Title: Highly Entangled Quantum Matter

Date: Mar 09, 2015 02:00 PM

URL: http://pirsa.org/15030117

Abstract: <p>The interplay of quantum mechanics and inter-particle interactions leads to enormously rich tapestry of quantum phases of matter. In this talk I will illustrate the unique synthesis offered by quantum entanglement on the landscape of quantum phases. I will especially discuss phases which do not show any kind of ordering even at the absolute zero temperature, two prime examples being spin liquids and quantum Hall phases. These phases are fascinating because they can exhibit extraordinary properties such as emergent fermions in a system composed only of bosons, or systems where the elementary excitations are neither fermions, nor bosons (â€œanyonsâ€•). I will discuss how quantum entanglement not only plays a crucial role in characterizing and diagnosing such quantum disordered phases, but in fact it also sheds light on their stability.</p>

Highly Entangled Quantum Matter

Tarun Grover
KITP, Santa Barbara

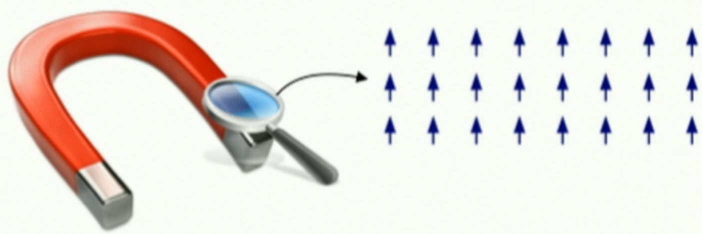
Outline

- Introduction to quantum liquids.
- Entanglement and topological Berry phase.
- Stability of quantum spin liquids.

Most Phases Order *in some way* as temperature is lowered

Crystals

Magnet



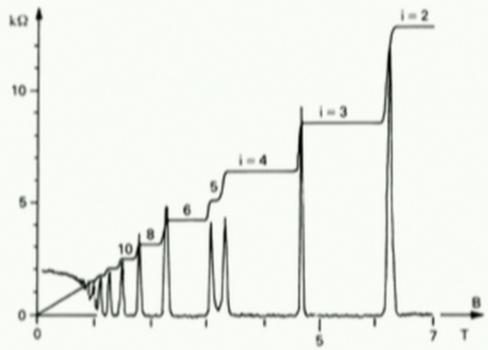
This Talk:

“Quantum Liquids” = No Symmetry breaking.

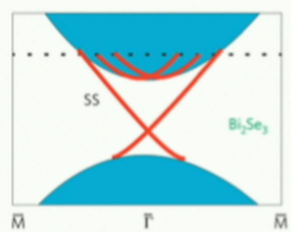
Do they exist?

Yes!

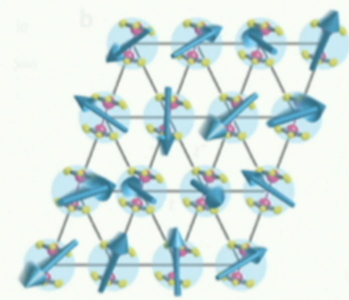
Several kinds of quantum liquids.



Quantum Hall



Topological Insulator



Quantum Spin-liquids

How to understand quantum liquids?

"Assume that a group of intelligent theoretical physicists had lived in closed buildings from birth such that they never had occasion to see any natural structures. What would they be able to predict from a fundamental knowledge of quantum mechanics? They probably would predict the existence of atoms, of molecules, of solid crystals, both metals and insulators, of gases, but most likely not the existence of liquids."

- Victor Weisskopf

How to understand quantum liquids?

Quantum entanglement provides the key.

What is Quantum Entanglement?

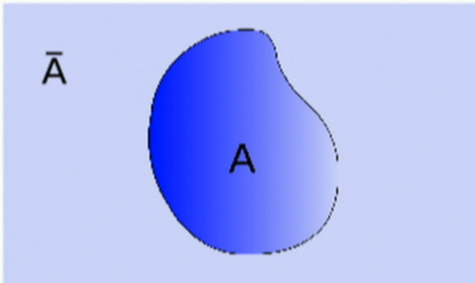
$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle\,|\downarrow\rangle - |\downarrow\rangle\,|\uparrow\rangle\right)$$ ("EPR singlet")

A measurement on the first spin affects the outcome of a subsequent measurement on the second spin.

The two spins are "entangled".

Quantum entanglement: Basics

- Divide system into two parts...



Reduced density-matrix for A:

$$\rho_A = \text{Trace}_{\bar{A}} |\psi\rangle\langle\psi|$$

- von Neumann entropy: $S = -\text{Trace}(\rho_A \log \rho_A)$

- S=0 if and and if $|\psi\rangle$ is a product state i.e.

$$|\psi\rangle = |\phi\rangle_A \otimes |\phi\rangle_{\bar{A}}$$

Entanglement of EPR Singlet

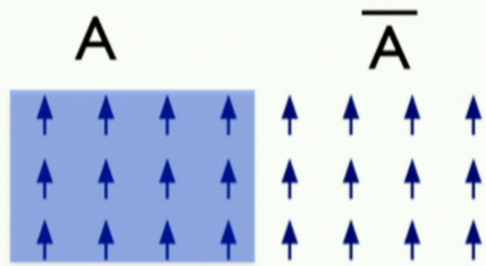
$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle\,|\downarrow\rangle - |\downarrow\rangle\,|\uparrow\rangle\right)$$

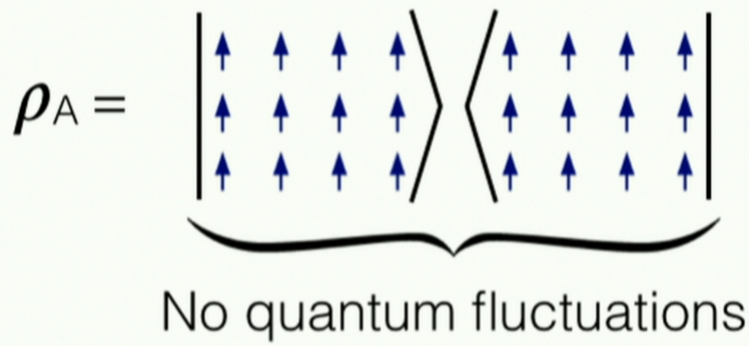
$$\rho_{\text{First Spin}} = \underbrace{\frac{1}{2}\left(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|\right)}_{\text{Two independent ways to quantum fluctuate}}$$

$$\Rightarrow \quad S = \log(2)$$

Maximum possible entanglement for two spin system.

Symmetry breaking = Low entanglement





How to generate quantum liquids?

One guess:

$$|\psi\rangle = \left|\begin{matrix}\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\\\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\\\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\end{matrix}\right\rangle + \left|\begin{matrix}\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\\\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\\\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\end{matrix}\right\rangle$$

"Schrodinger's Cat"

$\frac{1}{\sqrt{2}}\Big[\,\big|$  $\big\rangle + \big|$  $\big\rangle\,\Big]$

Very Unstable to tiny perturbations!

Unphysical

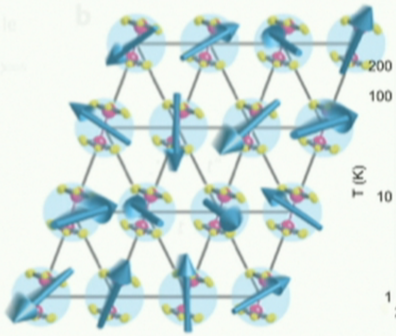
A Stable Version of Schrodinger's Cat

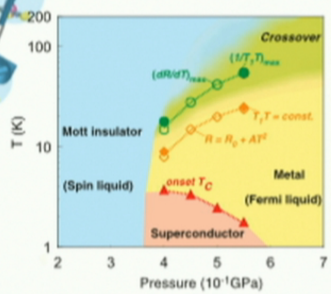
$\frac{1}{\sqrt{N}}\Big[\,\Big|$  $\Big\rangle + \Big|$  $\Big\rangle + \Big|$  $\Big\rangle$

$+ \Big|$  $\Big\rangle + \ldots + \Big|$  $\Big\rangle\,\Big]$

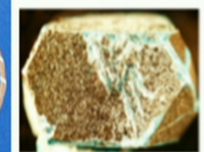
N = Number of Possible Moods of Cat >> 1.

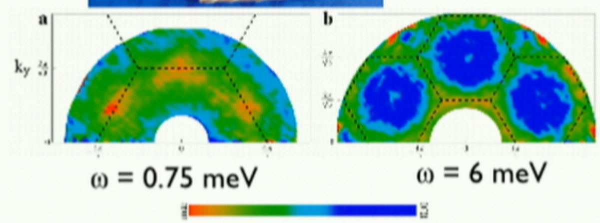
Quantum Spin-liquids= Stable Schrodinger Cats



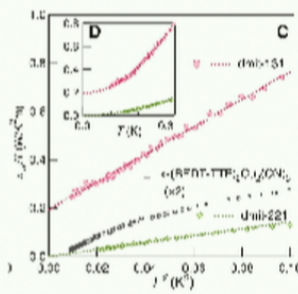


kappa-ET salts
Kanoda et al (2005)





$ZnCu_3(OD)_6Cl_2$
Han et al (2012)



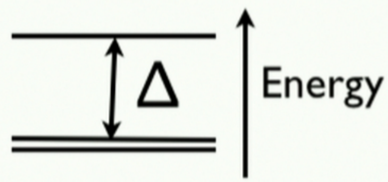
Et $Me_4Sb[Pd(dmit)_2]_2$
Yamashita et al (2010).

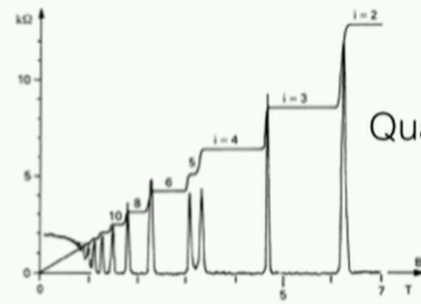
Quantum Spin-liquid =

Spin System with no order & strong quantum fluctuations.

Two types of Quantum liquids

Gapped

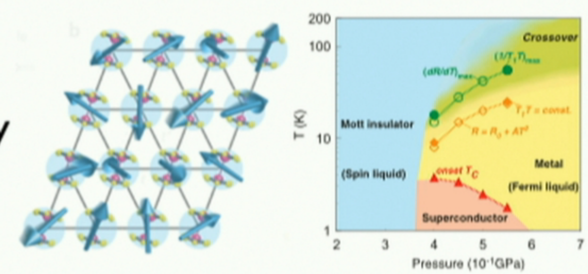




Quantum Hall

Gapless

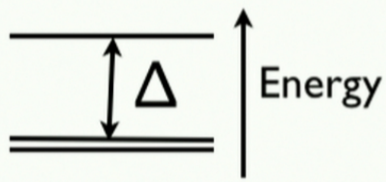
Less well understood.

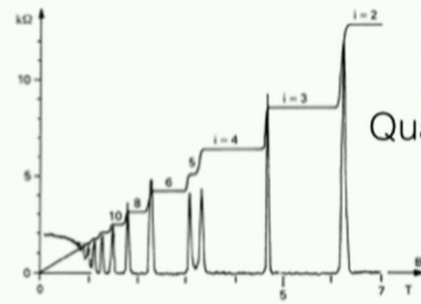


Triangular lattice organic compounds

Two types of Quantum liquids

Gapped





Quantum Hall

Gapless

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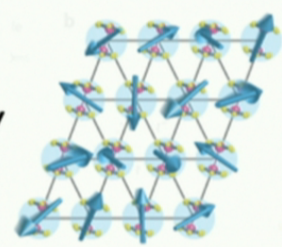
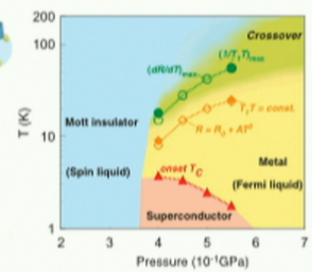


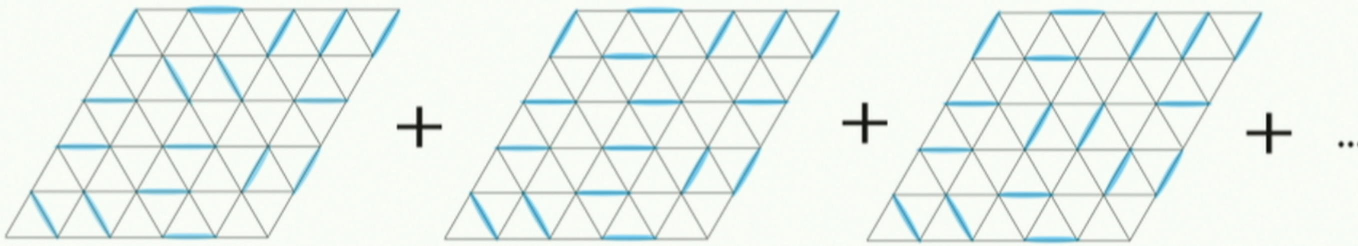


Triangular lattice organic compounds

A simple Gapped Spin Liquid

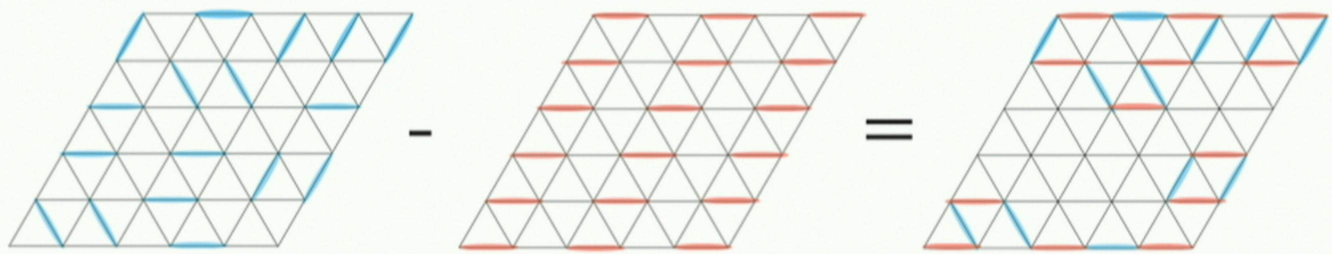
Tiling of EPR pairs on lattice

= $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ = EPR singlet



Physics of Gapped Spin-Liquids

Gapped Spin liquids = “Soup” of Closed loops



Physics of Gapped Spin-Liquids

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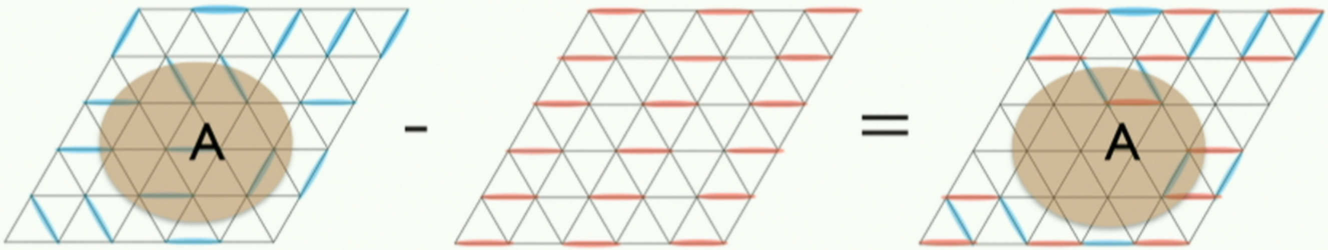
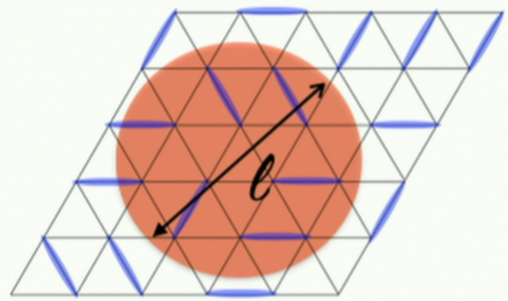


Closed loops ⇒ **Even number of intersections** at boundary of a subregion A

Entanglement entropy keeps track of <u>global</u> constraint on the number of intersections.

“Topological Entanglement Entropy”



“Boundary law” of entanglement entropy

$$S = \ell - \gamma + O(1/\ell)$$

γ = log(Number of constraints on the boundary of subregion A)

= topological invariant for gapped quantum liquids!

(Hamma, Ionicioiu, Zanardi 2005; Kitaev,Preskill 2006;Levin,Wen 2006):

$\gamma_1 \neq \gamma_2 \Rightarrow$ Distinct Phases!

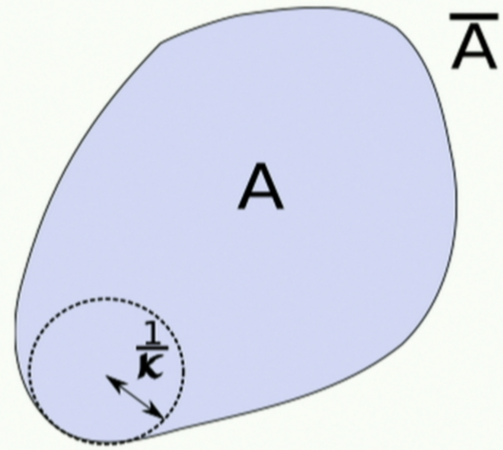
Topological Entanglement = Non-local Correlations

- Extract 'local' part of entanglement.

$$S_{\text{local}} = \int_{\partial A} F(\kappa, \partial_i \kappa)$$

$$S(A) = S(\bar{A}) \Rightarrow F(\kappa) \text{ even in } \kappa$$

$$\Rightarrow S_{local} = aL + b/L + c/L^3 + \ldots$$

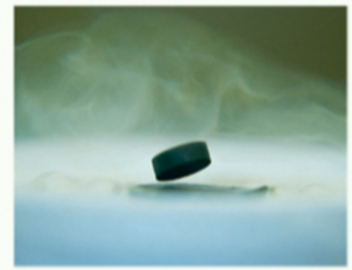


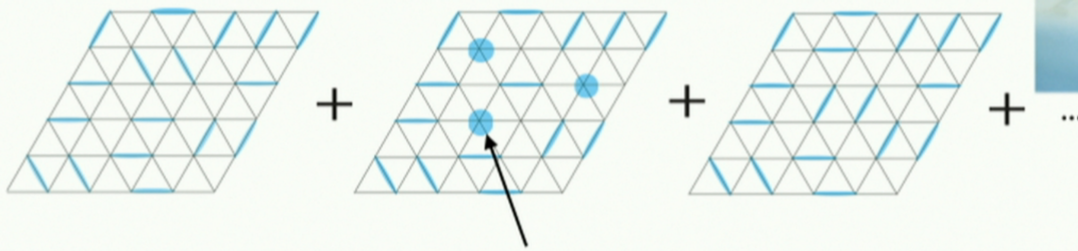
$\Rightarrow$ Topological Entanglement *must* come from non-local correlations!

Grover, Turner, Vishwanath 2011

From Superconductor to Quantum Spin-liquid

- Superconductor = condensate of EPR pairs

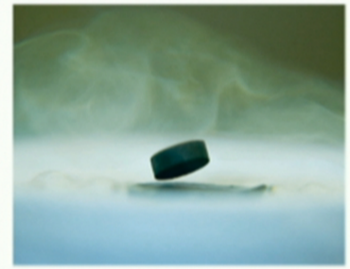


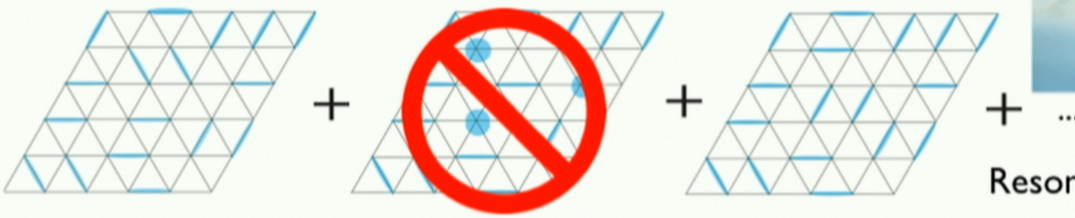


EPR singlet on the same site.

From Superconductor to Quantum Spin-liquid

•Superconductor = condensate of EPR pairs



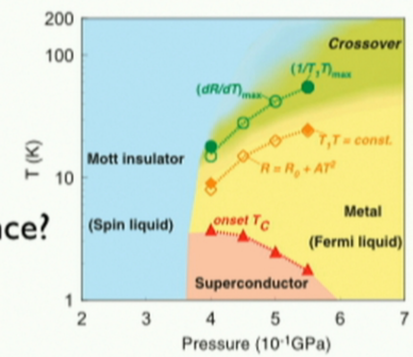


Resonating Valence Bond
"RVB"
(Anderson 1973)

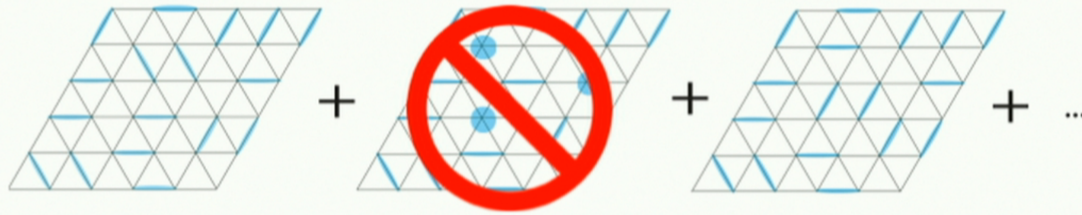
•No double occupancy ⇒ Spin-system

•Putative spin-liquid! (Anderson, Baskaran, Wen, Laughlin, Moessner, Sondhi, Ivanov, Senthil, Fisher, ...)

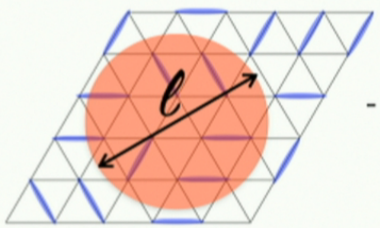
Coincidence?

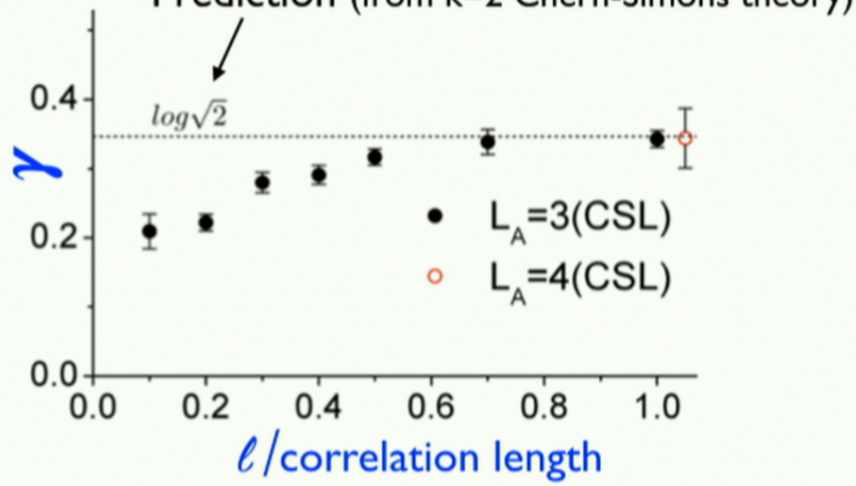


"Projected" Superconductor= Spin-liquid?



$S = \ell - \gamma + O(1/\ell)$





Zhang, Grover, Vishwanath 2011

Topological entanglement only partially characterizes a gapped quantum liquid.

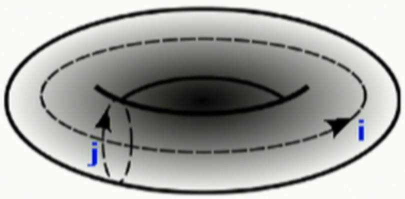
$\gamma_1 \neq \gamma_2 \Rightarrow$ Distinct Phases

But, Distinct Phases $\nRightarrow$ $\gamma_1 \neq \gamma_2$

Finer Characterization? **Anyons**

Entanglement and Topological Berry's Phase

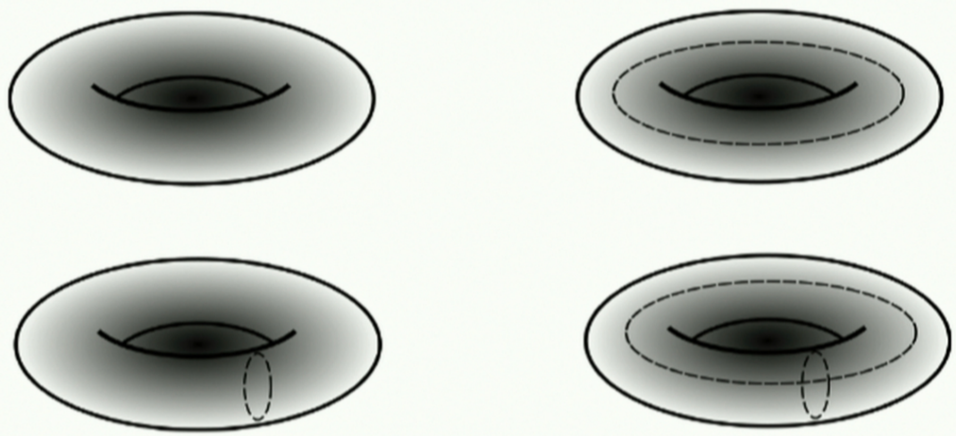
- Excitations of Gapped Quantum Liquids are Anyons
- Anyon = Neither boson, nor fermion.
- Berry phase upon encircling.



How to detect anyons, given the ground state?

Anyons $\Rightarrow$

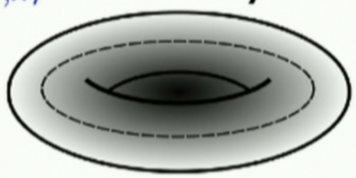
Degenerate ground states on torus...

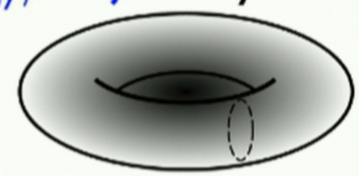


Mutual Statistics of anyons?

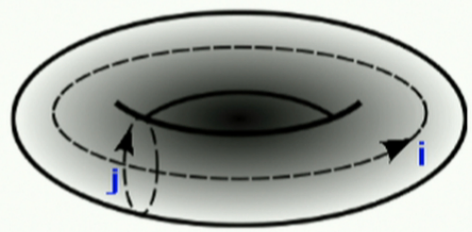
Mutual statistics S_{ij} = Berry phase of anyon *i* around *j*.

$|\Sigma_{i,x}\rangle =$ i'th anyon along $\hat{x}$ $|\Sigma_{j,y}\rangle =$ j'th anyon along $\hat{y}$





$$S_{ij} \propto {}_{i,x}\langle\Sigma|\Sigma\rangle_{j,y}$$



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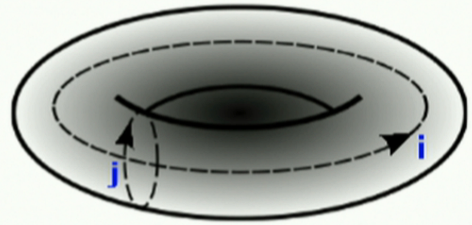
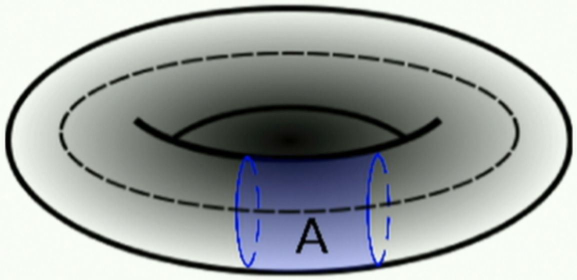


How to find states $|\Sigma\rangle_{i,\hat{n}}$ that correspond to

anyons winding around non-contractible loop along $\hat{n}$?

Key Observation



Anyon along $x \Rightarrow$

Minimum entanglement entropy for a non-contractible cut perpendicular to x.

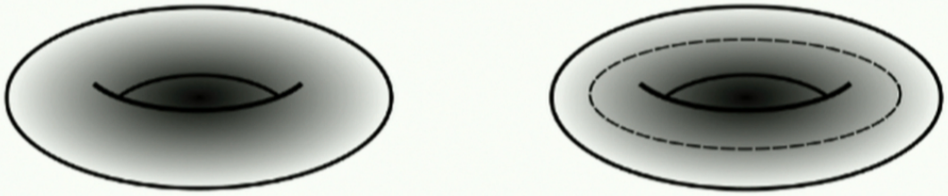
Intuition: Minimum Entanglement $\Rightarrow$ Maximum knowledge of anyonic content.

Zhang, Grover, Turner, Oshikawa, Vishwanath 2012

Example:

Anyons in a Spin Liquid

- Simplest spin-liquid: two ground states $|1\rangle$ and $|2\rangle$

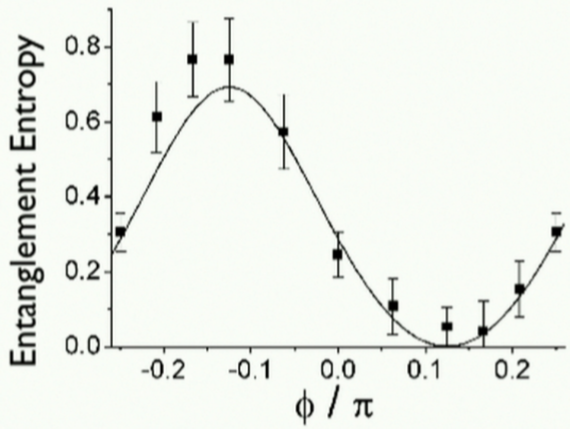


- Superpose: $|\Phi\rangle = cos(\phi)|1\rangle + sin(\phi)|2\rangle$

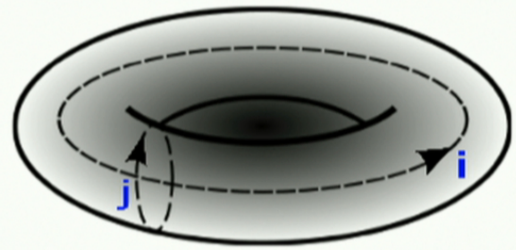
- Minimize entanglement entropy numerically to get quasiparticle states $|\Sigma\rangle_{i,\hat{n}}$

Zhang, Grover, Turner, Oshikawa, Vishwanath 2012

- Minimize entanglement entropy numerically to get quasiparticle states $|\Sigma\rangle_{i,\hat{n}}$



$S_{ij} \propto {}_{i,x}\langle\Sigma|\Sigma\rangle_{j,y}$



Numerical Result

S=

	Boson	Anyon
Boson	$1.1\ e^{2\pi i}$	$0.9\ e^{2\pi i}$
Anyon	$0.9\ e^{2\pi i}$	$1.1\ e^{i\pi}$

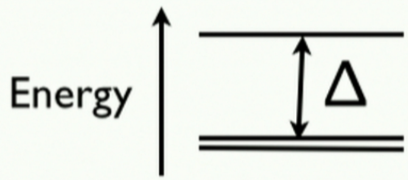
→ Anyon with $\theta = \frac{\pi}{2}$ self-statistics!

Zhang, Grover, Turner, Oshikawa, Vishwanath 2012

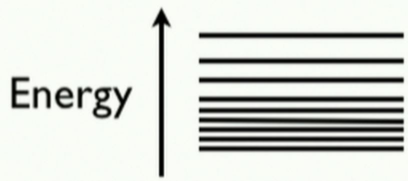
Outline

- Introduction to quantum liquids.
- Entanglement and topological Berry phase.
- Stability of quantum spin liquids.

Two types of Quantum liquids



Anyons,
Topological Entanglement.
Always stable



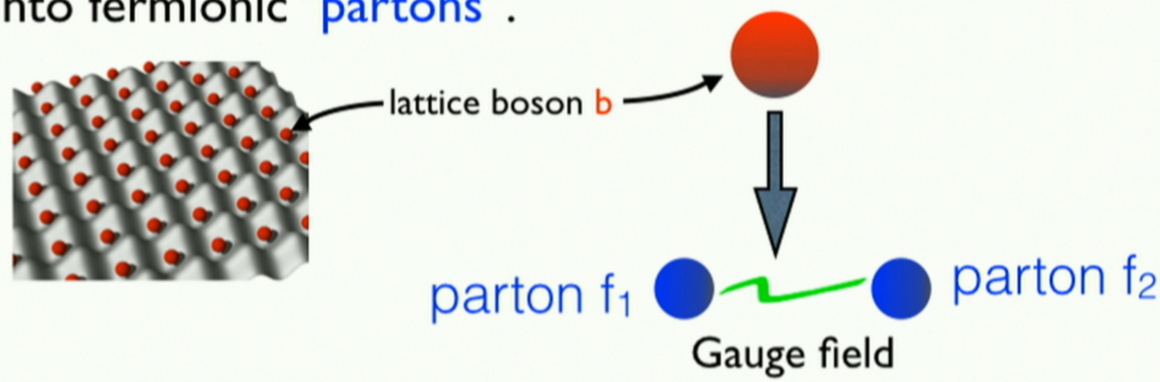
Less well understood.
Theoretical Description?
Stability?

Gapless Spin-liquids = Emergent gauge theories

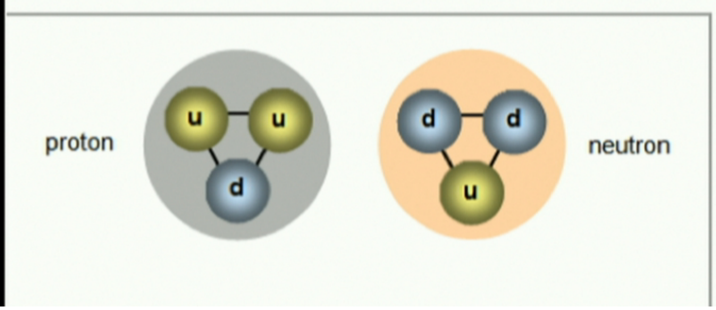
in quantum materials!

Why Emergent Gauge Structure?

- Quantum fluctuations may "fractionalize" spins/bosons into fermionic "partons".



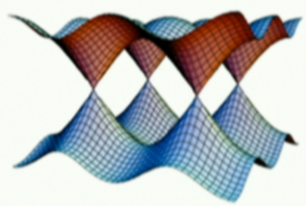
Read, Sachdev 1989; Wen 1990

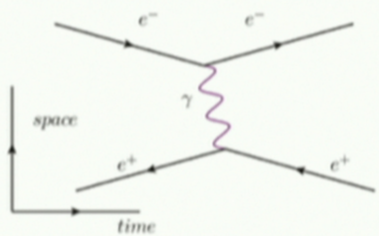


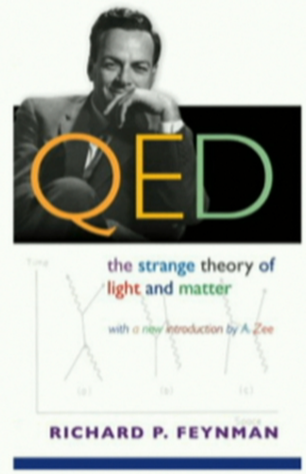
Focus: QED Spin-liquids

Emergent Dirac Fermions +

Emergent Photons in 2d





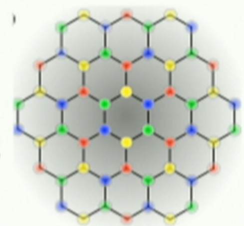


QED Spin-liquids: Theoretical & (Possibly) Experimental Sightings

Spin-orbital model on Honeycomb lattice

$$\mathcal{H} = \sum_{\langle i,j \rangle} (2\mathbf{S}_i \cdot \mathbf{S}_j + \frac{1}{2})(2\mathbf{T}_i \cdot \mathbf{T}_j + \frac{1}{2})$$

Corboz et al 2012

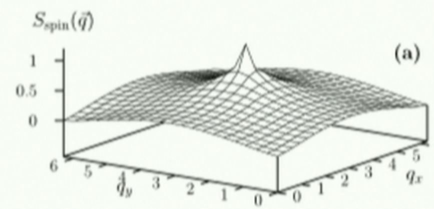


SU(N) Hubbard-Heisenberg Models

$$H = \frac{J}{N} \sum_{\langle \vec{i},\vec{j} \rangle} \sum_{\alpha,\beta} S_{\alpha,\beta,\vec{i}} S_{\beta,\alpha,\vec{j}}$$

Assaad 2004

$$S_{\alpha,\beta,\vec{i}} = c^{\dagger}_{\alpha,\vec{i}} c_{\beta,\vec{i}} - \frac{1}{N} \delta_{\alpha,\beta} \sum_{\gamma} c^{\dagger}_{\gamma,\vec{i}} c_{\gamma,\vec{i}}$$

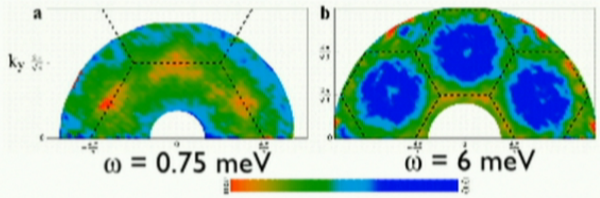


QED Spin-liquids: Theoretical & (Possibly) Experimental Sightings

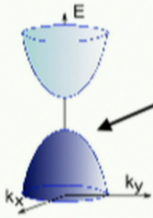
kagome lattice material "Herbertsmithite"

Han et al

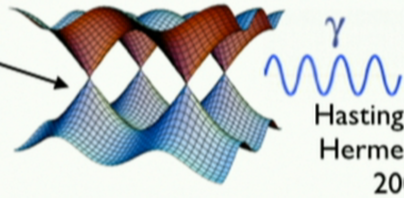
Structure factor S(k, ω)



Theoretical Proposals:



Spins fractionalize into fermions



Hastings 2000; Hermele et al 2008.

Gapped Spin-liquid

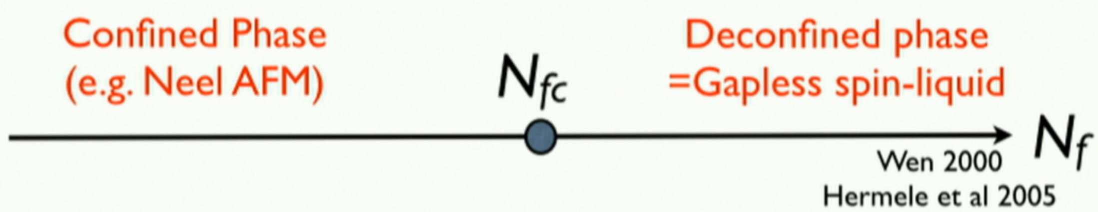
Sachdev 1993; Yan, Huse, White 2010; Jiang, Balents 2012; Rousochatzakis, Wan, Tchernyshyov, Mila 2013.

QED Spin-liquid

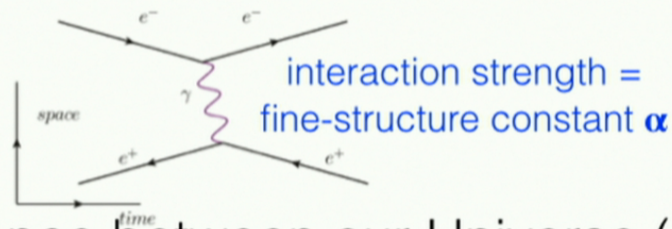
Phase Diagram of QED Spin-liquids?

$$\mathcal{L}_{QED-3} = \sum_{a=1}^{N_f} \overline{\psi}_a \left[-i\gamma_\mu \left(\partial_\mu + i a_\mu\right)\right] \psi_a + \frac{1}{2g^2} F_{\mu\nu} F_{\mu\nu}$$

N_f determined by parton band-structure.



Critical value of N_f above which spin-liquid stable?



Key difference between our Universe (3 space dimensions) & QED Spin liquids (2 space dimensions):

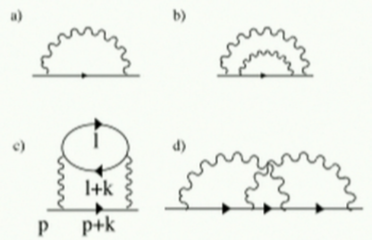
In 3d, $\alpha \cong 1/137$,

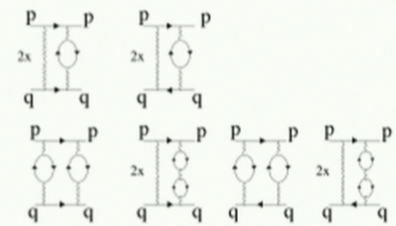
However, in 2d $\alpha \cong 1$.

Extremely Challenging to calculate anything!

How to determine Stability of QED Spin-liquids?

How to determine stability of *anything*?





$$\Gamma^{\omega}_{\alpha\beta,\alpha\beta}(p_F,q) = U(0) + i\int \frac{d^{D+1}l}{(2\pi)^{D+1}} U^2(|\mathbf{p}_F - \mathbf{l}|)\left(G_l G_{q-p_F+l} + G_l G_{q+p_F-l}\right)$$

$$-\delta_{\alpha\beta}\left[U(|\mathbf{q}-\mathbf{p}_F|) - i\int \frac{d^{D+1}l}{(2\pi)^{D+1}} \left[\{2U(|\mathbf{q}-\mathbf{p}_F|) - 2U(|\mathbf{q}-\mathbf{p}_F|)U(|\mathbf{p}_F-\mathbf{l}|)\} G_l G_{l+q-p_F} - U(|\mathbf{p}_F-\mathbf{l}|)U(|\mathbf{l}-\mathbf{q}|)G_l G_{q+p_F-l}\right]\right]$$

and $\delta\Sigma_{\rm pert}(\omega,\epsilon_{\mathbf{p}}) = \Sigma_{\rm pert}(\omega,\epsilon_{\mathbf{p}}) - \Sigma_{\rm pert}(0,0) = \delta\Sigma_1 + \delta\Sigma_2$ with

$$\delta\Sigma_1(\omega,\epsilon_{\mathbf{p}}) = \int d_{lq}\left[2G_l G_{k-p_F+l}\left\{U^2(|\mathbf{q}-\mathbf{p}_F|) - 2U(|\mathbf{p}_F-\mathbf{l}|)U(|\mathbf{q}-\mathbf{p}_F|)\right\} - G_l G_{q+p_F-l} U(|\mathbf{p}_F-\mathbf{l}|)U(|\mathbf{l}-\mathbf{q}|)\right](G_{k-\epsilon} - G_k)$$

$$\frac{\partial G_{\mathbf{p}}^{-1}}{\partial\omega} = \frac{1}{Z} = 1 - \frac{i}{2}\sum_{\alpha\beta}\int \Gamma^{\omega}_{\alpha\beta,\alpha\beta}(p_F,q)(G_q^2)_{\omega}\frac{d^{D+1}q}{(2\pi)^{D+1}},$$

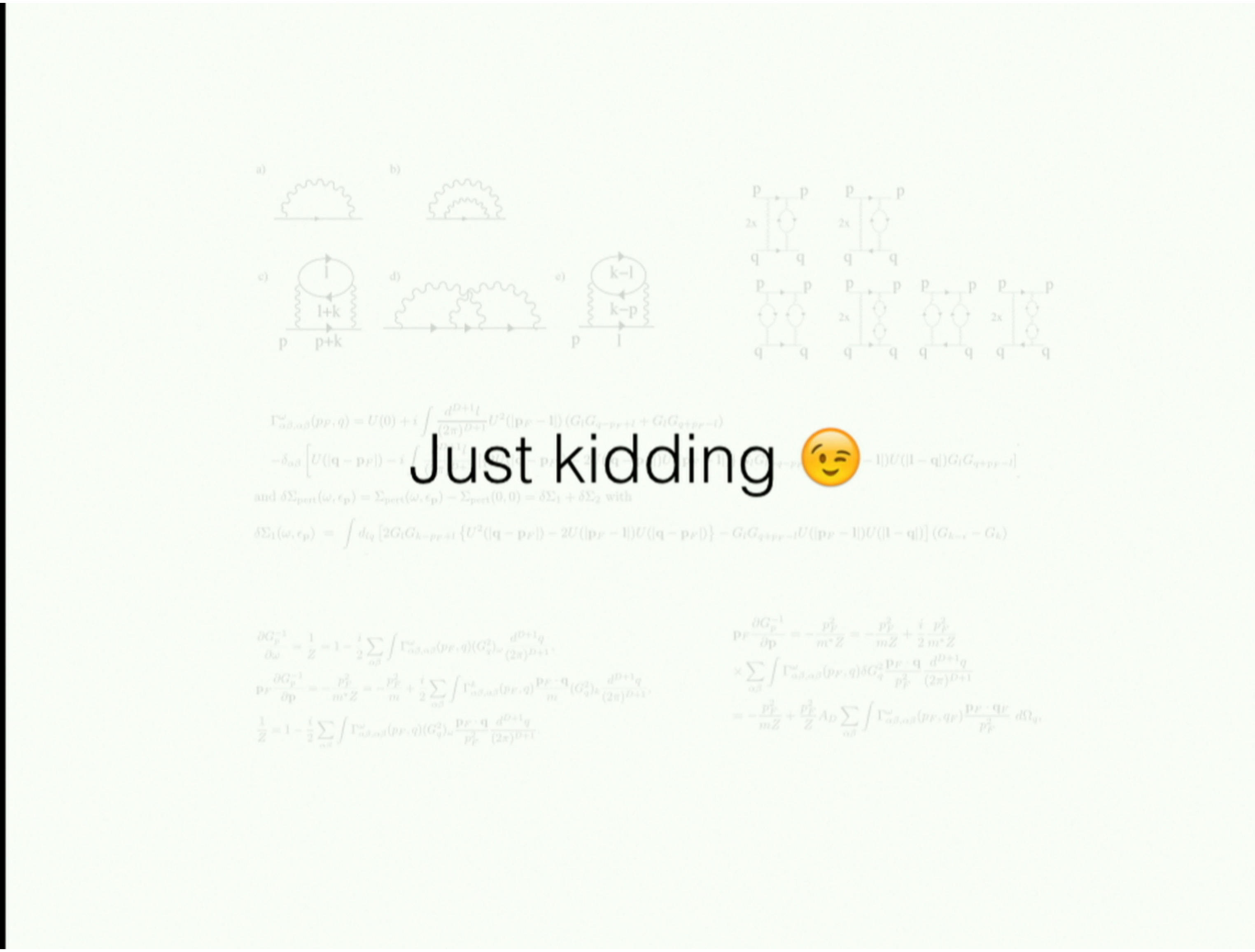
$$\mathbf{p}_F\frac{\partial G_{\mathbf{p}}^{-1}}{\partial\mathbf{p}} = -\frac{p_F^2}{m^* Z} = -\frac{p_F^2}{m} + \frac{i}{2}\sum_{\alpha\beta}\int \Gamma^{k}_{\alpha\beta,\alpha\beta}(p_F,q)\frac{\mathbf{p}_F\cdot\mathbf{q}}{m}(G_q^2)_k\frac{d^{D+1}q}{(2\pi)^{D+1}},$$

$$\frac{1}{Z} = 1 - \frac{i}{2}\sum_{\alpha\beta}\int \Gamma^{\omega}_{\alpha\beta,\alpha\beta}(p_F,q)(G_q^2)_{\omega}\frac{\mathbf{p}_F\cdot\mathbf{q}}{p_F^2}\frac{d^{D+1}q}{(2\pi)^{D+1}}.$$

$$\mathbf{p}_F\frac{\partial G_{\mathbf{p}}^{-1}}{\partial\mathbf{p}} = -\frac{p_F^2}{m^* Z} = -\frac{p_F^2}{mZ} + \frac{i}{2}\frac{p_F^2}{m^* Z}$$

$$\times\sum_{\alpha\beta}\int \Gamma^{\omega}_{\alpha\beta,\alpha\beta}(p_F,q)\delta G_q^2\frac{\mathbf{p}_F\cdot\mathbf{q}}{p_F^2}\frac{d^{D+1}q}{(2\pi)^{D+1}}$$

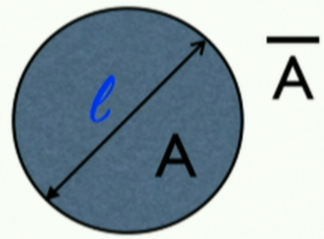
$$= -\frac{p_F^2}{mZ} + \frac{p_F^2}{Z}A_D\sum_{\alpha\beta}\int \Gamma^{\omega}_{\alpha\beta,\alpha\beta}(p_F,q_F)\frac{\mathbf{p}_F\cdot\mathbf{q}_F}{p_F^2}\,d\Omega_q,$$


Just kidding 😉

Stability of Phases and Entanglement

- Entanglement of 2D gapless CFTs and topological phases for circular bipartition:

$$S = \ell - \gamma + O(1/\ell)$$

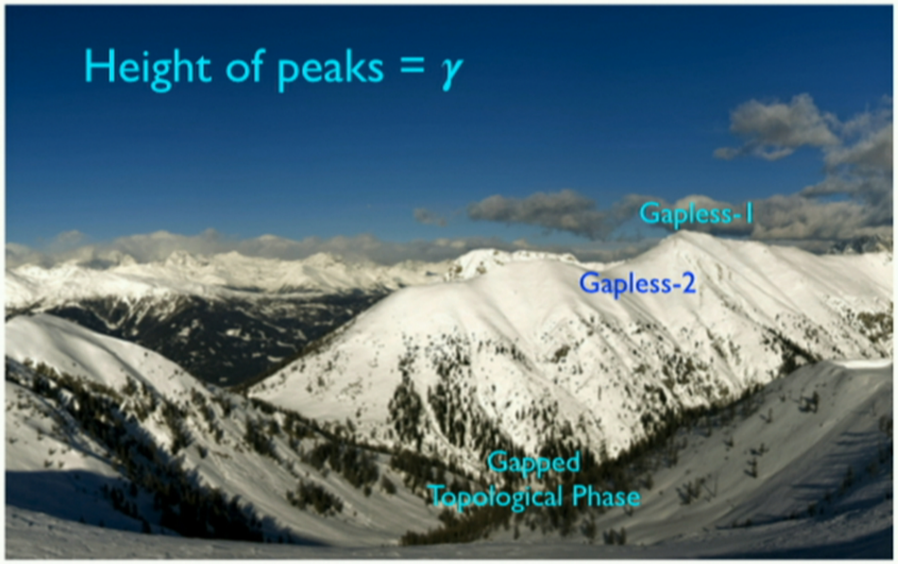


- If phase-1 unstable to phase-2, then $\gamma_1 \geq \gamma_2$.

"F-theorem" or "Entanglement Monotonicity"

Jafferis, Klebanov, Pufu, Safdi 2011; Casini, Huerta 2012; Casini, Huerta, Myers 2011; Casini, Huerta, Myers 2015.

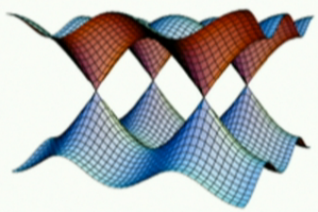
Landscape of Lorentz Invariant Theories in 2+1-D

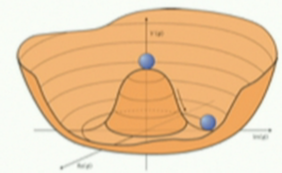


Gapless-2 cannot be unstable to Gapless-1 while the gapped topological phase is fully stable.

Example: Stability of Graphene

Can Graphene break all symmetries (valley+spin) spontaneously with infinitesimal interactions?





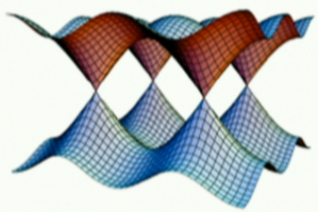
15 Goldstone modes

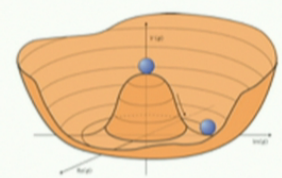
$\gamma_{\text{Dirac fermion}} \approx 0.22$

$\gamma_{\text{Scalar boson}} \approx 0.06$

Example: Stability of Graphene

Can Graphene break all symmetries (valley+spin) spontaneously with infinitesimal interactions?





15 Goldstone modes

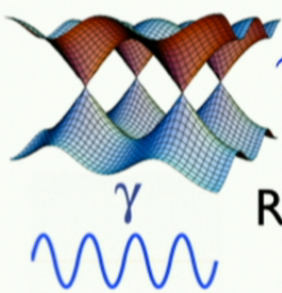
$\gamma_{\text{graphene}} = 4\gamma_{\text{Dirac}} \approx 0.88$ $\gamma_{\text{Goldstone}} = 15\gamma_{\text{Scalar boson}} \approx 0.90$

Not allowed!

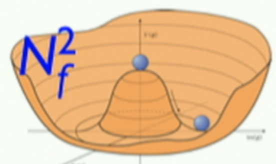
Stability of QED Spin-liquid via γ-theorem

Instability of QED-3 generates $N^2{}_f/2$ Goldstone modes.

But, these are too many to satisfy entanglement monotonicity when N_f large...



$\gamma_{QED-3} \propto N_f$ while $\gamma_{\text{Goldstone}} \propto N_f^2$

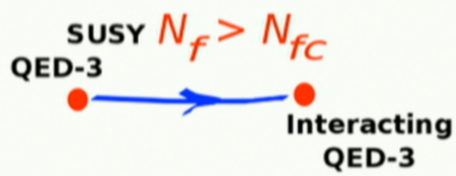


Rough estimate: $N_{fc} \simeq 2 \times \frac{\gamma_{\text{Free Dirac Fermion}}}{\gamma_{\text{Free Real Scalar}}} \simeq 8$

Grover 2012

Exact Upper bound on Stability by "Sandwiching"

Deform an auxiliary theory ("Supersymmetric QED", or SQED) to obtain QED.



$$\gamma_{SQED-3} \geq \gamma_{QED-3} + \gamma_{Dirac} \quad (1)$$

$$\gamma_{QED-3} \geq \gamma_{Goldstone} \quad (2)$$

Spin-liquid Stable for $N_f > 13$ Grover 2012

Improved bound by "Sandwiching"

Similar strategy, with some assumptions

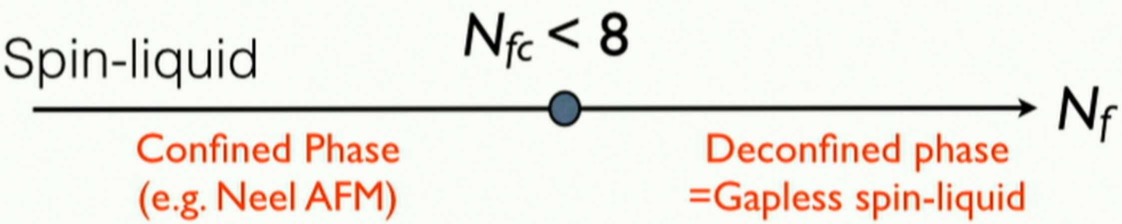
Spin-liquid Stable for $N_f > 7$

Grover 2012

Implications

Exact non-perturbative result for stability of strongly interacting, gapless spin-liquid.

QED-3 Spin-liquid



More generally, exact results on the phase diagram of 2+1-D abelian and non-abelian gauge theories.

Non-abelian version: $N_f \gtrsim 2N_c \frac{\gamma_{Dirac}}{\gamma_{scalar}}$

Grover 2012

Implications

$N_{fc} < 8$

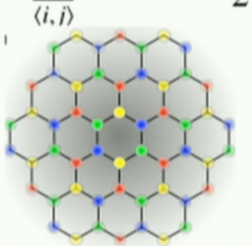
$\longrightarrow N_f$

Confined Phase
(e.g. Neel AFM)

Deconfined phase
=Gapless spin-liquid

Spin-orbital model on
Honeycomb lattice

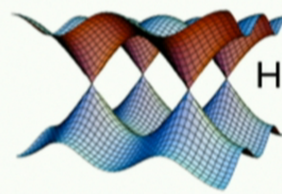
$$\mathcal{H} = \sum_{\langle i,j \rangle} (2\mathbf{S}_i \cdot \mathbf{S}_j + \frac{1}{2})(2\mathbf{T}_i \cdot \mathbf{T}_j + \frac{1}{2})$$



Corboz et al 2012

$N_f = 8$

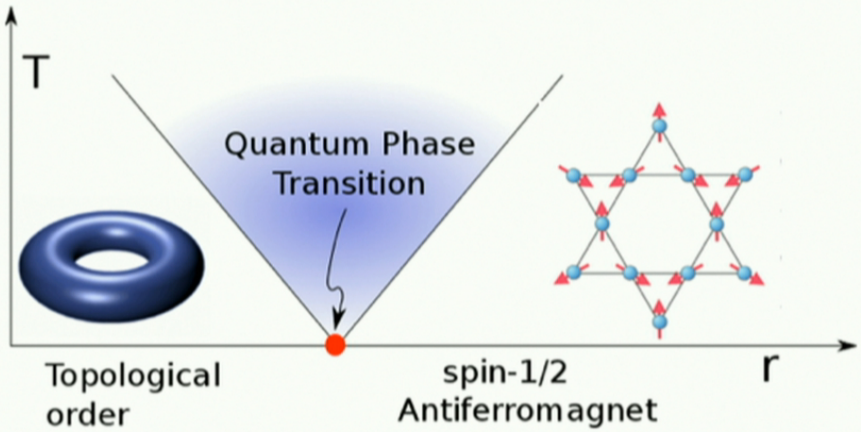
Dirac Spin-liquid on kagome
lattice



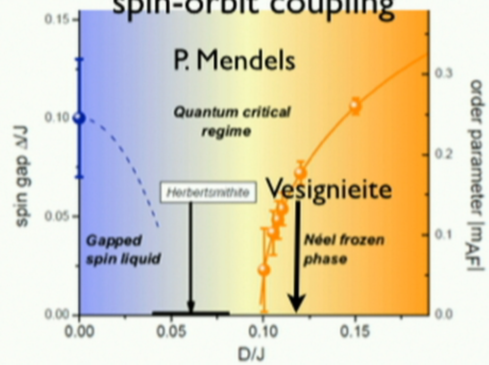
Hermele et al 2007

$N_f = 4$

Application II: Entanglement monotonicity & Quantum Phase Transitions



kagome phase diagram with spin-orbit coupling



Cepas et al 2008; Huh et al 2010

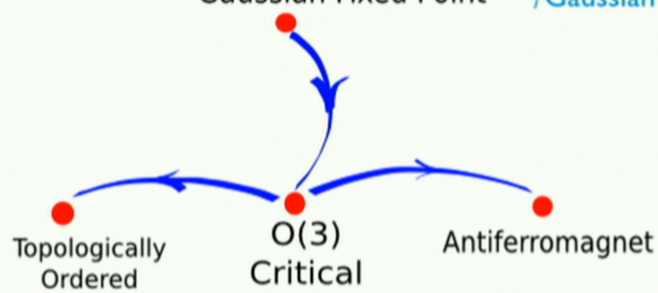
Question: Nature of transition?

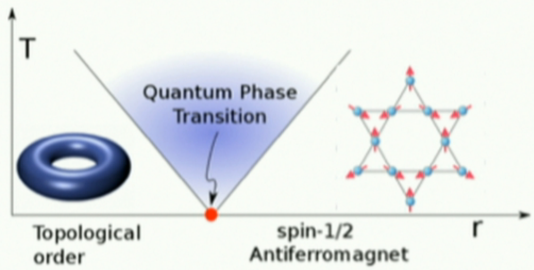
naive Landau-Ginzburg reasoning:
O(3) Wilson-Fisher.

A No-Go Theorem for Quantum Phase Transitions

RG flow <u>assuming</u> O(3) transition

Gaussian Fixed Point $\gamma_{\text{Gaussian}} = 3 \times \gamma_{\text{Scalar}} = 0.18$





$$\gamma_{\text{Topological}} = \log(D) \geq \log(\sqrt{2}) \approx 0.35$$

Contradiction with entanglement monotonicity!

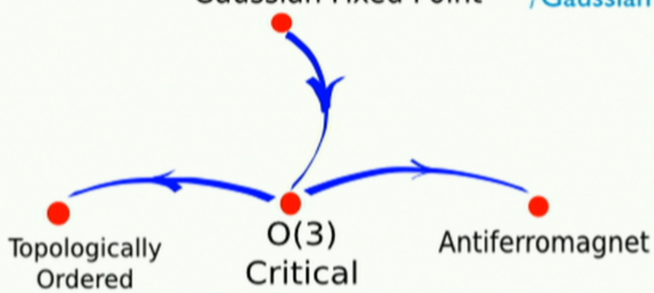
$\Rightarrow$ **O(3) Transition impossible.** TG 2012

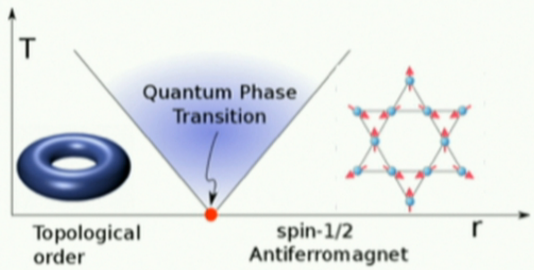
Obvious generalizations (SF $\longleftrightarrow$ FQH, Nematic $\longleftrightarrow$ $\mathbb{Z}_2$ Spin liquid ...)

A No-Go Theorem for Quantum Phase Transitions

RG flow assuming O(3) transition

Gaussian Fixed Point $\gamma_{\text{Gaussian}} = 3 \times \gamma_{\text{Scalar}} = 0.18$





$$\gamma_{\text{Topological}} = \log(D) \geq \log(\sqrt{2}) \approx 0.35$$

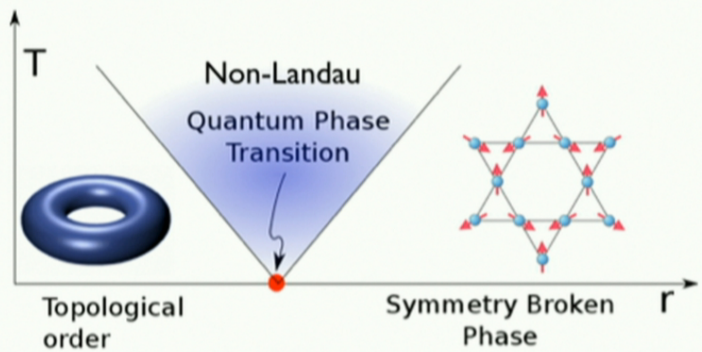
Contradiction with entanglement monotonicity!

$\Rightarrow$ **O(3) Transition impossible.** TG 2012

Obvious generalizations (SF $\longleftrightarrow$ FQH, Nematic $\longleftrightarrow$ $\mathbb{Z}_2$ Spin liquid ...)

Lesson

Phase transitions out of topologically ordered phases *necessarily* lie beyond Landau-Ginzburg paradigm

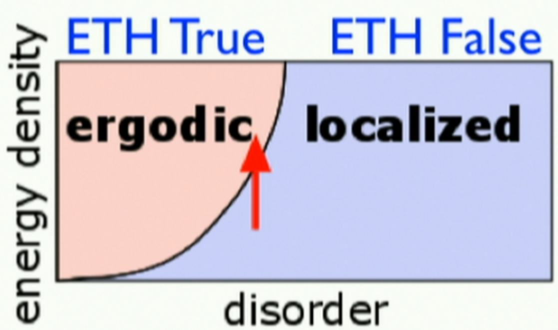


Consistent with field theoretic understanding of many such transitions, e.g., Spiral magnet↔ Spin-liquid (Senthil, Chubukov, Sachdev 1994); FQH ↔ Superfluid (Barkeshli, McGreevy 2011).

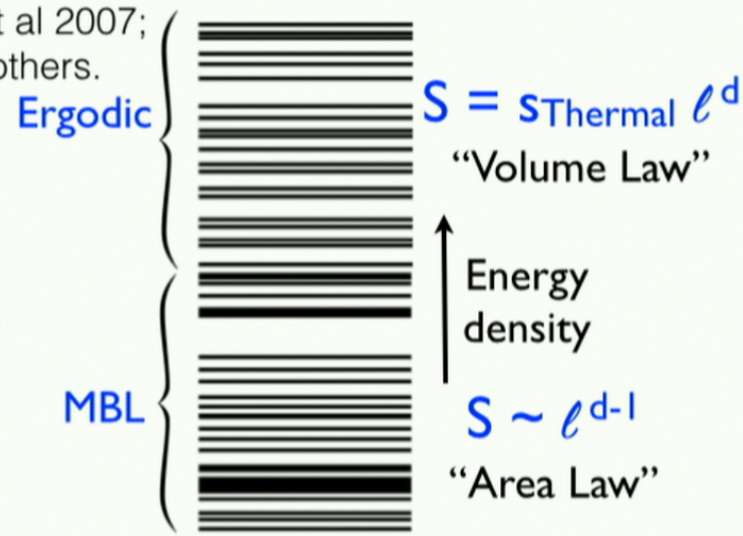
Application III: Many Body Localization Phase Transition

$$H = -\sum_{<ij>} \left(c_i^\dagger c_j + h.c. \right) + \sum_i \mu_i c_i^\dagger c_i + \sum_{<ij>} n_i n_j$$

Anderson 1958; Basko et al 2006; Huse et al 2007; Bauer et al 2013; Imbrie 2014 & many others.

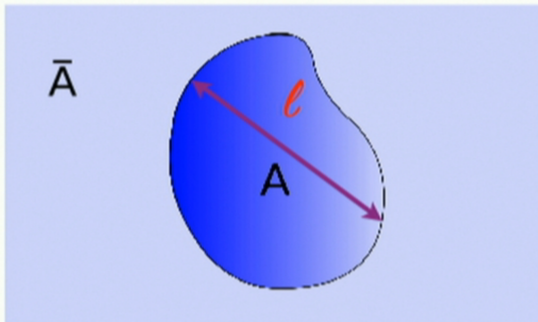


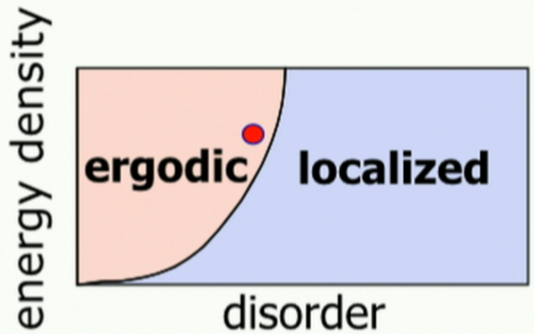
$= \mathrm{var}(\mu_i)$



Nature of transition?

Entanglement Scaling Close to MBL Transition



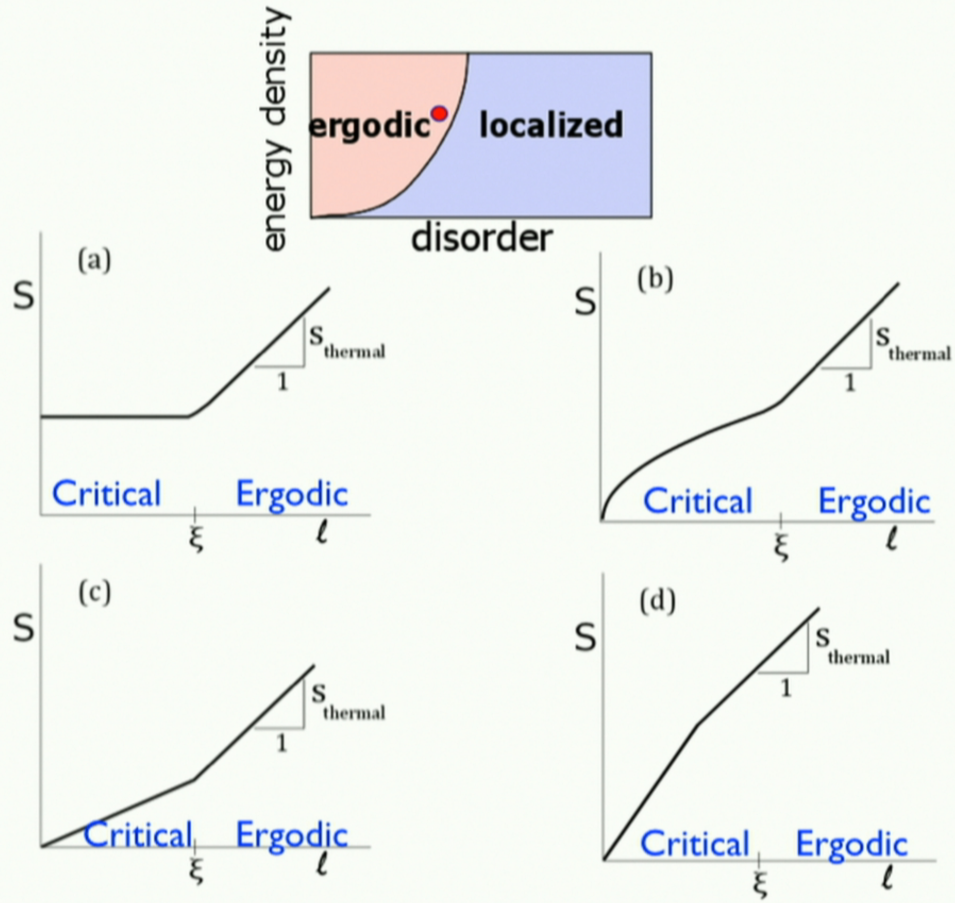


ξ = delocalization length   subsystem size ℓ

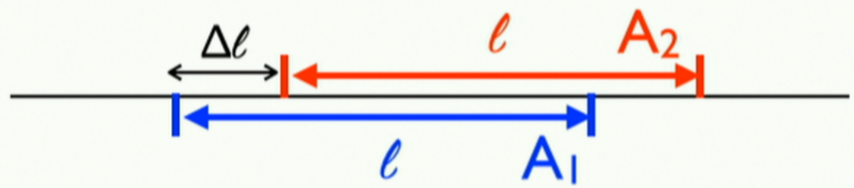
"universal entanglement" of critical point
↑
Unknown

Entanglement of ergodic phase
↑
Known

Critical Entanglement: Possible Scaling Behaviors.



Strong Subadditivity & Entanglement Scaling



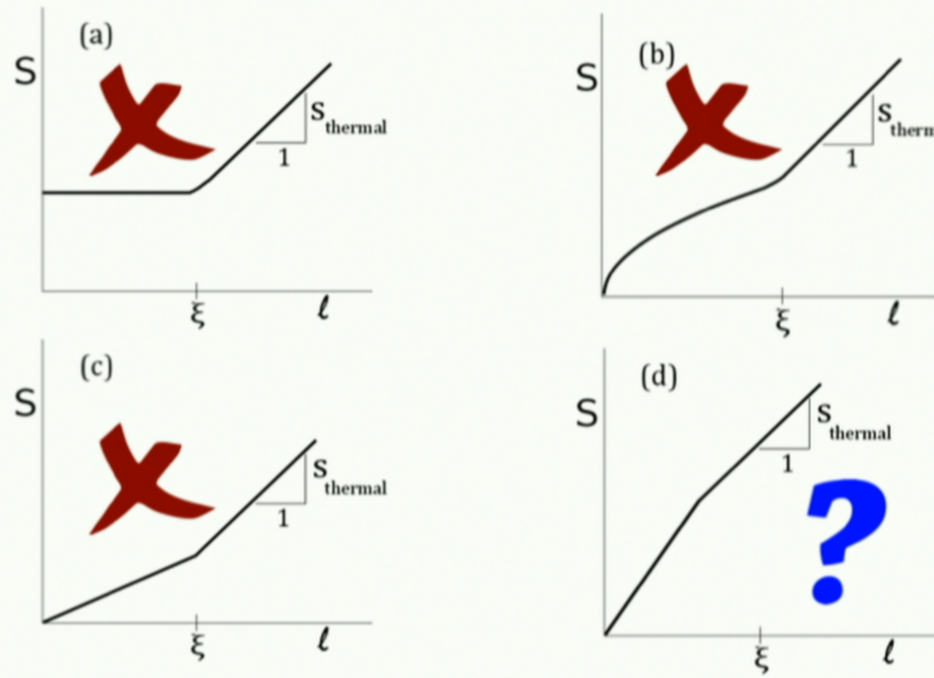
$$S(A_1) + S(A_2) - S(A_1 \cup A_2) - S(A_1 \cap A_2) \;\geq\; 0$$

$$\Rightarrow S(\ell) + S(\ell) - S(\ell + \Delta\ell) - S(\ell - \Delta\ell) \geq 0$$

$$\Rightarrow \qquad \boxed{\frac{\partial^2 S(\ell)}{\partial \ell^2} \leq 0}$$

Ruling out Possibilities via

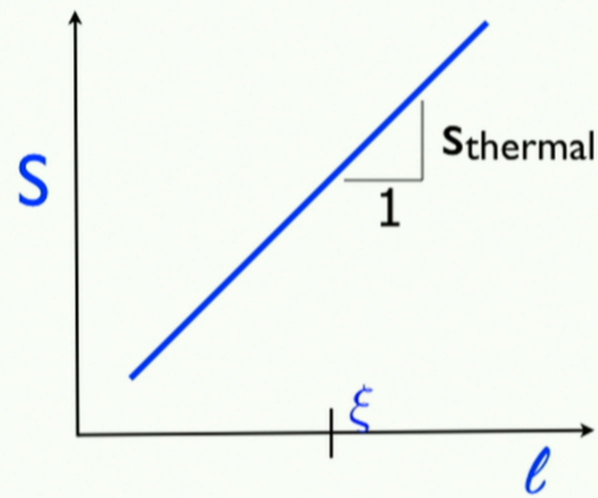
$$\frac{\partial^2 S(\ell)}{\partial \ell^2} \leq 0$$



TG 2014

Answer

Critical Entanglement $S = s_{Thermal}\,\ell \Rightarrow$ Ergodic!



TG 2014

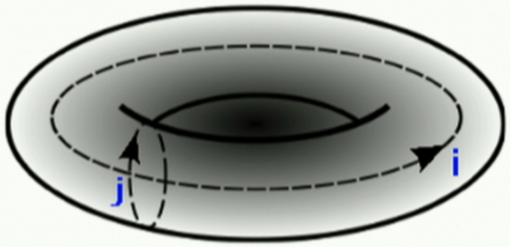
Outlook

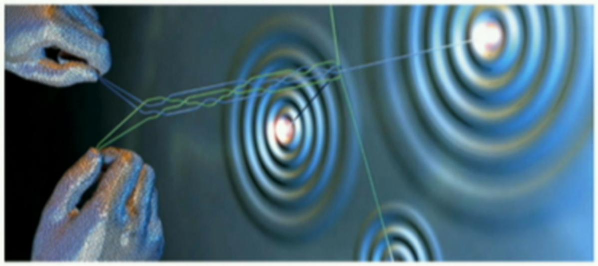
Quantum liquids are bizarre yet stable phases of matter.

They are…

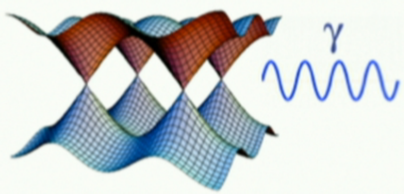
$\frac{1}{\sqrt{N}}\Big[\Big|$  $\Big\rangle + \Big|$  $\Big\rangle + \Big|$  $\Big\rangle + \ldots \Big]$

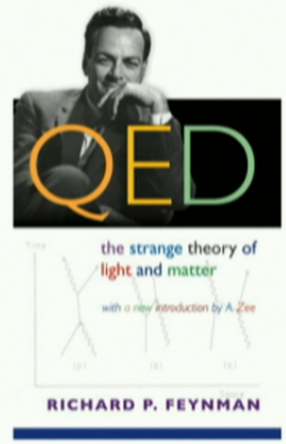
Stable Schrodinger's Cat.





Topological phases with prospects for quantum computation and cryptography.





Miniature Universes

Summary & Questions

- Quantum Entanglement = non-local order parameter for topological phases.
- Entanglement can extract topological Berry phase of anyons.
- Entanglement monotonicity allows one to predict when deconfinement *guaranteed to occur* in gapless strongly interacting gauge theories.
- Fractional statistics of anyons from a *single* ground state?
- Entanglement monotonicity for non-relativistic systems?