Title: Understanding Big Bang Singularities from Holography

Date: Mar 10, 2015 02:00 PM

URL: http://pirsa.org/15030094

Abstract: <p>Using holography, I will describe an approach for understanding the physics of a big bang singularity by translating the problem into the language of the dual quantum field theory. Certain two-point correlators in the dual field theory are sensitive to near-singularity physics in a dramatic way, and this provides an avenue for investigating how strong quantum gravity effects in string theory might modify the classical description of the big bang.</p>

Understanding Big Bang Singularities From Holography

Netta Engelhardt

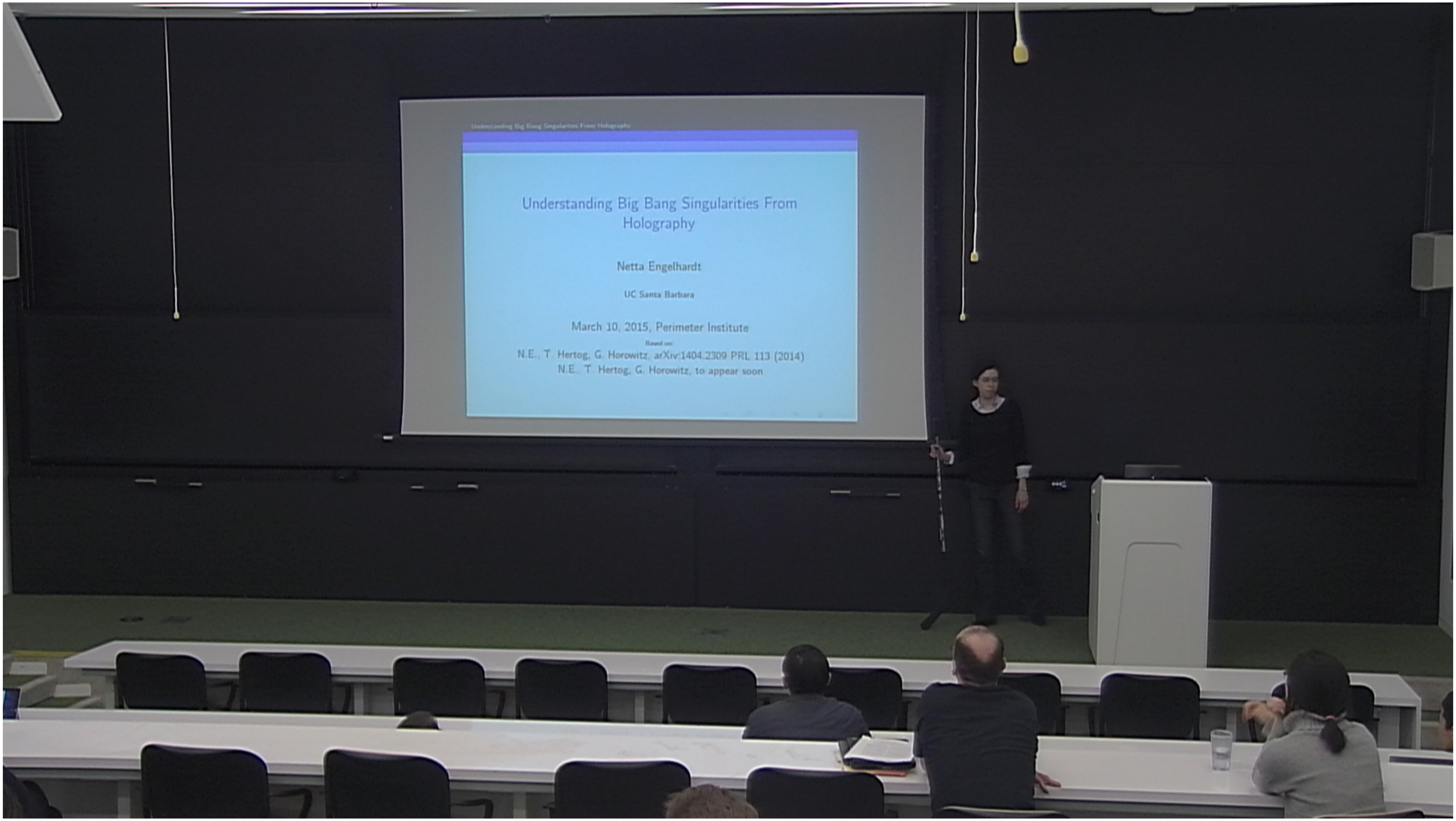
UC Santa Barbara

March 10, 2015, Perimeter Institute

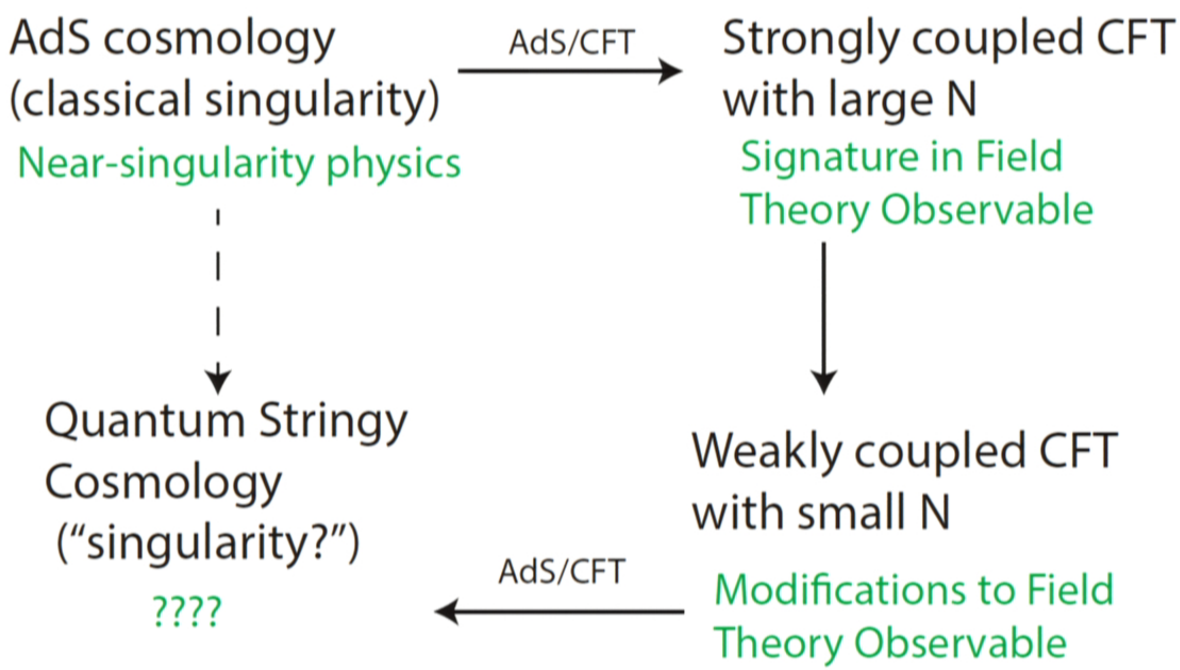
Based on:

N.E., T. Hertog, G. Horowitz, arXiv:1404.2309 PRL 113 (2014)

N.E., T. Hertog, G. Horowitz, to appear soon


Understanding Big Bang Singularities From Holography
Netta Engelhardt
UC Santa Barbara
March 10, 2015, Perimeter Institute
Based on:
N.E., T. Hertog, G. Horowitz, arXiv:1404.2309 PRL 113 (2014)
N.E., T. Hertog, G. Horowitz, to appear soon

AdS/CFT: Translating Quantum Gravity into QFT since 1997



What we want to do

- Construct an AdS cosmology with:
  1. a big bang (or crunch) bulk singularity
  2. Singularity-free conformal boundary (otherwise we translate one singularity into another)
  3. Bulk geodesics that reach the high curvature near-singularity region (insurance that the two-point function is sensitive to the singularity physics)
- Compute the two-point function in the strongly coupled CFT holographically
- Identify a feature of the two-point function which is a direct consequence of the bulk singularity
- Lower the coupling and N

What we want to do

- Construct an AdS cosmology with:
  1. a big bang (or crunch) bulk singularity
  2. Singularity-free conformal boundary (otherwise we translate one singularity into another)
  3. Bulk geodesics that reach the high curvature near-singularity region (insurance that the two-point function is sensitive to the singularity physics)
- Compute the two-point function in the strongly coupled CFT holographically
- Identify a feature of the two-point function which is a direct consequence of the bulk singularity
- Lower the coupling and N

What we want to do

- Construct an AdS cosmology with:
  1. a big bang (or crunch) bulk singularity
  2. Singularity-free conformal boundary (otherwise we translate one singularity into another)
  3. Bulk geodesics that reach the high curvature near-singularity region (insurance that the two-point function is sensitive to the singularity physics)
- Compute the two-point function in the strongly coupled CFT holographically
- Identify a feature of the two-point function which is a direct consequence of the bulk singularity
- Lower the coupling and N

What we want to do

- Construct an AdS cosmology with:
  1. a big bang (or crunch) bulk singularity
  2. Singularity-free conformal boundary (otherwise we translate one singularity into another)
  3. Bulk geodesics that reach the high curvature near-singularity region (insurance that the two-point function is sensitive to the singularity physics)
- Compute the two-point function in the strongly coupled CFT holographically
- Identify a feature of the two-point function which is a direct consequence of the bulk singularity
- Lower the coupling and N

What we want to do

- Construct an AdS cosmology with:
  1. a big bang (or crunch) bulk singularity
  2. Singularity-free conformal boundary (otherwise we translate one singularity into another)
  3. Bulk geodesics that reach the high curvature near-singularity region (insurance that the two-point function is sensitive to the singularity physics)
- Compute the two-point function in the strongly coupled CFT holographically
- Identify a feature of the two-point function which is a direct consequence of the bulk singularity
- Lower the coupling and N

What we want to do

- Construct an AdS cosmology with:
  1. a big bang (or crunch) bulk singularity
  2. Singularity-free conformal boundary (otherwise we translate one singularity into another)
  3. Bulk geodesics that reach the high curvature near-singularity region (insurance that the two-point function is sensitive to the singularity physics)
- Compute the two-point function in the strongly coupled CFT holographically
- Identify a feature of the two-point function which is a direct consequence of the bulk singularity
- Lower the coupling and N

Table of Contents

1. AdS Cosmologies and the Great Wall
2. Drama in the Two-Point Correlator
3. Interpretation
4. Summary and Open Questions

Pure AdS in different coordinate systems

- Global AdS:

$$ds^2 = -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\Omega^2$$

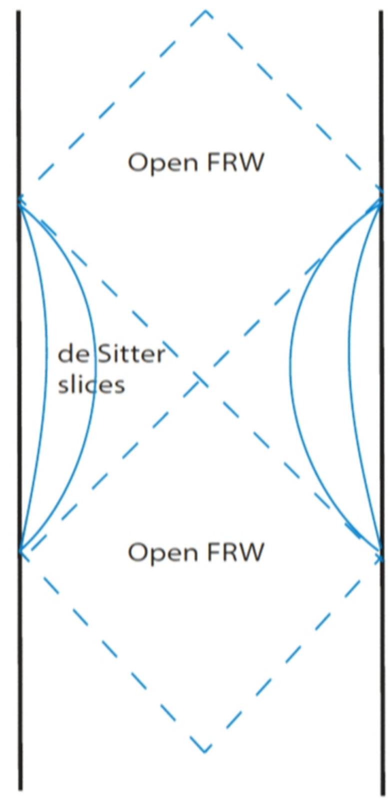
- Poincaré Patch:

$$ds^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right)$$

- dS and FRW patches:

$$ds^2 = dr^2 + \sinh^2 r\left(-d\tau^2 + \cosh^2\tau d\Omega^2\right)$$

$$ds^2 = -dt^2 + \sin^2 t\left(dx^2 + \sinh^2 x d\Omega^2\right)$$



Choice of Conformal Boundary

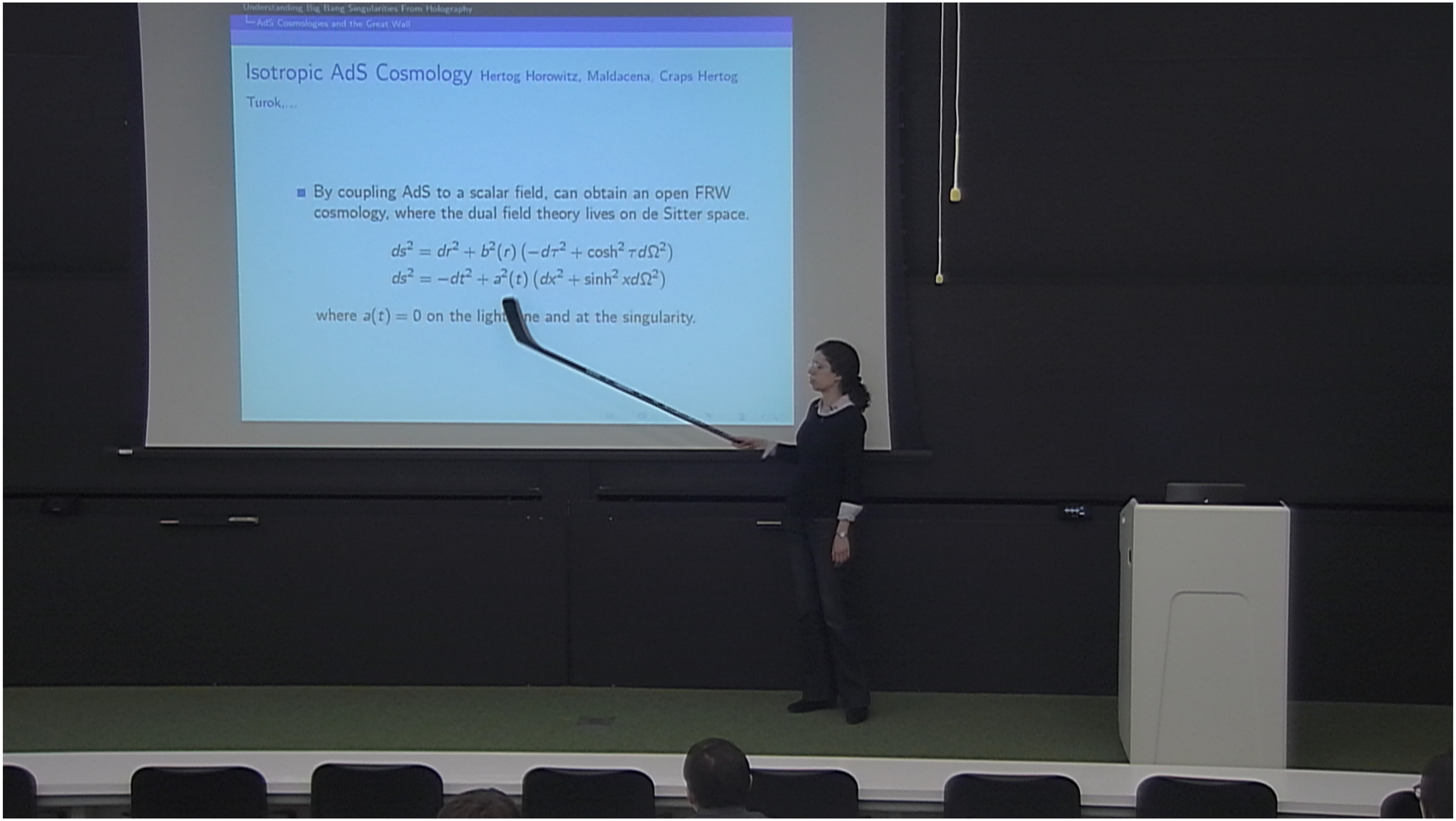
- Freedom in choice of conformal boundary, so long as it is conformal to a patch of the Einstein static universe
- Choose patch based on convenient choice of conformal boundary
- Einstein static universe for global AdS
- Flat Minkowski for Poincaré patch
- de Sitter for the dS slicing with FRW

Isotropic AdS Cosmology Hertog Horowitz, Maldacena, Craps Hertog Turok,...

- By coupling AdS to a scalar field, can obtain an open FRW cosmology, where the dual field theory lives on de Sitter space.

$$ds^2 = dr^2 + b^2(r)\left(-d\tau^2 + \cosh^2\tau d\Omega^2\right)$$
$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + \sinh^2 x d\Omega^2\right)$$

where $a(t) = 0$ on the lightcone and at the singularity.

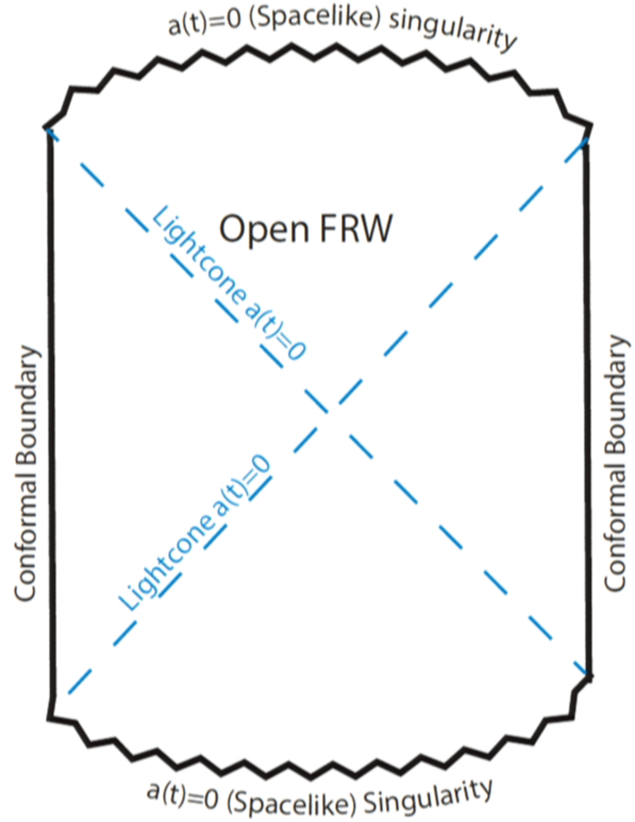

Understanding Big Bang Singularities From Holography
AdS Cosmologies and the Great Wall
Isotropic AdS Cosmology Hertog Horowitz, Maldacena, Craps Hertog Turok,...
By coupling AdS to a scalar field, can obtain an open FRW cosmology, where the dual field theory lives on de Sitter space.
$ds^2 = dr^2 + b^2(r)\left(-d\tau^2 + \cosh^2\tau d\Omega^2\right)$
$ds^2 = -dt^2 + a^2(t)\left(dx^2 + \sinh^2 x d\Omega^2\right)$
where $a(t) = 0$ on the light ne and at the singularity.

Isotropic AdS Cosmology Hertog Horowitz, Maldacena, Craps Hertog Turok,...

- By coupling AdS to a scalar field, can obtain an open FRW cosmology, where the dual field theory lives on de Sitter space.

$$ds^2 = dr^2 + b^2(r)\left(-d\tau^2 + \cosh^2\tau d\Omega^2\right)$$
$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + \sinh^2 x d\Omega^2\right)$$

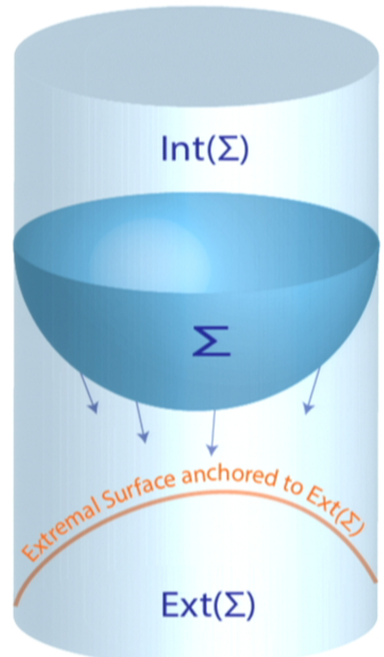
where $a(t) = 0$ on the lightcone and at the singularity.


a(t)=0 (Spacelike) singularity
Open FRW
Lightcone a(t)=0
Lightcone a(t)=0
Conformal Boundary
Conformal Boundary
a(t)=0 (Spacelike) Singularity

Theorem: Extremal Surface Barriers NE & A. Wall '13

- Spacetime with boundary (e.g. AdS)
- has codimension 1 "splitting surface" Σ
- Σ has $K_{\mu\nu} \leq 0$ (Normals to Σ converge outside Σ)

$\Rightarrow$ No spacelike extremal surface anchored outside of Σ can ever cross Σ.



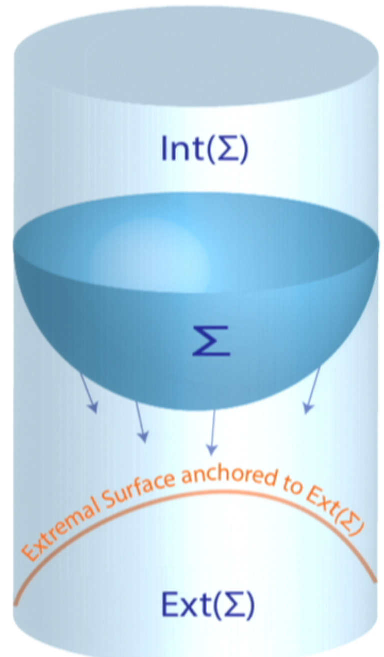
- Barriers with $K_{\mu\nu} < 0$ are stable under small perturbations.
- Example of barriers: stationary black hole horizons (e.g. Schwarzschild).

Theorem: Extremal Surface Barriers NE & A. Wall '13

- Spacetime with boundary (e.g. AdS)
- has codimension 1 "splitting surface" Σ
- Σ has $K_{\mu\nu} \leq 0$ (Normals to Σ converge outside Σ)

$\Rightarrow$ No spacelike extremal surface anchored outside of Σ can ever cross Σ.

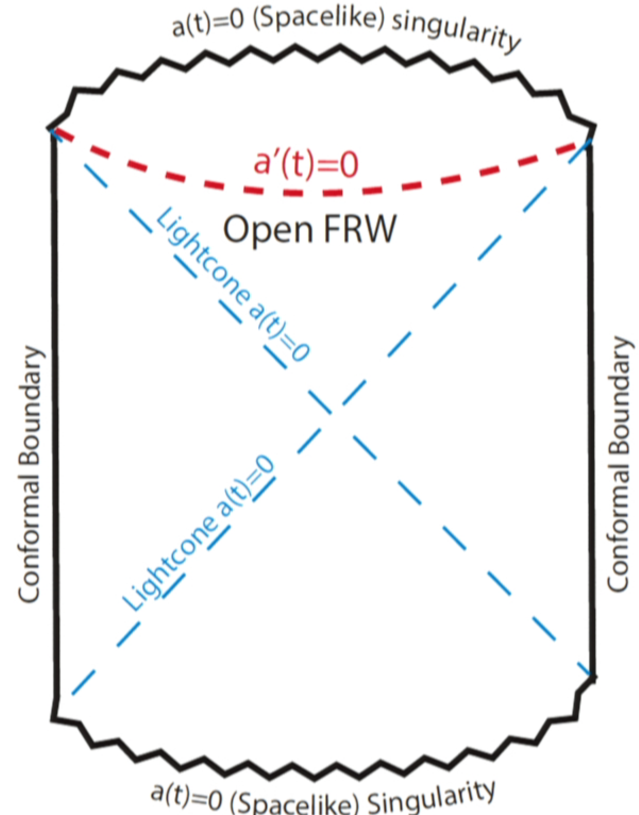
- Barriers with $K_{\mu\nu} < 0$ are stable under small perturbations.
- Example of barriers: stationary black hole horizons (e.g. Schwarzschild).



The Great Wall NE & A. Wall '13

On a surface with
$t = const$:

$$K_{ij} = a'(t) g_{ij}$$



Wishlist: Not satisfied by isotropic AdS cosmology

- Asymptotically AdS geometry with big bang/crunch that solves Einstein's Equations
- Conformal boundary metric has no curvature singularity
- ~~Spacelike, boundary-anchored geodesics probe the near-singularity region~~

Anisotropy to the Rescue

- Ricci-flat, vacuum, homogeneous, anisotropic geometry with an initial singularity:

$$ds^2 = -dt^2 + \sum_i t^{2p_i} dx_i^2$$

$$\sum_i p_i = 1 = \sum_i p_i^2$$

- $(d-2)$-parameter family of solutions in $(d+1)$-dimensions
- One exponent is always negative, except:
- If one $p_i = 1$, the others must vanish. This is the Milne Universe, and the singularity is a coordinate singularity.

Anisotropy to the Rescue

- Ricci-flat, vacuum, homogeneous, anisotropic geometry with an initial singularity:

$$ds^2 = -dt^2 + \sum_i t^{2p_i} dx_i^2$$

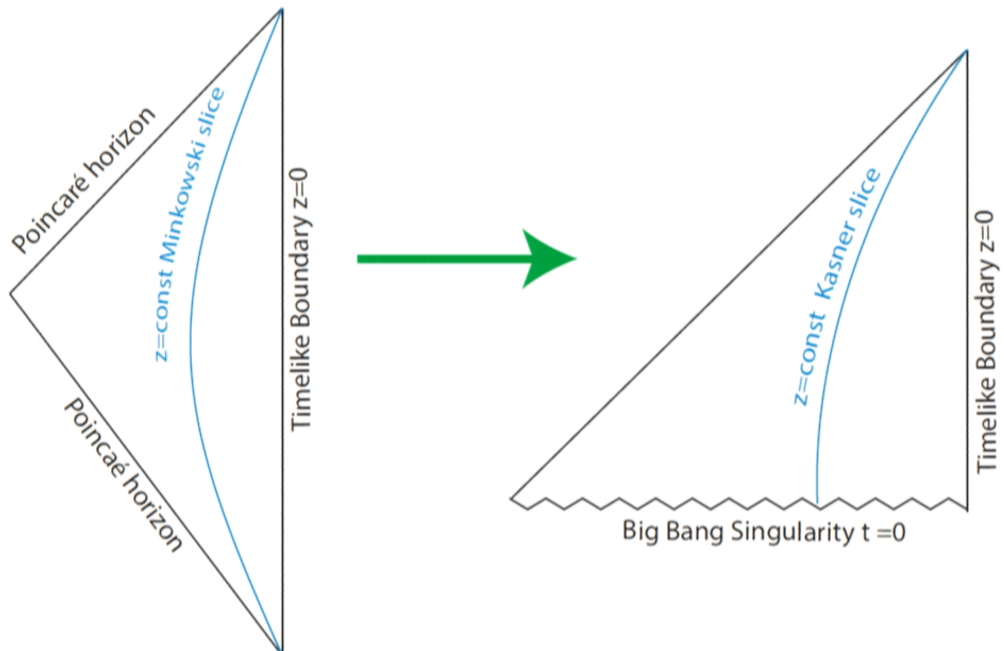
$$\sum_i p_i = 1 = \sum_i p_i^2$$

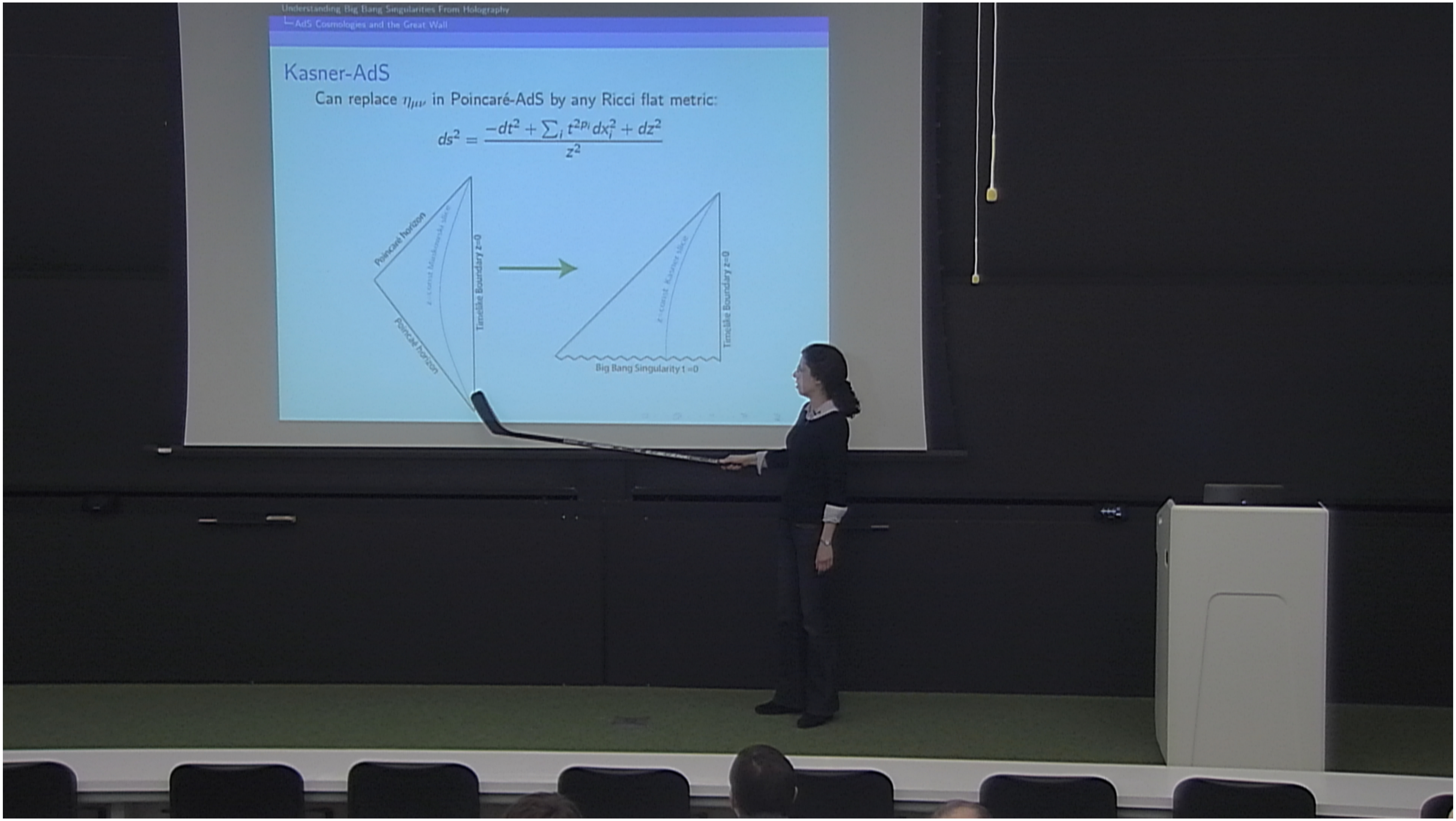
- $(d-2)$-parameter family of solutions in $(d+1)$-dimensions
- One exponent is always negative, except:
- If one $p_i = 1$, the others must vanish. This is the Milne Universe, and the singularity is a coordinate singularity.

Kasner-AdS

Can replace $\eta_{\mu\nu}$ in Poincaré-AdS by any Ricci flat metric:

$$ds^2 = \frac{-dt^2 + \sum_i t^{2p_i} dx_i^2 + dz^2}{z^2}$$



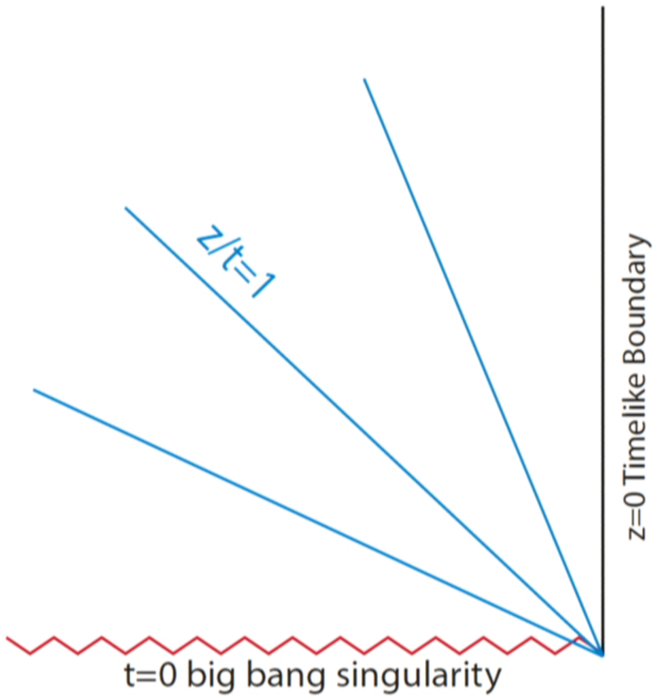

Understanding Big Bang Singularities From Holography
AdS Cosmologies and the Great Wall
Kasner-AdS
Can replace $\eta_{\mu\nu}$ in Poincaré-AdS by any Ricci flat metric:
$$ds^2 = \frac{-dt^2 + \sum_i t^{2p_i} dx_i^2 + dz^2}{z^2}$$
Poincaré horizon
Timelike Boundary z=0
Poincaré horizon
Timelike Boundary z=0
Big Bang Singularity t=0

A Warped Dilation

- Kasner-AdS has a nontrivial Killing symmetry:

$$t \to \lambda t$$
$$x_i \to \lambda^{1-p_i} x_i$$
$$z \to \lambda z$$

- Acts on surfaces of $z/t =$const
- Never relates points to singularity



Wishlist:

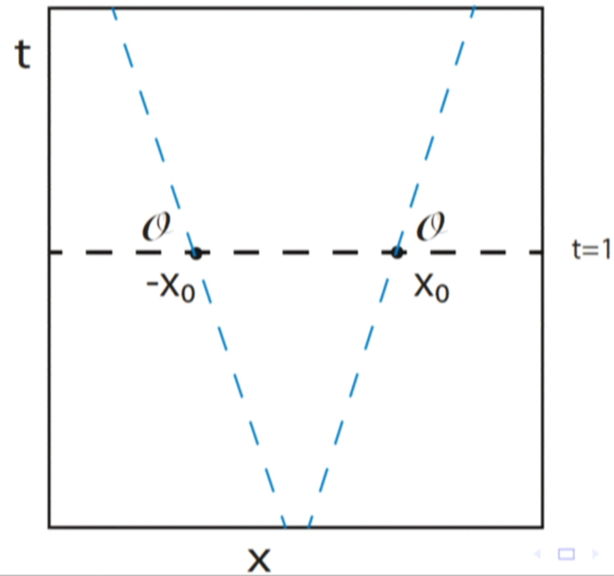
- Asymptotically AdS geometry with big bang/crunch that solves Einstein's Equations
- Conformal boundary metric has no curvature singularity
- Spacelike, boundary-anchored geodesics probe the near-singularity region

To do:

- Calculate the length of geodesics
- Identify the contribution of geodesics that probe near the singularity
- Isolate signature of near-singularity physics in two-point correlator

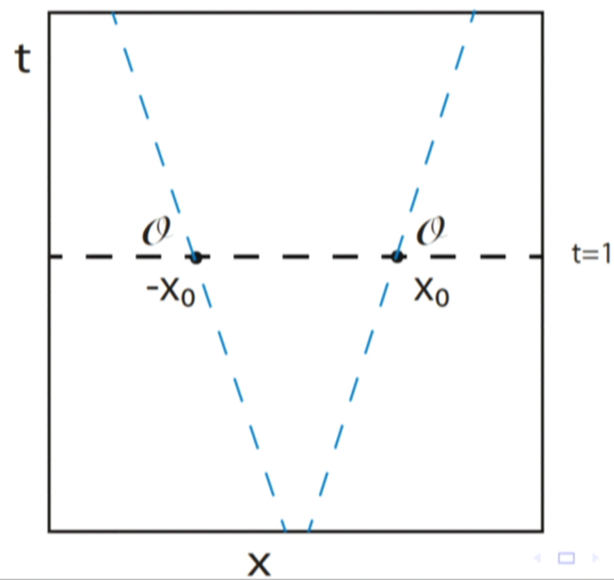
Setup

- Fix boundary endpoints to be separated in $x = x_1$, $x_{i \neq 1} = 0$, fix $t = 1$ ($\tau = 0$) at endpoints
- Boundary correlator can depend only on proper boundary separation $L_{bdy} = 2x_0 t_0^{1-p} = 2x_0$: can only expect special features at $L_{bdy} = 0$ or $L_{bdy} = L_{hor}$.



Setup

- Fix boundary endpoints to be separated in $x = x_1$, $x_{i\neq 1} = 0$, fix $t = 1$ ($\tau = 0$) at endpoints
- Boundary correlator can depend only on proper boundary separation $L_{bdy} = 2x_0 t_0^{1-p} = 2x_0$: can only expect special features at $L_{bdy} = 0$ or $L_{bdy} = L_{hor}$.



Behavior of Geodesics

- Geodesics propagate in three dimensions: (t, x, z)
- The geodesics turn around at $x = 0$. Parametrizing them in terms of x:

$$t''(x)t(x) = -p\left(t(x)^{2p} - 2t'(x)^2\right)$$

- So for $p < 0$, geodesics *are attracted to the singularity!*
- $p < 0$ geodesics probe high curvature region: how do they contribute to the two-point correlator?

Behavior of Geodesics

- Geodesics propagate in three dimensions: (t, x, z)
- The geodesics turn around at $x = 0$. Parametrizing them in terms of x:

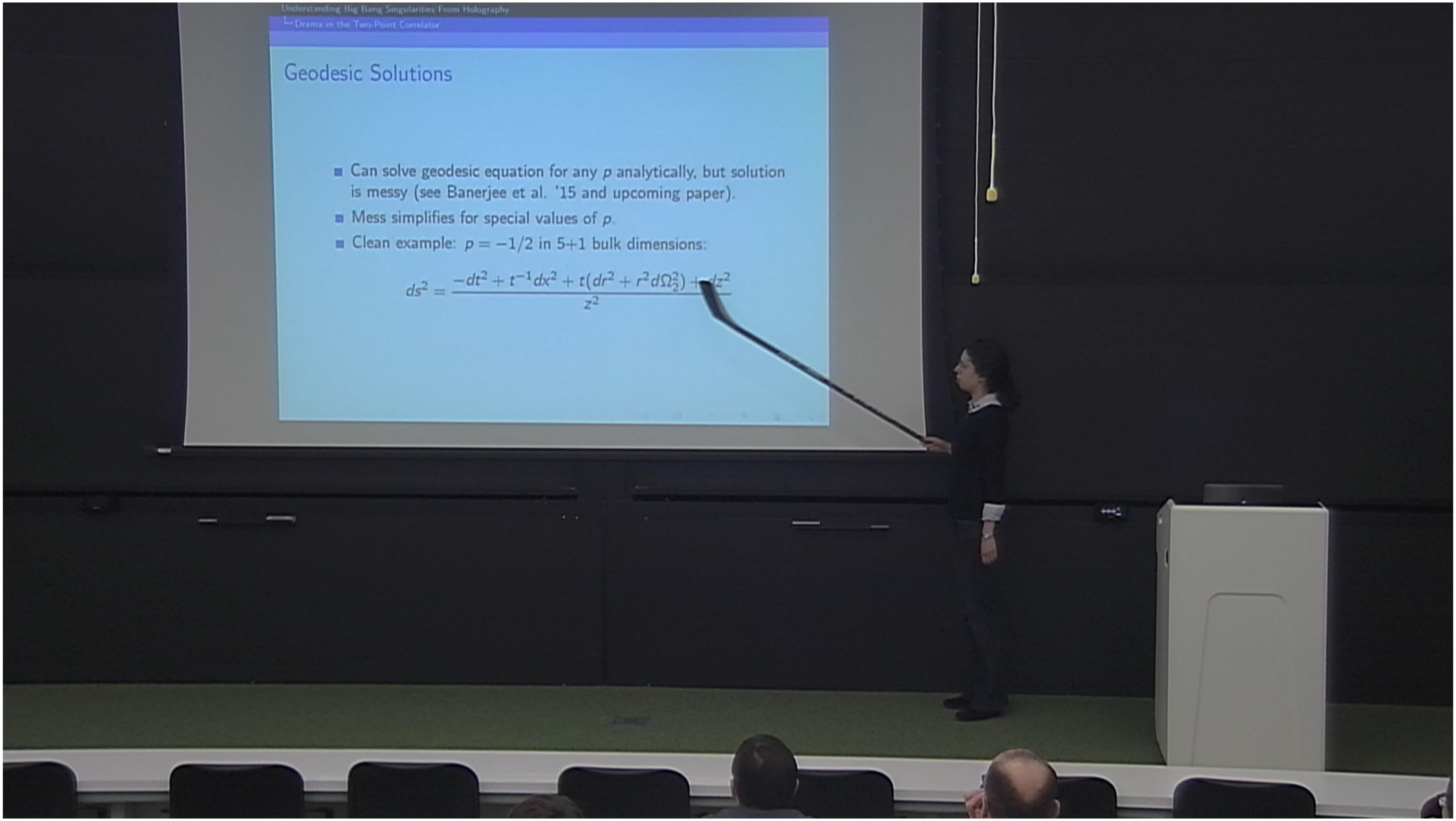
$$t''(x)t(x) = -p\left(t(x)^{2p} - 2t'(x)^2\right)$$

- So for $p < 0$, geodesics *are attracted to the singularity!*
- $p < 0$ geodesics probe high curvature region: how do they contribute to the two-point correlator?

Geodesic Solutions

- Can solve geodesic equation for any p analytically, but solution is messy (see Banerjee et al. '15 and upcoming paper).
- Mess simplifies for special values of p.
- Clean example: $p = -1/2$ in 5+1 bulk dimensions:

$$ds^2 = \frac{-dt^2 + t^{-1}dx^2 + t(dr^2 + r^2 d\Omega_2^2) + dz^2}{z^2}$$

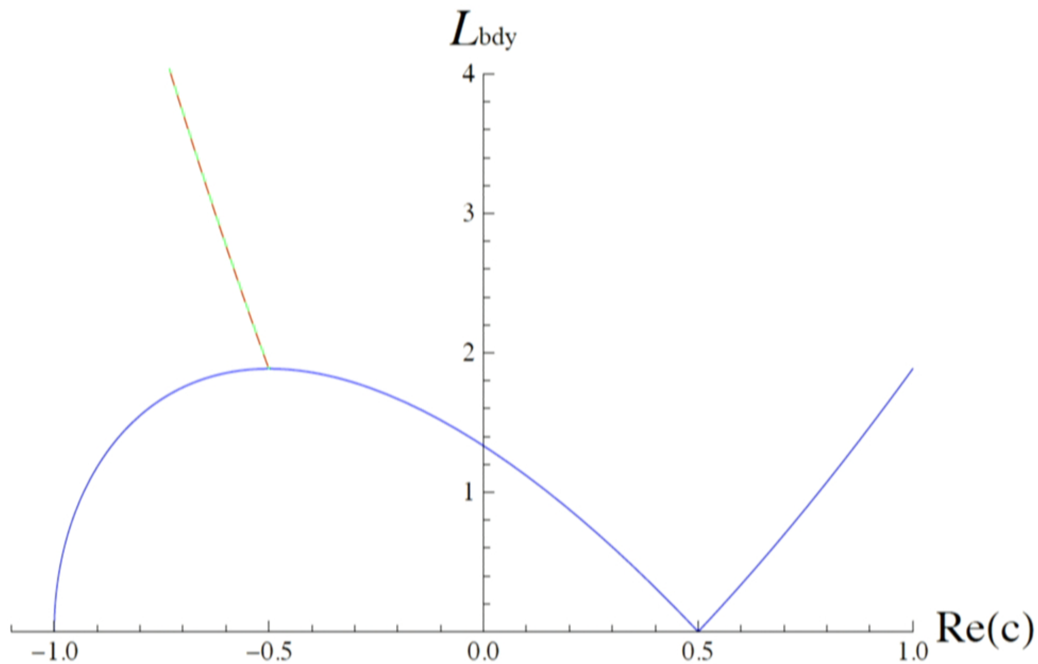

Understanding Big Bang Singularities From Holography
Drama in the Two-Point Correlator
Geodesic Solutions
Can solve geodesic equation for any p analytically, but solution is messy (see Banerjee et al. '15 and upcoming paper).
Mess simplifies for special values of p.
Clean example: $p = -1/2$ in 5+1 bulk dimensions:
$$ds^2 = \frac{-dt^2 + t^{-1}dx^2 + t(dr^2 + r^2 d\Omega_2^2) + dz^2}{z^2}$$

Geodesic Solutions

- Can solve geodesic equation for any p analytically, but solution is messy (see Banerjee et al. '15 and upcoming paper).
- Mess simplifies for special values of p.
- Clean example: $p = -1/2$ in 5+1 bulk dimensions:

$$ds^2 = \frac{-dt^2 + t^{-1}dx^2 + t(dr^2 + r^2 d\Omega_2^2) + dz^2}{z^2}$$

Boundary Separation at $p = -1/2$

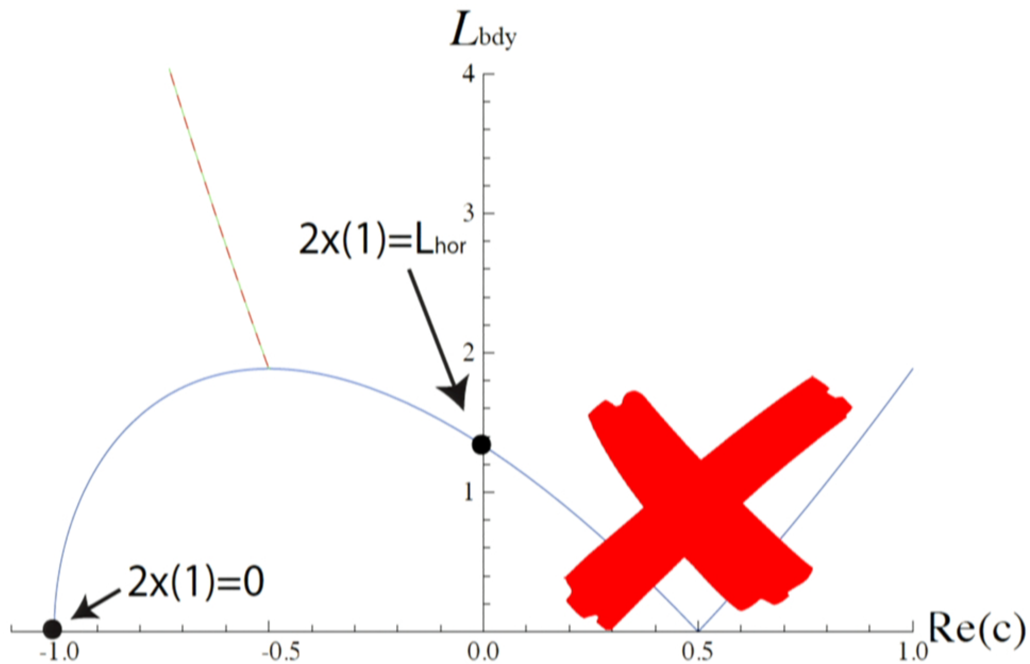


Bad Geodesics

$$x(t) = \pm\frac{2}{3}(-2c + t)\sqrt{c + t}$$

- Geodesics turn around at $t = -c$. When $c > 0$, the geodesics crash into the singularity.
- Geodesics with $c > 0$ not physical: probe non-analytic part of spacetime, cause a divergence in two-point correlator as the separation between the points approaches infinity; this divergence is on a spacelike surface, and is therefore unphysical

Boundary Separation at $p = -1/2$



The Two Point Correlator

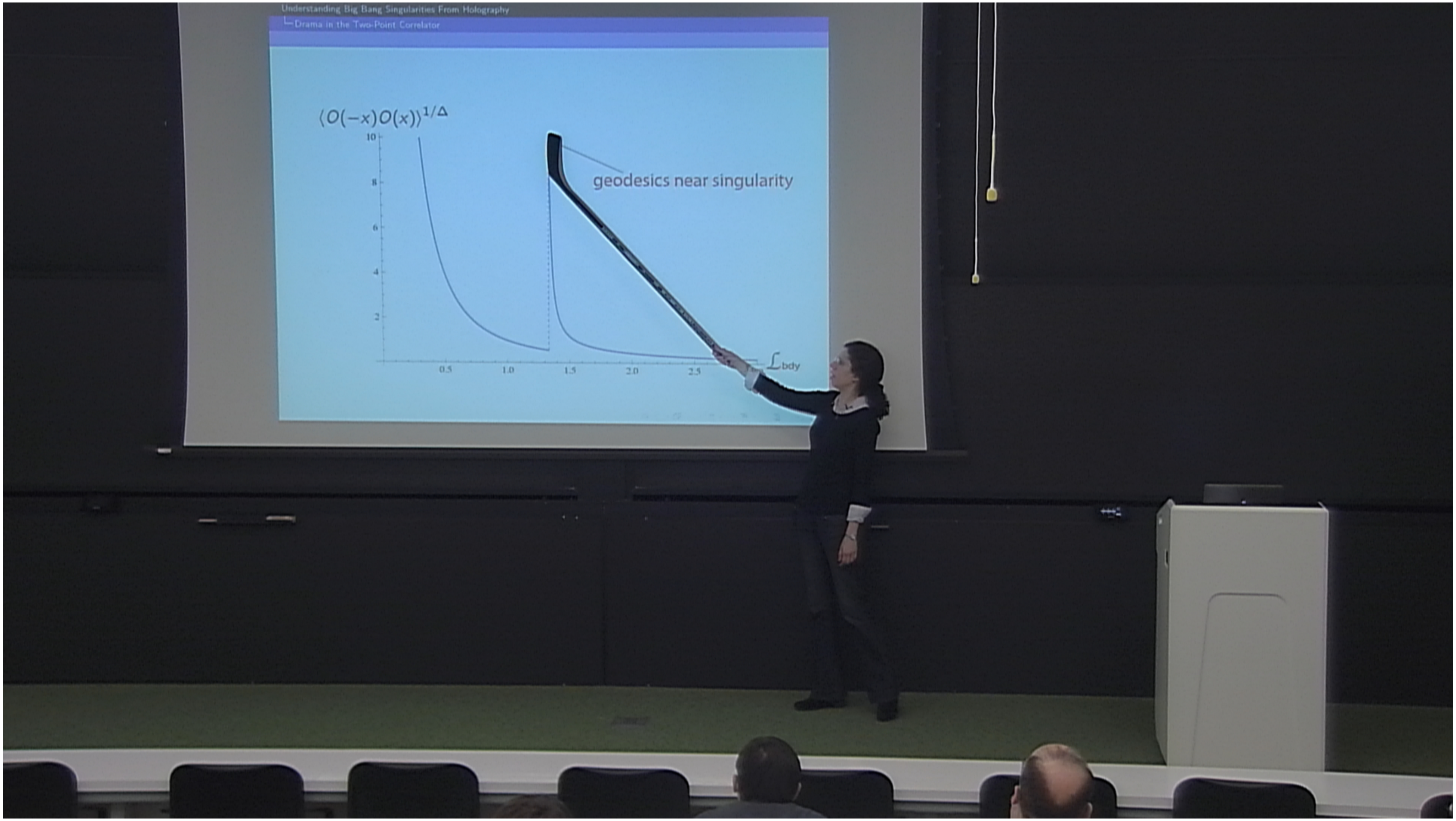
The regulated length is:

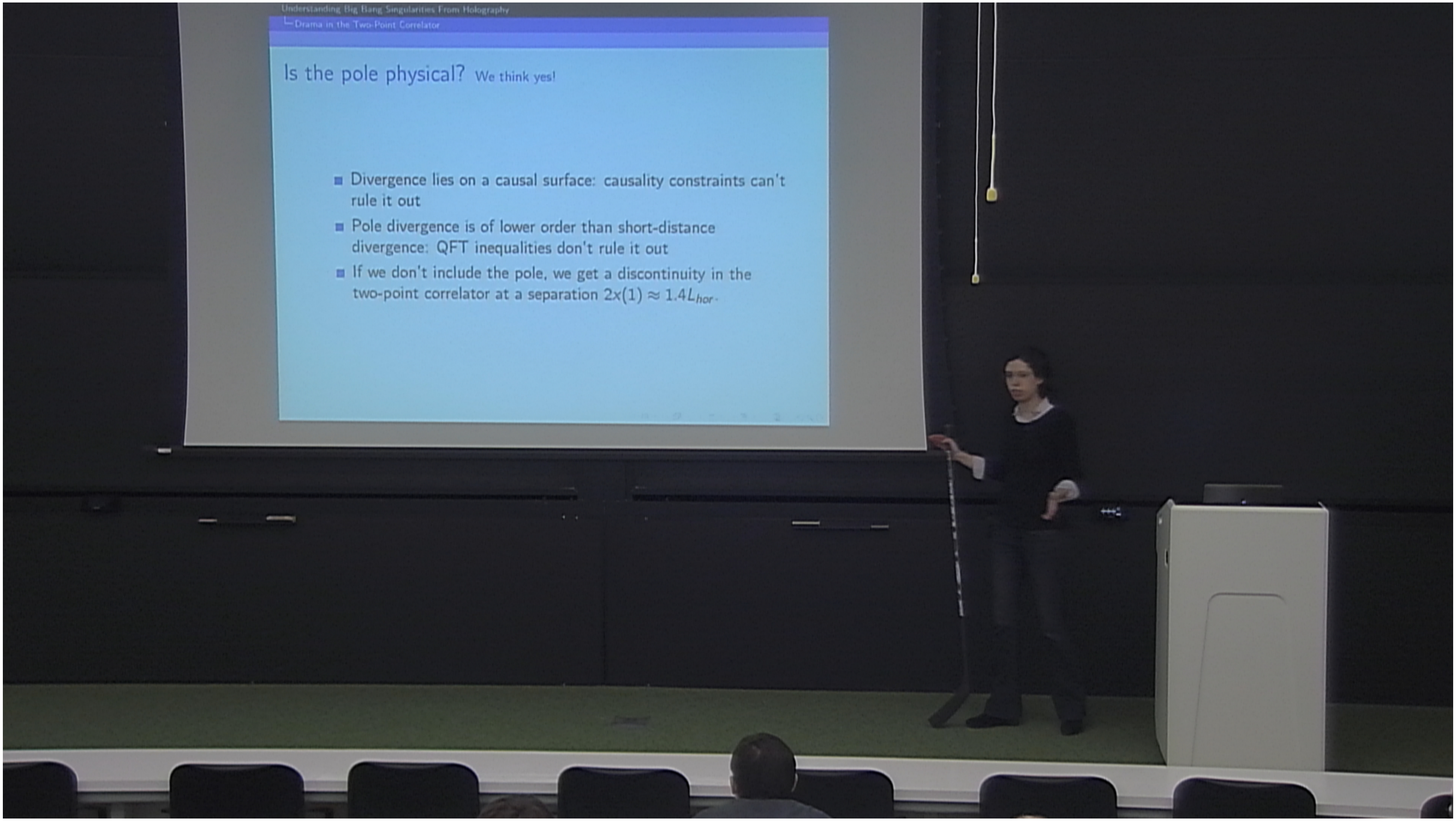
$$L_{reg} = \ln\left[-16c(1+c)\right]$$

With the two point correlator given by

$$\langle \mathcal{O}(-x(1))\mathcal{O}(x(1))\rangle = e^{-\Delta L_{reg}} = \left(\frac{1}{-16c(c+1)}\right)^{\Delta}$$

- Near $c \approx -1$, $\langle \mathcal{O}(-x(1))\mathcal{O}(x(1))\rangle \sim 1/L_{bdy}^{2\Delta}$
- Another divergence at $c = 0$!


Understanding Big Bang Singularities From Holography
Drama in the Two-Point Correlator
⟨O(−x)O(x)⟩^{1/Δ}
10
8
6
4
2
0.5
1.0
1.5
2.0
2.5
geodesics near singularity
L_bdy


Understanding Big Bang Singularities From Holography
Drama in the Two-Point Correlator
Is the pole physical? We think yes!
Divergence lies on a causal surface: causality constraints can't rule it out
Pole divergence is of lower order than short-distance divergence: QFT inequalities don't rule it out
If we don't include the pole, we get a discontinuity in the two-point correlator at a separation $2x(1) \approx 1.4L_{hor}$.

Pole ⇔ Singularity

- Can show that this pole *always* manifests for a two-point correlator with $p < 0$
- Kasner-AdS has a curvature singularity if and only if one of the p's is negative
- Whenever we have a curvature singularity, there exists a direction along which the two-point correlator diverges at L_{hor}
- The pole occurs if and only if there is a curvature singularity in the bulk!

Pole ⇔ Singularity

- Can show that this pole *always* manifests for a two-point correlator with $p < 0$
- Kasner-AdS has a curvature singularity if and only if one of the p's is negative
- Whenever we have a curvature singularity, there exists a direction along which the two-point correlator diverges at L_{hor}
- The pole occurs if and only if there is a curvature singularity in the bulk!

To do:

- Calculate the length of geodesics
- Isolate contribution to length from geodesics in near-singularity region
- Identify signature of near-singularity physics in two-point correlator

Momentum Space Correlator

- For $\Delta = 1$, in momentum space, with small perturbation in $p > 0$ directions:

$$\langle \mathcal{O}(-\mathbf{k})\mathcal{O}(\mathbf{k})\rangle = \frac{1}{(2\pi)^3}\left[\frac{1}{(k_1^2 + k^2)} - 2\cos(k_1 L)(\ln\epsilon) + \text{finite}\right]$$

where k_1 is the momentum in the $p < 0$ direction.

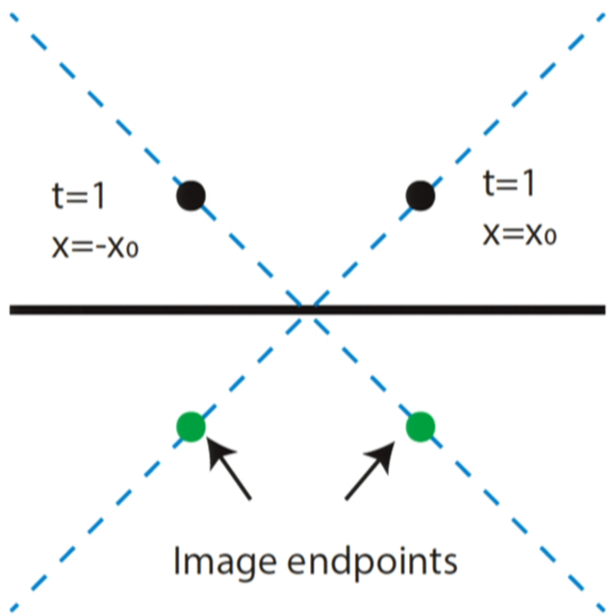
- But the term $-2\cos(k_1 L)\ln\epsilon$ comes from horizon-separation divergence.
- At large Δ, $\ln\epsilon$ will turn into a negative power of ϵ.

What does it mean?

- When $k_1 = (2n+1)\pi/L$, UV-cutoff dependence vanishes
- More generally, UV cutoff contribution oscillates with momentum.
- This doesn't seem possible at weak coupling – in favor of conclusion that stringy effects resolve the singularity

Possible BCFT Interpretation

- Horizon pole can be seen as lightcone singularity in a BCFT setup with Dirichlet boundary conditions:



Summary

- We've found a setup in which geodesics can probe arbitrarily far into large curvature regions
- The large curvature translates into a horizon-scale divergence in the dual CFT two-point function
- This pole is in the two-point correlator whenever the bulk singularity is a genuine curvature singularity
- In momentum space, pole corresponds to oscillations in the UV cutoff
- Pathological behavior at horizon scale seems unlikely for a weakly-coupled field theory