

Title: Kerr black holes with scalar hair

Date: Mar 05, 2015 01:00 PM

URL: <http://pirsa.org/15030115>

Abstract: <p>The Kerr metric of vacuum general relativity is expected to describe astrophysical black holes. Boson stars, on the other hand, are one of the simplest gravitating solitons, suggested as astrophysical compact objects, black holes mimickers and as dark matter candidates. Kerr black holes with scalar hair, found in [1], continuously interpolate between these two types of, per se, physically interesting solutions. I will describe the construction of these solutions and discuss theoretical, astrophysical and high energy physics aspects and challenges for Kerr black holes with scalar hair.

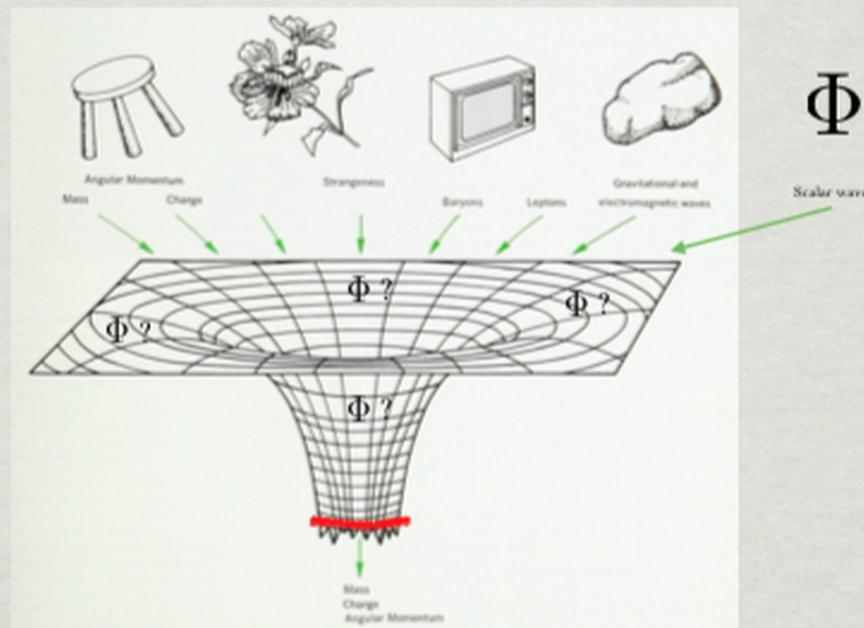
References:

[1] Kerr black holes with scalar hair, Carlos A. R. Herdeiro, Eugen Radu, Phys. Rev. Lett. 112 (2014) 221101; e-Print: arXiv:1403.2757

[2] A new spin on black hole hair, C. Herdeiro, E. Radu, International Journal of Modern Physics D 23 (2014) 1442014; e-Print: arXiv: 1405.3696;

[3] Construction and physical properties of Kerr black holes with scalar hair, C. Herdeiro and E. Radu, arXiv:1501.04319 [gr-qc]; Focus Issue on "Black holes and fundamental fields" to appear in Classical and Quantum Gravity</p>

Kerr black holes with scalar hair



Φ

C. Herdeiro
Departamento de Física da Universidade de Aveiro, Portugal

Perimeter Institute, Waterloo, Canada, March 5th 2015

based on

Physical Review Letters 112 (2014) 221101 (arXiv:1403:2757)
IJMPD 23 (2014) 1442014 (arXiv:1405:3696), honorable mention on GRF Awards 2014
To appear in a Focus Issue on Class. Quan. Grav. (arXiv:1501:04319)

Plan:

- 1) Motivation
- 2) Boson Stars
- 3) Scalar clouds around Kerr black holes
- 4) Kerr black holes with scalar hair
- 5) Properties/Phenomenology
- 6) Mechanism, extensions and pressing questions

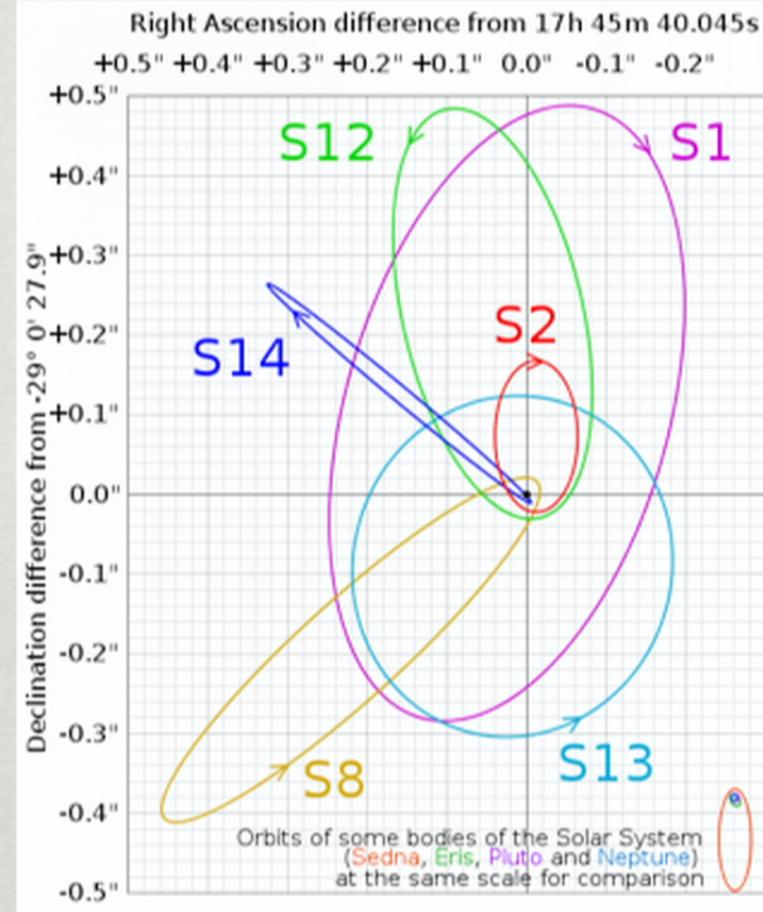
1) Motivation

There is
strong observational evidence
that extremely compact and massive
objects exist, which we call
black holes.

Sagittarius A*

From the motion
of star S2 (say), estimated mass
is 4.1 million solar masses

Ghez, A. M. et al., (2008) *Astrophysical Journal* 689 (2): 1044–1062



Eisenhauer, F. et al., (2005) *The Astrophysical Journal* (628): 246–259

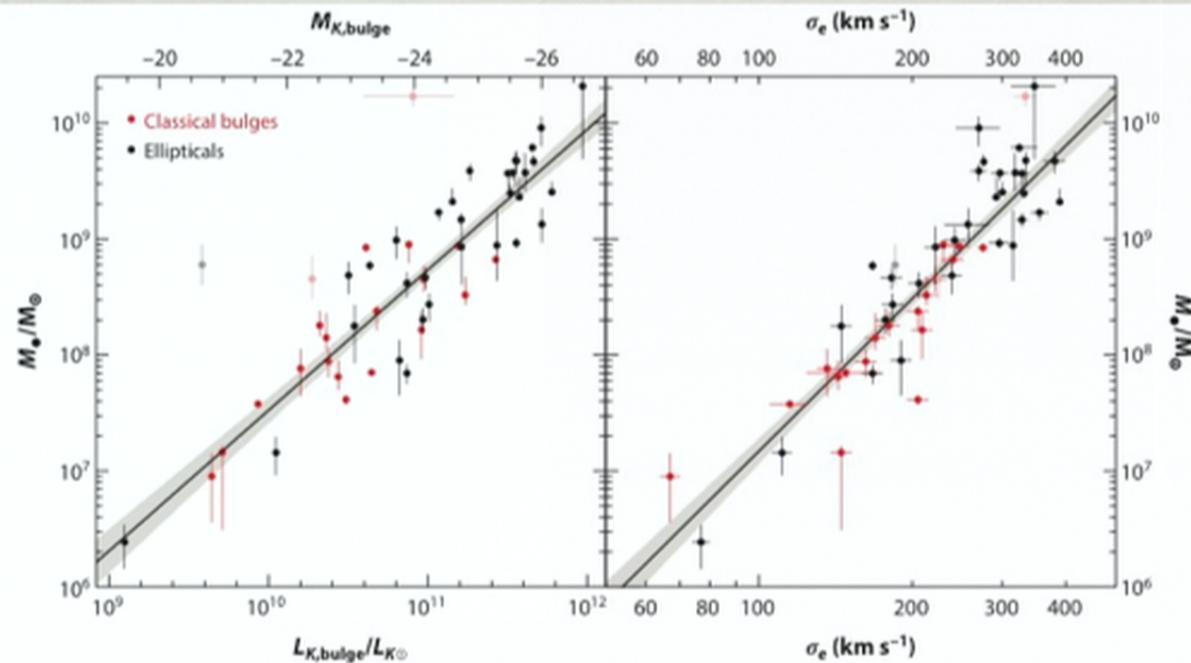


Figure 1.3 Observed correlations between supermassive black hole mass M_{\bullet} and (left) the infrared luminosity of the bulge of the host galaxy in units of solar luminosities (represented in the top axis by the absolute magnitude $M_{K,\text{bulge}}$), and (right) the velocity dispersion σ_e of the stars in the bulge. (Reprinted with permission from [40].)

Narayan and McClintock (2013), ArXiv:1312.6698

The first (stellar mass) black hole candidate: Cygnus X-1

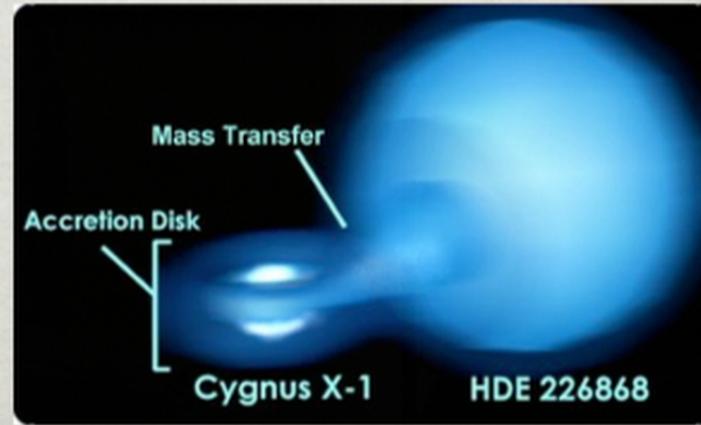
Estimated mass:
14.8 solar masses
(radius 44 kms)

Orosz, J. A. et al., *The Astrophysical Journal* 742 (2011) 84



Spin measurements:
Claim $a > 0.92$ at 3 sigma level.

Gou, L. et al., *Astrophysical Journal* 742 (2011) 85



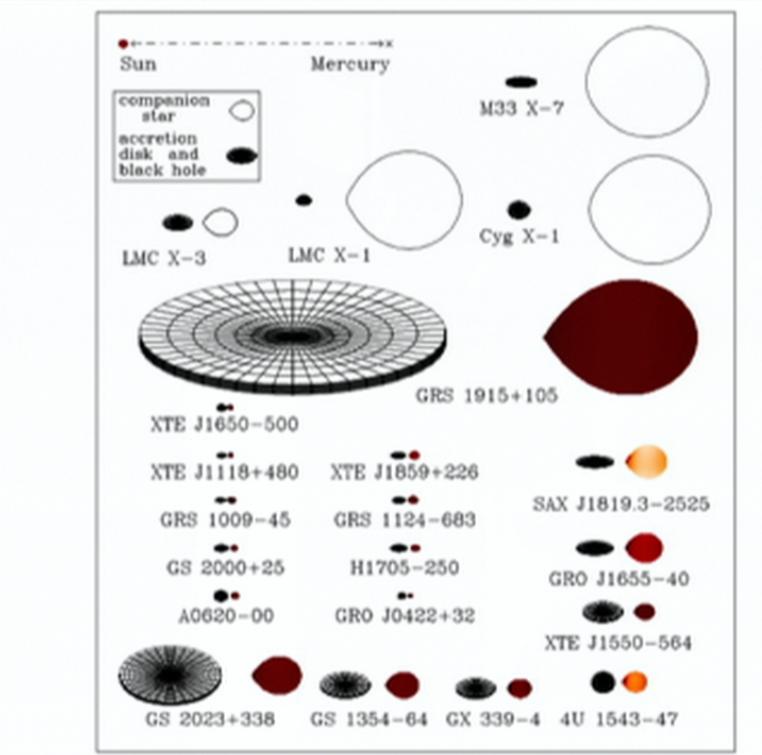


Figure 1.4 Sketches of 21 black hole binaries (see scale and legend in the upper-left corner). The tidally-distorted shapes of the companion stars are accurately rendered in Roche geometry. The black holes are located at the centers of the disks. A disk's tilt indicates the inclination angle i of the binary, where $i = 0$ corresponds to a system that is viewed face-on; e.g., $i = 21^\circ$ for 4U 1543-47 (bottom right) and $i = 75^\circ$ for M33 X-7 (top right). The size of a system is largely set by the orbital period, which ranges from 33.9 days for the giant system GRS 1915+105 to 0.2 days for tiny XTE J1118+480. Three systems hosting persistent X-ray sources — M33 X-7, LMC X-1 and Cyg X-1 — are located at the top. The other 18 systems are transient sources. (Figure courtesy of J. Orosz.)

Narayan and McClintock (2013),
ArXiv:1312.6698

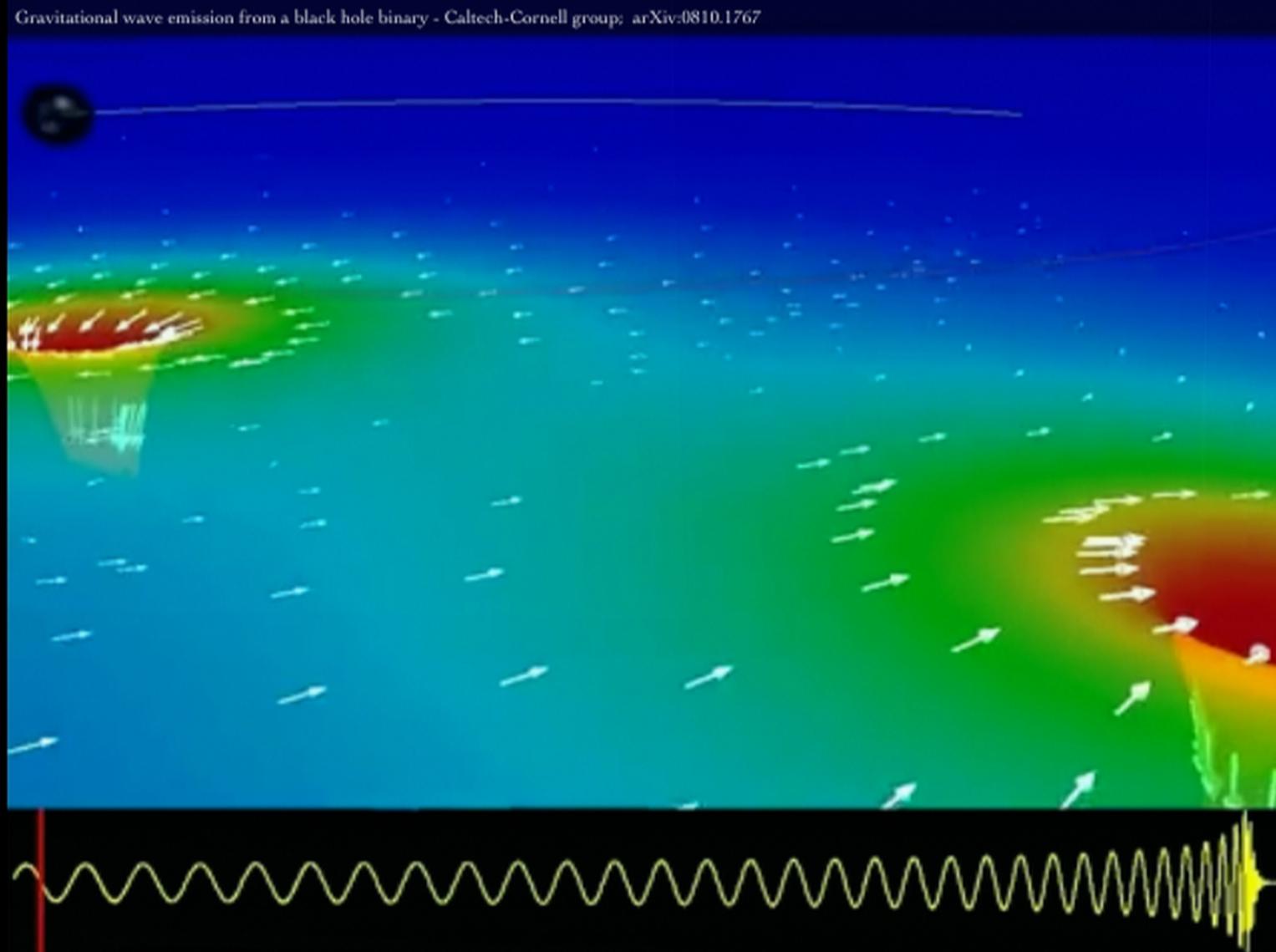
Will it be possible to
demonstrate
the existence of black holes in
the foreseeable future?



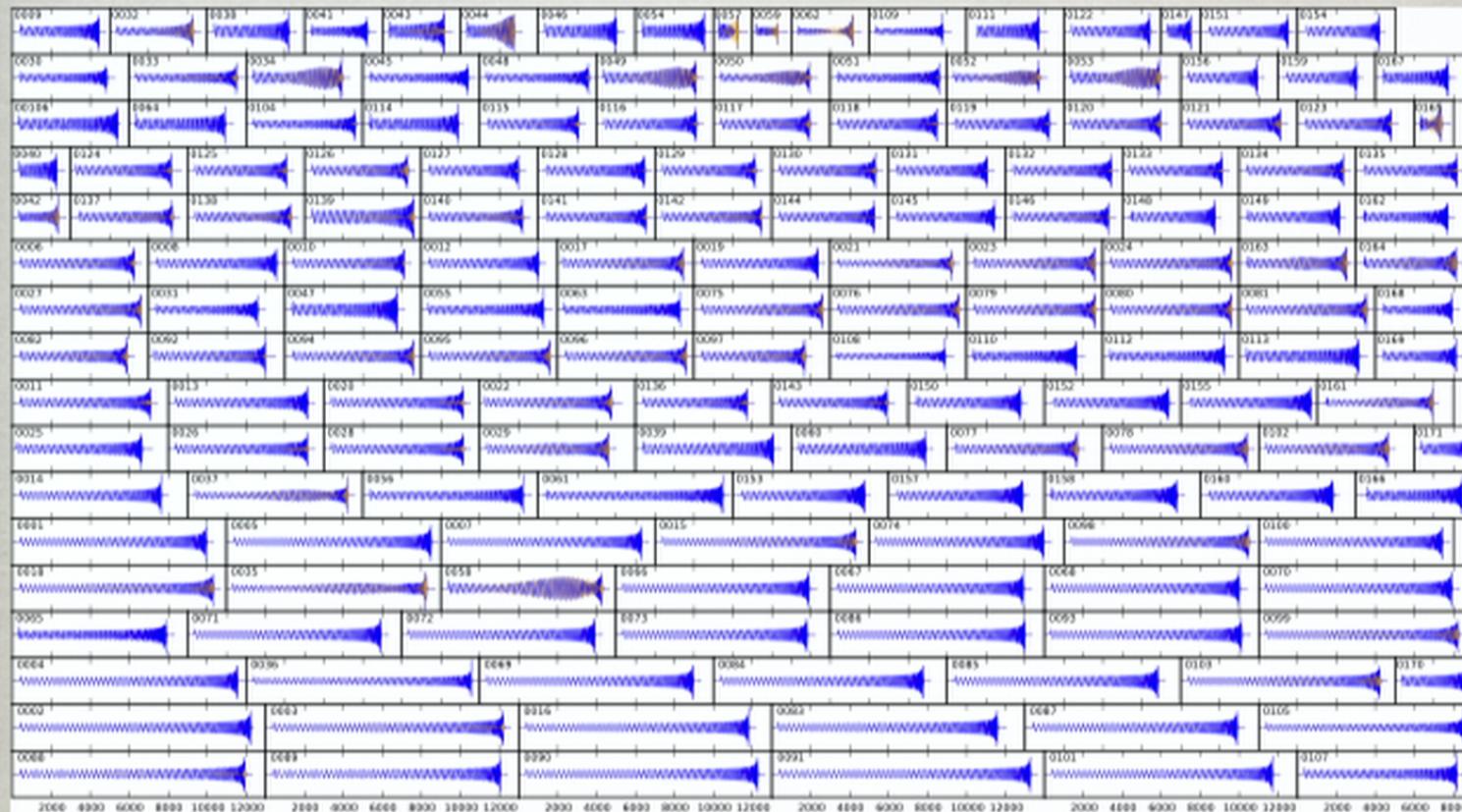
Shadow of a Kerr black hole



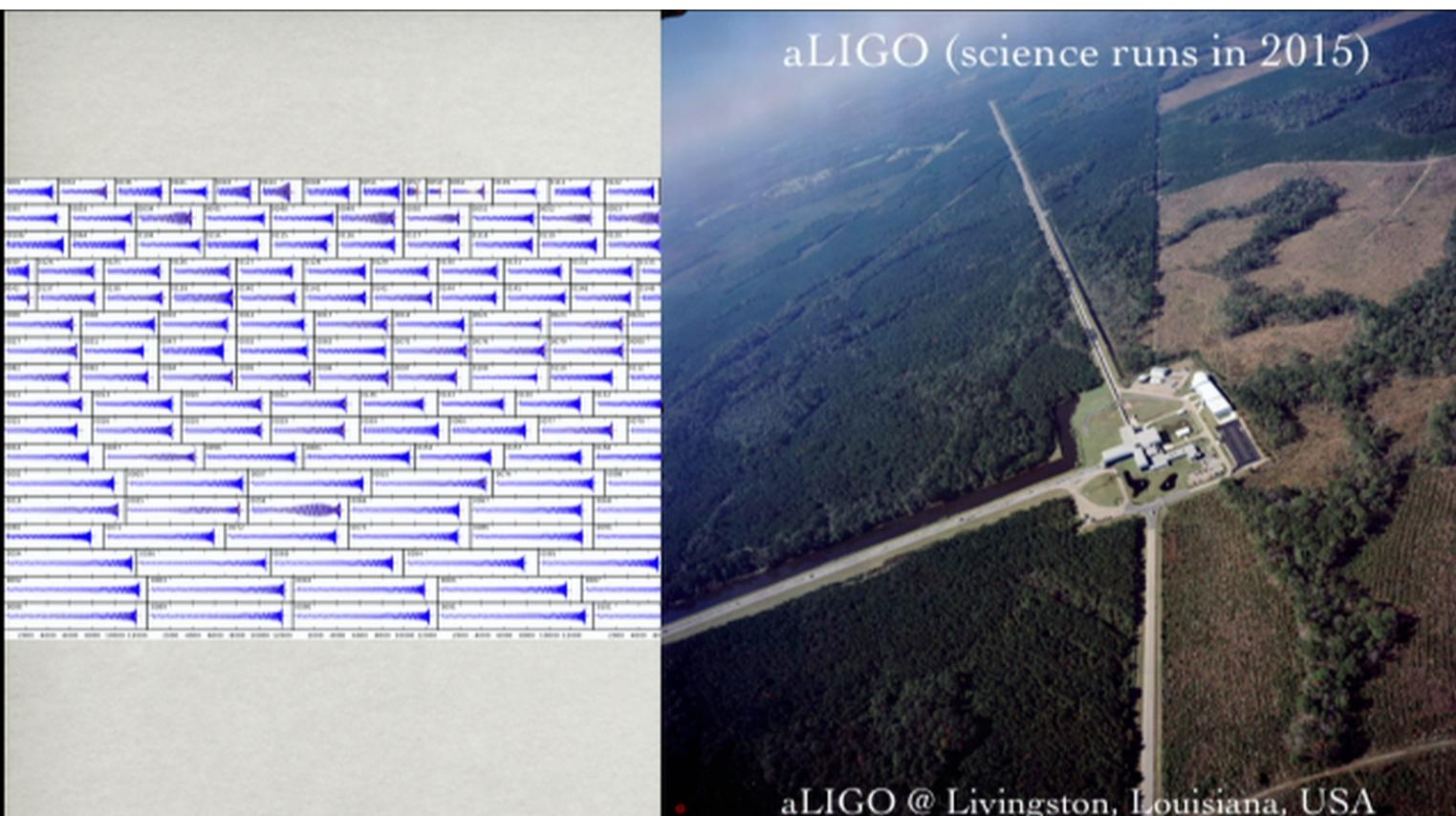
Different shadows ? E.g. boson stars Grandclement et al. 2015



Building waveform catalogues...



Mroué et al., arXiv:1304.6077



Different GW signatures?

e.g. Barausse, Palenzuela, Ponce, Lehner, 2015

GR paradigm based on the uniqueness theorems

e.g: Israel 1967, 1968; Carter 1970; Hawking 1972; Robinson 1975, 1977; and others

Overview: "Four decades of black hole uniqueness theorems" D. Robinson (2004, 2009)

GRAVITATIONAL FIELD OF A SPINNING MASS AS AN EXAMPLE OF ALGEBRAICALLY SPECIAL METRICS

Roy P. Kerr*

University of Texas, Austin, Texas and Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio
(Received 26 July 1963)

Goldberg and Sachs¹ have proved that the algebraically special solutions of Einstein's empty-space field equations are characterized by the existence of a geodesic and shear-free ray congruence, k_μ . Among these spaces are the plane-fronted waves and the Robinson-Trautman metrics² for which the congruence has nonvanishing divergence, but is hypersurface orthogonal.

where ζ is a complex coordinate, a dot denotes differentiation with respect to u , and the operator D is defined by

$$D = \dot{\partial}/\partial\zeta - \Omega\partial/\partial u.$$

P is real, whereas Ω and m (which is defined to be $m_1 + im_2$) are complex. They are all independent of the coordinate r . Δ is defined by

Carter-Robinson theorem:

An asymptotically-flat stationary and axi-symmetric vacuum spacetime that is non-singular on and outside an event horizon, is a member of the two-parameter Kerr family.

The assumption of axi-symmetry has since been shown to be unnecessary, i.e. for black holes, stationarity \Rightarrow axisymmetry (Hawking, Wald).

The “no-hair” idea

The collapse leads to a black hole endowed with mass and charge and angular momentum but, so far as we can now judge, no other adjustable parameters: “a black hole has no hair.” Make one black hole out of matter;

26 PHYSICS TODAY / JANUARY 1971

Ruffini, Wheeler (1971)

Box 33.1 A BLACK HOLE HAS NO “HAIR”

The following theorems come close to proving that *the external gravitational and electromagnetic fields of a stationary black hole (a black hole that has settled down into its “final” state) are determined uniquely by the hole’s mass M, charge Q, and intrinsic angular momentum S*—i.e., the black hole can have no “hair” (no other independent characteristics). For a detailed review, see Carter (1973).

Misner, Thorne, Wheeler (1973)

Original idea:
collapse leads to equilibrium black holes uniquely determined by M, J, Q -
asymptotically measured quantities subject to a Gauss law
and no other independent characteristics (hair)

Motivated by uniqueness theorems

Hairy black hole solutions exist (D=4, asymptotically flat):

Early example: Einstein-Yang-Mills theory

Bizón 1990; Kunzle and Masood-ul-Alam, 1990; Volkov and Gal'tsov, 1990

Other examples were obtained in: Einstein-Skyrme, Einstein-Yang-Mills-Dilaton, Einstein-Yang-Mills-Higgs, Einstein-non-Abelian-Proca, etc

Review by Bizón 1994; Volkov and Gal'tsov (1999)

‘Hair’ anchored on non-linearities of the field. Hard to have insights.

Suggests **mathematical** limitations of the ‘no-hair’ idea.

“The proliferation in the 1990s of stationary black hole solutions with hair of various sorts, may give the impression that the principle has fallen by the wayside. However, this is emphatically not the case for scalar field hair (...)

Mayo and Bekenstein (1996)

Timely to study alternatives to Kerr
that can parametrize phenomenological deviations.

We will suggest one such alternative, within General Relativity
minimally coupled to a complex, massive, scalar field.

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that can parametrize phenomenological deviations.

2) Boson Stars

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2) Boson Stars

Boson stars:

Kaup (1968); Ruffini and Bonazzola (1969)

Review: Liebling and Palenzuela (2012)

Einstein-Klein-Gordon theory:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R - \Phi^*_{;a} \Phi^{;a} - \mu^2 \Phi^* \Phi \right]$$

Rotating
boson stars:

Yoshida and Eriguchi (1997)

Schunck and Mielke (1998)

$$ds^2 = -e^{2F_0(r,\theta)} dt^2 + e^{2F_1(r,\theta)} (dr^2 + r^2 d\theta^2) + e^{2F_2(r,\theta)} r^2 \sin^2 \theta (d\varphi - W(r,\theta)dt)^2$$

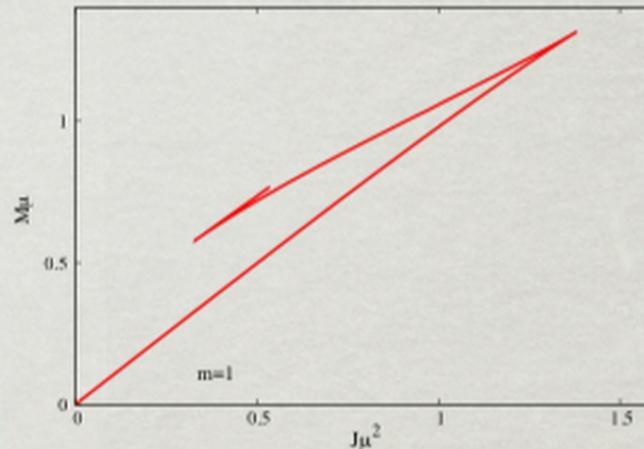
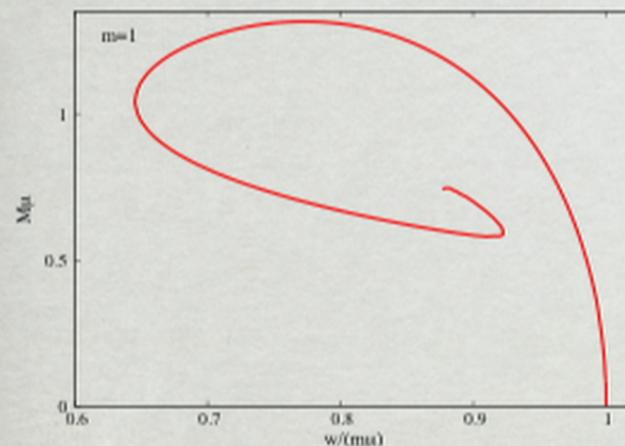
$$\Phi = \phi(r, \theta) e^{i(m\varphi - wt)}$$

Three input parameters: (w,m,n)

Solutions preserved
by a single helicoidal
Killing vector field:

$$\frac{\partial}{\partial t} + \frac{w}{m} \frac{\partial}{\partial \varphi}$$

Boson stars phase space (nodeless):



Conserved Noether charge:

$$Q = \int_{\Sigma} dr d\theta d\varphi j^t \sqrt{-g}$$

For rotating boson stars:

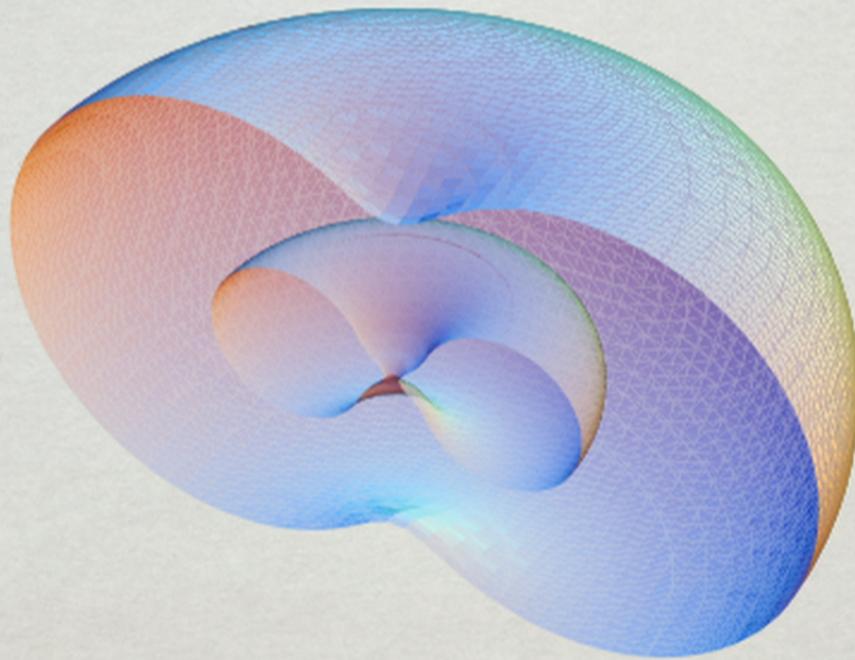
$$J = mQ$$

Schunck and Mielke (1998)

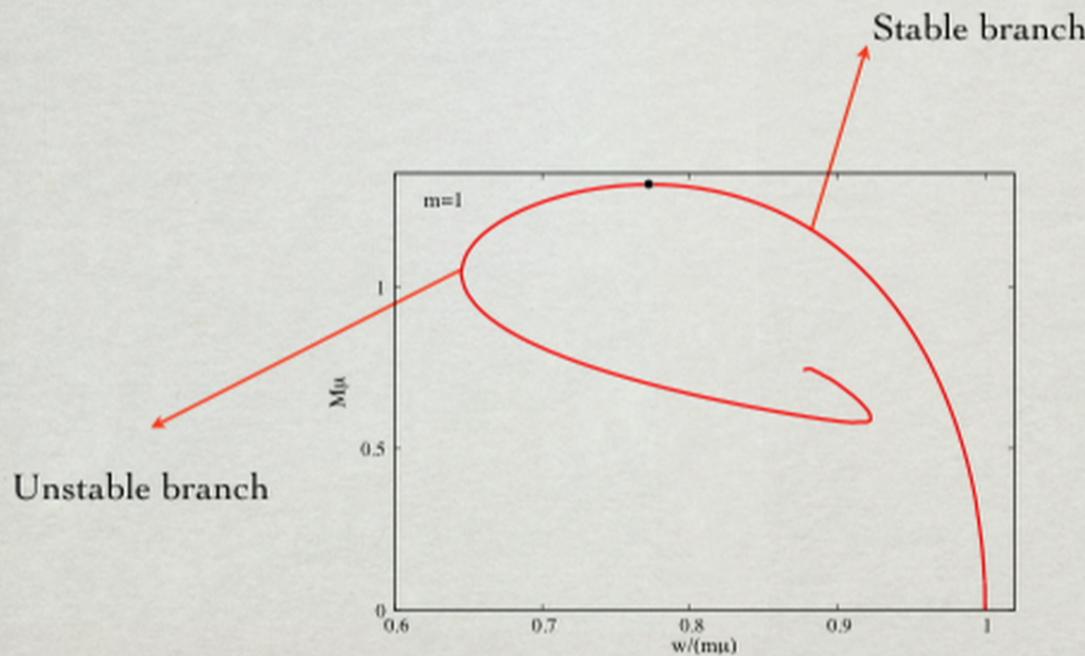
Convenient parameter:

$$q \equiv \frac{mQ}{J}$$

Surfaces of constant scalar energy density

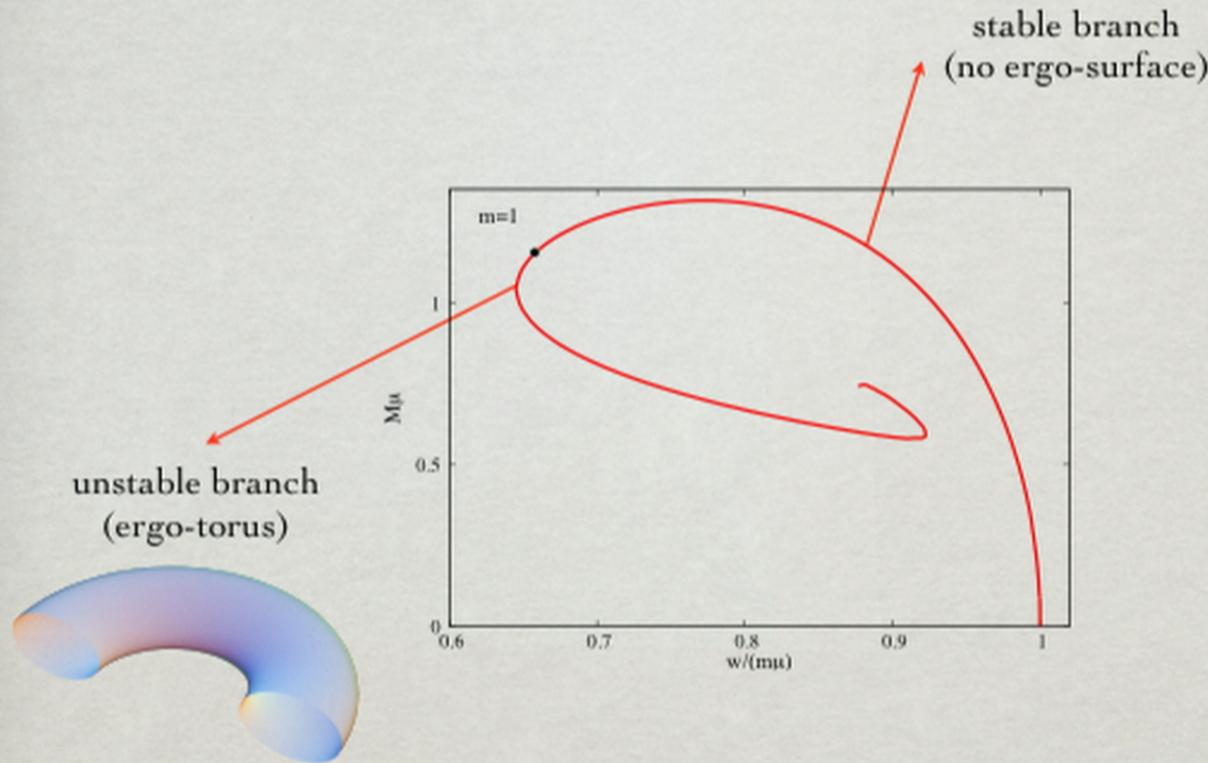


Perturbative stability:

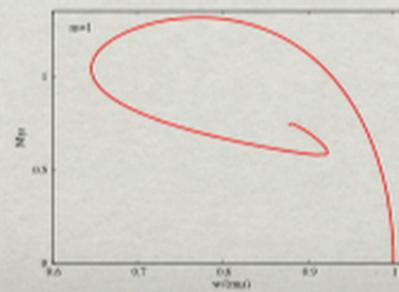
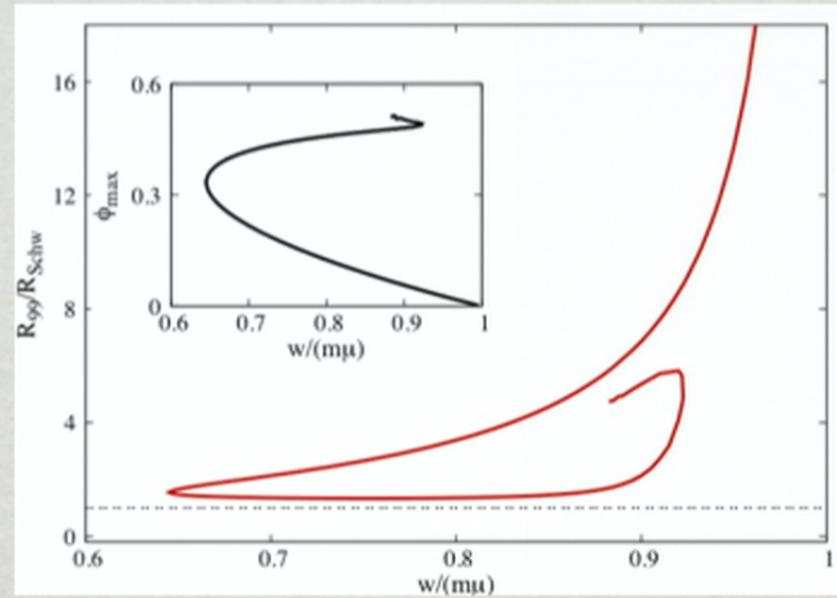


Similar to Fermionic Compact stars

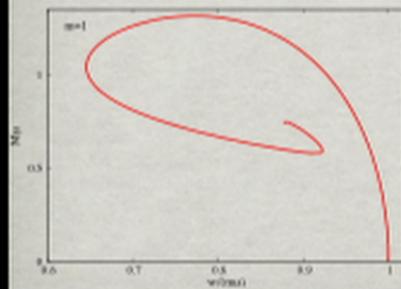
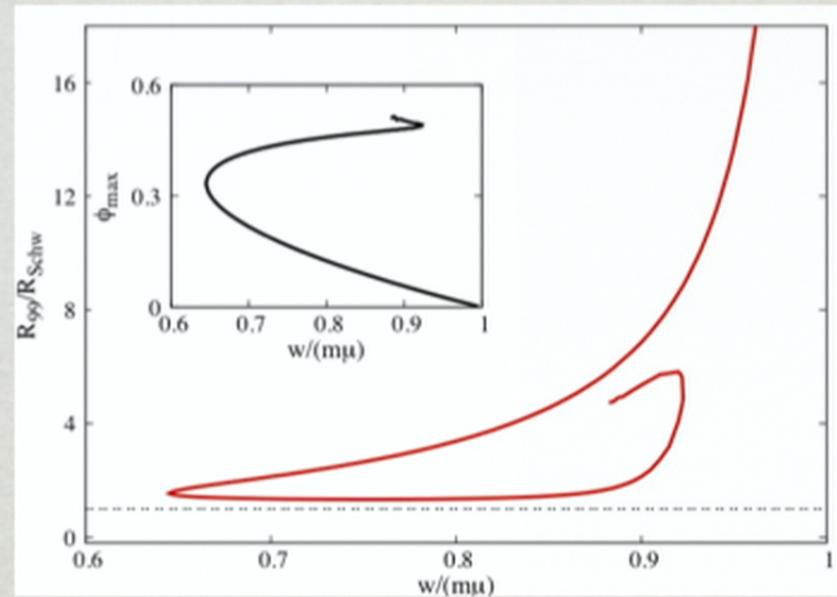
Superradiant instabilities:



Compactness:



Compactness:



Used to constrain scalar fields
with stellar kinematics

Amaro-Seone, Barranco, Bernal and Rezzolla 2010

3) Scalar clouds around Kerr black holes

Linear analysis: Klein-Gordon equation in Kerr

$$\square\Phi = \mu^2\Phi$$

$$\Phi = e^{-iwt} e^{im\varphi} S_{\ell m}(\theta) R_{\ell m}(r)$$

Radial Teukolsky equation: Teukolsky (1972); Brill et al. (1972)

$$\frac{d}{dr} \left(\Delta \frac{dR_{\ell m}}{dr} \right) = \left(a^2 w^2 - 2maw + \mu^2 r^2 + A_{\ell m} - \frac{K^2}{\Delta} \right) R_{\ell m}$$

$\Delta \equiv r^2 - 2Mr + a^2$
 $K \equiv (r^2 + a^2)w - am$

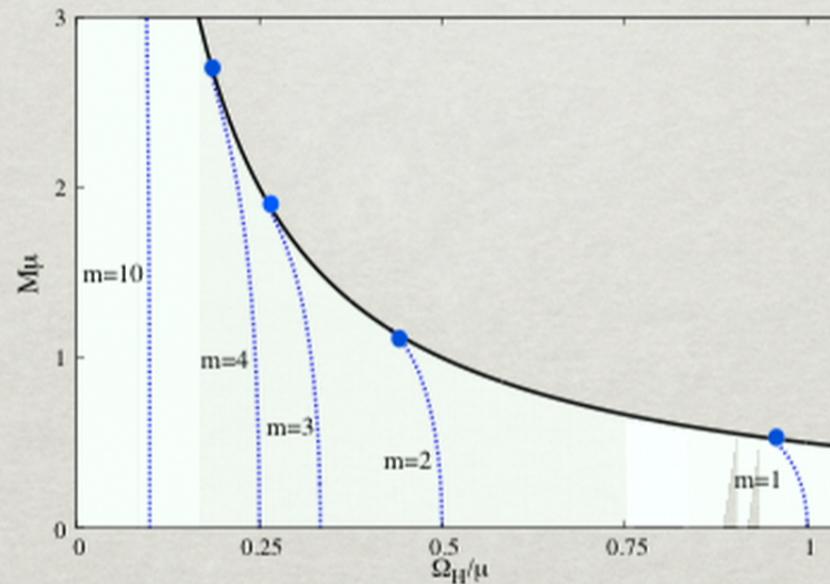
Generically one obtains *quasi-bound states*:

$\omega = \omega_R + i\omega_I$	critical frequency $w_c = m\Omega_H$	$w_I < 0$ if $w_R > w_c$	decay
		$w_I = 0$ if $w = w_c$	true bound states: <i>clouds</i>
		$w_I > 0$ if $w_R < w_c$	grow Press and Teukolsky (1972)

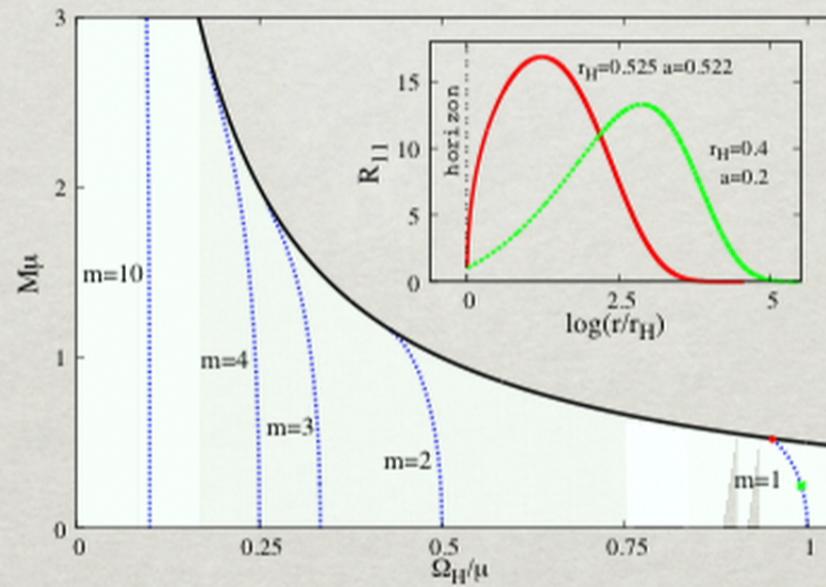
Klein-Gordon (linear) clouds around Kerr:

Damour, Deruelle and Ruffini (1976); Zouros and Eardley (1979); Detweiler (1980); Hod 2012;
(...); Yakov Shilapentokh-Rothman (2014)

Clouds for Kerr: discrete set labelled by (n, l, m) subject to one
quantization condition which yields BH mass,spin. Hod (2012)



Clouds radial profile



4) Kerr black holes with scalar hair

Einstein Klein-Gordon: non-linear setup

Ansatz:

$$ds^2 = -e^{2F_0(r,\theta)} \textcolor{red}{N} dt^2 + e^{2F_1(r,\theta)} \left(\frac{dr^2}{\textcolor{red}{N}} + r^2 d\theta^2 \right) + e^{2F_2(r,\theta)} r^2 \sin^2 \theta (d\varphi - W(r,\theta)dt)^2 \quad \textcolor{red}{N} = 1 - \frac{r_H}{r}$$

$$\Phi = \phi(r,\theta) e^{i(m\varphi - wt)}$$

Single KVF BH c.f.
Dias, Horowitz and Santos (2011)

Asymptotically:

$$g_{tt} = -1 + \frac{2M}{r} + \dots, \quad g_{\varphi t} = -\frac{2J}{r} \sin^2 \theta + \dots$$

$$\phi = f(\theta) \frac{e^{-\sqrt{\mu^2 - w^2}r}}{r} + \dots$$

take: $w < \mu$

Four input parameters: m, w, r_H, n

Near the horizon:

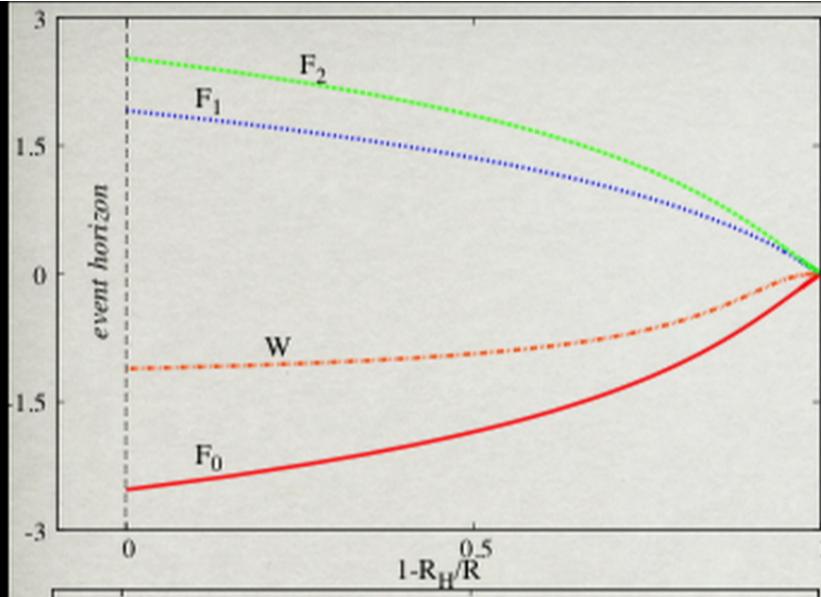
$$x \equiv \sqrt{r^2 - r_H^2}$$

$$F_i = F_i^{(0)}(\theta) + x^2 F_i^{(2)}(\theta) + \mathcal{O}(x^4)$$

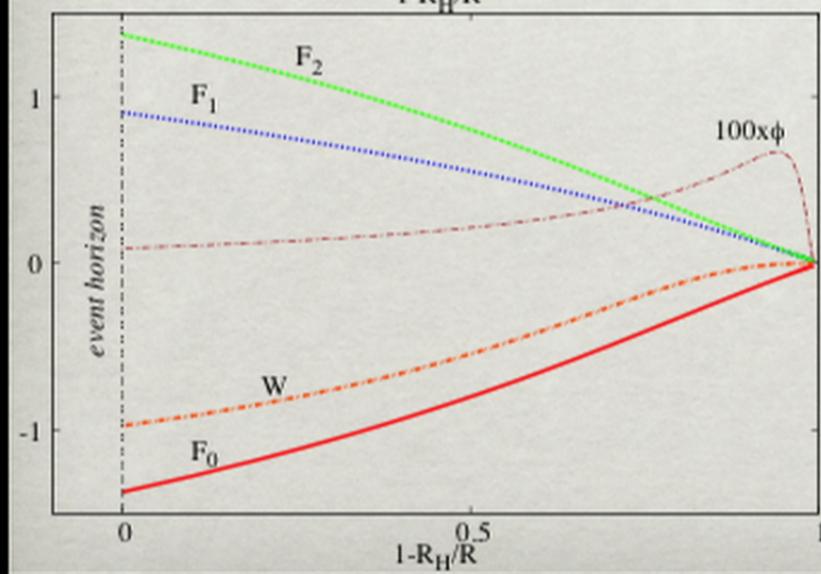
$$W = \Omega_H + \mathcal{O}(x^2)$$

$$\phi = \phi_0(\theta) + \mathcal{O}(x^2)$$

$$\text{take: } \Omega_H = \frac{w}{m}$$



Kerr black holes



Hairy black holes

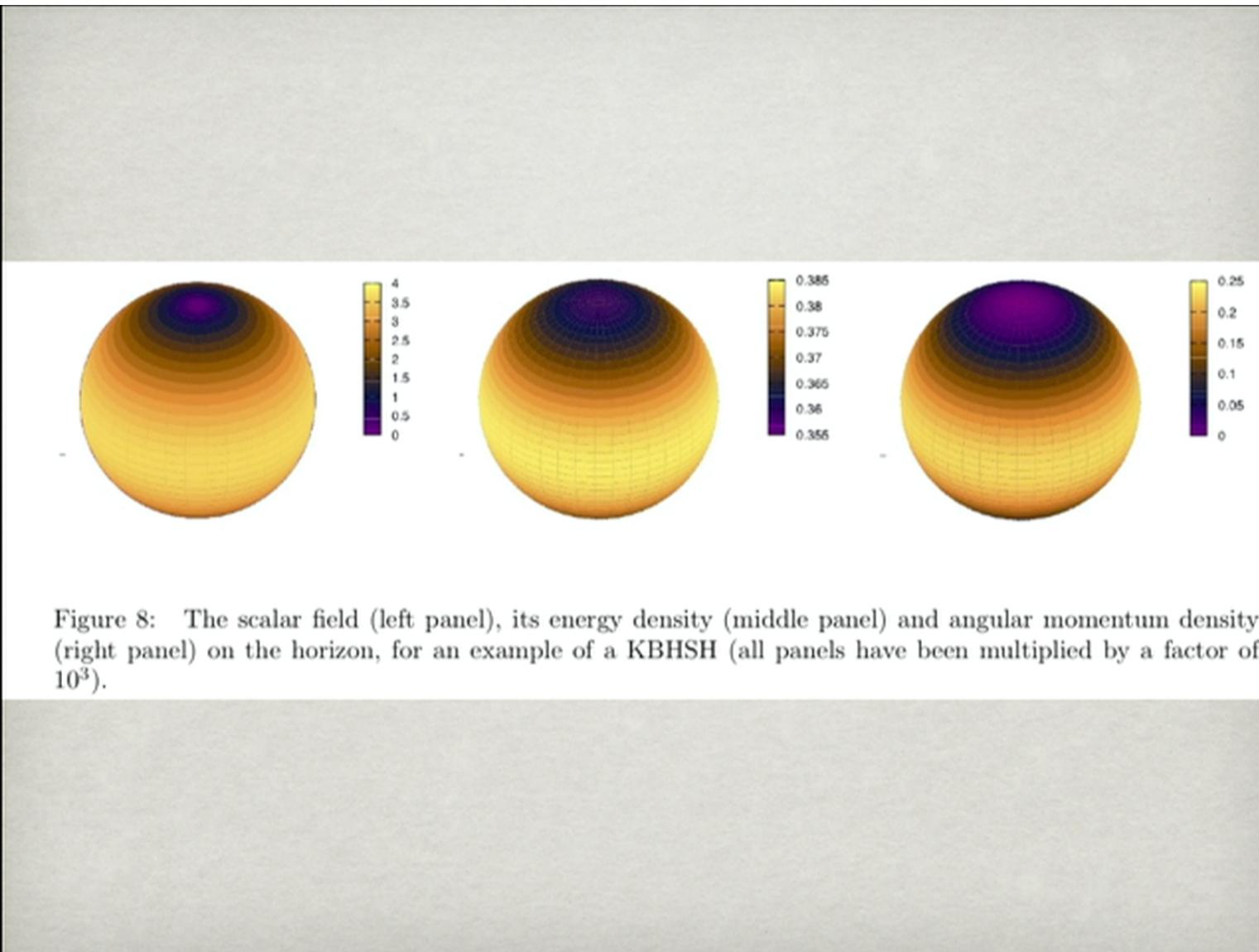
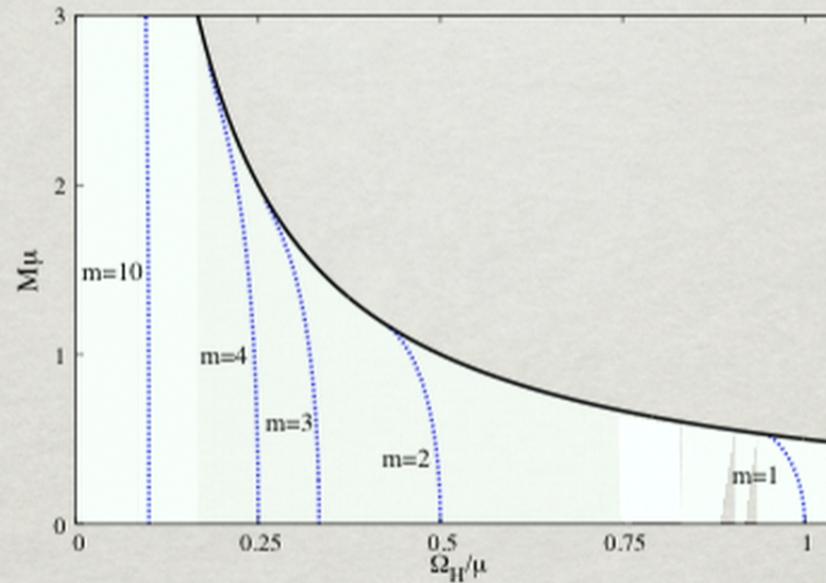
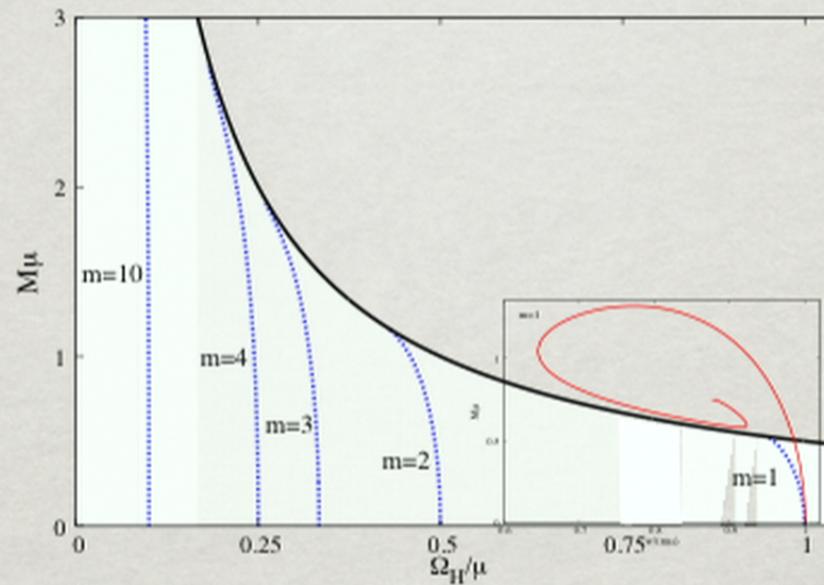


Figure 8: The scalar field (left panel), its energy density (middle panel) and angular momentum density (right panel) on the horizon, for an example of a KBHSH (all panels have been multiplied by a factor of 10^3).

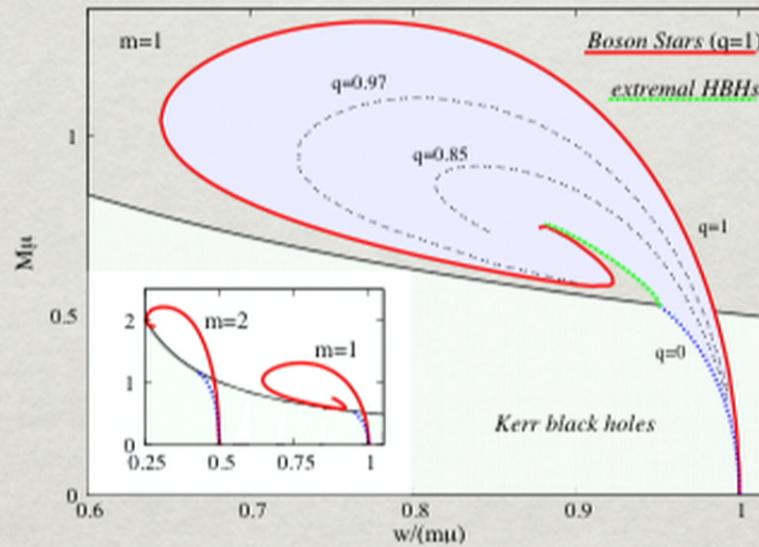
Hairy black holes phase space



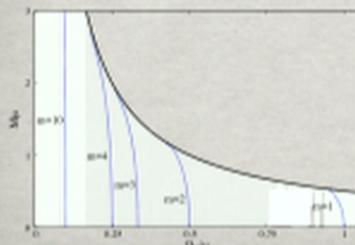
Hairy black holes phase space



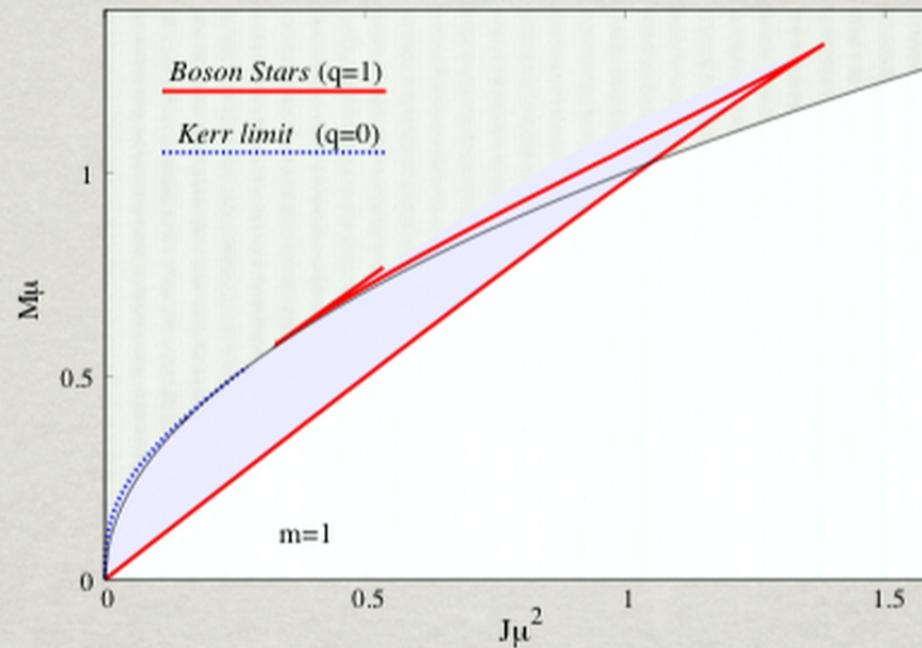
Hairy black holes phase space



$$q \equiv \frac{mQ}{J}$$



Hairy black holes phase space



- Can violate Kerr bound

Extremal BHs (Kerr and hairy)

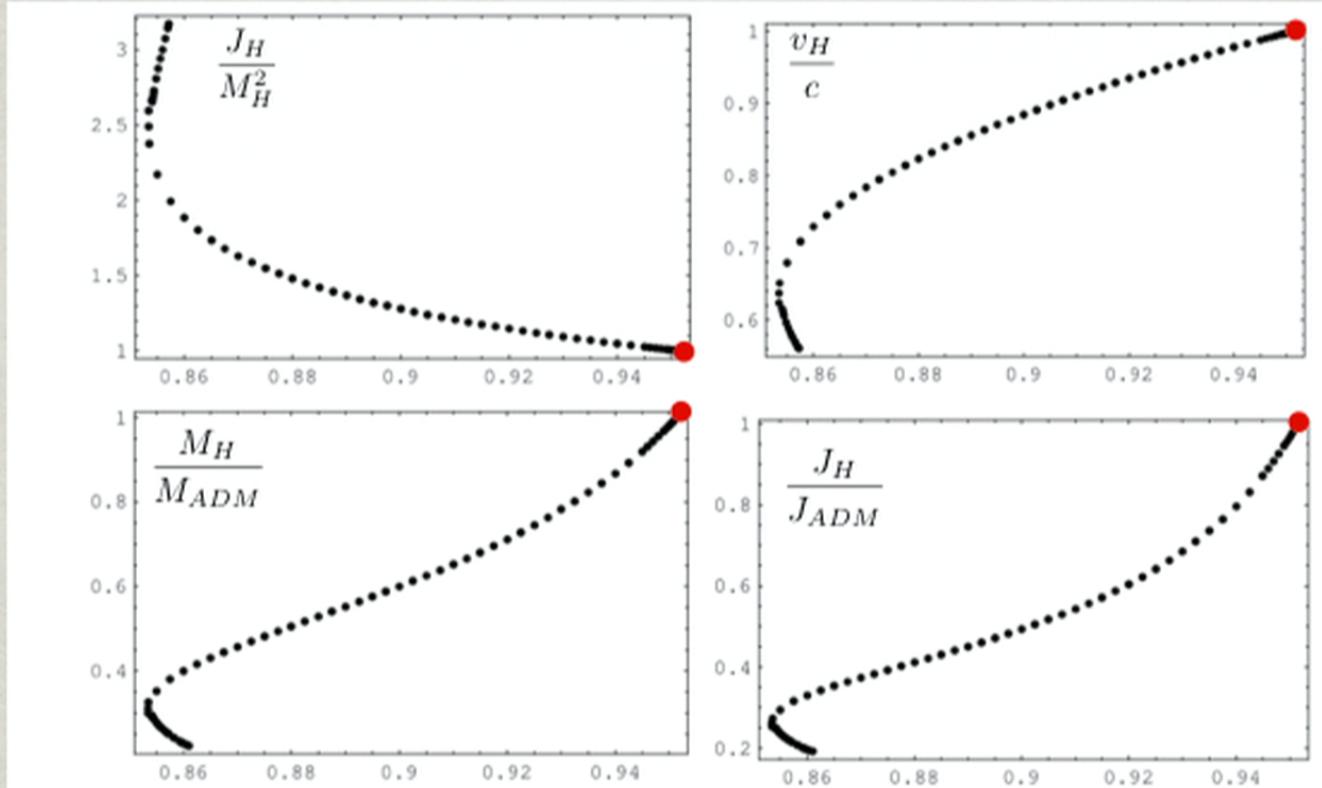


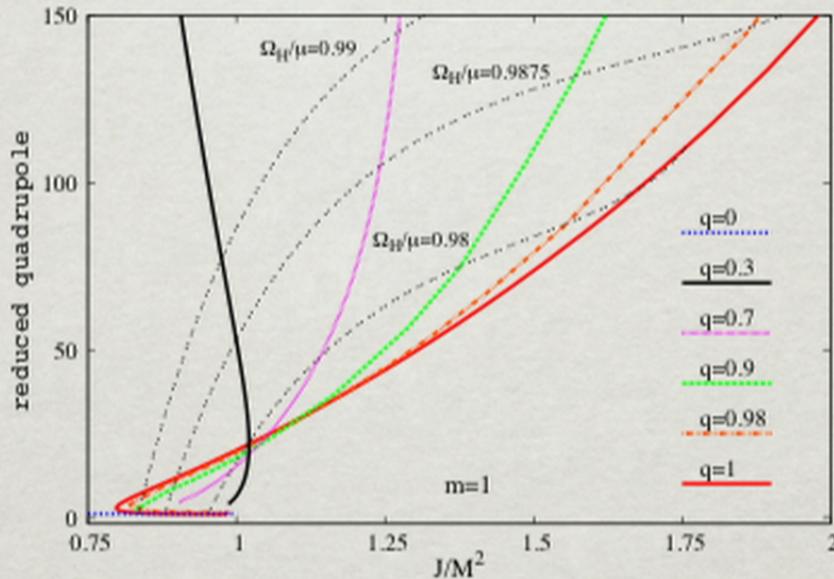
FIG. 2: Various quantities in terms of the scalar field frequency w for extremal KBHsSH: (top left panel) J_H/M_H^2 – observe the Kerr bound is saturated for vacuum Kerr and otherwise violated; (top right panel) horizon linear velocity – observe it is always smaller than unity and only saturated for vacuum Kerr; (bottom left panel) the ratio M_H/M_{ADM} ; (bottom right panel) the ratio J_H/J_{ADM} – observe that the relative contribution of the scalar field energy and angular momentum increases from the Kerr limit towards the centre of the spiral.

Herdeiro, Radu, in progress

Hairy black holes are more *star-like*

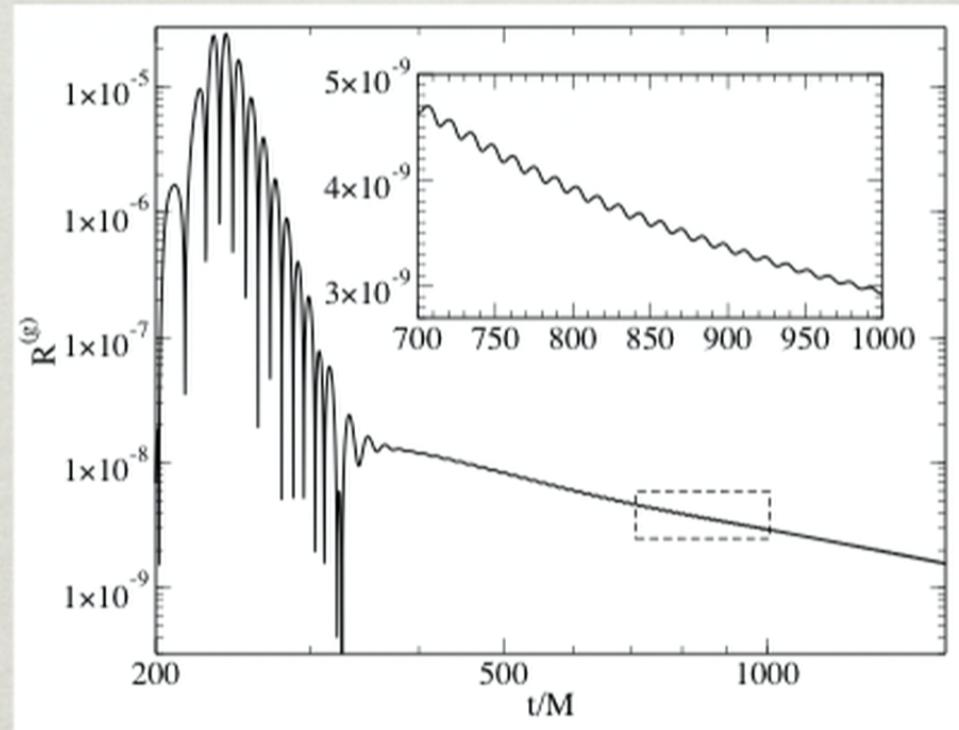
Geroch-Hansen quadrupole moment:

Geroch (1970); Hansen (1974); Pappas and Apostolatos (2012)



$$\text{reduced quadrupole} = \frac{\text{quadrupole}}{-J^2/M}$$

Wiggly tails: a gravitational wave signature?



Degollado and Herdeiro, PRD, 2014

Hairy black holes interpolate between Kerr and boson stars.

Can be seen at linear level: **do not** rely on non-linear effects.

Hairy black holes interpolate between Kerr and boson stars.

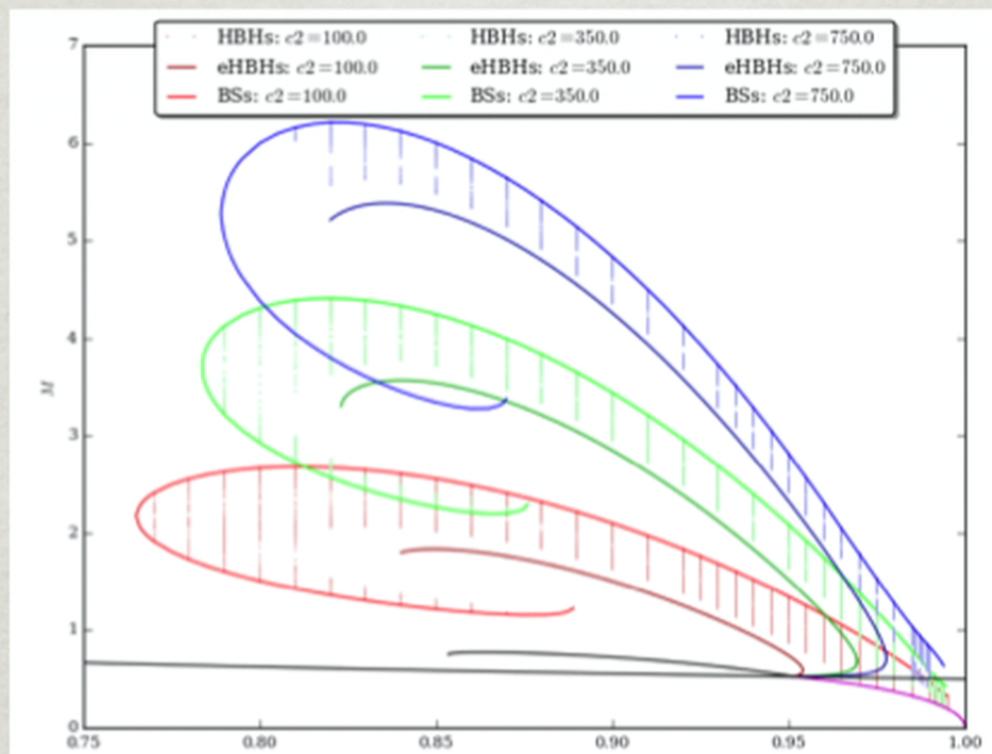
Can be seen at linear level: **do not** rely on non-linear effects.

From the perspective of **Kerr black holes**:
branching towards a new family of solutions due to superradiant instability.

Mechanism:

A (hairless) BH which is afflicted by the superradiant instability of a given field for which the energy-momentum tensor is time-independent, allows a hairy generalization with that field.

Extensions: e.g. including quartic self-interactions



Runarsson, Herdeiro, Radu, in progress

Stability:

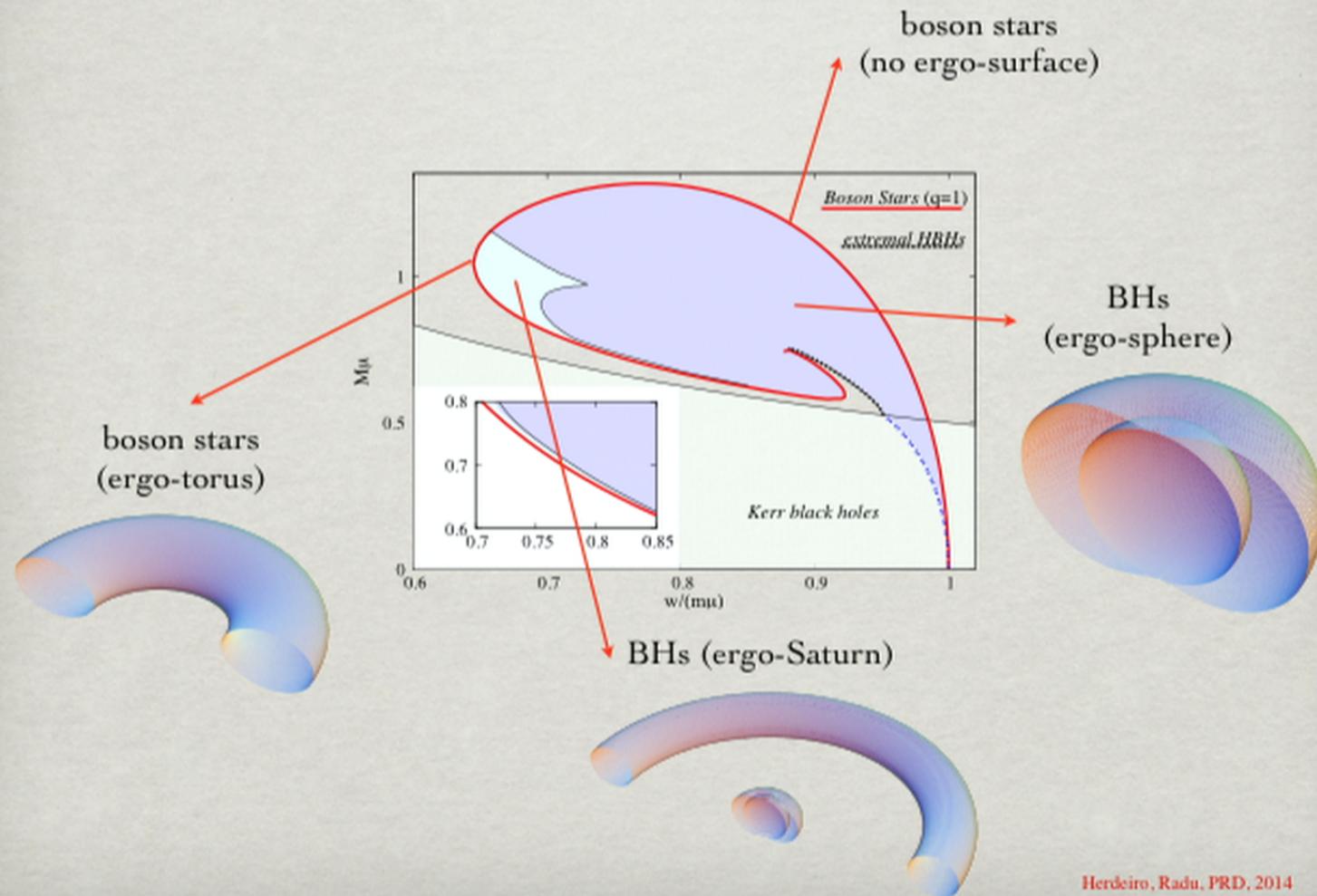
Mode, linear or non-linear? All difficult problems.

Absolute stability not mandatory for physical relevance
(ex: uranium!)

Stability:

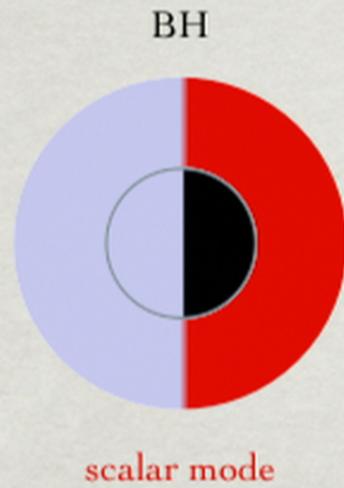
Mode, linear or non-linear? All difficult problems.

Ergo-surfaces:



Herdeiro, Radu, PRD, 2014

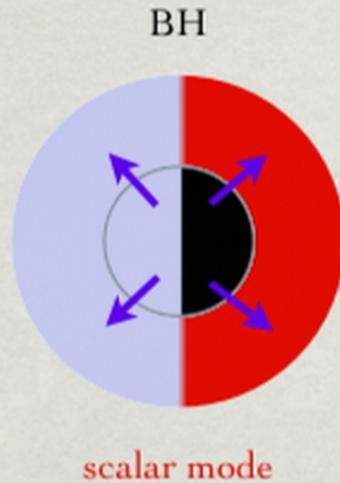
Stability: clouds



$$\frac{w}{m} < \Omega_H$$

Superradiant regime
black hole decreases angular velocity

Stability: clouds

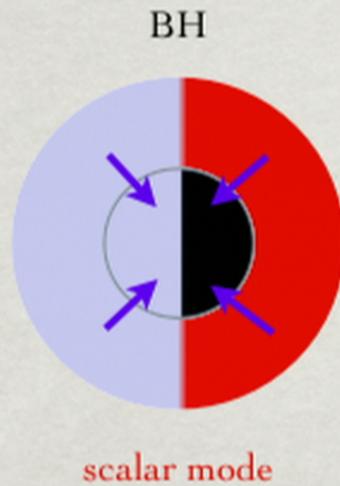


Transfer of rotational
energy from BH to
scalar cloud

$$\frac{w}{m} < \Omega_H$$

Superradiant regime
black hole decreases angular velocity

Stability: clouds



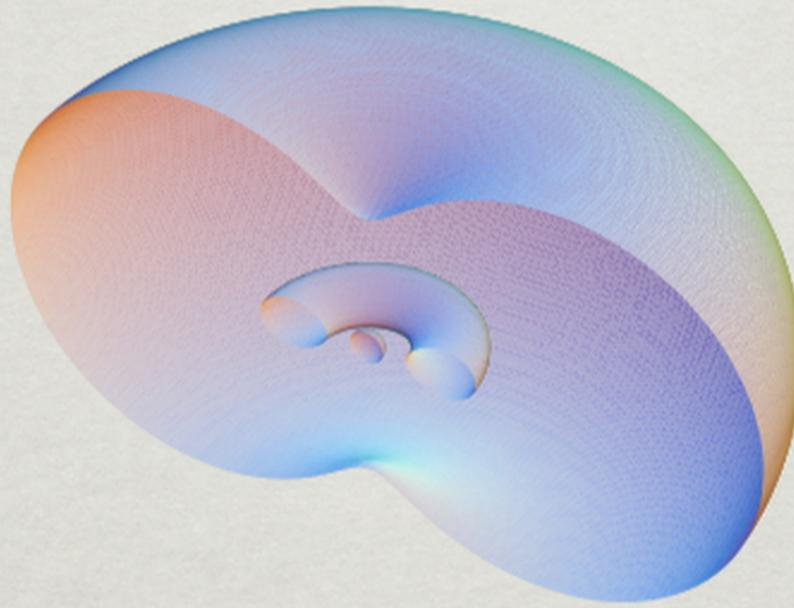
Transfer of rotational
energy from
scalar cloud to BH

$$\frac{w}{m} < \Omega_H$$

Superradiant regime
black hole decreases angular velocity

(Incomplete) To do list:

- More detailed analysis of stability (perturbations and time evolution)
 - Shadows of hairy black holes
 - Gravitational wave signals
 - Astrophysical constraints



Thank you for your
attention!