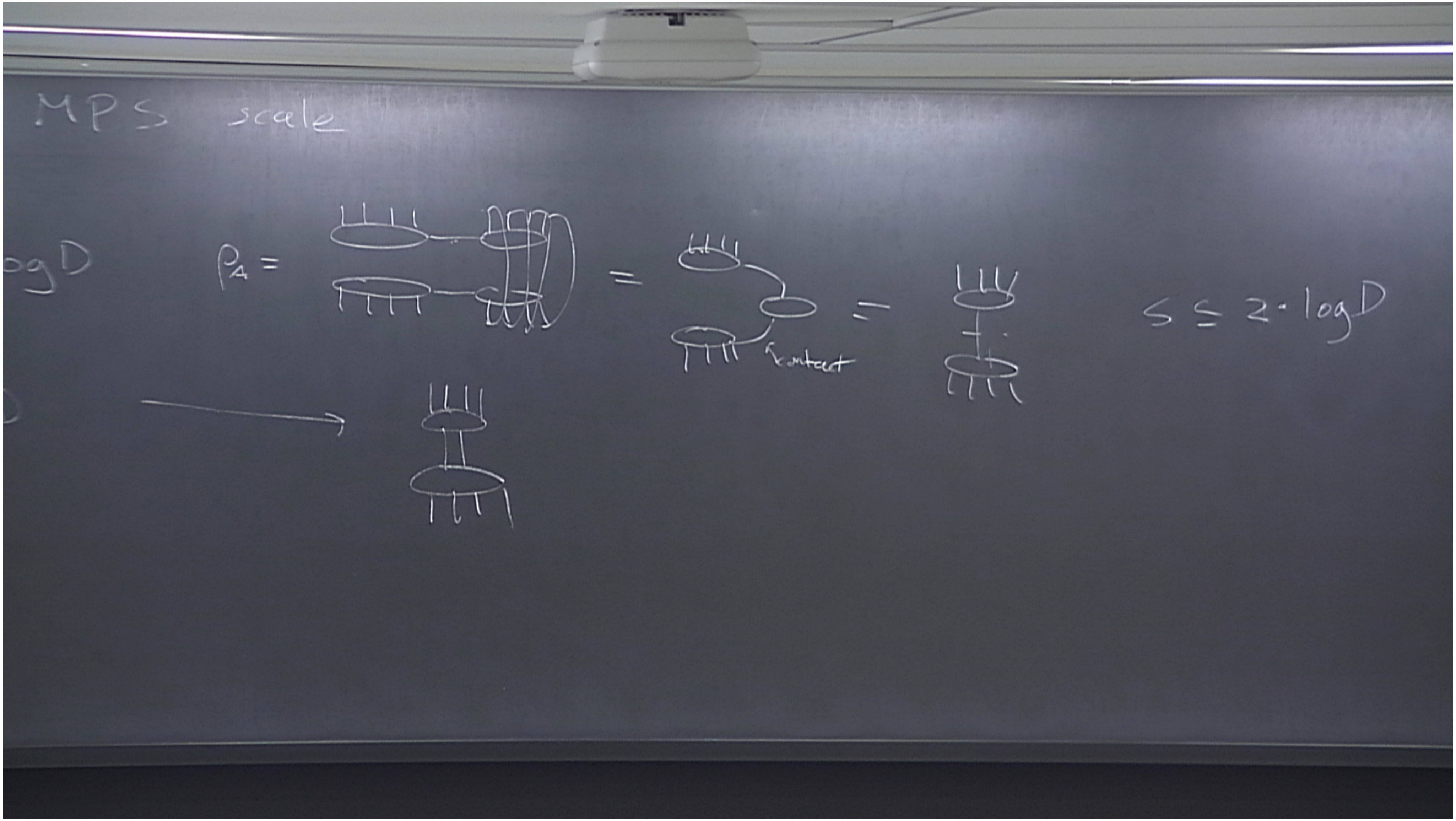


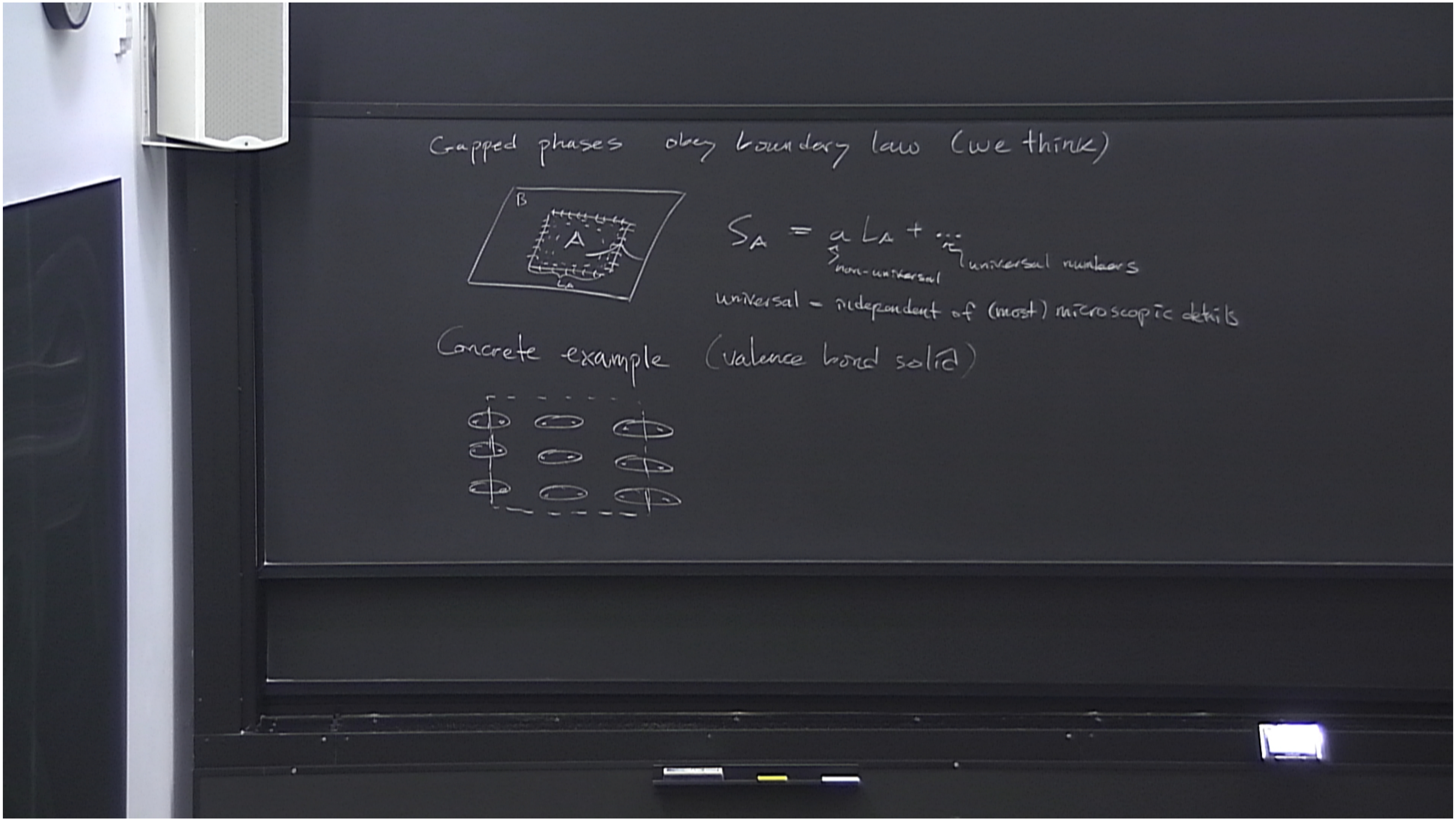
Entanglement - boundary law   how MPS scale

$S \le \log D$

$\rho_A =$

$S \le 2 \log D$


MPS scale
ogD
$\rho_A =$
=
=
contract
$S \le 2 \cdot \log D$



Gapped phases obey boundary law (we think)

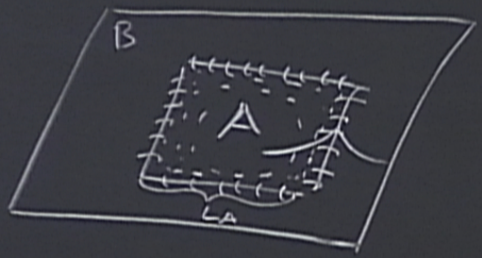
$$S_A = \underbrace{a}_{\text{non-universal}} L_A + \underbrace{\cdots}_{\text{universal numbers}}$$

universal = independent of (most) microscopic details

Concrete example (valence bond solid)


Gapped phases obey boundary law (area theorem)
Research into extensions of MPS ideas → 2d
tensor product states / projected entangled pair states (PEPS)
Concrete example (valence bond solid)

Gapped phases obey boundary law (we think)



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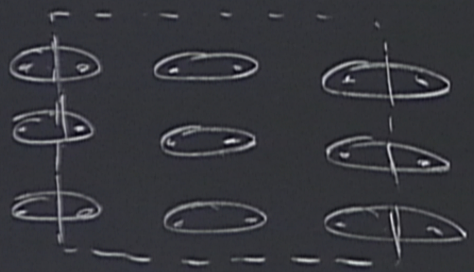
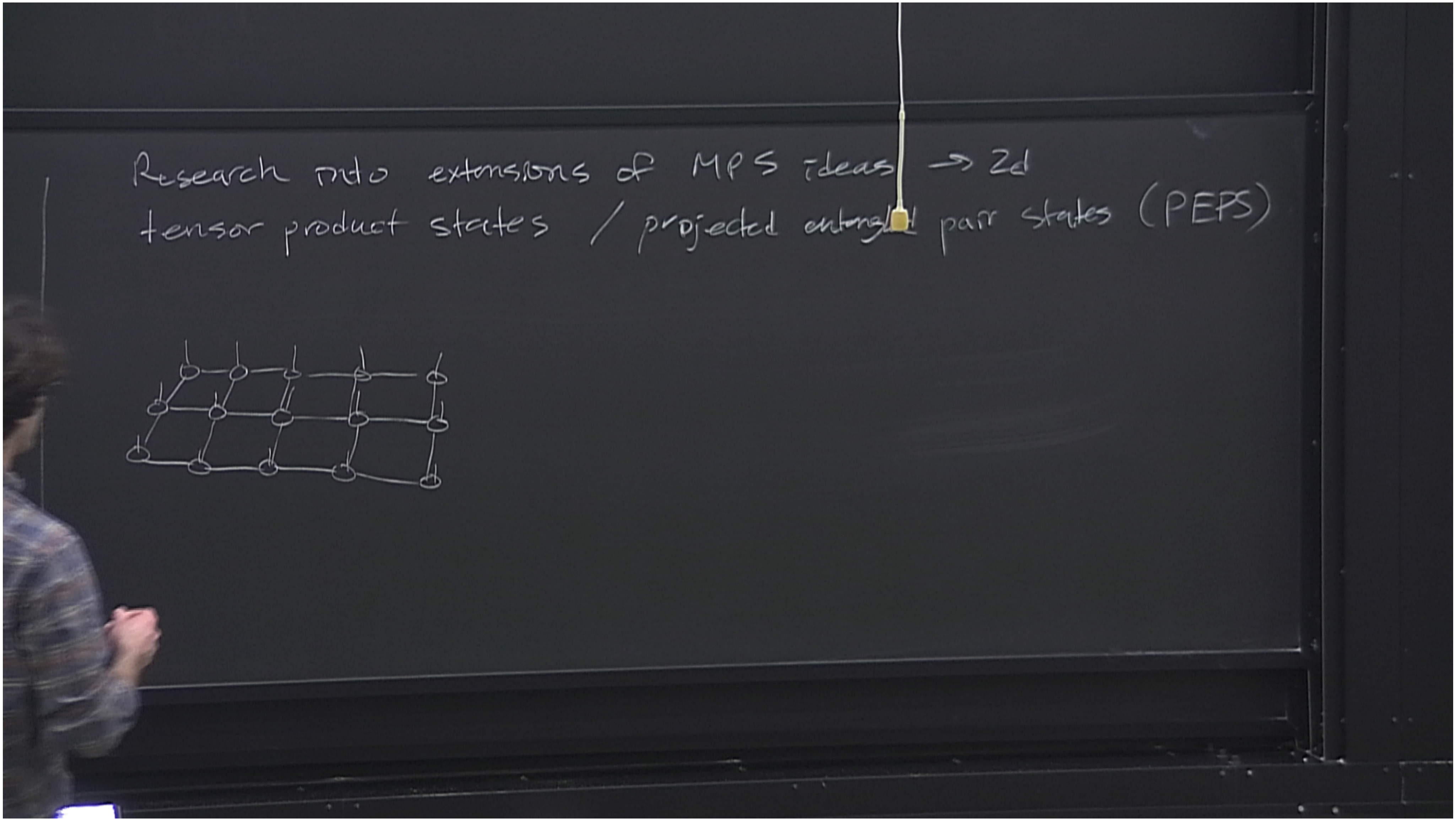
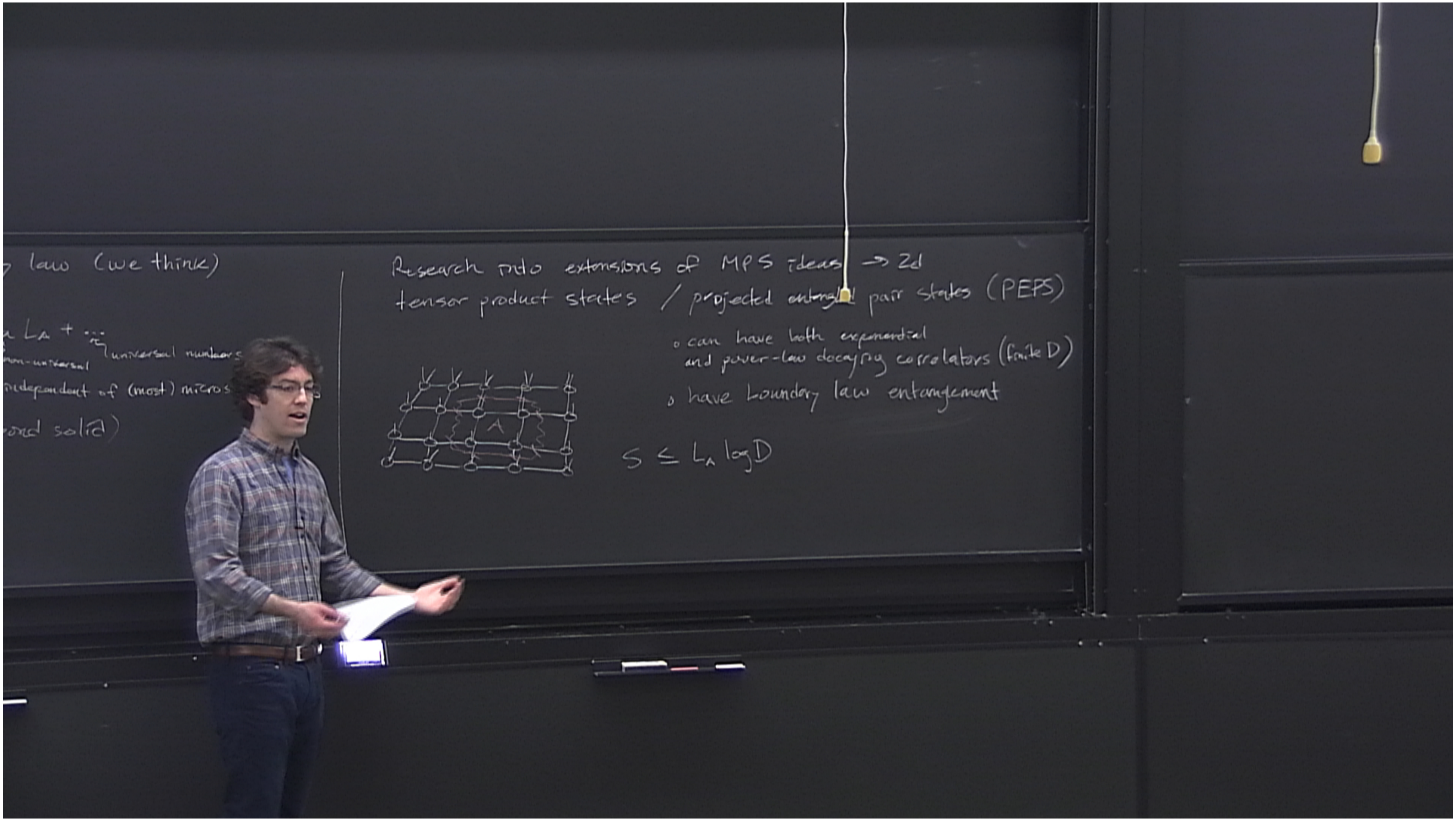
Concrete example (valence bond solid)





Research into extensions of MPS ideas → 2d

tensor product states / projected entangled pair states (PEPS)



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$L_A + \dots$

non-universal

universal numbers

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Research into extensions of MPS ideas → 2d

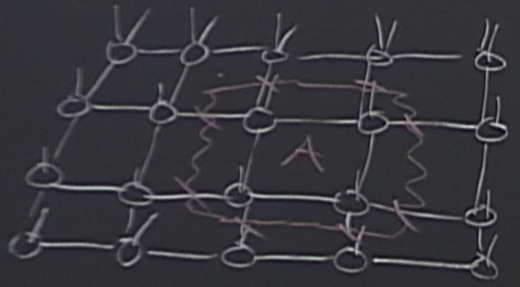
tensor product states / projected entangled pair states (PEPS)

- can have both exponential and power-law decaying correlators (finite D)
- have boundary law entanglement

$S \leq L_A \log D$

Research into extensions of MPS ideas → 2d

tensor product states / projected entangled pair states (PEPS)



- can have both exponential and power-law decaying correlators (finite D
- have boundary law entanglement
- not efficient to contract exactly

$S \leq L_A \log D$ (recall MPS D^3)

Research into extensions of MPS ideas → 2d

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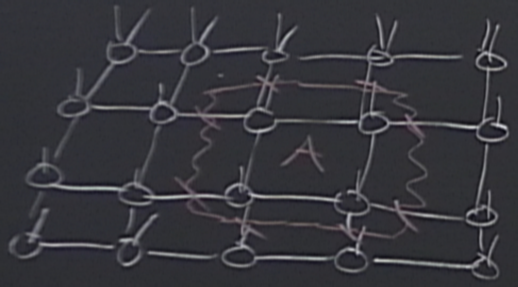

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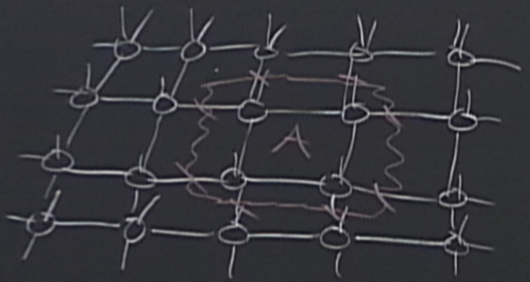
<u>Research</u> into extensions of MPS ideas → 2d

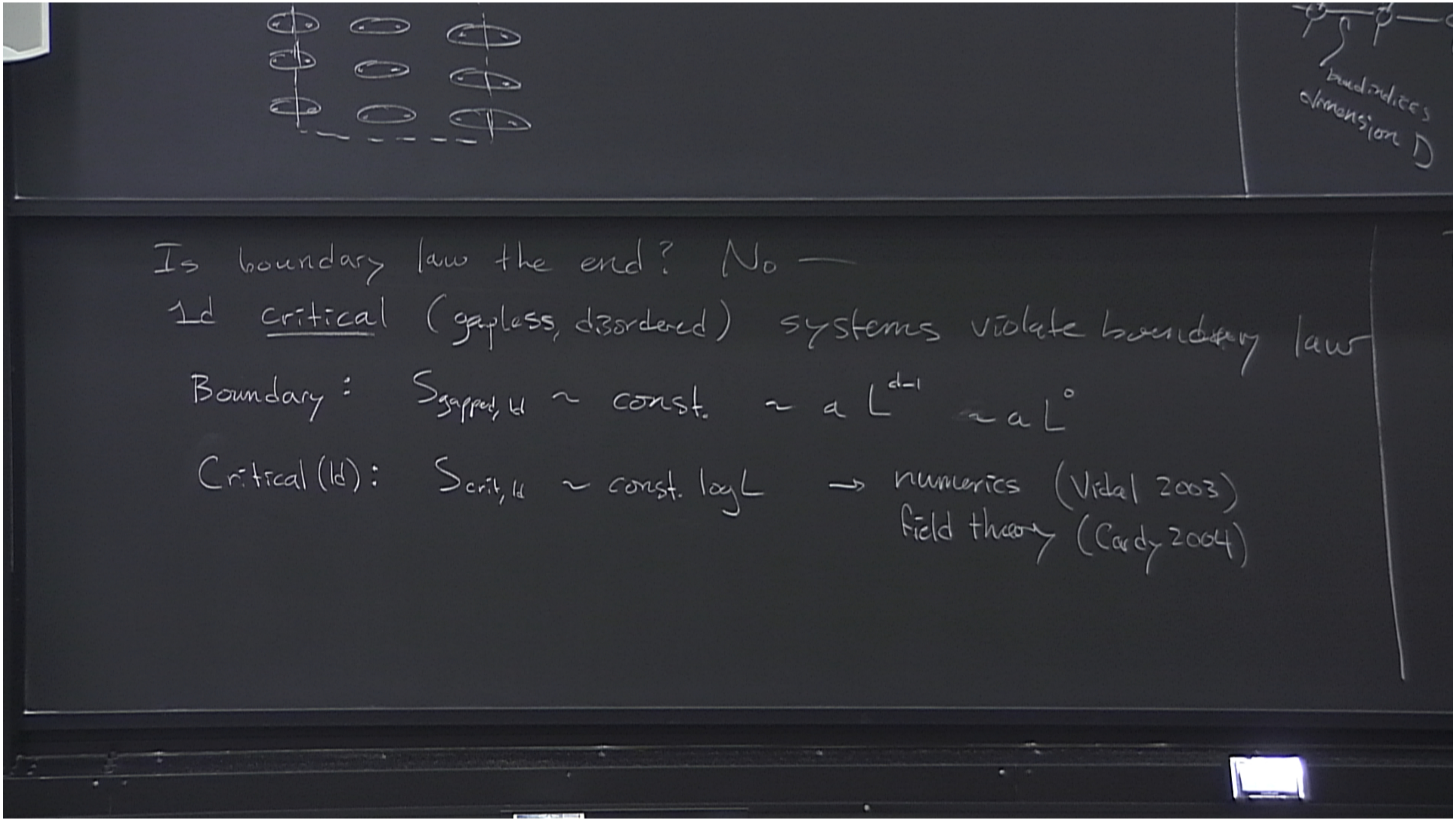
tensor product states / projected entangled pair states (PEPS)



- ∘ can have both exponential and power-law decaying correlators (finite D)
- ∘ have boundary law entanglement
- ∘ <u>not</u> efficient to contract exactly

$\boxed{S \leq P_A \log D}$ (recall MPS $\underline{\underline{\underline{D^3}}}$)

P_A = perimeter of A

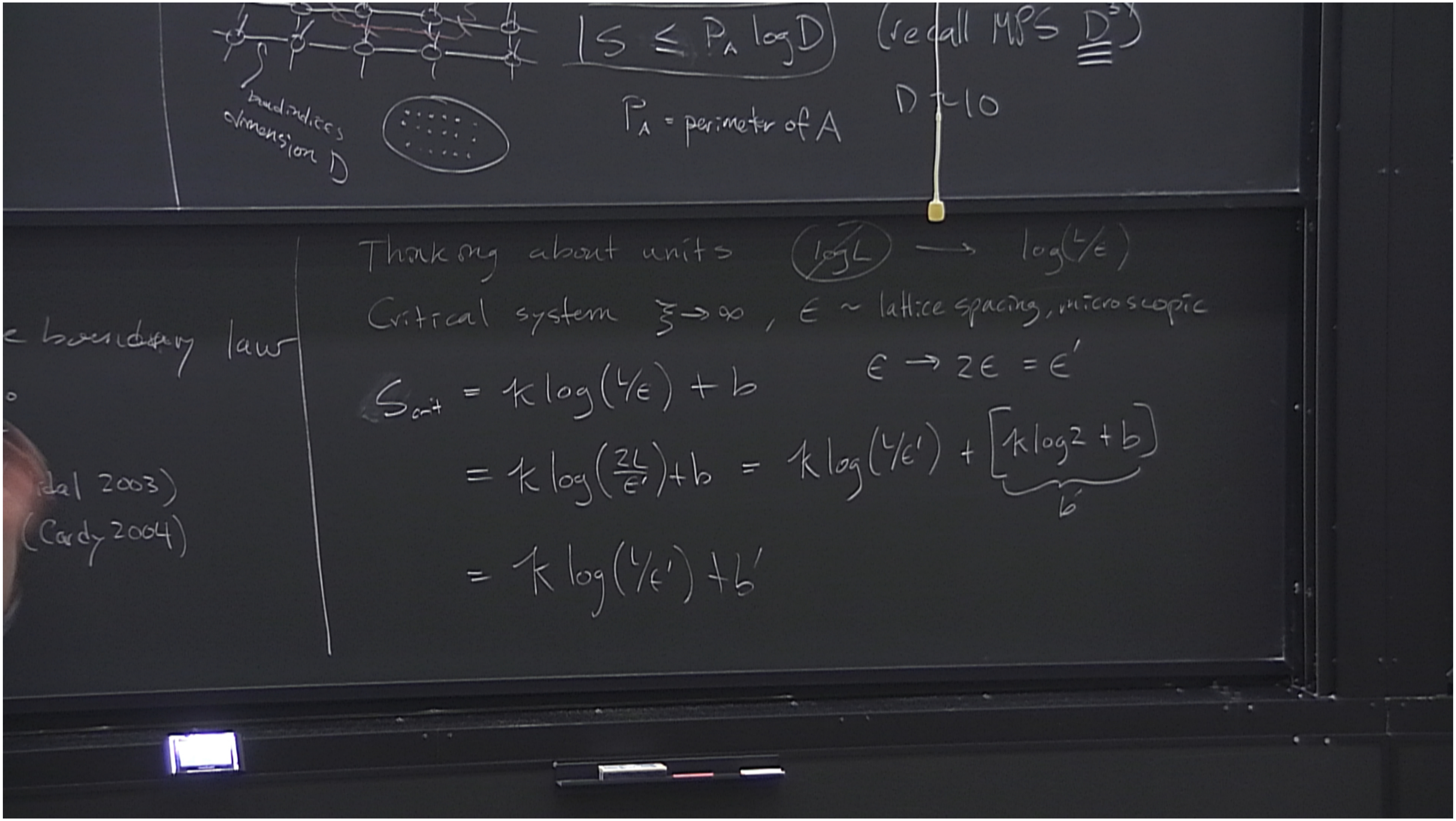


Is boundary law the end? No —

1d critical (gapless, disordered) systems violate boundary law

Boundary: $S_{\text{gapped, 1d}} \sim \text{const.} \sim a L^{d-1} \sim a L^{0}$

Critical (1d): $S_{\text{crit, 1d}} \sim \text{const.} \log L$ → numerics (Vidal 2003)
field theory (Cardy 2004)



$$\boxed{S \leq P_A \log D}$$ (recall MPS $\underline{D}^{3}$)

P_A = perimeter of A $D \sim 10$

e boundary law

o

idal 2003)

(Cardy 2004)

Thinking about units $\cancel{\log L} \longrightarrow \log(L/\epsilon)$

Critical system $\xi \to \infty$, ϵ ~ lattice spacing, microscopic

$$\epsilon \to 2\epsilon = \epsilon'$$

$$S_{crit} = k \log(L/\epsilon) + b$$

$$= k \log\left(\frac{2L}{\epsilon'}\right) + b = k \log(L/\epsilon') + \underbrace{\left[k \log 2 + b\right]}_{b'}$$

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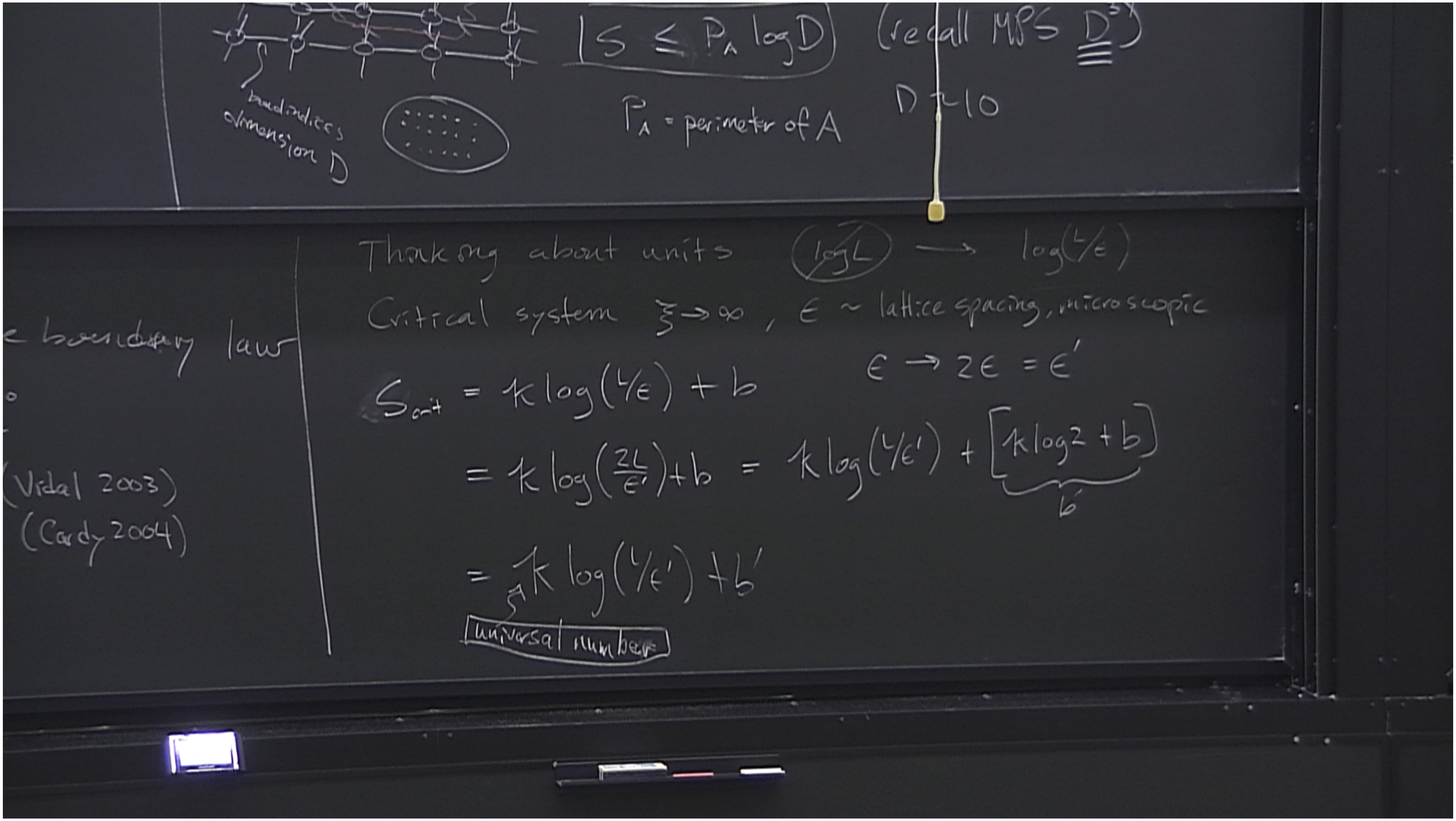

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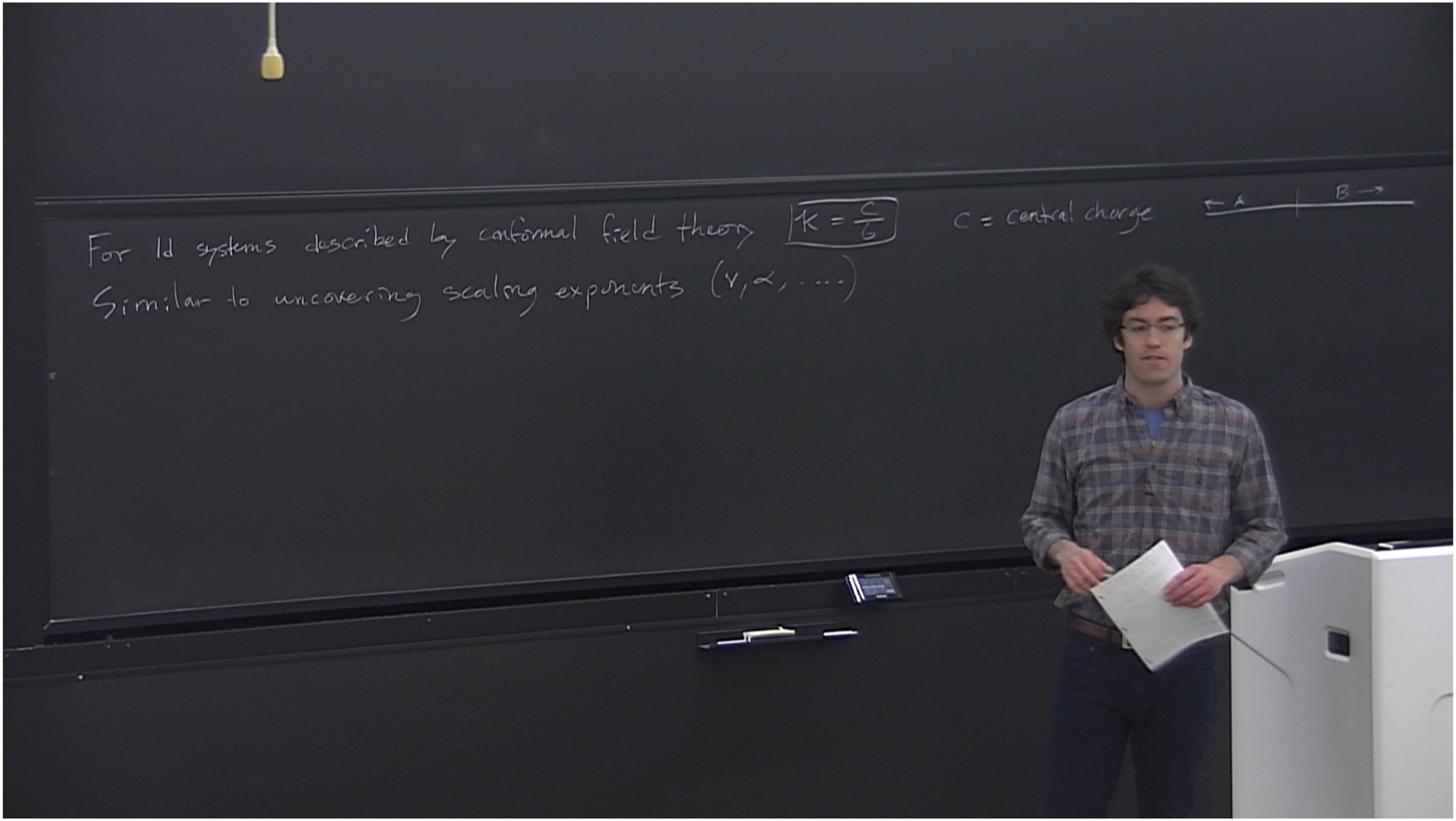
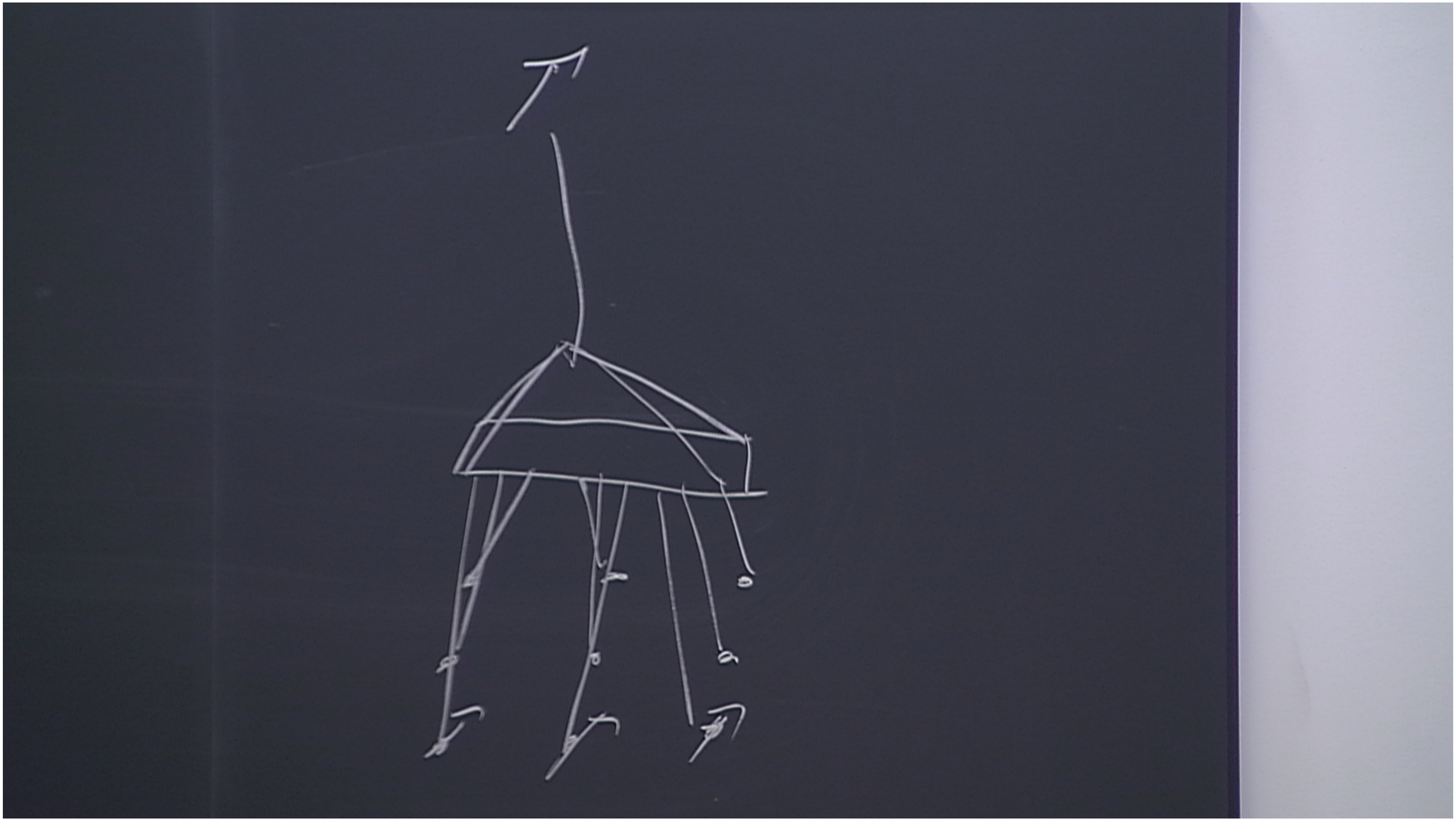
universal number



For 1d systems described by conformal field theory $\boxed{k = \frac{c}{6}}$ c = central charge

Similar to uncovering scaling exponents $(\nu, \alpha, \ldots)$



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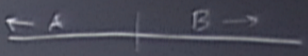
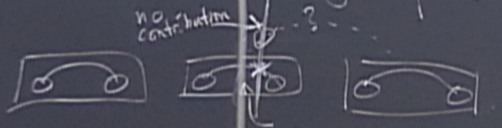


Similar to uncovering scaling exponents ($\nu, \alpha, \ldots$)

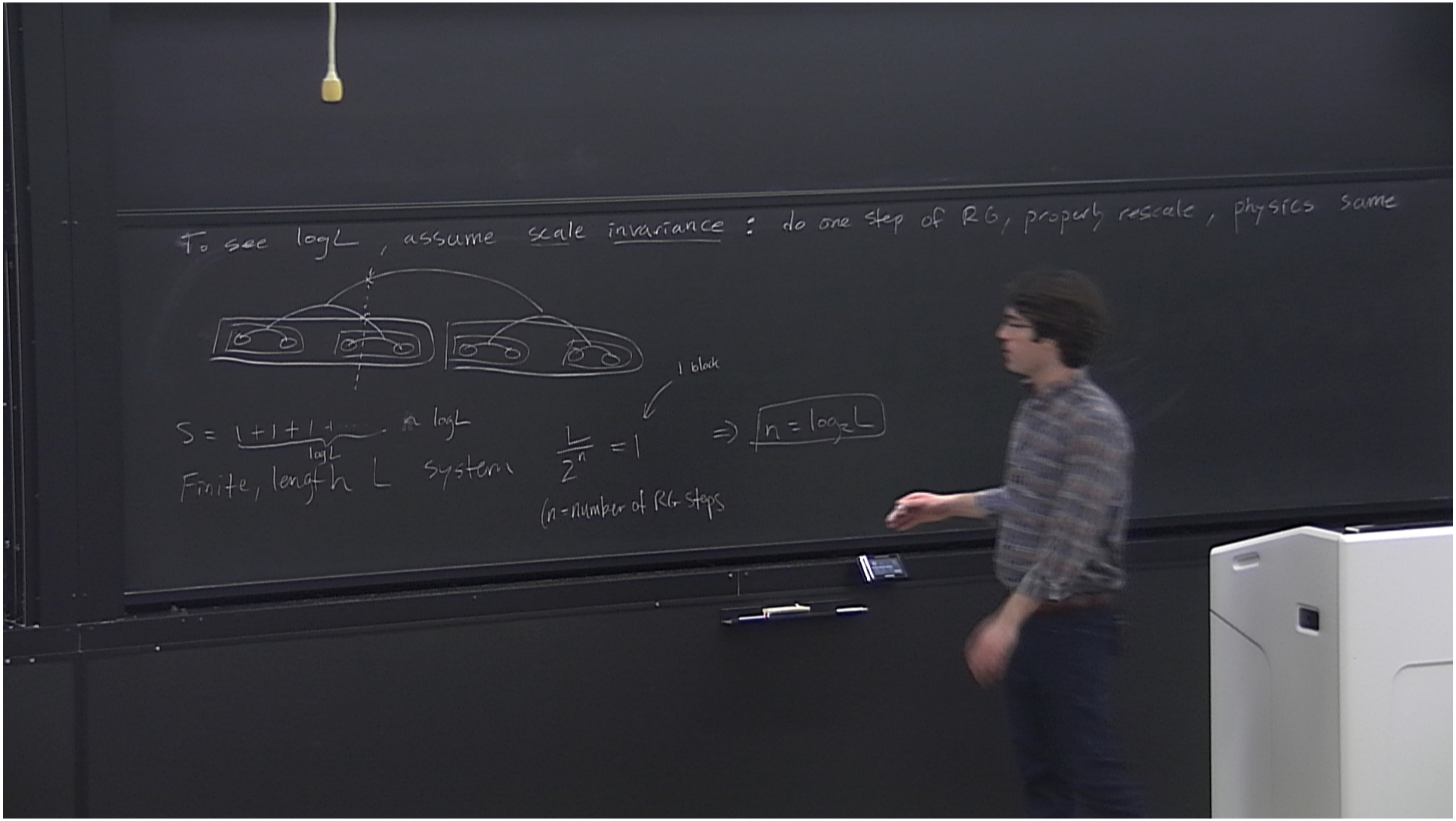
Why log L for critical 1d?

renormalization group thinking (block spin, real space RG)



$\longrightarrow$ $S \leq$ const. independent of global system size

contribution to entanglement at lattice scale



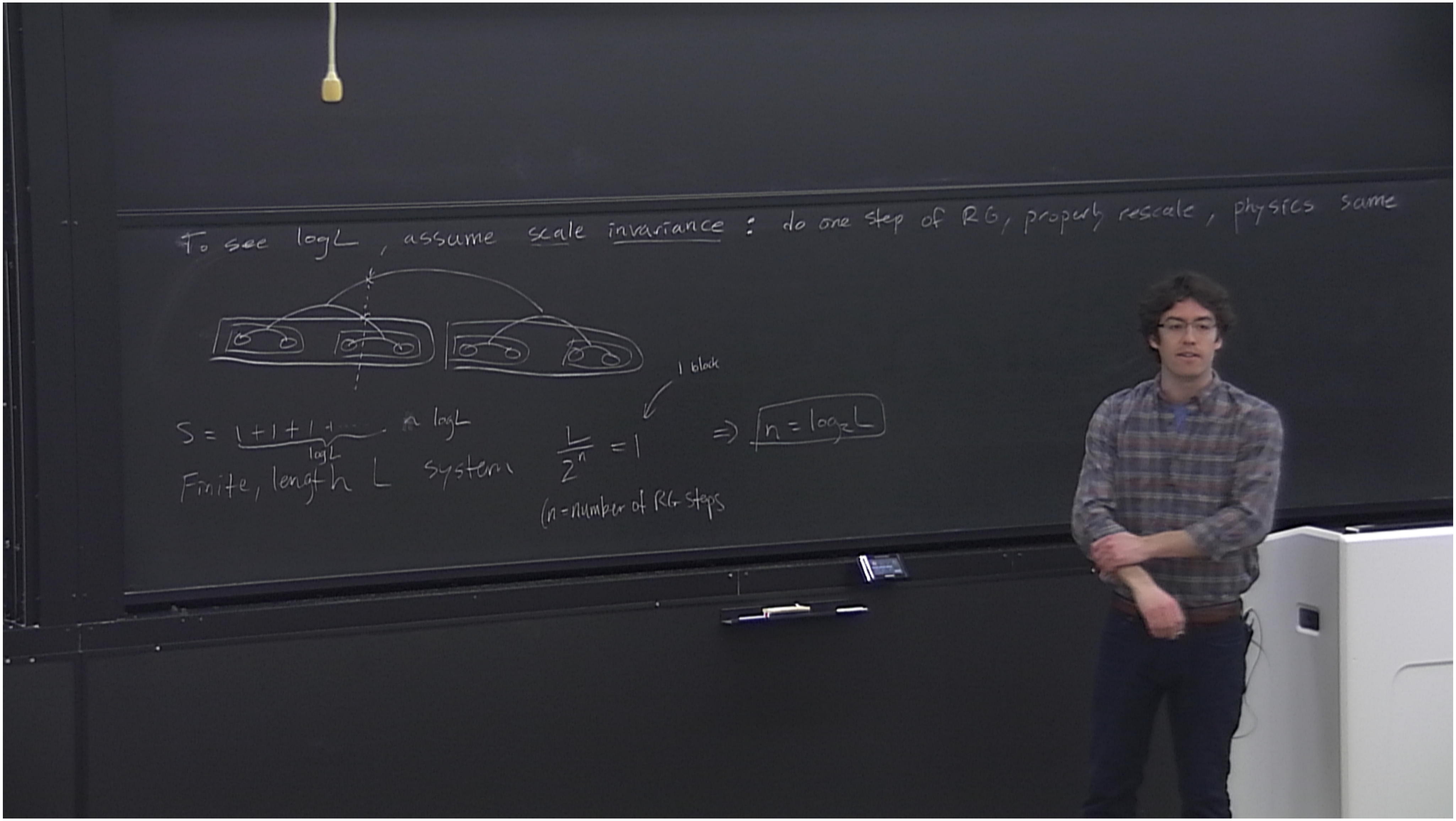
To see log L, assume scale invariance : do one step of RG, properly rescale, physics same

$S = \underbrace{1+1+1+\cdots}_{\log L} \sim \log L$

Finite, length L system

$\frac{L}{2^n} = \overset{\text{1 block}}{1} \Rightarrow \boxed{n = \log_2 L}$

(n = number of RG steps



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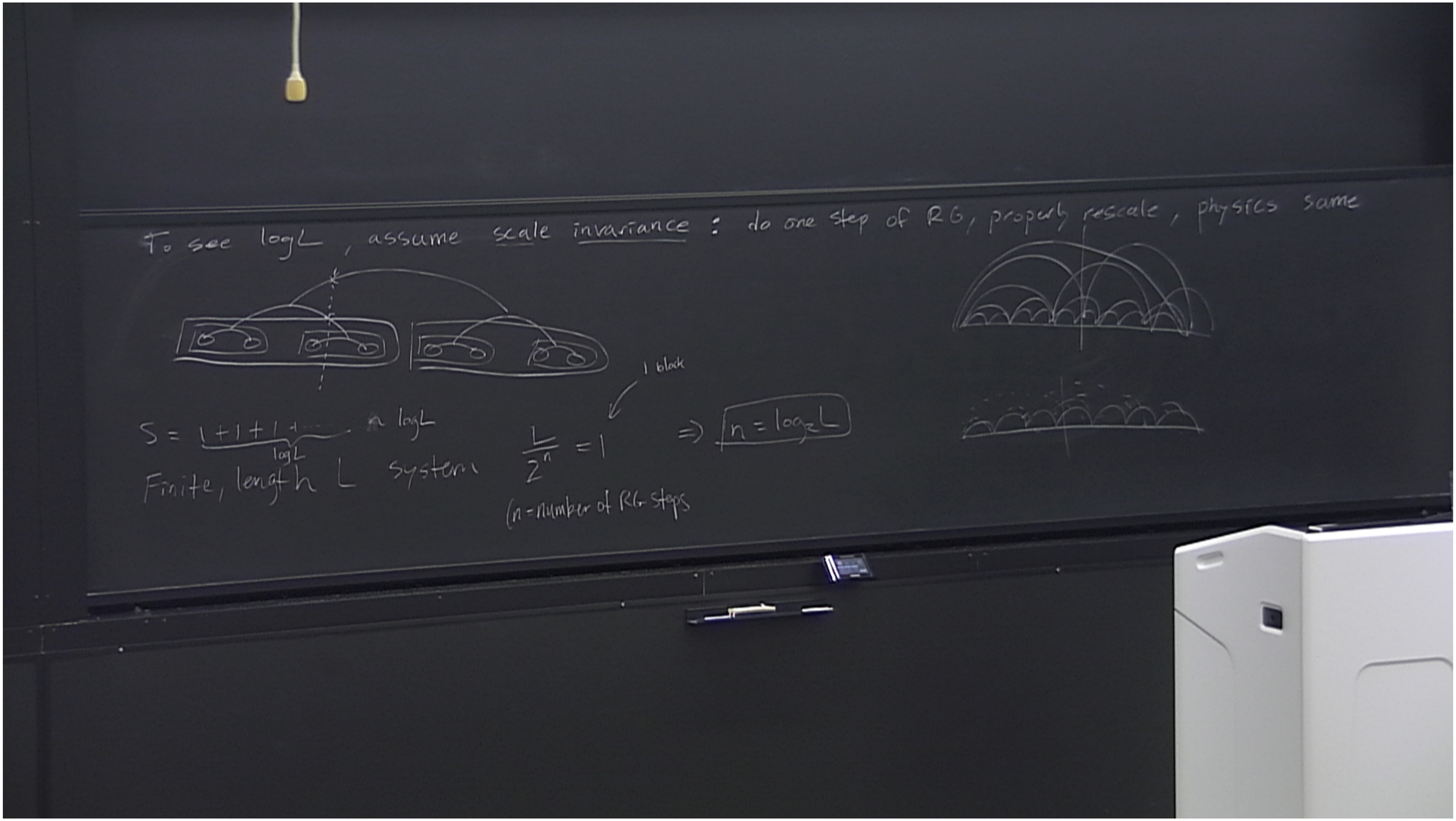

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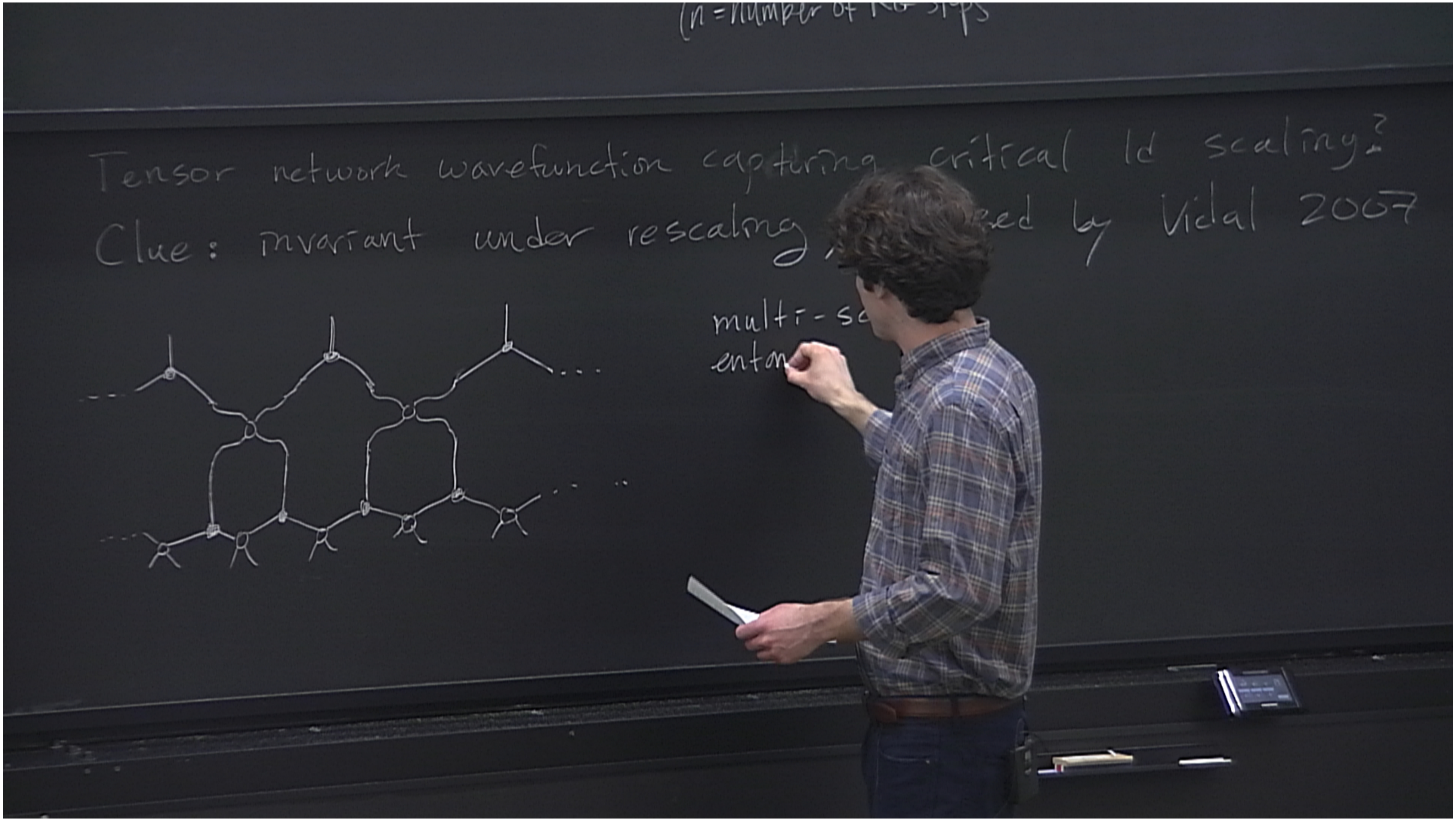

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Tensor network wavefunction capturing critical 1d scaling?

Clue: invariant under rescaling, [illegible] by Vidal 2007

multi-sc

entan

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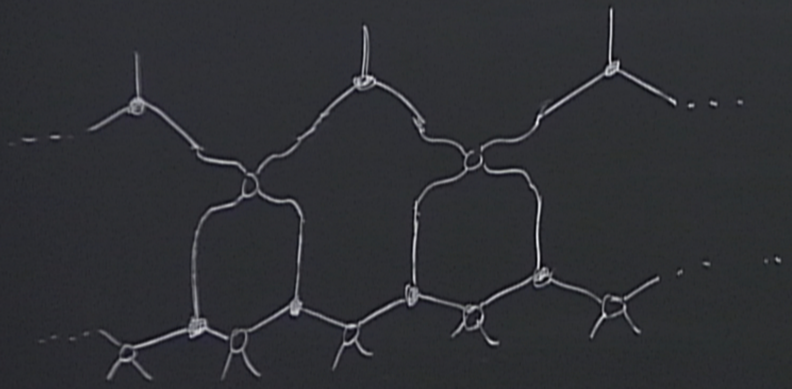
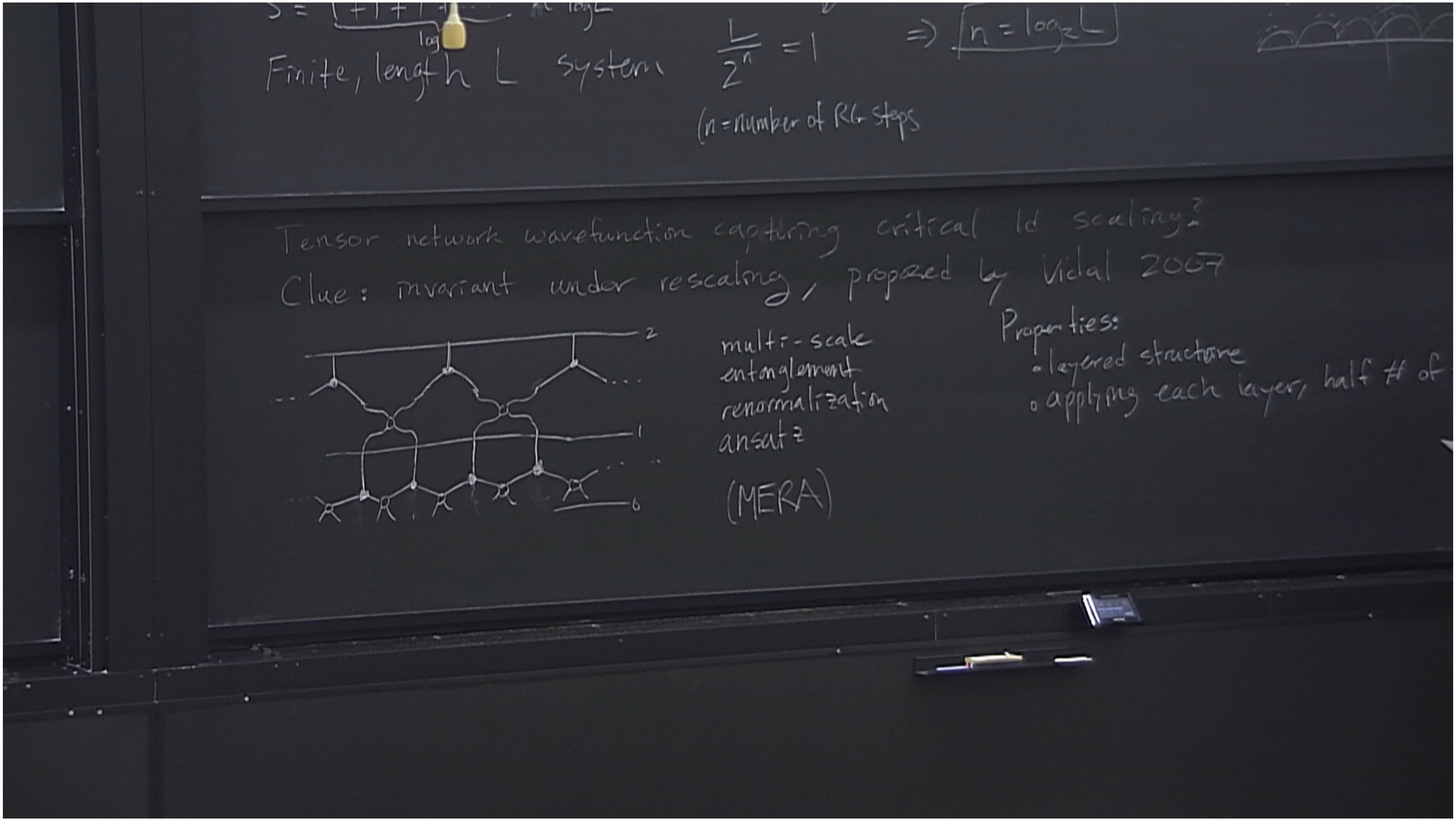


multi-scale
entanglement
renormalization
ansatz

(MERA)



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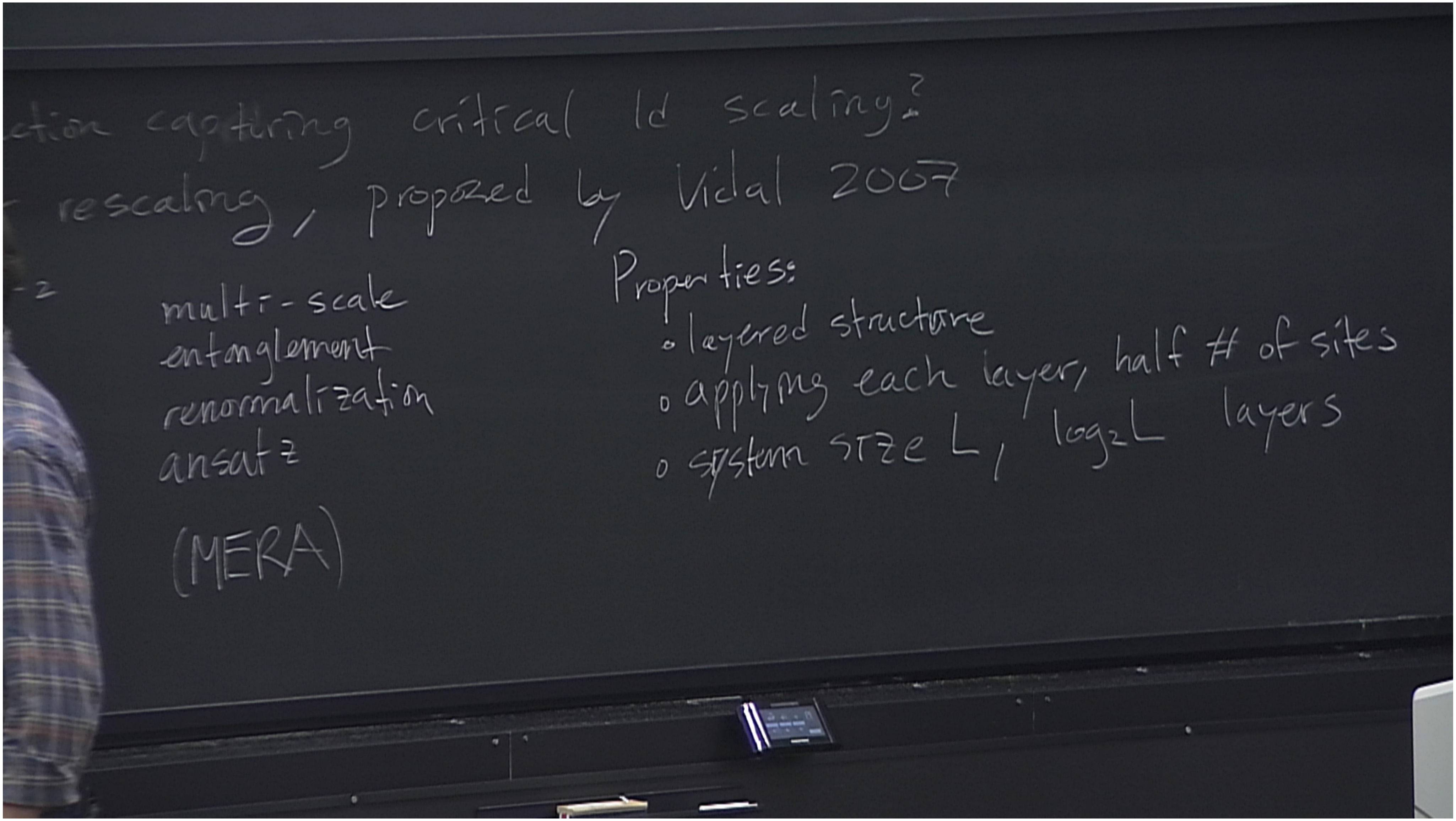
Properties:

- layered structure
- applying each layer, half # of



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rescaling, proposed by Vidal 2007

-2

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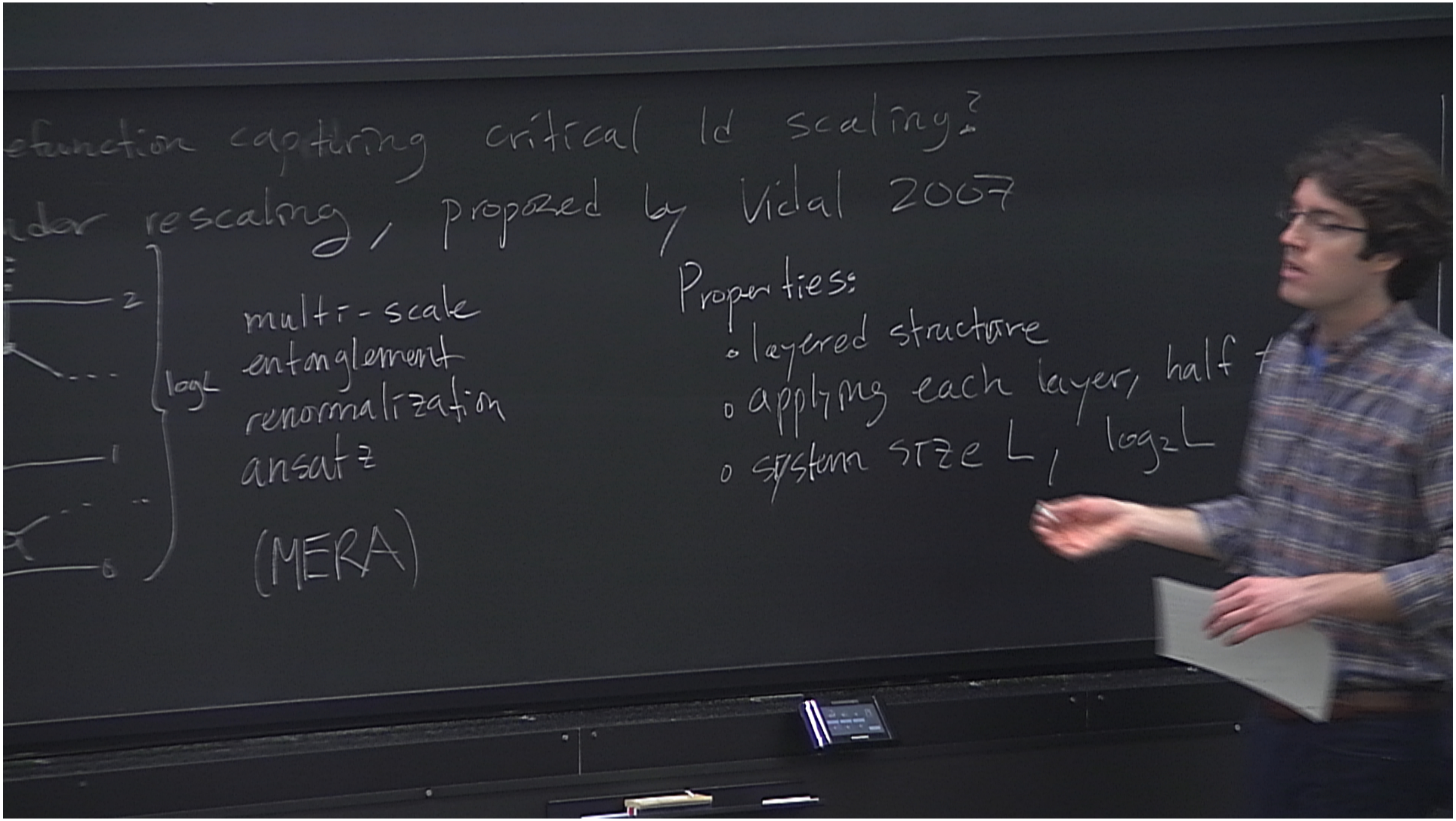
(MERA)

Properties:
- layered structure
- applying each layer, half # of sites
- system size L, $\log_2 L$ layers



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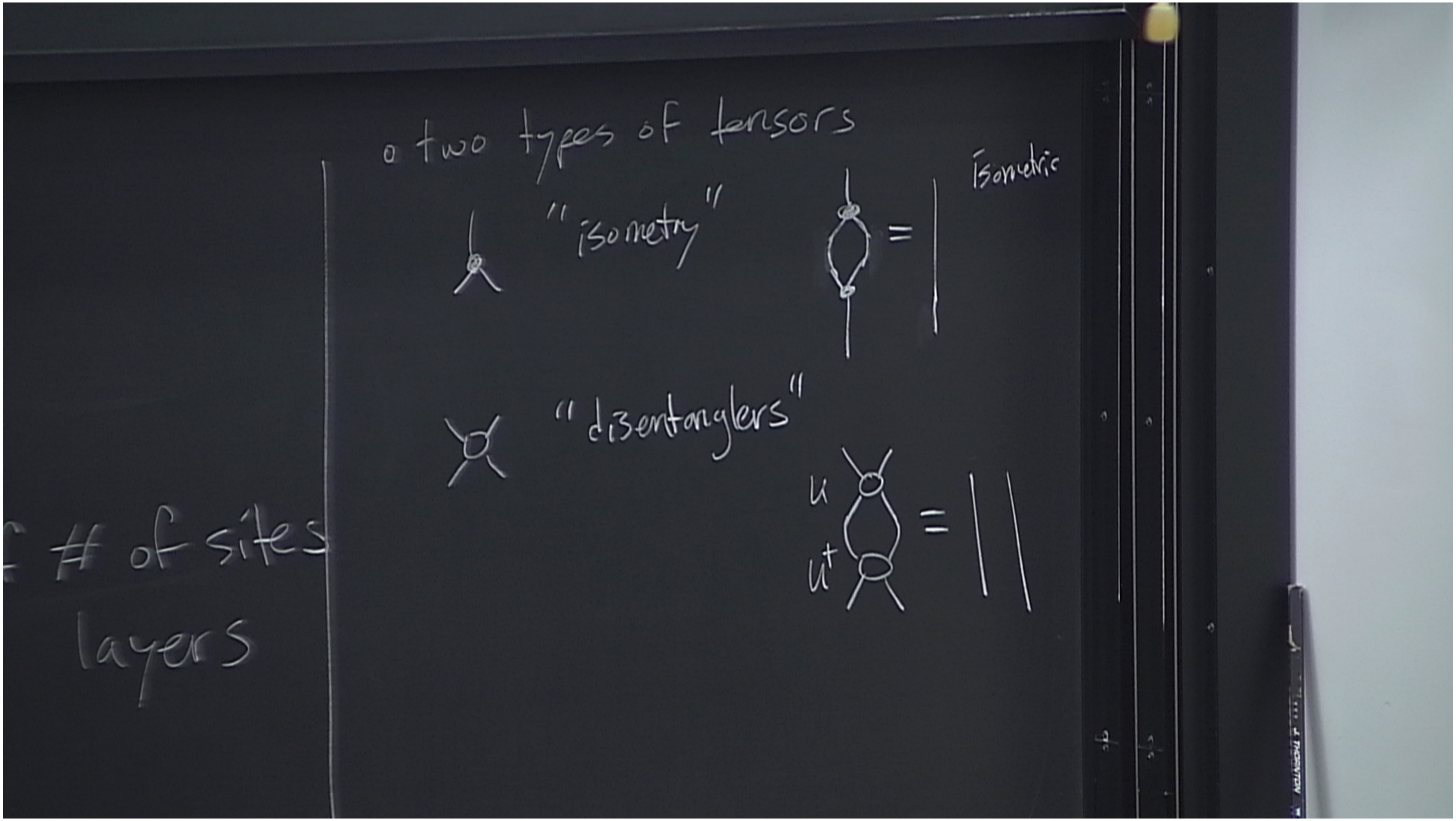

∘ two types of tensors

"isometry"

isometric

"disentanglers"

and power-law decaying correlators (finite D)

- have boundary law entanglement
- not efficient to contract exactly

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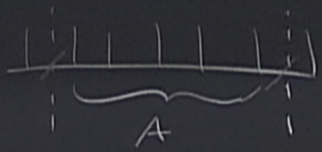
P_A = perimeter of A

$D \sim 10$



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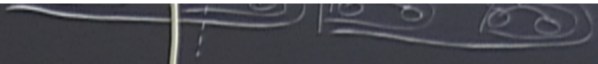

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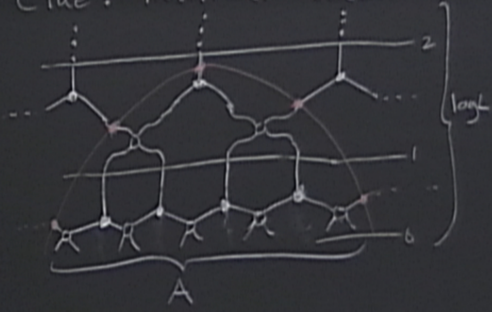

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(MERA)

Properties:

- layered st
- applying e
- system st

\# bonds cut (MERA) region A $\sim \log L_A$

$$S \leq (\log L_A)(\underbrace{\log D}_{\text{const.}}) \leq \text{const.} \log L_A$$

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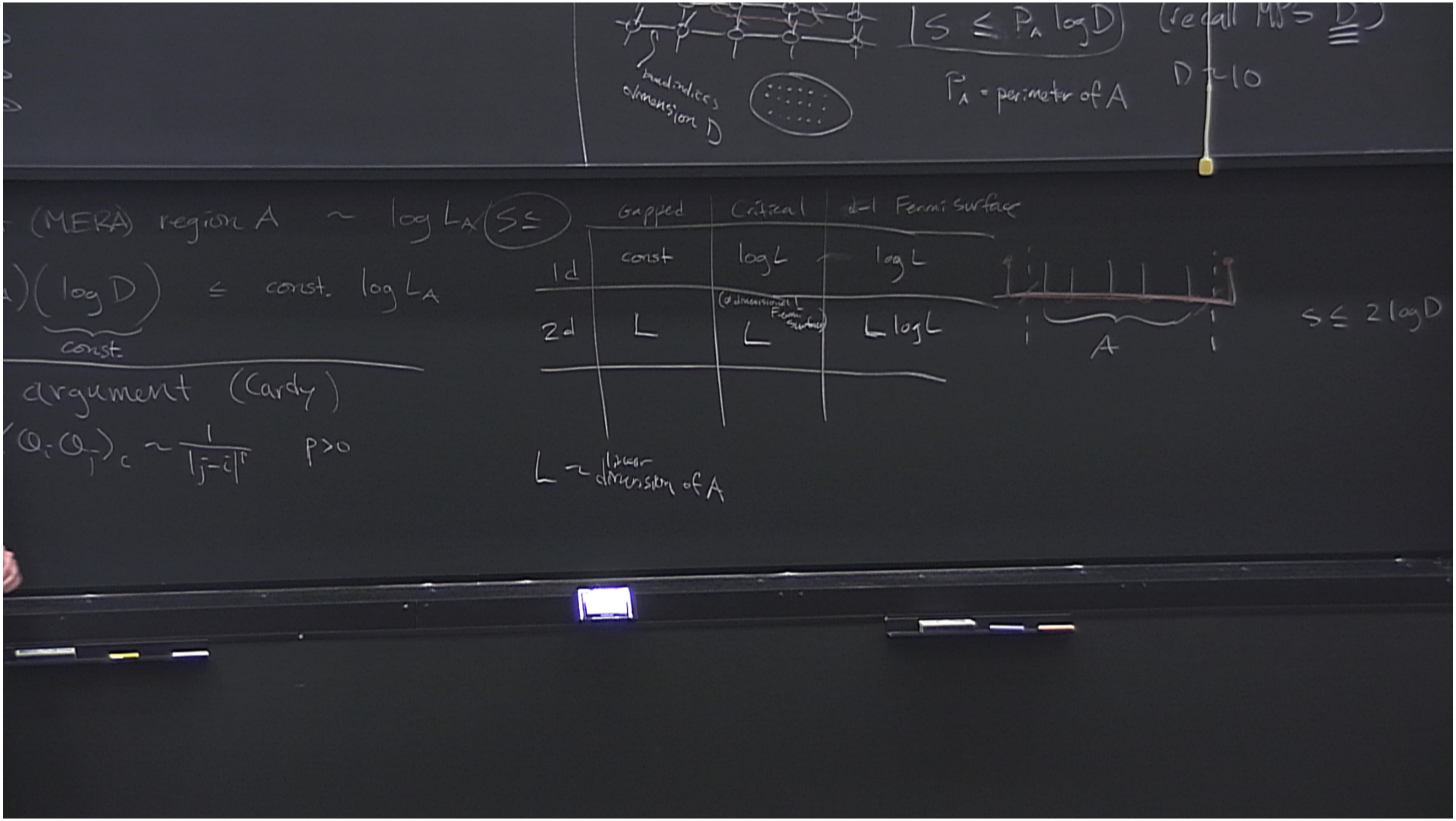
standard RG argument (Cardy)

$\rightarrow$ MERA $\langle O_i O_j \rangle_c \sim \frac{1}{|j-i|^p}$ $\quad p>0$



bond indices dimension D

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$D \sim 10$

(MERA) region A $\sim \log L_A$

$)\underbrace{(\log D)}_{\text{const.}} \le \text{const.} \log L_A$

argument (Cardy)

$\langle O_i O_j \rangle_c \sim \frac{1}{|j-i|^p}$ $p>0$

$\boxed{S \le}$	Gapped	Critical	d-1 Fermi surface
1d	const	$\log L$	$\log L$
2d	L	L (0 dimensional Fermi surface)	$L \log L$

$L \sim$ linear dimension of A

$S \le 2 \log D$