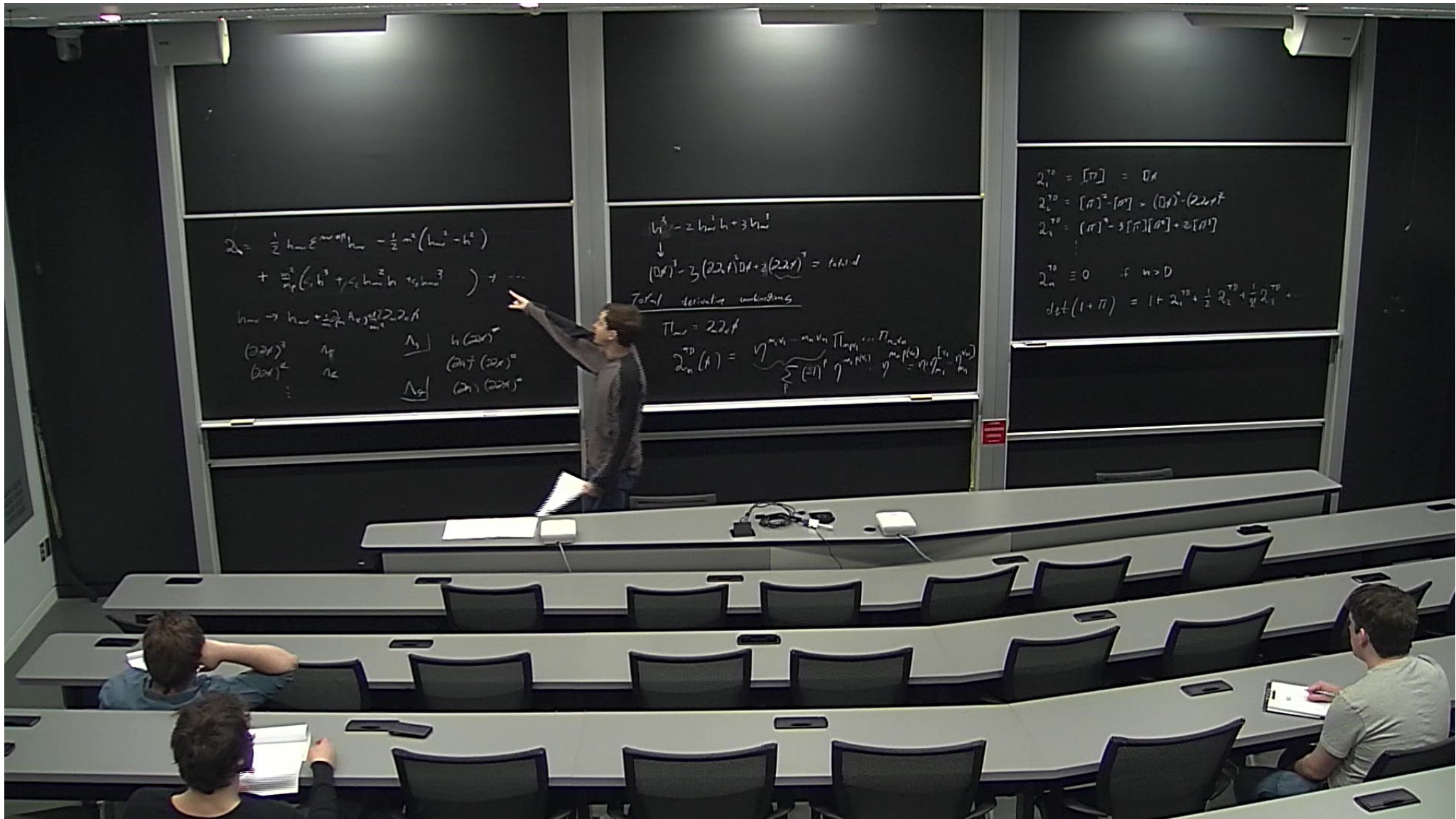


Title: Kurt Hinterbichler's third massive gravity lecture

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Abstract:



$$z_n = \frac{1}{2} h_n \varepsilon^{m+1} h_n - \frac{1}{2} \varepsilon^2 (h_n^2 - h^2)$$

$$+ \frac{\varepsilon^3}{6} (c_1 h^3 + c_2 h^2 h + c_3 h^3) + \dots$$

$$h_n \rightarrow h_n + \frac{\varepsilon}{2} \Lambda_0 \frac{d^2 z_n}{d h^2}$$

$(\partial z)^3$	Λ_1	Λ_2	$h(\partial z)^m$
$(\partial z)^2$	Λ_2		$(\partial z)^2 (\partial z)^m$
\vdots		Λ_0	$(\partial z) (\partial z)^m$

$$h^3 = 2 h^2 h + 3 h^2 h^2$$

$$\downarrow$$

$$(\partial z)^3 - 3 (\partial z)^2 (\partial z) + 3 (\partial z)^2 = \text{total}$$

Total derivative unknowns

$$T_{m+1} = 2 \partial z^m$$

$$z_n^{(m)}(h) = \sum_{i=1}^{m+1} \binom{m+1}{i} \eta^{m+1-i} \eta^{m+1-i} \eta^{m+1-i} \dots \eta^{m+1-i} \eta^{m+1-i} \eta^{m+1-i}$$

$$z_1^{(0)} = [z] = 0 \Delta$$

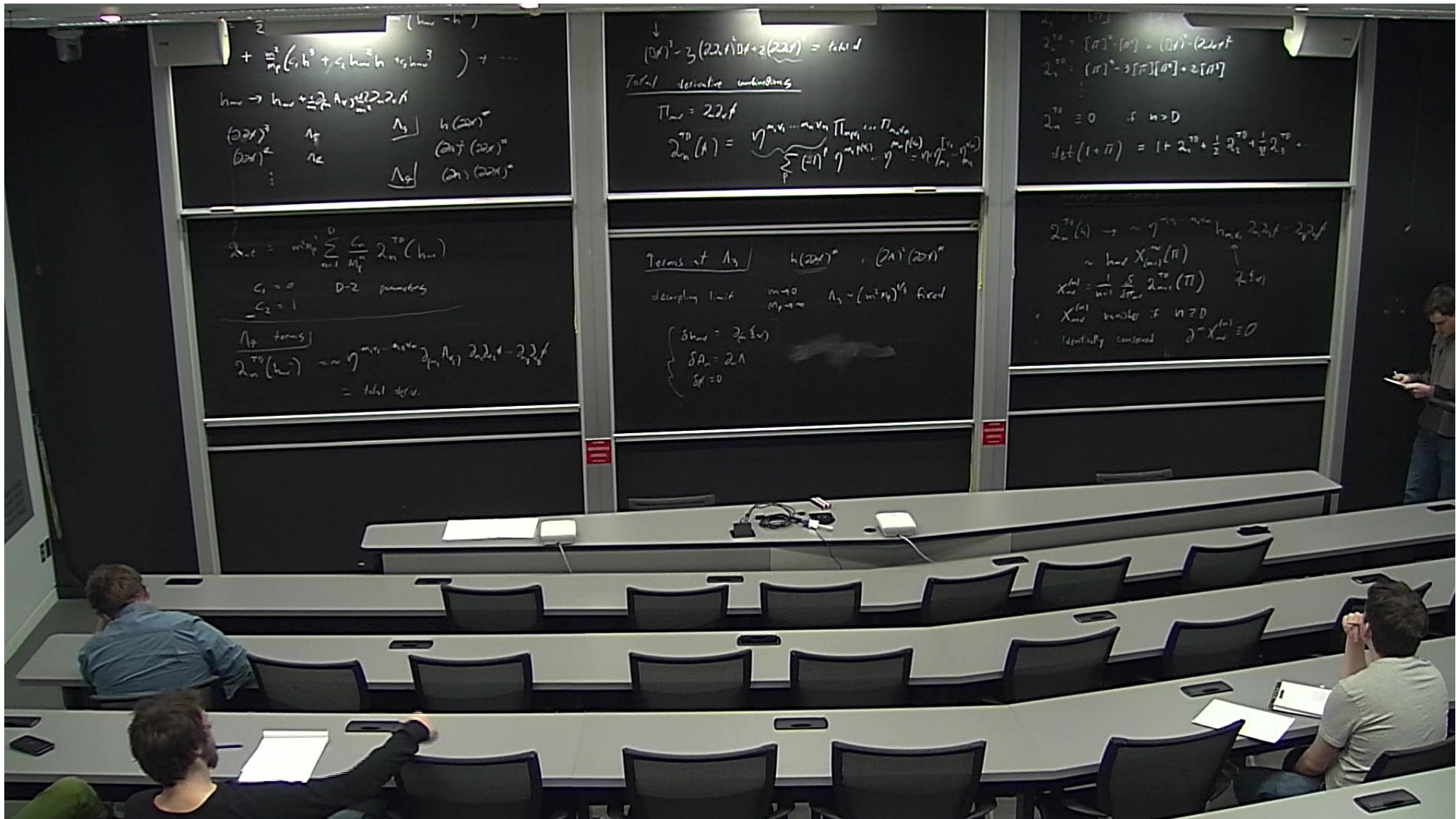
$$z_2^{(0)} = [z]^2 - [z] = (\partial z)^2 - 2 \partial z \eta^2$$

$$z_3^{(0)} = [z]^3 - 3 [z] [z] + 3 [z] \eta^3$$

$$\vdots$$

$$z_n^{(0)} \equiv 0 \quad \text{if } n > D$$

$$d_z (1 + \eta) = 1 + 2 z_1^{(0)} + \frac{1}{2} z_2^{(0)} + \frac{1}{6} z_3^{(0)} + \dots$$



$$+ \sum_{n=1}^{\infty} (c_n h^n + c_{n+1} h^{n+1} + c_{n+2} h^{n+2}) + \dots$$

$$h_{n+1} \rightarrow h_n + \frac{1}{2} \frac{d}{dt} \Lambda_n \frac{d}{dt} \Lambda_n^{-1} \frac{d}{dt} \Lambda_n$$

$(2\pi)^3$	Λ_1	Λ_2	$h(2\pi)^n$
$(2\pi)^2$	Λ_2		$(2\pi)^2 (2\pi)^n$
\vdots		Λ_3	$(2\pi)^3 (2\pi)^n$

$$Z_{tot} = \sum_{n=1}^D \frac{c_n}{M_n} Z_n^{TD}(h_{n+1})$$

$c_1 = 0$ $D=2$ parameters

$c_2 = 1$

Λ_1 terms

$$Z_n^{TD}(h_{n+1}) \sim \eta^{m_1 - m_2 n} \Lambda_n \Lambda_{n+1} Z_n \Lambda_n^{-1} - Z_n \Lambda_n^{-1}$$

= total deriv.

$$(D\pi)^3 - 3(2\pi)^2 \Lambda_1 \Lambda_2 + 2(2\pi)^2 = \text{total}$$

Total derivative conditions

$$\Pi_{tot} = 2\pi \Lambda_1$$

$$Z_n^{TD}(h) = \eta^{m_1 - m_2 n} \prod_{i=1}^{m_1} \Lambda_{n_i} \dots \prod_{i=1}^{m_2} \Lambda_{n_i}^{-1}$$

Terms of Λ_2 $h(2\pi)^n$ $(2\pi)^2 (2\pi)^n$

decoupling limit $m \rightarrow 0$ $M_n \rightarrow \infty$ $\Lambda_n \sim (m^2 M_n)^{1/2}$ fixed

$$\begin{cases} \delta h_{n+1} = Z_n \delta v \\ \delta \Lambda_n = 2\Lambda_1 \\ \delta \pi = 0 \end{cases}$$

$$Z_n^{TD} = [\pi]^{-1} [\sigma] = [\pi]^{-1} (2\pi)^2$$

$$Z_n^{TD} = [\pi]^{-1} - 3[\pi][\sigma] + 2[\sigma]^2$$

$$Z_n^{TD} \equiv 0 \quad \text{if } n > 0$$

$$\frac{d}{dt} (1 + \Pi) = 1 + 2Z_n^{TD} + \frac{1}{2} Z_n^{TD} + \frac{1}{2} Z_n^{TD}$$

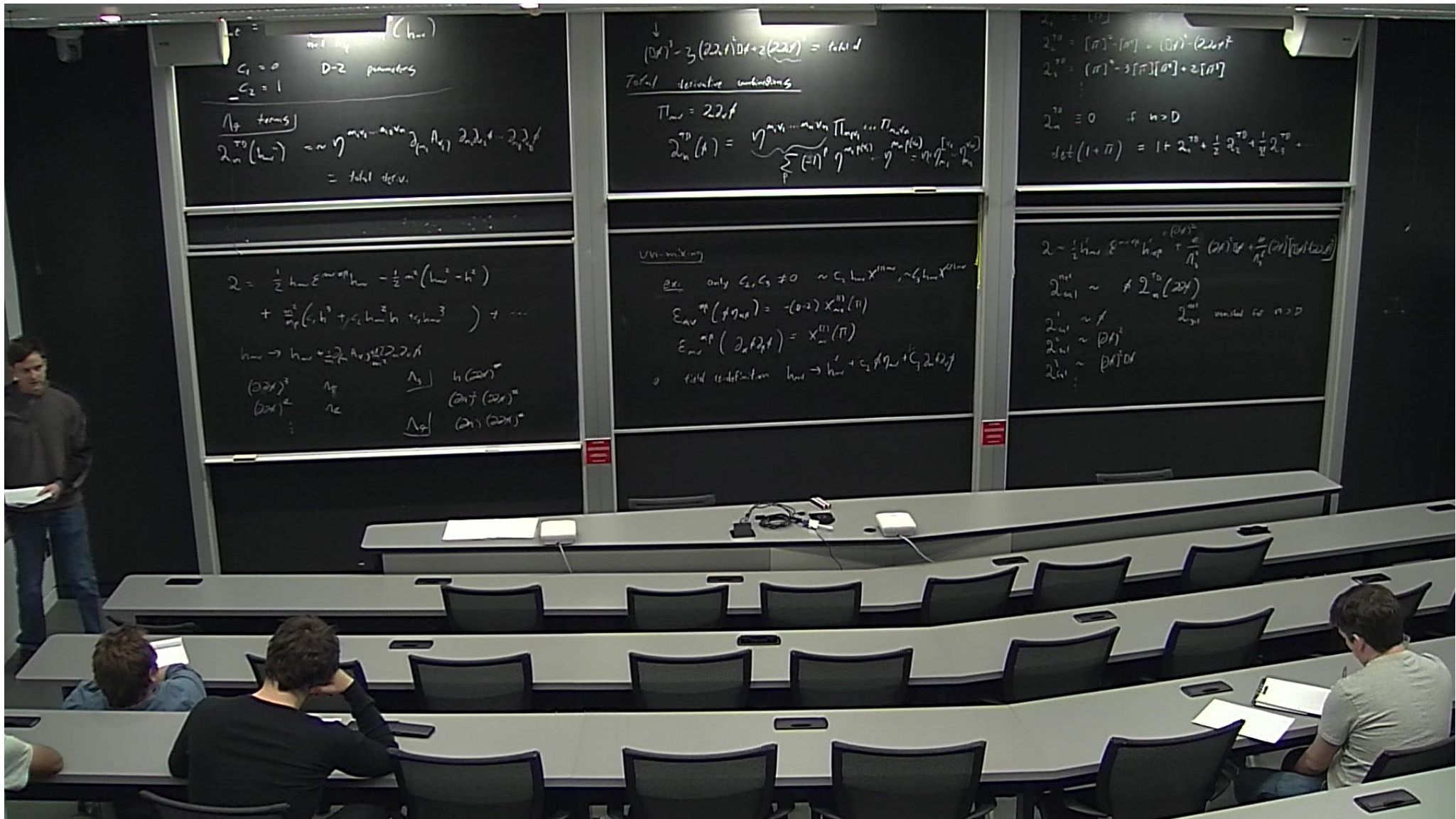
$$Z_n^{TD}(h) \rightarrow \sim \eta^{m_1 - m_2 n} h_{n+1} Z_n \Lambda_n^{-1} - Z_n \Lambda_n^{-1}$$

\sim had $X_{[m_1]}(\Pi)$

$$X_{[m_1]}^{(n)} = \frac{1}{n!} \frac{d^n}{d\Pi^n} Z_n^{TD}(\Pi) \quad Z_n^{TD}$$

$X_{[m_2]}^{(n)}$ vanishes if $n > 0$

identically vanishes $Z_n^{TD} \equiv 0$



$c_1 = 0$ $D=2$ parameters
 $c_2 = 1$
 Λ_T terms
 $\mathcal{L}_n^{TD}(h_{\mu\nu}) \sim \eta^{m_1 \dots m_n} \eta^{n_1 \dots n_n} \mathcal{L}_n(\Lambda_{\mu\nu}) \mathcal{L}_n(\Lambda_{\mu\nu}) \dots \mathcal{L}_n(\Lambda_{\mu\nu})$
 = total deriv.

$\mathcal{L} = \frac{1}{2} h_{\mu\nu} \partial^{\mu} \partial^{\nu} h_{\rho\sigma} - \frac{1}{2} m^2 (h_{\mu\nu}^2 - h^2)$
 $+ \frac{c_1}{m^2} (\partial_{\mu} h^{\mu}{}_{\nu})^2 + c_2 h_{\mu\nu} \partial^{\mu} \partial^{\nu} h_{\rho\sigma} + \dots$
 $h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_{\mu} \Lambda_{\nu} + \partial_{\nu} \Lambda_{\mu}$

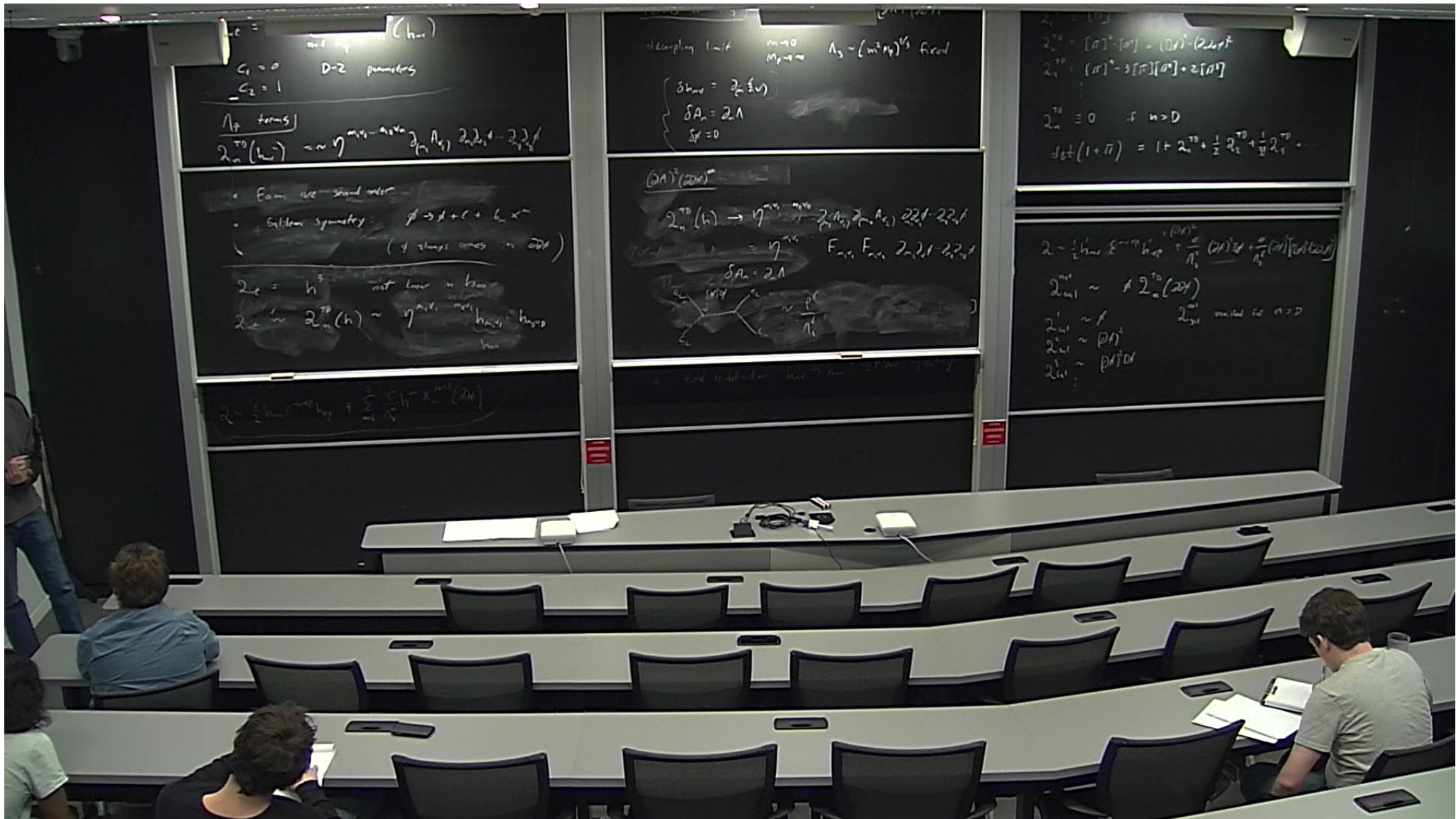
$(\partial \partial)^2$	Λ_T	Λ_L	$h(2\partial)^2$
$(\partial \partial)^2$	Λ_C		$(\partial \partial)^2 (2\partial)^2$
\vdots		Λ_T	$(\partial \partial)^2 (2\partial)^2$

$(\partial \partial)^2 - 3(2\partial \partial)^2 + 2(2\partial)^2 = \text{total d}$
Total derivative conditions
 $\Pi_{\mu\nu} = 2\partial \partial$
 $\mathcal{L}_n^{TD}(h) = \eta^{m_1 \dots m_n} \eta^{n_1 \dots n_n} \prod_{\mu_1 \nu_1} \dots \prod_{\mu_n \nu_n} \Pi_{\mu_i \nu_i}^{m_i n_i} \eta^{m_i n_i} \eta^{m_i n_i} \eta^{m_i n_i}$

VN-mixing
 ex. only $c_2, c_3 \neq 0 \sim c_2 h_{\mu\nu} X^{\mu\nu} \sim c_3 h_{\mu\nu} X^{\mu\nu}$
 $\mathcal{E}_{\mu\nu}^{\rho\sigma}(\partial \partial \eta_{\rho\sigma}) = -(D-2) X_{\mu\nu}^{\rho\sigma}(\Pi)$
 $\mathcal{E}_{\mu\nu}^{\rho\sigma}(\partial_{\alpha} \Lambda_{\rho\sigma}) = X_{\mu\nu}^{\rho\sigma}(\Pi)$
 a field redefinition $h_{\mu\nu} \rightarrow h_{\mu\nu} + c_2 \Lambda_{\mu\nu} + c_3 \partial_{\mu} \partial_{\nu} \Lambda$

$\mathcal{L}_n^{TD} = [\Pi]^{n-1} [\partial] = (\partial \partial)^{n-1} (2\partial \partial)$
 $\mathcal{L}_n^{TD} = (\partial \partial)^{n-1} [3[\Pi] [\partial] + 2[\partial \partial]]$
 $\mathcal{L}_n^{TD} \equiv 0 \quad \text{if } n > D$
 $\partial_{\mu} \partial^{\mu} (1 + \Pi) = 1 + 2\partial \partial + \frac{1}{2} \partial \partial^2 + \frac{1}{12} \partial \partial^3 \dots$

$\mathcal{L} = \frac{1}{2} h_{\mu\nu} \partial^{\mu} \partial^{\nu} h_{\rho\sigma} + \frac{m^2}{\Lambda^2} (\partial \partial)^2 h_{\mu\nu} + \frac{m^2}{\Lambda^2} (\partial \partial)^2 (h_{\mu\nu}^2)$
 $\mathcal{L}_{\text{total}} \sim \mathcal{L}_n^{TD}(2\partial \partial)$
 $\mathcal{L}_{\text{total}} \sim \mathcal{L}_n^{TD}$ $\mathcal{L}_{\text{total}}^{\text{mixed}}$ mixed for $n > D$
 $\mathcal{L}_{\text{total}}^1 \sim \mathcal{L}_n^{TD}$
 $\mathcal{L}_{\text{total}}^2 \sim (\partial \partial)^2$
 $\mathcal{L}_{\text{total}}^3 \sim (\partial \partial)^3$



$c_1 = 0$ $D=2$ parameters
 $c_2 = 1$
 [17 terms]
 $\lambda_n^{TD}(h) \sim \eta^{m_{n1} - a_n v_n} \lambda_{n1} \lambda_{n2} \dots \lambda_{nD}$

• From the second order
 • Galilean symmetry: $\phi \rightarrow \phi + c + b_n x^n$
 (if always comes in $2D$)
 $2_{\text{ex}} = h^2$ not lower in h
 $2_{\text{ex}} \sim \lambda_n^{TD}(h) \sim \eta^{m_{n1} - a_n v_n} h_{n1} h_{n2} \dots h_{nD}$

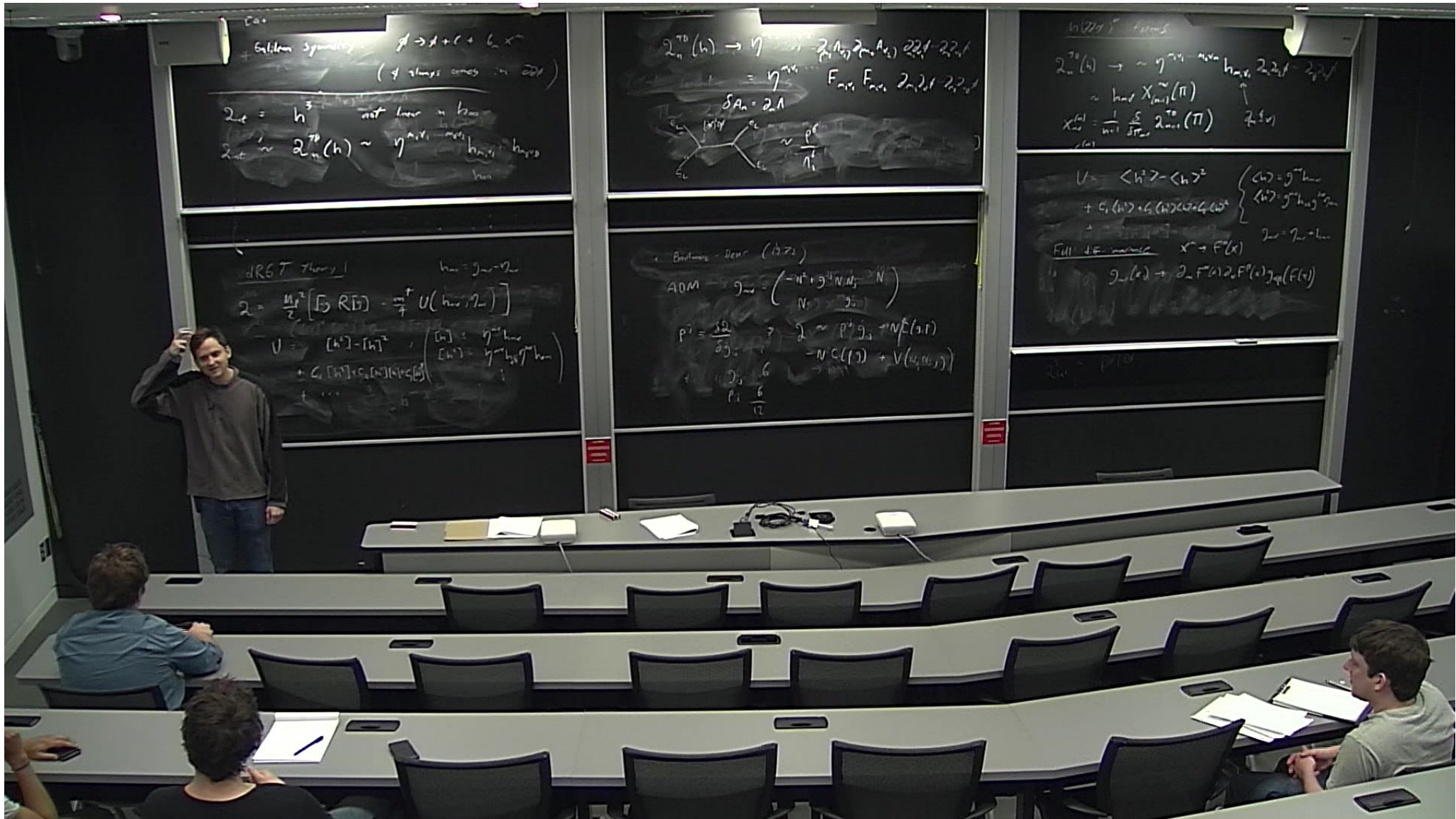
$2_{\text{ex}} = \frac{1}{2} h_{n1} h_{n2} + \sum_{i=1}^D \frac{1}{2} h_{ni}^2 \chi_{ni}^{(2)}(2h)$

decoupling limit $m \rightarrow 0$
 $M_P \rightarrow \infty$ $\Lambda_3 \sim (m^2 M_P^2)^{1/2}$ fixed
 $\delta h_{\mu\nu} = 2\lambda_{\mu\nu}$
 $\delta A_n = 2A$
 $\delta \psi = 0$

$(2A)^2 (2D)^2$
 $2_{\text{ex}}^{TD}(h) \rightarrow \eta^{m_{n1} - a_n v_n} \lambda_{n1} \lambda_{n2} \dots \lambda_{nD} 2_{2,1} 2_{2,1}$
 $= \eta^{m_{n1} - a_n v_n} F_{n1} F_{n2} \lambda_{n1} \lambda_{n2} \dots \lambda_{nD}$
 $\delta A_n = 2A$
 $\frac{p^2}{\Lambda_3^2}$

$2_{\text{ex}}^{TD} = [\pi]^\pi - [\sigma] = ([\pi]^\pi - 2\sigma) + \sigma$
 $2_{\text{ex}}^{TD} = (\pi)^\pi - 2[\pi][\sigma] + 2[\sigma]^2$
 $2_{\text{ex}}^{TD} \equiv 0$ if $n > D$
 $\delta_{\text{ex}}(1+\pi) = 1 + 2_{\text{ex}}^{TD} + \frac{1}{2} 2_{\text{ex}}^{TD} + \frac{1}{12} 2_{\text{ex}}^{TD}$

$2_{\text{ex}}^{TD} \sim \frac{1}{2} h_{n1} h_{n2} \dots h_{nD} + \frac{1}{\Lambda_3^2} (2A)^2 + \frac{1}{\Lambda_3^2} (2A)[\pi](2A)$
 $2_{\text{ex}}^{TD} \sim A 2_{\text{ex}}^{TD}(2D)$
 $2_{\text{ex}}^{TD} \sim \mathcal{O}(\Lambda_3^2)$
 $2_{\text{ex}}^{TD} \sim \mathcal{O}(\Lambda_3^2)$
 $2_{\text{ex}}^{TD} \sim \mathcal{O}(\Lambda_3^2)$



$\mathcal{L} \rightarrow \mathcal{L} + \epsilon \dot{x}$
 (4 steps comes in 2π)
 $Z_{cl} = \int \mathcal{L} \sim \int \dot{x}^2$ not linear in h
 $Z_{cl} \sim Z_{cl}^{(0)}(h) \sim \eta^{N+1} h_{m,v} h_{m,v}$

$Z_{cl}^{(0)}(h) \rightarrow \eta \dots \int \mathcal{L}_{m,v} \mathcal{L}_{m,v} \dots$
 $= \eta^{N+1} \dots F_{m,v} F_{m,v} \dots$
 $\delta A_h = 2A$
 $\sim \frac{\rho^6}{\eta^4}$

$Z_{cl}^{(0)}(h) \rightarrow \sim \eta^{N+1} h_{m,v} \dots$
 $\sim \int \mathcal{L}_{m,v}(\pi) \dots$
 $X_{m,v}^{(0)} = \frac{1}{N+1} \sum_{j=1}^N Z_{cl}^{(0)}(\pi) \dots$

dRGT theory
 $Z = \frac{M^2}{2} [\int \mathcal{L}(\mathbb{R}^4) - \frac{1}{4} U(h_{\mu\nu}, \gamma_{\mu\nu})]$
 $U = [h^2] - [h^2] \dots$
 $[h^2] = \eta^{2\alpha} h_{\mu\nu} h^{\mu\nu}$
 $[h^2] = \eta^{-2\alpha} h_{\mu\nu} h^{\mu\nu}$

Barlowe - Deur (1972)
 $AOM \rightarrow g_{\mu\nu} = \begin{pmatrix} -N^2 + g^2 N_{\mu\nu} & N \\ N & g_{\mu\nu} \end{pmatrix}$
 $P^2 = \frac{\delta Z}{\delta g_{\mu\nu}} \sim \dots$
 $\sim P^2 g_{\mu\nu} + N \mathcal{L}(g, T)$
 $= N C(\mathbb{R}^4) + V(h, \mathbb{R}^4)$

$U = \langle h^2 \rangle - \langle h^2 \rangle^2$
 $+ c_1 \langle h^2 \rangle + c_2 \langle h^2 \rangle^2 + c_3 \langle h^2 \rangle^3$
 $\left\{ \begin{array}{l} \langle h^2 \rangle = g^{\mu\nu} h_{\mu\nu} \\ \langle h^2 \rangle = g^{\mu\nu} h_{\mu\nu}^2 \dots \end{array} \right.$
 Full dF matrix $X^2 \rightarrow F^2(x)$
 $g_{\mu\nu}(x) \rightarrow \partial_{\mu} F^{\alpha}(x) \partial_{\nu} F^{\beta}(x) g_{\alpha\beta}(F(x))$