Title: Some Information for Assembling Space

Date: Mar 18, 2015 11:00 AM

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Abstract: <p>In holographic duality a gravitational spacetime emerges as an equivalent description of a lower-dimensional conformal field theory (CFT) living on the asymptotic boundary. Traditionally, the dimension not present in the CFT is interpreted in terms of its Renormalisation Group flow. In this talk I exploit the relation between boundary entanglement entropies and bulk minimal surfaces to define a quantitative framework for the holographic Renormalisation Group, in which quantum information theory plays a fundamental role. I will provide operational, quantitative, information-theoretic characterizations of several geometric concepts in asymptotically AdS3 spacetimes, including the radial direction and lengths of bulk curves.</p>

<p>Perhaps the biggest conceptual benefit of the information-theoretic framework for holographic Renormalisation Group is a quantitative correspondence between holography and the Multi-Scale Entanglement Renormalisation Ansatz (MERA). I also discuss prospects for understanding the near-horizon structure of black holes.</p>

Some information for assembling space

Bartłomiej Czech
Stanford University

Perimeter Institute, 18 March 2015


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Information in Black Hole physics

Quantum gravitational objects – **BLACK HOLES:**

- Large and massive ➔ gravitational
- Temperature ~ h ➔ quantum mechanical

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- Entropy ~ h^{-1} ➔ microscopic structure

Key feature of the problem – **INFORMATION PARADOX:**

1. Collapse a pure state to form a black hole
2. Wait until the black hole evaporates
3. All that's left is a mixed state – thermal radiation

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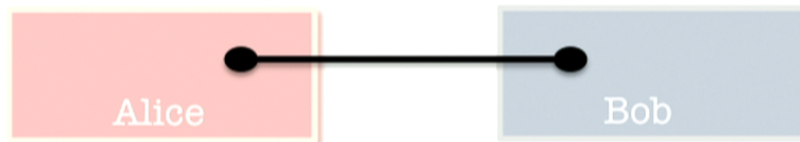
Information loss?

Information theory to prevent information loss

- Information ⟷ Entanglement between subsystems



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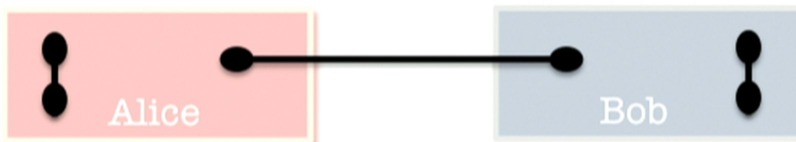

$|0\,0\rangle + |1\,1\rangle$


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|00> + |11>


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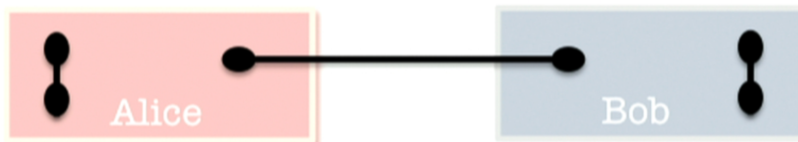

$|0\,0\rangle - |1\,1\rangle$

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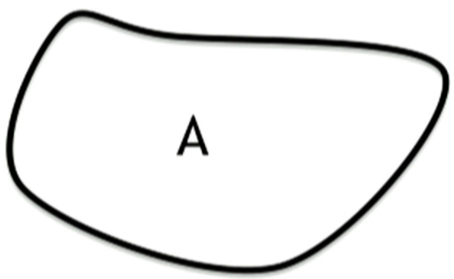

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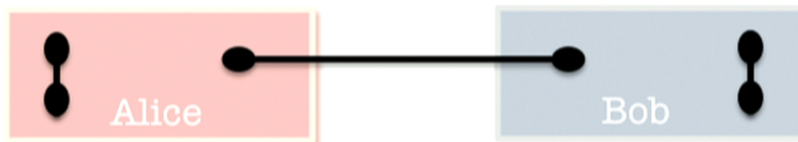

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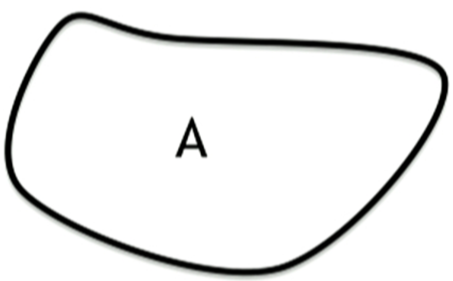
- Low energy states in local field theory obey the area law:
  - $S_{ent}(A) \sim Area(A)$ entanglement entropy



Bombelli et al., 1986; Holzhey et al., 1993; Srednicki, 1993

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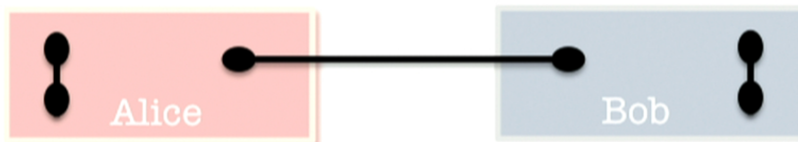

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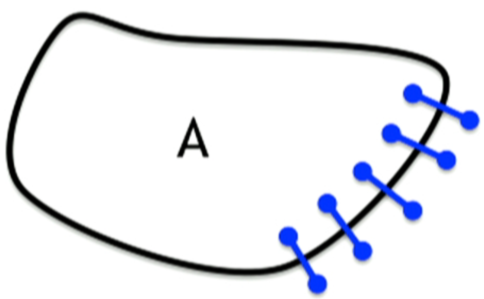

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➢ This quantity is divergent, because UV has infinitely many degrees of freedom

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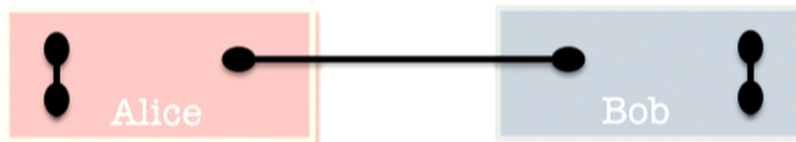

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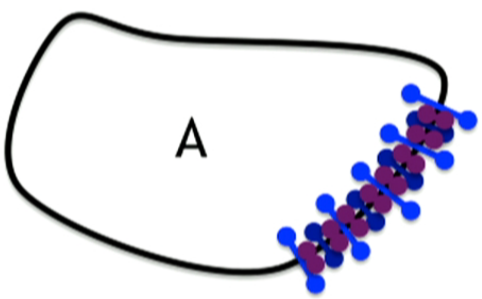

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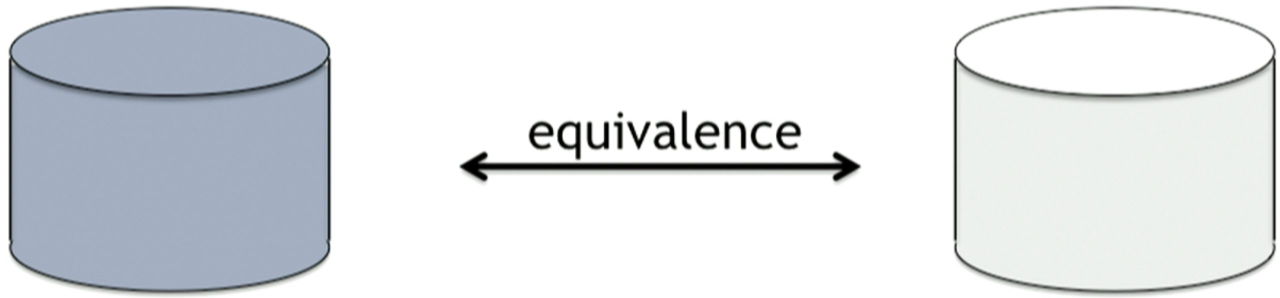
Entanglement Pattern → Map of Space

Van Raamsdonk, 2009; Bianchi-Myers 2012; Maldacena-Susskind, 2013

Quantitative framework: AdS/CFT correspondence



Gravity in AdS
(solid cylinder)

Conformal Field Theory (CFT)
on δAdS (hollow cylinder)

- States are asymptotically AdS geometries

- Degrees of freedom organized into N x N matrices

L_{AdS} (curvature radius) ~ $N^{\#}$ (matrix size)

Maldacena, 1997

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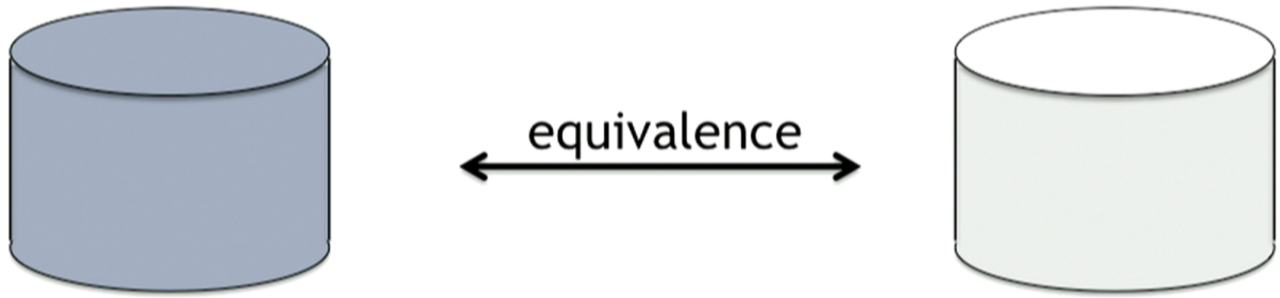

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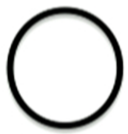
AdS/CFT correspondence: dictionary

Spatial slice of geometry (snapshot)		CFT state
	empty AdS *topologically trivial*	$\|0\rangle$ - pure state *entropy vanishes*

Global features of the state under the map:

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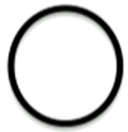 empty AdS *topologically trivial*	$\|0\rangle$ - pure state *entropy vanishes*
 black hole *horizon area*	$\mathcal{Z}^{-1}e^{-\beta H}$ - mixed state *thermal entropy*
 2-sided black hole (Einstein-Rosen bridge) *geometry extends past horizon, connecting the two boundaries*	$\mathcal{Z}^{-1/2}\sum_i e^{-\beta E_i/2}\|i\rangle_1 \otimes \|i\rangle_2$ - pure state in $\mathrm{CFT}_1 \otimes \mathrm{CFT}_2$ Maldacena, 2001 *entropy vanishes*

Global features of the state under the map:
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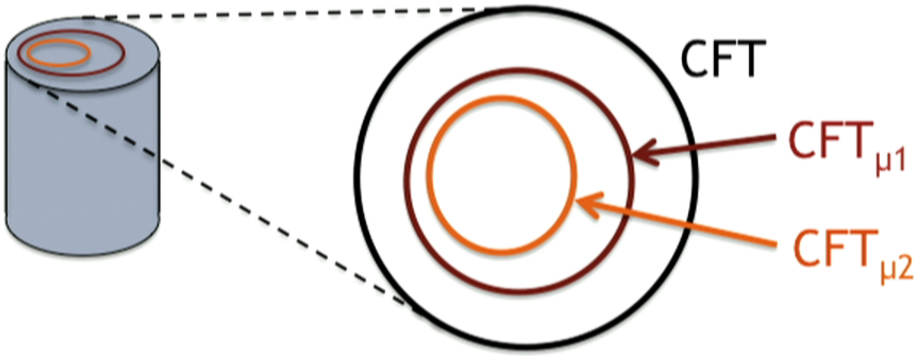
Local features of the geometry: holographic Renormalisation Group



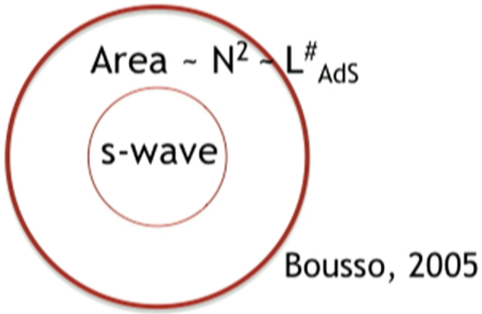
radial slices - define CFTs with cutoffs
radial direction – RG scale in CFT

How to make holographic RG quantitatively accurate?

- Coarse grain away all spatial dependence in CFT.

  This defines the s-wave sector of CFT, theory of N x N matrices.

  ➔ Entropy (s-wave sector) ~ N^2



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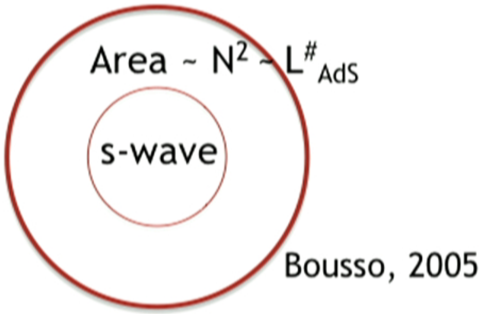

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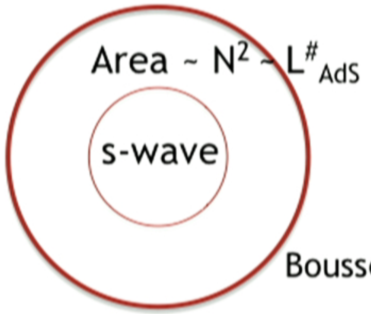
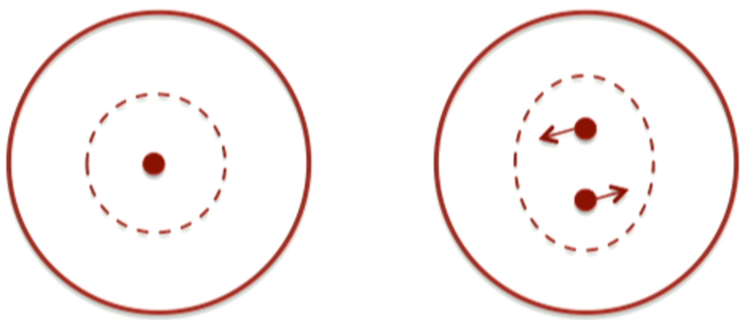


Bousso, 2005

- On sub-AdS scales, simple coarse-graining doesn't do the job

- Where to draw the appropriate radial slices?



- Describe two states in CFT_μ

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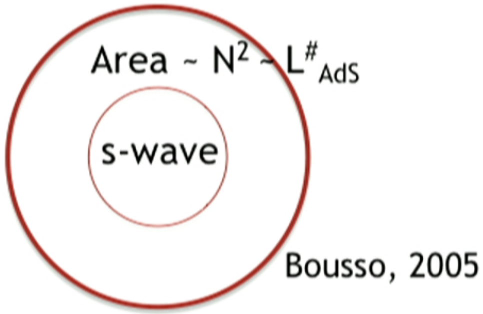

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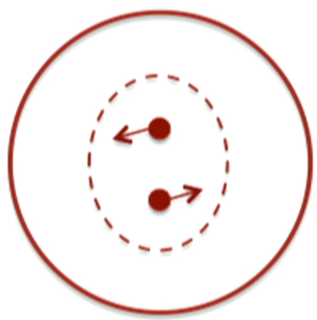

- Describe two states in CFT_μ
- Holographic RG must tell us how to coarse grain every CFT state individually

Toward a quantitative framework

Consider entanglement entropies
of boundary subsystems
of the dual field theory:

Ryu-Takayanagi, 2006

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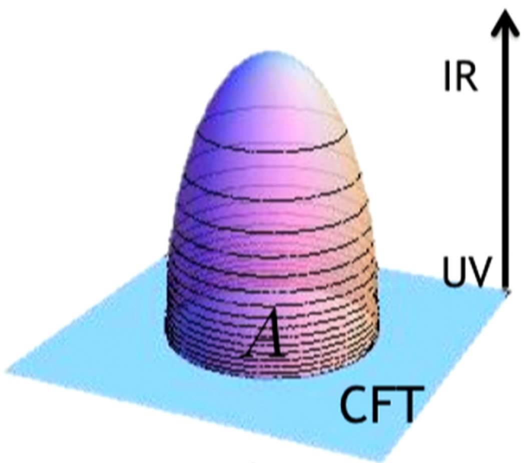
$$S(A) = \frac{\min \mathrm{Area}_{\to \partial A}}{4G}$$



snapshot at t=0

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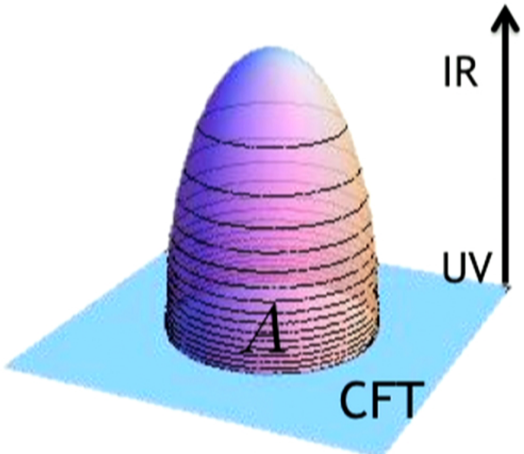

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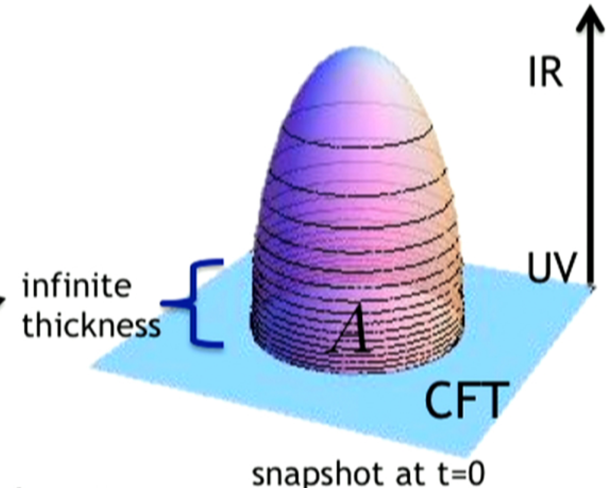
Toward a quantitative framework

Consider entanglement entropies of boundary subsystems of the dual field theory:

$$S(A) = \frac{\min \mathrm{Area}_{\to \partial A}}{4G}$$

- S_{ent} is divergent
- Divergence proportional to ∂A (reproduces area law on the boundary)
- Divergence arises because there are infinitely many degrees of freedom tucked at the boundary



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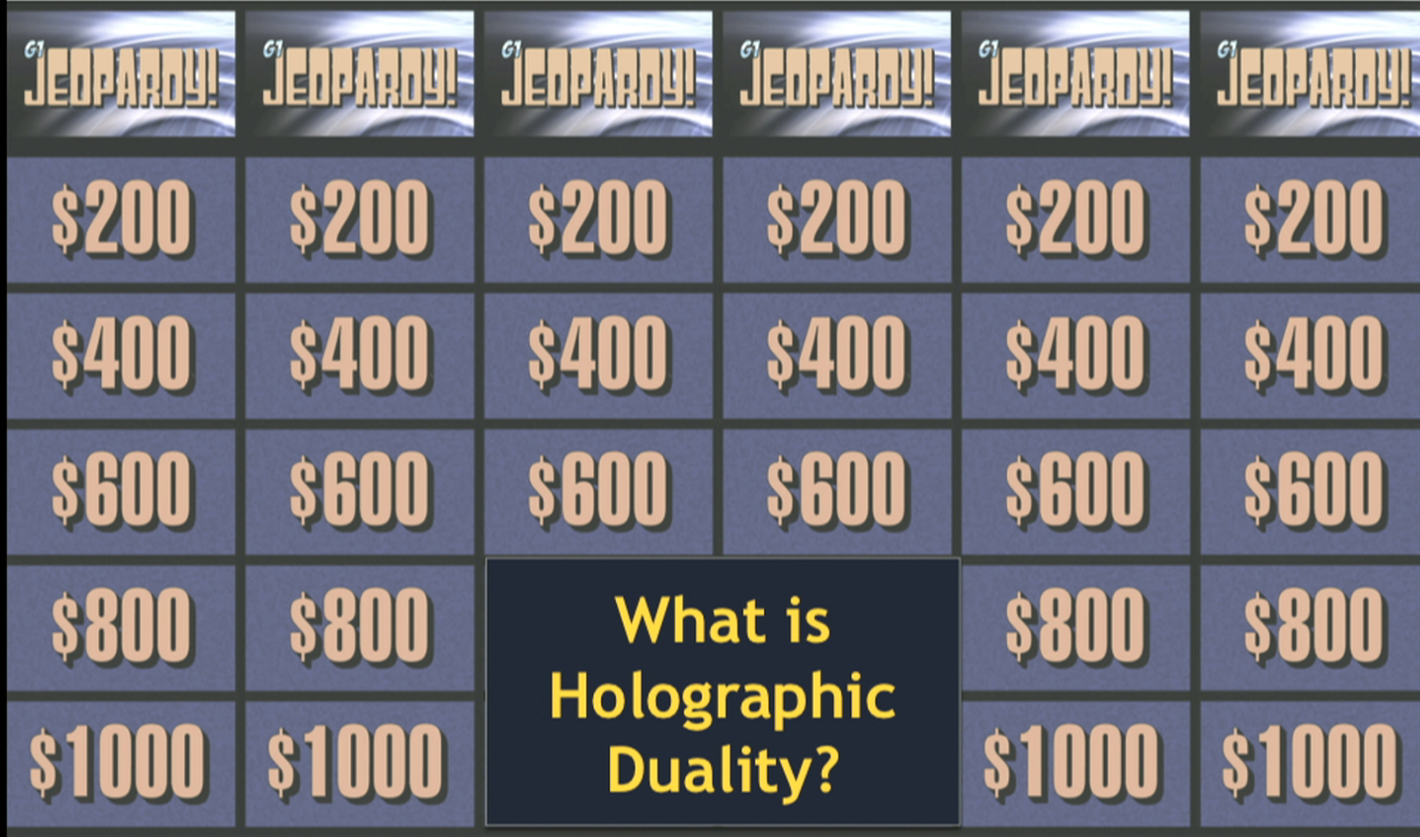
What question does holography answer?



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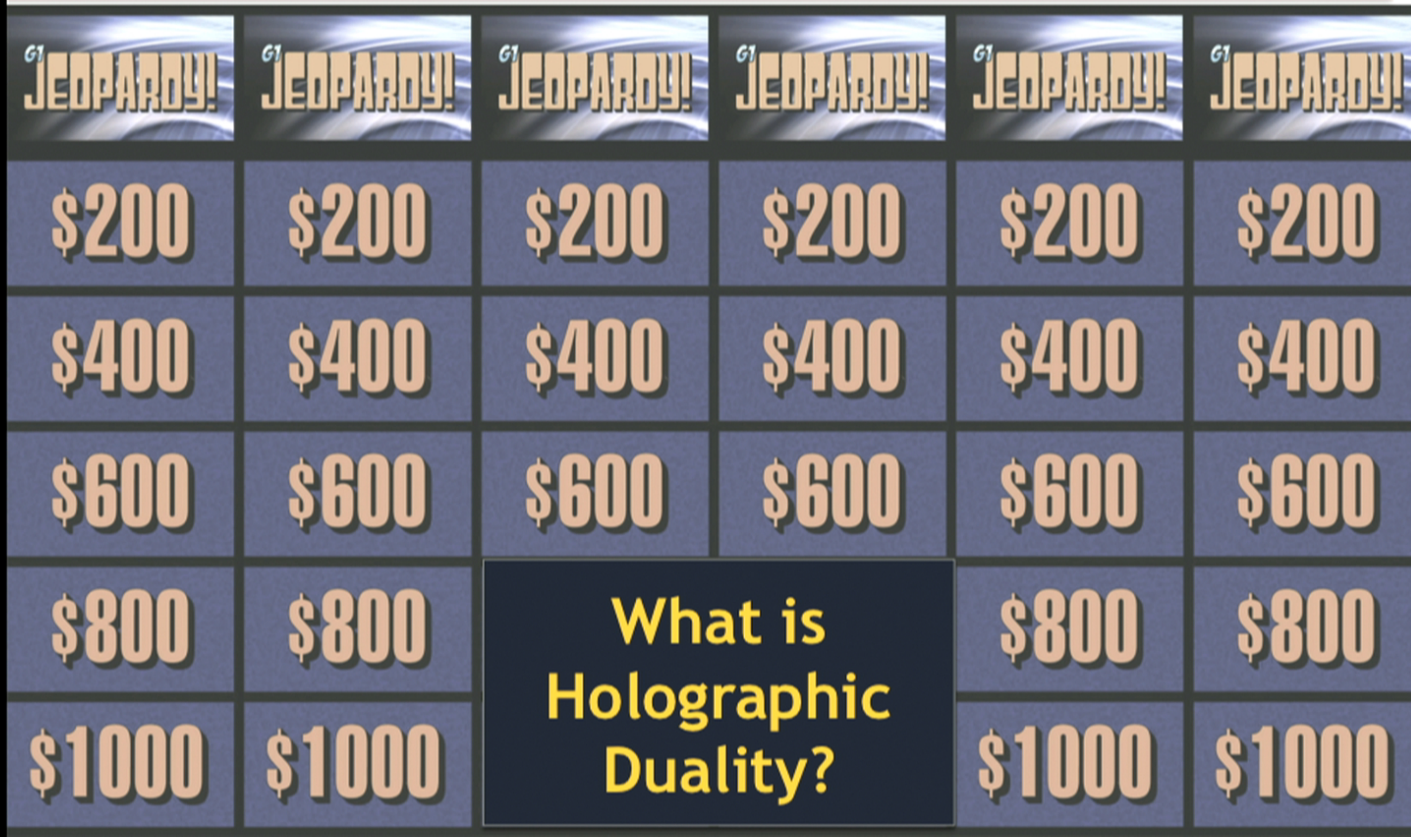

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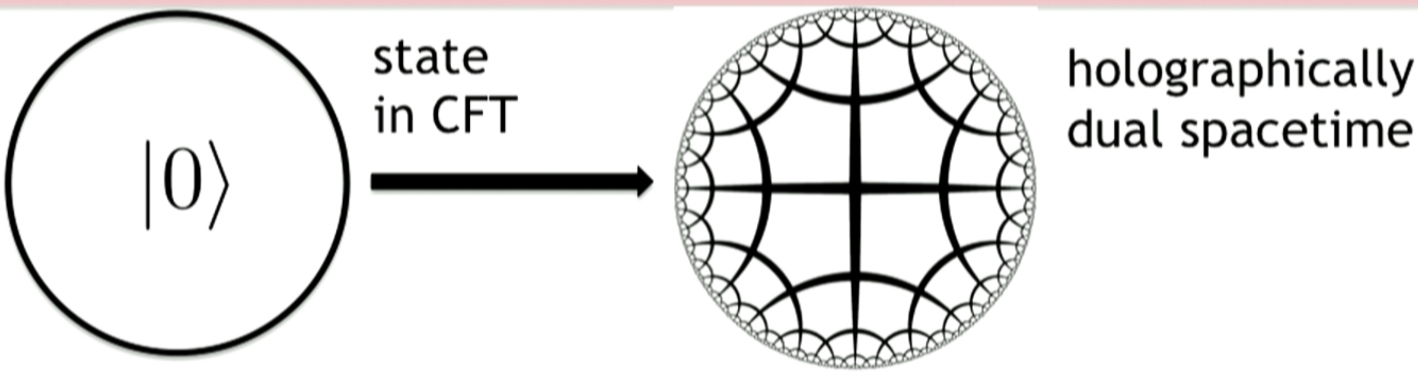

- map of entanglement of CFT state
- organized by scale
- helpful for understanding condensed matter systems

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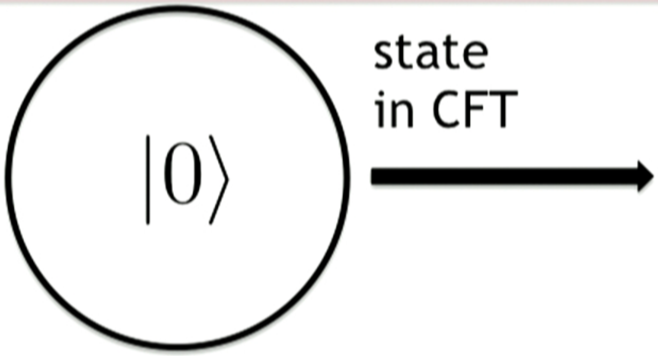
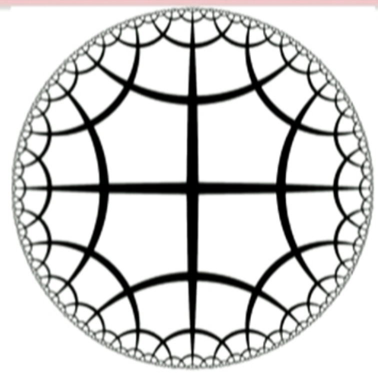


state in CFT



holographically dual spacetime

- map of entanglement of CFT state
- organized by scale
- helpful for understanding condensed matter systems

∴ Holography solves the same problems as tensor network ansätze, particularly **Multi-Scale Entanglement Renormalization Ansatz (MERA)** Vidal, 2006

∴ There should be a relation between holographic duality and tensor networks Swingle, 2009

State of the Union, circa 2012

Goal: Understand the bulk in AdS/CFT correspondence

- Pay attention to how quantum information / entanglement in the boundary theory is organized in the bulk

Entanglement Pattern
→ Map of Space

- The final picture must involve variations of scale on the boundary

- It should apply to every [reasonable] state, not just the vacuum

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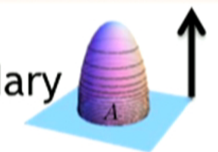

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- We should make a connection with tensor networks

Holographic RG made quantitative

- Restrict to AdS_3/CFT_2

Czech et al., 2013

Holographic RG made quantitative

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$$ds^2 = \frac{dr^2}{1+r^2} + r^2 d\theta^2$$

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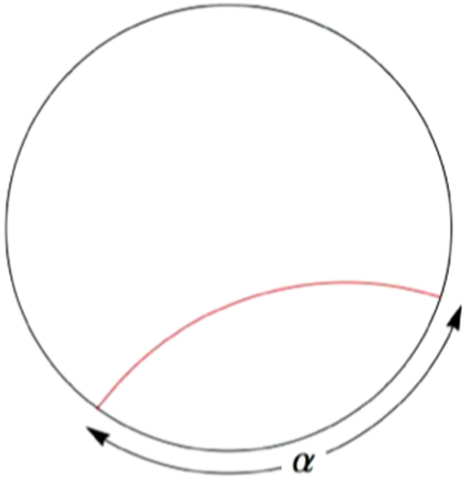
- Question: what is r in CFT?

- draw a minimal surface



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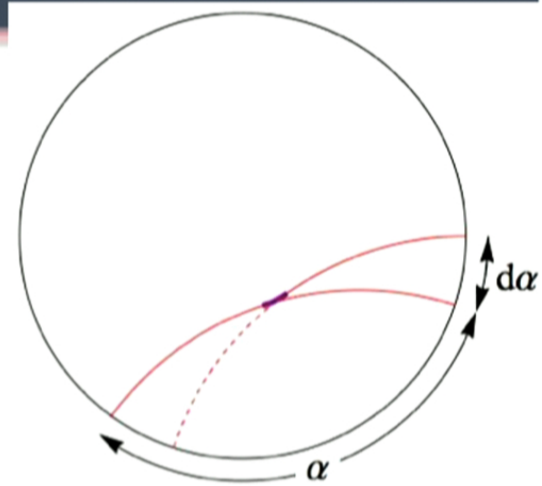

- draw a geodesic spanning an interval of size α
- draw a larger one, with α + dα
- if dα << 1, the new surface is the same at the old one plus **a flat top**

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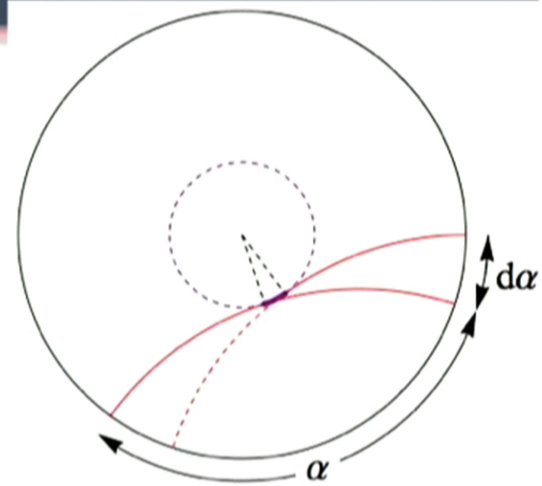

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- when you add such flat pieces, you must get 2πr

➡ $S_{\mathrm{ent}}(\alpha + d\alpha) - S_{\mathrm{ent}}(\alpha) = r\, d\alpha = S(d\alpha|\alpha)$ ← conditional entropy

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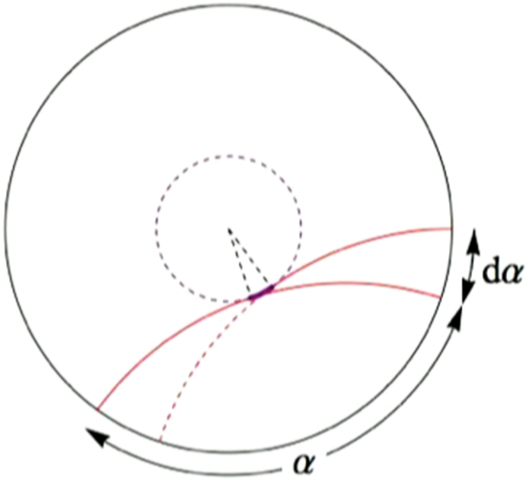

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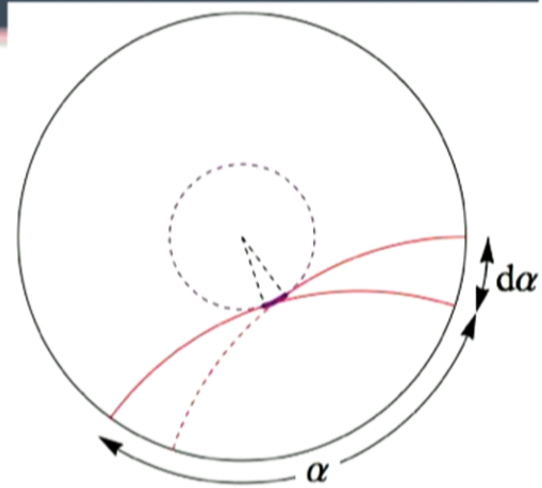

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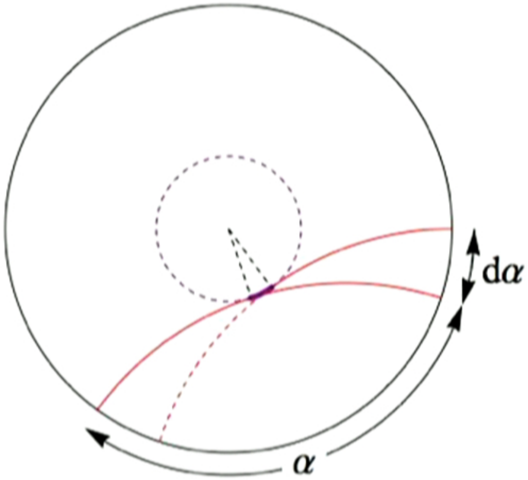

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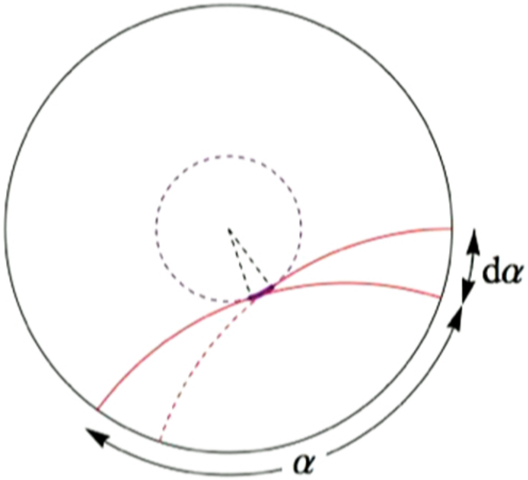

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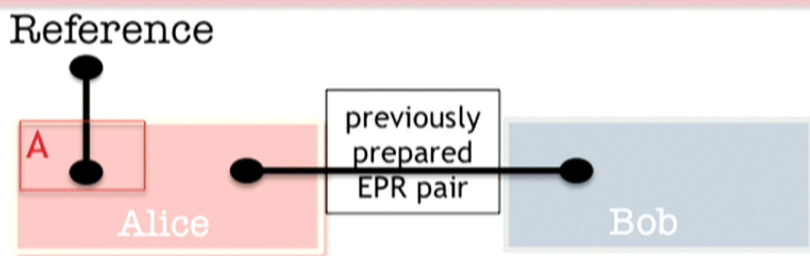
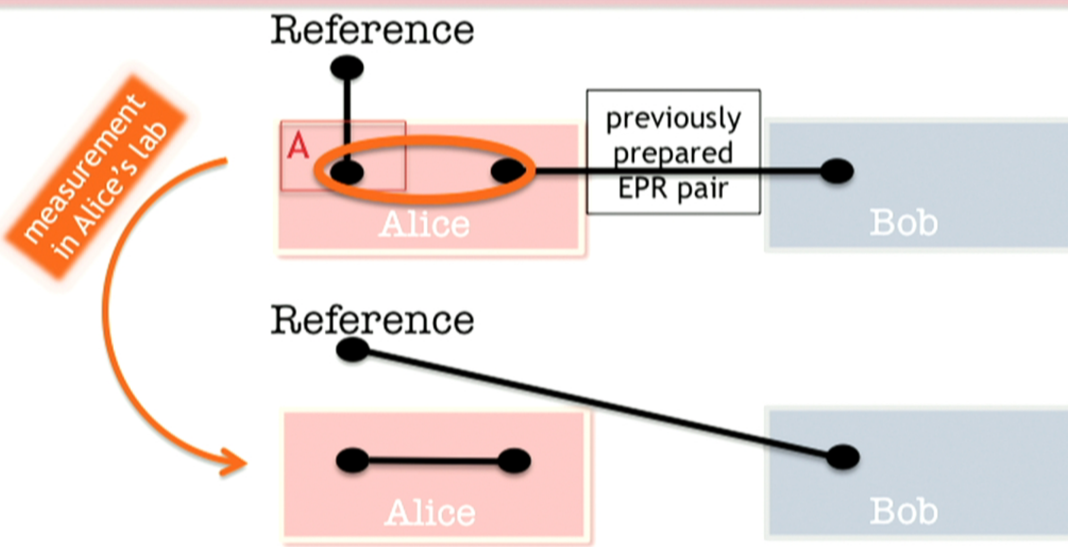
Entanglement Entropy – operationalised



Bennett et al., 1993

Entanglement Entropy – operationalised



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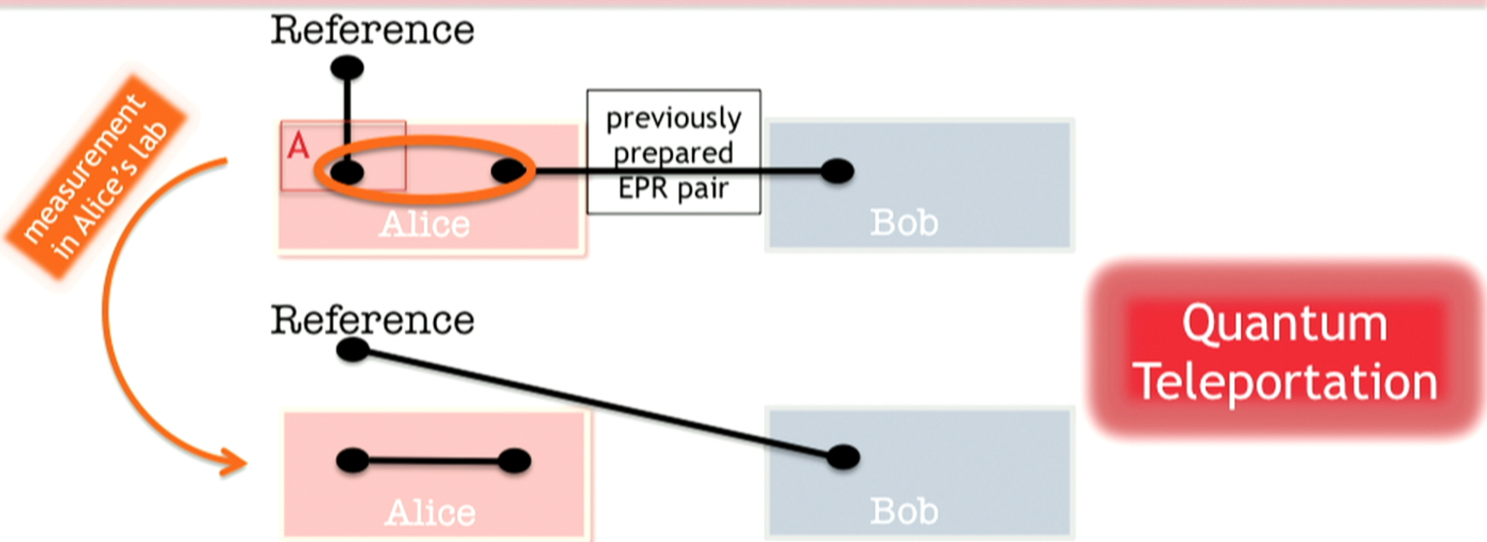


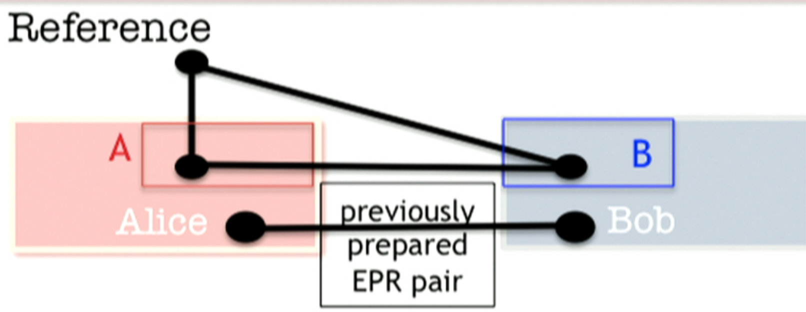
What happened?

- Alice made a local measurement
- The entanglement of "Reference" was transferred to Bob
- 1 pre-existing EPR pair was used ➔ currency for quantum communication

➡ **Cost of transfer = 1 EPR pair** = S(A)

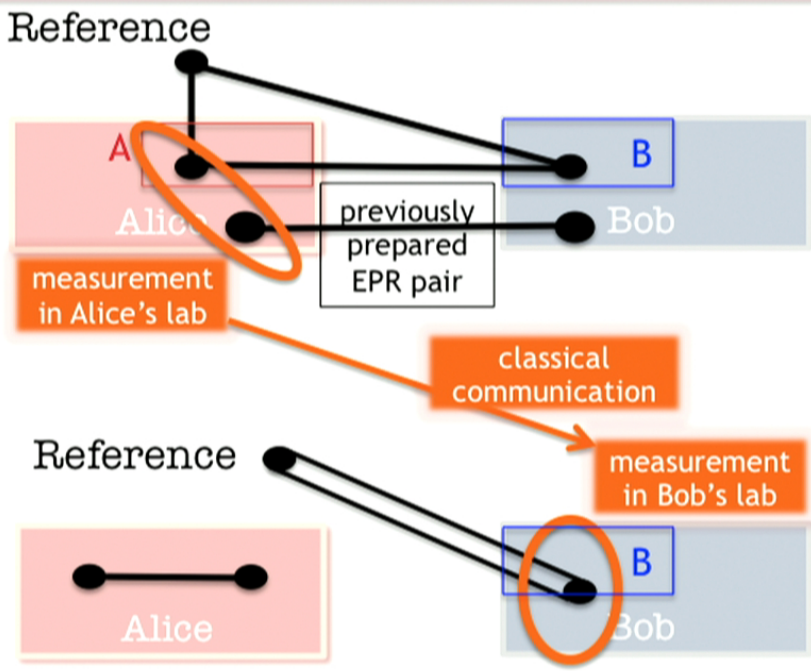
Bennett et al., 1993

Conditional Entropy - operationalised



Horodecki et al., 2005

Conditional Entropy - operationalised



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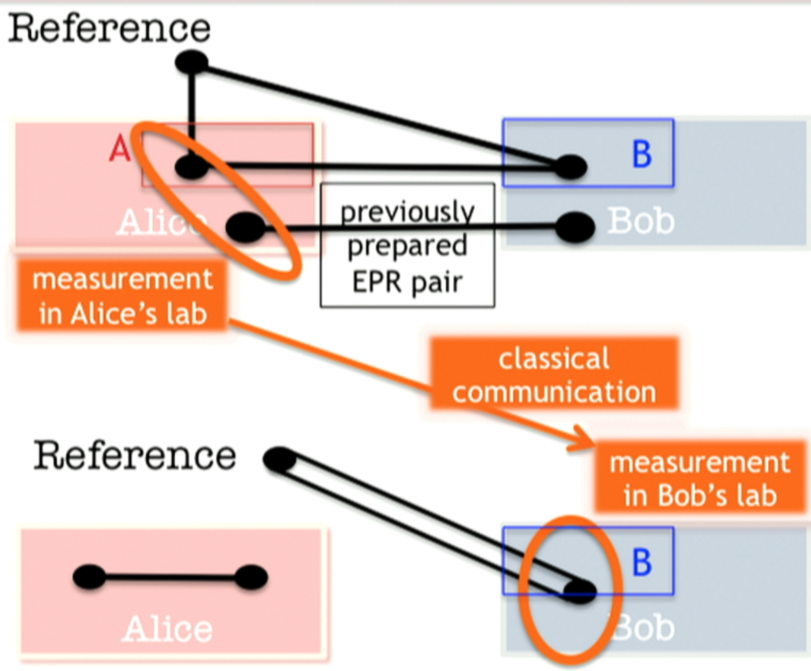
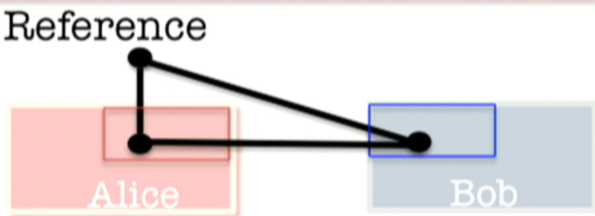


Horodecki et al., 2005

The meaning of the radial direction

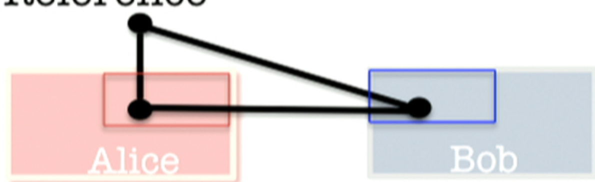


cost = S(A|B) = S(AB) - S(B)
"cost" of sending the missing data
to purify Reference

<u>Radial direction:</u>

Czech et al., 2014

The meaning of the radial direction



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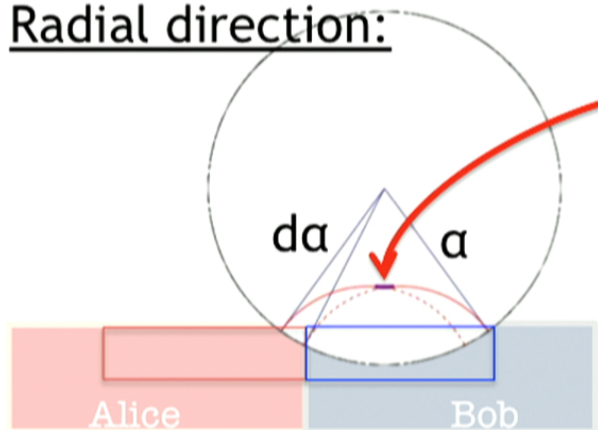

infrared correlations

"cost" of sending data about correlations at scales <u>between</u> α and α + dα:

$$r\,d\alpha = S(d\alpha|\alpha)$$
$$= S_{\text{ent}}(\alpha + d\alpha) - S_{\text{ent}}(\alpha)$$

Czech et al., 2014

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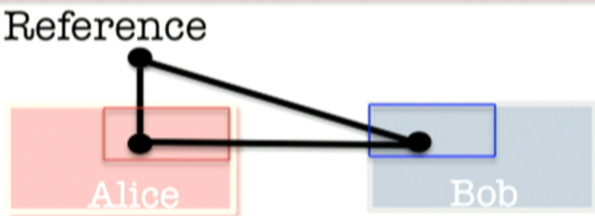

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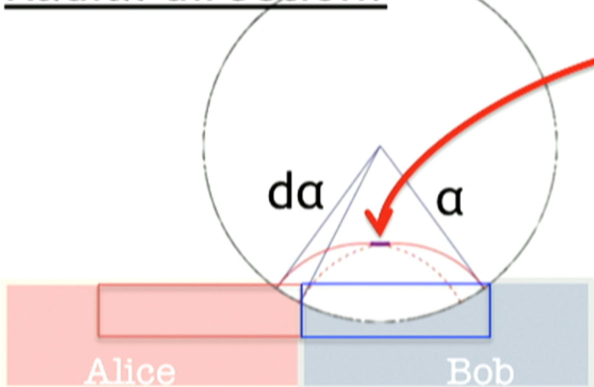

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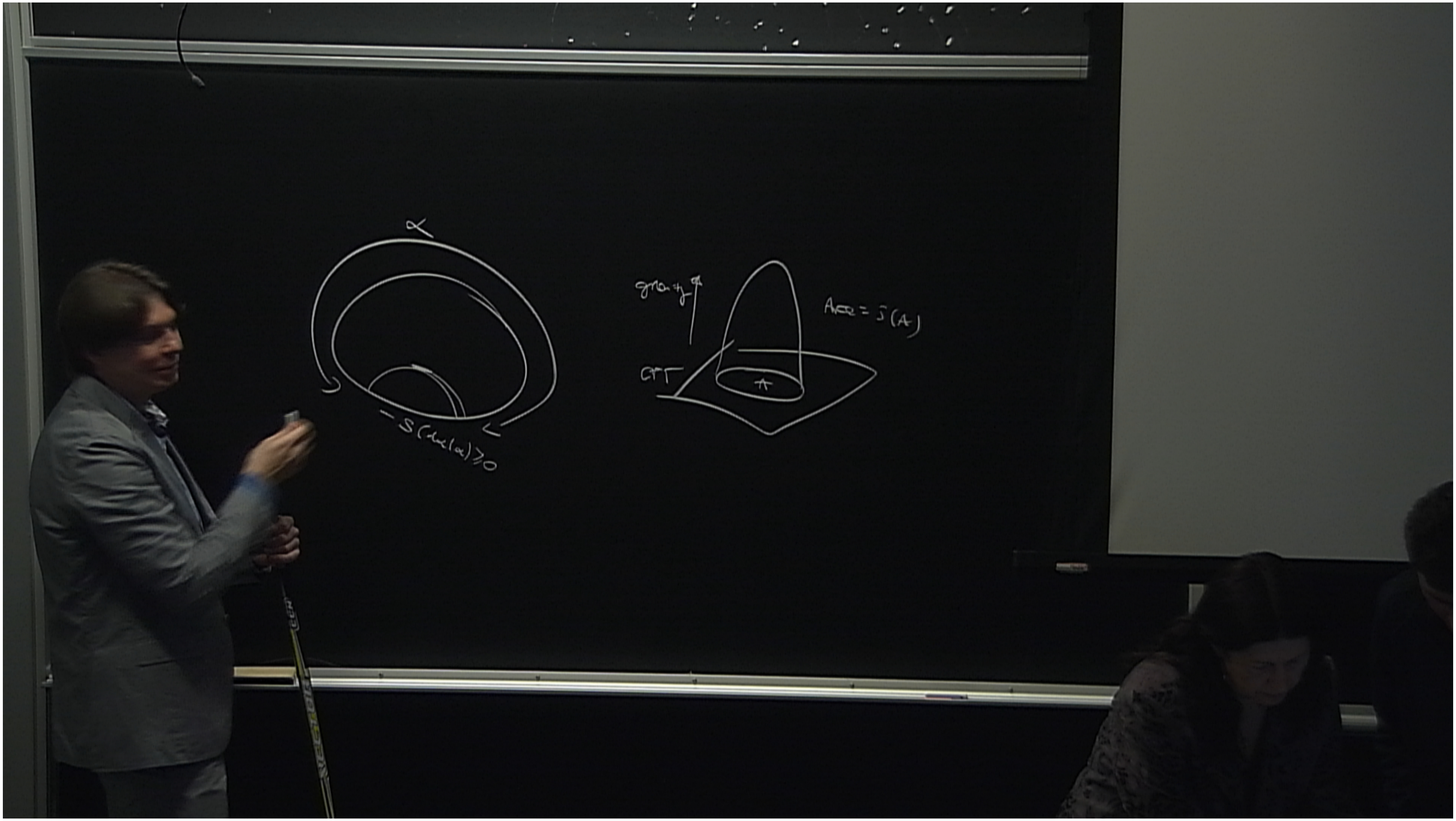
∴ Operational interpretation of the radial direction (RG) in quantum information theory

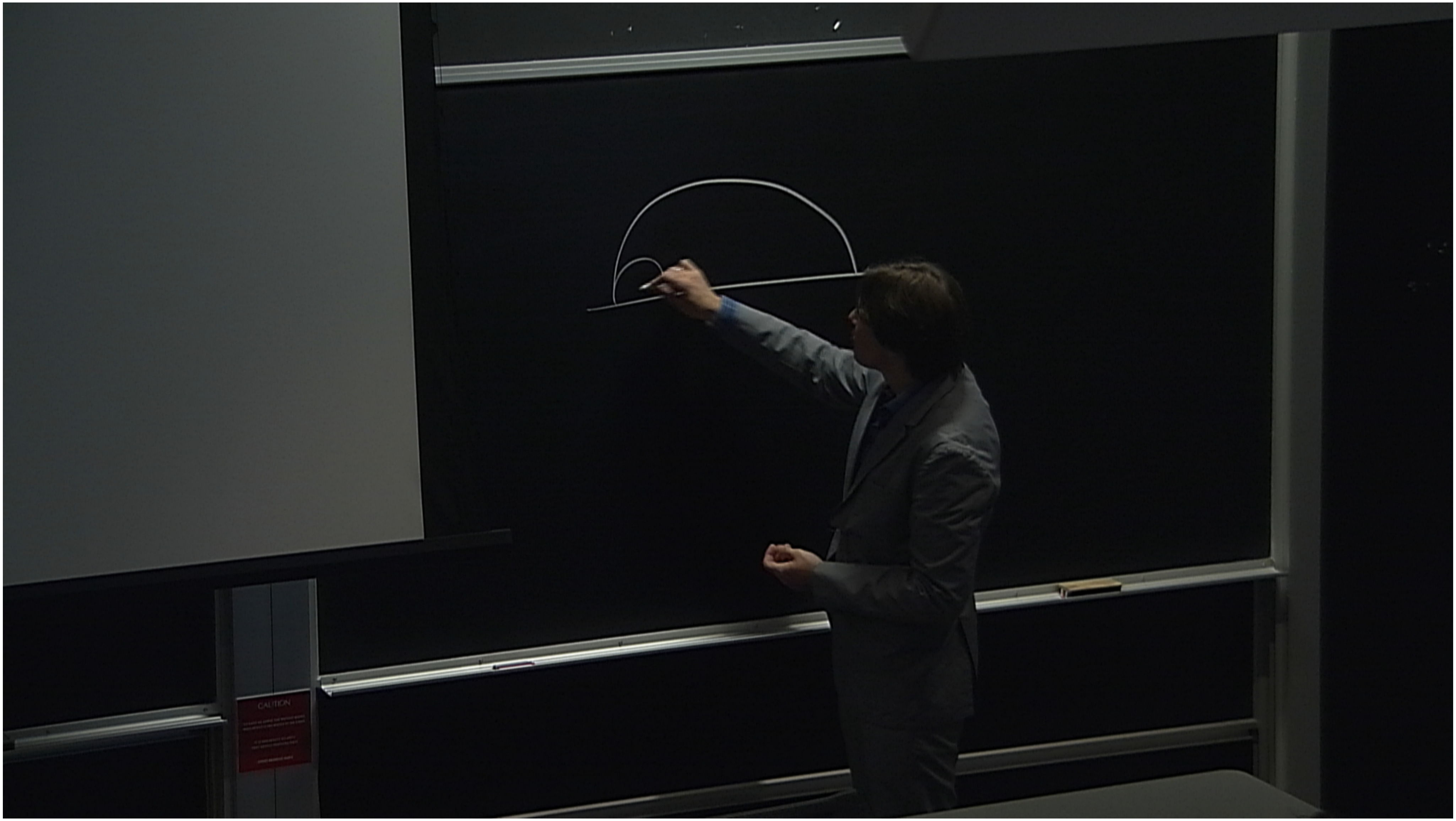
Czech et al., 2014

Measure lengths of curves

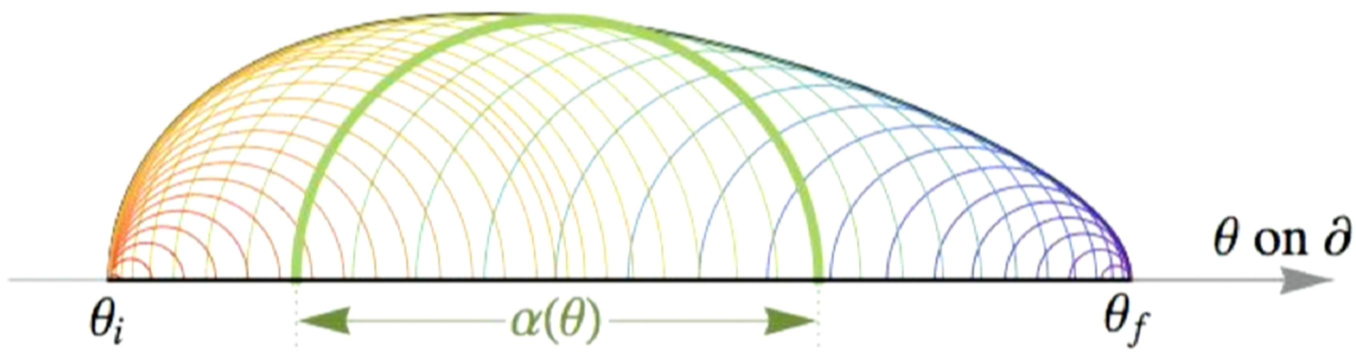
Czech et al., 2014



CFT



Measure lengths of curves



Czech et al., 2014

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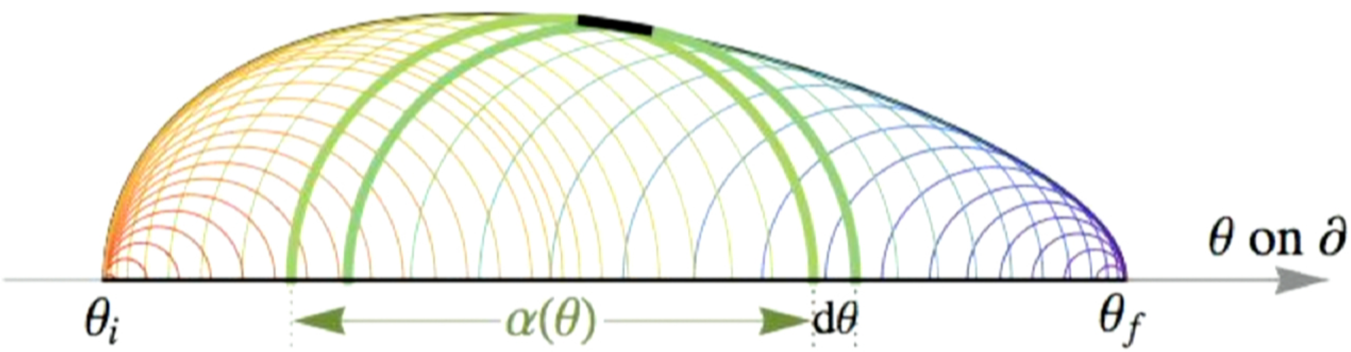

$$r\,d\theta = S\big(d\theta \,|\, \alpha(\theta)\big)$$

$$\text{length} = \int_{\theta_i}^{\theta_f} S\big(d\theta \,|\, \alpha(\theta)\big)$$

incremental cost of learning about an extra length on boundary conditioned on knowledge of α(θ)

∴ completely general statement, for any convex curve in any geometry obeying Ryu-Takayanagi

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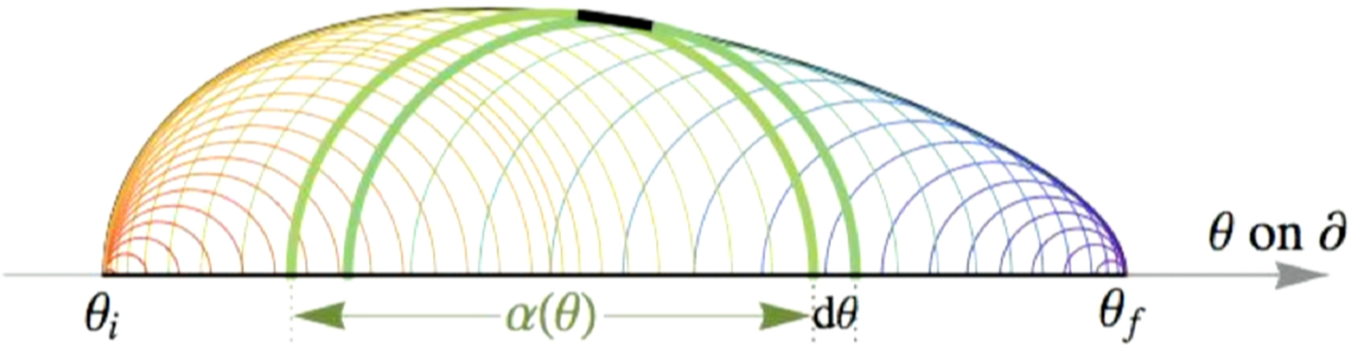
Measure lengths of curves



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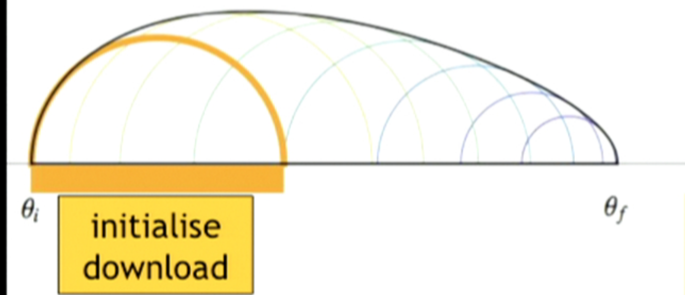
$\therefore$ completely general statement, for any convex curve in any geometry obeying Ryu-Takayanagi

Czech et al., 2014

Operational meaning of lengths – streaming protocols

$$\text{length} = \int_{\theta_i}^{\theta_f} S\big(d\theta \,|\, \alpha(\theta)\big)$$



$$S\big(\alpha(\theta_1) \,|\, \emptyset\big)$$

Czech et al., 2014

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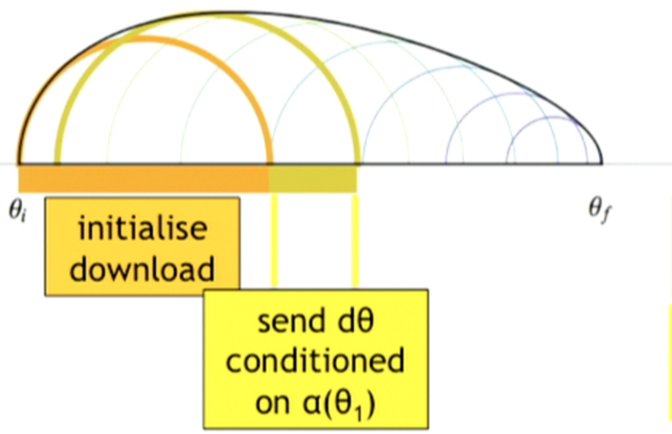

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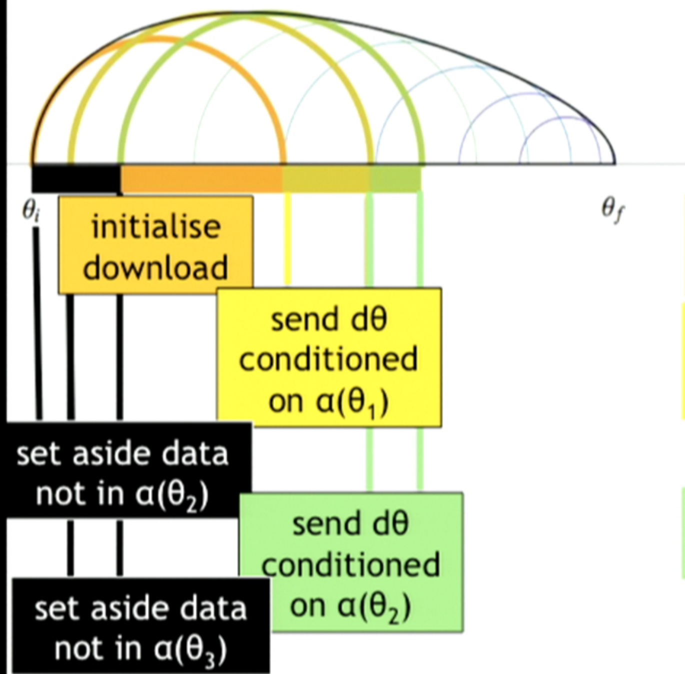

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$S\big(d\theta\,|\,\alpha(\theta_1)\big)$

$S\big(d\theta\,|\,\alpha(\theta_2)\big)$

Czech et al., 2014

Operational meaning of lengths – streaming protocols

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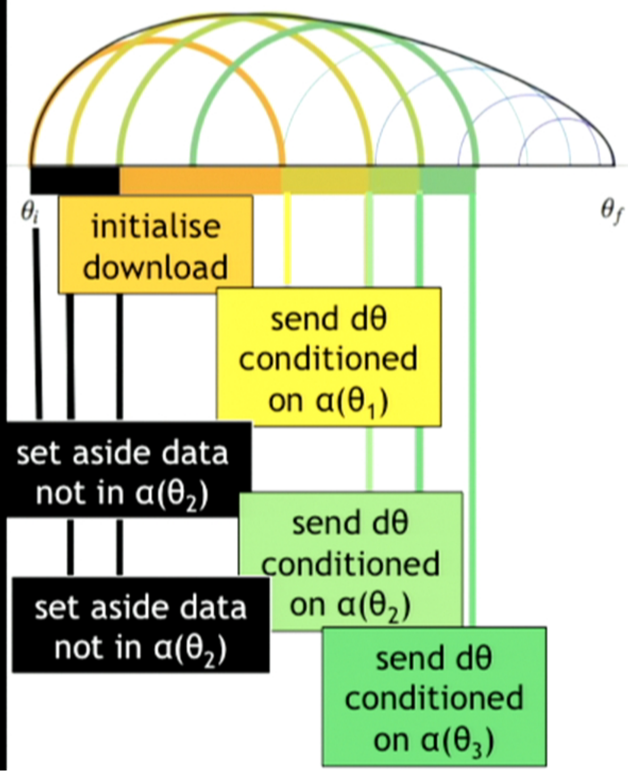
The cost of streaming a movie!



Why set aside the data?

- to make decoding computationally easier
- impose locality constraints on piecewise merging
- impose a location-dependent infrared cutoff

Czech et al., 2014

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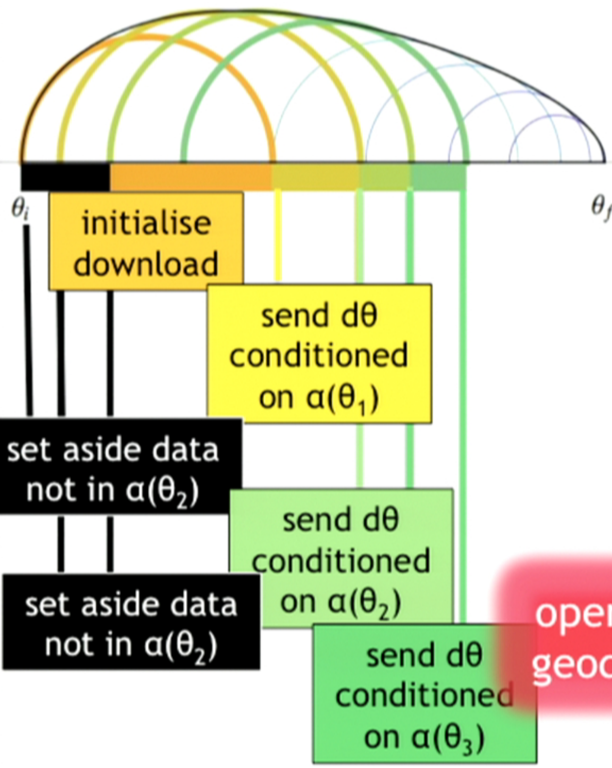

Why set aside the data?

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- impose locality constraints on piecewise merging
- impose a location-dependent infrared cutoff

What if you lift the infrared cutoffs altogether?

- cost = cost of teleportation = S(movie) = geodesic

∴ Locality constraints always drive cost higher
➔ longer curve

∴ No constraints ➔ geodesic

operational interpretation of the definition of the geodesic as the shortest curve connecting 2 points

Czech et al., 2014

State of the Union, circa 2012

Goal: Understand the bulk in AdS/CFT correspondence

- Pay attention to how quantum information / entanglement in the boundary theory is organized in the bulk ☑
- The final picture must involve variations of scale on the boundary ☑
- It should apply to every [reasonable] state, not just the vacuum ☑
- We should make a connection with tensor networks ☐
- Good to have: a potential explanation of black hole microstates ☐
- Be quantitative! ☑

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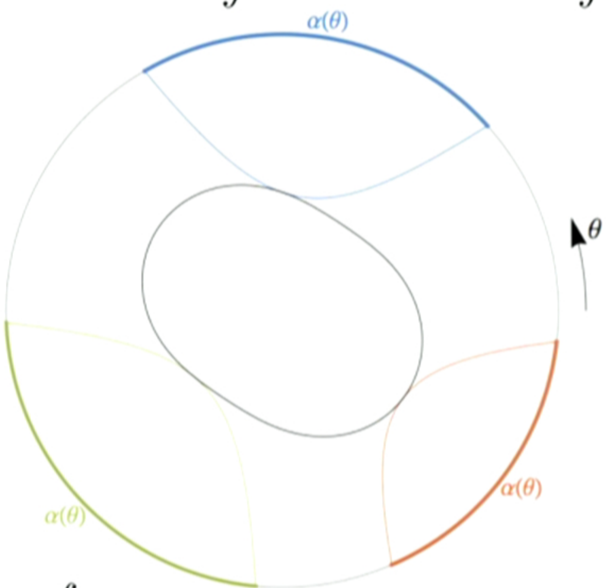
Toward Tensor Networks: A clever rewriting

$$\text{length} = \int S\big(d\theta \,|\, \alpha(\theta)\big) = \int d\theta \, \frac{dS_{\text{ent}}}{d\alpha}\Big|_{\alpha(\theta)}$$



$\int d\theta \, (\ldots)$

tangent geodesics

Czech et al., *in progress*

Toward Tensor Networks: A clever rewriting

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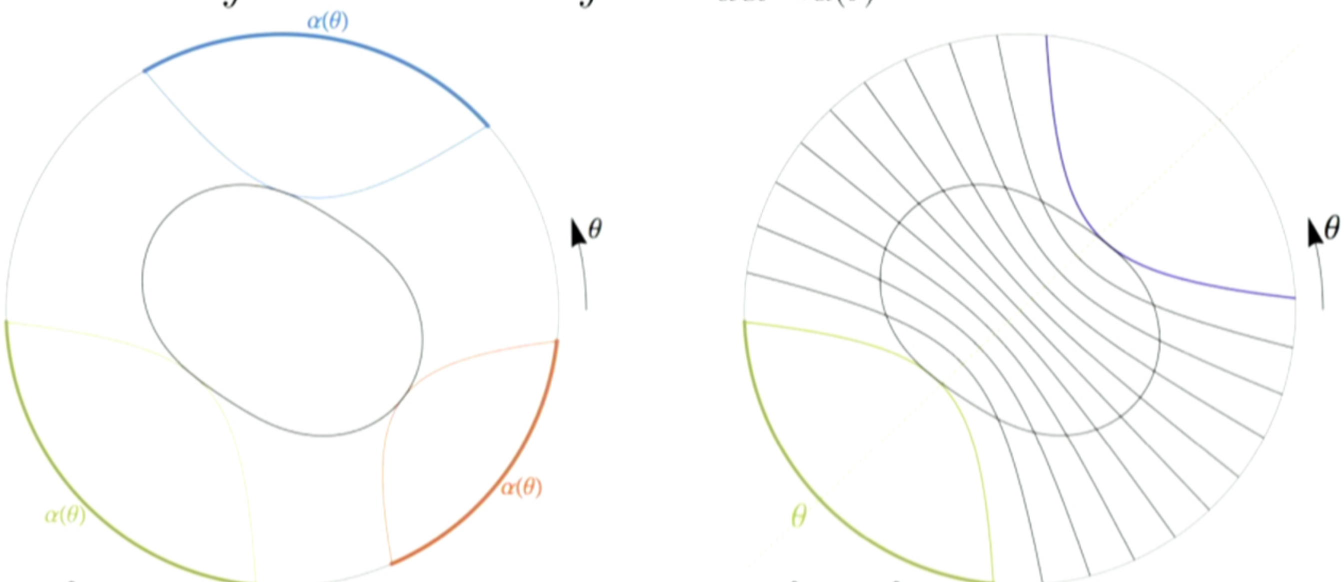


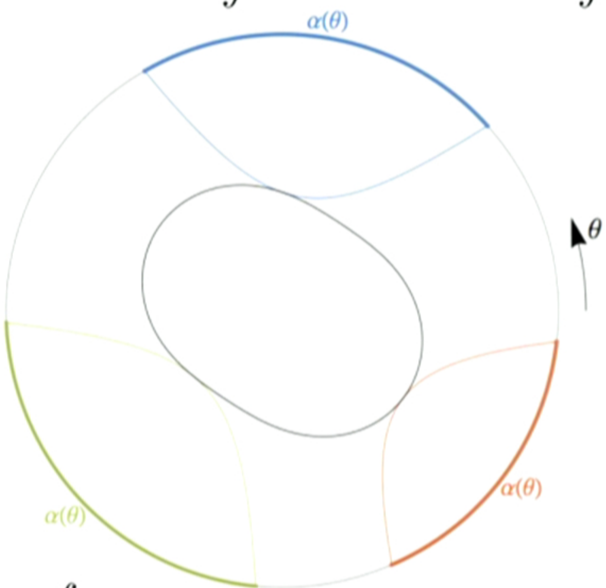
$\int d\theta \, (\ldots)$ → $\int d\theta \int d\alpha \, (\ldots)$

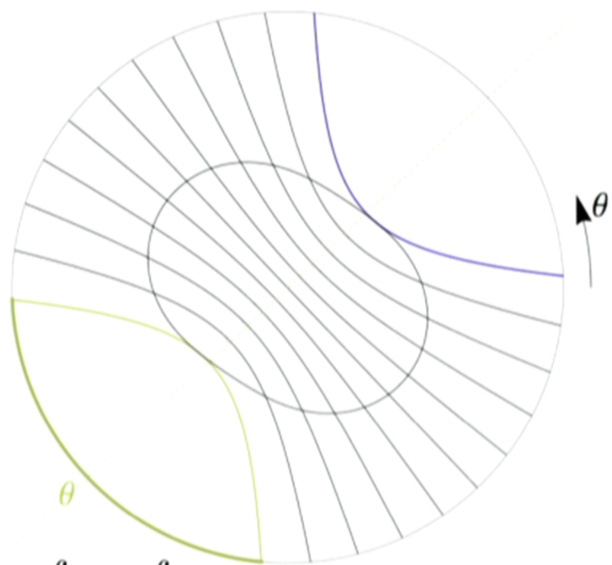
tangent geodesics → intersecting geodesics

Czech et al., *in progress*

Toward Tensor Networks: A clever rewriting

$$\text{length} = \int S\big(d\theta \,|\, \alpha(\theta)\big) = \int d\theta \, \frac{dS_{\text{ent}}}{d\alpha}\Big|_{\alpha(\theta)} = \frac{1}{2}\int d\theta \int d\alpha \left(-\frac{d^2 S_{\text{ent}}}{d\alpha^2}\right)$$





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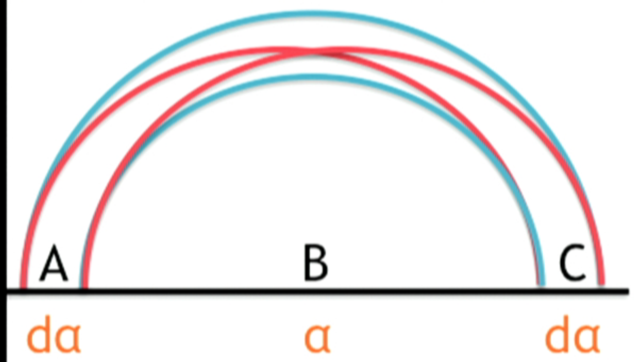
Czech et al., *in progress*

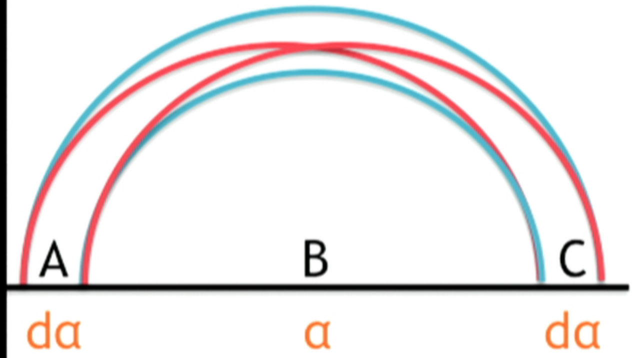
Strong Subadditivity of Entropy

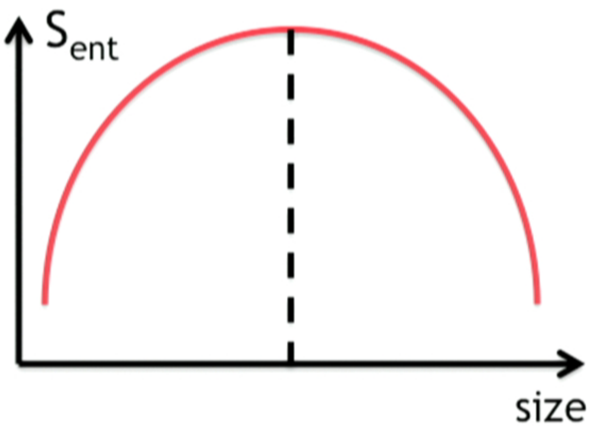


$S(AB) + S(BC) - S(B) - S(ABC) \geq 0$

Lieb-Ruskai, 1973

Strong Subadditivity of Entropy





S(AB) + S(BC) - S(B) - S(ABC) ≥ 0

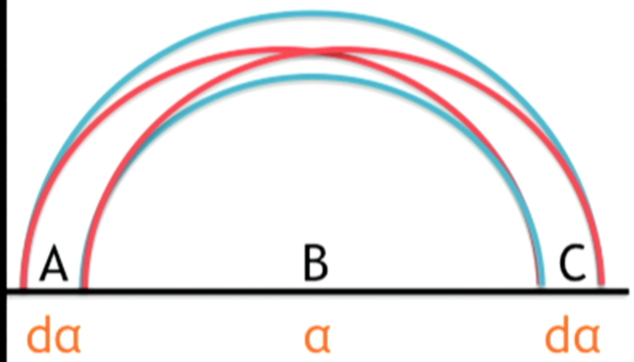
$$-\frac{d^2 S_{\text{ent}}}{d\alpha^2} \geq 0$$

Interpretations of SSA:

- Concavity of entropy
- Law of diminishing returns for information
- Information grows slower than system size
- States contain internal correlations

Lieb-Ruskai, 1973

Strong Subadditivity of Entropy



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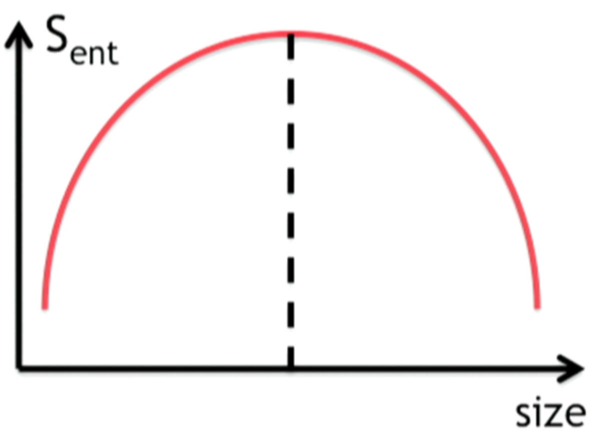

<u>Interpretations of SSA:</u>

- Concavity of entropy
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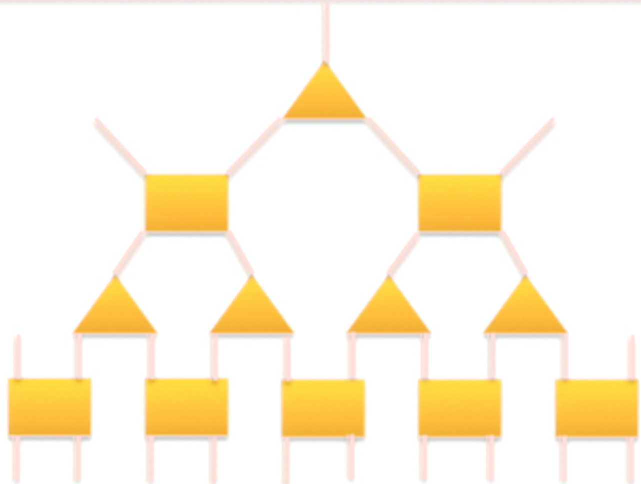
- Information grows slower than system size
- States contain internal correlations
- States are compressible:

Lieb-Ruskai, 1973

Multi-Scale Entanglement Renormalization Ansatz (MERA)



- Variational tensor network ansatz for finding ground states of critical systems

  Vidal, 2006

- Results from coarse-graining a tensor network representation of a path integral

  Vidal et al., 2015

Vidal, 2006

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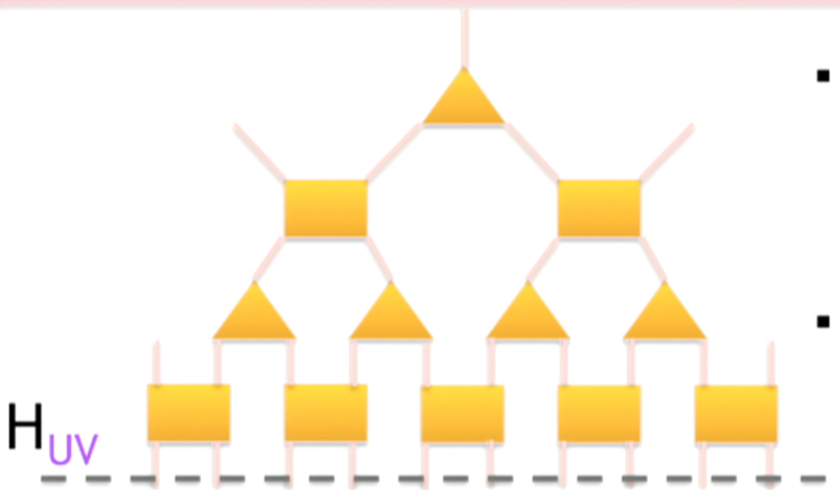

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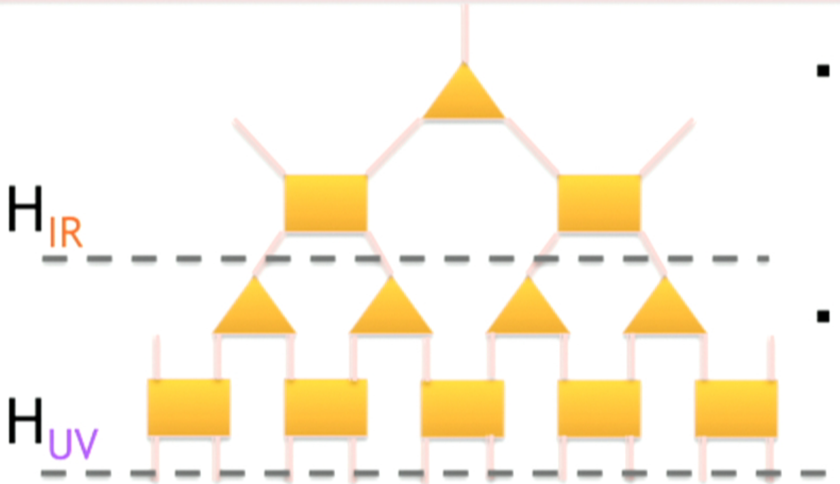

- Variational tensor network ansatz for finding ground states of critical systems

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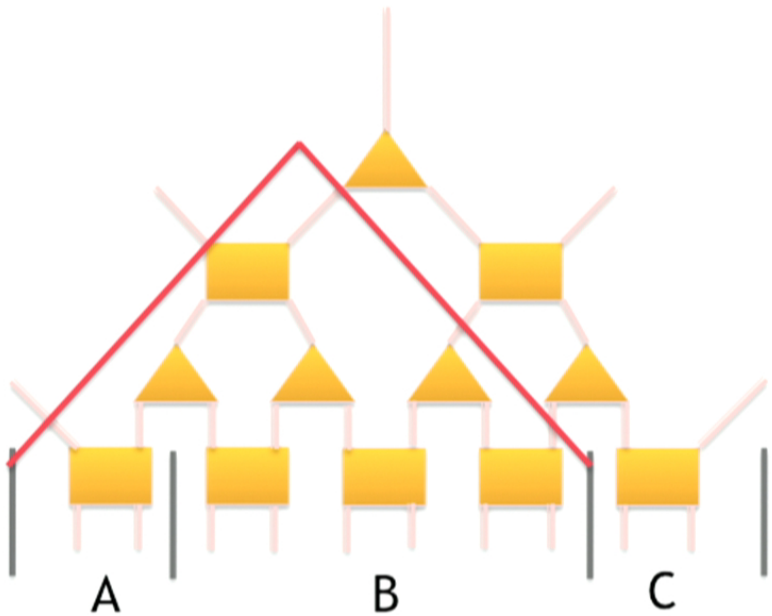
isometries distill entanglement at lower scales to map the state in H_{UV} to a smaller Hilbert space H_{IR} at larger scales

Vidal, 2006

Strong subadditivity in MERA

S(AB) + S(BC) - S(B) - S(ABC) ≥ 0



- Under the assumption of genericity, S(AB) is found by drawing a minimal cut
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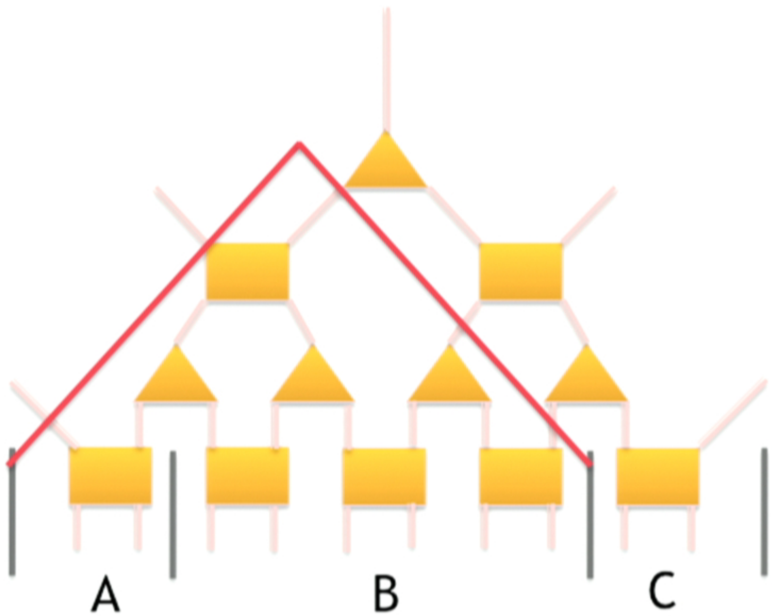

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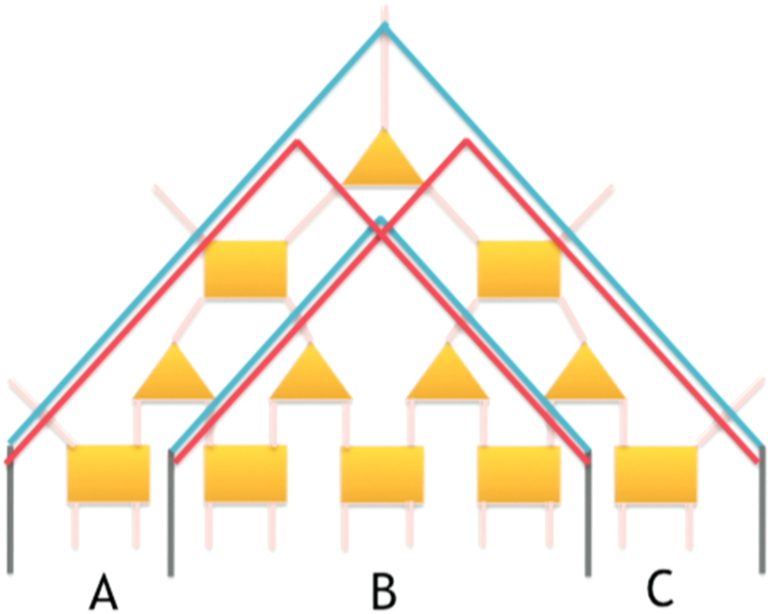

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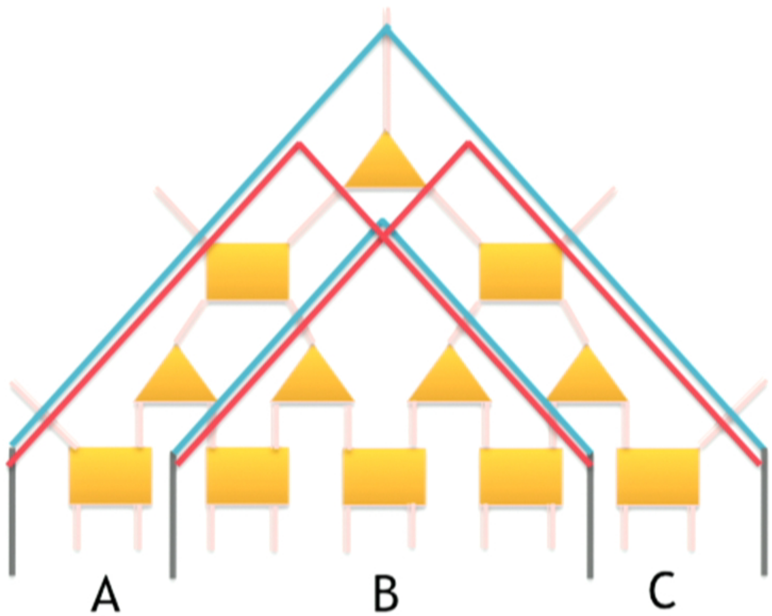

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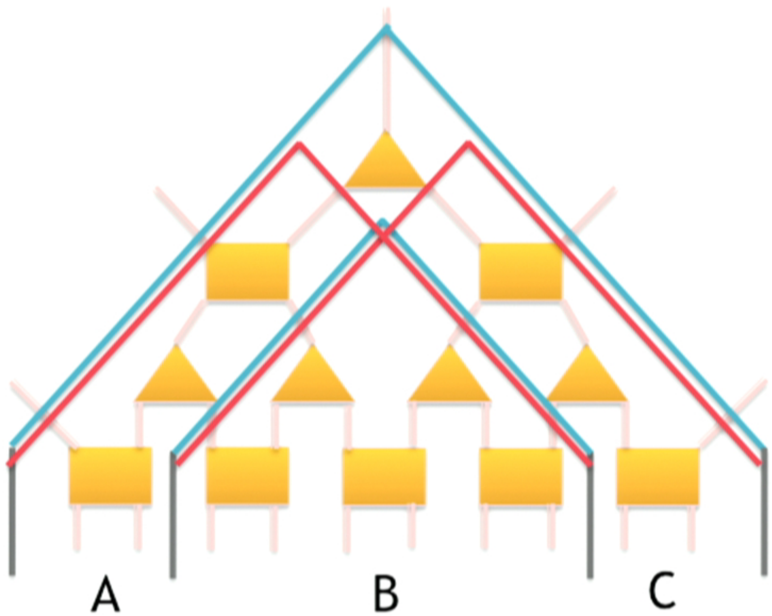

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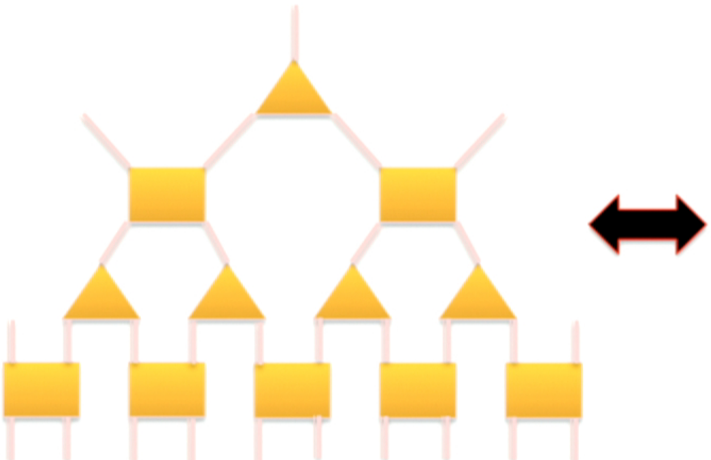
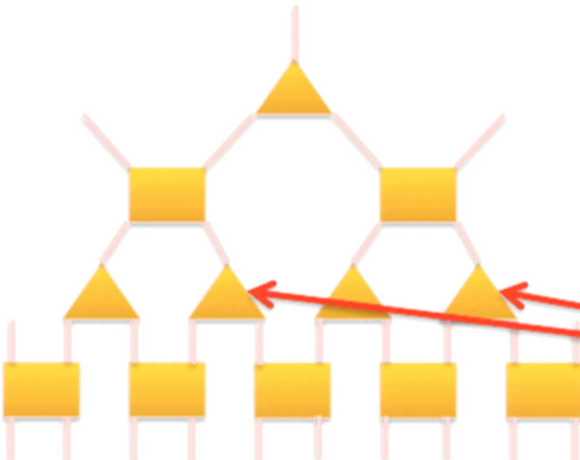
MERA vs. our length formula



$$\text{length} = \int \mathcal{D}(\text{geodesics})\,(\text{SSA})$$

Czech et al., *in progress*

MERA vs. our length formula



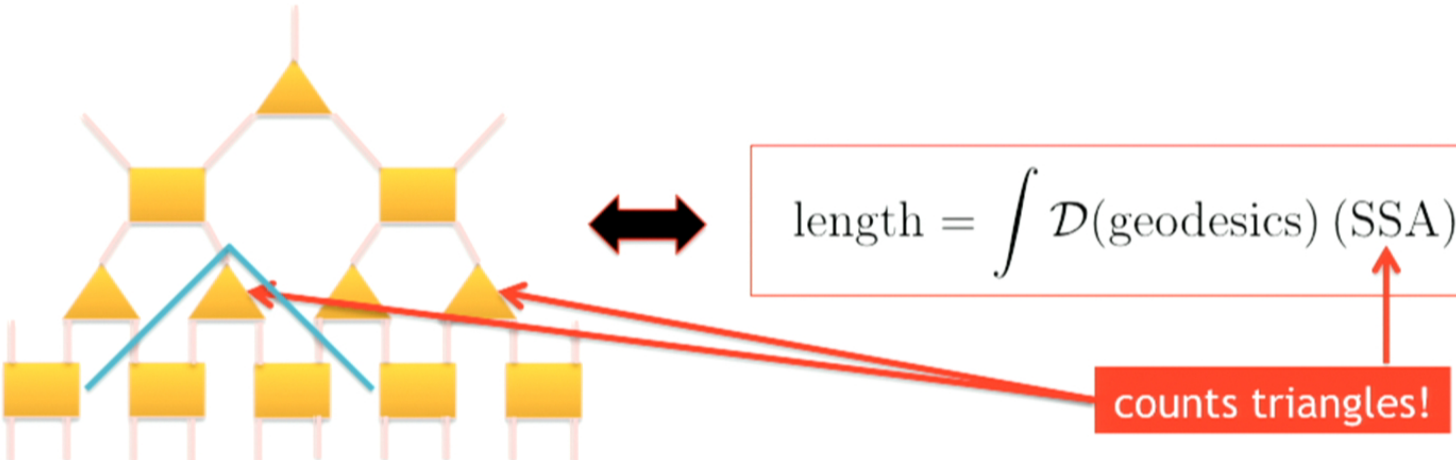
$$\text{length} = \int \mathcal{D}(\text{geodesics})\,(\text{SSA})$$

counts triangles!

length	= volume in MERA
geodesic in bulk	= point in MERA (and its causal cone)
d(vol) = D^2(geodesics)(SSA)	= #(Δ)

Czech et al., *in progress*

MERA vs. our length formula



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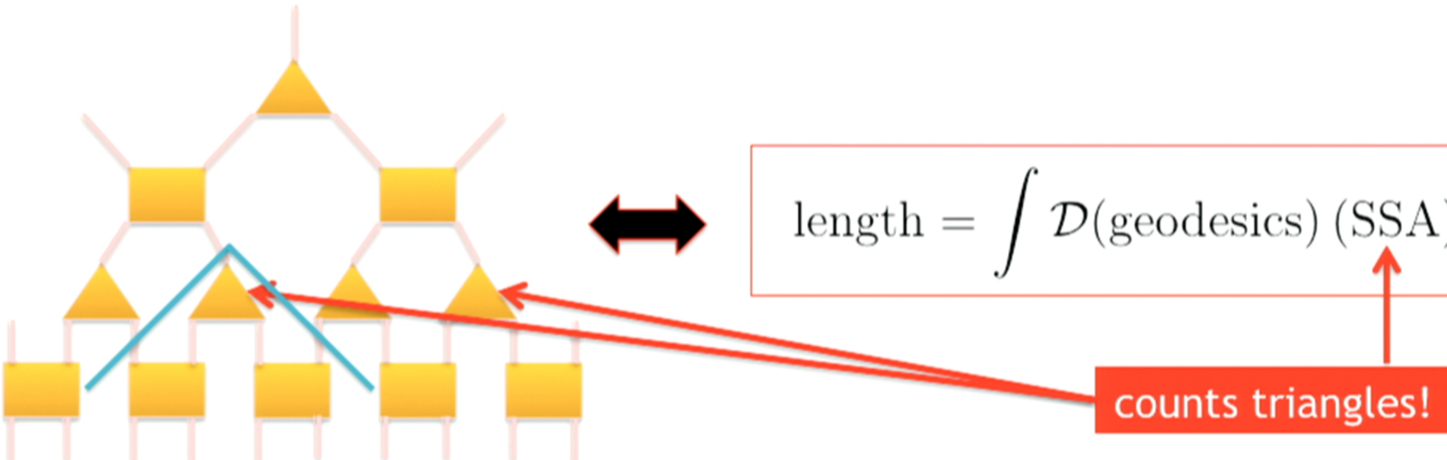

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Czech et al., *in progress*

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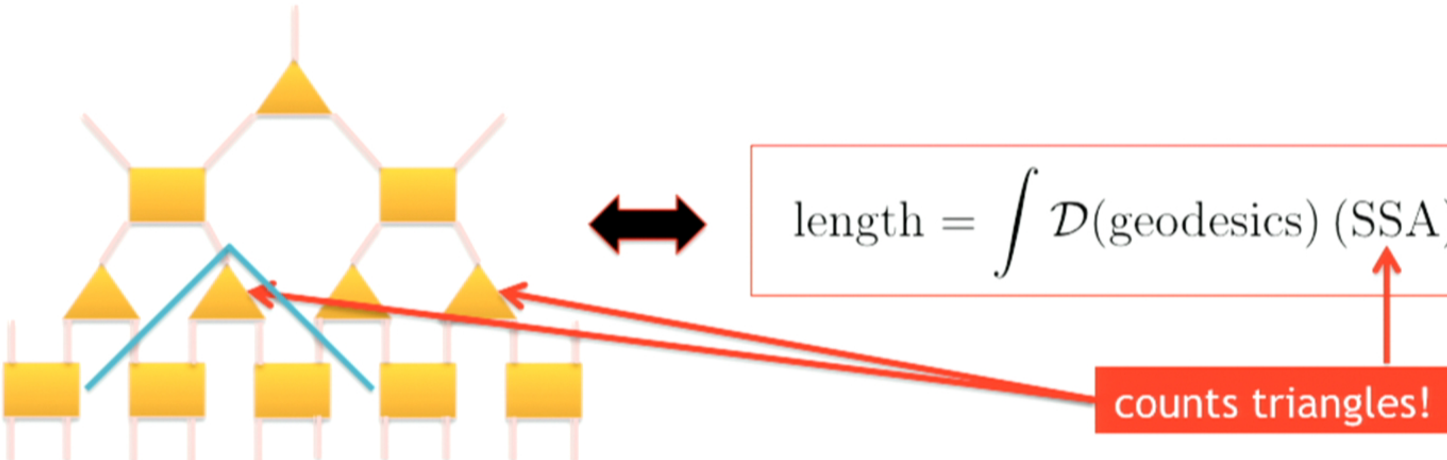

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c.f. my talk on 24 February

Czech et al., *in progress*

Example

$$\int_{\mathrm{MERA}} d(\#\Delta) = \mathrm{length} = \int \mathcal{D}(\mathrm{geodesics})\, (\mathrm{SSA})$$

Czech et al., *in progress*

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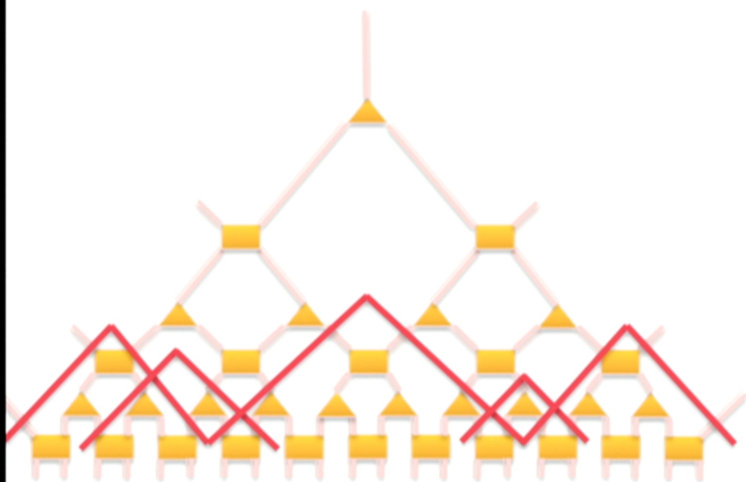

Czech et al., *in progress*

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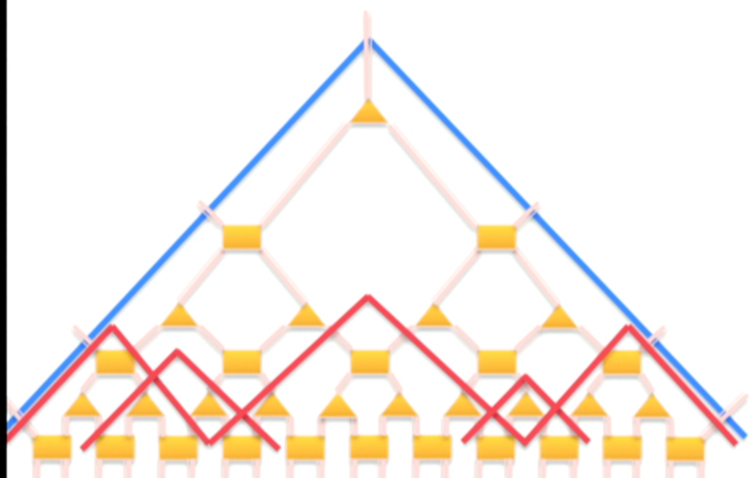
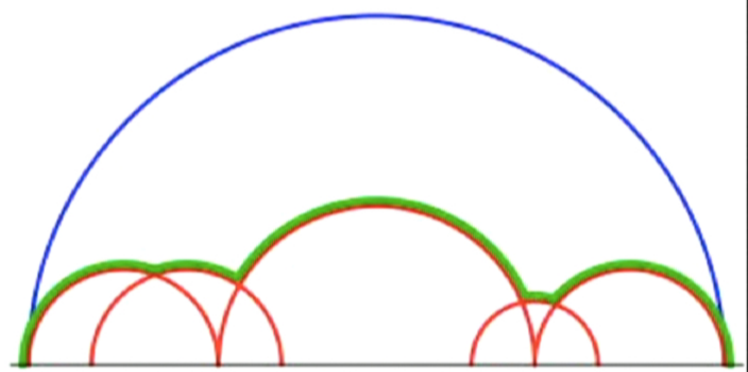
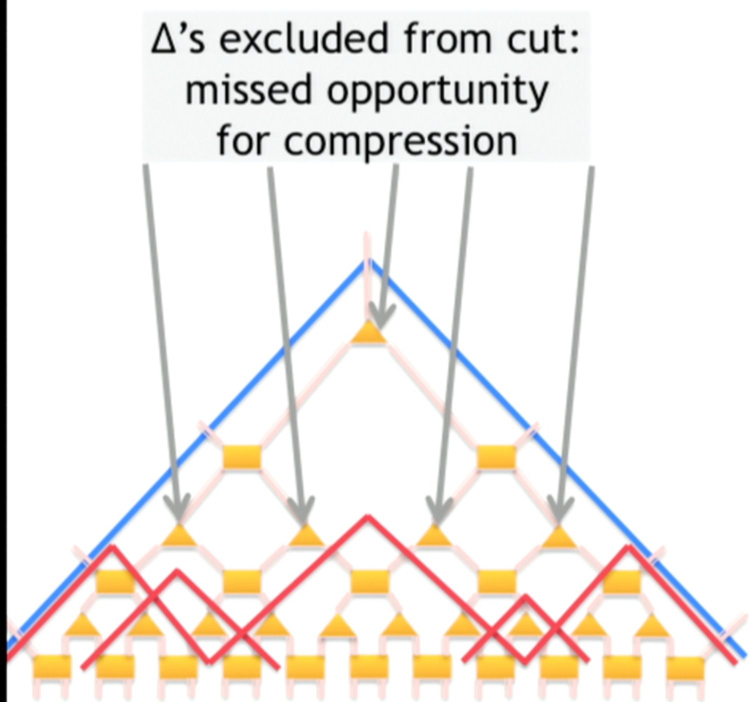


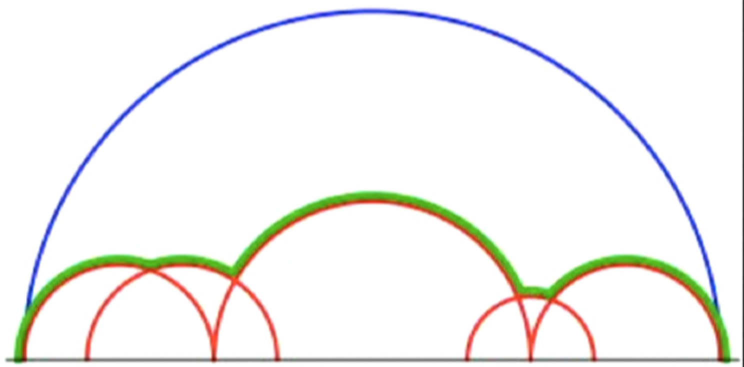


Czech et al., *in progress*

Example

$$\int_{\mathrm{MERA}} d(\#\Delta) = \mathrm{length} = \int \mathcal{D}(\mathrm{geodesics})\ (\mathrm{SSA})$$





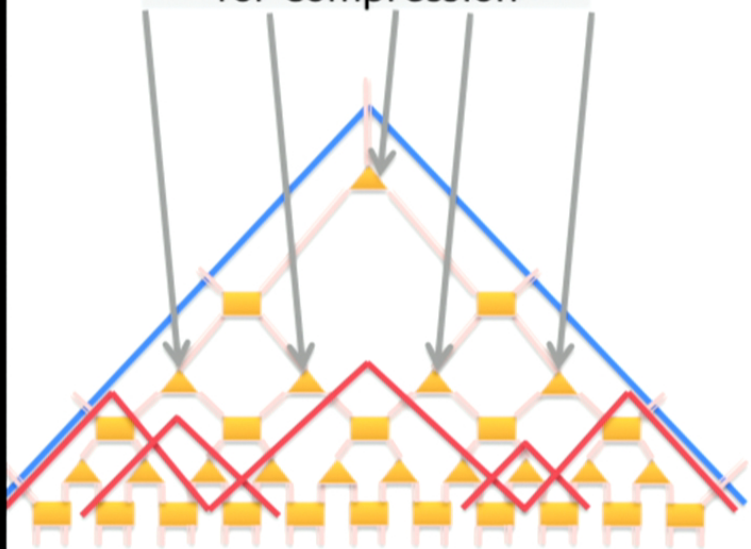
Czech et al., *in progress*

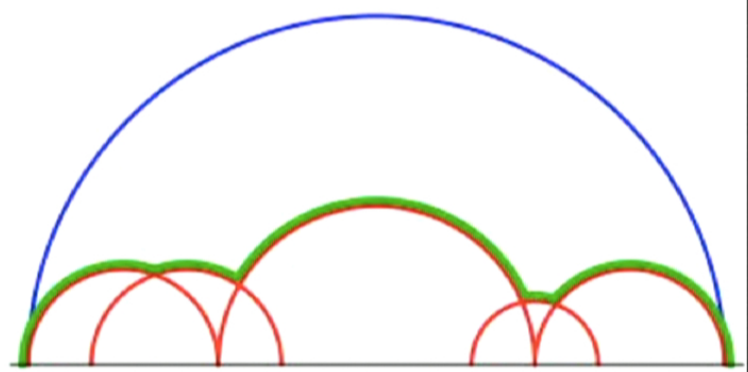
Example

$$\int_{\mathrm{MERA}} d(\#\Delta) = \text{length} = \int \mathcal{D}(\text{geodesics})\ (\text{SSA})$$

Δ's excluded from cut: missed opportunity for compression

= length - geodesic





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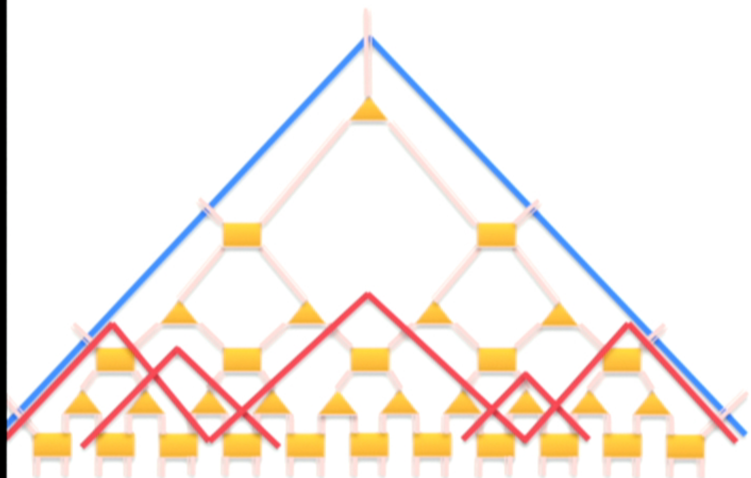
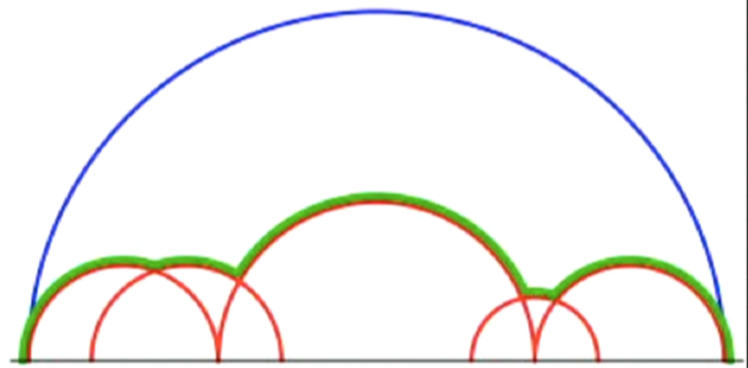
MERA vs our length formula - RECAP





- Quantitative connection between MERA and holographic RG

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MERA vs our length formula - RECAP

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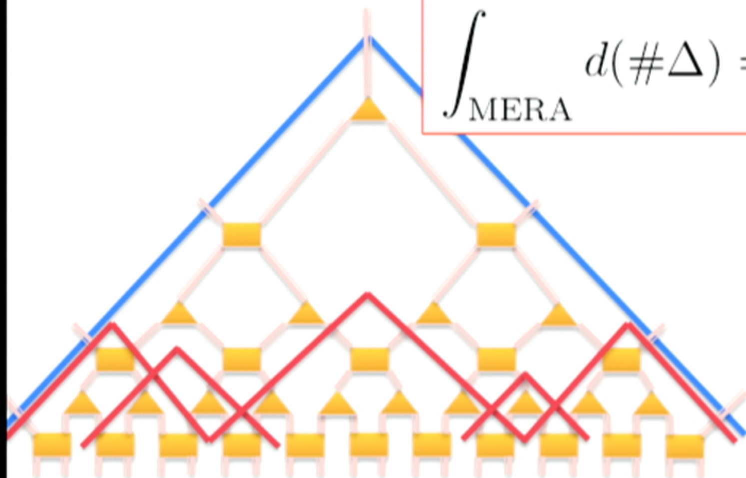

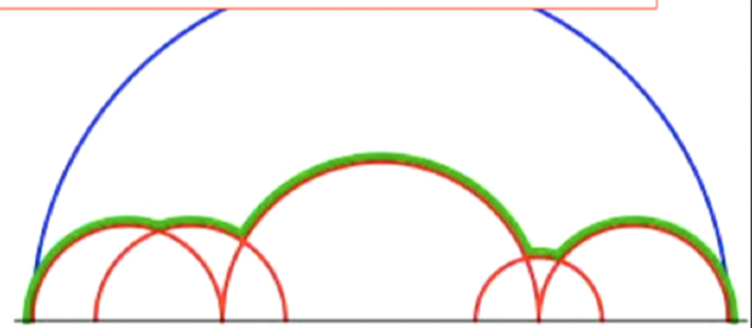

- Quantitative connection between MERA and holographic RG
- Uses quantum information theory directly
- MERA discretises the space of geodesics
  (equivalently the space of boundary intervals)

Czech et al., *in progress*

State of the Union, circa 2012

Goal: Understand the bulk in AdS/CFT correspondence

- Pay attention to how quantum information / entanglement in the boundary theory is organized in the bulk ☑
- The final picture must involve variations of scale on the boundary ☑
- It should apply to every [reasonable] state, not just the vacuum ☑
- We should make a connection with tensor networks ☑
- Good to have: a potential explanation of black hole microstates ☐
- Be quantitative! ☑

What to do next?

MEDIUM TERM:

- Study multi-boundary black hole solutions to understand multi-particle entanglement

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- Exploit relations between areas and geodesics / totally geodesic surfaces (integral geometry)—to understand the organisation of higher-dimensional holographic spacetimes

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MEDIUM TERM:

- Study multi-boundary black hole solutions to understand multi-particle entanglement
- Exploit relations between areas and geodesics / totally geodesic surfaces (integral geometry)—to understand the organisation of higher-dimensional holographic spacetimes
- Explore the relation between the space of geodesics and Guifre's Tensor Network Renormalisation better—to understand the path integral as preparing the state with a given entanglement structure

What to do next?

LONG TERM:

- How to describe a de Sitter spacetime?

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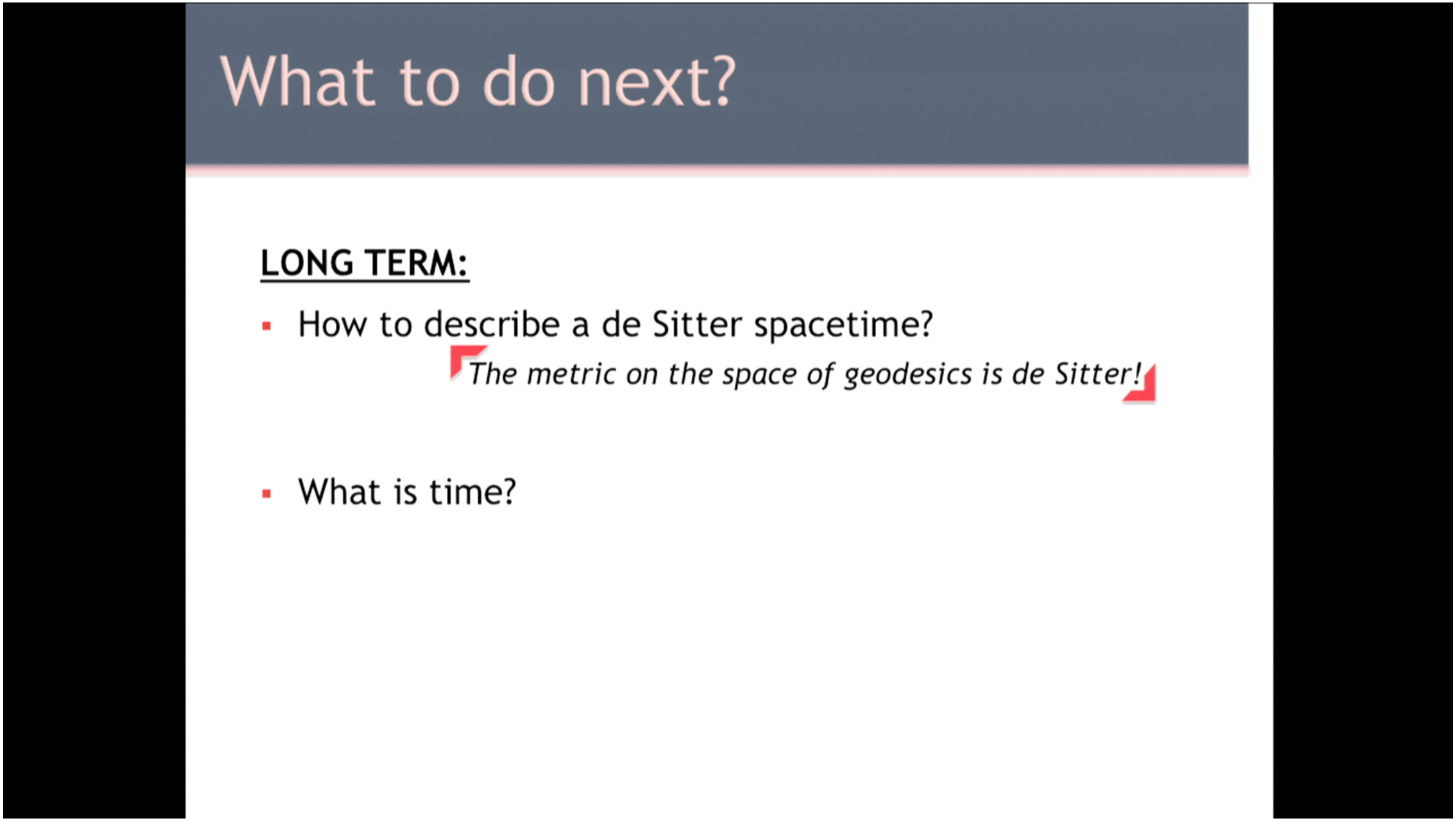
LONG TERM:

- How to describe a de Sitter spacetime?

The metric on the space of geodesics is de Sitter!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LONG TERM:
How to describe a de Sitter spacetime?
The metric on the space of geodesics is de Sitter!
What is time?

What to do next?

LONG TERM:

- How to describe a de Sitter spacetime?

  The metric on the space of geodesics is de Sitter!

- What is time?

  Einstein equation from entanglement entropies on all spatial slices

  Van Raamsdonk et al., 2013-2014

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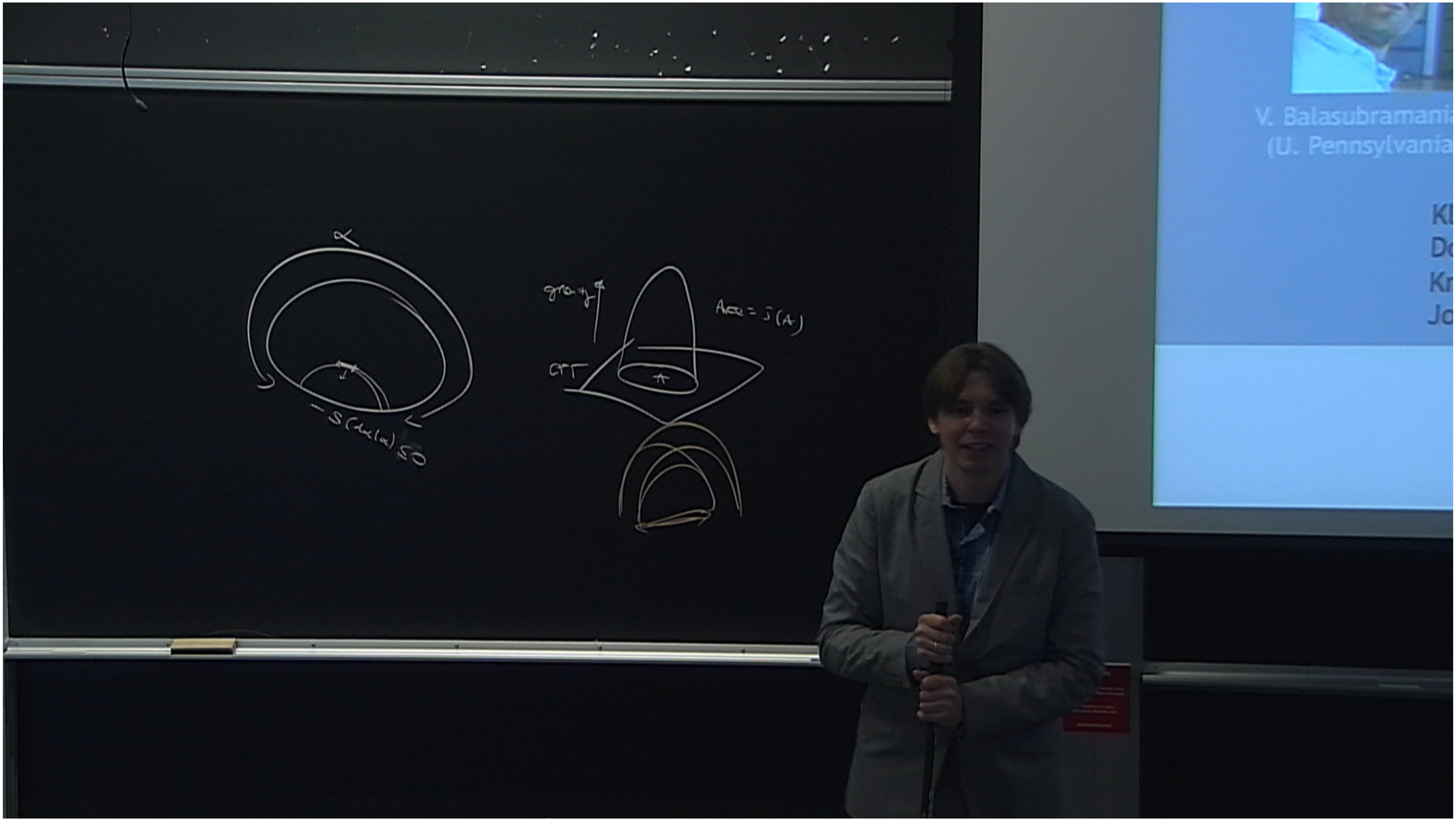
THANK YOU!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Area = S(A)
CFT
A