Title: The renormalization group: a tensor network perspective

Date: Mar 12, 2015 02:30 PM

URL: http://pirsa.org/15030091

Abstract: <p>A renormalization group transformation implements a scale transformation while resetting the UV cut-off, so that the theories before and after the RG transformation contain the same degrees of freedom, but with a modified action. In Wilson's original proposal, the cut-off is reset by integrating out the degrees of freedom between the old and new cut-off. In recent years we have learned that, alternatively, one can decouple these degrees of freedom by means of local unitary transformations (disentanglers). The result is a reversible renormalization group flow, and an efficient description of many-body ground states. In this talk, I will show you pretty drawings and will use fancy names, such as "multi-scale entanglement renormalization ansatz" (MERA). MERA is currently studied as a possible lattice realization of the AdS/CFT correspondence. I will keep it simple.</p>

INTERDISCIPLINARY QUANTUM GRAVITY SEMINAR, PERIMETER MARCH 12TH 2015

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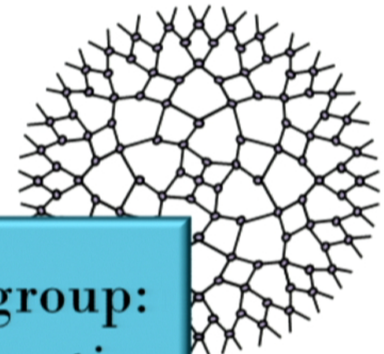
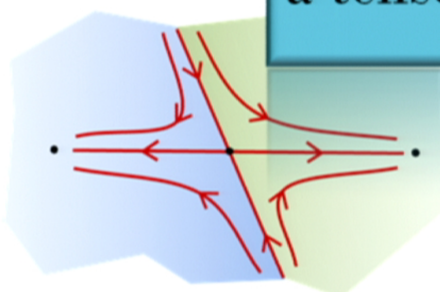
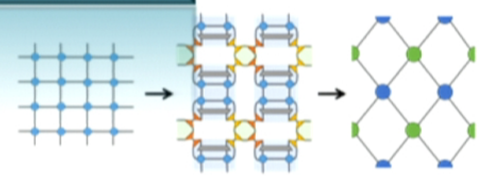


The renormalization group: a tensor network perspective





Guifre Vidal

PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS

SIMONS FOUNDATION

JOHN TEMPLETON FOUNDATION

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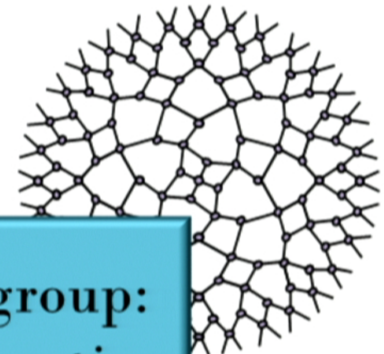

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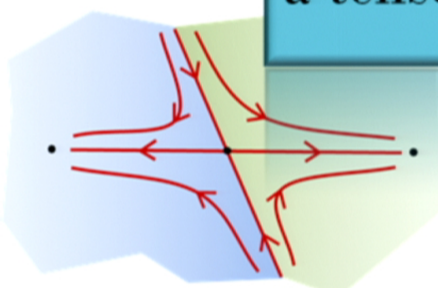

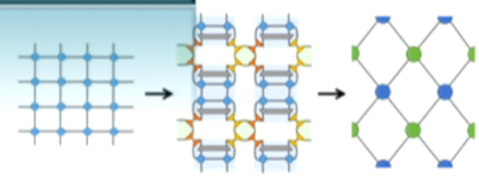

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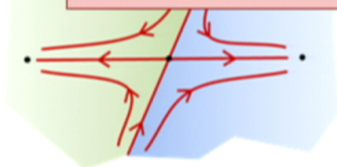
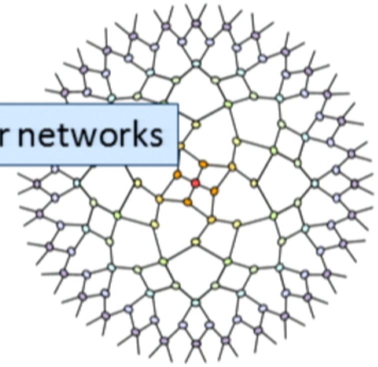
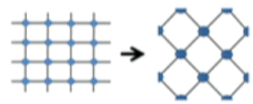
Renormalization group



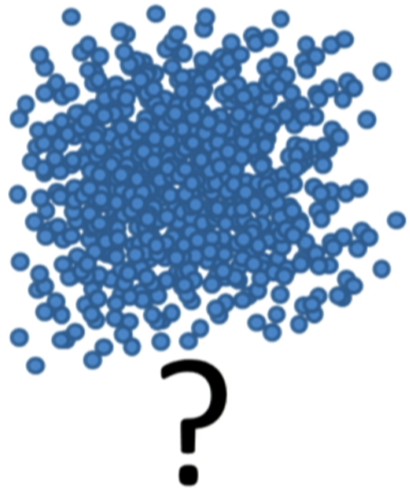
Tensor networks



Tensor network renormalization (TNR)



Emergent phenomena
in many-body systems



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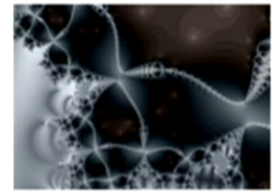
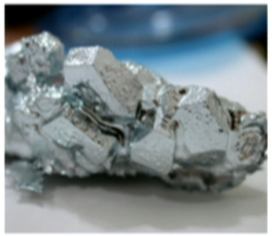
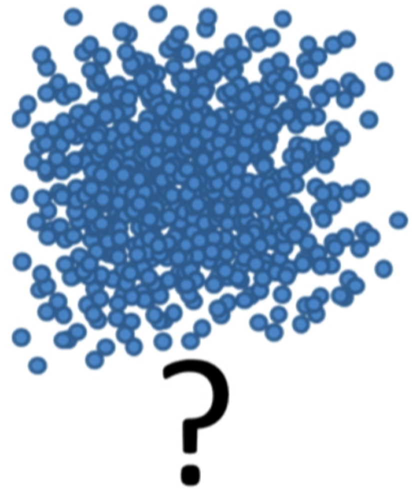
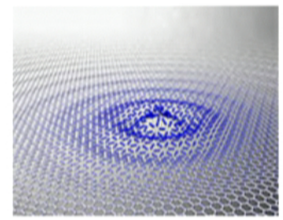


quantum criticality



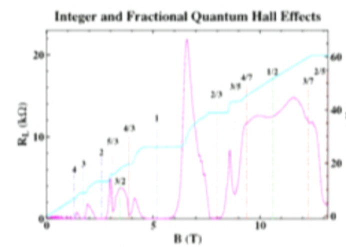
metal



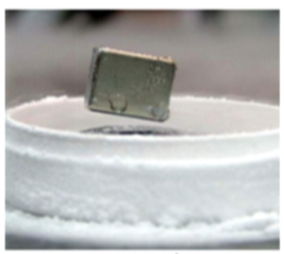


topological order

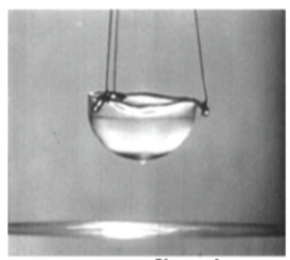
insulator



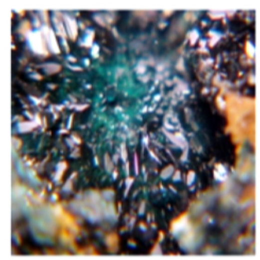
fractional quantum
Hall effect



superconductor



superfluid



spin liquid

The problem

we have

H

local Hamiltonian

we want

$|\Psi\rangle$

ground state

$\langle\Psi|o(x,t)o(0,0)|\Psi\rangle$

low energy,
collective excitations, ...

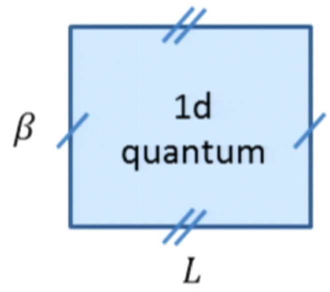
$Z = tr\ e^{-\beta H}$

Euclidean path integral

Example: 1d quantum Ising model

$$H_q^{1d} = \sum_i \sigma_i^x \sigma_j^x + \lambda \sum_i \sigma_i^z$$

$$Z(\lambda) = tr\, e^{-\beta\, H_q^{1d}}$$



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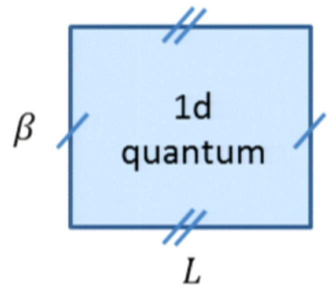

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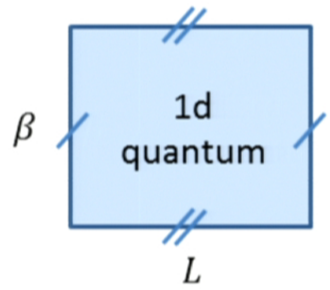

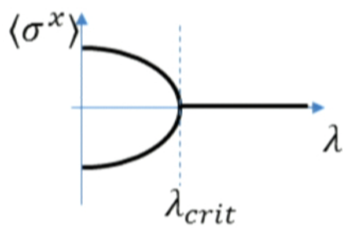

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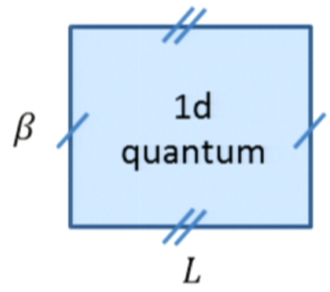
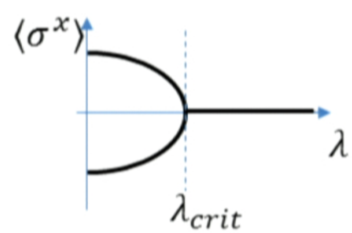


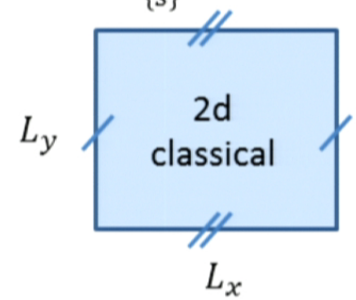


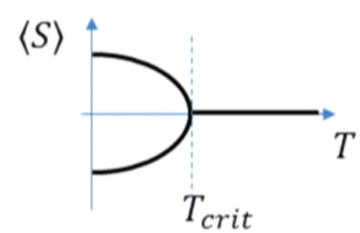
~

2d classical Ising model

$$H_{cl}^{2d} = \sum_{<i,j>} S_i S_j$$

$$Z(T) = \sum_{\{s\}} e^{-\frac{1}{T} H_{cl}^{2d}}$$





The Renormalization Group

$H[\vec{k}]$ $\vec{k} = (k_1, k_2, k_3, \cdots)$

Leo Kadanoff

Kenneth Wilson

The Renormalization Group

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$$H[k_1, k_2, 0, 0, \cdots] = k_1 \sum_i \sigma_i^x \sigma_j^x + k_2 \sum_i \sigma_i^z$$

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The Renormalization Group

$$H[\vec{k}] \longrightarrow H[\vec{k}(s)] \qquad \vec{k} = (k_1, k_2, k_3, \cdots)$$

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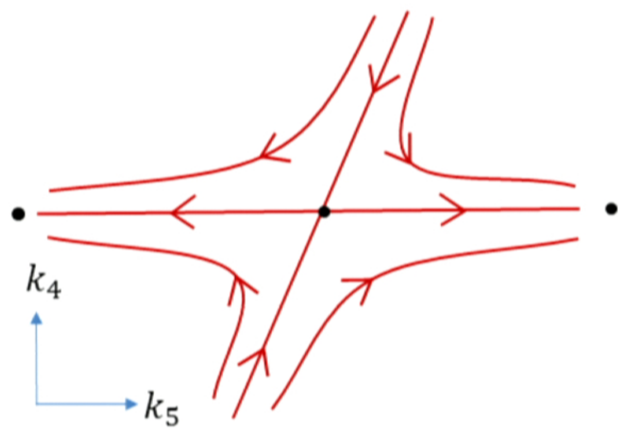
RG flow in the space of Hamiltonians



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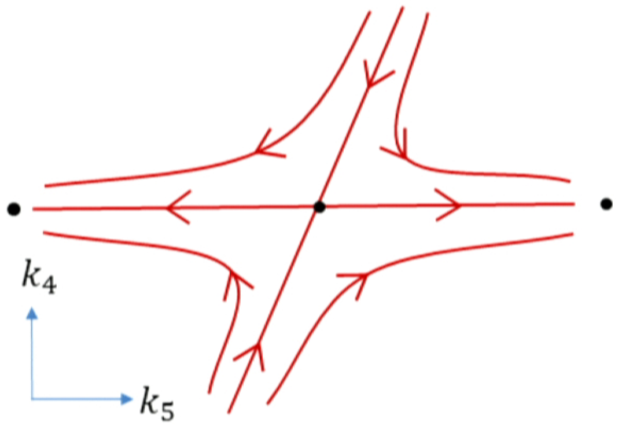

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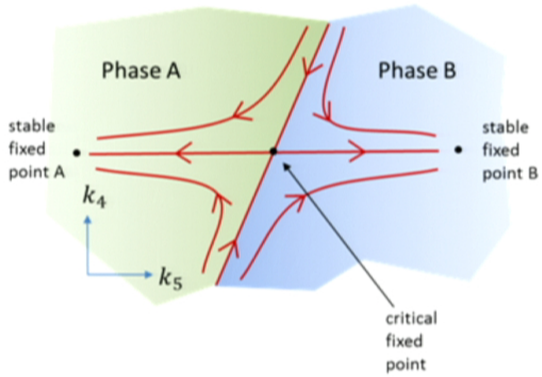

Change of scale?

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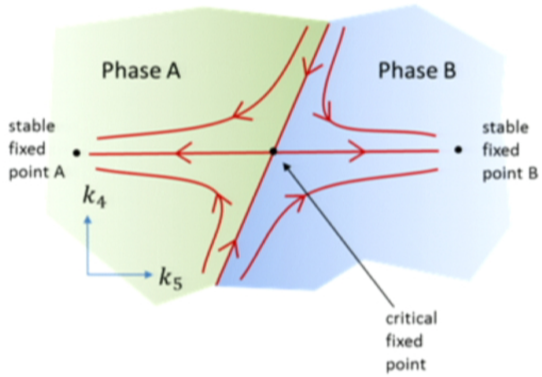

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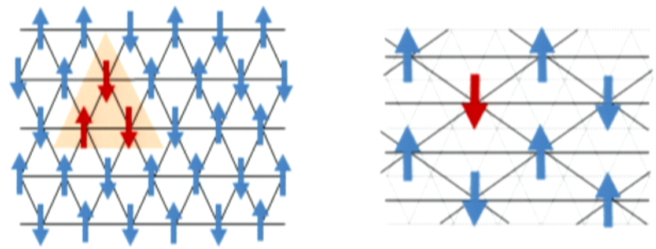

block spin

+ some rule: majority vote, etc

Change of scale?

coarse-graining transformation

Kenneth Wilson

$$Z = \int_{|p|\leq\Lambda} \mathcal{D}\phi e^{-H[\phi,\vec{k}]}$$

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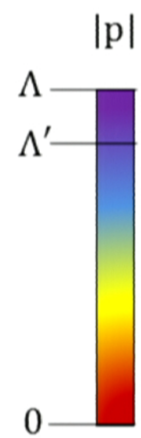
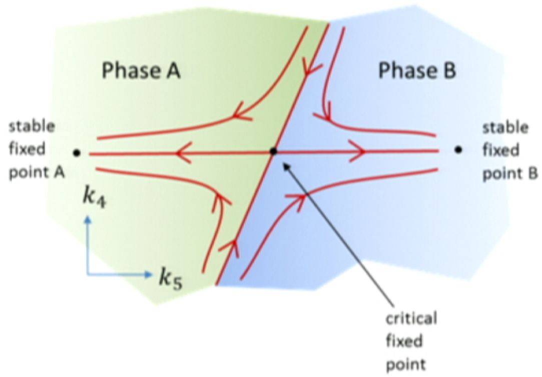


exact renormalization group equation (ERGE)

RG flow in the space of Hamiltonians



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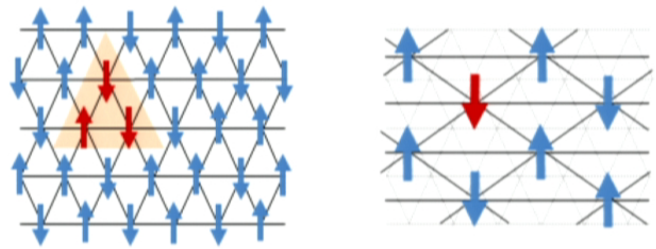

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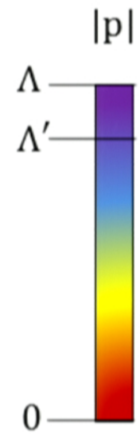

exact renormalization group equation (ERGE)

We would like (wish list)

- Non-perturbative RG approach $H \to H' \to H'' \to \cdots$ (local)
- Universal coarse-graining rules valid for a generic system
- Solve QCD ... !

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How far did we get ? (over the last 10 years)

- Reformulated the RG using quantum information tools/concepts (quantum circuits, entanglement)
- Efficient representation of ground state wave-functions $|\Psi\rangle$ (MERA)

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universal, non-perturbative, real-space RG approach!

TENSOR NETWORKS

Many-body wave-function of N spins

$$|\Psi\rangle = \sum_{i_1, i_2, \cdots, i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

2^N
parameters

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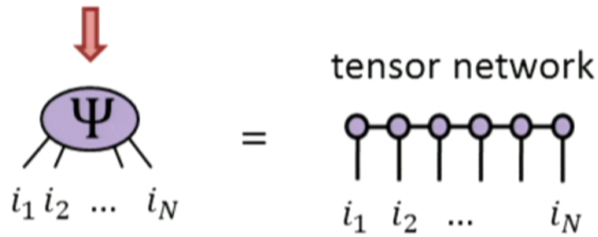
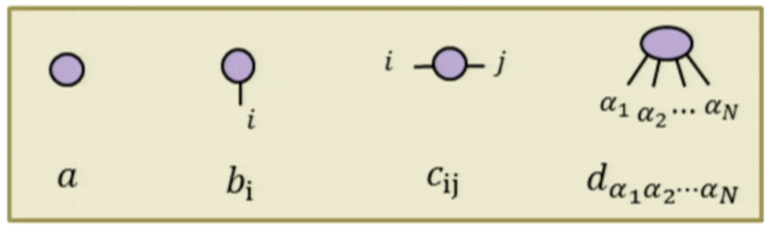


graphical notation



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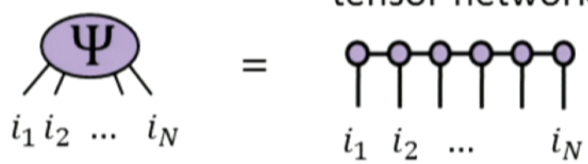

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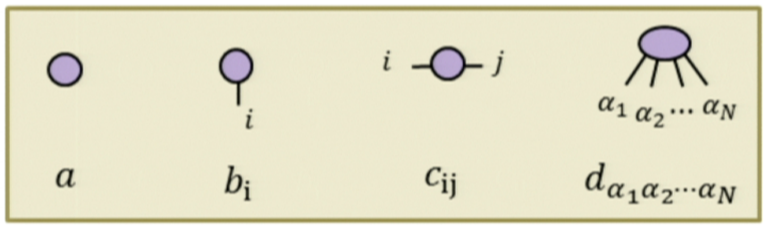

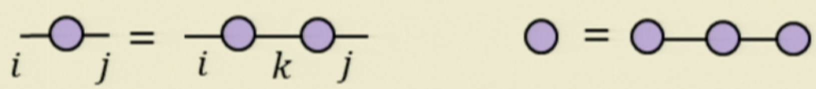

$$T_{ij} = \sum_k R_{ik} S_{kj}$$

$$a = \vec{y}^{\dagger} \cdot M \cdot \vec{x}$$

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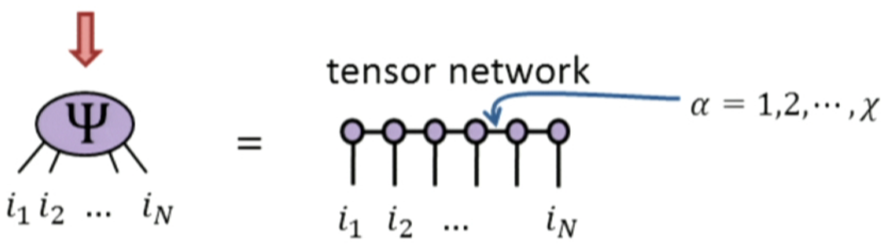
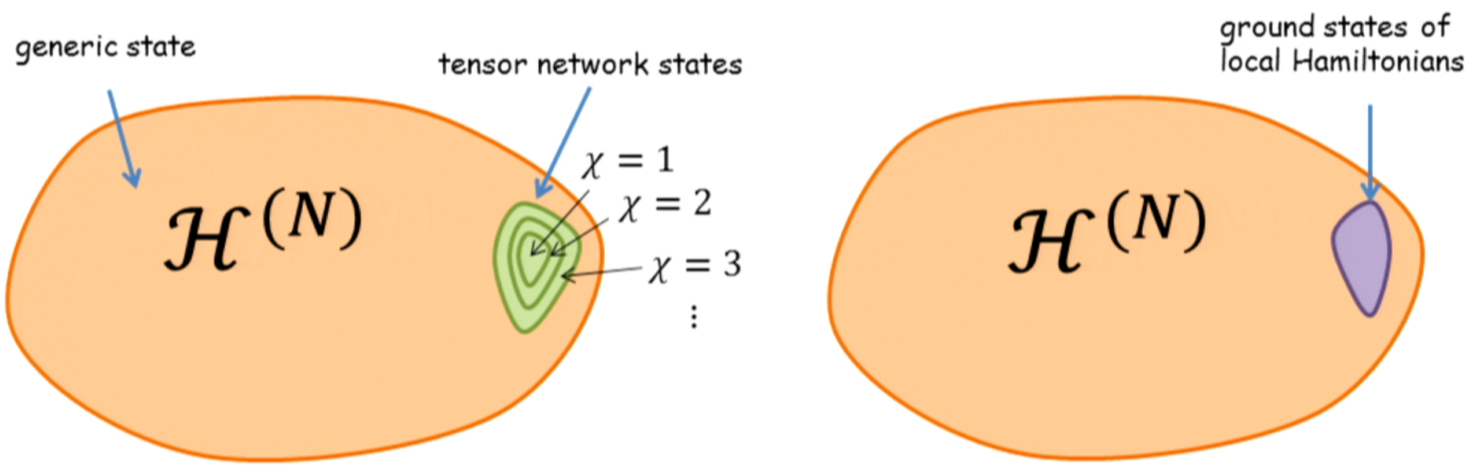


2^N parameters → $O(N2\chi^2)$ parameters

inefficient **efficient**

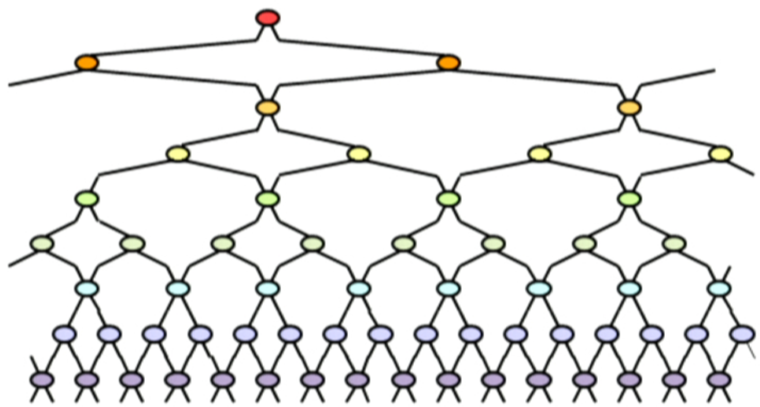


Example of tensor network

Vidal, 2006

Multi-scale entanglement renormalization ansatz

(MERA)



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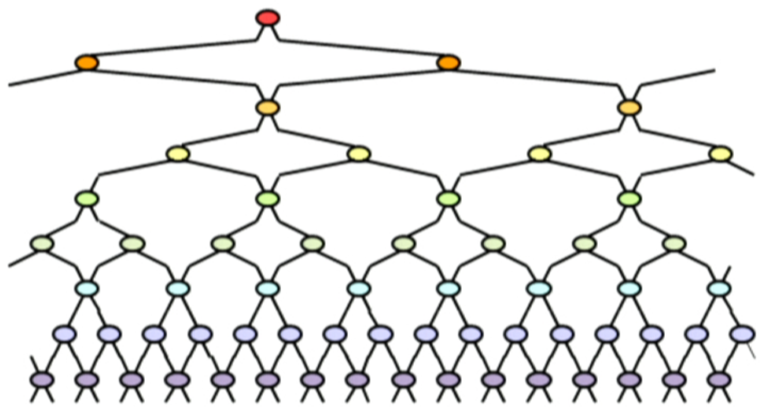
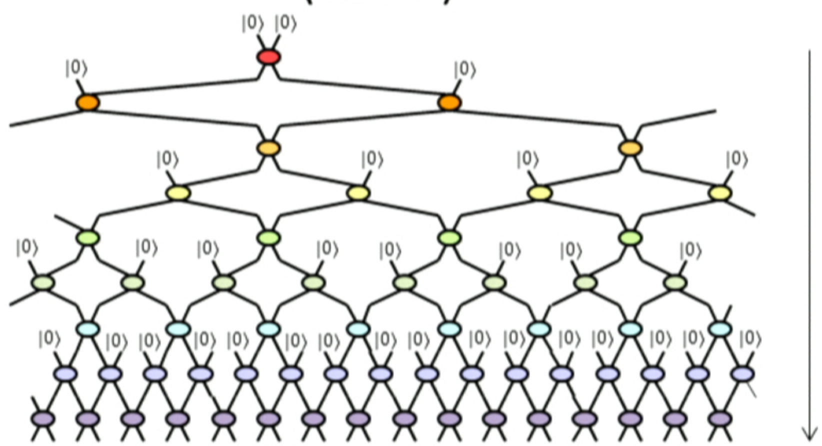


- Variational ansatz for 1d systems, which extends in space and scale
- Variational parameters for different scales
- It is secretly a **quantum circuit** and an **RG transformation**

Multi-scale entanglement renormalization ansatz

(MERA)

quantum circuit U



“time”

Multi-scale entanglement renormalization ansatz

(MERA)

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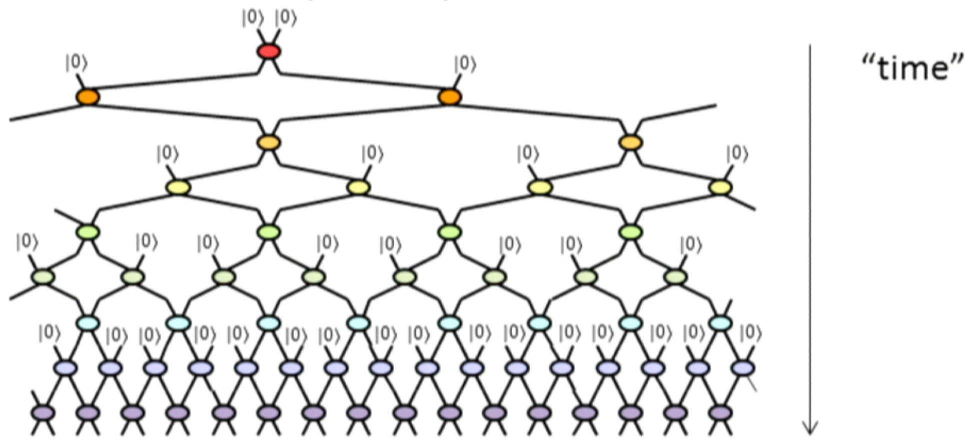
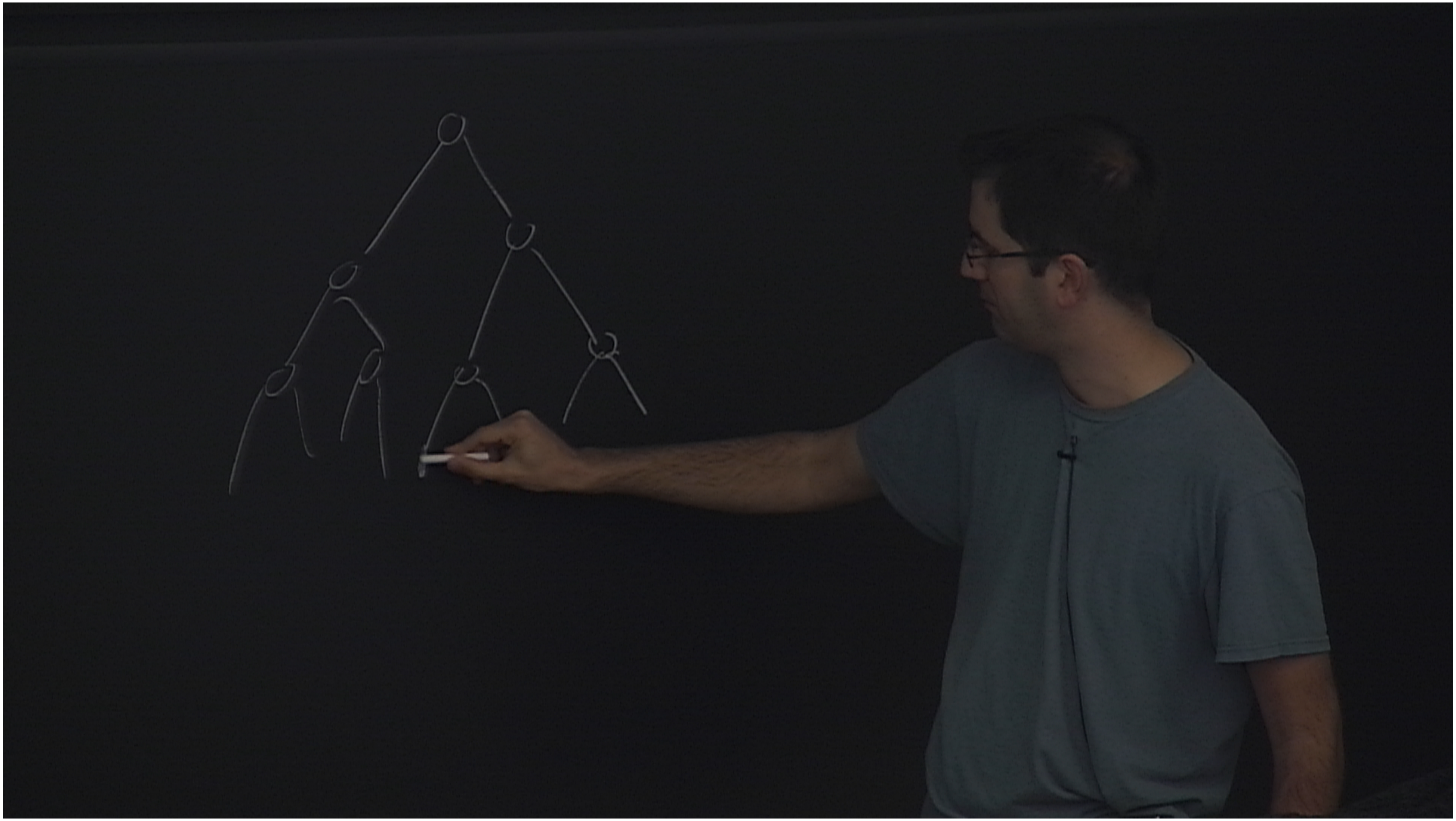


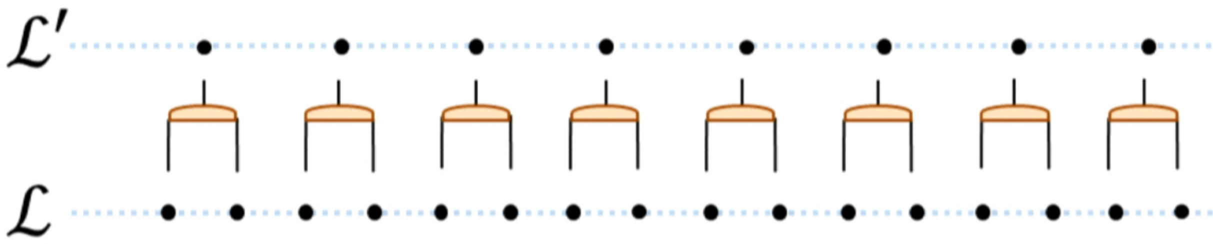
ground state $|\Psi\rangle = U \, |0\rangle \otimes |0\rangle \otimes \cdots \otimes |0\rangle$

Entanglement introduced by gates at different times (=scales)

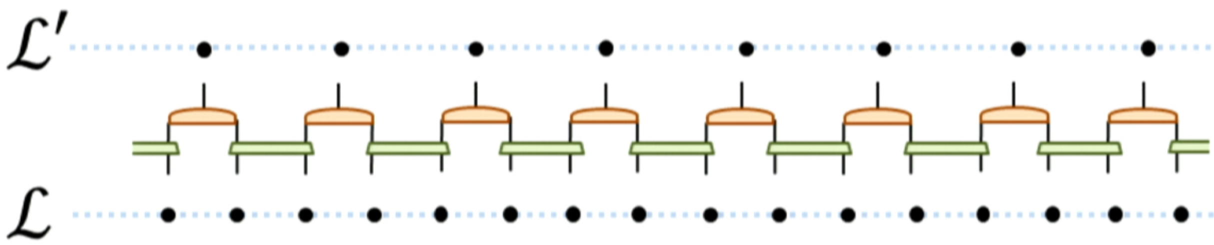


RG Transformation

Kadanoff (1966) + White (1992)
blocking variational optimization



Entanglement renormalization (2005)

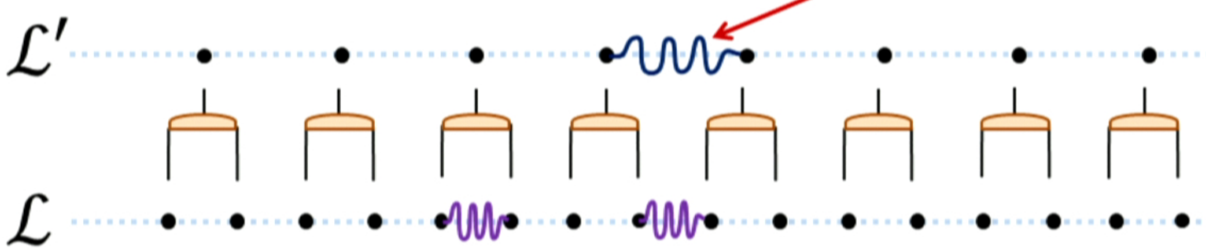


RG Transformation

Kadanoff (1966) + White (1992)

blocking variational optimization

failure to remove ***some*** short-range entanglement !



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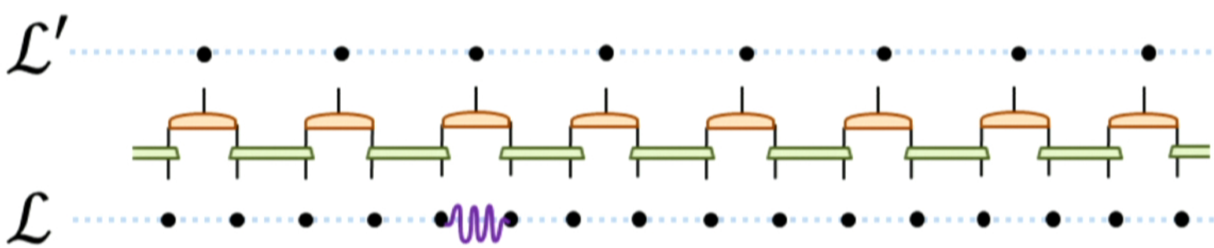
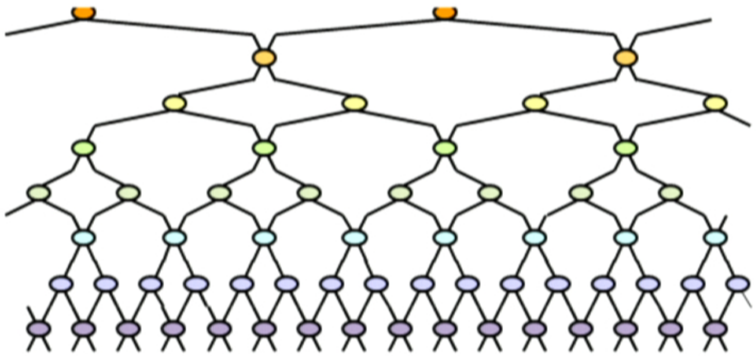
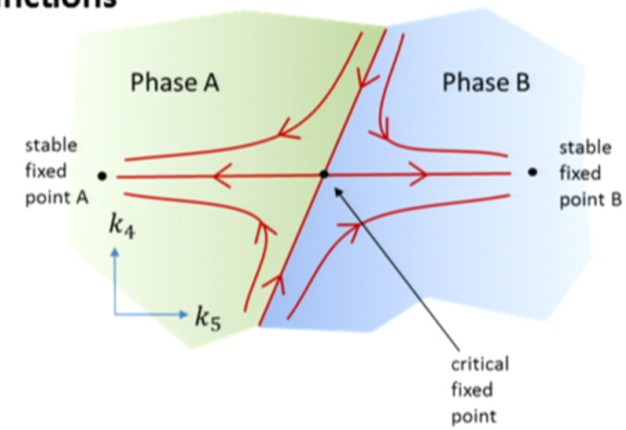
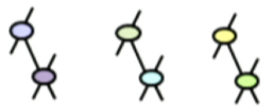


MERA -> RG flow in the space of ground state **wave-functions**





$$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \cdots \rightarrow |\Psi^{fp}\rangle$$

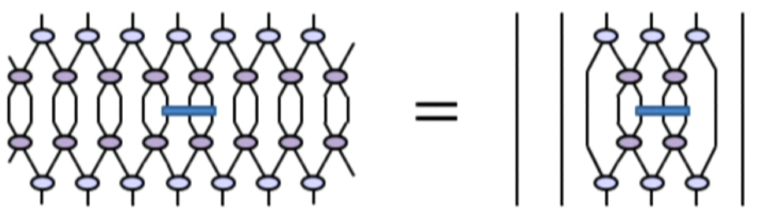


fixed-point wave-function

- topological order (2+1)
- quantum criticality (1+1)

MERA -> RG flow in the space of **Hamiltonians**

$$H \rightarrow H' \rightarrow H'' \rightarrow \cdots \rightarrow H^{fp}$$



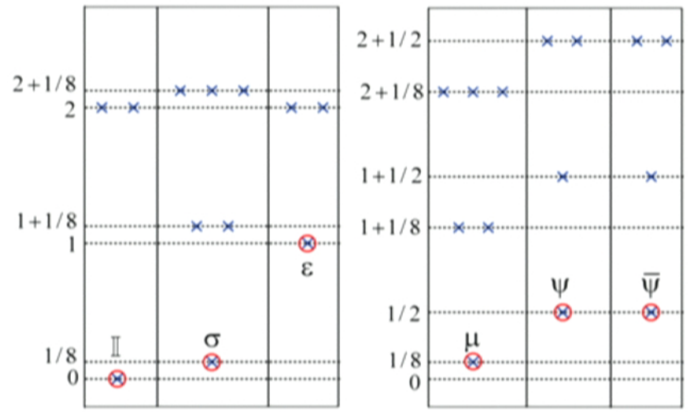
local operators are mapped into local operators !

Example: Critical quantum Ising model

Pfeifer, Evenbly, Vidal (2008)

Scaling dimensions of primary fields

		scaling dimension (exact)	scaling dimension (MERA)	error
identity	$\mathbb{I}$	0	0	----
spin	σ	0.125	0.124997	0.003%
energy density	ε	1	0.99993	0.007%
disorder	μ	0.125	0.1250002	0.0002%
fermions	μ	0.5	0.5	$<10^{-8}$ %
	ψ	0.5	0.5	$<10^{-8}$ %



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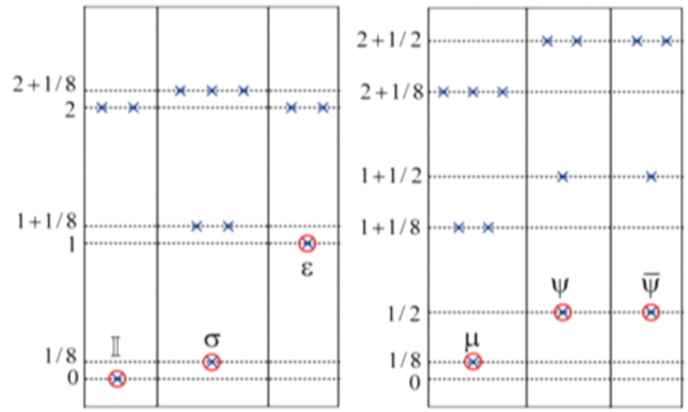

Operator product expansion (OPE) coefficients

$$C_{\epsilon\sigma\sigma} = \frac{1}{2} \qquad C_{\epsilon\mu\mu} = -\frac{1}{2} \qquad C_{\epsilon\psi\bar{\psi}} = i \qquad C_{\epsilon\bar{\psi}\psi} = -i \qquad C_{\psi\mu\sigma} = \frac{e^{-\frac{i\pi}{4}}}{\sqrt{2}} \qquad C_{\bar{\psi}\mu\sigma} = \frac{e^{\frac{i\pi}{4}}}{\sqrt{2}} \qquad (\pm 6\times 10^{-4})$$

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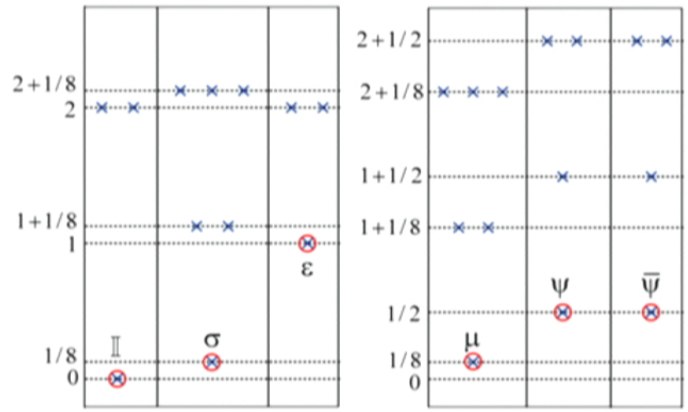

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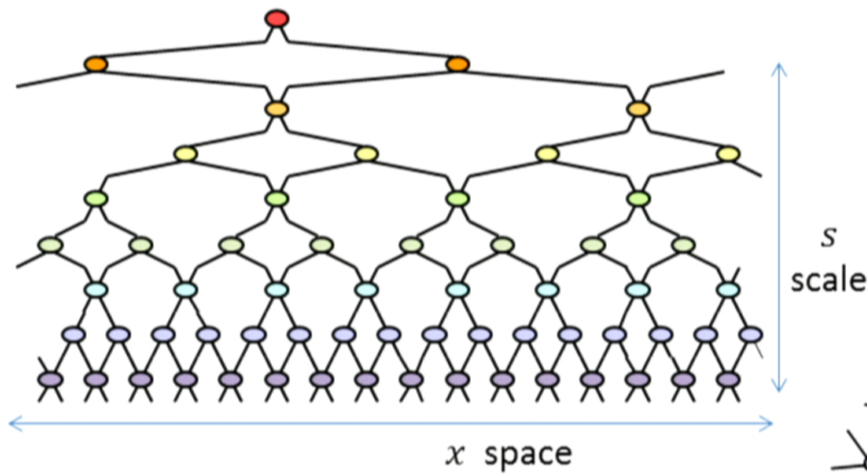
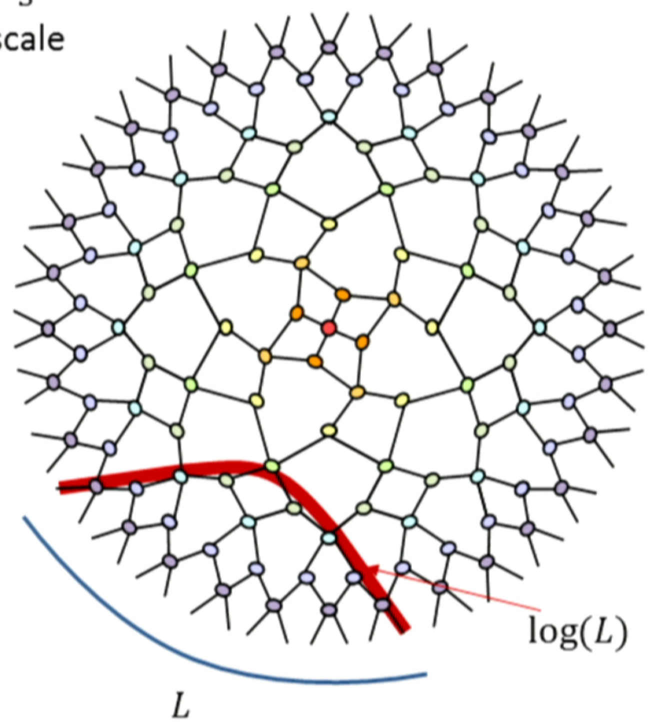
scale-invariant MERA
→ conformal data of a CFT:

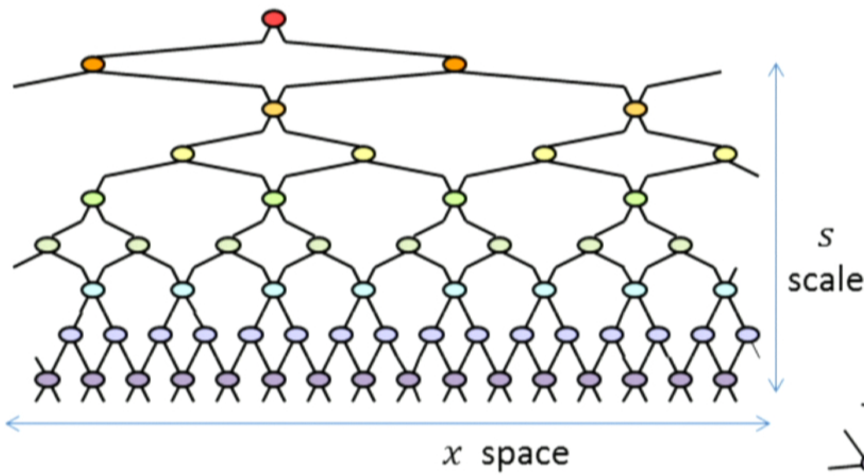
central charge	c
scaling dimensions	$\Delta_\alpha \equiv h_\alpha + \bar{h}_\alpha$
conformal spin	$s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
OPE	$C_{\alpha\beta\gamma}$

MERA and HOLOGRAPHY





MERA and HOLOGRAPHY



- entanglement entropy

$$S_L \approx \log(L)$$

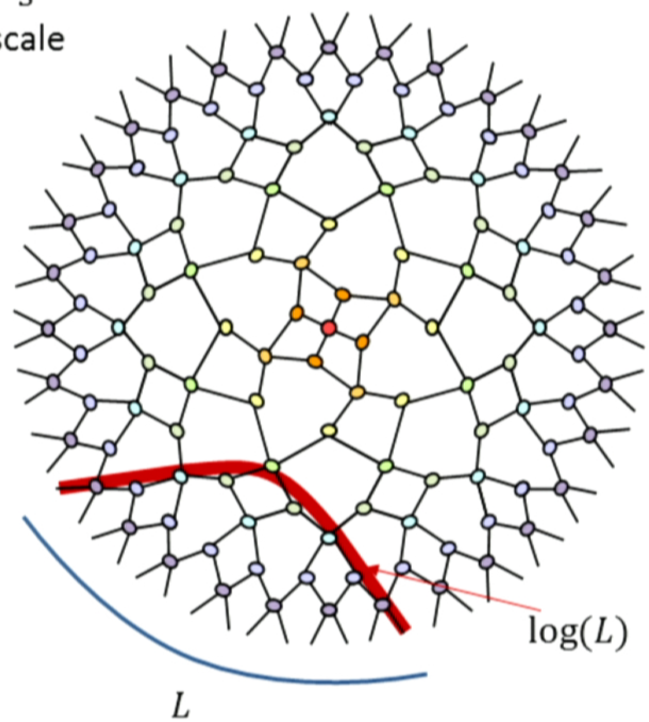
parallel to area of minimal surface in Ryu-Takayanagi

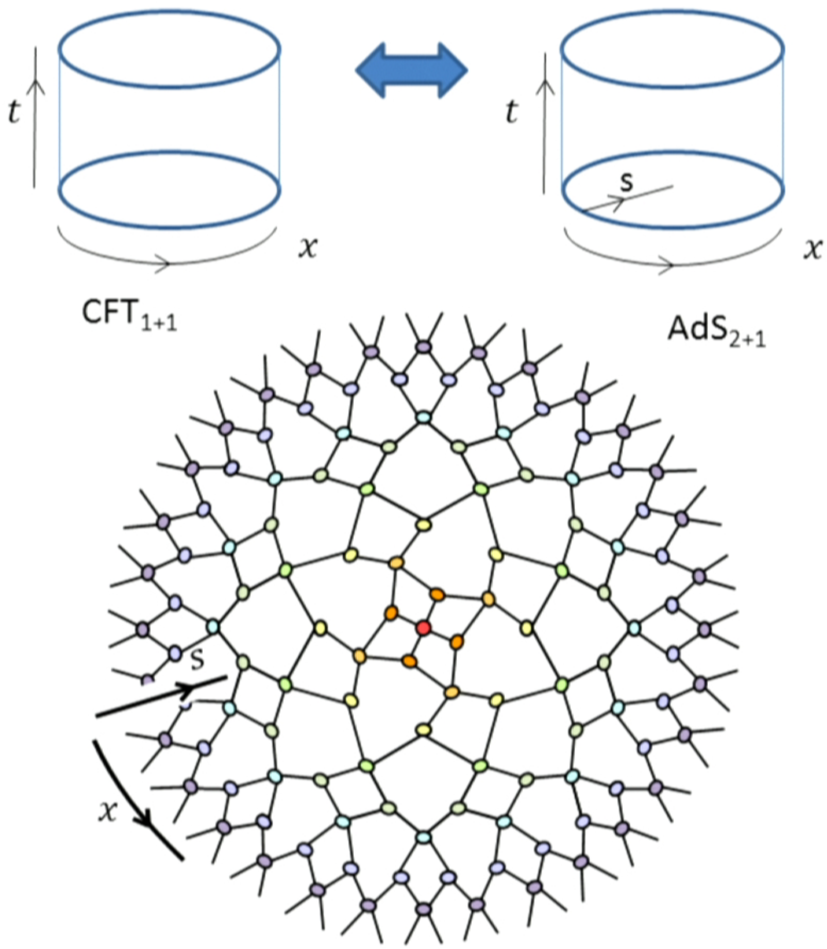
- two-point correlations

$$C(L) \approx L^{-2\Delta}$$

geodesic distance $D \approx \log(L)$ as in hyperbolic space

$$C(L) \approx e^{-D} = e^{-2\Delta \log(L)} = L^{-2\Delta}$$





Swingle 2009

Qi

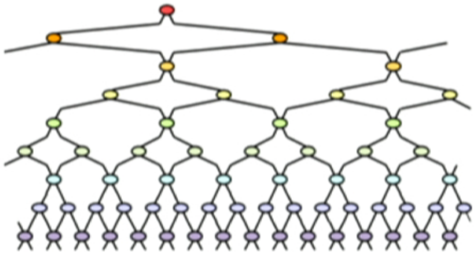
Hartman, Maldacena

Haegeman, Osborne,
Verschelde, Verstraete

Ryu, Takayanagi

Sully, Czech

Harlow, Yoshida, Pastawki, Preskill



So, MERA seems to work!

Great! However

- variational optimization is expensive; local minima.
- do we get the correct ground state?
- Euclidean path integrals / classical partition functions?

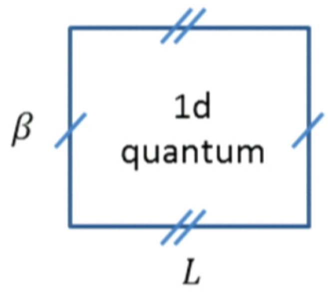
Tensor Network Renormalization

Evenbly, Vidal 2014-2015

GLEN EVENBLY

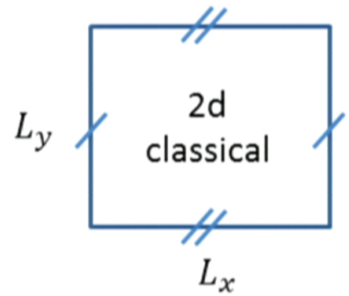
Euclidean path integral

$$Z(\lambda) = tr\, e^{-\beta H_q^{1d}}$$



Statistical partition function

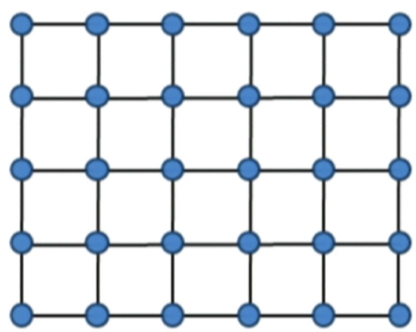
$$Z(T) = \sum_{\{s\}} e^{-\frac{1}{T} H_{cl}^{2d}}$$



$\sim$

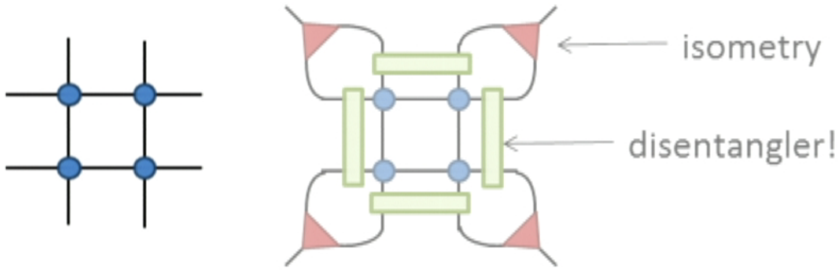
as a tensor network

$$Z =$$



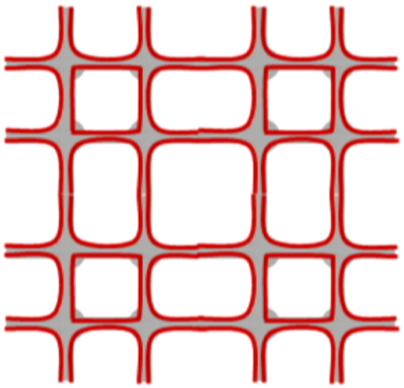
Tensor Network Renormalization (TNR)

Evenbly, Vidal 2014-2015



Tensor Network Renormalization (TNR)

CDL Tensor
(zero correlation length)



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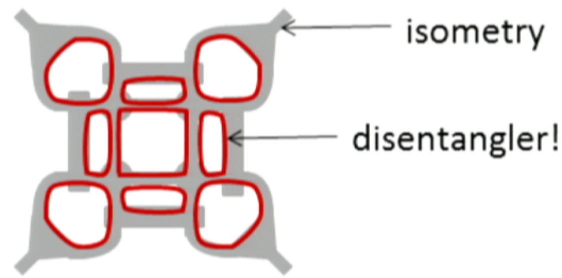

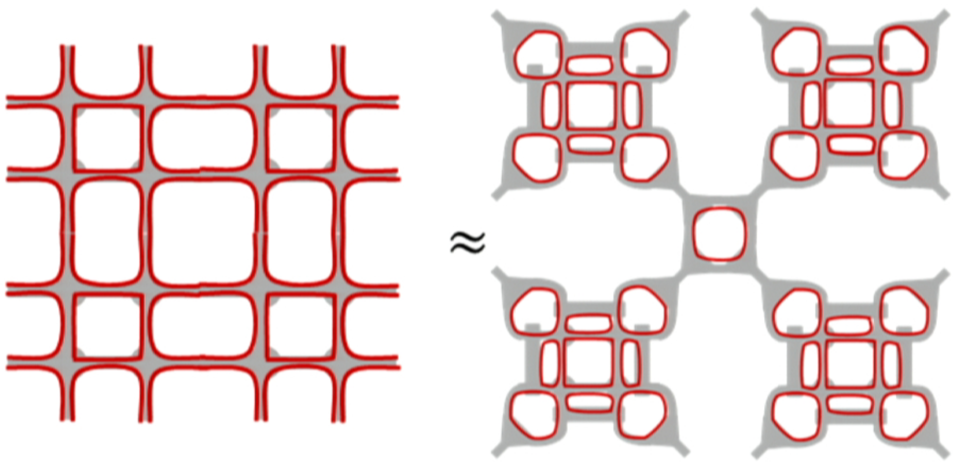

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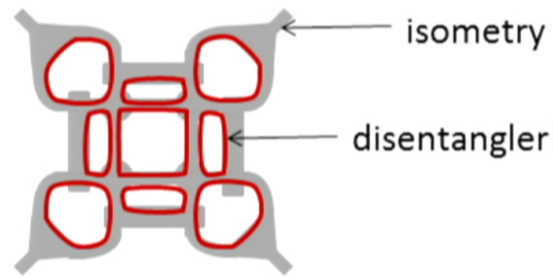
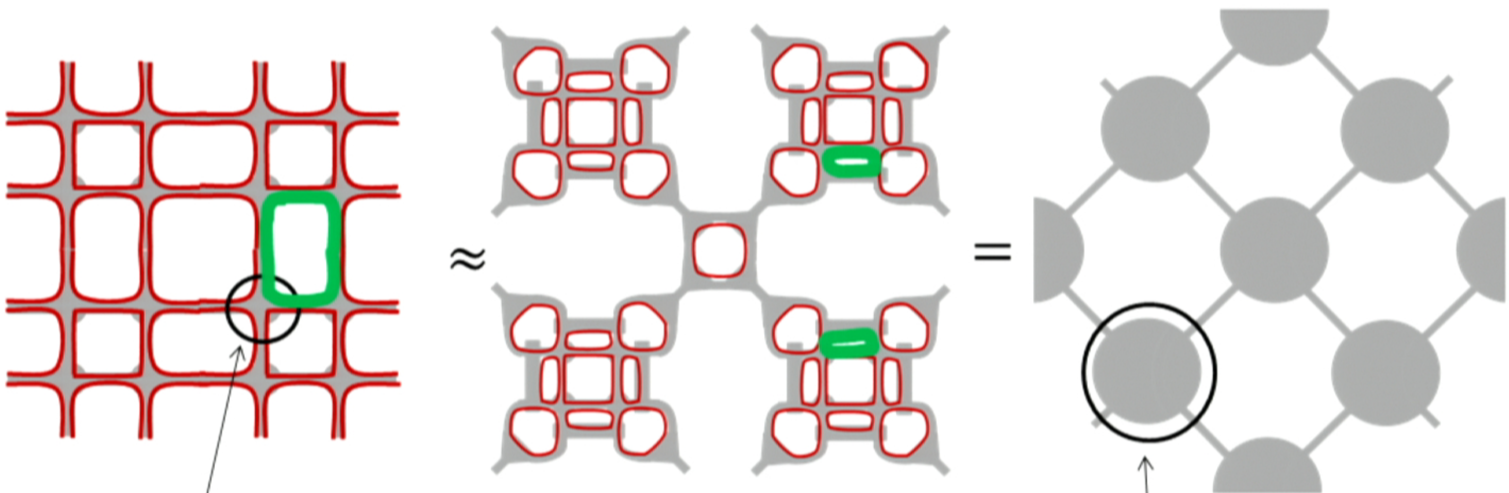


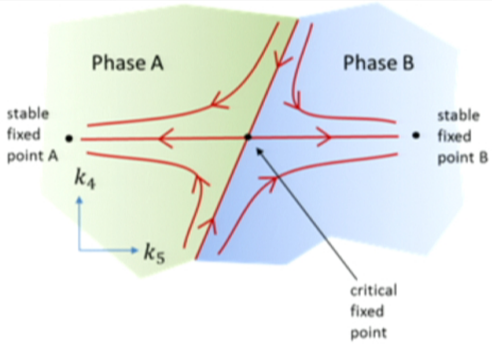


A removal of ***all*** short-range entanglement A'

[for CDL tensors, see also Gu and Wen 2009 Tensor Entanglement Filtering Renormalization (TEFR)]

TNR -> proper RG flow
Example: 2D classical Ising

$$A \to A' \to A'' \to \cdots \to A^{fp}$$



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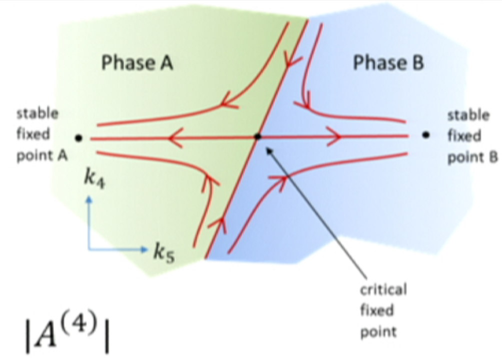

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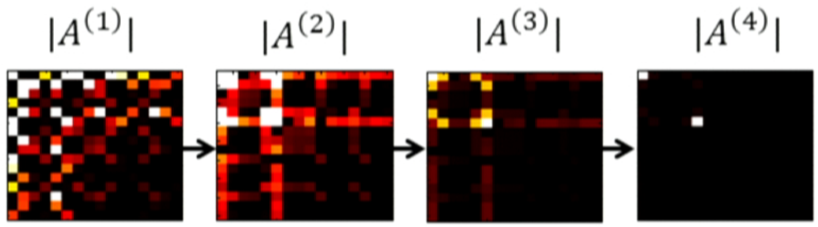
below critical

$T = 0.9\ T_c$



ordered (Z2) fixed point

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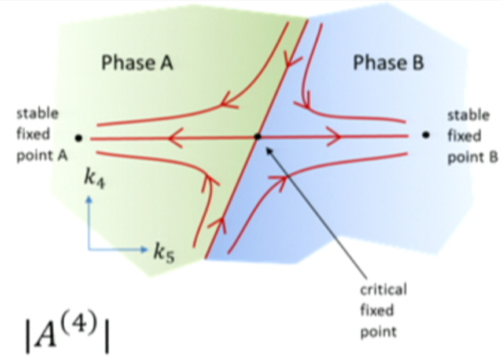

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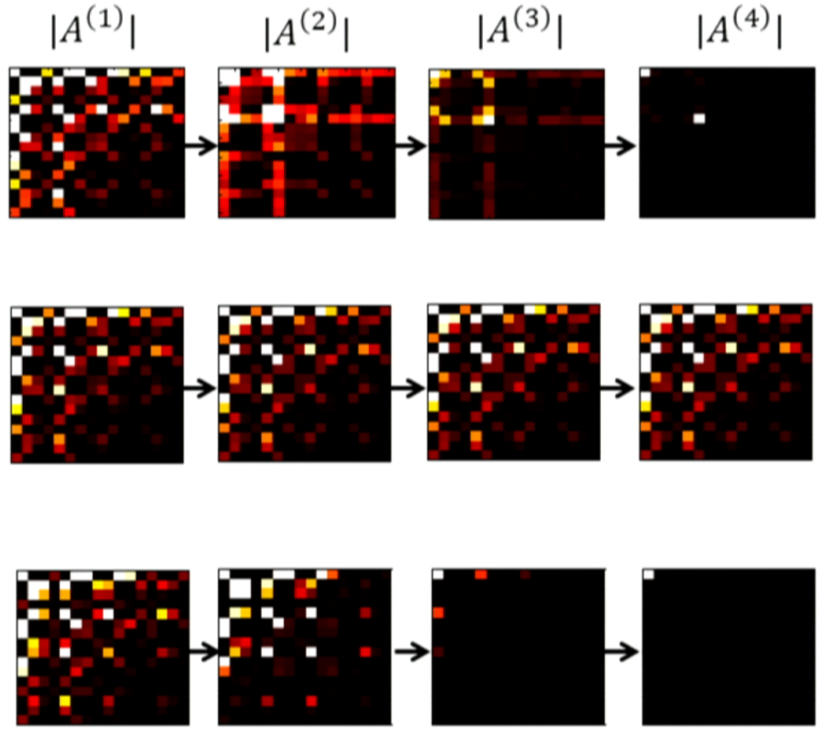


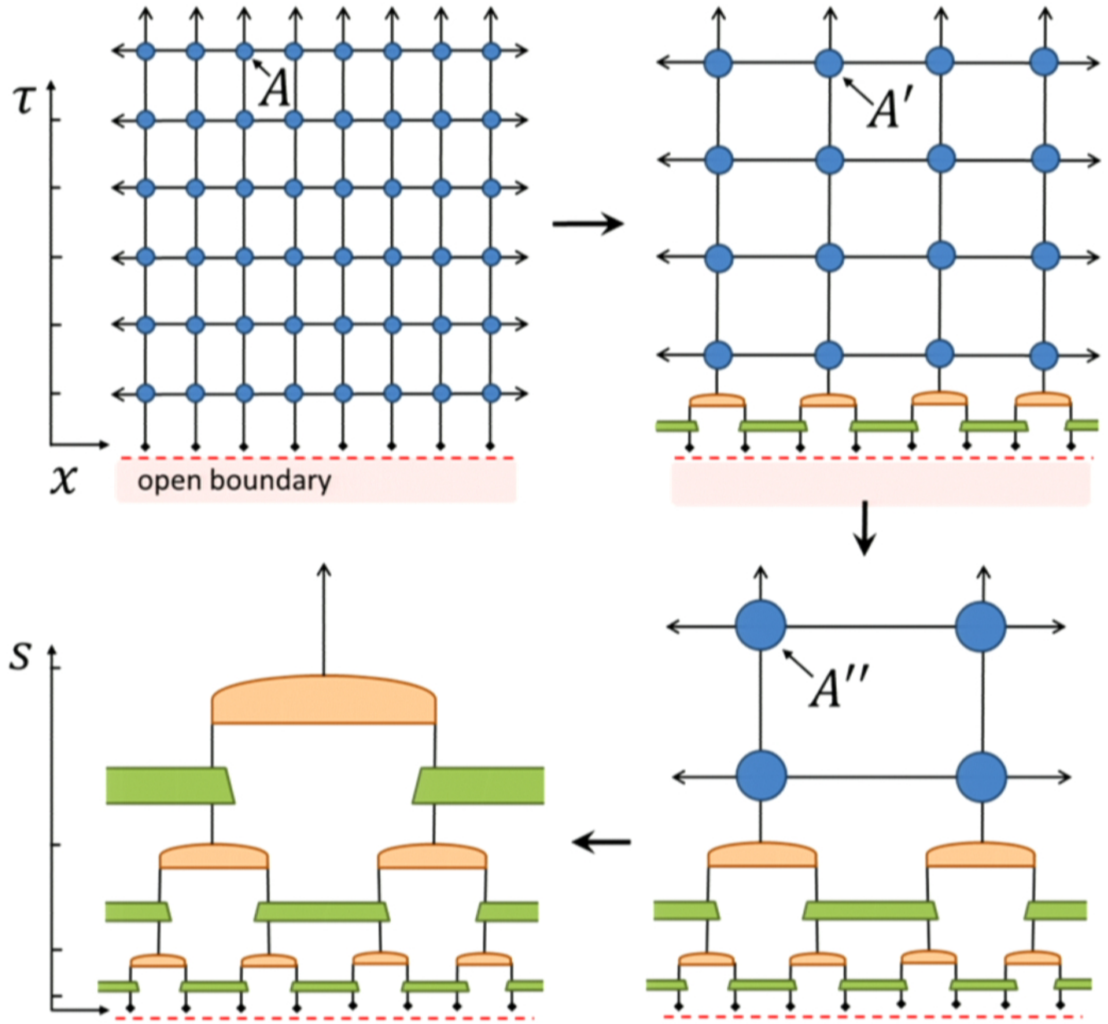
below critical
$T = 0.9\ T_c$
ordered (Z2) fixed point

critical
$T = T_c$
critical (scale-invariant) fixed point

above critical
$T = 1.1\ T_c$
disordered (trivial) fixed point

Evenbly, Vidal, 2015

TNR yields MERA



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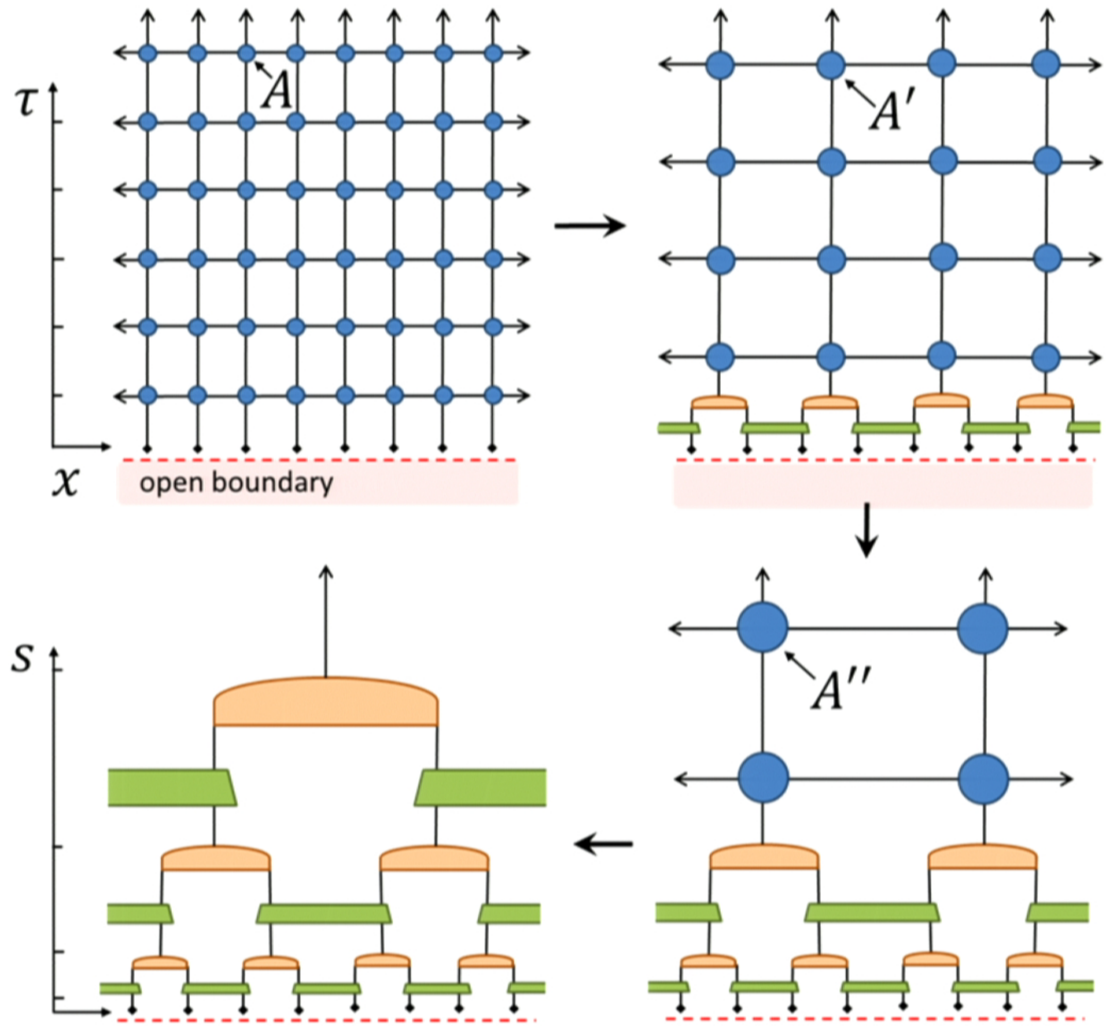
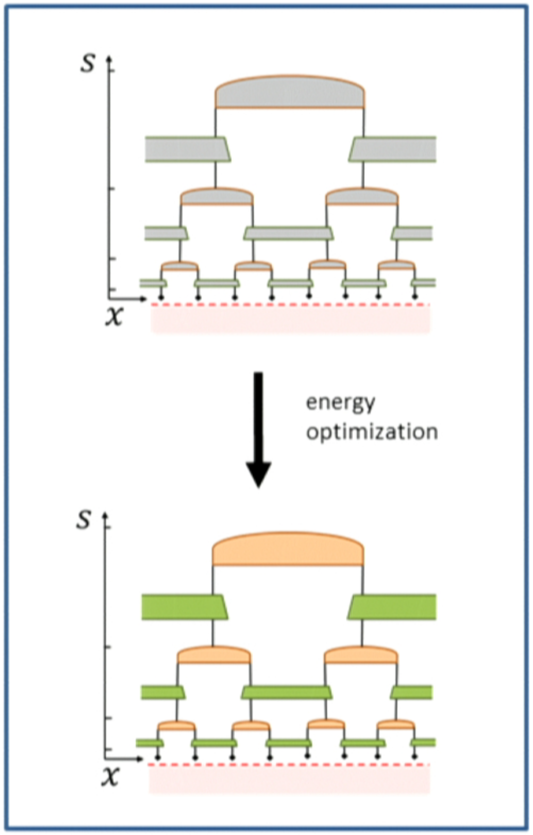
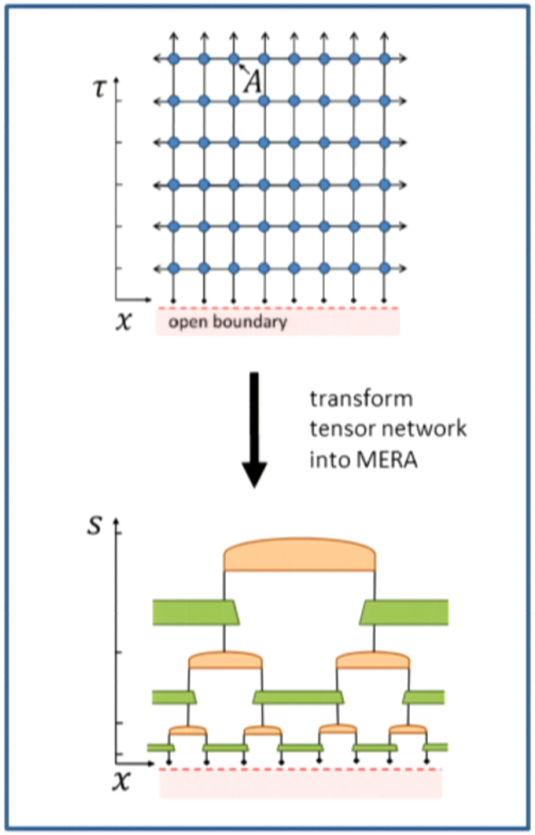
Evenbly, Vidal, 2015

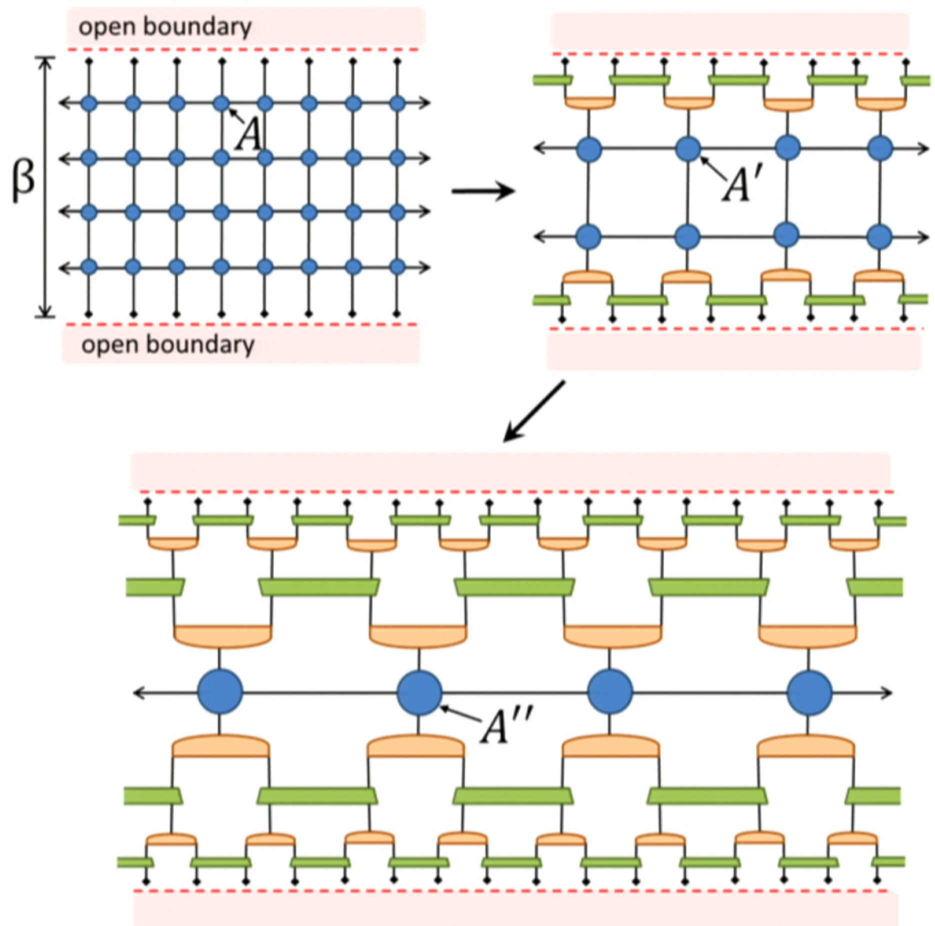


MERA = variational ansatz → MERA = by-product of TNR





extra bonus: MERA for a thermal state (or black hole in holography)



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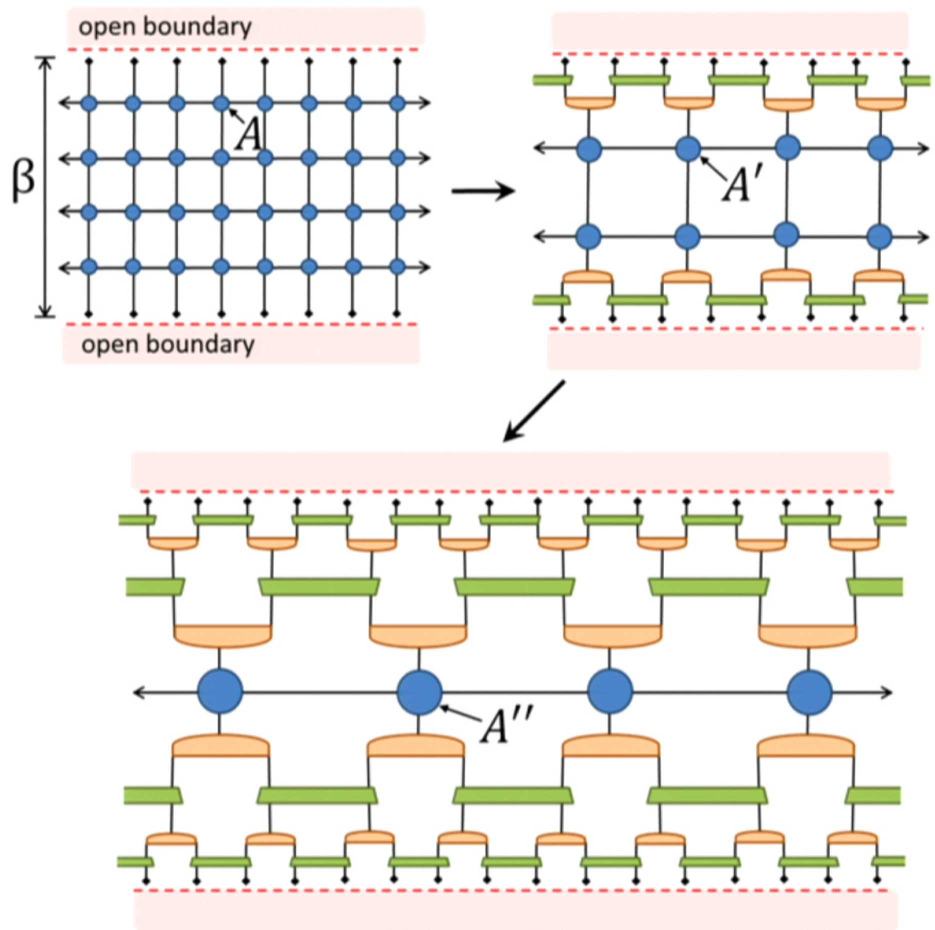

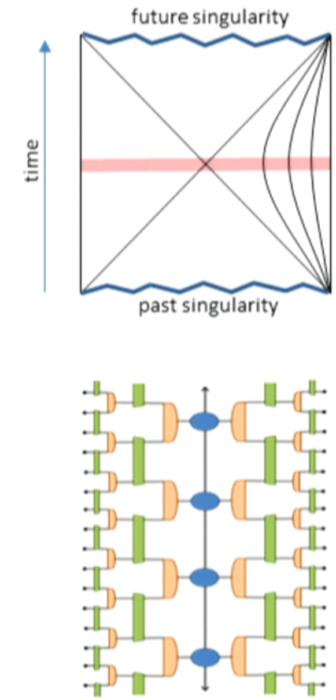

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  using quantum information tools/concepts
  (quantum circuits, entanglement)

- Efficient representation of ground states (MERA)
  -> toy model for holography

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	Tensors	*Ground states*	*Hamiltonians*
3 RG flows	$A \to A' \to A'' \to \cdots$	$\vert\Psi\rangle \to \vert\Psi'\rangle \to \vert\Psi''\rangle \to \cdots$	$H \to H' \to H'' \to \cdots$

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- Key ingredient: **removal of short-range entanglement**

- Very accurate in 1+1 dimensions (Ising model, etc)

What about 2+1, 3+1? (and QCD?)