Title: Bulk Locality and Quantum Error Correction in AdS/CFT

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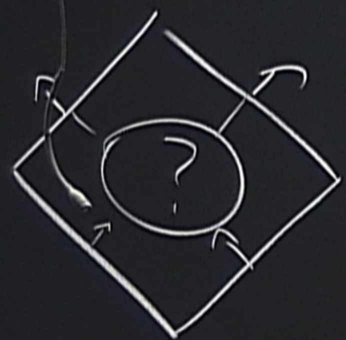
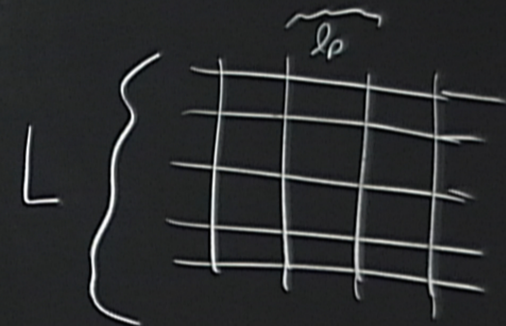
Abstract: <p>In this talk I will describe recent work with Almheiri and Dong, where we proposed a connection between the emergence of bulk locality in AdS/CFT and the theory of quantum error correction. Bulk notions such as Bogoliubov transformations, location in the radial direction, and the holographic entropy bound all have natural CFT interpretations in the language of quantum error correction. Time permitting, I will also discuss work in progress with Pastawski, Preskill, and Yoshida on a new class of stabilizer codes that explicitly realize many of the properties we argued the AdS/CFT error correcting code should have. </p>



$\langle \text{out} | \text{in} \rangle$

or

$$\langle \Omega | \mathcal{O}(x_1) \ldots \mathcal{O}(x_4) | \Omega \rangle$$



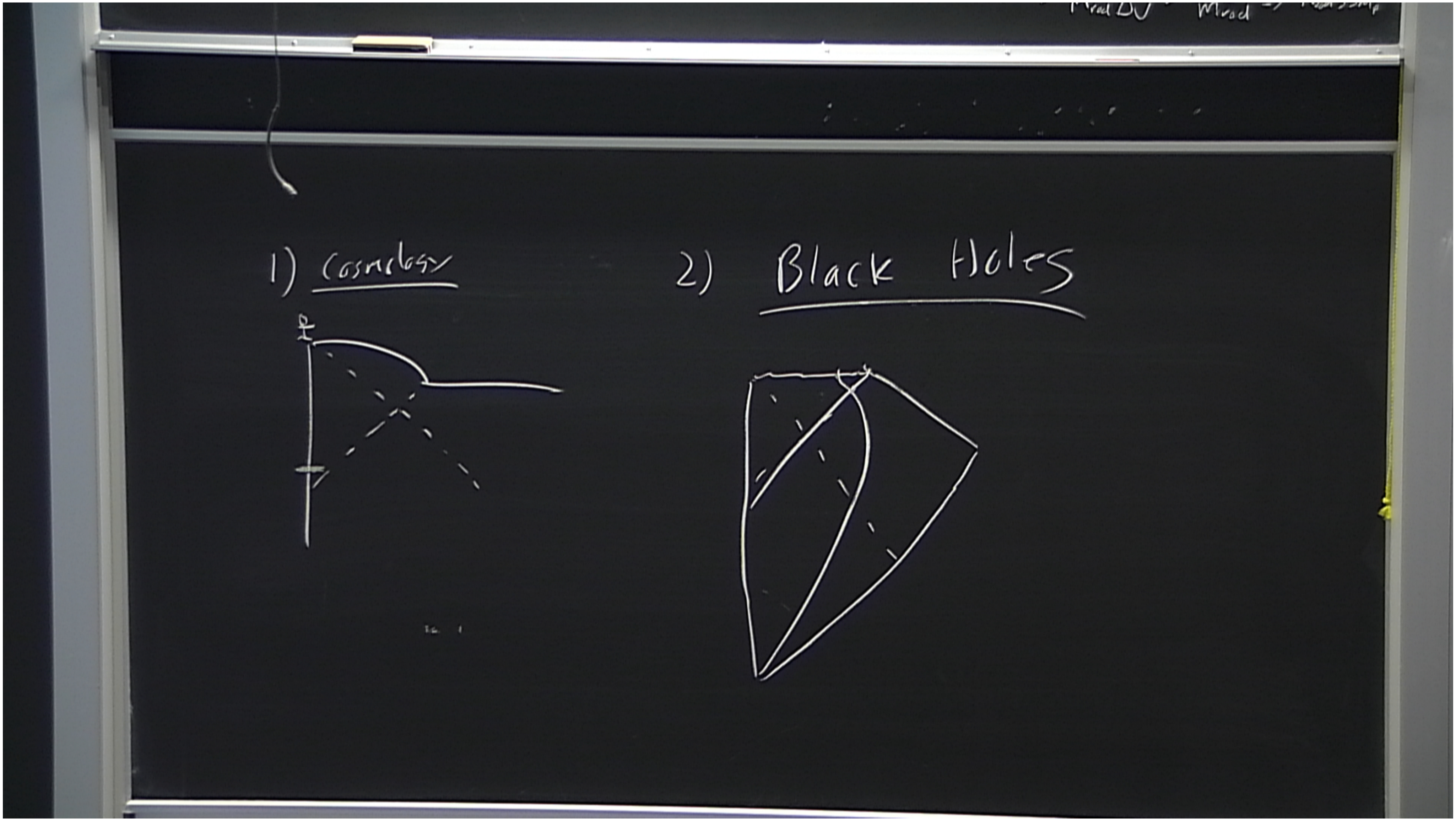

Not a BH

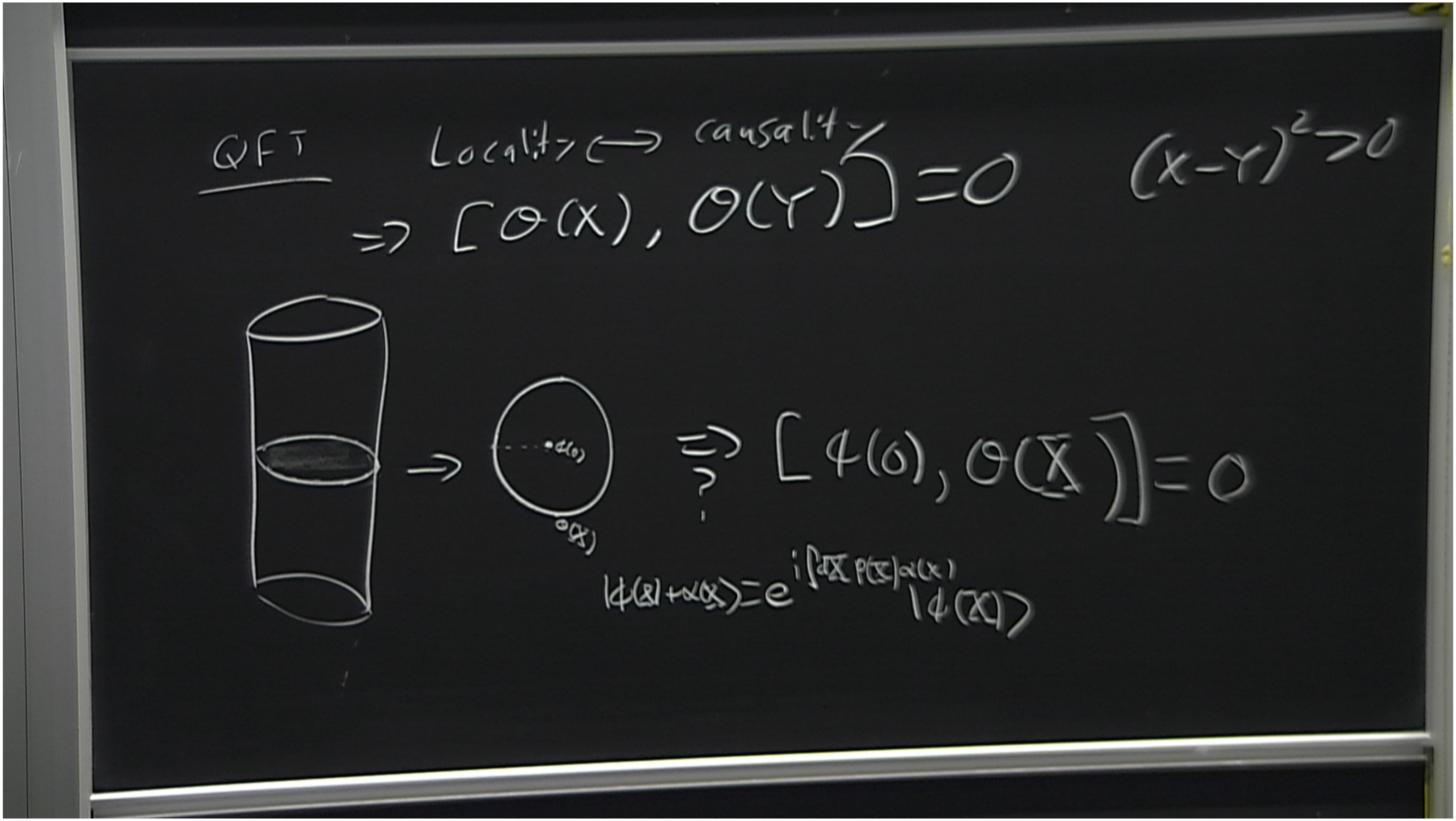
$$L > R_S \simeq l_p^2 M$$

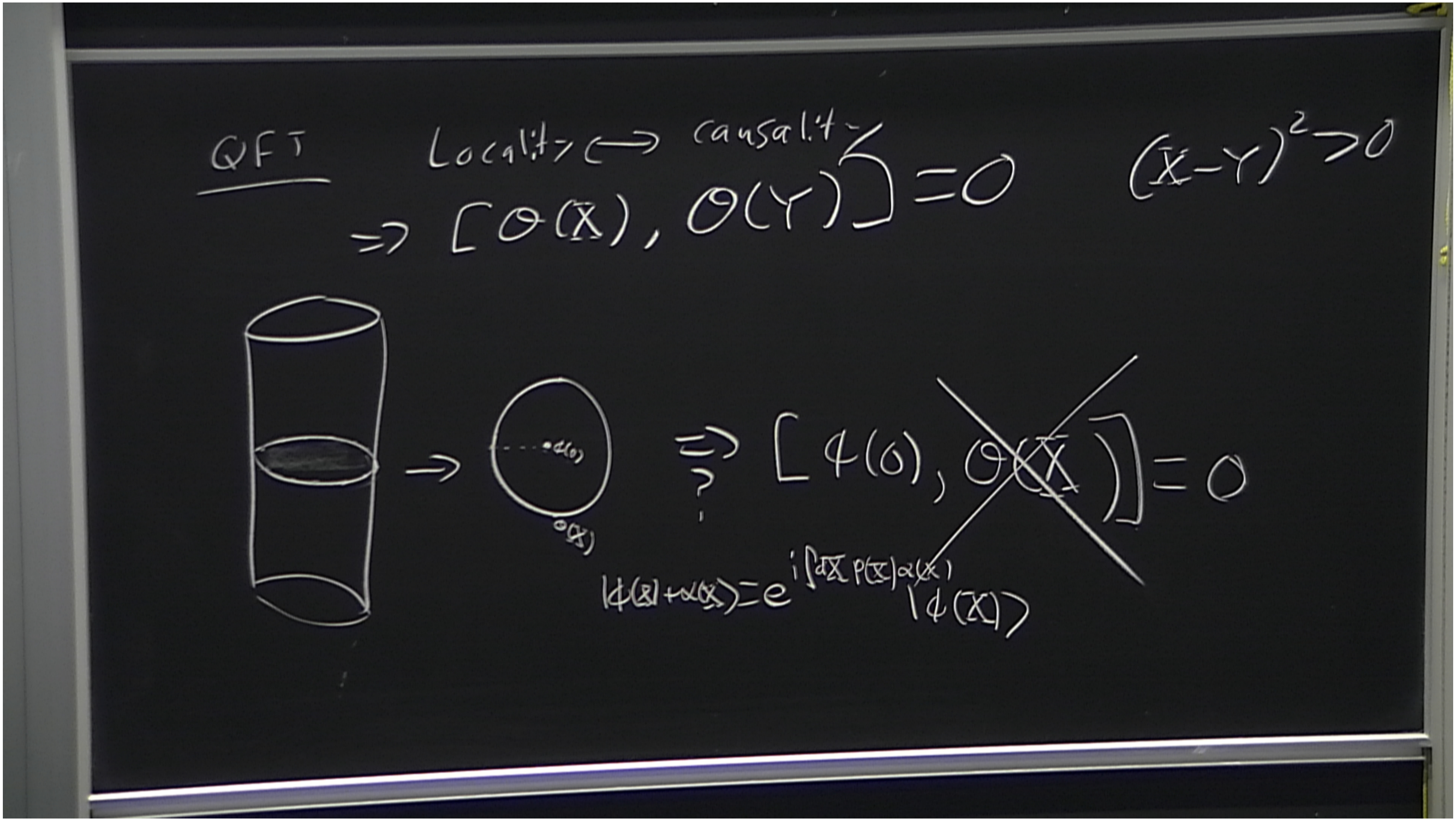
$$= l_p \left(\frac{L}{l_p}\right)^3 \left(\frac{M_{rad}}{m_p}\right)$$

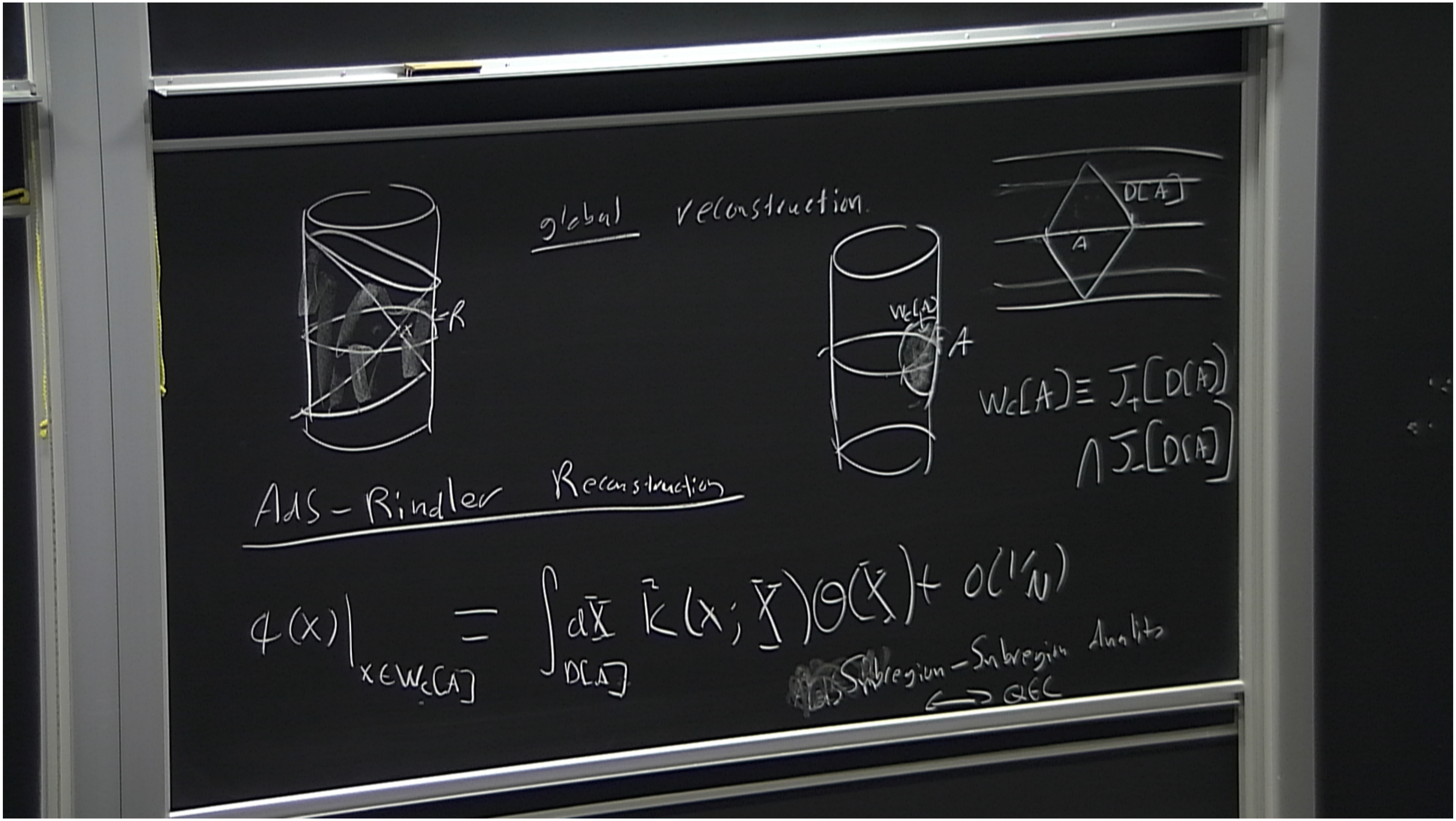
$$\Rightarrow \quad M_{rad} << M_p$$

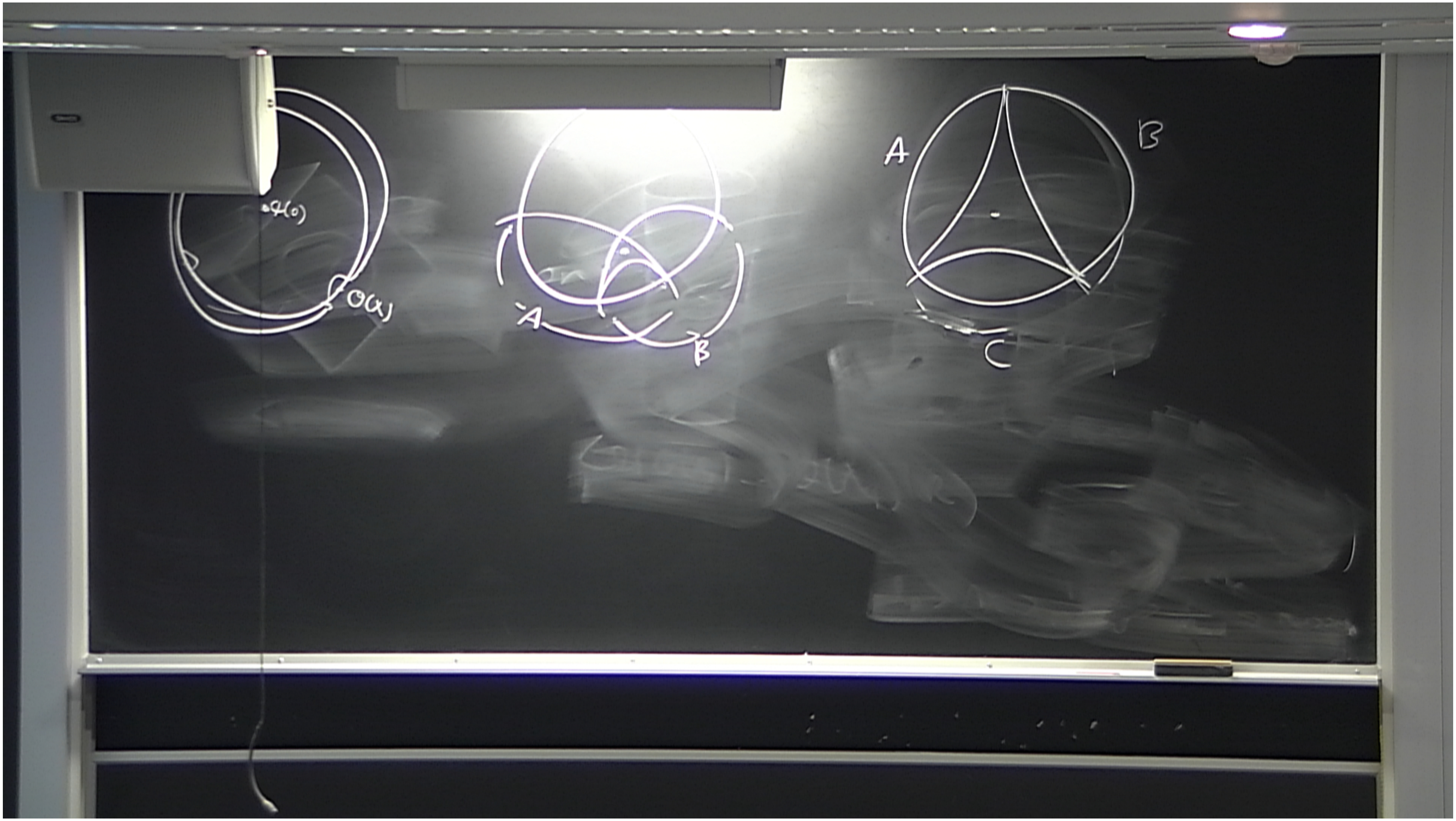
$$l_p > \Delta x > \frac{1}{M_{rad}\,\Delta V} > \frac{1}{M_{rad}} \Rightarrow M_{rad} >> M_p$$


1) Cosmology
t
2) Black Holes


QFT
Locality ⟷ causality
⇒ [O(x), O(y)] = 0
(x−y)² > 0
ϕ(0)
O(X)
⇒? [ϕ(0), O(X)] = 0
|ϕ(x)+α(x)⟩ = e^{i∫dX p(X)α(x)} |ϕ(X)⟩


QFT
Locality ⟷ causality
$\Rightarrow [\mathcal{O}(X), \mathcal{O}(Y)] = 0$
$(X-Y)^2 > 0$
$\phi(0)$
$\mathcal{O}(X)$
$\overset{?}{\Rightarrow} [\phi(0), \mathcal{O}(X)] = 0$
$|\phi(X)+\alpha(X)\rangle = e^{i\int dX\, p(X)\alpha(X)} |\phi(X)\rangle$


global reconstruction.
R
A
D[A]
W_C[A]
W_C[A] ≡ J_+[D(A)] ∩ J_-[D(A)]
AdS - Rindler Reconstruction
φ(x)|_{x∈W_C[A]} = ∫_{D[A]} dX̄ K̄(x; X̄) O(X̄) + O(1/N)
Subregion - Subregion Duality
⟷ QEC


O(x)
A
B
A
B
C

$$|\psi\rangle = \sum_{i=0}^{2} c_i |i\rangle$$

$$|\tilde{\psi}\rangle = \sum_{i=0} c_i |\tilde{i}\rangle$$

$$|\tilde{0}\rangle = \frac{1}{\sqrt{3}}\left(|000\rangle + |111\rangle + |222\rangle\right)$$

$$|\tilde{1}\rangle = \frac{1}{\sqrt{3}}\left(|012\rangle + |120\rangle + |201\rangle\right)$$

$$|\tilde{2}\rangle = \frac{1}{\sqrt{3}}\left(|021\rangle + |102\rangle + |210\rangle\right)$$

$$U_{12}|\tilde{i}\rangle = |i\rangle_1 |\chi\rangle_{23} \qquad |\chi\rangle = \frac{1}{\sqrt{3}}\left(|00\rangle + |11\rangle + |22\rangle\right)$$

$$U_{12}|\tilde{\psi}\rangle = |\psi\rangle_1 |\chi\rangle$$

Say $\tilde{O}|\tilde{i}\rangle = \sum_j (O)_{ji} |\tilde{j}\rangle$

$$\Rightarrow$$

$$O_{12} = U_{12}^\dagger O_1 U_{12} \qquad (O_{23}, O_{13})$$

$$\langle\tilde{\psi}|[\underset{\uparrow \atop O_{23}}{\tilde{O}}, X_1]|\tilde{\psi}\rangle$$

$\mathcal{O}_{23}$


$$ds^2 = -(r^2+1)dt^2 + \frac{dr^2}{r^2+1} + r^2 d\Omega^2$$

$$\underset{r\to\infty}{\sim} \quad r^2(-dt^2 + d\Omega^2)$$

states $\Longleftrightarrow$ States

$H \Longleftrightarrow H$

$$\lim_{r\to\infty} \phi(t,r,\Omega)\, r^\Delta = \mathcal{O}(t,\Omega)$$

$n$ total qubits

$k$ protected qubits

$\ell$ erased qubits $\equiv E$

$$|\tilde{\psi}\rangle_{E\bar{E}} = \sum_i C_i \, |\tilde{i}\rangle_{E\bar{E}}$$

$$\exists\, U_{\bar{E}} \Rightarrow U_{\bar{E}} |\tilde{\psi}\rangle = |\psi\rangle_{\bar{E}_1} |\chi\rangle_{\bar{E}_2 E}$$

1) $|\phi\rangle \equiv \frac{1}{2^{k/2}}\sum_i |i\rangle_R |\tilde{i}\rangle_{C\bar{E}}$

correctability $\Longleftrightarrow$ $\rho_{RE}(\phi) = \rho_R(\phi)\otimes\rho_E(\phi)$

2) $\forall O \;\exists\; O_{\bar{E}}$ s.t.

$$O_{\bar{E}}|\tilde{\psi}\rangle = \tilde{O}|\tilde{\psi}\rangle$$

$$O_{\bar{E}}^{\dagger}|\tilde{\psi}\rangle = \tilde{O}^{\dagger}|\tilde{\psi}\rangle$$

Page $\Rightarrow$ want $\frac{|R||E|}{|\bar{E}|} \ll 1 \Rightarrow n > k+2l$

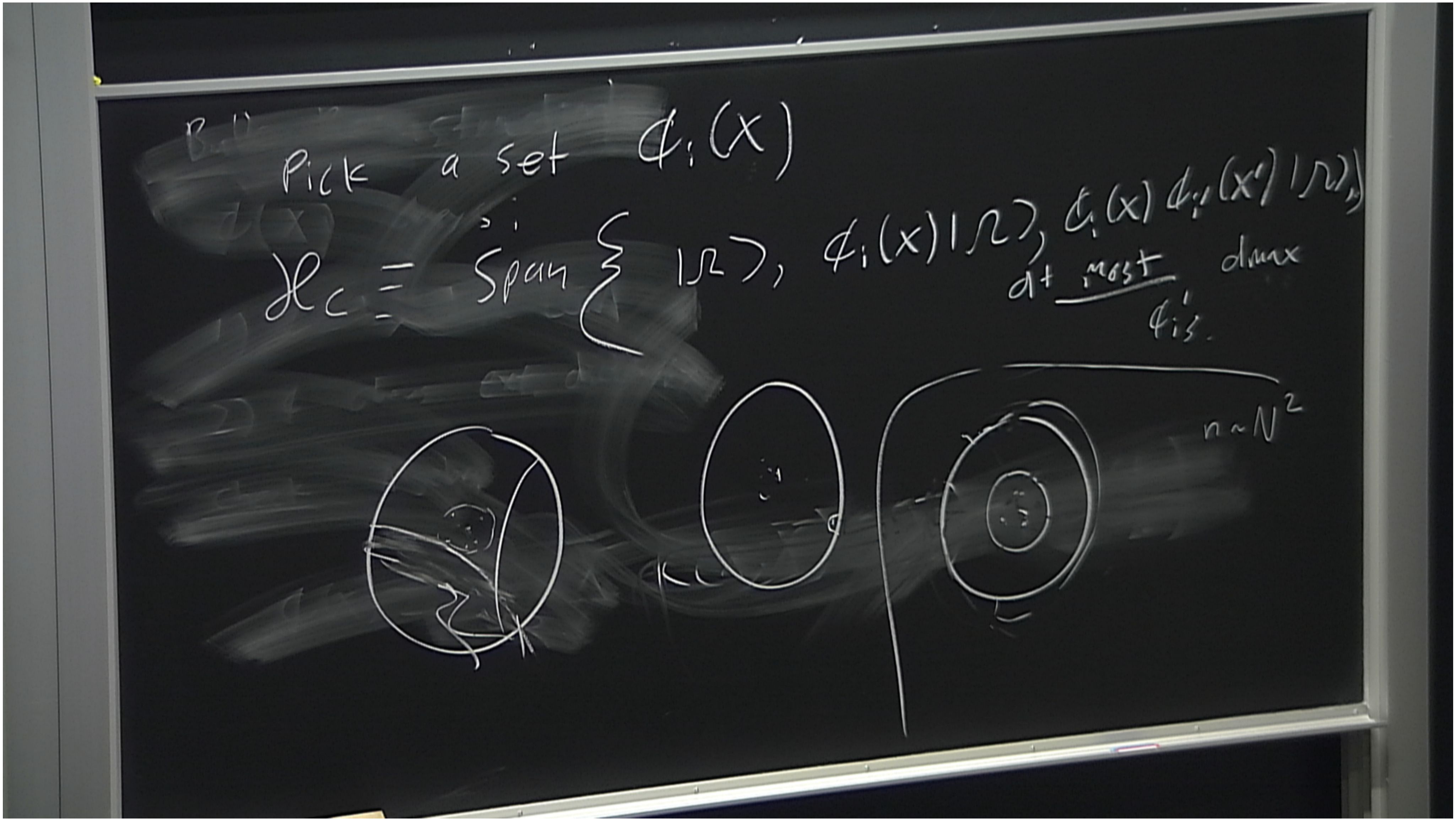
# Bulk Reconstruction

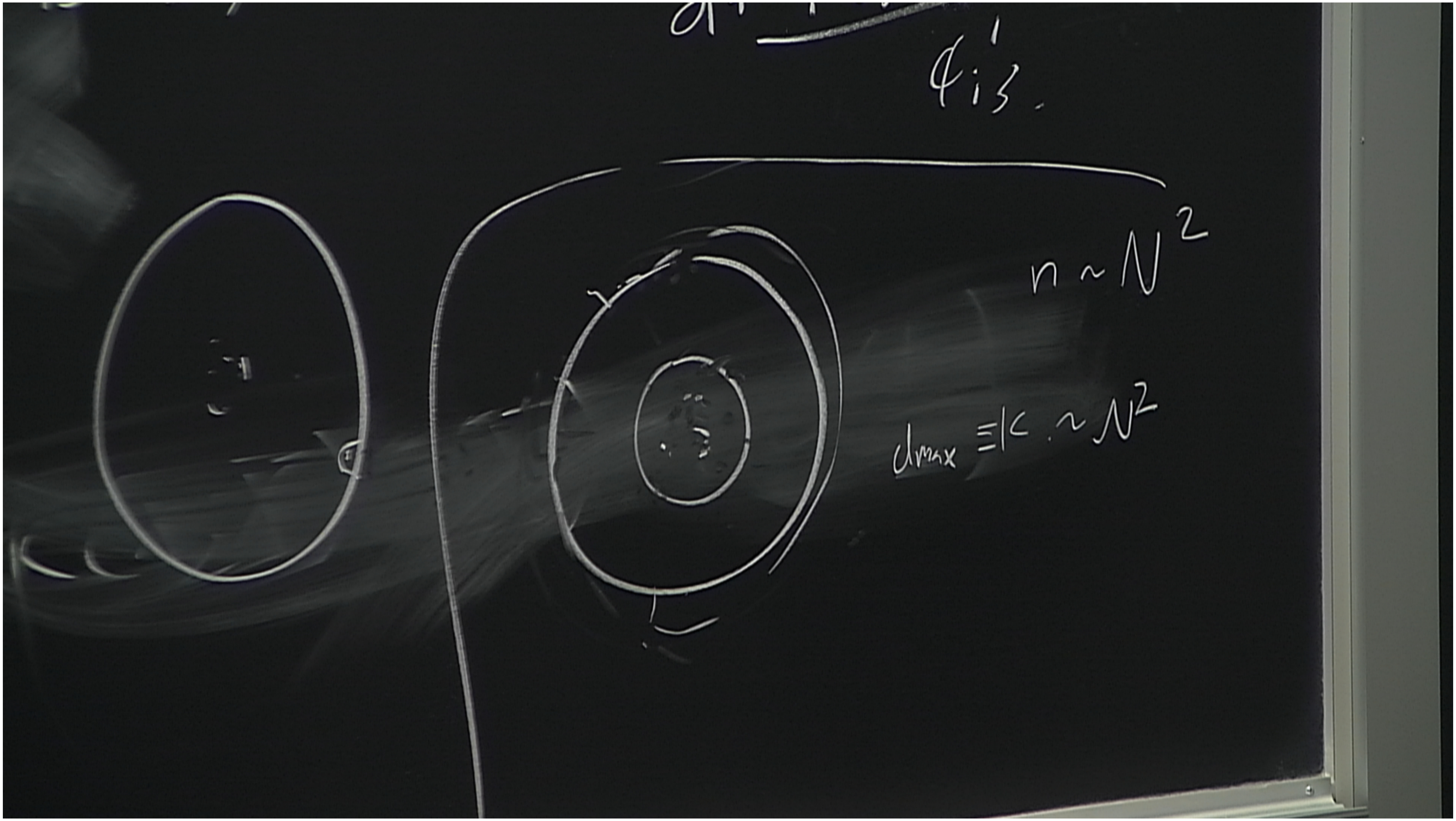
$\phi(x)$ obeys:

1) $\phi$ obey bulk EOM
2) $\phi$ obeys ext. dictionary

check Algebra

$$\phi(x) = \int_R d\bar{X}\ K(x;\bar{X})\,\mathcal{O}(\bar{X}) + \mathcal{O}\left(\frac{1}{N}\right)$$


Pick a set $\phi_i(x)$
$\mathcal{H}_C = \text{Span}\{ |\Omega\rangle, \phi_i(x)|\Omega\rangle, \phi_i(x)\phi_{i'}(x')|\Omega\rangle, \}$
at most dmax $\phi_i$'s.
$n \sim N^2$


n ~ N^2
dmax ≡ K ~ N^2