Title: Probabilistic General Relativity with Agency"

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Abstract:

Probabilistic General Relativity with Agency: An operational approach[1]

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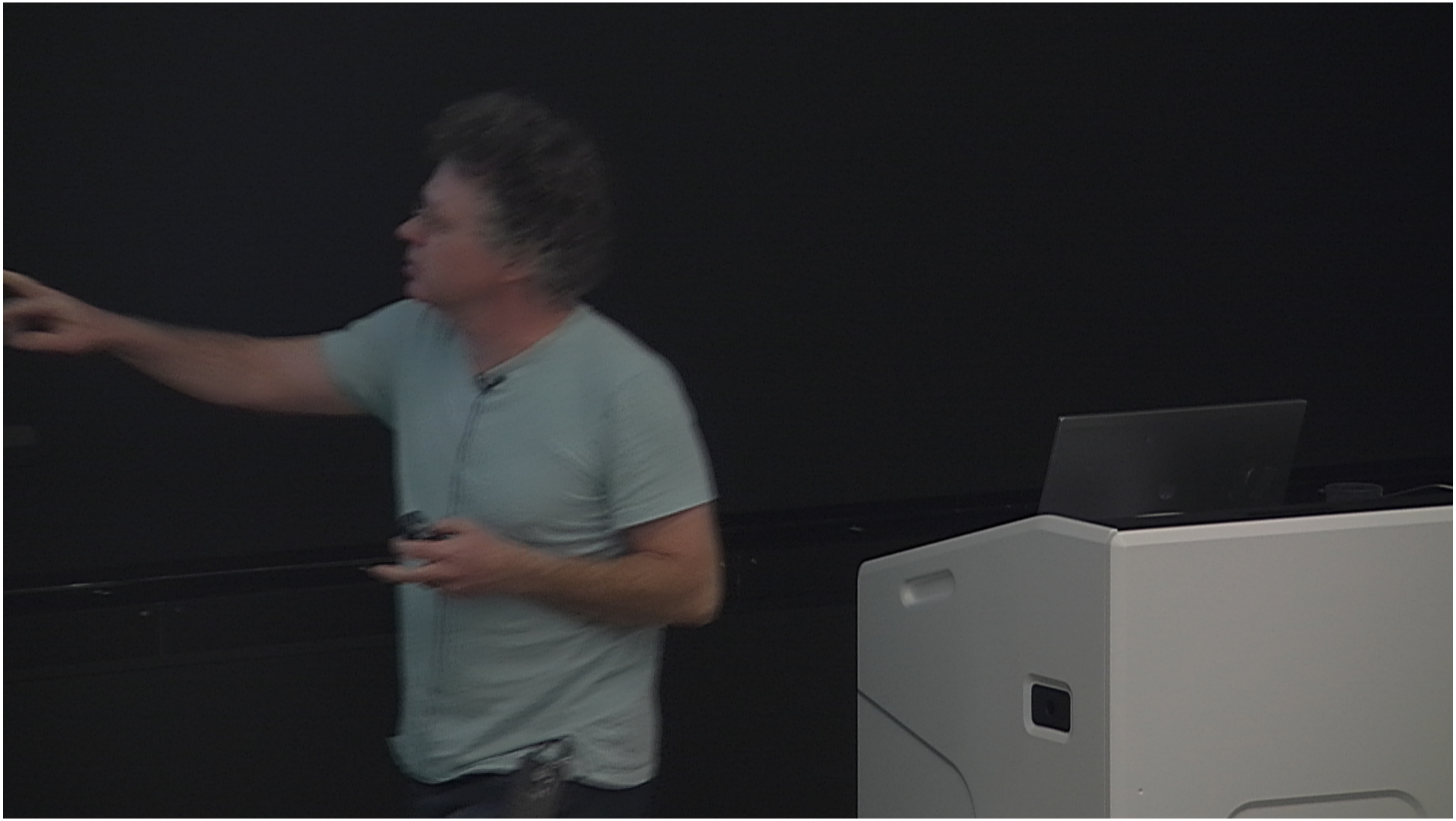
Perimeter Institute, Waterloo, Ontario, Canada

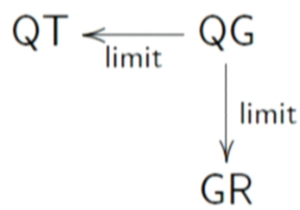
Please be at least as struck by the words *probabilistic* and *operational* as the word *agency*.

[1]Work in progress



The problem of quantum gravity

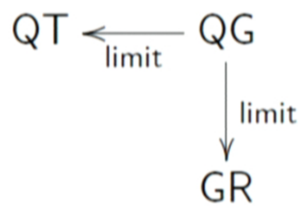
The problem of quantum gravity is to find a theory which reduces, in appropriate limits to quantum theory on the one hand and General Relativity on the other.



Further, we would want the theory to be experimentally verified in new situations where neither QT or GR apply (should such situations exist).

The plan of attack

Identify formulations of QT and GR that should appear as limit theories of QG and work out how to reverse arrows



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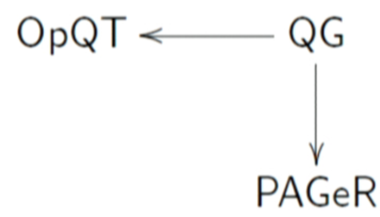

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- PAGeR: Probabilistic General Relativity with Agency

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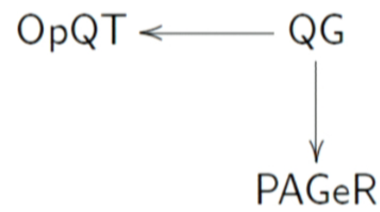

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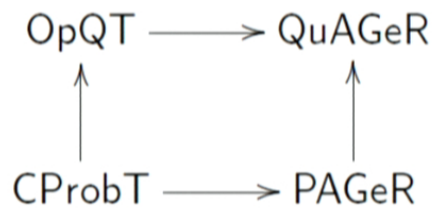

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- CProbT: Classical Probability theory

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$$\begin{array}{ccc} \text{OpQT} & \xrightarrow{\text{GRize}} & \text{QuAGeR} \\ \text{quantize} \uparrow & & \uparrow \text{quantize} \\ \text{CProbT} & \xrightarrow[\text{GRize}]{} & \text{PAGeR} \end{array}$$

- Quantization: simplex $\Longrightarrow$ curved convex set from a Hilbert space.
- GRization: fixed causal structure $\Longrightarrow$ fuzzy causal structure.

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Earlier work

This is part of an ongoing project (papers on the arXiv).

- 2005 The causaloid framework: A framework for probabilistic theories with indefinite causal structure
- 2010 The duotensor framework: A way to do probability theory in a manifestly covariant manner (for circuits but applicable to space-time).

Related work

- Chris Fuchs - Emphasized agent centric approach to Quantum Foundations (QBism)
- Samson Abramsky and Bob Coecke's categorical (pictorial) approach to quantum theory. This emphasizes compositionality.
- Generalized probability theories (going back to Mackey) - much recent work.
- Some space-time approaches to QT:
  - Quantum causal histories - Markopoulou; Dual point of view - Blute, Ivanov and Panangaden;
  - Aharonov and co-workers - multitime states.
  - General Boundary formulation - Oeckl;
  - Quantum combs - Chiribella, D'Ariano, and Perinotti; Oeckl's positive formulation.
  - Various axiomatic approaches to QT (LH, Dakic and Brukner, Masanes Müeller, CDP, ...) (in particular the tomographic locality axiom).
  - Leifer and Spekkens Quantum Bayesian Inference, more recent work by Henson, Lal, and Pusey.
  - Indefinite causal structure: Brukner, Oreshkov, Costa, Cerf

Operational Quantum Theory

- Unsung (working class) hero.
- Developed initially by Sudarshan, Ludwig, Krauss, and many others in 1960's.
- Agency built in (chose knob settings).
- Used to great effect in Quantum Information (if you want to prove a theorem it is good to know what is possible at most general level).
- Have
  - density matrices, ρ
  - completely positive maps $\$(\cdot)$
  - and POVMs, $\{\hat{P}_l\}$.

  where we calculate probabilities with the equation

  $$\text{prob}_l = \text{trace}(\hat{P}_l \$(\rho))$$

- It is wonderfully abstract and dry (enough physics has been removed that you can see the essential QT).
- If you think that all measurements are associated with Hermitian operators then you are just plain wrong.

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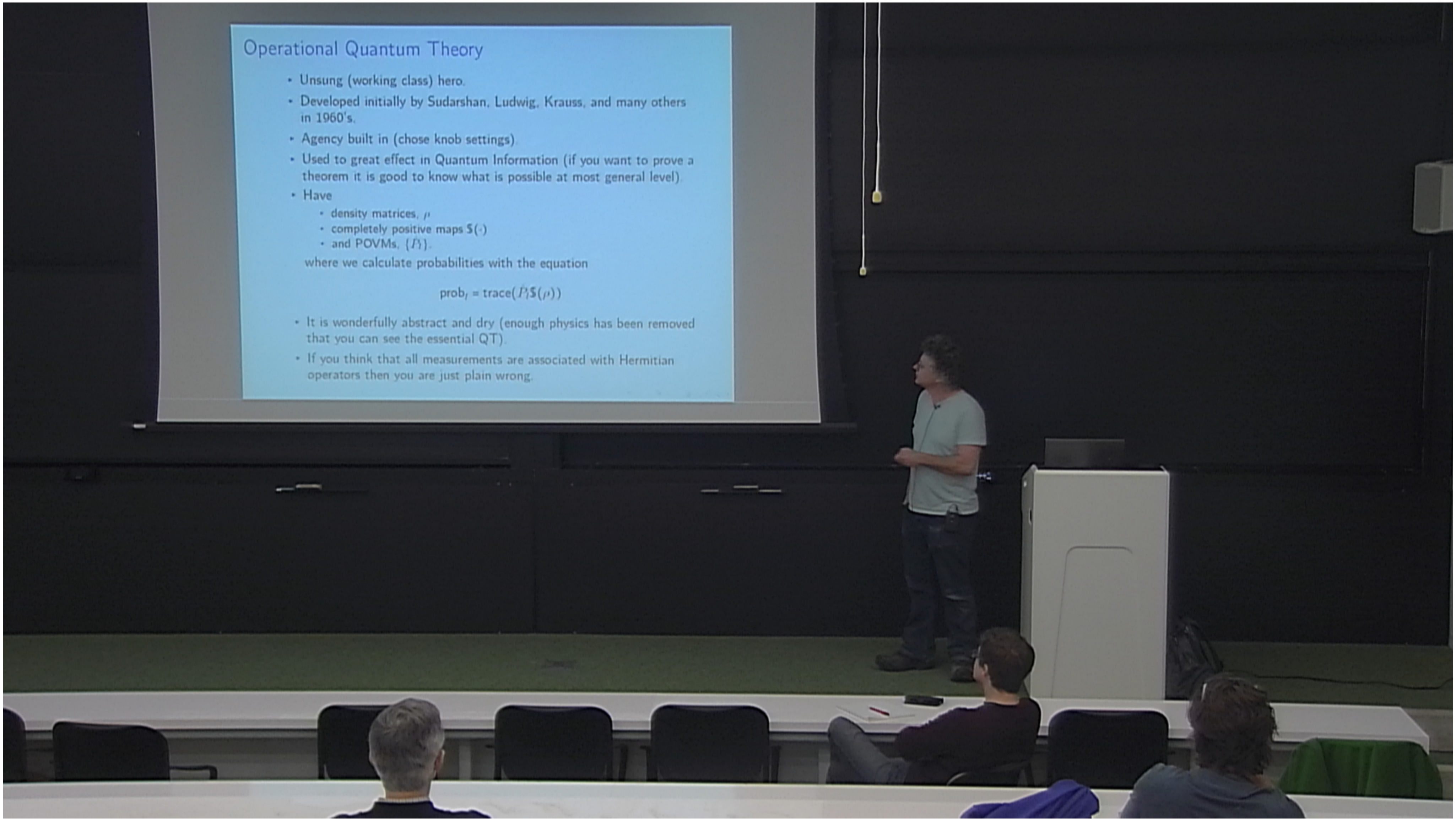
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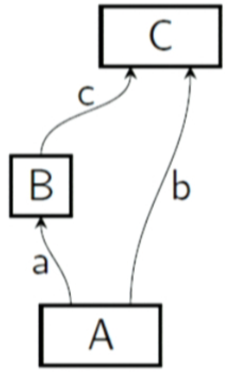
Modern incarnations of OpQT

Modern incarnations due to Chiribella-D'Ariano-Perinotti, LH, and Oeckl.
The Operator Tensor Formulation of QT



$$\Leftrightarrow \quad A^{a_1 b_2} B^{c_3}_{a_1} C_{b_2 c_3}$$

In quantum theory have

$$\text{Prob}(A^{a_1 b_2} B^{c_3}_{a_1} C_{b_2 c_3}) = \hat{A}^{a_1 b_2} \hat{B}^{c_3}_{a_1} \hat{C}_{b_2 c_3}$$

In the notation on the right, the repeated label indicates multiplication and partial trace over the appropriate part of the operator space.
Unlike old operational QT, this approach is manifestly covariant (it does not require a foliation of the circuit). Want to do something similar for GR.

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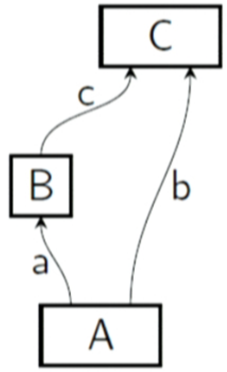

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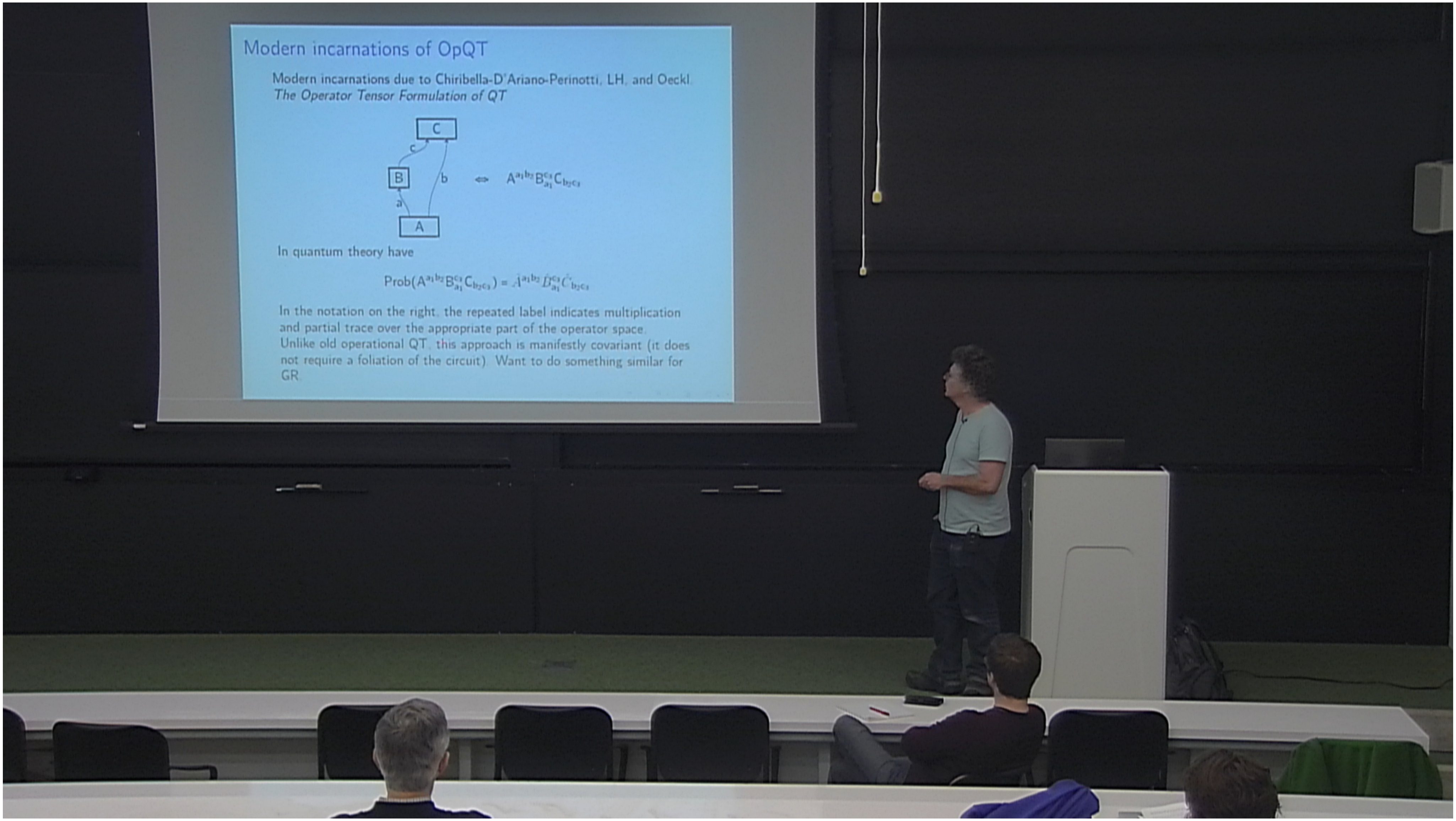
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PAGeR stands for Probabilistic General Relativity with Agency.

Homage to the Blackberry pager

The A in "PAGeR" is out of sequence but this is a selling point since PAGeR will have fuzzy causal structure.

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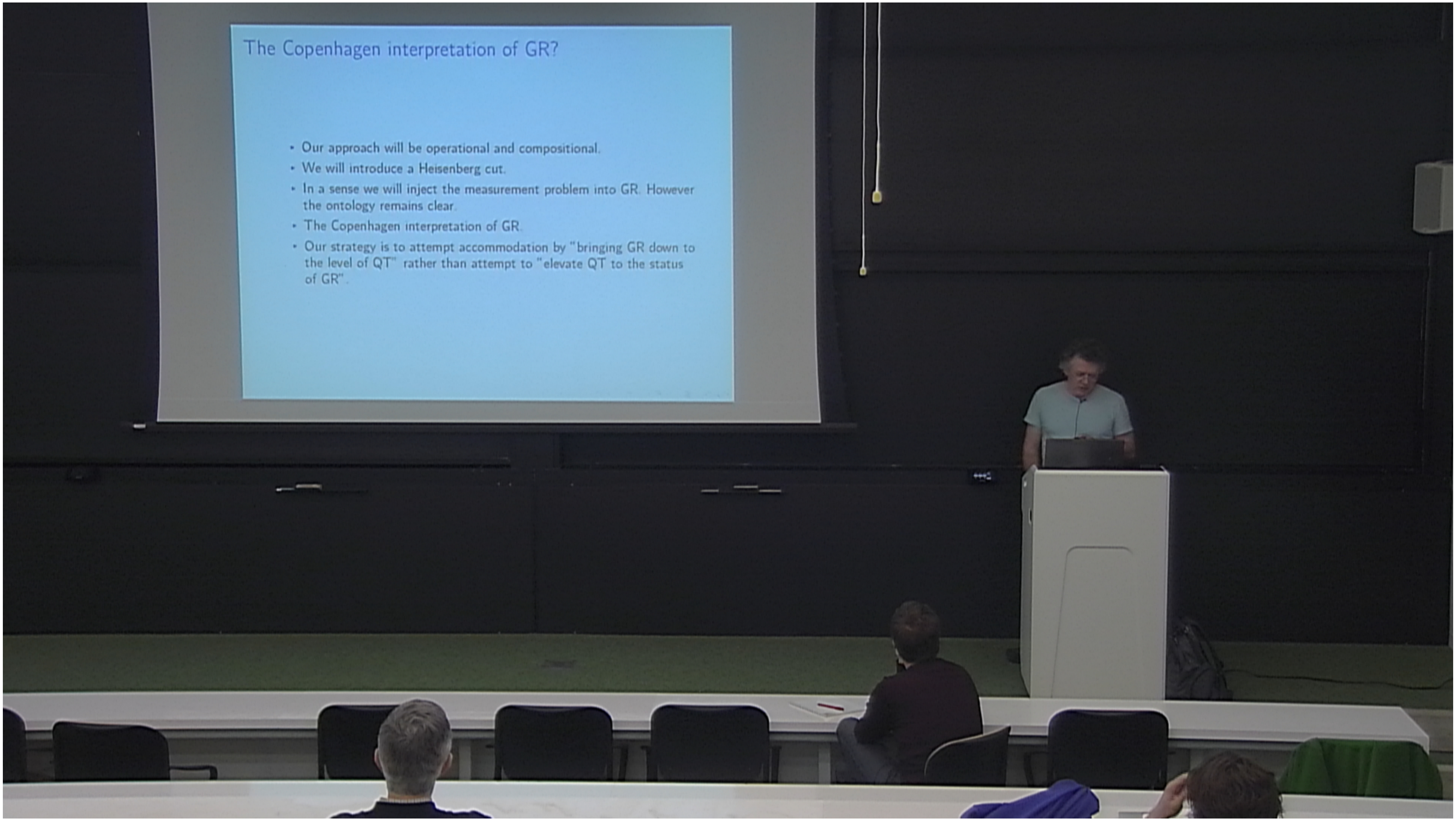
The Copenhagen interpretation of GR?

- Our approach will be operational and compositional.
- We will introduce a Heisenberg cut.
- In a sense we will inject the measurement problem into GR. However the ontology remains clear.
- The Copenhagen interpretation of GR.
- Our strategy is to attempt accommodation by "bringing GR down to the level of QT" rather than attempt to "elevate QT to the status of GR".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General Probability theories

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Standard General Relativity

We can represent a solution in GR by

$$\Psi = \left((p, \vec{\mathbf{\Phi}}(p)) : \forall p \in \mathcal{M}\right)$$

where $\vec{\mathbf{\Phi}} = (\mathbf{g}, \vec{\varphi})$ is a list of tensors associated with the various fields (metric and matter) for the physical situation we are considering.

Everything is built out of fields (test particles, clocks, ...).

This solution must satisfy a set of coupled PDE's

- The matter equations obtained from SR by comma to semicolon rule and $\eta_{\mu\nu}$ to $g_{\mu\nu}$.
- The Einstein field equations (since now we have $g_{\mu\nu}$).

Beables are given by functions on Ψ that are invariant under diffeomorphisms.

A big problem with beables is that they are nonlocal on the manifold. This makes it difficult to picture what the real physics is.

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The Westman Sonego formulation

An attempt to provide a diffeomorphism invariant presentation of GR.

Illustrate with their example. Consider where we have fields

- Metric, $g_{\mu\nu}$.
- Electromagnetic field, $F_{\mu\nu}$.
- Chemical currents, $j^{\mu}[a]$ (where a runs over the elements).

We can construct a multitude of scalars from these

$$g_{\mu\nu} j^{\mu}[a] j^{\nu}[b], \quad F_{\mu\nu} F^{\mu\nu}, \quad \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

Denote these scalars by list

$$\vec{\theta} = (\theta^{A} : A = 1 \text{ to } K)$$

where K is the number of scalars.

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Can define further WS scalars

$$j^A[a] = \frac{\partial \theta^A}{\partial x^\mu} j^\mu[a], \qquad g^{AB} = \frac{\partial \theta^A}{\partial x^\mu}\frac{\partial \theta^B}{\partial x^\nu} g^{\mu\nu}, \qquad F^{AB} = \frac{\partial \theta^A}{\partial x^\mu}\frac{\partial \theta^B}{\partial x^\mu} F^{\mu\nu}$$

Physics is messy. Hence, generically, $\vec{\theta}$ different for each $p \in \mathcal{M}$ (non-degenerate case). Then can invert these to get tensor fields back. Let $\vec{\vartheta}$ include scalars in $\vec{\theta}$ and the WS scalars. Can plot points in a solution, Ψ, into this WS-space.

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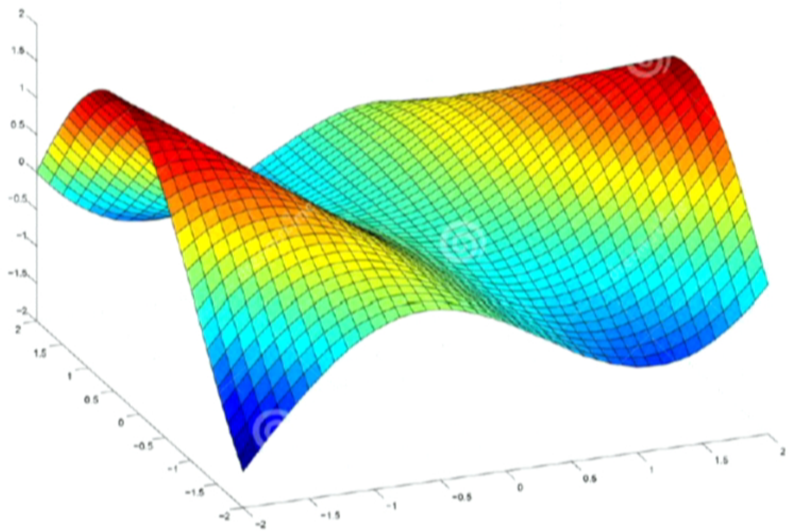
Generically, will get 4-dimensional surface (can pinch down to less than 4-dim, but cannot have dimension greater than 4).



This surface is invariant under diffeomorphisms.
The points in the surface are parameterized by the points in $\mathcal{M}$.
There is a problem, however, when we have degeneracy so still need standard GR.

Beables: observables and hidden variables

Heisenberg cut

$$(\text{beables}) = (\text{observables}) \times (\text{hidden variables})$$

What can we observe?

Assertion 1: *We can only directly observe coincidences between scalars having specified values.*

In particular, we *nominate* an ordered set of scalar fields from which we will build observables

$$\mathbf{S} = (S[k](x) : k = 1 \text{ to } K)$$

We call the space of possible $\mathbf{S}$ the *observable scalar coincidence space* (OSC space). This is the space we "live in" - the space we see.
Observables built out of points in OSC space.
Restores a kind of locality. Our observables are local in OSC space.

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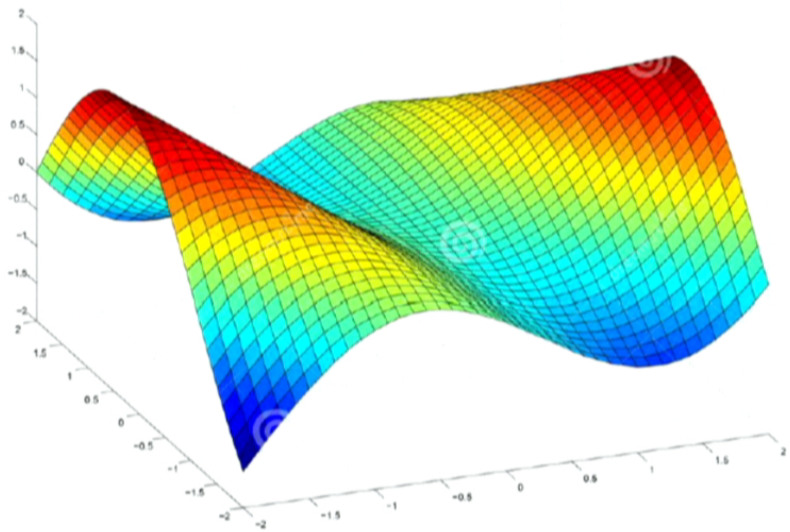

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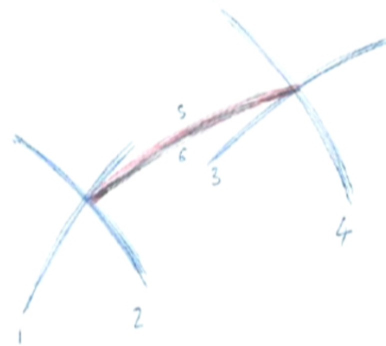
Hidden variables

Example - fluid blobs.

Extremal distance is a beable but not observable.



We can determine extremal distance from mixing in fifth blob.

In four blob example, extremal distance is a hidden variable.

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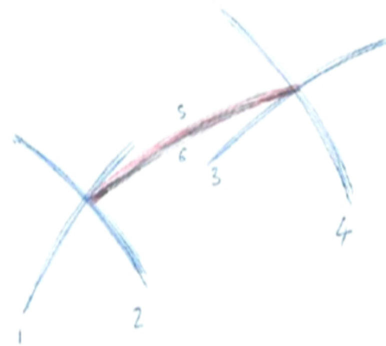

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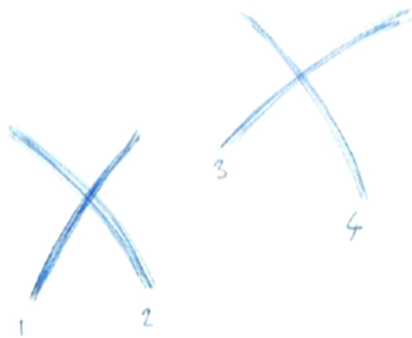

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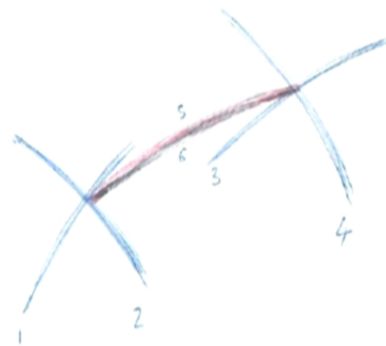

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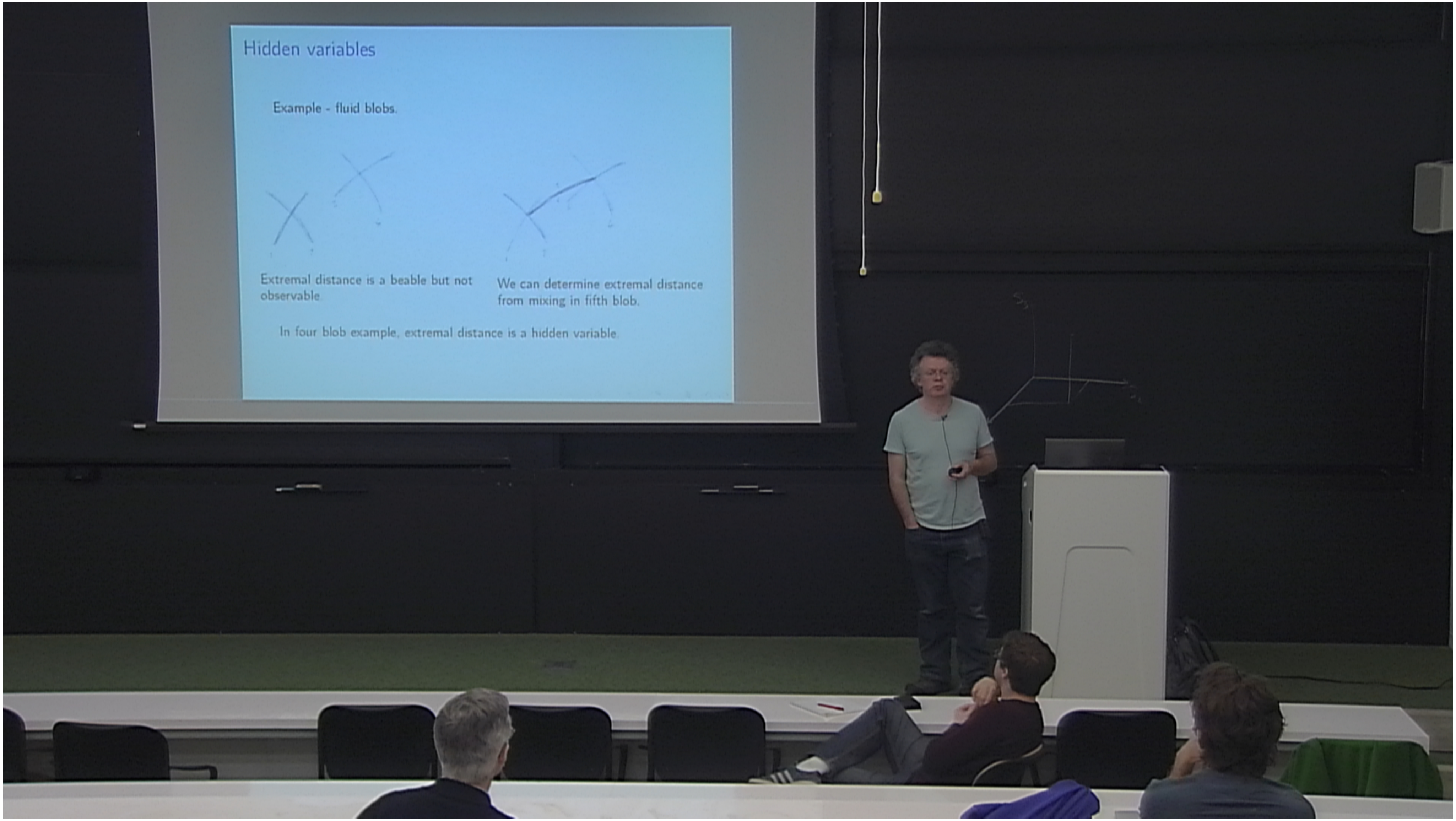
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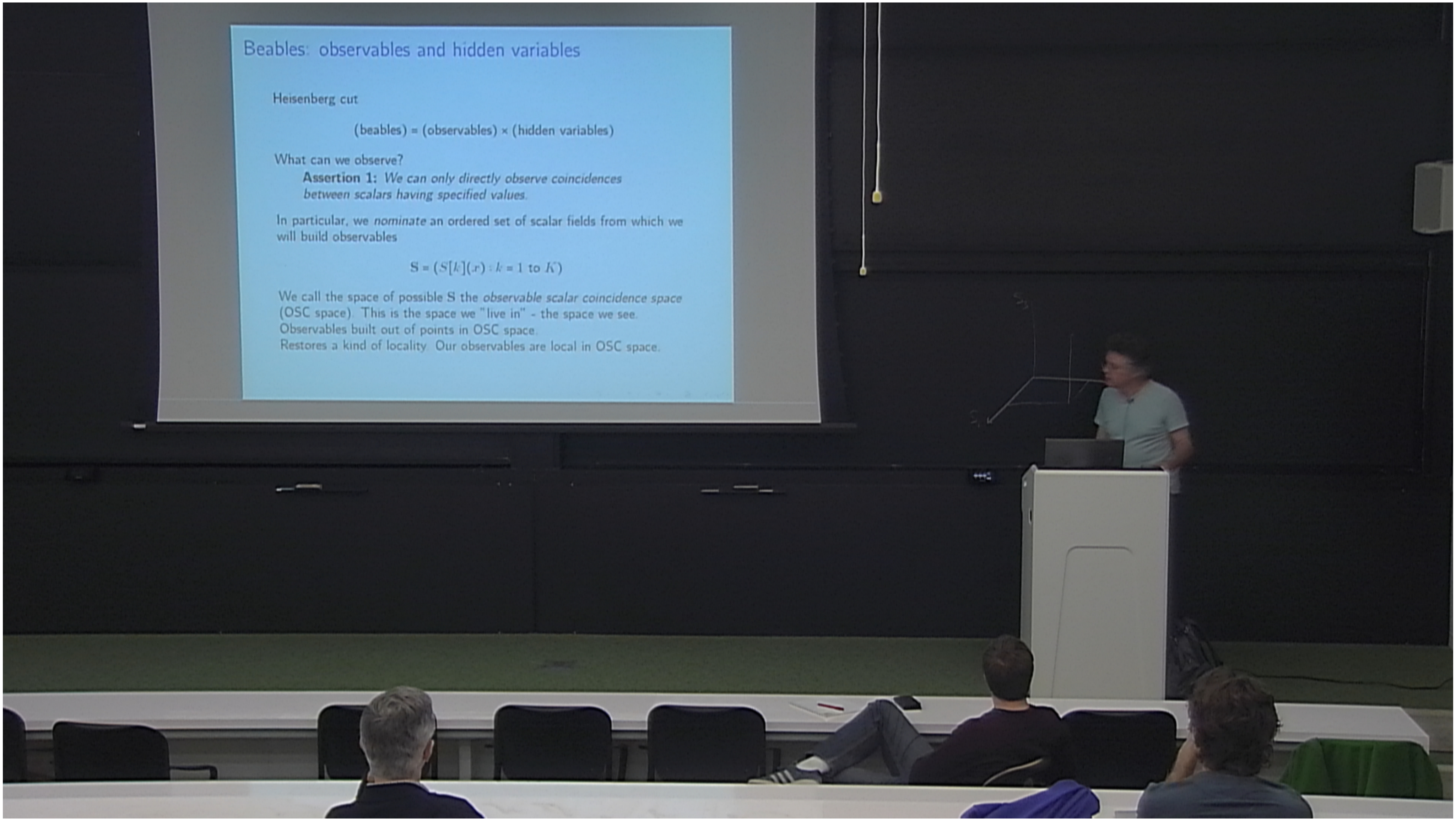
Beables: observables and hidden variables
Heisenberg cut
(beables) = (observables) × (hidden variables)
What can we observe?
Assertion 1: We can only directly observe coincidences between scalars having specified values.
In particular, we nominate an ordered set of scalar fields from which we will build observables
S = (S[k](x) : k = 1 to K)
We call the space of possible S the observable scalar coincidence space (OSC space). This is the space we "live in" - the space we see.
Observables built out of points in OSC space.
Restores a kind of locality. Our observables are local in OSC space.


Hidden variables
Example - fluid blobs.
Extremal distance is a beable but not observable.
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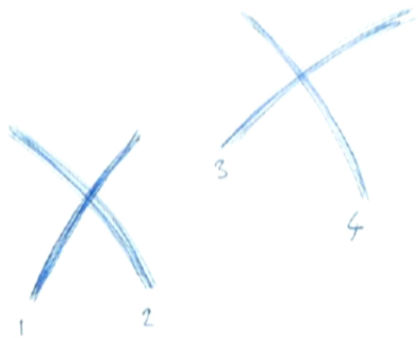

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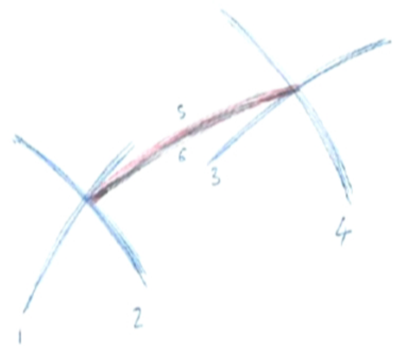

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Turning a solution inside out

- We write

$$\boldsymbol{\Phi}(p) \leftrightarrow (\mathbf{S}(p), \boldsymbol{\omega}_{\boldsymbol{\Phi}}(p))$$

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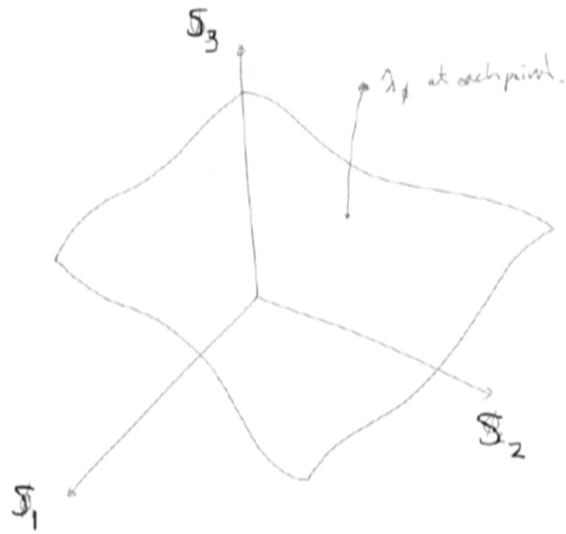
Inside out solution



$$\Psi = \left(\left(\mathbf{S}, \boldsymbol{\lambda}_{\boldsymbol{\Phi}}(\mathbf{S}) \right) : \forall \mathbf{S} \in \Gamma \right)$$

Γ is independent of diffeomorphisms but $\boldsymbol{\lambda}_{\boldsymbol{\Phi}}$ is not.

Introducing Agency

Common in physics: Knob settings, external forces, initial conditions. . . .

Conservative approach: agency corresponds to effective level of description. There are certain processes whereby below resolution effects are magnified to being above resolution that can be thought of as acts of agents.

Example. Fleet of spaceships (a dust) with sails and a wind (dust)

$$\nabla_\alpha T^{\mu\alpha}[\text{ship}] = G^\mu[\text{ship}] \qquad \nabla_\alpha T^{\mu\alpha}[\text{wind}] = G^\mu[\text{wind}] = -G^\mu[\text{ship}]$$

We could have

$$G^\mu[\text{ship}] = \chi^{\mu\alpha} U^\beta[\text{ship}] T_{\alpha\beta}[\text{wind}]$$

where the *agency field* $\chi^{\mu\alpha}$ depends on the orientation of the sail. If we knew $\chi^{\mu\alpha}$ could solve . . .

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Ontic Propositions in GR

These are logical statements that are true, or not, of what is happening inside the volume, Υ, of the OSC space.

We can form the basic ontological proposition

$$\mathsf{Prop}^{E}_{\Upsilon}[\tilde{\Psi}^{q}_{\Upsilon}|\boldsymbol{\chi}^{\mathbf{Y}}_{\Upsilon}]$$

This is the proposition that, in the region Υ of the OSC space, the beables under the purview of experiment E are fully described by $\tilde{\Psi}^{q}_{\Upsilon}$ given choice of agency strategy $\boldsymbol{\chi}^{\mathbf{Y}}_{\Upsilon}$ in Υ.

We can also form course-grained ontic propositions

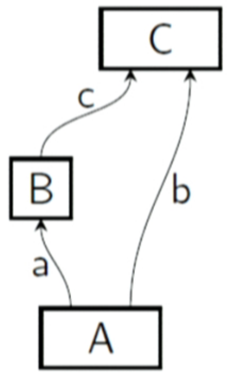
$$\mathsf{Prop}^{E}_{\Upsilon}[\{\tilde{\Psi}^{q}_{\Upsilon} : \forall\ q \in Q\}|\boldsymbol{\chi}^{\mathbf{Y}}_{\Upsilon}] := \bigoplus_{q\in Q} \mathsf{Prop}^{E}_{\Upsilon}[\tilde{\Psi}^{q}_{\Upsilon}|\boldsymbol{\chi}^{\mathbf{Y}}_{\Upsilon}]$$

This is the proposition that one (and only one) of the propositions labeled by q is true (here $\oplus$ stands for exclusive or).

Principle of General Compositionality

Principle of general compositionality: Any composite physical description can be mapped into a calculation having the same structure.

An example: The Operator Tensor Formulation of QT.



$$\Leftrightarrow \quad A^{a_1 b_2} B^{c_3}_{a_1} C_{b_2 c_3}$$

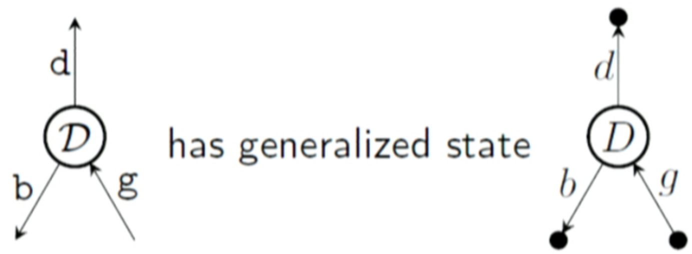
In quantum theory have

$$\text{Prob}(A^{a_1 b_2} B^{c_3}_{a_1} C_{b_2 c_3}) = \hat{A}^{a_1 b_2} \hat{B}^{c_3}_{a_1} \hat{C}_{b_2 c_3}$$

In the notation on the right, the repeated label indicates multiplication and partial trace over the appropriate part of the operator space.
We map components to *generalized states*.

Using the duotensor framework

Need to associate generalized state with every operational proposition using machinery of duotensor framework.



Here,

$$b = \text{cond}(\mathtt{b}), \quad d = \text{cond}(\mathtt{d}), \quad g = \text{cond}(\mathtt{g})$$

are the boundary conditions that have to be matched at the typing surfaces. The above returns a probability density associated with having these conditions be true at the typing boundaries.

Must have same probability density for boundary conditions mapped into one another by ϕ_σ.

If we have caps, then we also impose some distribution on those surfaces.

Two and a half roads to QuAGeR

1. Simplex to Hilbert space derived convex sets.
2. Just maybe, maybe, PAGeR reduces to OpQT (has many QT like properties). Then PAGeR would be QuAGeR.
3. Slightly modify foundations of PAGeR - break transitivity of coincidence assumption for example.

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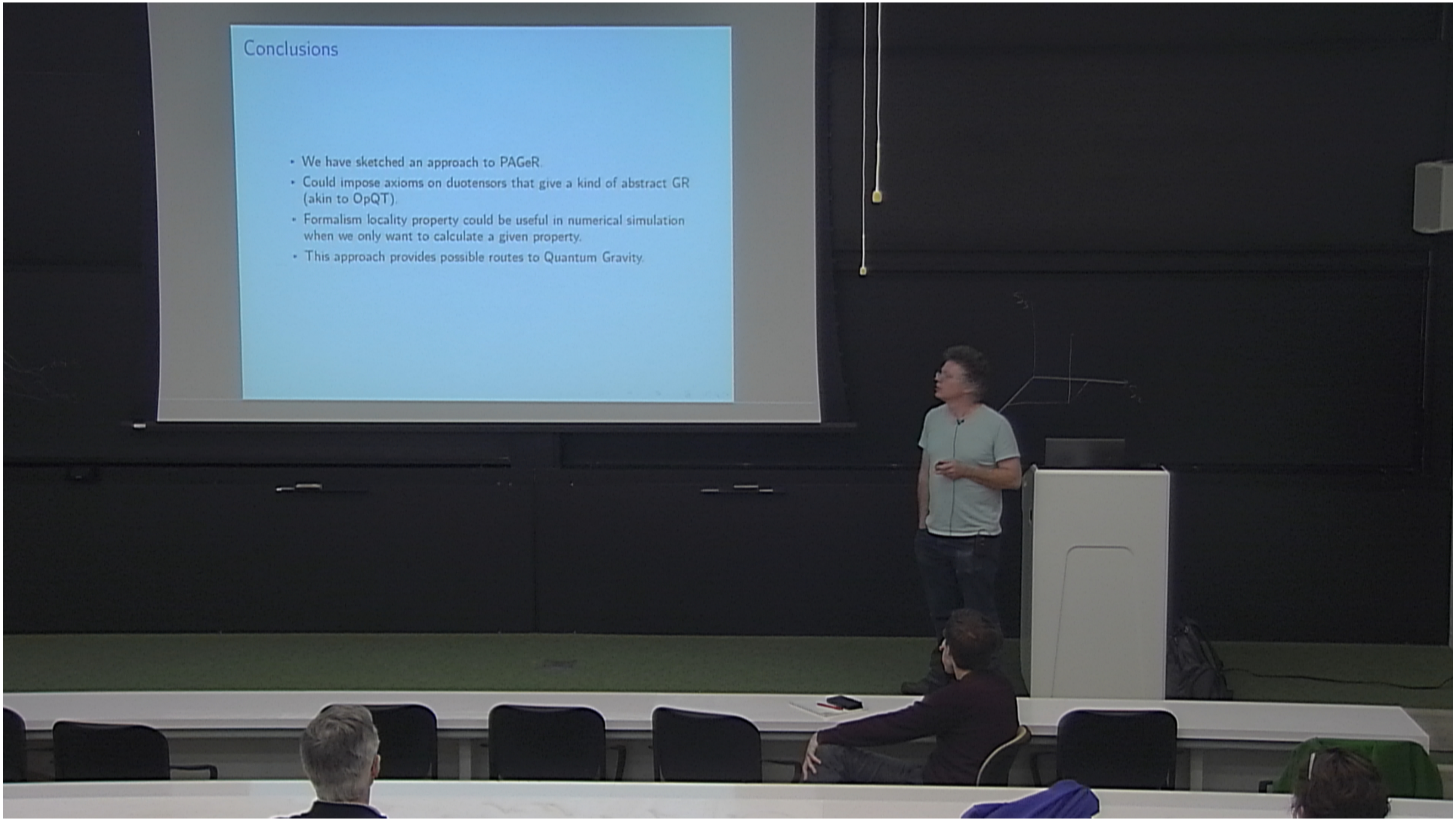
Conclusions

- We have sketched an approach to PAGeR.
- Could impose axioms on duotensors that give a kind of abstract GR (akin to OpQT).
- Formalism locality property could be useful in numerical simulation when we only want to calculate a given property.
- This approach provides possible routes to Quantum Gravity.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A generalized state is probability density for each basic *boundary defined ontological encapsulated proposition* consistent with the given operational encapsulated proposition.

What determines the particular probabilities is a matter for discussion (models of measurement apparatuses, statistical physics, . . .).