Title: Universal Aspects of Many-body Localization Phase Transition and Eigenstate Thermalization

Date: Mar 10, 2015 03:30 PM

URL: http://pirsa.org/15030113

Abstract: <p>Does a generic quantum system necessarily thermalize? Recent developments in disordered many-body quantum systems have provided crucial insights into this long-standing question. It has been found that sufficiently disordered systems may fail to thermalize leading to a 'many-body localized' phase. In this phase, the fundamental assumption underlying equilibrium statistical mechanics, namely, the equal likelihood for all states at the same energy, breaks down. A fundamental question is: what happens as the disorder becomes weaker so that one approaches the localization-delocalization transition? For example, does the system thermalize *at* the transition?

In this talk, I will show that very general considerations on the scaling of entanglement entropy close to the transition imply that at a continuous many-body localization transition, the system is necessarily ergodic.

Finally, I will present recent results on "eigenstate thermalization", a long standing hypothesis which posits that a single eigenstate hides within itself a thermal ensemble. In particular, I will discuss which class of operators do or do not satisfy eigenstate thermalization.</p>

Universal Aspects of Many Body Localization Transition & Eigenstate Thermalization

Tarun Grover

KITP

The Kavli Institute for Theoretical Physics
University of California, Santa Barbara

Gordon and Betty MOORE FOUNDATION


The

What this talk is about

Equilibrium Statistical Mechanics: one of the oldest and the most basic paradigms of physics.

Prof. Maxwell *on the Motions and Collisions*

PHILOSOPHICAL MAGAZINE

JANUARY 1860.

$$f(v) \propto v^2 e^{-\frac{mv^2}{kT}}$$

But, it breaks down dramatically in generic, strongly disordered quantum systems!

Main message

Quantum entanglement resolves some of the basic puzzles on the breakdown of statistical mechanics.

Focus: Phase transition between breakdown and restoration of statistical mechanics.

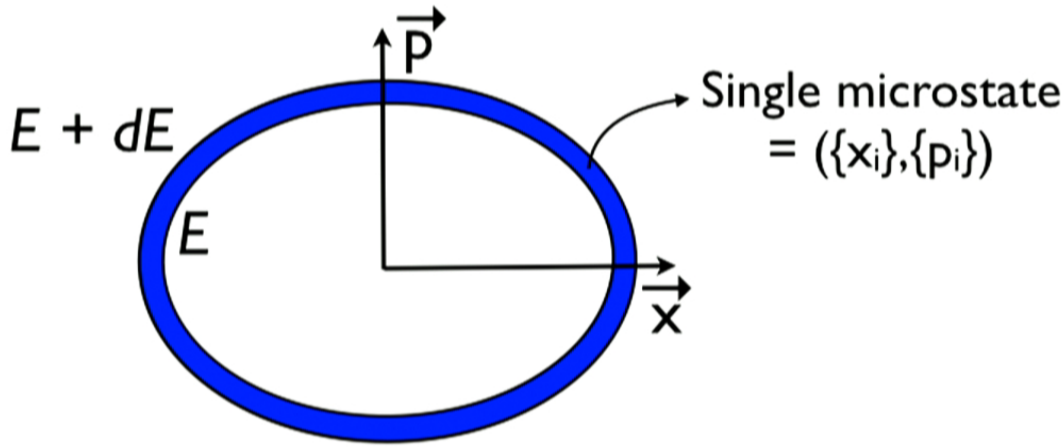
Outline

- Ergodicity in classical and quantum mechanics.
- Ergodicity breaking in disordered quantum systems.
- Many Body localization phase transition.

Ergodicity in Classical Statistical Mechanics

Hamiltonian = H(*x*,*p*) = $\sum_i \frac{p_i^2}{2m} + \sum_{ij} V(x_i - x_j) + \ldots$

= Constant ("Microcanonical Ensemble")

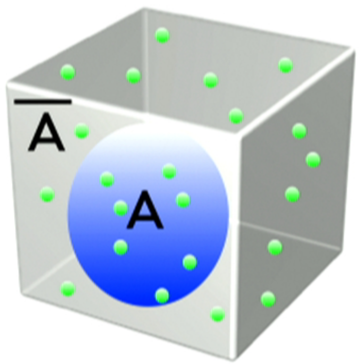


Ergodicity = All microstates equally likely.

Ergodicity ⇒

Microcanonical = Canonical

(Constant E) (Constant T)



$E = E_A + E_{\bar{A}} + E_{A\bar{A}} = \text{Constant}$

$V_A \ll V_{\bar{A}}$

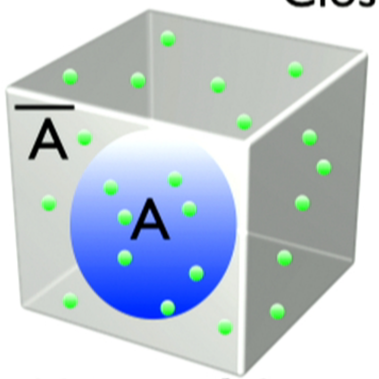
Probability $P(E_A) \propto e^{-E_A/T}$

A subsystem of a closed system looks thermal.

"Quantum Ergodicity":

Eigenstate Thermalization Hypothesis

Closed Quantum System in Eigenstate $|\psi\rangle$



I $\langle\psi|O|\psi\rangle = \frac{\mathrm{tr}(\,Oe^{-\beta H})}{\mathrm{tr}(\,e^{-\beta H})}$

Microcanonical Canonical

Unique β for a given $|\psi\rangle$: Solve above Eq. for $O = H$.

Entanglement Entropy = Thermal Entropy

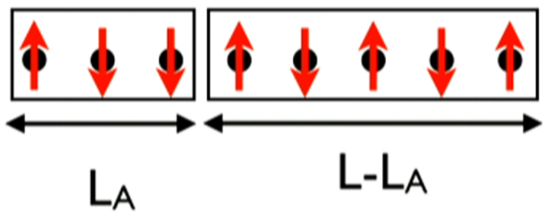
II $S(|\psi\rangle_\beta) = s_{\mathrm{thermal}}(\beta)V_A$

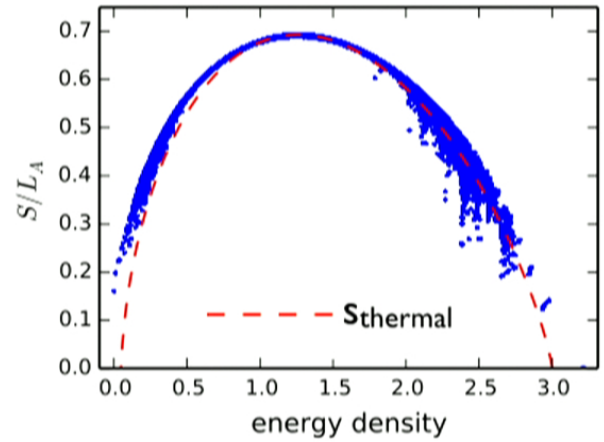
Originally conjectured to work for any generic quantum system.

Deutsch 1991; Srednicki 1994 Deutsch 2009

Entanglement Entropy = Thermal Entropy

$H = \sum_i \left(-\sigma_i^z \sigma_{i+1}^z + h_x \sigma_i^x + h_z \sigma_i^z\right)$





$S(|\psi\rangle_\beta) = s_{\text{thermal}}(\beta) V_A$ Garrison, TG 2015

"Volume Law of Entanglement Entropy"

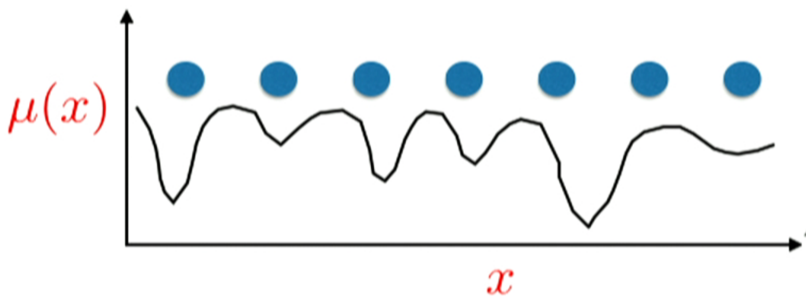
Contrast: "Area Law" for ground states.

Deutsch 2009: Weakly perturbed Integrable system

2D CFTs at large c: Asplund, Bernamonti, Galli, Hartman 2014; Fitzpatrick, Kaplan, Walters 2015

Anderson Localization

$$H = -\sum_{<ij>} \left(c_i^\dagger c_j + h.c. \right) + \sum_i \mu_i c_i^\dagger c_i$$



Absence of Diffusion in Certain Random Lattices

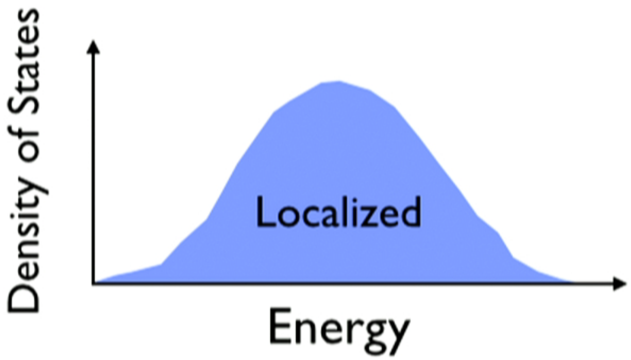
P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

Structure of many-body eigenstates?

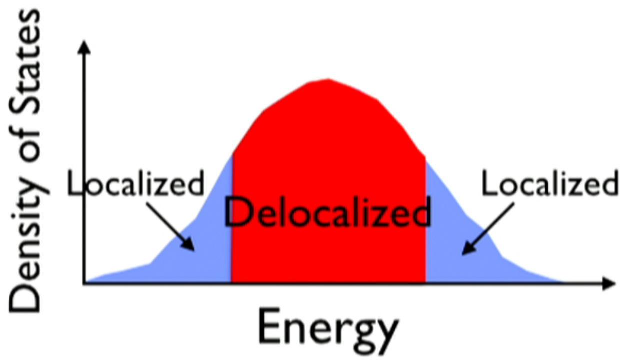
Are they ergodic?

Anderson Localization

Single Particle Eigenstates



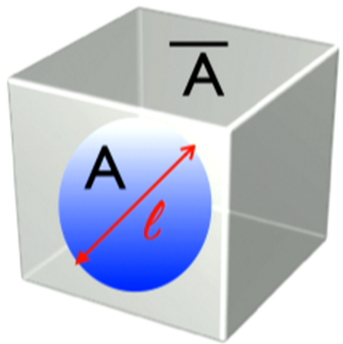
1d, 2d at any disorder,
3d at strong disorder

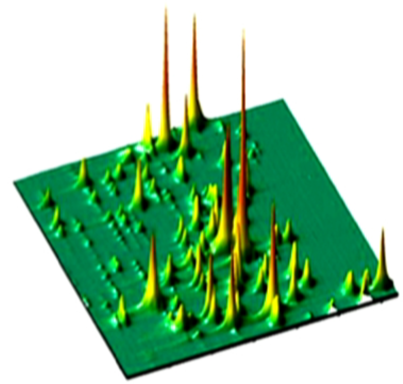


3d at weak disorder

Anderson Localization:

Breakdown of Quantum Ergodicity





Finite energy density eigenstates of Ergodic Phases: $S \sim \ell^{d}$

Ground States of Ergodic Phases: $S \sim \ell^{d-1}$

Finite energy density eigenstates of Anderson localized Phase: $S \sim \ell^{d-1}$

<u>Excited states behave like ground states:</u>

Dramatic Breakdown of Ergodicity!

Many Body Localization:

Ergodicity Breaking *despite* interactions

$$H = -\sum_{<ij>}\left(c_i^\dagger c_j + h.c.\right) + \sum_i \mu_i c_i^\dagger c_i + \sum_{<ij>} n_i n_j$$

Claim (Basko, Aleiner, Altshuler 2005):

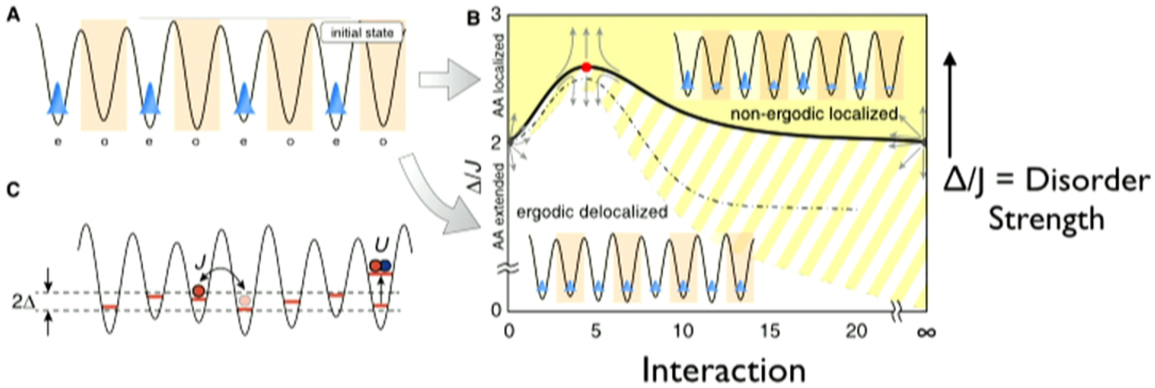
At strong disorder, many-body eigenstates stay localized

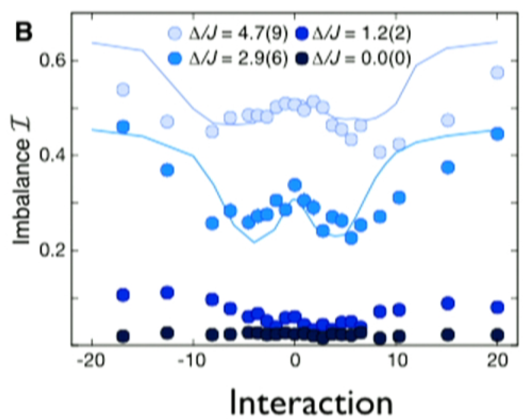
Intuition:

Unperturbed eigenstates at a given energy sufficiently different from each other. Interactions ineffectual.

Oganesyan, Huse 2007; Pal, Huse 2010; Bauer et al 2013; Abanin et al 2014; Vidal et al 2014; Imbrie 2014, and many others.

Experimental Realization?

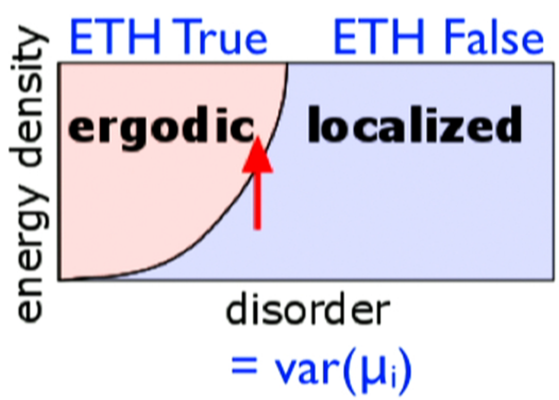


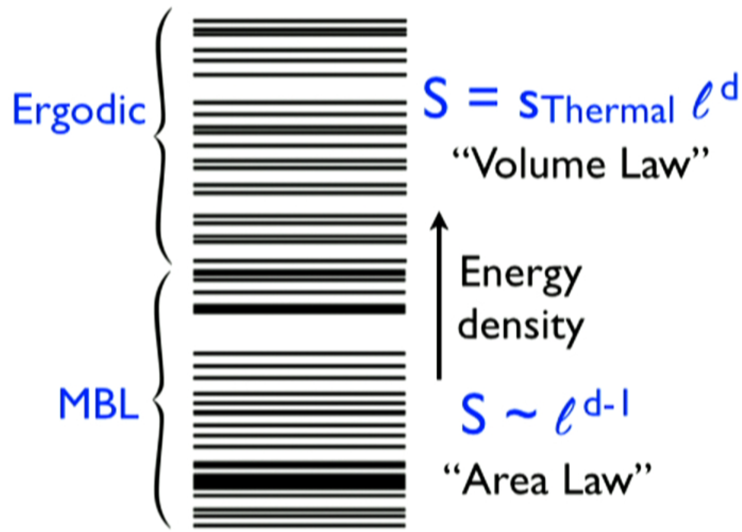


I. Bloch et al 2015

Phase diagram

$$H = -\sum_{<ij>}\left(c_i^\dagger c_j + h.c.\right) + \sum_i \mu_i c_i^\dagger c_i + \sum_{<ij>} n_i n_j$$





Nature of transition?

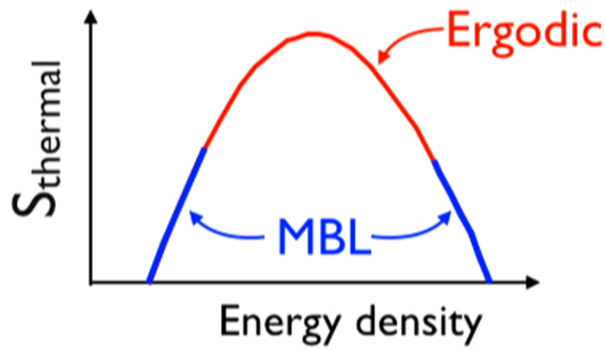
Outline

- Ergodic … um mechan…
- Ergodic … d quantum systems…
- Many Body localization phase transition.

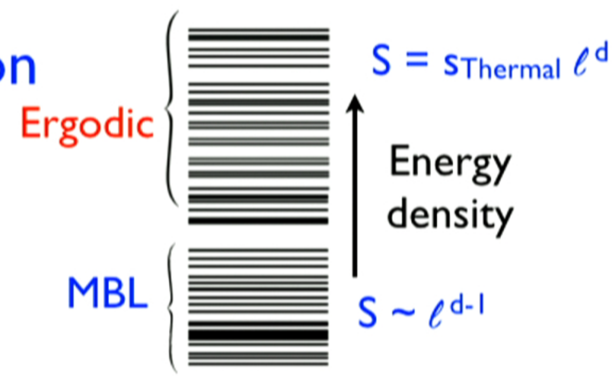
TG, arXiv:1405.1471

MBL transition Vs Any Other transition

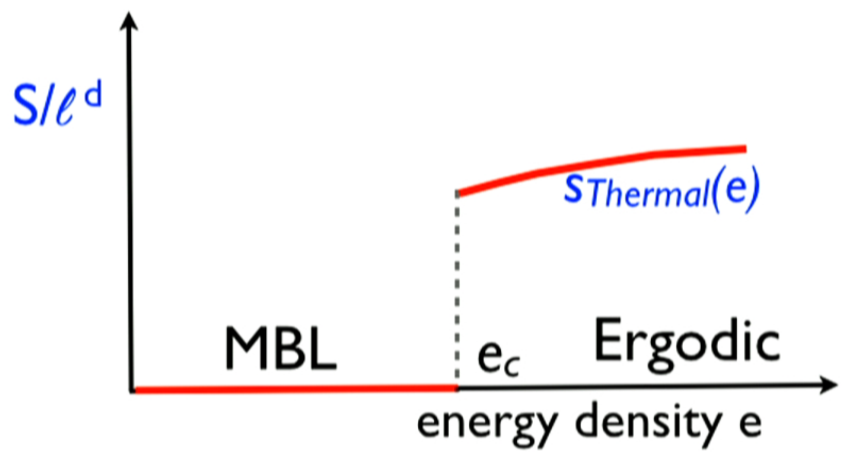
- No signature in thermodynamics. Analytic $S_{Thermal}$.



- Quantum phase transition at "finite temperature".



Can transition be continuous?



Volume law coefficient "jumps" across the transition.

Order Parameter for MBL Transition?

Conductivity Vs Entanglement Entropy

Conductivity

Pros:

- Experimentally accessible.
- Well studied in Anderson localization.

Cons:

- Limited to systems with conserved particle/energy.
- DC conductivity hard to calculate, even numerically.
- Sensitive to rare region effects (e.g. Griffiths phase).

Entanglement Entropy

Pros:

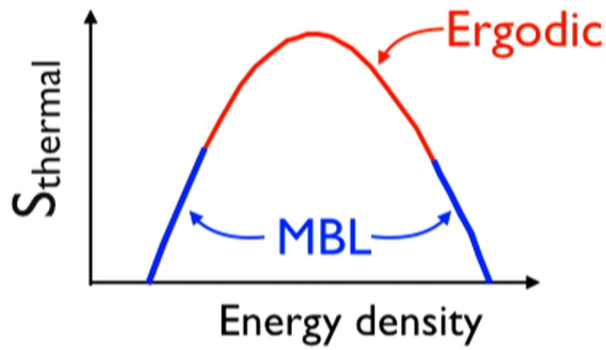
- Faithful diagnostic of ergodicity.
- Works for all known phase transitions. Not limited to systems with conservation laws.
- Not sensitive to rare region effects.
- Theoretical results on entanglement have indirect experimental consequences.

Cons:

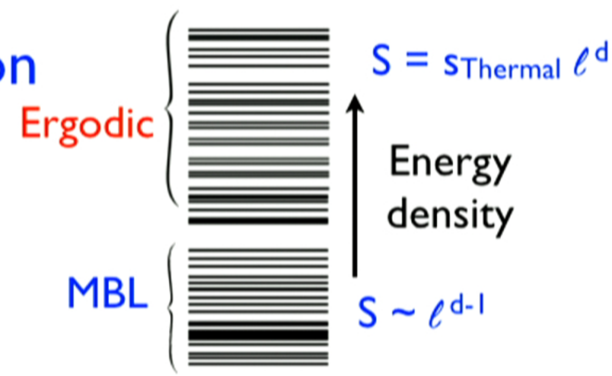
- As yet, experimentally inaccessible.

MBL transition Vs Any Other transition

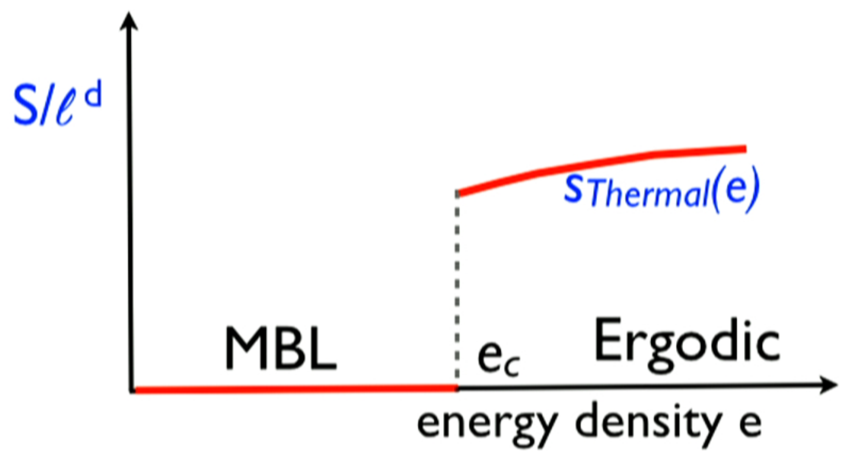
- No signature in thermodynamics. Analytic $S_{Thermal}$.



- Quantum phase transition at "finite temperature".

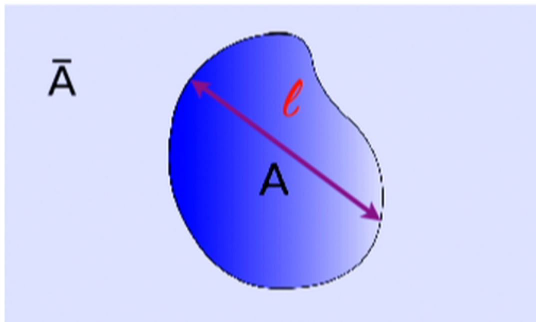


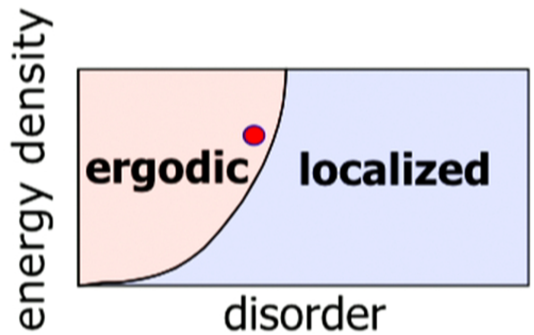
Can transition be continuous?



Volume law coefficient "jumps" across the transition.

Entanglement Scaling Close to MBL Transition





ξ = delocalization length

subsystem size ℓ

"universal entanglement" of critical point

↑

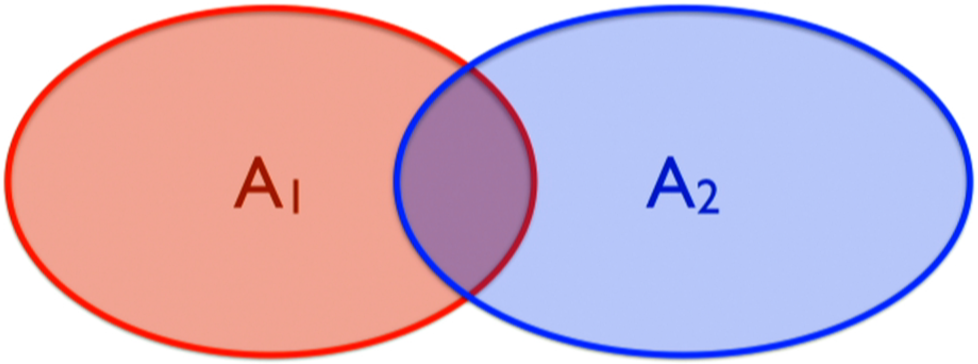
Unknown

Entanglement of ergodic phase

↑

Known

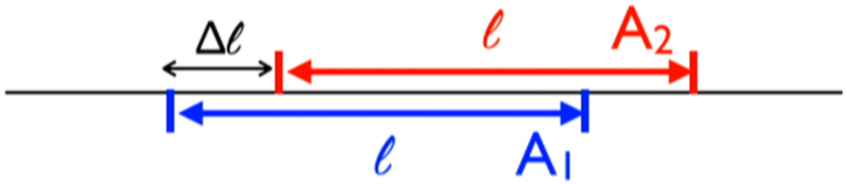
Our toolbox: Strong Subadditivity constraint on Entanglement



$$S(A_1) + S(A_2) - S(A_1 \cup A_2) - S(A_1 \cap A_2) \geq 0$$

Lieb, Ruskai 1973

Strong Subadditivity & Entanglement Scaling



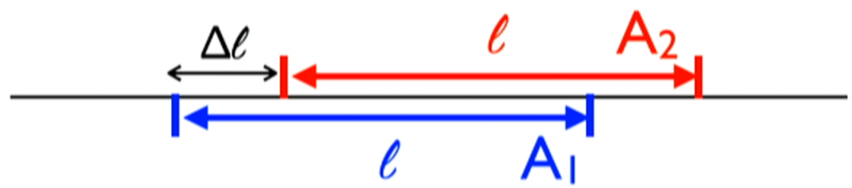
$$S(A_1) + S(A_2) - S(A_1 \cup A_2) - S(A_1 \cap A_2) \geq 0$$

$$\Rightarrow S(\ell) + S(\ell) - S(\ell + \Delta\ell) - S(\ell - \Delta\ell) \geq 0$$

$$\Rightarrow \quad \frac{\partial^2 S(\ell)}{\partial \ell^2} \leq 0$$

Hirata, Takayanagi 2007

Strong Subadditivity & Entanglement Scaling



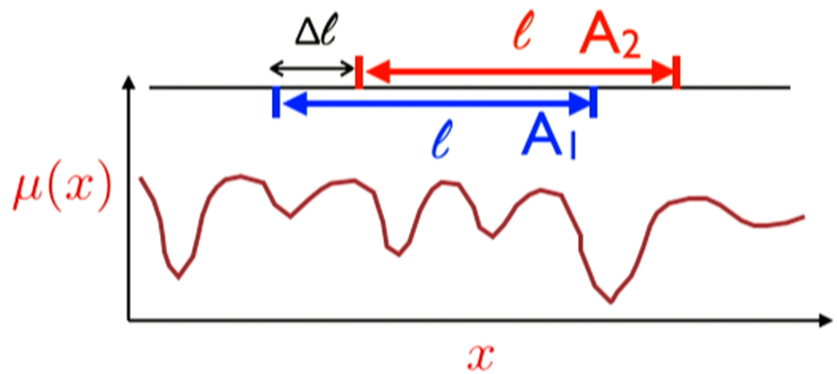
$$S(A_1) + S(A_2) - S(A_1 \cup A_2) - S(A_1 \cap A_2) \geq 0$$

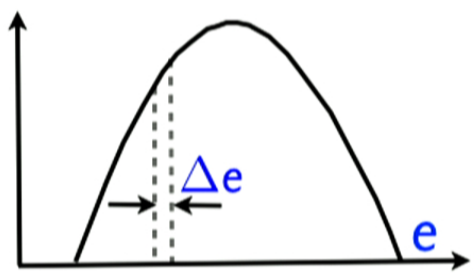
$$\Rightarrow S(\ell) + S(\ell) - S(\ell + \Delta\ell) - S(\ell - \Delta\ell) \geq 0$$

$$\Rightarrow \quad \boxed{\frac{\partial^2 S(\ell)}{\partial \ell^2} \leq 0}$$

Hirata, Takayanagi 2007

Inequality with Disorder?

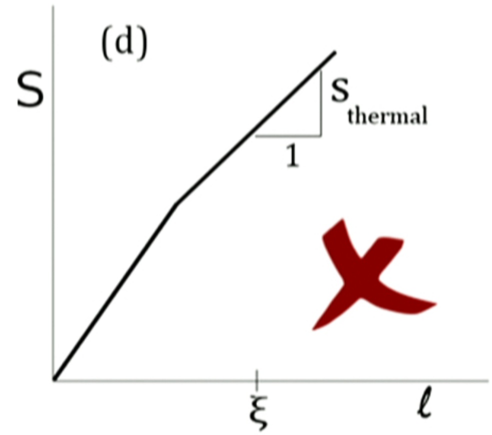




$$S(\ell, e) = \lim_{\Delta e \to 0} \lim_{V \to +\infty} \sum_{\mathcal{D}} P(\mathcal{D}) \left(\frac{\sum_{e'=e-\Delta e/2}^{e'=e+\Delta e/2} S_{\mathcal{D}}(\ell, e')}{\mathcal{N}_{\mathcal{D}}} \right)$$

$$\frac{\partial^2 S(\ell, e)}{\partial \ell^2} \leq 0$$

TG 2014



Not allowed because $s \leq s_{Thermal}$

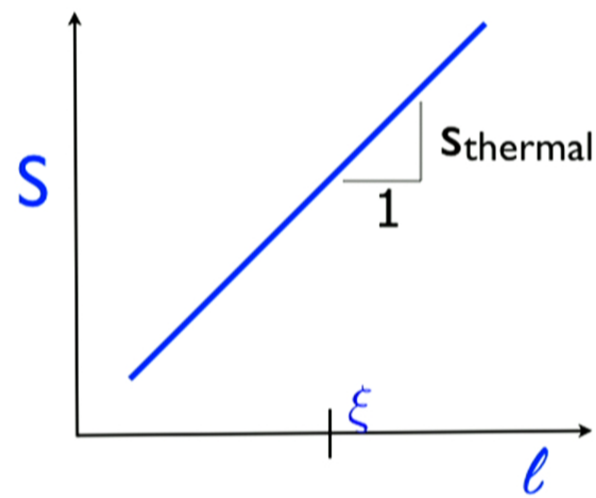
Follows from positivity of "Relative Entropy"

$$\mathrm{tr}\,(\rho_1 \log \rho_1) - \mathrm{tr}\,(\rho_1 \log \rho_2) \geq 0 \qquad \forall \rho_1, \rho_2$$

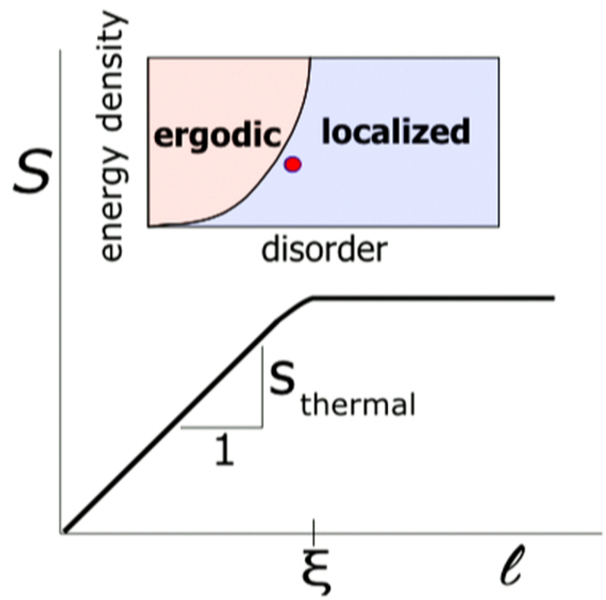
TG 2014

Answer

Critical Entanglement $S = s_{Thermal}\,\ell \Rightarrow$ Ergodic!

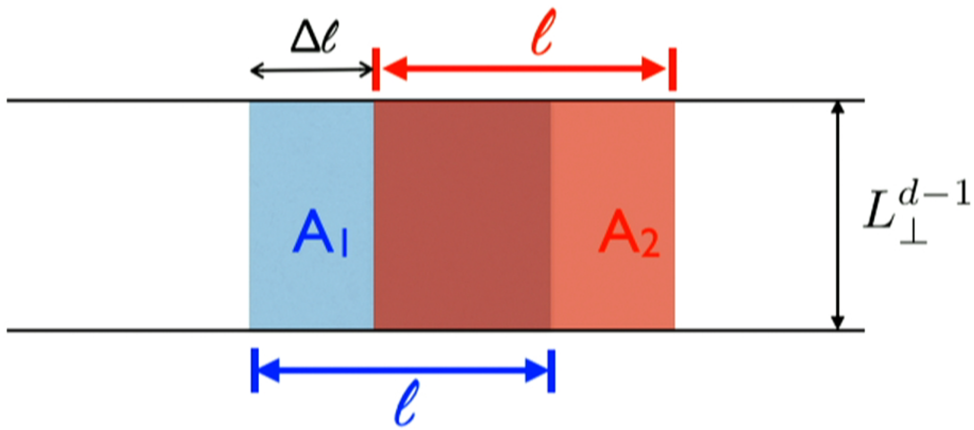


Approaching Transition from the MBL Side



Transition continuous because area-law coefficient diverges from the MBL side!

Generalization to Higher Dimensions



$$\frac{\partial^2 S(\ell, L_\perp)}{\partial \ell^2} \leq 0$$

Same conclusion. Critical point ergodic.

Consequences

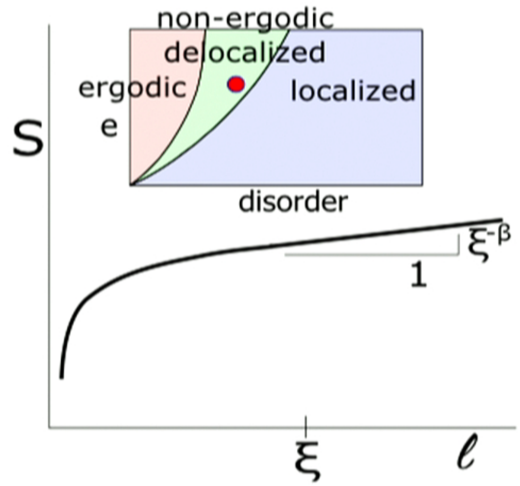
Predictions of equilibrium statistical mechanics valid AT the transition.

$$\langle\psi|O|\psi\rangle = \frac{\mathrm{tr}\left(Oe^{-\beta H}\right)}{\mathrm{tr}\left(e^{-\beta H}\right)}$$

Long time evolution of a state with critical energy density should yield a thermal state.

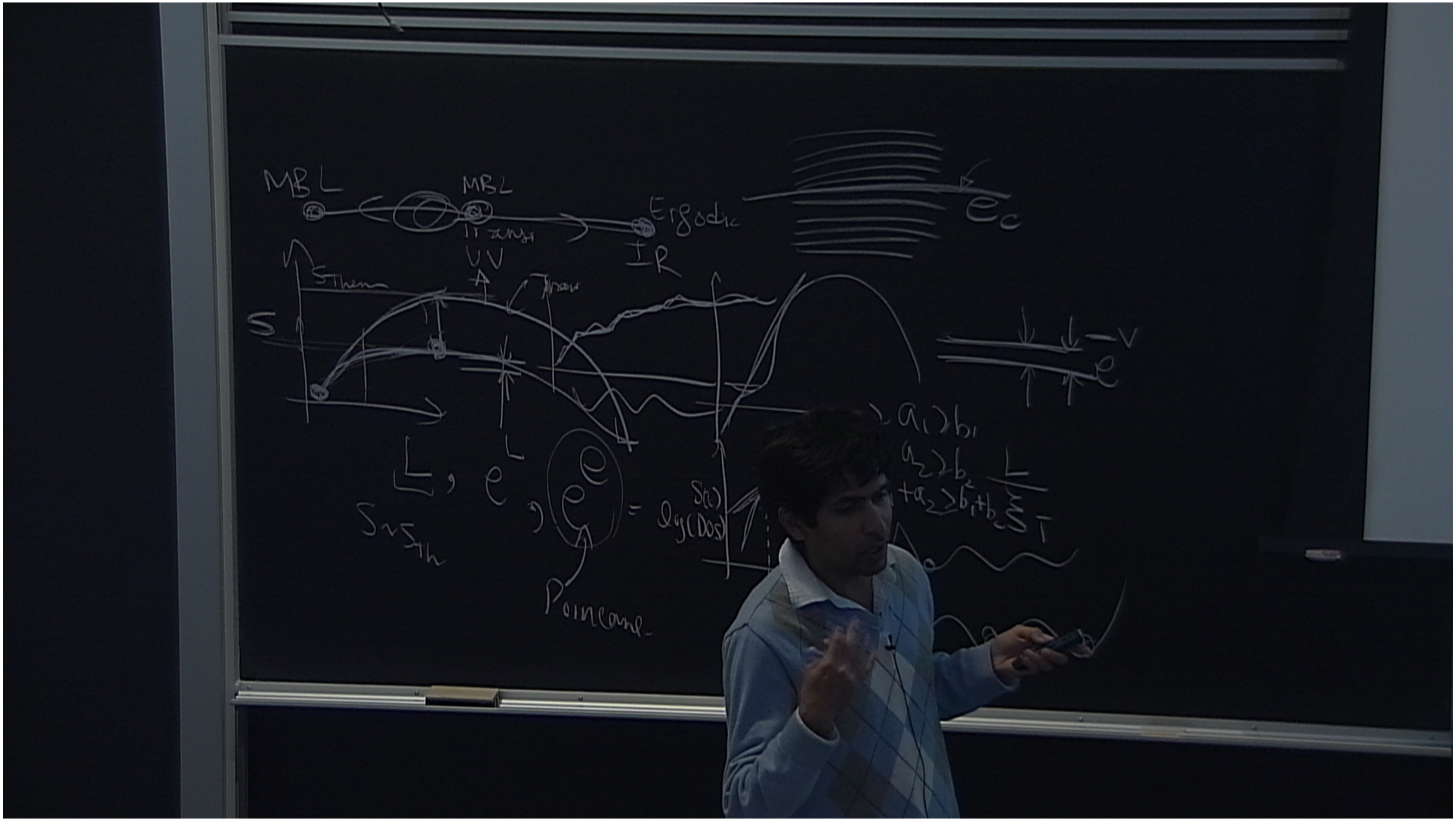
$$\mathrm{tr}_{\bar{A}} \lim_{t\to\infty} e^{iHt}|\psi_0\rangle\ \langle\psi_0|e^{-iHt} \propto e^{-\beta H_A}$$

Alternate Possibilities?



An intermediate non-ergodic delocalized phase?

Could it be first order?


MBL
MBL
Ergodic
UV
IR
S
e_c
Poincare

Consider a finite subsystem A of an infinite system.

$$\langle\psi|O|\psi\rangle = \frac{\mathrm{tr}\left(O e^{-\beta H} \right)}{\mathrm{tr}\left(e^{-\beta H} \right)}$$

We conjecture that above equation satisfied for **all** operators *O* with support only in region A.

$$\Rightarrow \quad \boxed{\rho_A(|\psi\rangle_\beta) = \rho_{A,\mathrm{th}}(\beta)}$$

where $\rho_{A,\mathrm{th}}(\beta) = \dfrac{\mathrm{tr}_{\overline{A}}\left(e^{-\beta H}\right)}{\mathrm{tr}\left(e^{-\beta H}\right)}$

"thermal reduced density matrix"

Consequence #1:

Thermodynamics using a single eigenstate.

$$\rho_A(|\psi\rangle_\beta) = \rho_{A,\mathrm{th}}(\beta)$$

would imply that Renyi entropies encode
free energy at various temperatures.

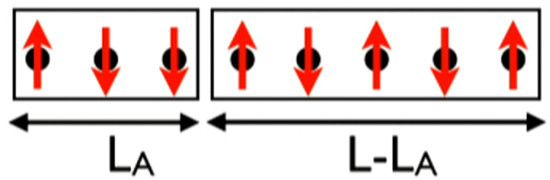
$$S_n(|\psi\rangle_\beta) \;=\; \frac{n}{n-1} V_A \beta \left(f(n\beta) - f(\beta)\right)$$

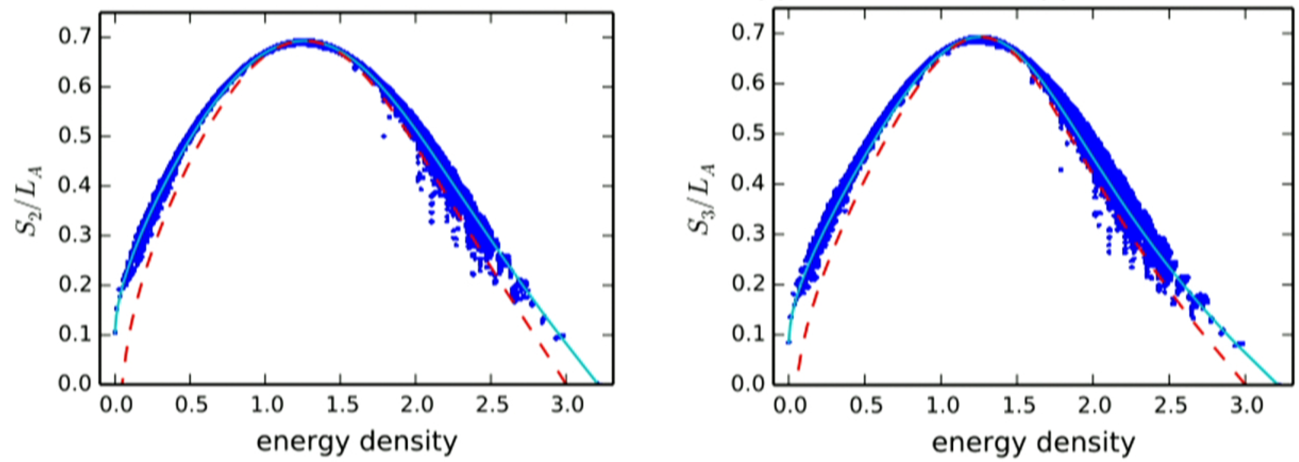
(holds only to leading order due to conical singularity)

$\rho_{A,\mathrm{th}}(\beta)$ studied extensively using analytical methods (e.g. Holzhey, Larsen, Wilczek; Cardy, Calabrese) and Numerical techniques (e.g. Singh, Hastings, Kallin, Melko)

Numerical check on the conjecture

$H = \sum_i \left(-\sigma_i^z \sigma_{i+1}^z + h_x \sigma_i^x + h_z \sigma_i^z\right)$





Blue dots • = Renyi Entropy density S_n/L_A of individual eigenstates

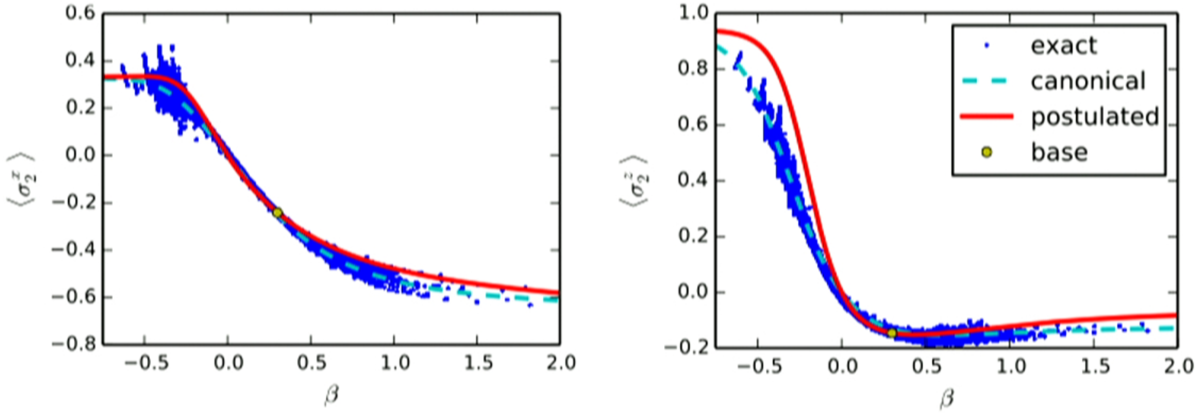
red dashed line $= -\frac{1}{n-1}\frac{\log(\mathrm{tr}\rho^n_{A,th})}{L_A} = \frac{n}{n-1}\beta(f(n\beta) - f(\beta))$

+ subleading corrections due to conical singularity.

cyan line $= \frac{n}{n-1}\beta\left(f(n\beta) - f(\beta)\right)$ in a system with open boundary conditions.

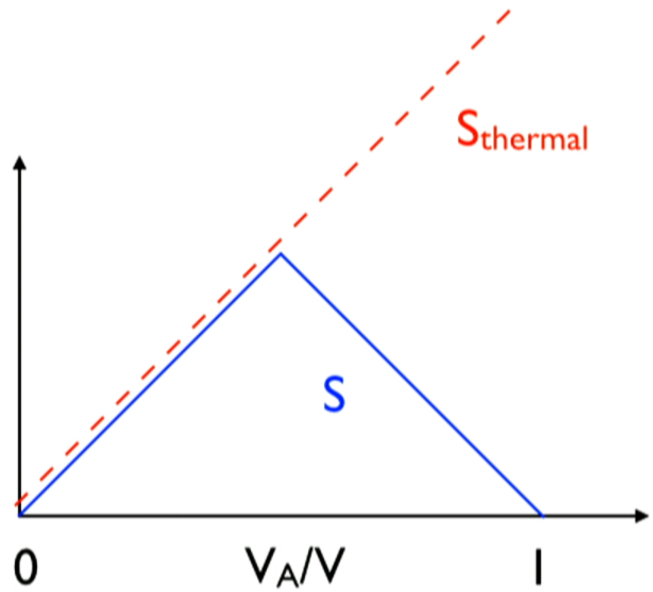
Consequence #2:

Expectation value of observables at all temperatures using a single eigenstate.



$$\langle O(x)O(y)\rangle_{n\beta} = \frac{\mathrm{tr}_A\left(\rho_A^n(|\psi\rangle_\beta)O(x)O(y)\right)}{\mathrm{tr}_A\left(\rho_A^n(|\psi\rangle_\beta)\right)}$$

+ corrections of order $e^{-x/\xi_T}, e^{-y/\xi_T}$



For random pure states, entanglement entropy equals thermal entropy only when $V_A/V < 1/2$.

Lubkin1978,;Lloyd, Pagels 1988; Page 1993

Physical Content of Conjectured ETH

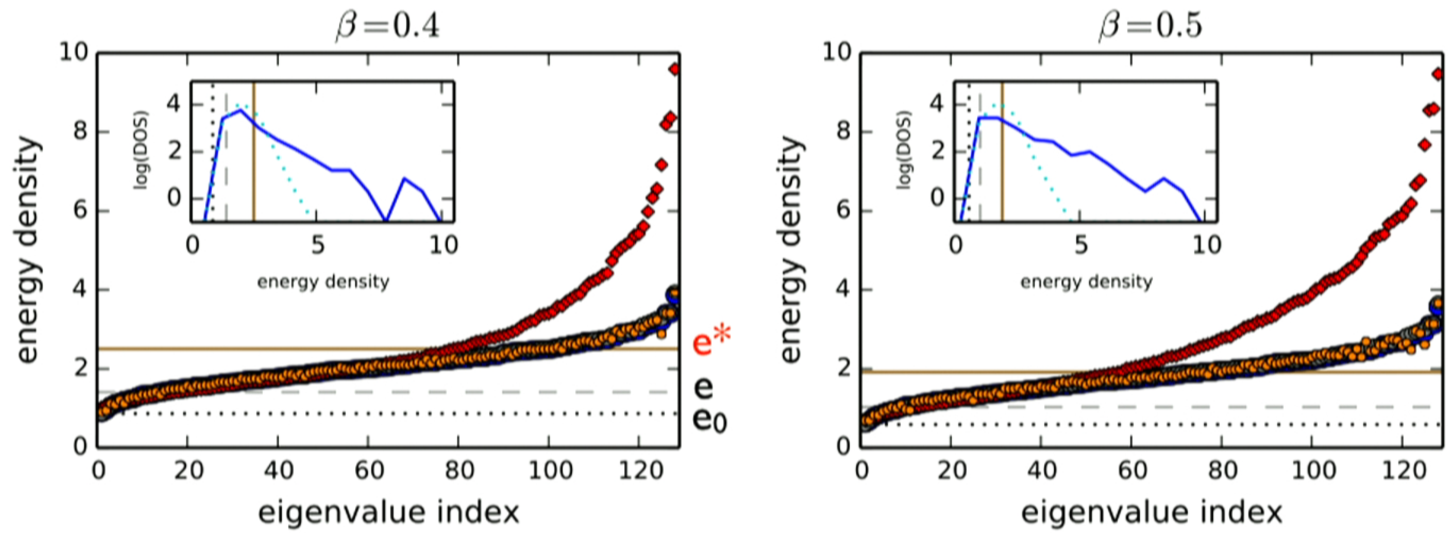
$$\rho_A(|\psi\rangle_\beta) = \frac{\mathrm{tr}_{\overline{A}}\left(e^{-\beta H}\right)}{\mathrm{tr}\left(e^{-\beta H}\right)} \approx \frac{e^{-\beta H_A}}{\mathrm{tr}_A\left(e^{-\beta H_A}\right)}$$

$$|\psi\rangle_\beta = \sum_i \sqrt{\lambda_i}|u_i\rangle \otimes |v_i\rangle$$

$\lambda_i \propto e^{-\beta E_{A,i}}$ $|u_i\rangle$ = Approximate Eigenstate of H_A

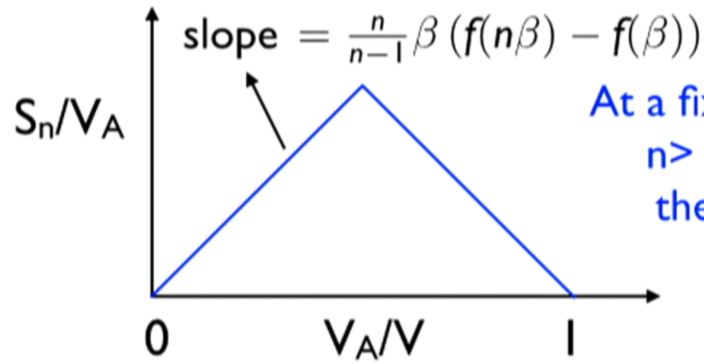
Rare quantum fluctuations of energy in a subsystem allow one to access properties at various energy densities

Breakdown of ETH!

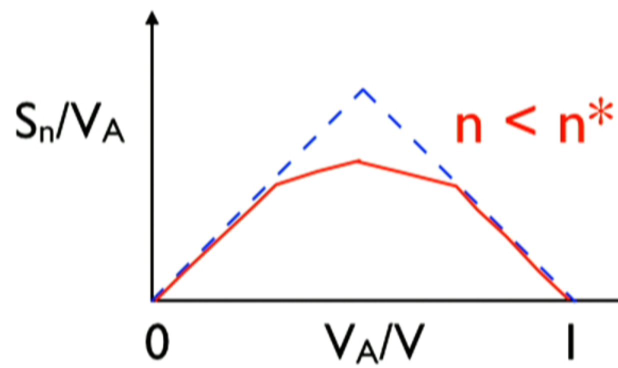


- $\frac{-1}{L_A \beta} \log[\rho_{A,\mathrm{th}}(\beta)]$
- $\frac{1}{L_A}[H_A + c_A]$
- $\frac{-1}{L_A \beta} \log[\rho_A(|\psi\rangle_\beta)]$
- $\frac{1}{L_A}[\langle u_i | H_A | u_i \rangle + c_A]$

Consequences for Renyi Entropies

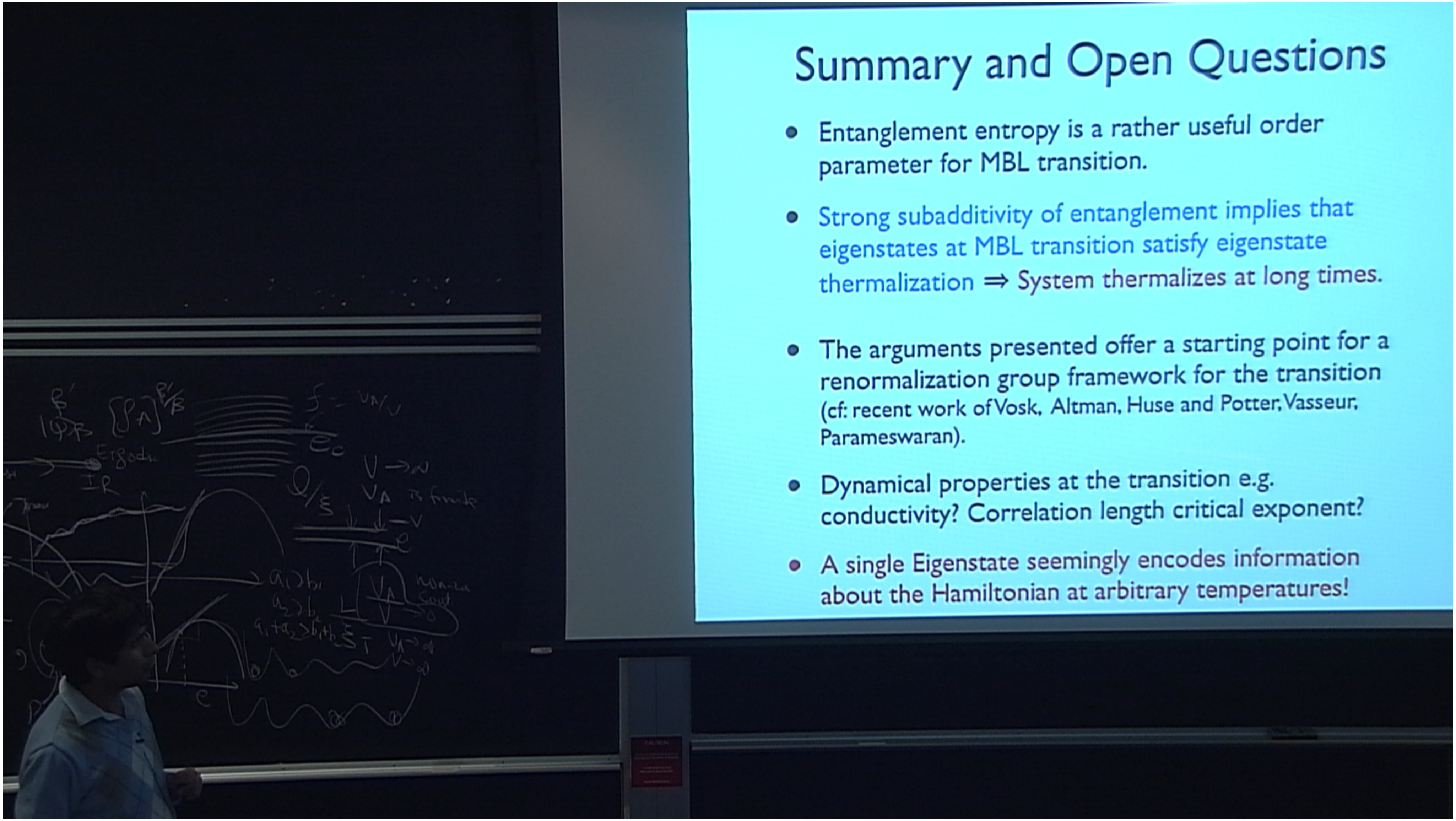


At a fixed V_A/V, Renyi entropies for $n > n^*$ continue to satisfy the the analog of "Page Curve".



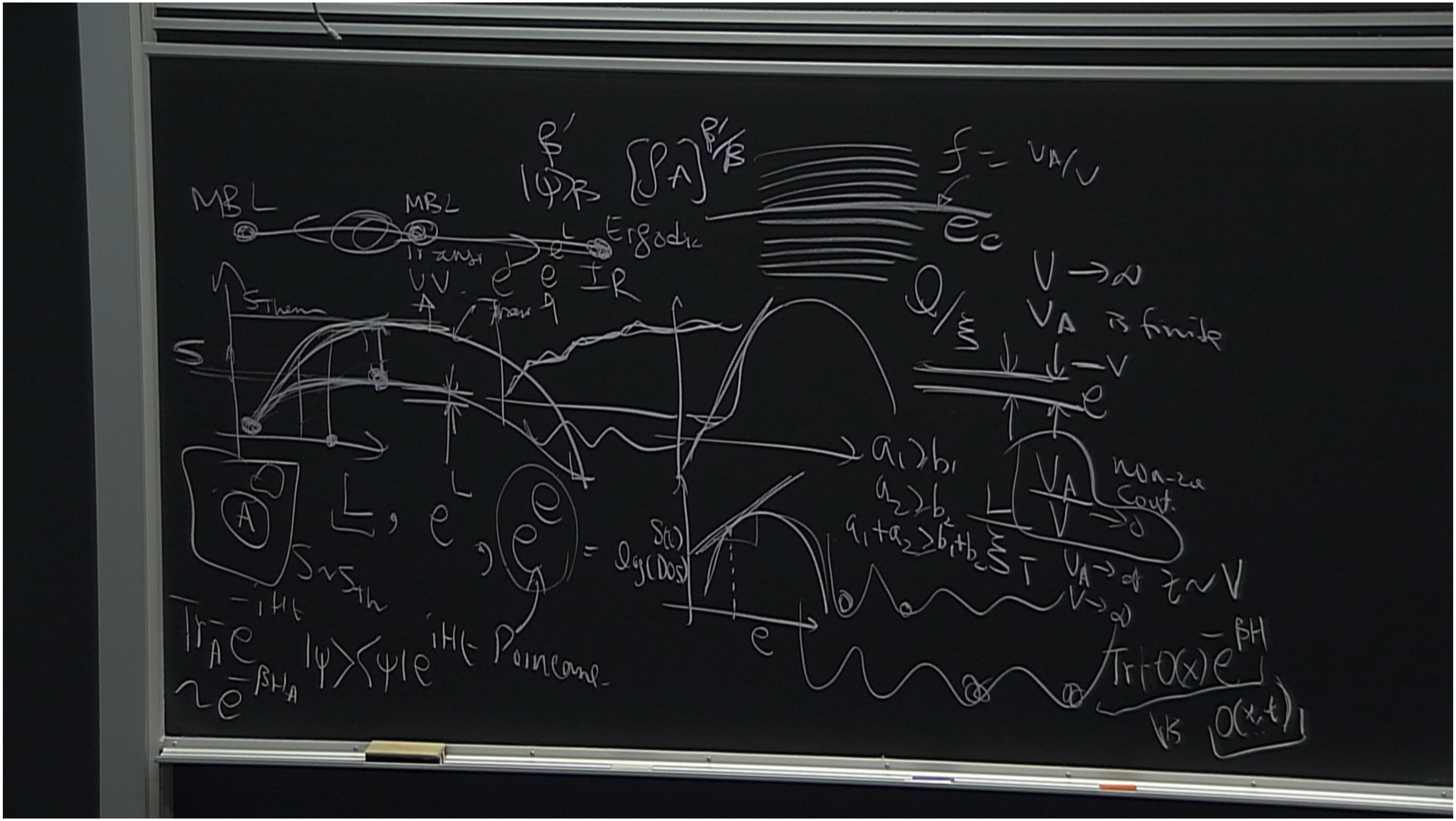
n^* determined by solving $e((n\beta)^{-1})) = e^*$ for $V_A/V = 1/2$

For CFTs in d dimensions $n^* = (1/2)^{1/(d+1)}$


Summary and Open Questions
Entangelement entropy is a rather useful order parameter for MBL transition.
Strong subadditivity of entanglement implies that eigenstates at MBL transition satisfy eigenstate thermalization ⇒ System thermalizes at long times.
The arguments presented offer a starting point for a renormalization group framework for the transition (cf: recent work of Vosk, Altman, Huse and Potter, Vasseur, Parameswaran).
Dynamical properties at the transition e.g. conductivity? Correlation length critical exponent?
A single Eigenstate seemingly encodes information about the Hamiltonian at arbitrary temperatures!

Summary and Open Questions

- Entanglement entropy is a rather useful order parameter for MBL transition.
- Strong subadditivity of entanglement implies that eigenstates at MBL transition satisfy eigenstate thermalization $\Rightarrow$ System thermalizes at long times.
- The arguments presented offer a starting point for a renormalization group framework for the transition (cf: recent work of Vosk, Altman, Huse and Potter, Vasseur, Parameswaran).
- Dynamical properties at the transition e.g. conductivity? Correlation length critical exponent?
- A single Eigenstate seemingly encodes information about the Hamiltonian at arbitrary temperatures!


MBL
MBL
Ergodic
IR
f = VA/V
V → ∞
VA is finite
non-zero const.
a1 > b1
a2 > b2
a1+a2 > b1+b2
S(E) = log(DOS)
Poincare