Title: THE SM HIGGS VACUUM INSTABILITY, INFLATION AND THE FATE OF OUR UNIVERSE

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Abstract: The presence of an instability in the Standard Model Higgs potential may have important implications for inflation and the viability of our Universe. In particular, if the Hubble scale during inflation is comparable to (or larger than) the instability scale of the potential, quantum fluctuations in the Higgs field will lead to the Higgs sampling the unstable part of the potential during inflation. However, to correctly study transitions to the unstable regime and determine the significance for the resulting universe requires addressing a number of subtleties. I will discuss these subtleties and a variety of possible approaches to studying Higgs evolution during inflation. By considering both (1) the evolution of Higgs fluctuations in the Gaussian approximation and (2) a perturbative calculation of the fluctuation two-point correlation function, I aim to elucidate how to address these issues in a consistent, physical way. The insight provided by these approaches will set the scene for studying Higgs fluctuations via the Fokker-Planck (FP) approach, which captures the non-Gaussian nature of the field. As such, it provides information about the rareâ€"but, in terms of the fate of our Universe, potentially extremely importantâ€"patches that experience particularly large fluctuations.

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THE SM HIGGS VACUUM INSTABILITY, INFLATION AND THE FATE OF OUR UNIVERSE

Jack Kearney



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Higgs During Inflation

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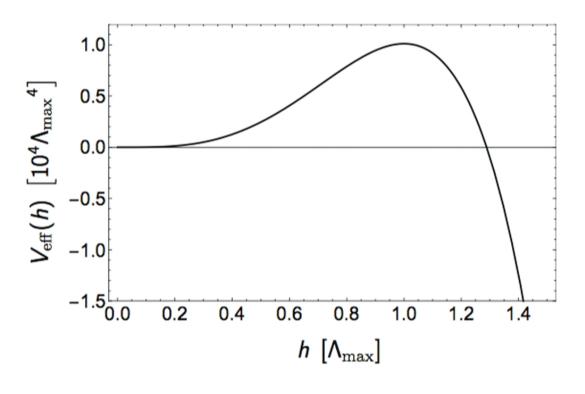
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The SM Higgs Potential is Unstable 200 Jack Kearney (Fermilab) Higgs During Inflation 2 / 38 March 24, 2015

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In the SM, Higgs quartic coupling $\lambda(\mu)$ runs negative at scales $\mu > \Lambda_I$



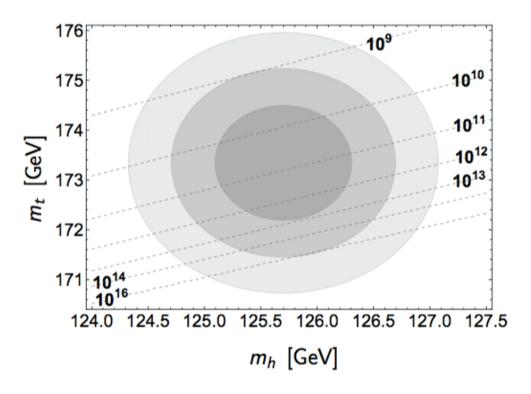
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What is the instability scale?



 Λ_I in GeV. Contours show $(1,2,3)\sigma \Rightarrow 10^9$ GeV $\lesssim \Lambda_I \lesssim 10^{16}$ GeV at 2σ .

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What are the implications of this instability? 200 Jack Kearney (Fermilab) Higgs During Inflation 5 / 38 March 24, 2015

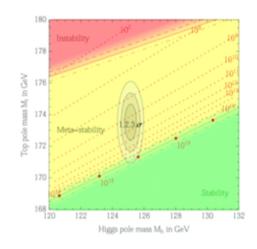
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Our Universe is metastable

Electoweak vacuum can decay via Coleman-De Luccia instanton...

e.g., Sher [Phys.Rept. 179, 273 (1989)], Casas, Espinosa, Quiros [hep-ph/9409458]

...but lifetime exceeds age of Universe for measured m_h , m_t .



Buttazzo et al. [1307.3536]

So, phenomenological implications today are limited.

e.g., New physics not necessarily required to stabilize Higgs potential.

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But...what about during inflation!?

Light scalar fields experience quantum fluctuations $\delta h \sim \frac{H}{2\pi}$ in de Sitter (dS) space due to expansion ($H \equiv$ Hubble during inflation).

 $H \gtrsim \Lambda_I \Rightarrow$ unstable regime sampled during inflation. What are the implications of such transitions for our Universe, NP?

Particularly relevant if we observe $r \sim 0.1 \Rightarrow H \sim 10^{14} \text{GeV}$.

Espinosa, Giudice, Riotto [0710.2484], Kobakhidze & Spencer-Smith [1301.2846], Enqvist, Meriniemi, Nurmi [1306.4511], Fairbairn & Hogan [1403.6786]...

In order to answer this question, we need to (correctly) study the evolution of the Higgs field and its fluctuations during inflation.

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Really?

Plenty of study of light scalar field fluctuations during inflation.

- ullet e.g., inflaton fluctuations, $\phi(t,\mathbf{x})=ar{\phi}(t)+\delta\phi(t,\mathbf{x})$
- "Slow-roll" \Rightarrow fluctuations $\delta \phi(t, \mathbf{x})$ approximately massless.
- Produce local ($\sim H^{-1}$) inhomogeneities in energy density, curvature.

$$\langle \delta \phi^2 \rangle \Rightarrow \langle (\delta \rho / \rho)^2 \rangle, \langle \mathcal{R}^2 \rangle \Rightarrow \langle (\delta T_{\rm CMB} / T_{\rm CMB})^2 \rangle$$

Similar story should hold for Higgs fluctuations, $\delta h(t, \mathbf{x})$.

Local variation in Higgs vev, energy density.

Note: vev in a Hubble patch ≡ sum over superhorizon modes.



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So what makes the Higgs special/tricky/especially tricky?

- 4 How do we appropriately calculate Higgs field evolution and transitions to unstable regime in an inflationary background?
 - Higgs dynamics governed by both dS space and $V(h) \approx \frac{\lambda}{4} h^4$.
 - Not entirely "light:" V(h) dominates for

$$h \gtrsim h_{\rm classical} \equiv \left(\frac{3}{-2\pi\lambda}\right)^{1/3} H.$$



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So what makes the Higgs special/tricky/especially tricky?

- 4 How do we appropriately calculate Higgs field evolution and transitions to unstable regime in an inflationary background?
- ② How does h behavior influence inflation and our Universe?
 - Runaway direction in V ⇒ large -ve energy density in Higgs field.
 "True" vacuum nucleation vs. backreaction and AdS-like crunching?
 - But causally disconnected patches free to evolve independently.



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Case Study in Confusion: The Hawking-Moss Calculation

Field excited to top of potential barrier with

$$\mathbb{P} = A \exp \left[-rac{8\pi\Delta V}{3H^4}
ight], \qquad \Delta V = V(\Lambda_{ ext{max}}) - V(0),$$

subsequently rolls down to "true vacuum."

PROS: Gauge invariant! Physical!

Cons: So what if any single patch transitions to $h \ge \Lambda_{\text{max}}$?

Inflation: causally disconnected patches evolve independently

For $h \gtrsim \Lambda_{\text{max}}$, patch dynamics, ρ still inflaton dominated if $H \gtrsim \Lambda_{\text{max}}$.

For $h \gg \Lambda_{\text{max}}$, defects due to, e.g., backreaction should be diluted.

After: could be stabilized by reheating (if h not too large)

Prefactor A for $H^4 \gtrsim \Delta V$?



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So, need to study full evolution and distribution of Higgs vev fluctuations during inflation.

- Toy model in Hartee-Fock/Gaussian approximation See how fluctuations evolve, consider implications for Universe
- Perturbative calculation of correlation function Connect to stochastic HF approach, incorporate SM
- Sokker-Planck Equation Incorporate non-Gaussianity, study most pathological patches

Hook, JK, Shakya, Zurek [1404.5953] JK, Yoo, Zurek [1503.05193]

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Goals for (I)

Study how fluctuations evolve in unstable potential, and

② Consider implications for resulting universe.



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Field evolution in dS space

Equation of Motion in dS:

$$\ddot{h} + 3H\dot{h} - \left(\frac{\vec{\nabla}}{a}\right)^2 h + V'(h) = 0$$

- Take $V(h) = \frac{\lambda}{4}h^4$ with $\lambda < 0$,
- Decompose $h(t, \mathbf{x}) = \bar{h}(t) + \delta h(t, \mathbf{x})$ with $\bar{h}(t) = \bar{h}(0) = 0$.

Mode equation

$$\ddot{\delta h}_k + 3H\dot{\delta h}_k + \left\{ \left(\frac{k}{a}\right)^2 + 3\lambda \left\langle \delta h^2(t) \right\rangle \right\} \delta h_k = 0$$



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Superhorizon Fluctuation Two-Point Correlation Function

$$\left\langle \delta h^2(t) \right\rangle = \int_{k=1/L}^{k=\epsilon a H} \frac{d^3 k}{(2\pi)^3} \left| \delta h_k(t) \right|^2$$

- Superhorizon modes only
 - Subhorizon (UV) contributions cancelled by "local" counterterms
 - Dominant effects on superhorizon physics reabsorbed into renormalization—will return to this in (II)
- 2 IR cutoff
 - Region of space over which $\bar{h}(0) = 0$ is a good approximation, i.e.

$$L^{-1} = a_0 H$$

where a_0 is scale factor at onset of inflation.



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For $|\lambda| \left< \delta h^2(t) \right> \ll H^2$ and slow-roll,

$$rac{d}{dt}\left\langle \delta h^2(t)
ight
angle = -rac{2\lambda}{H}\left\langle \delta h^2(t)
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angle^2 + rac{H^3}{4\pi^2}$$

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$$\frac{d}{dt}\left\langle\delta h^2(t)\right\rangle = -\frac{2\lambda}{H}\left\langle\delta h^2(t)\right\rangle^2 + \frac{H^3}{4\pi^2}$$

SOLUTION:

$$\left\langle \delta h^2(t) \right
angle = rac{1}{\sqrt{-2\lambda}} rac{H^2}{2\pi} an \left(\sqrt{-2\lambda} rac{\mathcal{N}}{2\pi}
ight)$$

where $\mathcal{N} = Ht$.

Unstable potential accelerates growth of fluctuations relative to

$$\left\langle \delta h^2(t) \right\rangle = \frac{H^2}{4\pi^2} \mathcal{N} \qquad (\lambda = 0)$$

• In fact, diverges in finite time! $\mathcal{N}_{\max} = \frac{\pi^2}{\sqrt{-2\lambda}}$ \checkmark



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What are the implications of this divergence?

$$\delta h \sim \sqrt{HM_P} \Rightarrow \rho_\phi + \rho_h \sim 0 \text{ (or } \delta \rho/\rho \sim 1).$$

Moreover, "backreaction"—patch rapidly evolves to a singularity (in fact, this occurs soon after the patch leaves the slow-roll regime).

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Typical size of fluctuation characterized by $\langle \delta h^2(t) \rangle$...so such large fluctuations rare until $\langle \delta h^2(t) \rangle$ starts to diverge.

Resulting rare defects diluted.

However, once $\langle \delta h^2(t) \rangle$ does get large, significant proportion of patches will be fluctuating to backreacting regime.

When inflation ends, resulting universe exhibits large inhomogeneities, defects. Inconsistent with small perturbations in our Universe.

If collapsing patches come to dominate, entire space may become unstable, see Sekino, Shenker, Susskind [1003.1374].

So, in HF approximation, \mathcal{N}_{\max} is bound on \mathcal{N} . \checkmark

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Comments

① $\mathcal{N} < \mathcal{N}_{\mathrm{max}}$ necessary, but not sufficient...

Unstable patches present at end of inflation still need to be stabilized!

Assumed massless modes and slow-roll...

Only violated once $\mathcal{N} \sim \mathcal{N}_{\max}$.

3 No regulation of (unphysical) divergence

e.g., should throw away backreacting patches (or those exiting slow-roll)? Fortunately proportion only significant once $\mathcal{N} \sim \mathcal{N}_{\rm max}$.

- Field treated as Gaussian stochastic variable Non-Gaussianity relevant for most unstable (diverging, crunching) patches. Hence, can significantly impact inflationary scenario—see (III).
- \bigcirc Constant λ

Let's turn to this now in approach (II).



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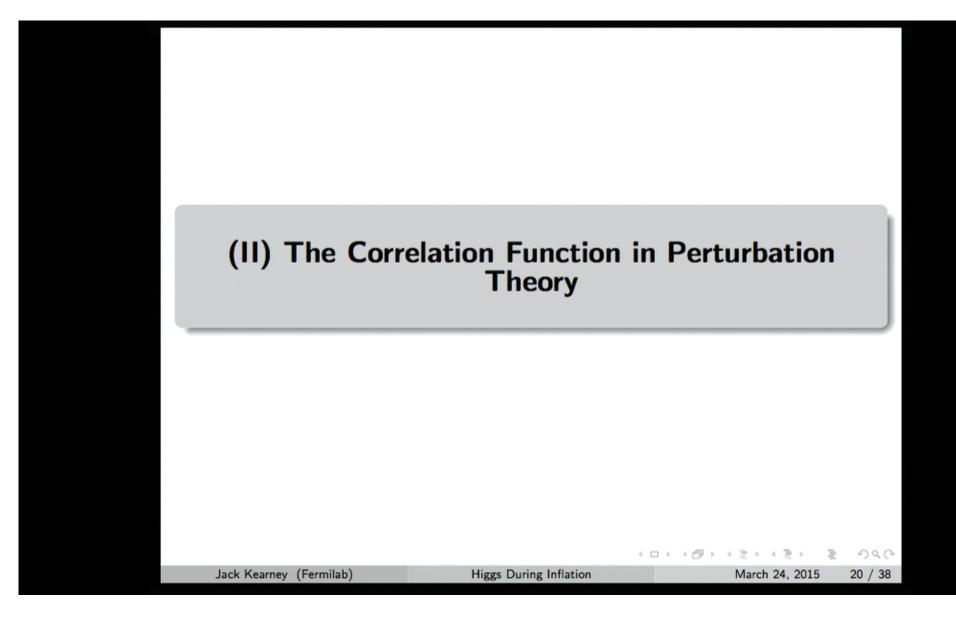
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Goals for (II)

Understand how a stochastic approach such as HF captures results of a "more traditional" perturbative calculation, and

② Elucidate how to extend toy model to incorporate rest of SM.

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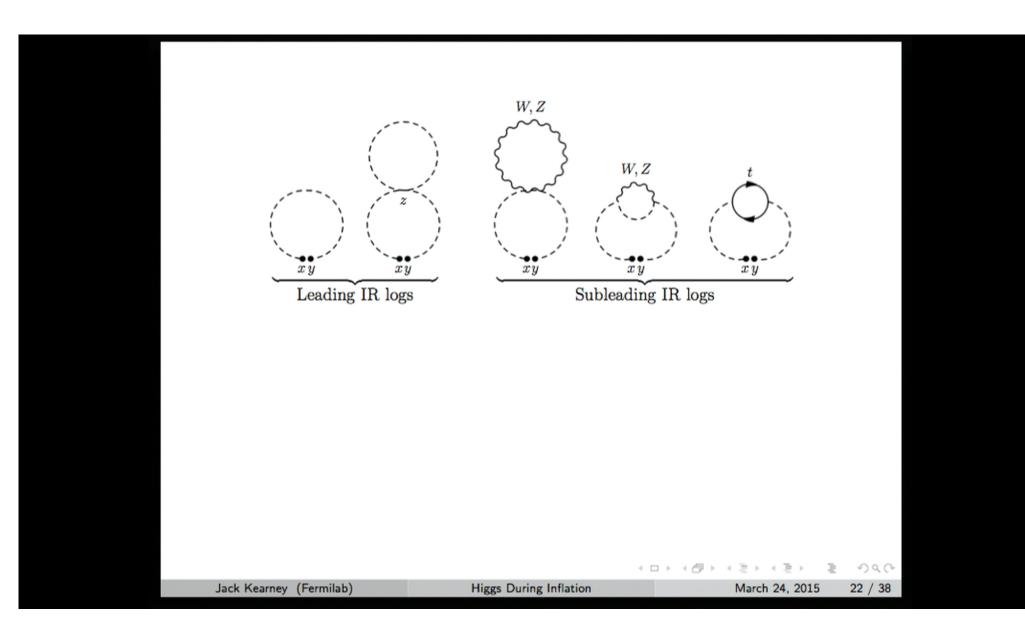
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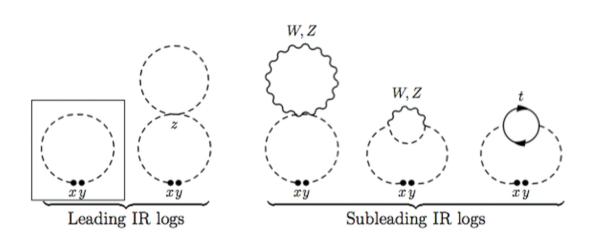
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Calculate first diagram, take "coincident limit" $|\mathbf{x} - \mathbf{y}| \approx (aH)^{-1}$.

Leading IR behavior given by

$$\left\langle \delta h^2(t) \right\rangle pprox \frac{H^2 \mathcal{N}}{4\pi^2} + \dots$$

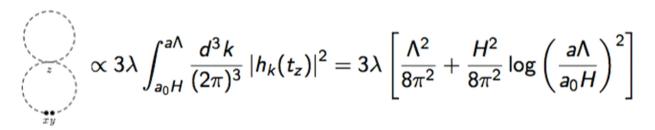
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One-loop correction with UV (& IR) cutoff



Two important types of terms

UV: Divergences as in Minkowski space (with H relevant energy scale), cancelled by local counterterms

$$\delta m^2(\mu) = -3\lambda(\mu) \frac{\Lambda^2}{8\pi^2}, \qquad 12\delta \xi = -\frac{3\lambda(\mu)}{4\pi^2} \log\left(\frac{\Lambda^2}{\mu^2}\right)$$

fixing renormalization conditions. $\mu = H$ resums logs.

② IR: logs contribute to growth of correlator, $\log \frac{a}{a_0} = \mathcal{N}$.



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Where do the all-important IR logarithms come from?

Light, minimally-coupled scalar wave functions unsuppressed outside horizon, so (t, k) integrals produce IR logarithms.

Growth of correlator enhanced (for $\lambda < 0$) by scalar loops

$$\left\langle \delta h^2(t) \right\rangle pprox rac{H^2 \mathcal{N}}{4\pi^2} - rac{\lambda H^2 \mathcal{N}^3}{24\pi^2} + \dots$$

PT breaks down for $\mathcal{N}>\pi\sqrt{\frac{6}{|\lambda|}}\gtrsim\mathcal{N}_{\max}!$ Moreover, for $\sqrt{-\lambda}\mathcal{N}\ll 1$,

$$\left\langle \delta h^2(t) \right\rangle_{\mathrm{HF}} pprox rac{H^2 \mathcal{N}}{4\pi^2} - rac{\lambda H^2 \mathcal{N}^3}{24\pi^2} + \dots$$

So stochastic approach resums leading IR logarithms. ✓

See, e.g., Tsamis, Woodard [gr-qc/0505115], Garbrecht, Rigopoulos, Zhu [1310.0367]

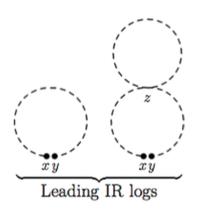


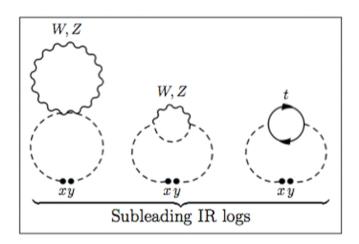
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So what about the rest of the Standard Model?





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So what about the rest of the Standard Model?

Transverse gauge bosons, fermions damped outside horizon \Rightarrow do not directly contribute to leading IR logarithms...

Leading contributions calculated including only scalar loops

...but high-energy subhorizon modes do see local (flat) spacetime!

• Generate usual logarithms of form $\log \left(\frac{\mu^2}{H^2}\right)$.

e.g., Veff in dS space: Herranen, Markkanen, Nurmi, Rajantie [1407.3141]

• Choose $\mu \approx H$ to control PT, resum large logarithms.

So $\lambda = RG$ -improved SM quartic evaluated at $\mu = H$, $\lambda(\mu = H)$. \checkmark

Gauge-invariant, physical!



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One Subtelty: Would-be GBs?

$$\mathcal{H} = rac{1}{\sqrt{2}} \left(egin{array}{c} \chi_1 + i \chi_2 \ ar{h} + \delta h + i \chi_3 \end{array}
ight)$$

 χ_i eaten for $\langle \mathcal{H}^{\dagger} \mathcal{H} \rangle \neq 0$, but light for $g^2 \langle \mathcal{H}^{\dagger} \mathcal{H} \rangle \lesssim H^2$.

If remain light,

$$\langle \chi_i^2 \rangle \approx \langle \delta h^2 \rangle \qquad \Rightarrow \qquad \lambda \to 2\lambda,$$

But, this is violated before PT breaks down [i.e., contributions at $\mathcal{O}(\lambda g^2)$].

"Actual" SM limit in Gaussian approximation:

$$rac{\pi^2}{2\sqrt{-\lambda(H)}}\!\lesssim\!\mathcal{N}_{
m max}\!\lesssimrac{\pi^2}{\sqrt{-2\lambda(H)}}$$



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The Fokker-Planck Equation

Calculates $P(\delta h, t) \equiv$ probability to observe δh in a patch at time t

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \delta h} \left[\frac{V'(\delta h)}{3H} P + \frac{H^3}{8\pi^2} \frac{\partial P}{\partial \delta h} \right]$$

Related to correlation functions via

$$\langle \delta h^n(t) \rangle = \int d\delta h (\delta h)^n P(\delta h, t)$$

Advantage relative to HF? Incorporate non-Gaussianity.

Lessons learned from (I) & (II):

$$V(h) = \frac{\lambda}{4}h^4$$
 with $\lambda = \lambda(H)$

should capture leading IR divergent behavior for SM Higgs.

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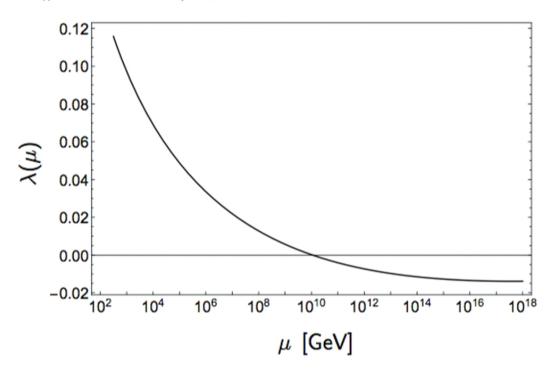
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Quick Aside: What is $\lambda(H)$?

e.g., for $m_h = 125.7 \text{ GeV}$, $m_t = 173.34 \text{ GeV}$,



In the SM, find $-0.015 \lesssim \lambda(H) \lesssim -0.005$.

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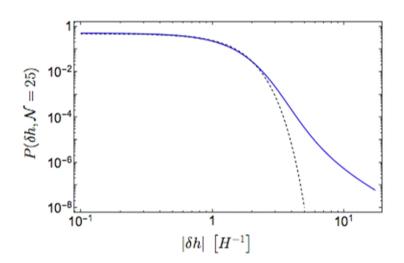
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"A Tale in the Tails:" the Impact of NG

Unstable patches with $\delta h \gtrsim \delta h_{\rm classical} \equiv \left(\frac{3}{-2\pi\lambda}\right)^{1/3} H$ quickly roll away



Taking
$$\lambda(H) = -0.01 \ (\Rightarrow \delta h_{\rm cl} \approx 4H)$$
:

Fokker-Planck

Hartree-Fock with

$$\left<\delta h^2(t)
ight> = rac{1}{\sqrt{-2\lambda}}rac{H^2}{2\pi} an\left(\sqrt{-2\lambda}rac{\mathcal{N}}{2\pi}
ight)$$

These patches give large contributions to correlation functions

Divergence of $\langle \delta h^2(t) \rangle \not\Rightarrow$ large inhomogeneities necessarily...



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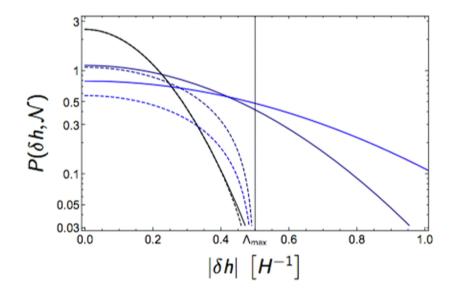
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An Aside on Boundary Conditions

Inappropriately truncating FP solution can artifically suppress P, P_{Λ} .

e.g., for $H = 2\Lambda_{\text{max}}$



Dotted assumes

$$P(|\delta h| \ge \Lambda_{\max}, \mathcal{N}) = 0$$

For
$$\mathcal{N} = 1, 5, 10$$
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So what can we learn from FP about our Universe?

Depends on the behavior of the unstable patches!

ASSUMPTION 1: All unstable patches rapidly but benignly crunch.

What proportion of patches survive in EW vacuum at end of inflation?

$$P_{\Lambda} \equiv \int_{-\Lambda_{
m max}}^{\Lambda_{
m max}} d\delta h \, P(\delta h, t_e)$$

Espinosa, Giudice, Riotto [0710.2484]

Correctly computing P_{Λ} reveals you can always inflate long enough to replace lost patches even if $H \gg \Lambda_{\rm max}$

Hook, JK, Shakya, Zurek [1404.5953]

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But was that the right/a reasonable assumption?

- ① Patches with $|\delta h| \gtrsim \Lambda_{\rm max}$ don't necessarily crunch rapidly until they exit slow-roll for $|\delta h| \gtrsim \delta h_c \equiv \left(\frac{3}{-\lambda}\right)^{1/2} H$,
- ② Patches that crunch at the end of inflation are not necessarily benign—they leave undiluted defects.



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Assumption 2: Patches exiting slow-roll always crunch rapidly.

What proportion surviving space is becoming unstable?

$$f_{\mathcal{N}} \equiv rac{\int_{-\delta h_c}^{\delta h_c} d\delta h \left\{ P(\delta h, \mathcal{N}) - P(\delta h, \mathcal{N} - 1)
ight\}}{\int_{-\delta h_c}^{\delta h_c} d\delta h P(\delta h, \mathcal{N} - 1)}.$$

At end of inflation, if unstable patches with $\Lambda_{\text{max}} \leq |\delta h| \leq \delta h_c$ are stabilized by reheating, what is minimum level of inhomogeneity/defects?

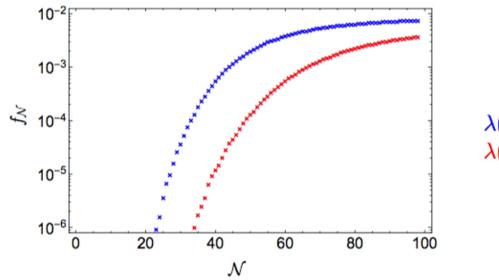
JK, Yoo, Zurek [1503.05193]



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$$\lambda(H) = -0.010$$

 $\lambda(H) = -0.005$

$\lambda(H)$	$\mathcal{N}_{\mathrm{max,HF}}$	$f_{N} = 10^{-5}$		$f_{N} = 10^{-3}$		$f_{\mathcal{N}}=10^{-2}$	
						$\mathcal{N}_{ ext{FP}}$	$\mathcal{N}_{\mathrm{HF}}$
-0.005	$70 \lesssim \mathcal{N}_{ m max} \lesssim 99$	40	95	60	96	-	97
-0.010	$49 \lesssim \mathcal{N}_{\rm max} \lesssim 70$	27	66	44	67	_	68
-0.015	$40 \lesssim \mathcal{N}_{\mathrm{max}} \lesssim 57$	22	53	35	55	86	55

N.B. Assuming slow-roll breaks down before backreaction occurs.

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Bounding High-Scale Inflation?

If our Universe requires $f_{\mathcal{N}} < f_{\mathcal{N}}^{\mathrm{crit}}$, must have $\mathcal{N} < \mathcal{N}_{\mathrm{crit}}$

Analogous to treating $\mathcal{N}_{\mathrm{max},\mathrm{HF}}$ as bound.



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Bounding High-Scale Inflation?

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Analogous to treating $\mathcal{N}_{\mathrm{max,HF}}$ as bound.

e.g., if $f_N \sim \mathcal{O}(0.5)$ required for space as a whole to crunch, instability never terminates inflation.

Consistent with results of Hook, JK, Shakya, Zurek [1404.5953]

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Higgs During Inflation

March 24, 2015

Transitions to the unstable regime do not appear to abort inflation.

- Unstable patches $(\delta h > \Lambda_I)$ can still inflate for a significant period
- Really unstable patches likely crunch rapidly...but causally-disconnected patches evolve independently, and inflation dilutes any resulting defects
- As proportion of rapidly crunching patches remains $\ll \mathcal{O}(1)$, do not suspect space as a whole to become unstable
- ... Inflation can proceed, perhaps producing our Universe.

HOWEVER, at the end of inflation

- unstable patches will be present—must be stabilized by reheating s.t. do not nucleate and destroy EW vacuum (as patches come back into causal contact)
- Moreover, some patches crunch will crunch so rapidly they cannot to be stabilized Do not necessarily need to be...but instability (re)generates inhomogeneities!
- Extent to which this is a problem depends very much on nature/behavior of resulting defects
- :. Instability does imply bounds on reheating, post-inflationary evolution...

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So the instability

- does not prevent high-scale inflation from occurring
- ② likely generates defects, but these do not necessarily make the resulting universe look inhomogeneous or unlike ours

As such,

The SM Higgs instability is not necessarily incompatible with our Universe.[†]

[†] But more study of defects produced at the end of inflation, reheating, Higgs dynamics during post-inflationary epoch &c. definitely warranted...

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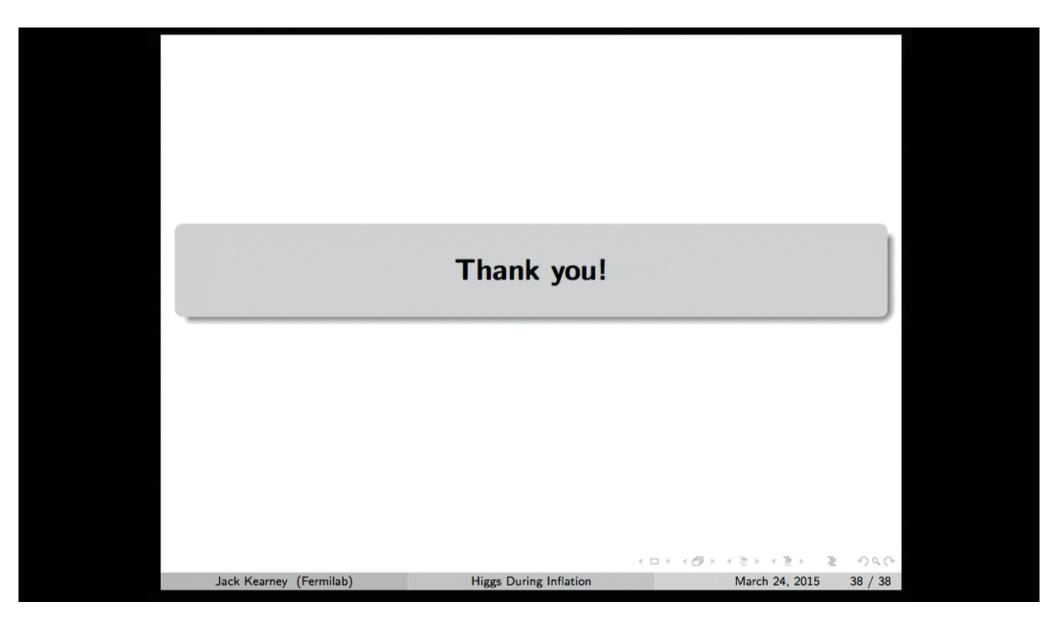
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