Title: Separability of Bosonic Systems

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Abstract: <p>We study the separability of quantum states in bosonic system. Our main tool here is the "separability witnesses", and a connection between "separability witnesses" and a new kind of positivity of matrices--- "Power Positive Matrices" is drawn. Such connection is employed to demonstrate that multi-qubit quantum states with Dicke states being its eigenvectors is separable if and only if two related Hankel matrices are positive semidefinite. By employing this criterion, we are able to show that such state is separable if and only if it's partial transpose is non-negative, which confirms the conjecture in [Wolfe, Yelin, Phys. Rev. Lett. (2014)]. Then, we present a class of bosonic states in dâŠ—d system such that for general d, determine its separabilityNP-hard although verifiable conditions for separability is easily derived in case d=3,4.</p>

Separability of bosonic System

$$\mathcal{H}_1 \otimes \mathcal{H}_2 \cdots \otimes \mathcal{H}_n$$

$$\rho = \sum p_i \, |\alpha_{1i}\rangle\langle\alpha_{1i}| \otimes |\alpha_{2i}\rangle\langle\alpha_{2i}| \cdots \otimes |\alpha_{ni}\rangle\langle\alpha_{ni}|$$

$\dim(H_i) = 2$

$n = 2$

$d \otimes d$

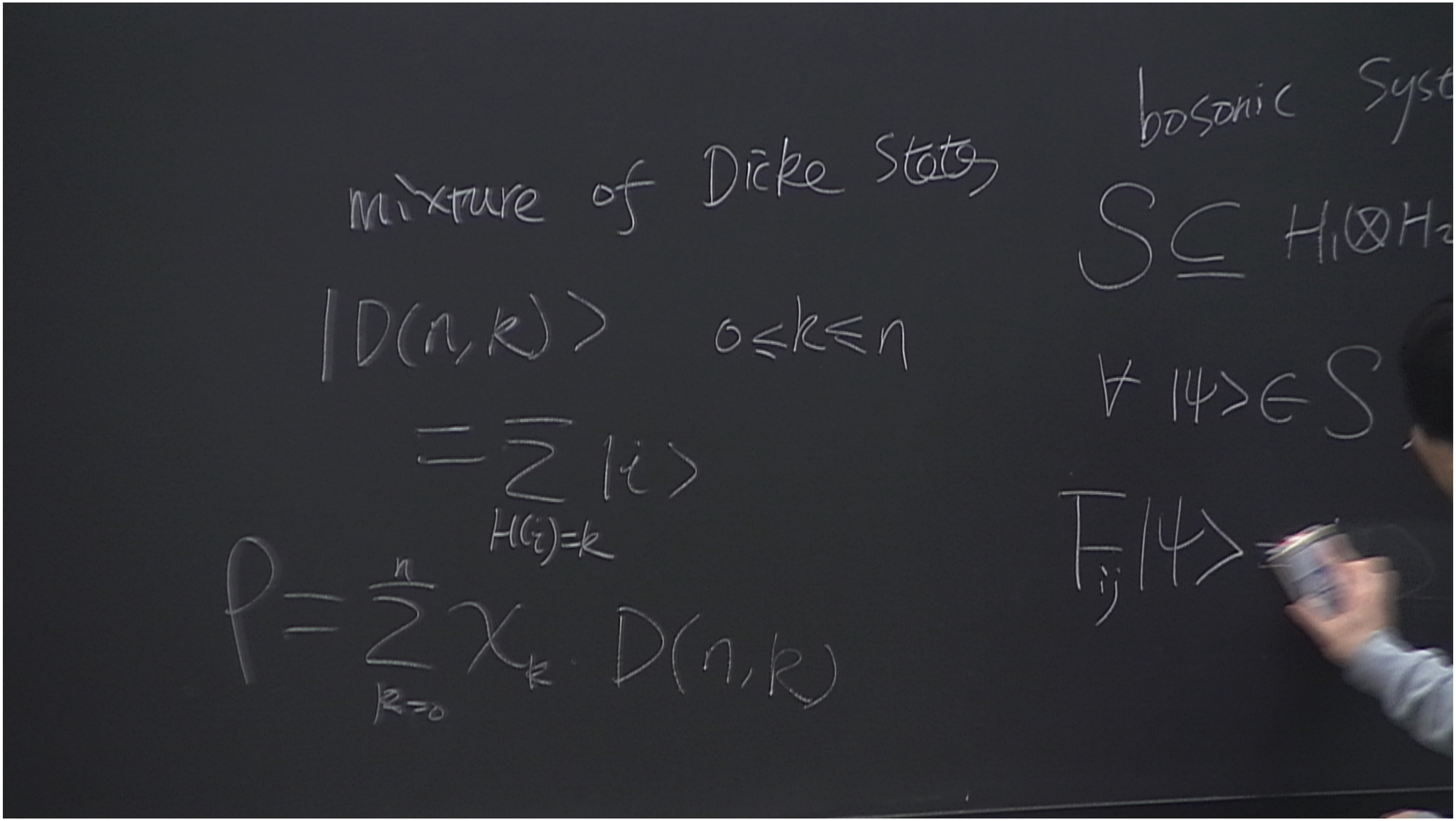
bosonic System

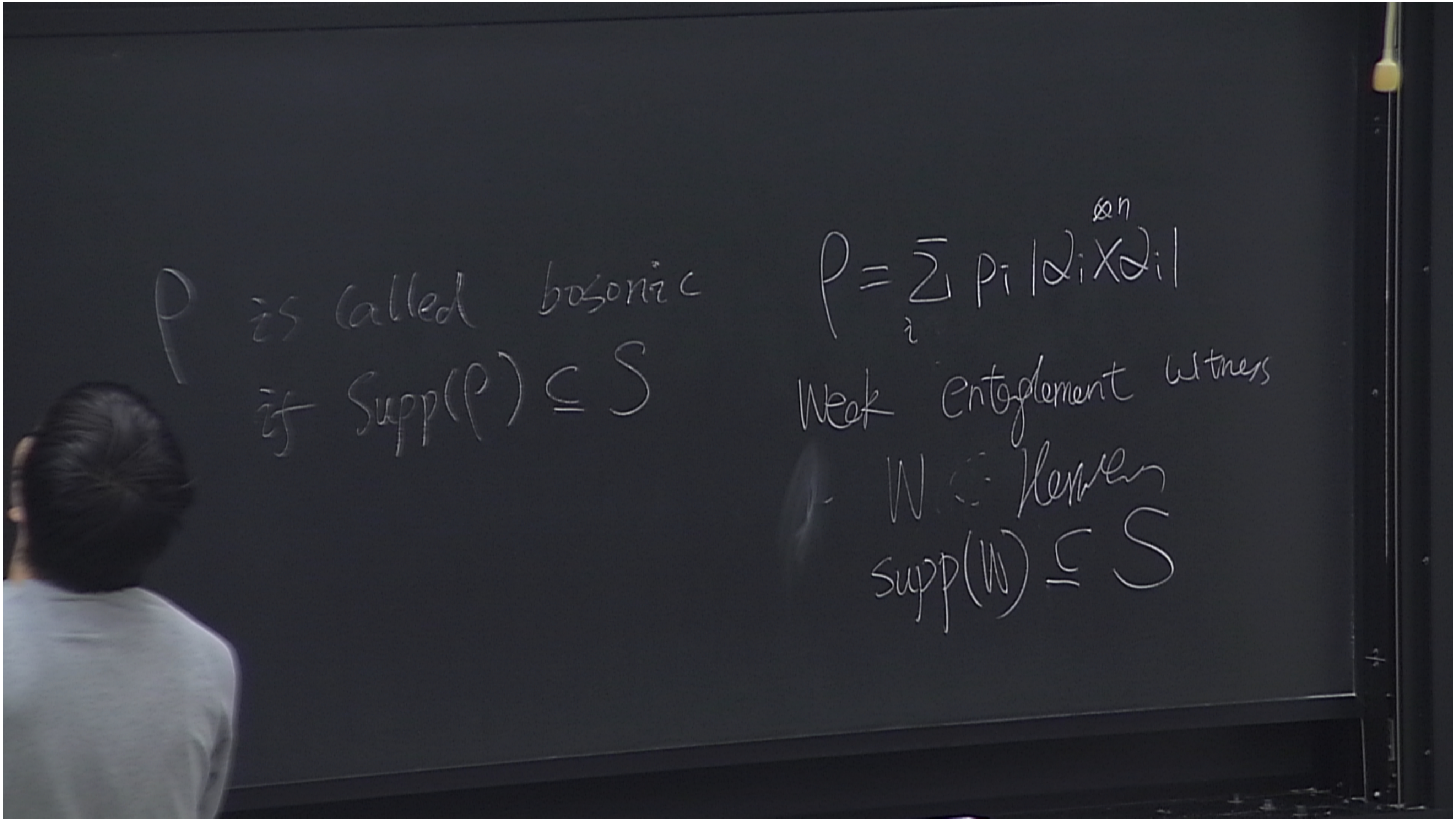
$S \subseteq H_1 \otimes H_2 \cdots \otimes H_n$

$\forall\ |\psi\rangle \in S,\ 1 \le i, j \le n$

$F_{ij} |\psi\rangle = |\psi\rangle$

ρ is called bosonic if $\mathrm{Supp}(\rho) \subseteq S$


mixture of Dicke States
$|D(n,k)\rangle$ $0 \leq k \leq n$
$= \sum_{H(i)=k} |i\rangle$
$\rho = \sum_{k=0}^{n} x_k \cdot D(n,k)$
bosonic Syst
$S \subseteq H_1 \otimes H_2$
$\forall |\psi\rangle \in S$
$F_{ij} |\psi\rangle =$


ρ is called bosonic if $\mathrm{Supp}(\rho) \subseteq S$
$\rho = \sum_i p_i |\alpha_i\rangle\langle\alpha_i|^{\otimes n}$
Weak entanglement witness
W Hermitian
$\mathrm{supp}(W) \subseteq S$

$$W = \sum u_{k,l} \, |\widetilde{D(n,k)}\rangle\langle\widetilde{D(n,l)}|$$

$$\langle D(n,k) | \widetilde{D(n,k)} \rangle = 1 \qquad W = \sum \mu_{k,l} |\widetilde{D(n,k)}\rangle\langle\widetilde{D(n,l)}|$$

$$i = i_1 \cdots i_n \in \{0,1\}^n$$

$$H(i) = \# 1$$

$\forall$ qubit U mixture of Dicke state

$$U^{\otimes n} \rho U^{\dagger \otimes n} = \rho$$

$$\langle \widetilde{D(n,k)} | \widetilde{D(n,k)} \rangle = 1 \qquad W = \sum u_{k,l} | \widetilde{D(n,k)} \rangle \langle \widetilde{D(n,l)} | \qquad \forall \text{ qubi-}$$

$i_n \in \{0,1\}^n$

$$U^{\otimes n} W U^{\dagger \otimes n} = W$$

$$W = \sum u_{k,k} | \widetilde{D(n,k)} \rangle \langle \widetilde{D(n,k)} |$$

$\langle \widetilde{D(n,k)} \rangle = 1$ $\quad W = \sum u_{k,l} |\widetilde{D(n,k)}\rangle\langle\widetilde{D(n,l)}|$

$\forall$ qubit

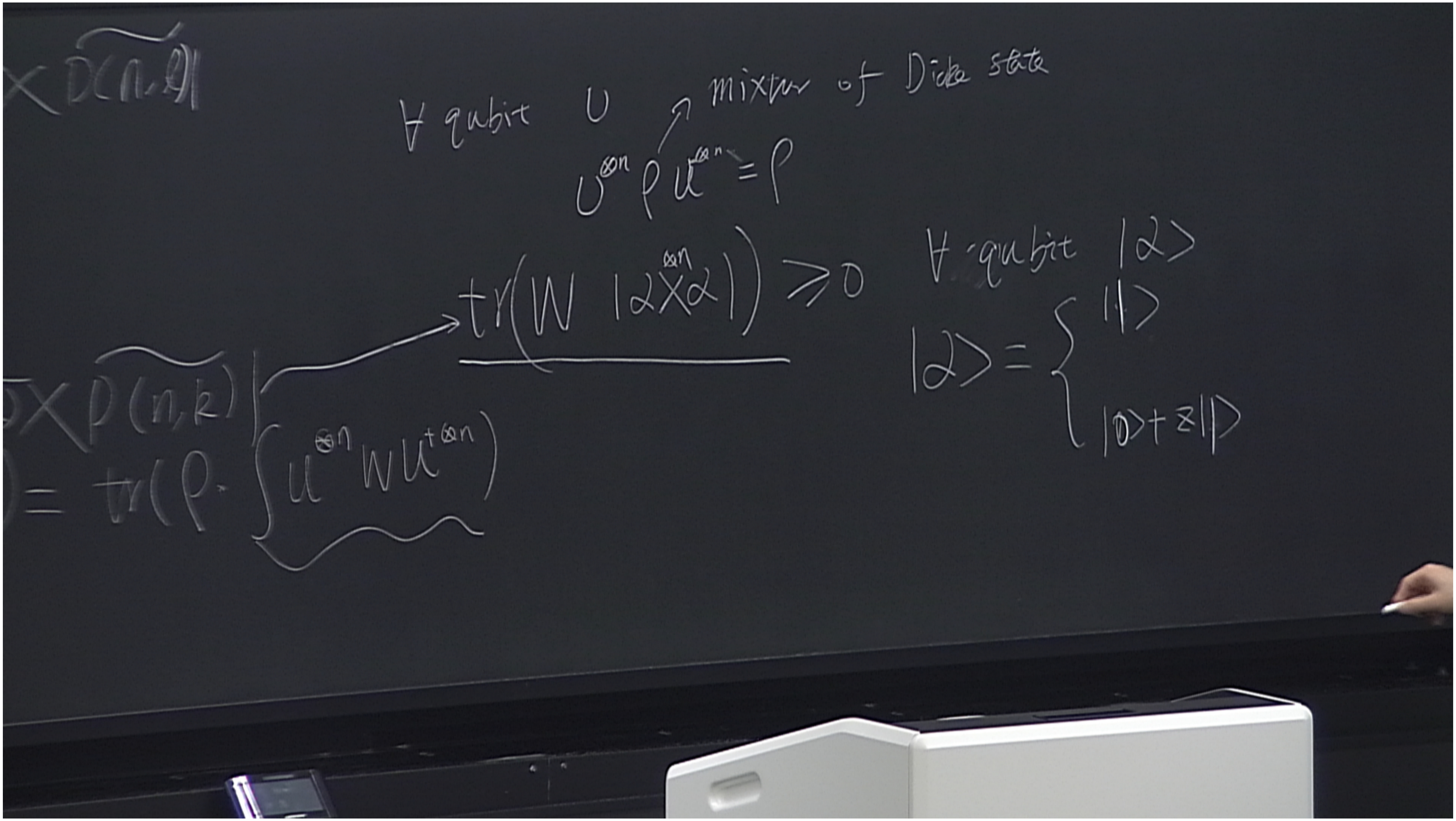
$\in \{0,1\}^n$

$$u^{\otimes n} W u^{\dagger \otimes n} = W$$

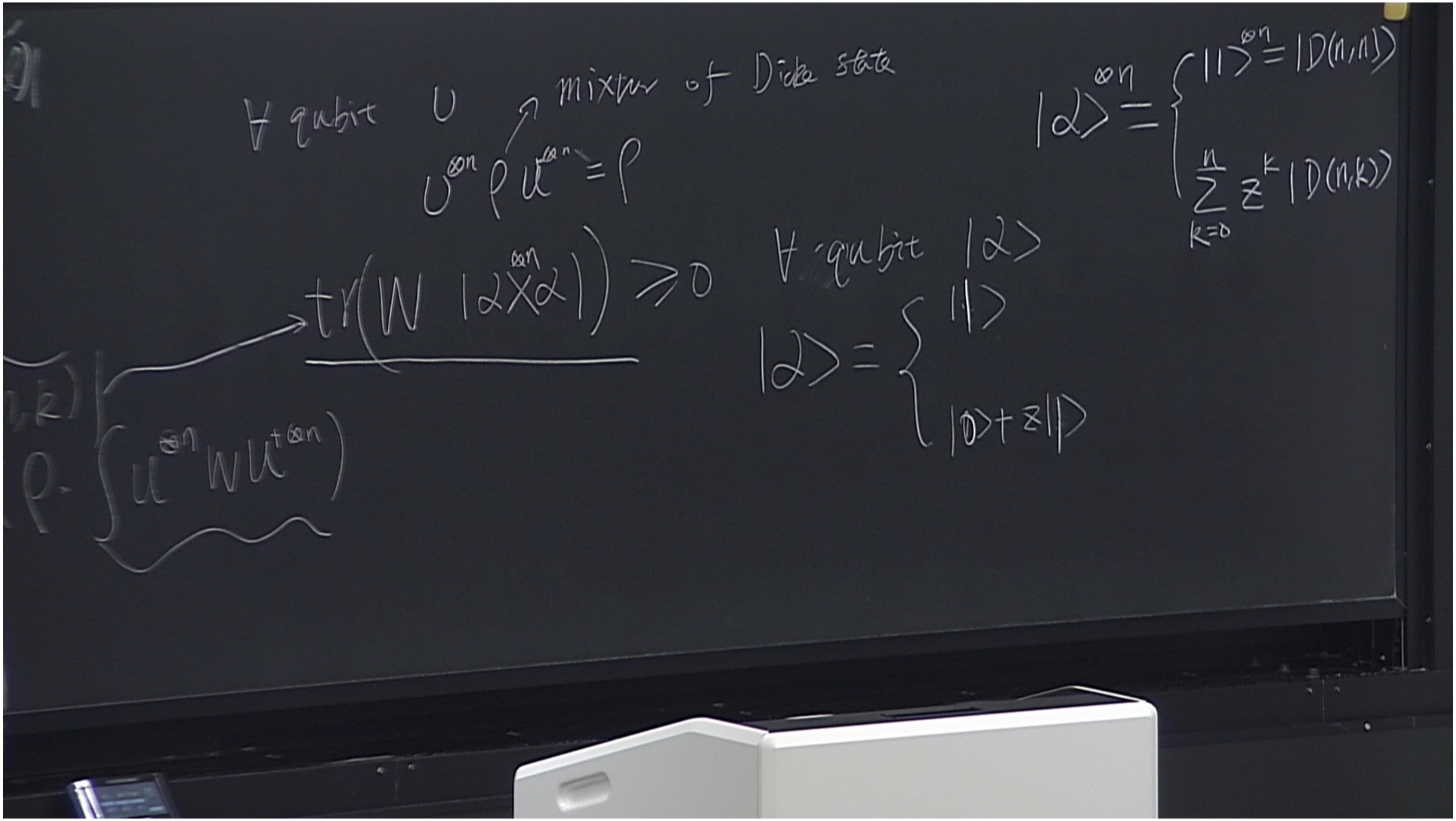
$$W = \sum u_{k,k} |\widetilde{D(n,k)}\rangle\langle\widetilde{D(n,k)}|$$

$\mathrm{tr}(W$

$$\mathrm{tr}(\rho \cdot W) = \mathrm{tr}(\rho \cdot u^{\otimes n} W u^{\dagger \otimes n}) = \mathrm{tr}(\rho \cdot \int u^{\otimes n} W u^{\dagger \otimes n})$$



$\forall$ qubit U
mixture of Dicke state
$U^{\otimes n} \rho U^{\dagger \otimes n} = \rho$
$\mathrm{tr}(W\, |\alpha\rangle\langle\alpha|^{\otimes n}) \geq 0$
$\forall$ qubit $|\alpha\rangle$
$|\alpha\rangle = \begin{cases} |1\rangle \\ |0\rangle + z|1\rangle \end{cases}$
$\mathrm{tr}(\rho \cdot \int U^{\otimes n} W U^{\dagger \otimes n})$


∀ qubit U
mixtur of Dicke state
$U^{\otimes n} \rho U^{\dagger \otimes n} = \rho$
$\mathrm{tr}(W |\alpha\rangle\langle\alpha|^{\otimes n}) \geq 0$ ∀ qubit $|\alpha\rangle$
$|\alpha\rangle = |1\rangle$
$|0\rangle + z|1\rangle$
$|\alpha\rangle^{\otimes n} = |1\rangle^{\otimes n} = |D(n,n)\rangle$
$\sum_{k=0}^{n} z^k |D(n,k)\rangle$
$\rho - \int U^{\otimes n} W U^{\dagger \otimes n})$

$$\mathrm{tr}\left(W\,|\alpha\rangle\langle\alpha|^{\otimes n}\right)$$

$$= \sum u_{k,k} \cdot |z|^{2k}$$

$$\mathrm{tr}\left(W\,|\alpha\rangle\langle\alpha|^{\otimes n}\right)$$

$$= \sum u_{k,k} \cdot |z|^{2k} \geq 0$$

$$= P_u(|z|^2)$$

$$P_a(x) = \sum u_{k,k} \cdot x^k \geq 0 \quad \forall\, x \geq 0$$

Set of entangled witnesses

$Q^2(x)$ $\Downarrow$ $\deg Q \leq \frac{n}{2}$

$x \cdot R^2(x)$ $\deg R \leq \frac{n-1}{2}$

$$\mathrm{tr}(W\rho) = \sum u_{k,k} \cdot \chi_k \geq 0$$

$\forall$ all entanglement W

$$\mathrm{tr}(W\,|\alpha\rangle\langle\alpha|^{\otimes n}) = \sum u_{k,k} \cdot |z|^{2k} \geq$$

$$= P_u(|z|^2)$$

$$P_u(x) = \sum u_{k,k} \cdot x^k \geq 0 \quad \forall x \in \mathbb{R},\ x \geq 0$$

$$= \sum Q_i^2(x) + \sum x \cdot R_i^2(x)$$

$$\rho = \sum x_k |D(n,k)\rangle\langle D(n,k)|$$

is separable

$$\begin{bmatrix} x_1 & x_{m_1} \\ x_{m_1} & x_{2m_1-1} \end{bmatrix} \geq 0 \qquad m_1 = \left[\frac{n-1}{2}\right]$$

$$\begin{bmatrix} x_0 & x_{m_0} \\ x_{m_0} & x_{2m_0} \end{bmatrix} \geq 0 \qquad m_0 = \left[\frac{n}{2}\right]$$

$\Longleftarrow$

Set

$\mathrm{tr}(W\rho$

$= \sum u_{k,k}$

$\forall$ all

$$Q^2(x) = \sum u_{k,k} \cdot x^k$$

$$Q(x) = \sum_{i=0}^{m_0} \beta_i \cdot x^i \qquad \beta_i \in \mathbb{R}$$

$$Q^2(x) = \sum_{0 \le i,j \le m_0} \beta_i \beta_j \cdot x^{i+j}$$

$$u_{k,k} = \sum_{i+j=k} \beta_i \beta_j$$

$$\mathrm{tr}(W\cdot\rho) \geq 0$$

$$\sum_k M_{k,k}\cdot x_k \geq 0$$

$$\beta = \begin{pmatrix}\beta_0\\ \vdots \\ \beta_{m_0}\end{pmatrix} \in \mathbb{R}^{m_0} \qquad = \sum_k \sum_{\substack{i+j=k\\ 0\leq i,j\leq m_0}} \beta_i\beta_j\cdot x_k \geq 0$$

$$= \beta^T M_0\, \beta \geq 0$$

$$Q^2(x) = \sum M_{k,k}\cdot x^k$$

$$Q(x) = \sum_{i=0}^{m_0} \beta_i\cdot x^i \qquad \beta_i \in \mathbb{R}$$

$$Q^2(x) = \sum_{0\leq i,j\leq m_0} \beta_i\beta_j\cdot x^{i+j}$$

$$M_{k,k} = \sum_{i+j=k} \beta_i\beta_j$$

ρ $\quad 1 \ldots n$

is separable

if $\rho^{\Gamma_A} \geq 0$

$A = \{1 \ldots m_0\}$

$tr(W \cdot \rho) \geq 0$

$\sum M_{k,k} \cdot \alpha_k \geq 0$

$\beta = \begin{pmatrix} \beta_0 \\ \vdots \\ \beta_m \end{pmatrix} \in \mathbb{R}^{m_0} = \sum_k \sum_{\substack{i+j=k \\ 0 \le i,j \le m_0}} \beta_i \beta_j \cdot \alpha_k \geq$

$= \beta^T M_0 \beta \geq 0$

bosonic System

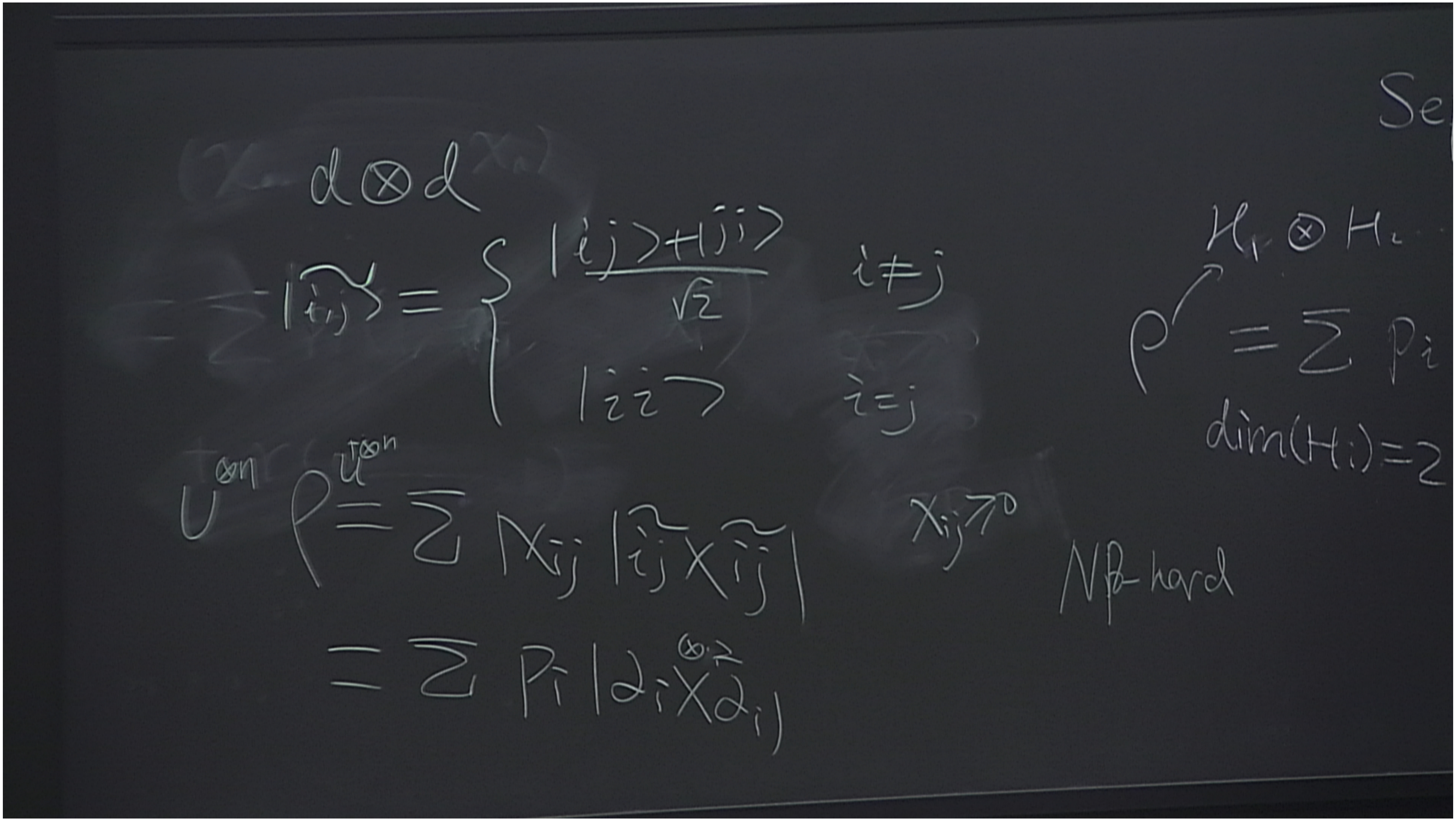
$$\rho = \sum \alpha_{k,l} \, |D(n,k)\rangle\langle D(n,l)|$$

$$= \sum p_i \, |\alpha\rangle\langle\alpha|^{\otimes n} \qquad z_i \in \mathbb{C}$$

$\otimes \, |\alpha_{ni}\rangle\langle\alpha_{ni}|$

$$\Rightarrow \chi = (\chi_{k,l}) = \sum p_i \begin{pmatrix} 1 \\ z_i \\ \vdots \\ z_i^{n-1} \end{pmatrix} \overline{(1, z_i, \ldots, z_i^{n-1})}$$

$$+ \gamma \begin{pmatrix} 0 & & 0 \\ & & \vdots \\ 0 & & 1 \end{pmatrix}$$


$d \otimes d$
$|\widetilde{ij}\rangle = \begin{cases} \frac{|ij\rangle + |ji\rangle}{\sqrt{2}} & i \neq j \\ |ii\rangle & i = j \end{cases}$
$U^{\otimes n} \rho U^{\dagger \otimes n}$
$\rho = \sum x_{ij} |\widetilde{ij}\rangle\langle\widetilde{ij}|$
$x_{ij} \geq 0$
$= \sum p_i |\alpha_i\rangle\langle\alpha_i|^{\otimes 2}$
$H_1 \otimes H_2$
$\rho = \sum p_i$
$\dim(H_i) = 2$
NP-hard
Se

$d \otimes d$

$$|\widetilde{ij}\rangle = \begin{cases} \frac{|ij\rangle + |ji\rangle}{\sqrt{2}} & i \neq j \\ |ii\rangle & i = j \end{cases}$$

$U^{\otimes n} \quad \rho \, U^{\dagger \otimes n}$

$$\rho = \sum X_{ij} |\widetilde{ij}\rangle\langle\widetilde{ij}|$$

$$= \sum p_i |\alpha_i\rangle\langle\alpha_i|^{\otimes 2}$$

$X = (X_{ij})_{d \times d}$ is CP

Se

$H_1 \otimes H_2$

$\rho = \sum p_i$

$\dim(H_i) = 2$