Title: Galileons and their generalizations

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Abstract: <p>Galileons are higher-derivative effective field theories with curious properties which have attracted much recent interest among cosmologists. I will review their origins, their properties, their generalizations, and some recent developments.</p>

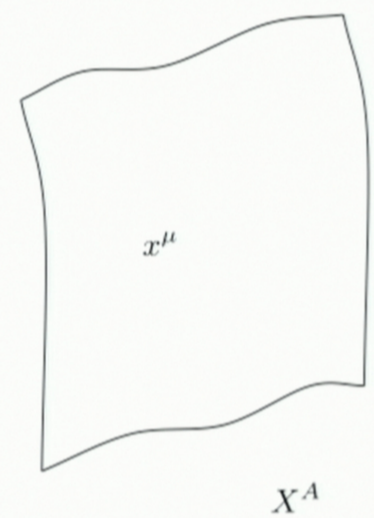
# Galileon theories

- Effective field theories (non-renormalizable)
- Non-linearly realized symmetries (spontaneous breaking)
- Higher-derivative, yet ghost-free
- Regimes in which non-linearities are important and quantum effects are not

# Origin of modern galileons: The DGP model

Dvali, Gabadadze, Porrati (2000)

3 brane (3+1 dimensions) in a 5-d bulk



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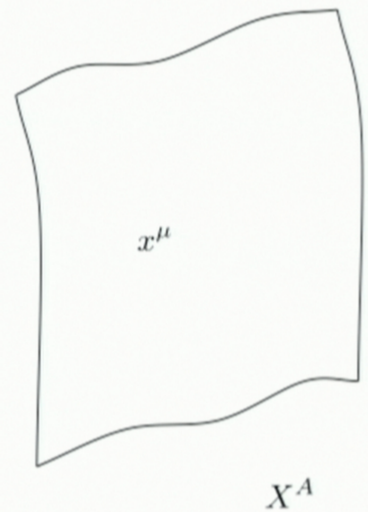
Einstein-Hilbert action on the brane and in the bulk:

$$S = \frac{M_5^3}{2}\int d^5X\sqrt{-G}R(G) + \frac{M_4^2}{2}\int d^4x\sqrt{-g}R(g) + S_M$$



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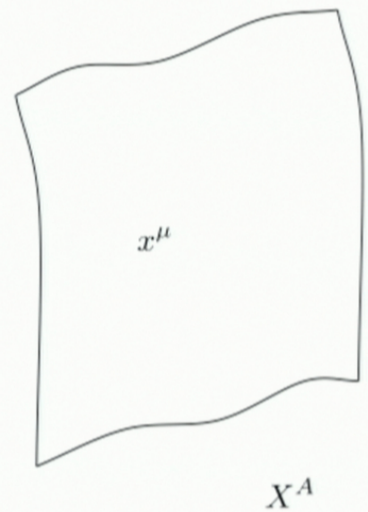
Gravity localized over a length-scale: $r_c \sim \frac{1}{m} \sim \frac{M_4^2}{M_5^3}$



# DGP: 4-d effective action

Luty, Porrati, Rattazzi (2003)
Nicolis, Rattazzi (2004)

Integrate out the extra dimension:

Brane-bending mode

$$\frac{1}{\Lambda^3}(\partial\pi)^2\Box\pi + \text{other stuff} \subset \mathcal{L}_4$$

Horrible non-local stuff suppressed by scales higher than $\Lambda$

Strong coupling scale $\quad \Lambda = \frac{M_5^2}{M_4} \sim (m^2 M_4)^{1/3} \sim (10^3\ \text{km})^{-1}$

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Decoupling limit:

$M_4 \to \infty, \quad M_5 \to \infty, \quad \Lambda$ fixed

other stuff $\to 0$

# Properties of the Pi lagrangian

Higher derivative lagrangian: $\mathcal{L} = (\partial\pi)^2\Box\pi$

- Equations of motion are still second order:

$$\mathcal{E} = \frac{\delta\mathcal{L}}{\delta\pi} = (\partial_\mu\partial_\nu\pi)^2 - (\Box\pi)^2$$

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No additional DOF.

- Symmetry under shifts of the field and its derivative: $\pi(x) \to \pi(x) + c + b_\mu x^\mu$

Lagrangian changes by a total derivative, equations of motion are invariant.

Like Galilean transformations in particle mechanics: $q(t) \to q(t) + c + vt$

$$\mathcal{L} = \frac{1}{2}\dot{q}^2$$

# Galileons

Nicolis, Rattazzi, Trincherini (2008)

4 possible terms in 4 dimensions: $\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi, \quad [\ldots] = Tr(\ldots)$

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\begin{aligned}
\mathcal{L}_2 &= -\frac{1}{2}(\partial\pi)^2 \,, \\
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$$\begin{aligned}
\mathcal{E}_2 &= \Box\pi \\
\mathcal{E}_3 &= (\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2 \\
\mathcal{E}_4 &= (\Box\pi)^3 - 3\,\Box\pi(\partial_\mu\partial_\nu\pi)^2 + 2(\partial_\mu\partial_\nu\pi)^3 \\
\mathcal{E}_5 &= (\Box\pi)^4 - 6(\Box\pi)^2(\partial_\mu\partial_\nu\pi)^2 + 8\,\Box\pi(\partial_\mu\partial_\nu\pi)^3 + 3\left[(\partial_\mu\partial_\nu\pi)^2\right]^2 - 6(\partial_\mu\partial_\nu\pi)^4
\end{aligned}$$

# Spherical solutions: Vainshtein Mechanism

Screening through kinetic non-linearities

(Vainshtein 1972 Arkani-Hamed, Georgi, Schwartz 2003
Deffayet, Dvali, Gabadadze & Vainshtein 2002;
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$\pi$-lagrangian from DGP

$$\mathcal{L} = -3(\partial\pi)^2 - \frac{1}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{1}{M_{Pl}}\pi T$$

Scale of non-linearities

Trace of matter stress tensor

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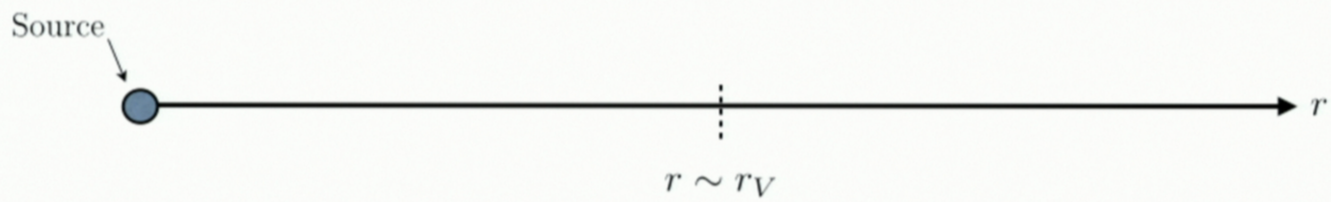


Solution around point source of mass M:

$$\pi(r) = \begin{cases} \sim \Lambda^3 R_V^{3/2}\sqrt{r} + const. & r \ll R_V \\ \sim \Lambda^3 R_V^3 \frac{1}{r} & r \gg R_V \end{cases}$$

Vainshtein radius: $R_V \equiv \frac{1}{\Lambda}\left(\frac{M}{M_{Pl}}\right)^{1/3}$

Non-linearity become important at the Vainshtein radius

## Suppressing the 5-th force: Vainshtein Mechanism

Nicolis, Rattazzi (2004)

5-th force on a test particle, relative to gravity:

$$\frac{F_\phi}{F_{\rm Newton}} = \frac{\hat{\phi}'(r)/M_P}{M/(M_P^2 r^2)} = \begin{cases} \sim \left(\frac{r}{r_V^{(3)}}\right)^{3/2} & r \ll r_V^{(3)}, \\ \sim 1 & r \gg r_V^{(3)}. \end{cases}$$

$\hat{\phi} = \Phi + \varphi, \quad T = T_0 + \delta T$

$$-3(\partial\varphi)^2 + \frac{2}{\Lambda^3}\left(\partial_\mu\partial_\nu\Phi - \eta_{\mu\nu}\Box\Phi\right)\partial^\mu\varphi\partial^\nu\varphi - \frac{1}{\Lambda^3}(\partial\varphi)^2\Box\varphi + \frac{1}{M_4}\varphi\delta T$$

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$$\sim \left(\frac{r_V^{(3)}}{r}\right)^{3/2}$$

Kinetic terms are enhanced, which means that, after canonical normalization, the coupling to $\delta$T is suppressed. The non-linear coupling scale is also raised.

This is known as a Screening mechanism

# The effective field theory

Non-renormalizable: effective theory with a cutoff $\Lambda$. Must include all terms compatible with galilean symmetry, suppressed by powers of the cutoff

$$\mathcal{L} \sim (\partial\pi)^2 + \frac{1}{\Lambda^{3n}}(\partial\pi)^2(\partial\partial\pi)^n + \frac{1}{\Lambda^{m+3n-4}}\partial^m(\partial\partial\pi)^n$$

Galileon terms $\alpha_{cl} \equiv \dfrac{\partial\partial\pi}{\Lambda^3}$ Terms with at least two derivatives per field $\alpha_q \equiv \dfrac{\partial^2}{\Lambda^2}$

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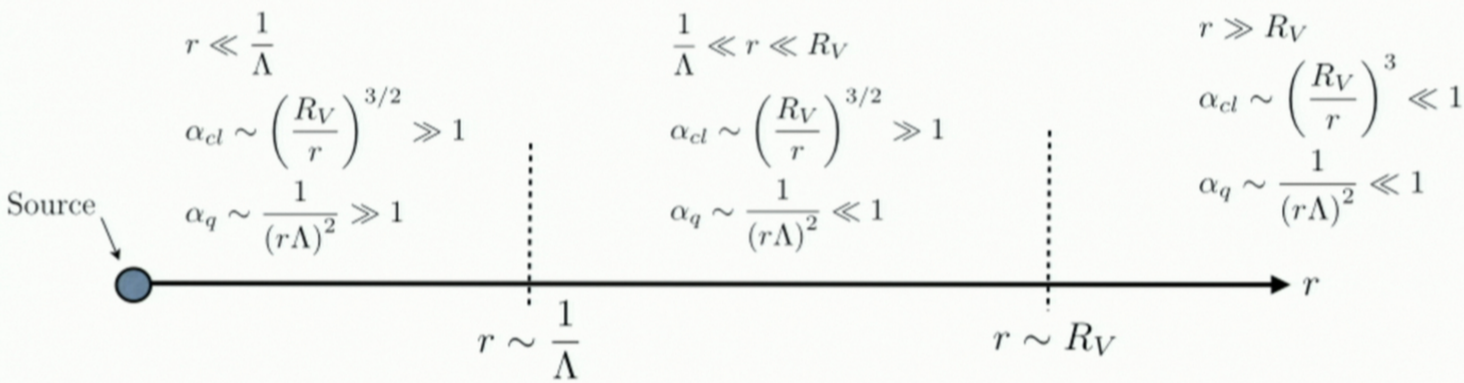

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Terms with at least two derivatives per field $\alpha_q \equiv \dfrac{\partial^2}{\Lambda^2}$

$r \ll \dfrac{1}{\Lambda}$

$\alpha_{cl} \sim \left(\dfrac{R_V}{r}\right)^{3/2} \gg 1$

$\alpha_q \sim \dfrac{1}{(r\Lambda)^2} \gg 1$

$\dfrac{1}{\Lambda} \ll r \ll R_V$

$\alpha_{cl} \sim \left(\dfrac{R_V}{r}\right)^{3/2} \gg 1$

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$r \gg R_V$

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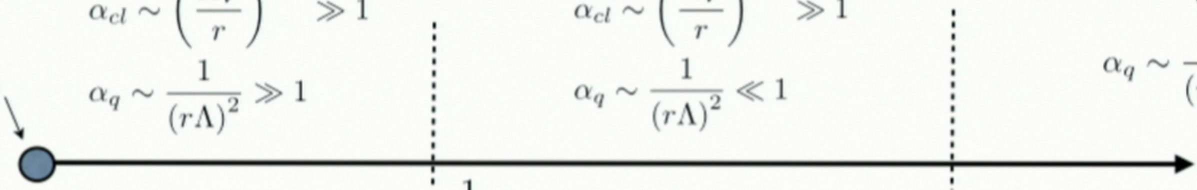
Source



$r$

$r \sim \dfrac{1}{\Lambda}$ $\qquad$ $r \sim R_V$

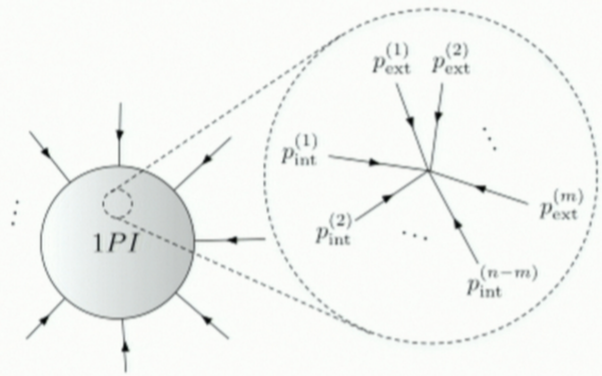
"Sweet spot"

# Quantum corrections

Luty, Porrati, Ratazzi (2003)
Nicolis, Rattazzi (2004)
KH, Mark Trodden, Dan Weseley (2010)
de Rham, Gabadadze, Heisenberg, Pirtskhalava (2012)

Non renormalization theorem: the galileon terms receive no quantum corrections!

Can show that an n-point contribution to the quantum effective action contains at least 2n powers of external momenta, so it can't renormalize the Galilean term which has only 2n-2 derivatives.




$$\mathcal{L}_{n+1} \sim \eta^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_n\nu_n}\left(\pi_{\rm ext}\partial_{\mu_1}\partial_{\nu_1}\pi_{\rm ext}\cdots\partial_{\mu_{m-1}}\partial_{\nu_{m-1}}\pi_{\rm ext}\partial_{\mu_m}\partial_{\nu_m}\pi_{\rm int}\cdots\partial_{\mu_n}\partial_{\nu_n}\pi_{\rm int}\right).$$

$$\mathcal{L}_{n+1} \sim \eta^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_n\nu_n}\left(\pi_{\rm ext}\partial_{\mu_1}\partial_{\nu_1}\pi_{\rm ext}\cdots\partial_{\mu_{m-1}}\partial_{\nu_{m-1}}\pi_{\rm ext}\partial_{\mu_m}\partial_{\nu_m}\left[\pi_{\rm int}\cdots\partial_{\mu_n}\partial_{\nu_n}\pi_{\rm int}\right]\right).$$

Also: a mass term is not renormalized

$$\mathcal{L} \sim (\partial\pi)^2 + m^2\pi^2 + \frac{1}{\Lambda^{3n}}(\partial\pi)^2(\partial\partial\pi)^n + \frac{1}{\Lambda^{m+3n-4}}\partial^m(\partial\partial\pi)^n$$

Not renormalized, no hierarchy problem

# Appearance in ghost-free massive gravity

de Rham, Gabadadze, Tolley (2011)

Parameters $\begin{cases} \text{graviton mass} \quad m \\ \text{Planck mass} \quad M_P \\ \text{two additional dimensionless parameters} \end{cases}$

$$\frac{M_P^2}{2}\int d^4x\sqrt{-g}\left[R - \frac{m^2}{4}\sum_{n=2}^{4}\beta_n S_n\left(1-\sqrt{g^{-1}\eta}\right)\right]$$

Characteristic Polynomials

$$S_n(M) = \frac{1}{n!(D-n)!}\,\tilde{\epsilon}_{A_1 A_2\cdots A_D}\,\tilde{\epsilon}^{B_1 B_2\cdots B_D}\,M^{A_1}_{\ B_1}\cdots M^{A_n}_{\ B_n}\,\delta^{A_{n+1}}_{\ B_{n+1}}\cdots\delta^{A_D}_{\ B_D}$$

Introduce Stükelberg fields: $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu + 2\,\partial_\mu\partial_\nu\phi$

$h_{\mu\nu}$ 5 DOF $\xrightarrow{\text{relativistic limit } m\to 0}$ $\begin{cases} h_{\mu\nu} \sim \text{helicity } \pm 2 & \text{2 DOF} \\ A_\mu \sim \text{helicity } \pm 1 & \text{2 DOF} \\ \phi \sim \text{helicity } 0 & \text{1 DOF} \end{cases}$

# Leading operators

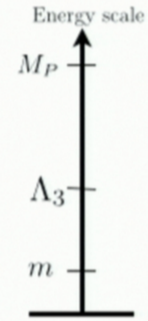
de Rham, Gabadadze (2010)

The leading operators carry the scale $\Lambda_3 \equiv (M_P m^2)^{1/3}$ $\sim \frac{\hat{h}(\partial^2\hat{\phi})^n}{M_P^{n+1} m^{2n+2}}$

Explicitly:

$$\frac{1}{2}\hat{h}_{\mu\nu}\mathcal{E}^{\mu\nu,\alpha\beta}\hat{h}_{\alpha\beta} - \frac{1}{2}\hat{h}^{\mu\nu}\left[-4X^{(1)}_{\mu\nu}(\hat{\phi}) + \frac{4(6c_3-1)}{\Lambda_3^3}X^{(2)}_{\mu\nu}(\hat{\phi}) + \frac{16(8d_5+c_3)}{\Lambda_3^6}X^{(3)}_{\mu\nu}(\hat{\phi})\right] + \frac{1}{M_P}\hat{h}_{\mu\nu}T^{\mu\nu}$$

X tensors:

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&\vdots
\end{aligned}$$

They have the following properties, which ensures that the decoupling limit is ghost free

$$\partial^\mu X^{(n)}_{\mu\nu} = 0$$

$X^{(n)}_{ij}$ has at most two time derivatives,

$X^{(n)}_{0i}$ has at most one time derivative,

$X^{(n)}_{00}$ has no time derivatives.

# Diagonalized leading operators

Diagonalize: $\hat{h}_{\mu\nu} \to \hat{h}_{\mu\nu} + \hat{\phi}\hat{h}_{\mu\nu} + \frac{2(6c_3-1)}{\Lambda_3^3}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi}$

$$\frac{1}{2}\hat{h}_{\mu\nu}\mathcal{E}^{\mu\nu,\alpha\beta}\hat{h}_{\alpha\beta} - \frac{8(8d_5+c_3)}{\Lambda_3^6}\hat{h}^{\mu\nu}\hat{X}^{(3)}_{\mu\nu} + \frac{1}{M_P}\hat{h}_{\mu\nu}T^{\mu\nu}$$
$$-3(\partial\hat{\phi})^2 + \frac{6(6c_3-1)}{\Lambda_3^3}(\partial\hat{\phi})^2\Box\hat{\phi} - 4\frac{(6c_3-1)^2-4(8d_5+c_3)}{\Lambda_3^6}(\partial\hat{\phi})^2\left([\hat{\Pi}]^2-[\hat{\Pi}^2]\right)$$
$$-\frac{40(6c_3-1)(8d_5+c_3)}{\Lambda_3^9}(\partial\hat{\phi})^2\left([\hat{\Pi}]^3-3[\hat{\Pi}^2][\hat{\Pi}]+2[\hat{\Pi}^3]\right)$$
$$+\frac{1}{M_P}\hat{\phi}T + \frac{2(6c_3-1)}{\Lambda_3^3 M_P}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi}T^{\mu\nu}.$$

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Longitudinal mode is described by Galileon interactions:

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Longitudinal mode is described by Galileon interactions:

There is now in general a conformal and a disformal coupling to matter

# Galileons as Wess-Zumino terms

Algebra of galileon symmetries: $Gal(3,1|1)$

Garret Goon, KH, Austin Joyce, Mark Trodden (2012)

Ordinary Poincare transformations

$$P_\mu, J_{\mu\nu} \qquad C, \ B_\mu$$

shift symmetry   galileon boost symmetry

$$[P_\mu, B_\nu] = \eta_{\mu\nu} C \ , \qquad [J_{\rho\sigma}, B_\nu] = \eta_{\rho\nu} B_\sigma - \eta_{\sigma\nu} B_\rho$$

+ Poincare algebra

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$$\pi \to \pi + c + b_\mu x^\mu$$

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$$[P_\mu, B_\nu] = \eta_{\mu\nu} C \,, \qquad [J_{\rho\sigma}, B_\nu] = \eta_{\rho\nu} B_\sigma - \eta_{\sigma\nu} B_\rho$$

$+$ Poincare algebra

$$\pi \to \pi + c + b_\mu x^\mu$$

Symmetry breaking pattern: $Gal(3,1|1) \to iso(3,1)$

Coset construction: a procedure to write the most general lagrangian invariant under some given symmetry breaking pattern.

Coleman, Wess, Zumino (1969)

# Galileons as Wess-Zumino terms

Algebra of galileon symmetries: $Gal(3,1|1)$

Garret Goon, KH, Austin Joyce, Mark Trodden (2012)

Ordinary Poincare transformations

$$P_\mu, J_{\mu\nu} \qquad C, \; B_\mu$$

shift symmetry galileon boost symmetry

$$[P_\mu, B_\nu] = \eta_{\mu\nu} C \; , \qquad [J_{\rho\sigma}, B_\nu] = \eta_{\rho\nu} B_\sigma - \eta_{\sigma\nu} B_\rho$$

$+$ Poincare algebra

$$\begin{aligned}
\mathcal{L}_2 &= -\frac{1}{2}(\partial\pi)^2 \; , \\
\mathcal{L}_3 &= -\frac{1}{2}(\partial\pi)^2[\Pi] \; , \\
\mathcal{L}_4 &= -\frac{1}{2}(\partial\pi)^2\left([\Pi]^2 - [\Pi^2]\right) \; , \\
\mathcal{L}_5 &= -\frac{1}{2}(\partial\pi)^2\left([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]\right) \; .
\end{aligned}$$

$$\pi \to \pi + c + b_\mu x^\mu$$

Symmetry breaking pattern: $Gal(3,1|1) \;\to\; iso(3,1)$

Coset construction: a procedure to write the most general lagrangian invariant under some given symmetry breaking pattern.

Coleman, Wess, Zumino (1969)

The galileons are missed by this procedure: they are invariant only up to a total derivative.

# Wess-Zumino-Witten terms

Wess, Zumino (1971)
Witten (1983)

Chiral lagrangian describes interactions of low energy pions.

$$U(x) = e^{B(x)} \in SU(3), \quad B(x) = -i\sum_{I=1}^{8} \pi^I(x)\lambda_I$$

Spontaneous symmetry breaking $\mathrm{SU(3)_R \times SU(3)_L \to SU(3)_D}$

$$U(x) \to LU(x)R^\dagger$$

Most general low energy invariant Lagrangian:

$$\mathcal{L} = f^2\,\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U) + L_4\left[\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U)\right]^2 + L_5\left[\mathrm{tr}(\partial_\mu U^\dagger \partial_\nu U)\right]^2 + \cdots$$

There is also a term which is only invariant up to a total derivative: the Wess-Zumino-Witten term

$$\sim \lambda \int_{B_5} d^5x\ \epsilon^{ijklm} Tr\left[U^\dagger\partial_i U\ U^\dagger\partial_j U\ U^\dagger\partial_k U\ U^\dagger\partial_l U\ U^\dagger\partial_m U\right]$$

Coupling is quantized for topological reasons

Small field

$$\lambda \int_{B_5} d^5x\ \epsilon^{ijklm} Tr\left[\partial_i B \partial_j B \partial_k B \partial_l B \partial_m B\right] \to \lambda \int d^4x\ \epsilon^{\mu\nu\alpha\beta} Tr\left[B\partial_\mu B\partial_\nu B\partial_\alpha B\partial_\beta B\right]$$

WZW term has fewer derivatives per field than the other terms, and is not renormalized. Topological quantization of the coupling gives a deeper reason for the non-renormalization theorem.

## Galileons as Wess-Zumino terms

Garret Goon, KH, Austin Joyce, Mark Trodden (2012)

The galileons are boundary values of invariant 5-forms which can't be written as the derivative of invariant 4-forms

Problem in Lie algebra cohomology

Galileons are classified by non-trivial elements of $H^5\left(\mathfrak{Gal}(3+1,1), \mathfrak{so}(3,1)\right)$

$$\omega_1^{\text{wz}} = \epsilon_{\mu\nu\rho\sigma}\, \omega_C \wedge \omega_P^\mu \wedge \omega_P^\nu \wedge \omega_P^\rho \wedge \omega_P^\sigma$$
$$\omega_2^{\text{wz}} = \epsilon_{\mu\nu\rho\sigma}\, \omega_C \wedge \omega_B^\mu \wedge \omega_P^\nu \wedge \omega_P^\rho \wedge \omega_P^\sigma$$
$$\omega_3^{\text{wz}} = \epsilon_{\mu\nu\rho\sigma}\, \omega_C \wedge \omega_B^\mu \wedge \omega_B^\nu \wedge \omega_P^\rho \wedge \omega_P^\sigma$$
$$\omega_4^{\text{wz}} = \epsilon_{\mu\nu\rho\sigma} \omega_C \wedge \omega_B^\mu \wedge \omega_B^\nu \wedge \omega_B^\rho \wedge \omega_P^\sigma$$
$$\omega_5^{\text{wz}} = \epsilon_{\mu\nu\rho\sigma} \omega_C \wedge \omega_B^\mu \wedge \omega_B^\nu \wedge \omega_B^\rho \wedge \omega_B^\sigma$$

# Generalizations of the galileons

The galileon idea is quite general: can be generalized in many directions and occurs in many contexts

(Review: KH & Mark Trodden arXiv:1104.2088)

# Other kinds of galileons: relativistic DBI galileons

de Rham, Tolley (2010)

The internal galilean symmetry can be made relativistic:

$$\delta\pi = c + b_\mu x^\mu \quad \longrightarrow \quad \delta\pi = c + b_\mu x^\mu - b^\mu \pi \partial_\mu \pi$$

Combines with spacetime Poincare(3,1) to form Poincare(4,1)

$$\delta\pi = -\omega^\mu{}_\nu x^\nu \partial_\mu \pi - \epsilon^\mu \partial_\mu \pi + c + \underbrace{b_\mu x^\mu - b^\mu \pi \partial_\mu \pi}$$

Ordinary boosts and translations     5th translation and boosts

Poincare(3,1) subgroup is represented linearly, the rest is not, so we have spontaneous breaking Poincare(4,1)$\rightarrow$ Poincare(3,1)

D possible terms in D dimensions which give second order EOM. Recover galileons in the small field limit.

$$\mathcal{L}_2 = -\sqrt{1+(\partial\pi)^2}\ ,$$

DBI kinetic term

$$\mathcal{L}_3 = -[\Pi] + \gamma^2 [\pi^3]\ ,$$

$$\mathcal{L}_4 = -\gamma\left([\Pi]^2 - [\Pi^2]\right) - 2\gamma^3\left([\pi^4] - [\Pi][\pi^3]\right)\ ,$$

$$\mathcal{L}_5 = -\gamma^2\left([\Pi]^3 + 2[\Pi^3] - 3[\Pi][\Pi^2]\right) - \gamma^4\left(6[\Pi][\pi^4] - 6[\pi^5] - 3\left([\Pi]^2 - [\Pi^2]\right)[\pi^3]\right)$$

$$\gamma = \frac{1}{\sqrt{1+(\partial\pi)^2}} \qquad \Pi_{\mu\nu} \equiv \partial_\mu\partial_\nu\pi, \quad [\Pi^n] \equiv Tr(\Pi^n), \quad [\pi^n] \equiv \partial\pi\cdot\Pi^{n-2}\cdot\partial\pi$$

# Conformal Galileons

Nicolis, Rattazzi, Trincherini (2008)

$$\delta\pi = c \rightarrow \delta\pi = c - cx^\mu \partial_\mu \pi$$

$$\delta\pi = b_\mu x^\mu \;\; \rightarrow \;\; \delta\pi = b_\mu x^\mu + \partial_\mu \pi \left( \frac{1}{2} b^\mu x^2 - (b \cdot x) x^\mu \right)$$

Combines with spacetime Poincare(3,1) to form conformal group SO(4,2)

Spontaneous breaking SO(4,2)$\rightarrow$ Poincare(3,1)

Again, D possible terms in D dimensions which give second order EOM

$$\begin{aligned}
\mathcal{L}_2 &= -\frac{1}{2} e^{-2\hat{\pi}} (\partial\hat{\pi})^2 \\
\mathcal{L}_3 &= \frac{1}{2} (\partial\hat{\pi})^2 \Box\hat{\pi} - \frac{1}{4} (\partial\hat{\pi})^4 \\
\mathcal{L}_4 &= \frac{1}{20} e^{2\hat{\pi}} (\partial\hat{\pi})^2 \left( 10([\hat{\Pi}]^2 - [\hat{\Pi}^2]) + 4((\partial\hat{\pi})^2 \Box\hat{\pi} - [\hat{\phi}]) + 3(\partial\hat{\pi})^4 \right) \\
\mathcal{L}_5 &= e^{4\hat{\pi}} (\partial\hat{\pi})^2 \Big[ \frac{1}{3} ([\hat{\Pi}]^3 + 2[\hat{\Pi}^3] - 3[\hat{\Pi}][\hat{\Pi}^2]) + (\partial\hat{\pi})^2 ([\hat{\Pi}]^2 - [\hat{\Pi}^2]) \\
&\quad + \frac{10}{7} (\partial\hat{\pi})^2 ((\partial\hat{\pi})^2 [\hat{\Pi}] - [\hat{\phi}]) - \frac{1}{28} (\partial\hat{\pi})^6 \Big]
\end{aligned}$$

# Conformal Galileons: $L_3$ is Wess-Zumino

$$\mathcal{L}_3 \;=\; \frac{1}{2}(\partial\hat{\pi})^2\Box\hat{\pi} - \frac{1}{4}(\partial\hat{\pi})^4$$

- "Dilaton effective action" from proof of the a-theorem in 4-d — Komargodski, Schwimmer (2011)

- The basis of "galilean genesis" scenario as an alternative to inflation, and its generalizations

Creminelli, Nicolis, Trincherini (2010)

KH, Justin Khoury (2011)

# Covariant galileons

Deffayet, Esposito-Farese, Gilles, Vikman (2009)

Deffayet, Deser, Esposito-Farese (2009)

Coupling the galileons to curved space: $\partial \to \nabla$

- Minimal coupling ruins the second-order equations of motion.

- Adding non-minimal terms restores second order equations:

$$\mathcal{L}_2 = \sqrt{-g}\left(\pi_{;\lambda}\pi^{;\lambda}\right)$$

Non-minimal terms

$$\mathcal{L}_3 = \sqrt{-g}\left(\pi_{;\lambda}\pi^{;\lambda}\right)\Box\pi$$

$$\mathcal{L}_4 = \sqrt{-g}\left(\pi_{;\lambda}\,\pi^{;\lambda}\right)\left[2\,(\Box\pi)^2 - 2\,(\pi_{;\mu\nu}\,\pi^{;\mu\nu}) - \frac{1}{2}\,(\pi_{;\mu}\,\pi^{;\mu})\,R\right]$$

$$\mathcal{L}_5 = \sqrt{-g}\left(\pi_{;\lambda}\,\pi^{;\lambda}\right)\left[(\Box\pi)^3 - 3\,(\Box\pi)\,(\pi_{;\mu\nu}\,\pi^{;\mu\nu}) + 2\left(\pi_{;\mu}{}^{\nu}\,\pi_{;\nu}{}^{\rho}\,\pi_{;\rho}{}^{\mu}\right) - 6\,(\pi_{;\mu}\,\pi^{;\mu\nu}\,G_{\nu\rho}\,\pi^{;\rho})\right]$$

# Covariant galileons

Deffayet, Esposito-Farese, Gilles, Vikman (2009)

Deffayet, Deser, Esposito-Farese (2009)

Coupling the galileons to curved space: $\partial \to \nabla$

- Minimal coupling ruins the second-order equations of motion.

- Adding non-minimal terms restores second order equations:

Non-minimal terms

$$\mathcal{L}_2 = \sqrt{-g}\left(\pi_{;\lambda}\pi^{;\lambda}\right)$$

$$\mathcal{L}_3 = \sqrt{-g}\left(\pi_{;\lambda}\pi^{;\lambda}\right)\Box\pi$$

$$\mathcal{L}_4 = \sqrt{-g}\left(\pi_{;\lambda}\,\pi^{;\lambda}\right)\left[2\left(\Box\pi\right)^2 - 2\left(\pi_{;\mu\nu}\,\pi^{;\mu\nu}\right) - \frac{1}{2}\left(\pi_{;\mu}\,\pi^{;\mu}\right)R\right]$$

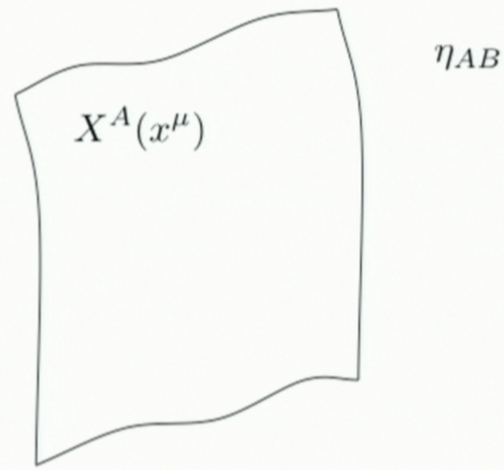
$$\mathcal{L}_5 = \sqrt{-g}\left(\pi_{;\lambda}\,\pi^{;\lambda}\right)\left[\left(\Box\pi\right)^3 - 3\left(\Box\pi\right)\left(\pi_{;\mu\nu}\,\pi^{;\mu\nu}\right) + 2\left(\pi_{;\mu}{}^{\nu}\,\pi_{;\nu}{}^{\rho}\,\pi_{;\rho}{}^{\mu}\right) - 6\left(\pi_{;\mu}\,\pi^{;\mu\nu}\,G_{\nu\rho}\,\pi^{;\rho}\right)\right]$$

- Breaks Galilean symmetry (but preserves shift symmetry)

# Probe brane construction

de Rham, Tolley (2010)

3-brane embedded in 5-d Minkowski




Action should be invariant under bulk Poincare, and reparametrizations of the brane worldsheet:

$$\begin{aligned}\delta_P X^A &= \omega^A{}_B X^B + \epsilon^A \\ \delta_g X^A &= \xi^\mu \partial_\mu X^A\end{aligned}$$

Fix gauge with a "unitary" gauge:

$$X^\mu(x) = x^\mu, \quad X^5(x) \equiv \pi(x)$$

Poincare transformations do not preserve the gauge $\delta_P X^\mu = \omega^\mu{}_\nu x^\nu + \omega^\mu{}_5 \pi + \epsilon^\mu$

but gauge can be restored with a compensating gauge transformation with $\xi^\mu = -\omega^\mu{}_\nu x^\nu - \omega^\mu{}_5 \pi - \epsilon^\mu$

Combined transformation is

$$\delta_{P'}\pi = \delta_P \pi + \delta_g \pi = -\omega^\mu{}_\nu x^\nu \partial_\mu \pi - \epsilon^\mu \partial_\mu \pi + \omega^5{}_\mu x^\mu - \omega^\mu{}_5 \pi \partial_\mu \pi + \epsilon^5$$

P(4,1)→P(3,1) symmetry breaking

# Probe brane construction

Actions are constructed from diff invariants of the intrinsic quantities on the brane:

Induced metric $\quad g_{\mu\nu} \equiv \frac{\partial X^A}{\partial x^\mu}\frac{\partial X^B}{\partial x^\nu}\eta_{AB} \xrightarrow[\text{gauge } X^\mu(x)=x^\mu]{} g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu\pi\partial_\nu\pi$

extrinsic curvature $K_{\mu\nu}$

covariant derivative $\nabla_\mu$

intrinsic curvature $R^\rho{}_{\sigma\mu\nu}$

Most general action with relativistic DBI symmetry:

$$S = \int d^4x\,\sqrt{-g}F\left(g_{\mu\nu}, \nabla_\mu, R^\rho{}_{\sigma\mu\nu}, K_{\mu\nu}\right)\Big|_{g_{\mu\nu}=\eta_{\mu\nu}+\partial_\mu\pi\partial_\nu\pi}$$

Small field limit will have galilean symmetry.

Example: DBI term

$$\int d^4x\,\sqrt{-g} \to \int d^4x\,\sqrt{1+(\partial\pi)^2} \to \int d^4x\,\frac{1}{2}(\partial\pi)^2$$

# Probe brane construction

Possible terms in four dimensions:

$$
\begin{aligned}
\mathcal{L}_2 &= \sqrt{-g} \\
\mathcal{L}_3 &= \sqrt{-g}K \\
\mathcal{L}_4 &= \sqrt{-g}\,R \\
\mathcal{L}_5 &= \sqrt{-g}\,\mathcal{K}_{GB}
\end{aligned}
$$

Bulk Einstein-Hilbert

Bulk Gauss-Bonnet

Flat brane and bulk

$$g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu\pi\partial_\nu\pi$$

DBI galileons

$$
\begin{aligned}
\mathcal{L}_2 &= -\sqrt{1+(\partial\pi)^2} \\
\mathcal{L}_3 &= -[\Pi] + \gamma^2\,[\pi^3] \\
\mathcal{L}_4 &= -\gamma\left([\Pi]^2 - [\Pi^2]\right) - 2\gamma^3\left([\pi^4] - [\Pi]\,[\pi^3]\right) \\
\mathcal{L}_5 &= -\gamma^2\left([\Pi]^3 + 2\,[\Pi^3] - 3\,[\Pi]\,[\Pi^2]\right) \\
&\quad -\gamma^4\left(6\,[\Pi]\,[\pi^4] - 6\,[\pi^5] - 3\left([\Pi]^2 - [\Pi^2]\right)[\pi^3]\right)
\end{aligned}
$$

Small field limit

regular galileons

$$
\begin{aligned}
\mathcal{L}_2 &= -\frac{1}{2}(\partial\pi)^2\,, \\
\mathcal{L}_3 &= -\frac{1}{2}(\partial\pi)^2[\Pi]\,, \\
\mathcal{L}_4 &= -\frac{1}{2}(\partial\pi)^2\left([\Pi]^2 - [\Pi^2]\right)\,, \\
\mathcal{L}_5 &= -\frac{1}{2}(\partial\pi)^2\left([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]\right)\,.
\end{aligned}
$$

AdS bulk, flat brane
Small derivative limit

Conformal galileons

$$
\begin{aligned}
\mathcal{L}_2 &= -\frac{1}{2}e^{-2\hat\pi}(\partial\hat\pi)^2 \\
\mathcal{L}_3 &= \frac{1}{2}(\partial\hat\pi)^2\Box\hat\pi - \frac{1}{4}(\partial\hat\pi)^4 \\
\mathcal{L}_4 &= \frac{1}{20}e^{2\hat\pi}(\partial\hat\pi)^2\left(10([\hat\Pi]^2 - [\hat\Pi^2]) + 4((\partial\hat\pi)^2\Box\hat\pi - [\hat\phi]) + 3(\partial\hat\pi)^4\right) \\
\mathcal{L}_5 &= e^{4\hat\pi}(\partial\hat\pi)^2\Big[\frac{1}{3}([\hat\Pi]^3 + 2[\hat\Pi^3] - 3[\hat\Pi][\hat\Pi^2]) + (\partial\hat\pi)^2([\hat\Pi]^2 - [\hat\Pi^2]) \\
&\quad + \frac{10}{7}(\partial\hat\pi)^2((\partial\hat\pi)^2[\hat\Pi] - [\hat\phi]) - \frac{1}{28}(\partial\hat\pi)^6\Big]
\end{aligned}
$$

# Probe brane construction

Possible terms in four dimensions:

$$
\begin{aligned}
\mathcal{L}_2 &= \sqrt{-g} \\
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\end{aligned}
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Bulk Einstein-Hilbert

Bulk Gauss-Bonnet

Flat brane and bulk

$$g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu\pi\partial_\nu\pi$$

DBI galileons

$$
\begin{aligned}
\mathcal{L}_2 &= -\sqrt{1+(\partial\pi)^2} \\
\mathcal{L}_3 &= -[\Pi] + \gamma^2\,[\pi^3] \\
\mathcal{L}_4 &= -\gamma\left([\Pi]^2 - [\Pi^2]\right) - 2\gamma^3\left([\pi^4] - [\Pi]\,[\pi^3]\right) \\
\mathcal{L}_5 &= -\gamma^2\left([\Pi]^3 + 2\,[\Pi^3] - 3\,[\Pi]\,[\Pi^2]\right) \\
&\quad -\gamma^4\left(6\,[\Pi]\,[\pi^4] - 6\,[\pi^5] - 3\left([\Pi]^2 - [\Pi^2]\right)[\pi^3]\right)
\end{aligned}
$$

Small field limit

regular galileons

$$
\begin{aligned}
\mathcal{L}_2 &= -\frac{1}{2}(\partial\pi)^2\,, \\
\mathcal{L}_3 &= -\frac{1}{2}(\partial\pi)^2[\Pi]\,, \\
\mathcal{L}_4 &= -\frac{1}{2}(\partial\pi)^2\left([\Pi]^2 - [\Pi^2]\right)\,, \\
\mathcal{L}_5 &= -\frac{1}{2}(\partial\pi)^2\left([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]\right)\,.
\end{aligned}
$$

AdS bulk, flat brane
Small derivative limit

General brane metric
Small field limit

Conformal galileons

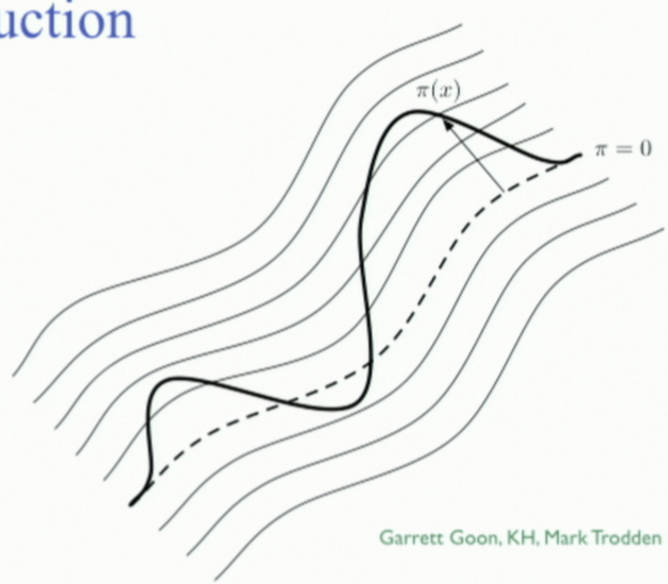
$$
\begin{aligned}
\mathcal{L}_2 &= -\frac{1}{2}e^{-2\hat\pi}(\partial\hat\pi)^2 \\
\mathcal{L}_3 &= \frac{1}{2}(\partial\hat\pi)^2\Box\hat\pi - \frac{1}{4}(\partial\hat\pi)^4 \\
\mathcal{L}_4 &= \frac{1}{20}e^{2\hat\pi}(\partial\hat\pi)^2\left(10([\hat\Pi]^2 - [\hat\Pi^2]) + 4((\partial\hat\pi)^2\Box\hat\pi - [\hat\phi]) + 3(\partial\hat\pi)^4\right) \\
\mathcal{L}_5 &= e^{4\hat\pi}(\partial\hat\pi)^2\Big[\frac{1}{3}([\hat\Pi]^3 + 2[\hat\Pi^3] - 3[\hat\Pi][\hat\Pi^2]) + (\partial\hat\pi)^2([\hat\Pi]^2 - [\hat\Pi^2]) \\
&\quad + \frac{10}{7}(\partial\hat\pi)^2((\partial\hat\pi)^2[\hat\Pi] - [\hat\phi]) - \frac{1}{28}(\partial\hat\pi)^6\Big]
\end{aligned}
$$

covariant galileons

$$
\begin{aligned}
\mathcal{L}_2 &= \sqrt{-g}\,(\pi_{;\lambda}\pi^{;\lambda}) \\
\mathcal{L}_3 &= \sqrt{-g}\,(\pi_{;\lambda}\pi^{;\lambda})\,\Box\pi \\
\mathcal{L}_4 &= \sqrt{-g}\,(\pi_{;\lambda}\pi^{;\lambda})\left[2\,(\Box\pi)^2 - 2\,(\pi_{;\mu\nu}\pi^{;\mu\nu}) - \frac{1}{2}\,(\pi_{;\mu}\pi^{;\mu})\,R\right] \\
\mathcal{L}_5 &= \sqrt{-g}\,(\pi_{;\lambda}\pi^{;\lambda})\left[(\Box\pi)^3 - 3\,(\Box\pi)\,(\pi_{;\mu\nu}\pi^{;\mu\nu}) + 2\,({\pi_{;\mu}}^{\nu}{\pi_{;\nu}}^{\rho}{\pi_{;\rho}}^{\mu}) - 6\,(\pi_{;\mu}\pi^{;\mu\nu}G_{\nu\rho}\pi^{;\rho})\right]
\end{aligned}
$$

# The general construction

General bulk metric: $G_{AB}(X)$




Garrett Goon, KH, Mark Trodden (2011)

Gauge is fixed with respect to a chosen foliation

$$X^\mu(x) = x^\mu, \quad X^5(x) \equiv \pi(x)$$

If the bulk metric has Killing vectors: $\delta_K X^A = a^i K_i^A(X) + a^I K_I^A(X)$

Galileons will have a corresponding global symmetry: $(\delta_K + \delta_{g,\mathrm{comp}})\pi = -a^i k_i^\mu(x)\partial_\mu \pi + a^I K_I^5(x,\pi) - a^I K_I^\mu(x,\pi)\partial_\mu \pi$

Possible maximally symmetric galileons correspond to possible foliations of maximally symmetric spaces by maximally symmetric hypersurfaces.

# More general backgrounds

Garrett Goon, KH, Mark Trodden (2011)
Burrage, de Rham, Heisenberg (2011)

Possible maximally symmetric galileons correspond to possible foliations of maximally symmetric spaces by maximally symmetric hypersurfaces:

| Ambient space: | Brane space: AdS | Brane space: Flat | Brane space: dS |
|---|---|---|---|
| AdS | AdS galileons | Conformal galileons | dS galileons type I |
| Flat | X | Original galileons | dS galileons type II |
| dS | X | X | dS galileons type III |

# dS/AdS galileons

The symmetric galileon in curved space acquires a potential

de Sitter:

Tachyonic mass term (fixed by symmetry)

$$\mathcal{L}_2 = -\frac{1}{2}\sqrt{-g}\left((\partial\hat{\pi})^2 - \frac{4}{L^2}\hat{\pi}^2\right),$$

$$\mathcal{L}_3 = \sqrt{-g}\left([\hat{\pi}^3] - \frac{3}{L^2}(\partial\hat{\pi})^2\hat{\pi} + \frac{4}{L^4}\hat{\pi}^3\right),$$

$$\mathcal{L}_4 = \sqrt{-g}\left[-\frac{1}{2}(\partial\hat{\pi})^2\left([\hat{\Pi}]^2 - [\hat{\Pi}^2] + \frac{1}{2L^2}(\partial\hat{\pi})^2 + \frac{6}{L^2}\hat{\pi}[\hat{\Pi}] + \frac{18}{L^4}\hat{\pi}^2\right) + \frac{6}{L^6}\hat{\pi}^4\right],$$

$$\vdots$$

$$\delta_+\pi = \frac{1}{u}\left(u^2 - y^2\right), \quad \delta_-\pi = -\frac{1}{u}, \quad \delta_i\pi = \frac{y_i}{u}$$

Anti de Sitter:

Normal mass term

$$\mathcal{L}_2 = -\frac{1}{2}\sqrt{-g}\left((\partial\hat{\pi})^2 + \frac{4}{L^2}\hat{\pi}^2\right),$$

$$\mathcal{L}_3 = \sqrt{-g}\left([\hat{\pi}^3] + \frac{3}{L^2}(\partial\hat{\pi})^2\hat{\pi} + \frac{4}{L^4}\hat{\pi}^3\right),$$

$$\mathcal{L}_4 = \sqrt{-g}\left[-\frac{1}{2}(\partial\hat{\pi})^2\left([\hat{\Pi}]^2 - [\hat{\Pi}^2] - \frac{1}{2L^2}(\partial\hat{\pi})^2 - \frac{6}{L^2}\hat{\pi}[\hat{\Pi}] + \frac{18}{L^4}\hat{\pi}^2\right) - \frac{6}{L^6}\hat{\pi}^4\right],$$

$$\vdots$$

$$\delta_+\pi = \frac{1}{u}\left(u^2 + x^2\right), \quad \delta_-\pi = -\frac{1}{u}, \quad \delta_i\pi = \frac{x_i}{u}$$

# dS/AdS galileons

The symmetric galileon in curved space acquires a potential

de Sitter:

Tachyonic mass term (fixed by symmetry)

$$
\begin{aligned}
\mathcal{L}_2 &= -\frac{1}{2}\sqrt{-g}\left((\partial\hat{\pi})^2 - \frac{4}{L^2}\hat{\pi}^2\right), \\
\mathcal{L}_3 &= \sqrt{-g}\left([\hat{\pi}^3] - \frac{3}{L^2}(\partial\hat{\pi})^2\hat{\pi} + \frac{4}{L^4}\hat{\pi}^3\right), \\
\mathcal{L}_4 &= \sqrt{-g}\left[-\frac{1}{2}(\partial\hat{\pi})^2\left([\hat{\Pi}]^2 - [\hat{\Pi}^2] + \frac{1}{2L^2}(\partial\hat{\pi})^2 + \frac{6}{L^2}\hat{\pi}[\hat{\Pi}] + \frac{18}{L^4}\hat{\pi}^2\right) + \frac{6}{L^6}\hat{\pi}^4\right], \\
&\vdots
\end{aligned}
$$

$$\delta_+\pi = \frac{1}{u}\left(u^2 - y^2\right), \quad \delta_-\pi = -\frac{1}{u}, \quad \delta_i\pi = \frac{y_i}{u}$$

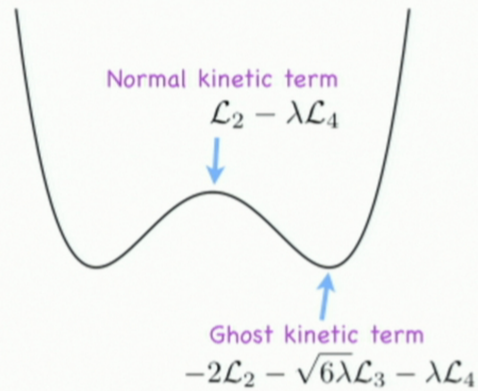
Anti de Sitter:

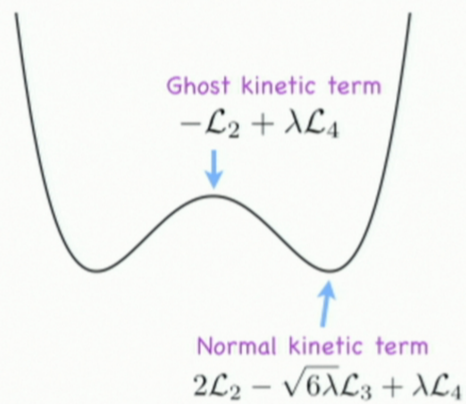
Normal mass term

$$
\begin{aligned}
\mathcal{L}_2 &= -\frac{1}{2}\sqrt{-g}\left((\partial\hat{\pi})^2 + \frac{4}{L^2}\hat{\pi}^2\right), \\
\mathcal{L}_3 &= \sqrt{-g}\left([\hat{\pi}^3] + \frac{3}{L^2}(\partial\hat{\pi})^2\hat{\pi} + \frac{4}{L^4}\hat{\pi}^3\right), \\
\mathcal{L}_4 &= \sqrt{-g}\left[-\frac{1}{2}(\partial\hat{\pi})^2\left([\hat{\Pi}]^2 - [\hat{\Pi}^2] - \frac{1}{2L^2}(\partial\hat{\pi})^2 - \frac{6}{L^2}\hat{\pi}[\hat{\Pi}] + \frac{18}{L^4}\hat{\pi}^2\right) - \frac{6}{L^6}\hat{\pi}^4\right], \\
&\vdots
\end{aligned}
$$

$$\delta_+\pi = \frac{1}{u}\left(u^2 + x^2\right), \quad \delta_-\pi = -\frac{1}{u}, \quad \delta_i\pi = \frac{x_i}{u}$$

Imposing $Z_2$ symmetry



# FRW Cosmological backgrounds

Garrett Goon, KH, Mark Trodden (2011)

The brane can be a general FRW space-time with arbitrary scale factor $a(t)$

$$
\begin{aligned}
\mathcal{L}_2 &= -\sqrt{-f}\frac{1}{\gamma}, \\
\mathcal{L}_3 &= \sqrt{-f}\Big[-\langle\Pi\rangle + \frac{1}{2}\langle f'\rangle + \gamma^2\left(\langle\pi\Pi\pi\rangle + \frac{1}{2}\langle\pi f'\pi\rangle\right)\Big], \\
\mathcal{L}_4 &= \sqrt{-f}\Big[-\frac{1}{2}\langle\pi f'\pi\rangle^2\gamma^3 - \langle f'\rangle\langle\pi\Pi\pi\rangle\gamma^3 - 2\langle\pi\Pi^2\pi\rangle\gamma^3 + 2\langle\pi\Pi\pi\rangle\langle\Pi\rangle\gamma^3 \\
&\quad - \frac{1}{2}\langle f'\rangle\langle\pi f'\pi\rangle\gamma^3 + \langle\Pi\rangle\langle\pi f'\pi\rangle\gamma^3 - \frac{\langle f'\rangle^2\gamma}{4} - \langle\Pi\rangle^2\gamma + \frac{\langle f'^2\rangle\gamma}{4} \\
&\quad - \langle\Pi f'\rangle\gamma + \langle f'\rangle\langle\Pi\rangle\gamma + \langle\Pi^2\rangle\gamma + \frac{\langle\pi f'^2\pi\rangle\gamma}{2}\Big], \\
\mathcal{L}_5 &= \sqrt{-f}\Big[3\langle\pi\Pi\pi\rangle\langle\Pi\rangle^2\gamma^4 + \frac{3}{4}\langle f'\rangle\langle\pi f'\pi\rangle^2\gamma^4 - \frac{3}{2}\langle\Pi\rangle\langle\pi f'\pi\rangle^2\gamma^4 + \frac{3}{4}\langle f'\rangle^2\langle\pi\Pi\pi\rangle\gamma^4 \\
&\quad - \frac{3}{4}\langle f'^2\rangle\langle\pi\Pi\pi\rangle\gamma^4 + 3\langle\Pi f'\rangle\langle\pi\Pi\pi\rangle\gamma^4 + 6\langle\pi\Pi^3\pi\rangle\gamma^4 + 3\langle f'\rangle\langle\pi\Pi^2\pi\rangle\gamma^4 \\
&\quad - 3\langle f'\rangle\langle\pi\Pi\pi\rangle\langle\Pi\rangle\gamma^4 - 6\langle\pi\Pi^2\pi\rangle\langle\Pi\rangle\gamma^4 - 3\langle\pi\Pi\pi\rangle\langle\Pi^2\rangle\gamma^4 + \frac{3}{8}\langle f'\rangle^2\langle\pi f'\pi\rangle\gamma^4 \\
&\quad + \frac{3}{2}\langle\Pi\rangle^2\langle\pi f'\pi\rangle\gamma^4 - \frac{3}{8}\langle f'^2\rangle\langle\pi f'\pi\rangle\gamma^4 + \frac{3}{2}\langle\Pi f'\rangle\langle\pi f'\pi\rangle\gamma^4 \\
&\quad - \frac{3}{2}\langle f'\rangle\langle\Pi\rangle\langle\pi f'\pi\rangle\gamma^4 - \frac{3}{2}\langle\Pi^2\rangle\langle\pi f'\pi\rangle\gamma^4 - \frac{3}{2}\langle\pi\Pi\pi\rangle\langle\pi f'^2\pi\rangle\gamma^4 \\
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&\quad + \frac{\langle f'\rangle^3\gamma^2}{8} - \langle\Pi\rangle^3\gamma^2 + \frac{3}{2}\langle f'\rangle\langle\Pi\rangle^2\gamma^2 - \frac{3}{8}\langle f'\rangle\langle f'^2\rangle\gamma^2 + \frac{\langle f'^3\rangle\gamma^2}{4} \\
&\quad + \frac{3}{2}\langle f'\rangle\langle\Pi f'\rangle\gamma^2 - \frac{3\langle\Pi f'^2\rangle\gamma^2}{2} - \frac{3\langle\Pi f'\Pi f'\pi\rangle\gamma^2}{2} - \frac{3}{4}\langle f'\rangle^2\langle\Pi\rangle\gamma^2 \\
&\quad + \frac{3}{4}\langle f'^2\rangle\langle\Pi\rangle\gamma^2 - 3\langle\Pi f'\rangle\langle\Pi\rangle\gamma^2 - 2\langle\Pi^3\rangle\gamma^2 - \frac{3}{2}\langle f'\rangle\langle\Pi^2\rangle\gamma^2 \\
&\quad + 3\langle\Pi\rangle\langle\Pi^2\rangle\gamma^2 + 3\langle\pi f'\pi\rangle\gamma^2 - \frac{3}{4}\langle f'\rangle\langle\pi f'^2\pi\rangle\gamma^2 + \frac{3}{2}\langle\Pi\rangle\langle\pi f'^2\pi\rangle\gamma^2 + \frac{3\langle\pi f'^3\pi\rangle\gamma^2}{4}\Big].
\end{aligned}
$$

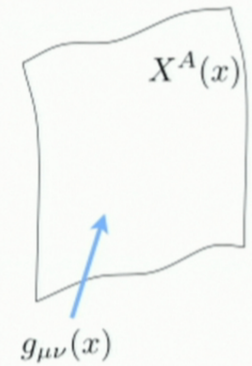
$$
\begin{aligned}
\delta_{v_i}\pi &= \frac{1}{2}x^i\dot a\int dt\,\frac{\dot H}{H^3 a} - \frac{x^i\left(a - \dot a\pi + \dot a^2\int dt\,\frac{\dot H}{H^3 a}\right)}{2\dot a - 2\pi\ddot a}\dot\pi \\
&\quad + \left[\frac{x^i x^i\dot a^2 + 1}{4\dot a^2} + \frac{\int dt\,\frac{\dot H}{H^3 a}}{2a - 2\pi\dot a}\right]\partial_i\pi - \sum_{j\neq i}\left[-\frac{x^i x^j}{2}\partial_j\pi + \frac{x^j x^j}{4}\partial_i\pi\right], \\
\delta_{k_i}\pi &= x^i\dot a\left(\frac{\dot a\dot\pi}{\dot a - \pi\ddot a} - 1\right) - \frac{\partial_i\pi}{a - \pi\dot a}, \\
\delta_q\pi &= \frac{\dot\pi\dot a^2}{\pi\ddot a - \dot a} + \dot a, \\
\delta_u\pi &= \frac{x^2\dot a^2 - 1}{4\dot a} - \frac{x^2\dot a^2 + 1}{4\dot a - 4\pi\ddot a}\dot\pi + \frac{1}{2a - 2\pi\dot a}\sum_i x^i\partial_i\pi, \\
\delta_s\pi &= -\dot a\int dt\,\frac{\dot H}{H^3 a} + \frac{\left(a - \dot a\pi + \dot a^2\int dt\,\frac{\dot H}{H^3 a}\right)\dot\pi}{\dot a - \pi\ddot a} - \sum x^i\partial_i\pi,
\end{aligned}
$$

Invariant under these non-linear symmetries

# Coupling to ghost-free massive gravity

Gregory Gabadadze, KH, Justin Khoury, David Pirtskhalava, Mark Trodden (2012)

By adding a dynamical metric to the brane world-volume, we obtain a theory of massive gravity coupled to galileons:




- Massive scalar/tensor theory
- Preserves the galileon symmetries (now acts non-trivially on the metric)
- Still ghost-free

Decoupling limit now has galileon-like theories with infinite number of terms:

$$M_P^2 m^2 \left[ \frac{1}{2} \left( \partial^\mu \partial_\nu \phi - \delta^\mu{}_\nu \Box \phi \right) \frac{1}{\delta^\nu{}_\lambda - \partial^\nu \partial_\lambda \phi} \partial^\lambda \pi \partial_\mu \pi - \frac{1}{2} \det \left( \delta^\alpha{}_\beta - \partial^\alpha \partial_\beta \phi \right) \frac{1}{\left( \delta^\mu{}_\nu - \partial^\mu \partial_\nu \phi \right)^2} \partial^\nu \pi \partial_\mu \pi \right] + \text{massive gravity terms}$$

Longitudinal mode of massive graviton

Galileon scalar field

# Higher co-dimensions/multi-field galileons

Deffayet, Deser, Esposito-Farese (2010)
Padilla, Saffin, Zhou (2010)
KH, Mark Trodden, Dan Wesely (2010)

N-field model can be constructed from a 3-brane embedded in 4+N dimensional Minkowski

$$X^\mu(x) = x^\mu, \quad X^I(x) \equiv \pi^I(x)$$

KH, Mark Trodden, Dan Wesely (2010)

$\eta_{AB}$

$X^A(x^\mu)$

Induced metric $g_{\mu\nu} \equiv \frac{\partial X^A}{\partial x^\mu}\frac{\partial X^B}{\partial x^\nu}\eta_{AB} \xrightarrow[\text{gauge } X^\mu(x)=x^\mu]{} g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \pi^I \partial_\nu \pi_I$

Most general action with relativistic DBI symmetry:

extrinsic curvature $K^i_{\mu\nu}$

covariant derivative $\nabla_\mu$

intrinsic curvature $R^\rho{}_{\sigma\mu\nu}$

Twist connection $\beta^j_{\mu i}$

Curvature of normal bundle $R^i{}_{j\mu\nu}$

$$S = \int d^4x\, \sqrt{-g} F\left(g_{\mu\nu}, \nabla_\mu, R^i{}_{j\mu\nu}, R^\rho{}_{\sigma\mu\nu}, K^i_{\mu\nu}\right)\Big|_{g_{\mu\nu}=\eta_{\mu\nu}+\partial_\mu\pi^I\partial_\nu\pi_I}$$

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Brane Einstein-Hibert term gives the unique multi-galileon term:

$$\int d^4x\, \sqrt{-g}\,(-a_2 + a_4 R) \rightarrow \int d^4x\, \left[-a_2\, \frac{1}{2}\partial_\mu\pi^I\partial^\mu\pi_I + a_4\, \partial_\mu\pi^I\partial_\nu\pi^J\left(\partial_\lambda\partial^\mu\pi_J\partial^\lambda\partial^\nu\pi_I - \partial^\mu\partial^\nu\pi_I\Box\pi_J\right)\right]$$

# p-form galileons

Deffayet, Deser, Esposito-Farese (2010)

Actions for p-form fields, with purely second order equations of motion: $\omega_{\lambda\mu\nu\ldots} = \partial_{[\lambda} A_{\mu\nu\ldots]}$

$$\eta^{\mu\alpha\,\nu\beta\ldots}\omega_{\mu\nu\ldots}\omega_{\alpha\beta\ldots}(\partial_\rho\omega_{\gamma\delta\ldots}\ \ldots)(\partial_\epsilon\omega_{\sigma\tau\ldots}\ \ldots).$$

Galilean symmetry: $\delta A_{\mu\nu\cdots} = c_{\mu\nu\cdots}$

$\delta\omega_{\mu\nu\lambda\cdots} = b_{\mu\nu\lambda\cdots}$

Only works for even p, and in certain dimensions. First non-trivial case is 2 forms in 7 dimensions:

$$\begin{aligned}\mathcal{L} = \quad & -9(\omega^{\mu}{}_{\nu\rho,\sigma}\omega^{\sigma\tau\varphi}\omega_{\tau\varphi\mu,\chi}\omega^{\chi\nu\rho}) - 18(\omega^{\mu}{}_{\nu}{}^{\rho}\omega_{\mu\sigma}{}^{\tau}\omega^{\varphi\chi\nu,\sigma}\omega_{\varphi\chi\tau,\rho}) \\ & -36(\omega^{\mu\nu\rho}\omega_{\rho\sigma\tau}\omega_{\mu\nu\varphi}{}^{,\sigma}\omega^{\tau\varphi\chi}{}_{,\chi}) + 6(\omega_{\mu\nu\rho}\omega^{\mu\nu\rho,\sigma}\omega_{\sigma\varphi\chi}\omega^{\varphi\chi\tau}{}_{,\tau}) + 18(\omega_{\mu\nu}{}^{\rho}\omega^{\mu\nu\sigma}\omega_{\varphi\chi\rho,\sigma}\omega^{\varphi\chi\tau}{}_{,\tau}) \\ & -3(\omega^{\mu\nu\lambda}\omega_{\rho\sigma\tau,\lambda})^2 - 9(\omega^{\mu\nu\rho}\omega_{\rho\sigma\tau,\lambda})^2 + 18(\omega_{\mu\nu\rho}\omega^{\rho\sigma\tau}{}_{,\tau})^2 + 9(\omega^{\mu\nu\rho}\omega_{\mu\nu\sigma,\tau})^2 \\ & -9(\omega_{\mu\nu\rho}\omega^{\mu\nu\sigma}{}_{,\sigma})^2 - (\omega^{\mu\nu\rho}\omega_{\mu\nu\rho,\sigma})^2 + (\omega^2)(\omega_{\mu\nu\rho,\sigma})^2 - 3(\omega^2)(\omega^{\mu\nu\rho}{}_{,\rho})^2.\end{aligned}$$

Can also have mixed form-degrees, multi-field forms: Species labels

$$\eta^{\mu\alpha\,\nu\beta\ldots}\omega^{I}_{\mu\nu\ldots}\omega^{J}_{\alpha\beta\ldots}(\partial_\rho\omega^{K}_{\gamma\delta\ldots}\ \ldots)(\partial_\epsilon\omega^{L}_{\sigma\tau\ldots}\ \ldots)$$

Example: cubic coupling of a scalar to electromagnetism in 4 dimensions

$$\mathcal{L} = 4F^{\mu\rho}F^{\nu}{}_{\rho}\pi_{,\mu\nu} - 2F^2\,\Box\pi$$

$$\begin{aligned}(F_{\mu\nu,\rho})^2 - 2(F^{\mu\nu}{}_{,\nu})^2 &= 0, \\ F^{\lambda\mu,\nu}\pi_{,\mu\nu} + F^{\mu\nu}{}_{,\nu}\pi^{,\lambda}{}_{\mu} - F^{\lambda\mu}{}_{,\mu}\Box\pi &= 0.\end{aligned}$$

# How general are second order equations?

Farlie, Govaerts, Morozov (1991)

Start with a lagrangian which is a function only of first derivatives of a field $\pi$:

$$\mathcal{L}_0 = F(\partial\pi)$$

Let $\mathcal{E}$ be the Euler lagrangian operator that gives the equations of motion

$$\mathcal{E} = \frac{\partial}{\partial\pi} - \partial_\mu \frac{\partial}{\partial(\partial_\mu\pi)} + \partial_\mu\partial_\nu \frac{\partial}{\partial(\partial_\mu\partial_\nu\pi)} - \cdots$$

The following hierarchy gives lagrangians with purely second order equations:

$$\begin{aligned}
&\mathcal{L}_0 \\
&\mathcal{L}_1 = \mathcal{L}_0\mathcal{E}\mathcal{L}_0 \\
&\mathcal{L}_2 = \mathcal{L}_0\mathcal{E}\mathcal{L}_0\mathcal{E}\mathcal{L}_0 \\
&\mathcal{L}_3 = \mathcal{L}_0\mathcal{E}\mathcal{L}_0\mathcal{E}\mathcal{L}_0\mathcal{E}\mathcal{L}_0 \\
&\vdots
\end{aligned}$$

$\leftarrow$ Euler hierarchy

Terminates after D steps in D dimensions. Last equation has invariance under $\pi \to f(\pi)$

$$F(\partial\pi) = (\partial\pi)^2 \to \text{galileons}$$
$$F(\partial\pi) = \sqrt{1+(\partial\pi)^2} \to \text{DBI galileons}$$

## Horndenski theory

Horndeski (1974)

Deffayet, Deser, Esposito-Farese (2009)

Most general scalar-tensor lagrangian with at most second order equations of motion for both tensor and scalar.

4 general functions of $\phi$ and $X \equiv (\partial\phi)^2$

$K,\ G^{(1)},\ G^{(2)},\ G^{(3)}$

$$\begin{aligned}
\mathcal{L}^{(0)} &= K\left(X,\phi\right), \\
\mathcal{L}^{(1)} &= G^{(1)}\left(X,\phi\right)\Box\phi, \\
\mathcal{L}^{(2)} &= G^{(2)}_{,X}\left(X,\phi\right)\left[\left(\Box\phi\right)^2-\left(\nabla_\mu\nabla_\nu\phi\right)^2\right]+R\,G^{(2)}\left(X,\phi\right), \\
\mathcal{L}^{(3)} &= G^{(3)}_{,X}\left(X,\phi\right)\left[\left(\Box\phi\right)^3-3\Box\phi\left(\nabla_\mu\nabla_\nu\phi\right)^2+2\left(\nabla_\mu\nabla_\nu\phi\right)^3\right]-6G_{\mu\nu}\nabla^\mu\nabla^\nu\phi\,G^{(3)}\left(X,\phi\right).
\end{aligned}$$

# Another path to galileons: soft limits in EFT

Classification of effective field theories of a single scalar

Cheung, Kampf, Novotny, Trnka (2014)

$\rho =$ Number of derivatives per field: $\quad \mathcal{L}_{(\rho)} = (\partial\phi)^2 F(\partial^m \phi^n) \qquad \rho = m/n$

$$\mathcal{A}_n \sim p^{\rho(n-2)+2}$$

$\sigma =$ Order of the soft limit: $\quad \mathcal{A} \sim \mathcal{O}(z^\sigma) \quad \text{as} \quad p_1^\mu \to z p_1^\mu$

Look for theories with *enhanced* soft limits (i.e. not obvious from power counting)

$\rho = 0 \qquad$ Trivial free theory $\qquad (\partial\phi)^2 F(\phi) \to (\partial\phi')^2$

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Look for theories with *enhanced* soft limits (i.e. not obvious from power counting)

$\rho = 0$ Trivial free theory $\quad (\partial\phi)^2 F(\phi) \to (\partial\phi')^2$

$\rho = 1$ $P(X)$ theory, $\quad X \equiv (\partial\phi)^2$

$\sigma = 2$ soft limit $\quad \Rightarrow \quad$ DBI $\quad \mathcal{L} \sim \sqrt{1 + (\partial\phi)^2}$

# A special galileon

Yes, the quartic galileon has a higher shift symmetry:

KH, Austin Joyce (2015)

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{12\Lambda^6}(\partial\phi)^2\left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right]$$

$$\delta\phi = s_{\mu\nu}x^\mu x^\nu + \frac{1}{\Lambda^6}s^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$$

Symmetric traceless constant tensor

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Symmetric traceless constant tensor

9 new symmetries close with the 10 Poincare
+ 5 galileon to form a 24 dimensional algebra

$$[P_\mu, S_{\nu\lambda}] = \eta_{\mu\nu}B_\lambda + \eta_{\mu\lambda}B_\nu - \frac{2}{D}B_\mu\eta_{\nu\lambda}\,,$$

$$[B_\mu, S_{\nu\lambda}] = -\alpha\Big(\eta_{\mu\nu}P_\lambda + \eta_{\mu\lambda}P_\nu - \frac{2}{D}P_\mu\eta_{\nu\lambda}\Big),$$

$$[S_{\mu\nu}, S_{\lambda\sigma}] = \alpha\left(\eta_{\mu\lambda}J_{\nu\sigma} + \eta_{\nu\lambda}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\lambda} + \eta_{\mu\sigma}J_{\nu\lambda}\right),$$

What is this algebra? Geometric interpretation?

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Freddy and co. came across this theory from completely different angle: they have exact tree S-matrix Freddy Cachazo, Song He, Ellis Ye Yuan (2014)

$$\mathcal{M}_n = \int \frac{d^n\sigma}{\mathrm{vol}\,SL(2,\mathbb{C})}\prod_a{}'\delta\Big(\sum_{\substack{b=1\\b\neq a}}^{n}\frac{s_{ab}}{\sigma_a-\sigma_b}\Big)(\mathrm{Pf}'A)^4$$

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# Higher shift symmetries

KH, Austin Joyce (2014)

Wess-Zumino terms for higher shift symmetries

$$\text{e.g. } N{=}2 \qquad \phi \longmapsto \phi + c + b_i x^i + S_{ij} x^i x^j$$

WZ terms have fewer than 3 derivatives per field:

$$\begin{aligned}
&\mathcal{L}_1 \sim \phi \\
&\mathcal{L}_2 \sim (\nabla^2\phi)^2 \\
&\mathcal{L}_3 \sim \frac{1}{2}\nabla^4\phi(\nabla^2\phi)^2 + \nabla^2\phi(\nabla_i\nabla_j\nabla_k\phi)^2 \\
&\quad \vdots
\end{aligned}$$

EOM are 4-th order (lower than the expected 6-th order):

$$\text{e.g.} \qquad \frac{\delta\mathcal{L}_3}{\delta\phi} \sim (\nabla_i\nabla_j\nabla_k\nabla_l\phi)^2 - 2(\nabla^2\nabla_i\nabla_j\phi)^2 + (\nabla^4\phi)^2$$

# Condensed matter application

Griffin, Grosvenor, Horava, Yan (2013)

Non-relativistic theories with a higher-order dispersion relation

$$\omega^2(\vec{k}) \sim a_2\vec{k}^2 + a_4\vec{k}^4 + \cdots$$

$$\mathcal{L} \sim \dot{\phi}^2 + a_2(\nabla\phi)^2 + a_4(\nabla\phi)^4 + \cdots$$

Can $a_2$ be naturally small, so that $\omega^2 \sim k^4$ ?

Yes: enhanced higher shift symmetry when $a_2 = 0$ :

$$\phi \longmapsto \phi + c + b_i x^i + S_{ij} x^i x^j$$

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Generalize to a higher dispersion relation $\omega^2 \sim k^{2N}$

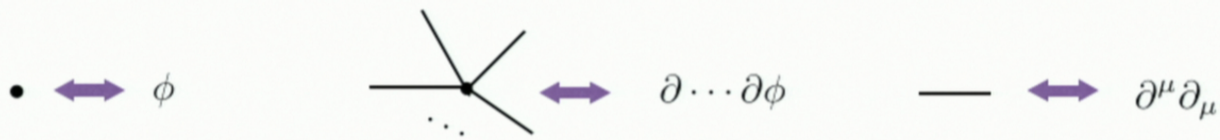
$$\phi \longmapsto \phi + c^{(0)} + c_i^{(1)} x^i + c_{ij}^{(2)} x^i x^j + \cdots + c_{i_1 \cdots i_N}^{(N)} x^{i1} \cdots x^{i_N}$$

$$\mathcal{L} \sim \dot{\phi}^2 + a_{2N} (\nabla\phi)^{2N} + \cdots$$

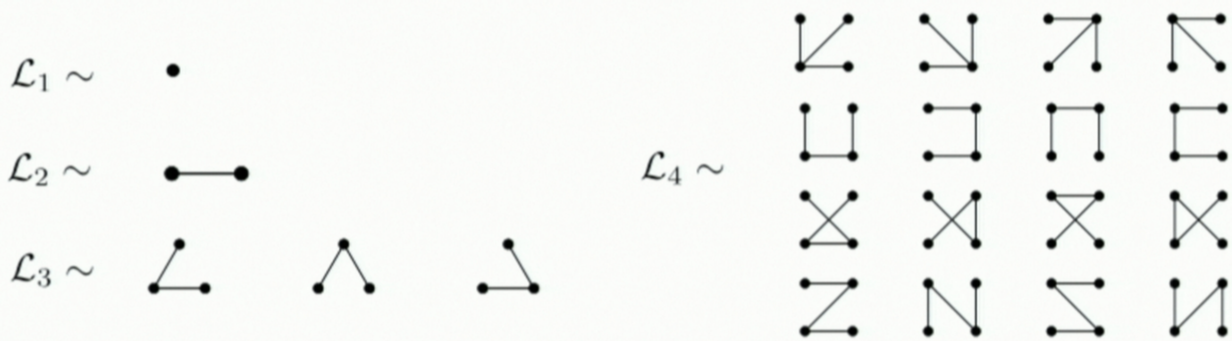
# Graph theory construction

Associate a graph to any scalar term:

Griffin, Grosvenor, Horava, Yan (2014)




The original galileons are the equal weight sums of all connected tree graphs:



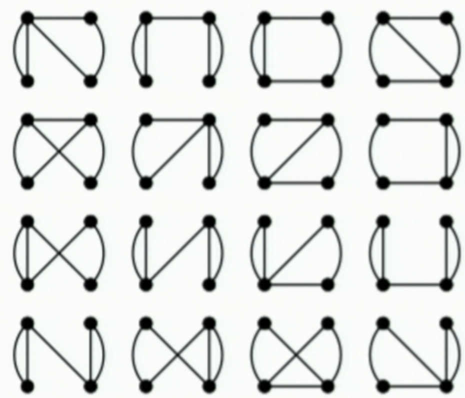

$\vdots$

# Graph theory construction

Griffin, Grosvenor, Horava, Yan (2014)

Higher order shift galileons are obtained from loop graphs

e.g. quartic term with 10 derivatives invariant under $N$=2 shifts:



$\sim \partial^{10}\phi^4$

# Topics I didn't touch

- Galileon duality  de Rham, Fasiello, Tolley (2013)

- UV properties/strong coupling regime/asymptotic fragility/Superluminality/Non-locality

- UV completion not a local field theory (toy model of quantum gravity)  Keltner, Tolley (2015)

- Classicalization  Dvali, Gomez (2010-present)

## Summary

- Galileon terms: ghost-free higher derivative scalar theories with extended symmetries and many nice properties. It is interesting that they exist at all.

- Connected to higher-dimensional geometry

- They show up in many places (decoupling limits of DGP, ghost-free massive gravities, dilaton effective actions...), and admit many generalizations

- Probably some deeper geometrical/topological reason for their existence