Title: Explorations in Condensed Matter-13

Date: Apr 01, 2015 10:15 AM

URL: http://pirsa.org/15040002

Abstract:

→ MPS → exponentially decaying (connected) $\langle O_i O_j \rangle - \langle O_i \rangle \langle O_j \rangle \sim e^{\cdots}$

→ MPS appropriate ground states gapped 1d systems

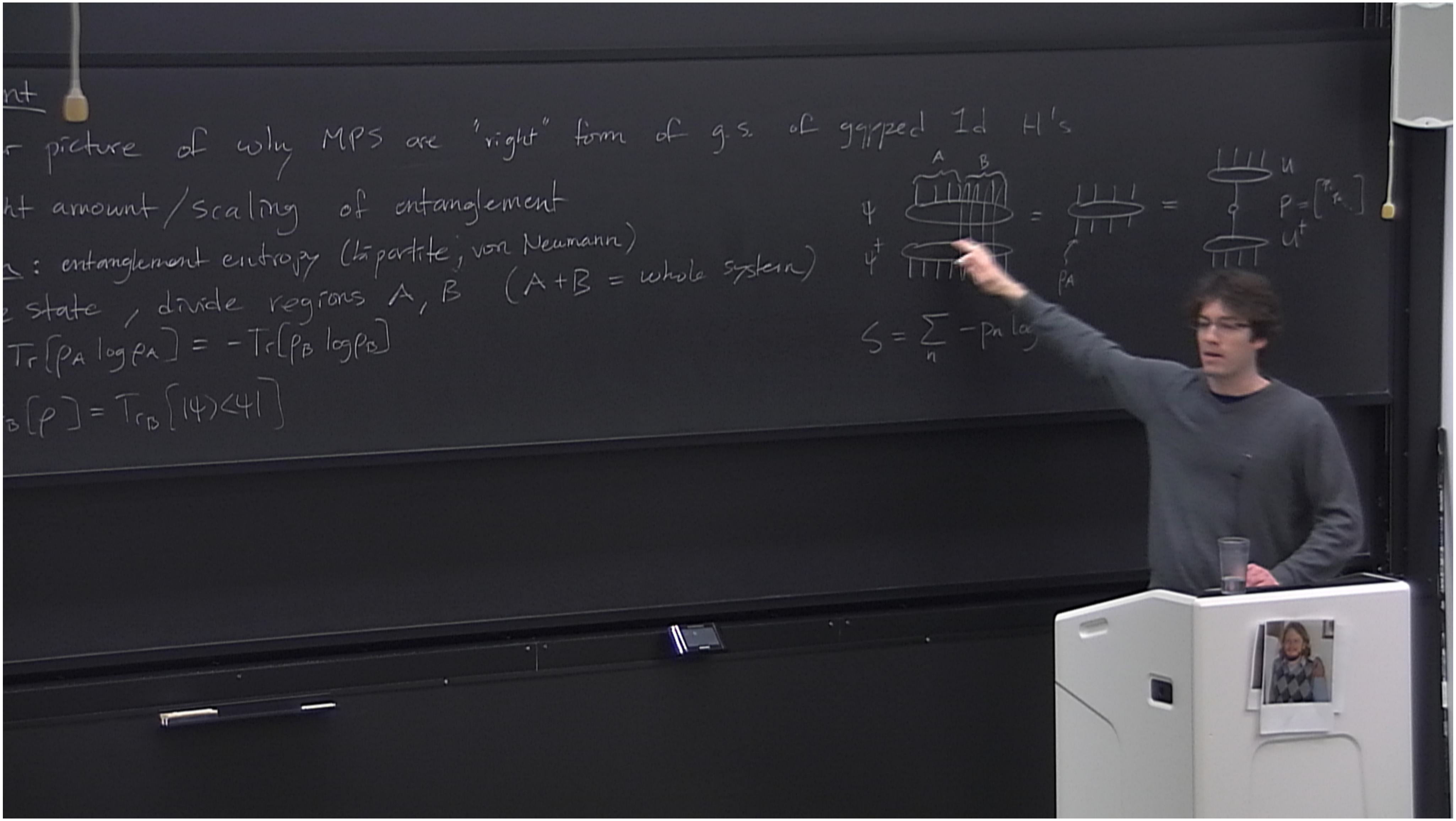
correlation length $\xi = \frac{1}{\Delta}$

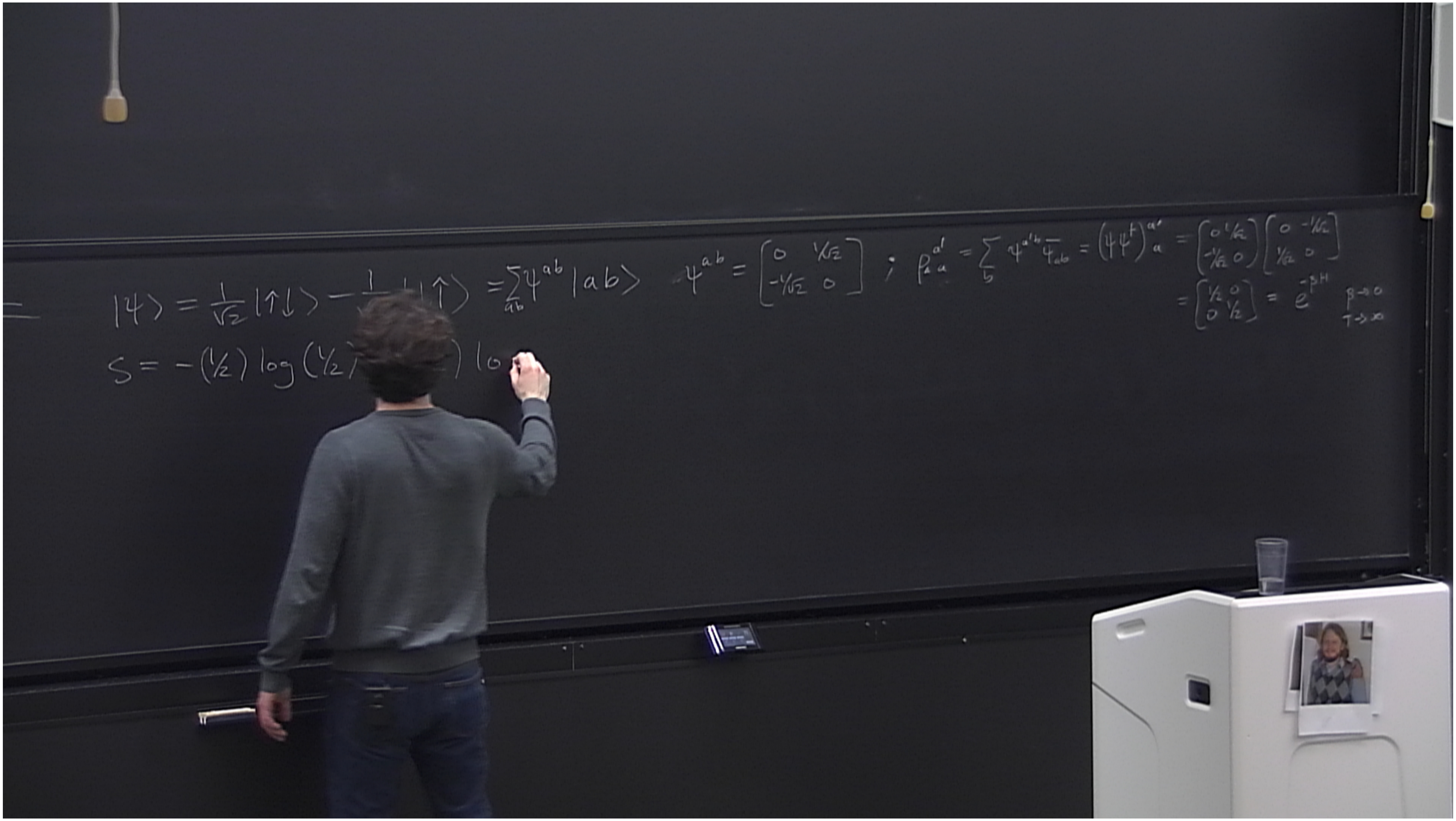
Theorem (Hastings 2007)

Local 1d Hamiltonian, finite energy gap, unique ground state Δ

$\exists$ MPS bond dimension D, $\rho > 0$ (independent of D)

such that $\left| \langle \psi_{mps} | \hat{O} | \psi_{mps} \rangle - \langle \psi | \hat{O} | \psi \rangle \right| \leq c_O D^{-\rho}$


nt
picture of why MPS are "right" form of g.s. of gapped 1d H's
ht amount / scaling of entanglement
: entanglement entropy (bipartite, von Neumann)
state, divide regions A, B (A + B = whole system)
Tr[ρA log ρA] = - Tr[ρB log ρB]
B[ρ] = TrB[|ψ⟩⟨ψ|]
A
B
ψ
ψ†
ρA
u
ρ =
u†
S = Σ -pn log


|ψ> = 1/√2 |↑↓> − 1/√2 |↓↑> = Σ_ab ψ^ab |ab>
ψ^ab = [0 1/√2; −1/√2 0]
S = −(1/2) log (1/2)

Ex 1

$$|\psi\rangle = \frac{1}{\sqrt{2}}|\uparrow\downarrow\rangle - \frac{1}{\sqrt{2}}|\downarrow\uparrow\rangle = \sum_{ab}\psi^{ab}|ab\rangle \qquad \psi^{ab} = \begin{bmatrix} 0 & 1/\sqrt{2} \\ -1/\sqrt{2} & 0 \end{bmatrix} \; ; \; \rho_A{}^{a'}{}_a = \sum_b \psi^{a'b}\bar{\psi}_{ab} = (\psi\psi^\dagger)^{a'}{}_a = \begin{bmatrix} 0 \\ -1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}$$

$$S = -(1/2)\log(1/2) - (1/2)\log(1/2) \longrightarrow \text{max entanglement} \quad \rho \text{ equal eigenvalues}$$

$$= -\log(1/2) = \log 2 \qquad D\begin{bmatrix} 1/D & 1/D & 1/D & \dots \end{bmatrix}$$

Ex 2

$$|\psi\rangle = \psi^{ab}|ab\rangle \qquad \psi^{ab} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \qquad |\psi\rangle = \frac{1}{2}\left(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle\right)$$

$$\rho_A{}^{a'}{}_a = (\psi\psi^\dagger)^{a'}{}_a = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\underbrace{\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}}_{U}\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}_{\rho}\underbrace{\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}}_{U^\dagger}\frac{1}{\sqrt{2}} \qquad S = -(1)\log(1) = 0$$

→ MPS $\overset{}{\underset{D}{\circ\!-\!\circ\!-\!\circ\!-\!\circ\!-\!\circ\!-\!\circ}}$ → exponentially decaying (connected) $\langle O_i O_j \rangle - \langle O_i \rangle \langle$

→ MPS appropriate ground states gapped 1d systems

correlation length $\xi = \frac{1}{\Delta}$

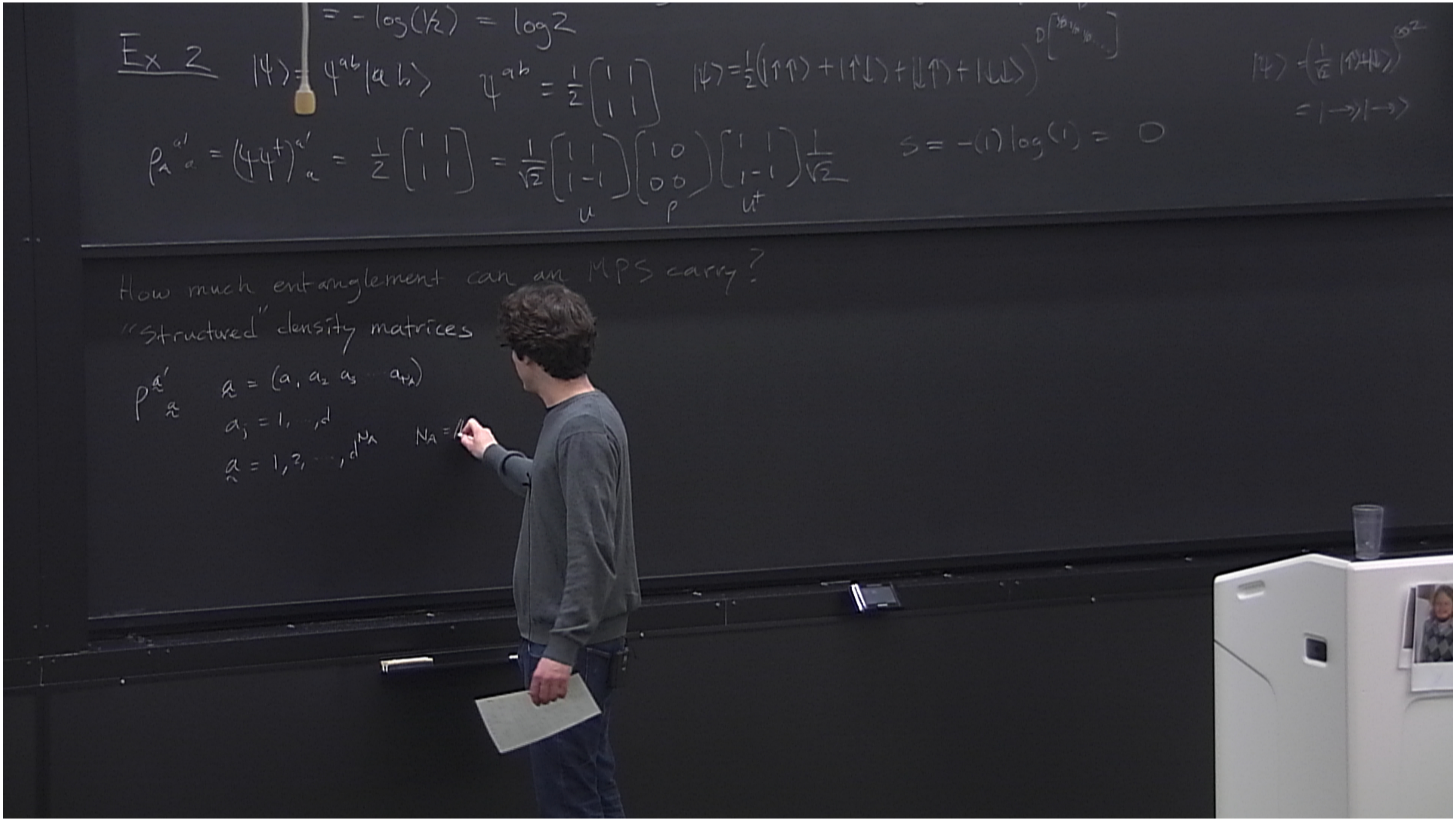
Theorem (Hastings 2007)

Local 1d Hamiltonian, finite energy gap, unique ground state

$\exists$ MPS bond dimension D, $\rho > 0$ (independent of D)

such that $|\langle \psi_{mps} | \hat{O} | \psi_{mps} \rangle - \langle \psi | \hat{O} | \psi \rangle| \leq c_{O} D^{-\rho}$

$\hat{O}$ local operator


= - log(1/2) = log2
Ex 2
|ψ> = ψ^{ab}|a b>
ψ^{ab} = 1/2 (1 1 / 1 1)
|ψ> = 1/2(|↑↑> + |↑↓> + |↓↑> + |↓↓>)
|ψ> = (1/√2 (|↑>+|↓>))^{⊗2}
= |→>|→>
ρ_A^{a'}_a = (ψψ^†)^{a'}_a = 1/2 (1 1 / 1 1) = 1/√2 (1 1 / 1 -1)(1 0 / 0 0)(1 1 / 1 -1) 1/√2
U ρ U^†
S = -(1) log(1) = 0
How much entanglement can an MPS carry?
"structured" density matrices
ρ^{a'}_{a}
a = (a_1, a_2, a_3 ... a_{N_A})
a_j = 1, ..., d
a = 1, 2, ..., d^{N_A}
N_A =

$= -\log(1/2) = \log 2$

Ex 2 $|\psi\rangle = \psi^{ab}|ab\rangle \qquad \psi^{ab} = \frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} \qquad |\psi\rangle = \frac{1}{2}\left(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle\right)$

$D\left[{}^{1/D}\ {}^{1/D}\ {}^{1/D} \ldots\right]$

$|\psi\rangle = \left(\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)\right)^{\otimes 2}$

$= |\rightarrow\rangle|\rightarrow\rangle$

$$\rho_A{}^{a'}{}_{a} = (\psi\psi^\dagger)^{a'}{}_{a} = \frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} = \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}}_{U}\underbrace{\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}}_{\rho}\underbrace{\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\frac{1}{\sqrt{2}}}_{U^\dagger}$$

$S = -(1)\log(1) = 0$

How much entanglement can an MPS carry?

"Structured" density matrices

$\rho^{\underset{\sim}{a}'}{}_{\underset{\sim}{a}} \qquad \underset{\sim}{a} = (a_1, a_2, a_3 \ldots a_{N_A})$

$a_j = 1, \ldots, d$

$\underset{\sim}{a} = 1, 2, \ldots, \underset{\underset{A}{\shortparallel}}{d^{N_A}}$ $\quad N_A = \#$ sites in A

assume $\rho_A{}^{\underset{\sim}{a}'}{}_{\underset{\sim}{a}} = \sum_{i=1}^{D} t_i^{\underset{\sim}{a}'}\, v_{i\underset{\sim}{a}} =$ a_1', a_2', a_3', a_4' — O —D— O — a_1, a_2, a_3, a_4

$D < A$

$v_{i\underset{\sim}{a}} = (\vec{v}_i)_{\underset{\sim}{a}} = i$ vectors each dimension A

$\sum_{\underset{\sim}{a}} v_{i\underset{\sim}{a}}\, w^{\underset{\sim}{a}} = 0$

$A - D$

generally $\mathrm{span}(\vec{v}_i) = D$ dimensional space

Kernel/null space $v_{i\underset{\sim}{a}}$ $(A-D)$ dimensional $\Rightarrow v_{i\underset{\sim}{a}}$ rank D

ρ_A also has rank D

$= -\log(1/2) = \log 2$

Ex 2 $|\psi\rangle = \psi^{ab}|ab\rangle \qquad \psi^{ab} = \frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$ $|\psi\rangle = \frac{1}{2}\left(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle\right)$

$|\psi\rangle = \left(\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)\right)^{\otimes 2}$

$= |\rightarrow\rangle|\rightarrow\rangle$

$\rho_A{}^{a'}{}_a = (\psi\psi^\dagger)^{a'}{}_a = \frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} = \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}}_{U}\underbrace{\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}}_{\rho}\underbrace{\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\frac{1}{\sqrt{2}}}_{U^\dagger}$ $S = -(1)\log(1) = 0$

How much entanglement can an MPS carry?

"Structured" density matrices

$\rho^{\underset{\sim}{a}'}{}_{\underset{\sim}{a}}$ $\underset{\sim}{a} = (a_1, a_2, a_3 \ldots a_{N_A})$

$a_j = 1, \ldots, d$

$\underset{\sim}{a} = 1, 2, \ldots, \underbrace{d^{N_A}}_{A}$

$N_A = \#$ sites in A

assume $\rho_A{}^{\underset{\sim}{a}'}{}_{\underset{\sim}{a}} = \sum_{i=1}^{D} t_i^{\underset{\sim}{a}'} v_{i\underset{\sim}{a}} =$

$D < A$

$\sum_{\underset{\sim}{a}} v_{i\underset{\sim}{a}} w^{\underset{\sim}{a}} = 0$

basis of A-D of these

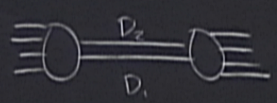
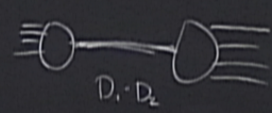
$v_{i\underset{\sim}{a}} = (\vec{v}_i)_{\underset{\sim}{a}}$ = i vectors each dimension A

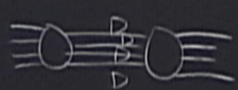
generally span$(\vec{v}_i) = D$ dimensional space

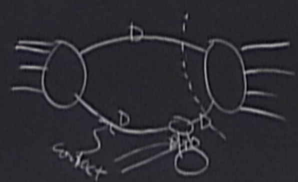
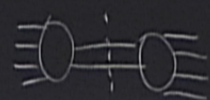
Kernel/null space v_{ia} $A-D$ dimensional $\Rightarrow v_{i\underset{\sim}{a}}$ rank D

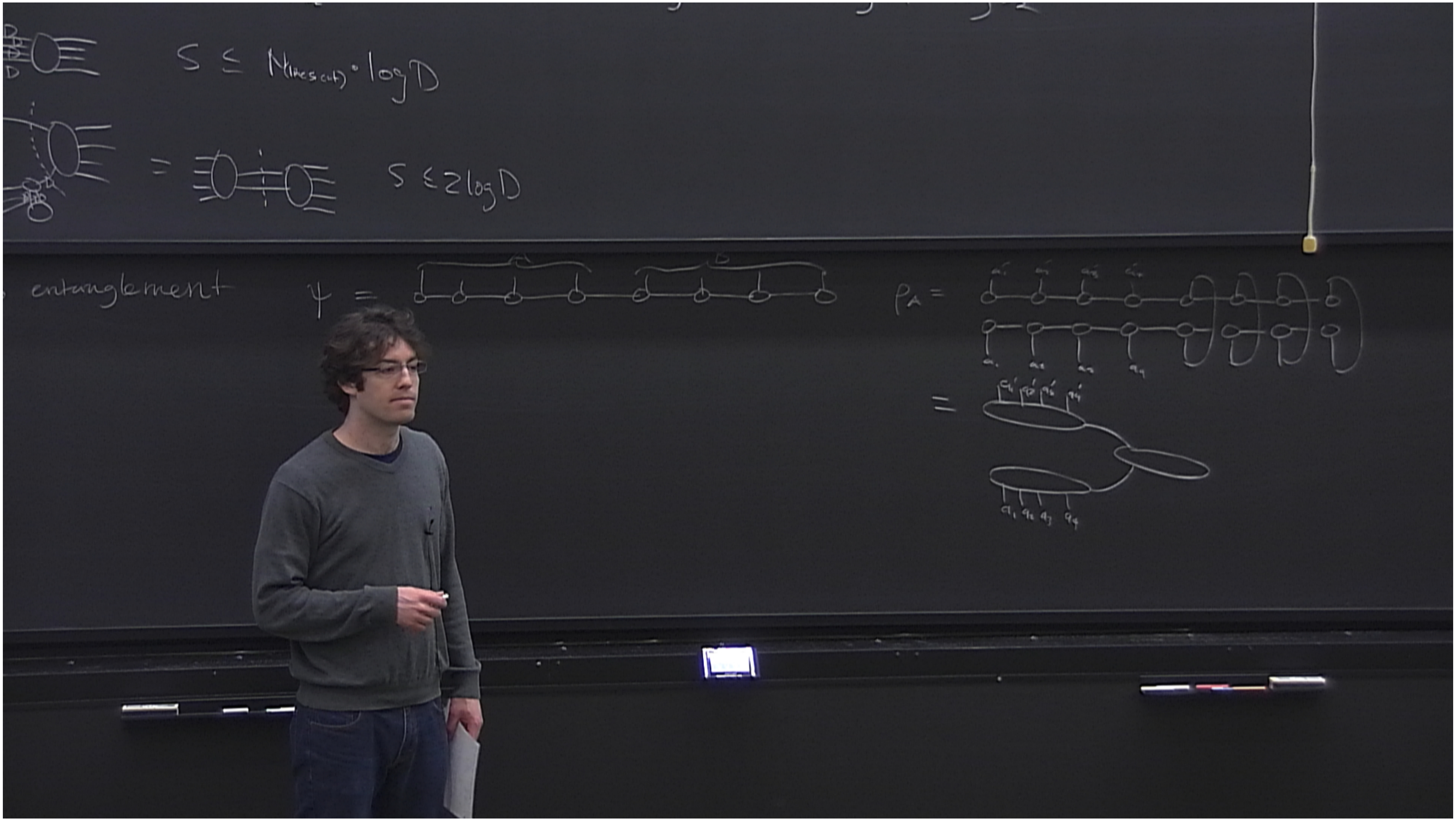
$\boxed{\rho_A \text{ also has rank } D}$

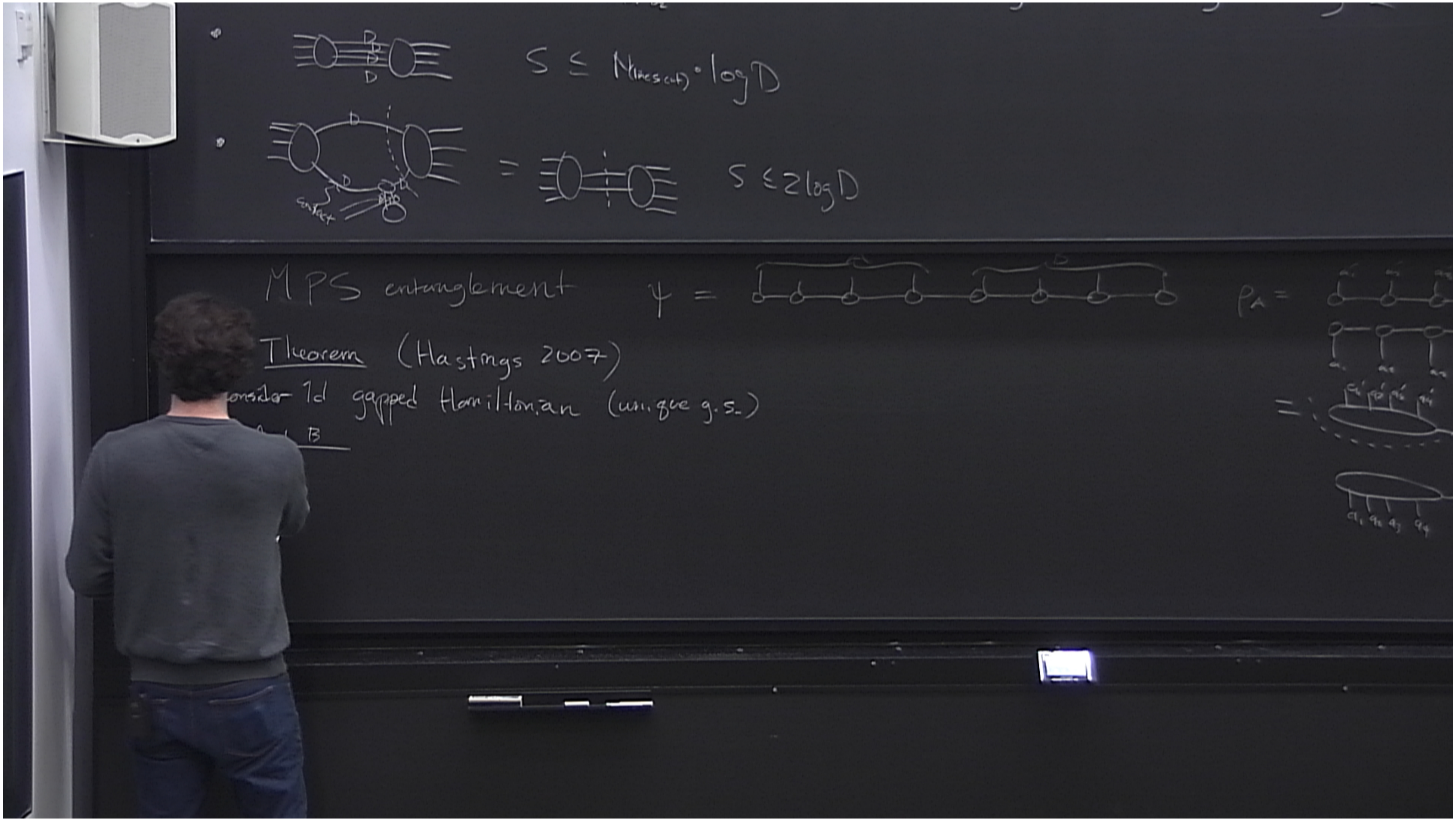
- most entangled case $\rho = \begin{array}{c} \overbrace{}^{D} \\ D\left\{\begin{pmatrix} 1/D & & & & \\ & 1/D & & & \\ & & \ddots & & \\ & & & 0 & \\ & & & & 0\,0 \end{pmatrix}\right. \end{array}$ rank D ρ_A $S[\rho_A] \le \log D$

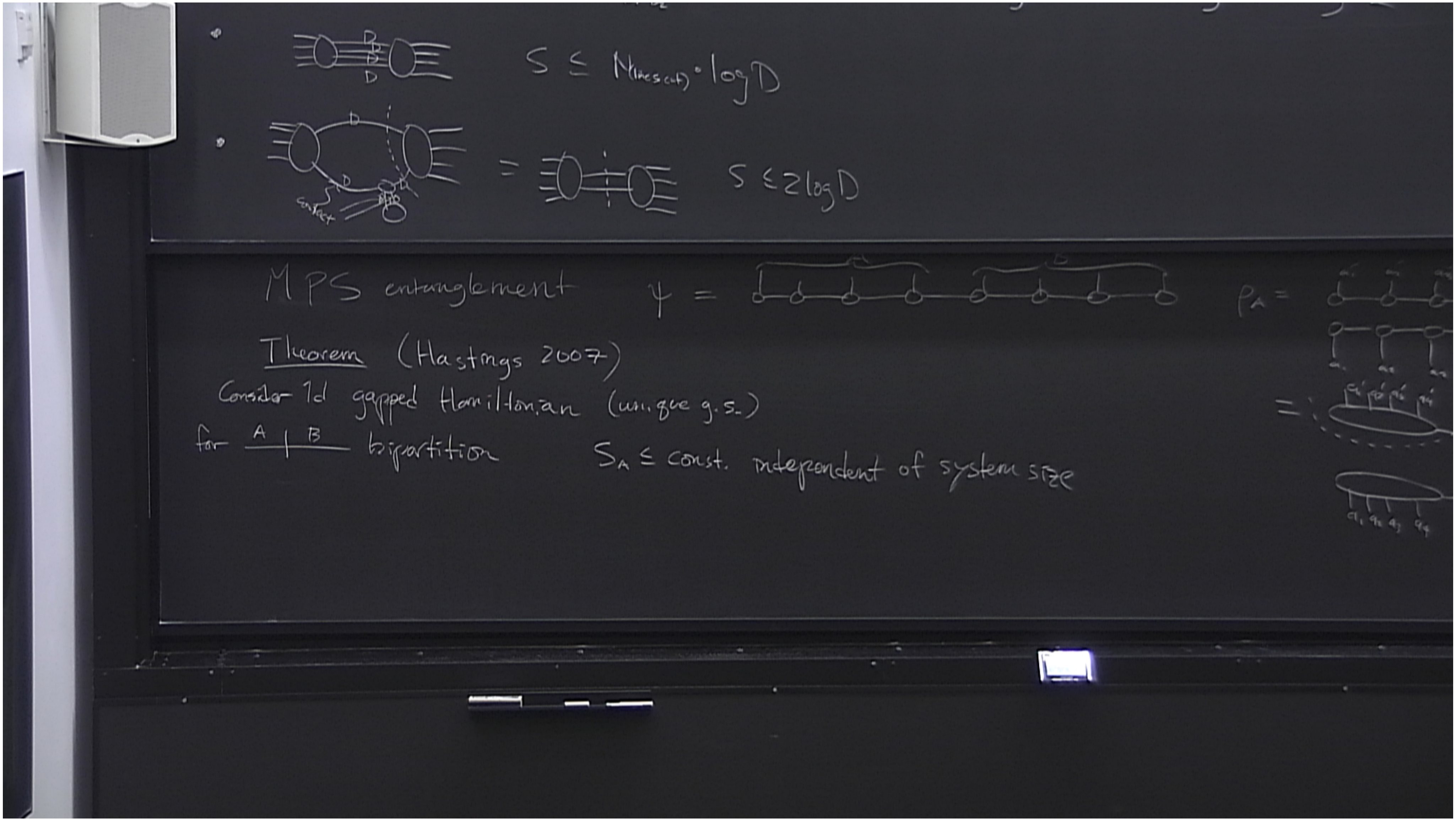
-  =  $S[\rho] \le \log(D_1 \cdot D_2) = \log D_1 + \log D_2$

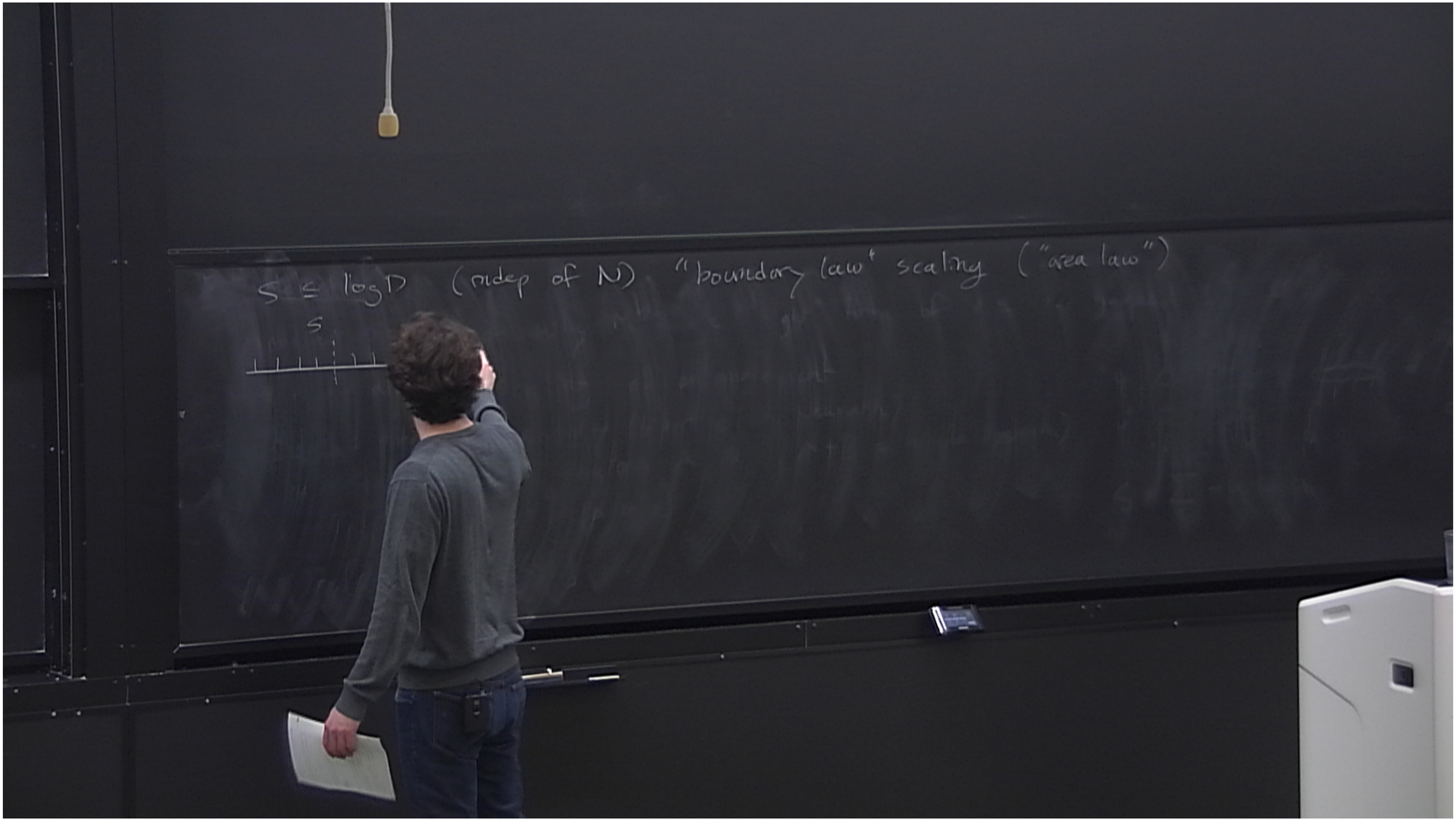
-  $S \le N_{(\text{lines cut})} \cdot \log D$

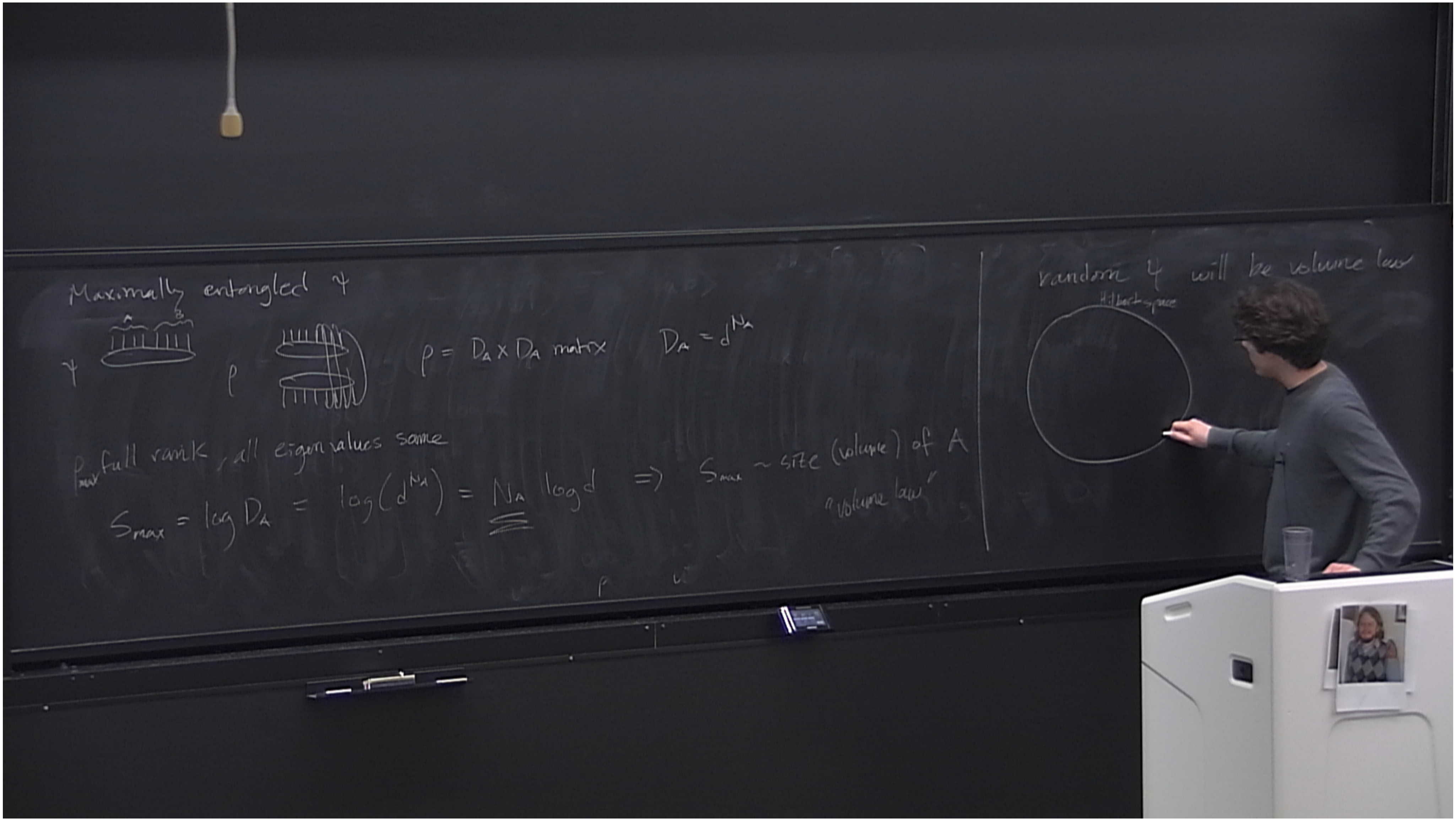
-  =  $S \le 2 \log D$


S ≤ N(lines cut) · log D
S ≤ 2 log D
entanglement
ψ =
ρA =
a1 a2 a3 a4


S ≤ N_lines cut · log D
S ≤ 2 log D
MPS entanglement
ψ =
ρA =
Theorem (Hastings 2007)
consider 1d gapped Hamiltonian (unique g.s.)


S ≤ N lines cut · log D
S ≤ 2 log D
MPS entanglement
ψ =
ρA =
Theorem (Hastings 2007)
Consider 1d gapped Hamiltonian (unique g.s.)
for A | B bipartition
SA ≤ const. independent of system size


S ≤ log D (indep of N) "boundary law" scaling ("area law")
S


Maximally entangled ψ
A
B
ψ
ρ
ρ = D_A × D_A matrix
D_A = d^{N_A}
ρ_max full rank, all eigenvalues same
S_max = log D_A = log(d^{N_A}) = N_A log d
=> S_max ~ size (volume) of A
"volume law"
random ψ will be volume law
Hilbert space