Title: Quantum theory from rules on information acquisition

Date: Mar 17, 2015 03:30 PM

URL: http://pirsa.org/15030086

Abstract: <p>The last decade has seen a wave of characterizations of quantum theory using the formalism of generalized probability theory.</p>

<p>In this talk, I will introduce a novel operational approach to characterizing and reconstructing quantum theory which puts an observerâ€™s information acquisition -- rather than the probability structure â€“ centre stage. In particular, we consider an observer interrogating a system with binary questions and explain how an elementary set of rules governing the observerâ€™s acquisition of information about the system leads to qubit quantum theory. The derivation is constructive, elucidating, among other things, the origin of entanglement, monogamy and more generally the correlation structure. This approach also yields a new characterization of pure states in terms of â€˜conserved informational chargesâ€™ which, in turn, define the unitary group.</p>

Quantum theory from rules on information acquisition

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Perimeter Institute

QF seminar
17 March 2015

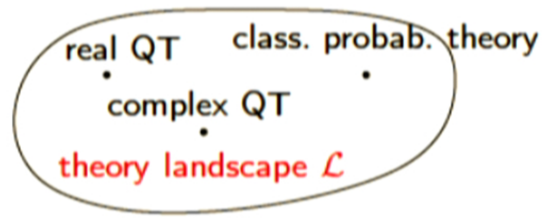
based on:
PH arXiv:1412.8323
PH, C. Wever (to appear Mar. 2015)

(Re-)constructing QT

axiomatization of QT with some basic set of postulates

steps:

1. define landscape $\mathcal{L}$ of theories
2. formulate axioms for QT within $\mathcal{L}$?
3. derive quantum formalism

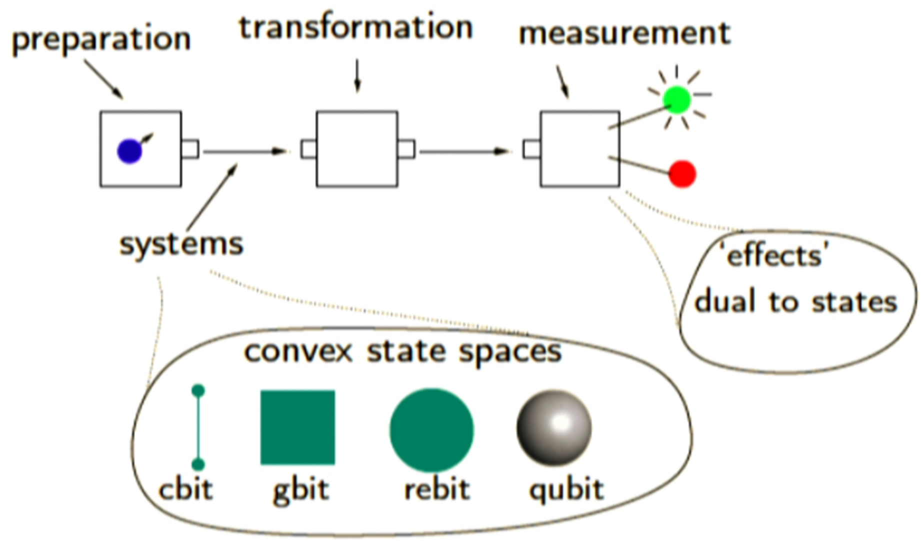


Why?

1. Give operational sense to usual textbook axioms (why $\mathcal{H}$, $\otimes$, $\mathbb{C}$, U...?)
2. new structural insights?
3. Better understand QT within larger context
4. Often voiced: will clarify interpretation of QT [Rovelli, Fuchs,...]
   $\Rightarrow$ hope thus far not realized (e.g., GPTs interpretationally neutral)

(Re-)constructing QT II

- usually: $\mathcal{L}$ = 'generalized probability theories' (GPT)
- operational axioms, primacy on probability
- wave of QT reconstructions within GPT framework ['01-'14 Hardy, Dakic, Brukner, Masanes, Müller, D'Ariano, Chiribella, Perinotti......]



Why another (re-)construction of QT?

QT governs observer's info acquisition [Brukner, Zeilinger, Rovelli, Spekkens, Fuchs,......]
Here: derive QT from this perspective

advantage: 1. 'simpler' axioms on relation between O and S
2. novel perspective, emphasizes information acquisition and close to Relational QM [for RQM see Rovelli, Smerlak]
3. yields constructive derivation

disadvantage: landscape $\mathcal{L}$ smaller than for GPTs

Outline for the remainder

Table of contents

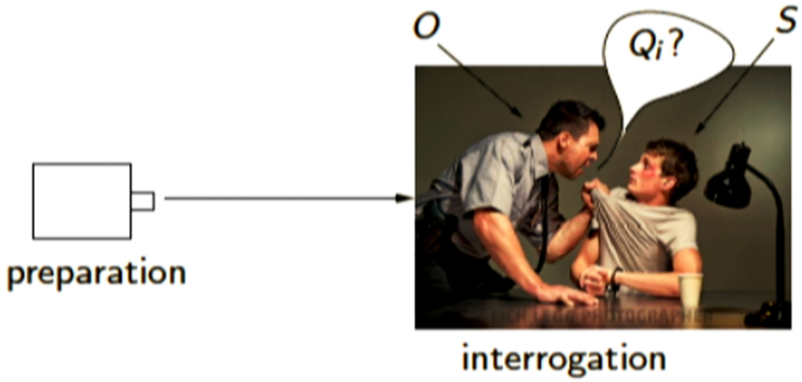
1. Landscape of inference theories and tool box
2. Postulates
3. Strategy
4. Summary of reconstruction steps
5. Conclusions

Specifying the landscape of inference theories [PH '14]

focus: information acquisition of observer O about system S

premise: speak only about info that O has access to (purely epistemic approach)

Setup: O interrogates (ensemble of) S with *binary* questions Q_i, $i = 1, \ldots$



Basic ingredients:

$\mathcal{Q}$: set of binary Qs that O may ask S

Σ: set of all possible answer statistics (every prep. to produce specific answer statistic)

- assume: O has tested identical S sufficiently often to 'know' set Σ

Specifying the landscape of inference theories II [PH '14]

- Bayesian viewpoint: for specific S, O assigns probabilities p_i to Q_i accord. to his info about
  1. Σ
  2. particular S

- p_i encode all O can say about $S \Rightarrow$ state of S (rel. to O): collection of p_i $\Rightarrow$ state space: Σ (to be convex)
- assume: $\exists$ state of 'no information' $p_i = \frac{1}{2} \; \forall i$

Q_i, Q_j are:

independent if, relative to state of no information of S, answer to only Q_i gives O no information about answer to Q_j (and vice versa) $\Rightarrow p(Q_i, Q_j) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ factorizes

compatible if O may know answers to both simultaneously $\Rightarrow \exists$ state s.t. p_i, p_j simultaneously $0, 1$

complementary if knowledge of Q_i disallows O to know Q_j at the same time (and vice versa) $\Rightarrow p_i = 0, 1$, then $p_j = 1/2 \; \forall$ states

Specifying the landscape of inference theories III [PH '14]

- assumption: state parametrized by max. set of pairwise indep. Q_i

$$\vec{P}_{O\to S} = \begin{pmatrix} p_1 \\ \vdots \\ p_D \end{pmatrix}, \quad p_i \text{ prob. that } Q_i = \text{'yes'}$$

$\Rightarrow \{Q_1, \ldots, Q_D\}$ informationally complete

- ansatz: O's info about Q_i: $0 \leq \alpha(p_i) \leq 1$ bit $\Rightarrow$ total info:

$$I_{O\to S}(\vec{P}_{O\to S}) = \sum_{i=1}^{D} \alpha(p_i)$$

(explicit measure later from principles)

- Defn.: composite system S_{AB} composed of S_A, S_B if

$$\mathcal{Q}_{AB} = \mathcal{Q}_A \cup \mathcal{Q}_B \cup \{\text{logical connectives } Q_A * Q_B,\ Q_{A,B} \in \mathcal{Q}_{A,B}\}$$

- Specker's principle: n Q_i pairwise compatible $\Rightarrow$ mutually compatible
- require: O not permitted to make consistent statements about logical connectives of complementary Qs

Rules for informational relation $O \to S$ for N qubits [PH '14]

(first two motivated from Rovelli, Zeilinger, Brukner)

P1: (limited information) "O can acquire maximally $N \in \mathbb{N}$ independent bits of information about S at the same time."
$\exists\, Q_i,\ i = 1, \ldots, N$ (mutually) independent compatible

P2: (complementarity) "O can always get up to N new (independent) bits of information about S. Whenever O asks a new question he experiences no net loss of information."

P3: (completeness) "O's info about S can be distributed over all Q's in any way consistent with P1 and P2."

P4: (preservation) "O's total amount of information about S preserved between interrogations".

P5: (time evolution) Time evolution of $\vec{P}_O$ is continuous

P6: (locality) "O can determine state of composite system by only interrogating its constituents."

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$\exists\, Q_i'$, $i = 1, \ldots, N$ (mutually) independent compatible but $Q_i, Q_{j=i}'$ complementary

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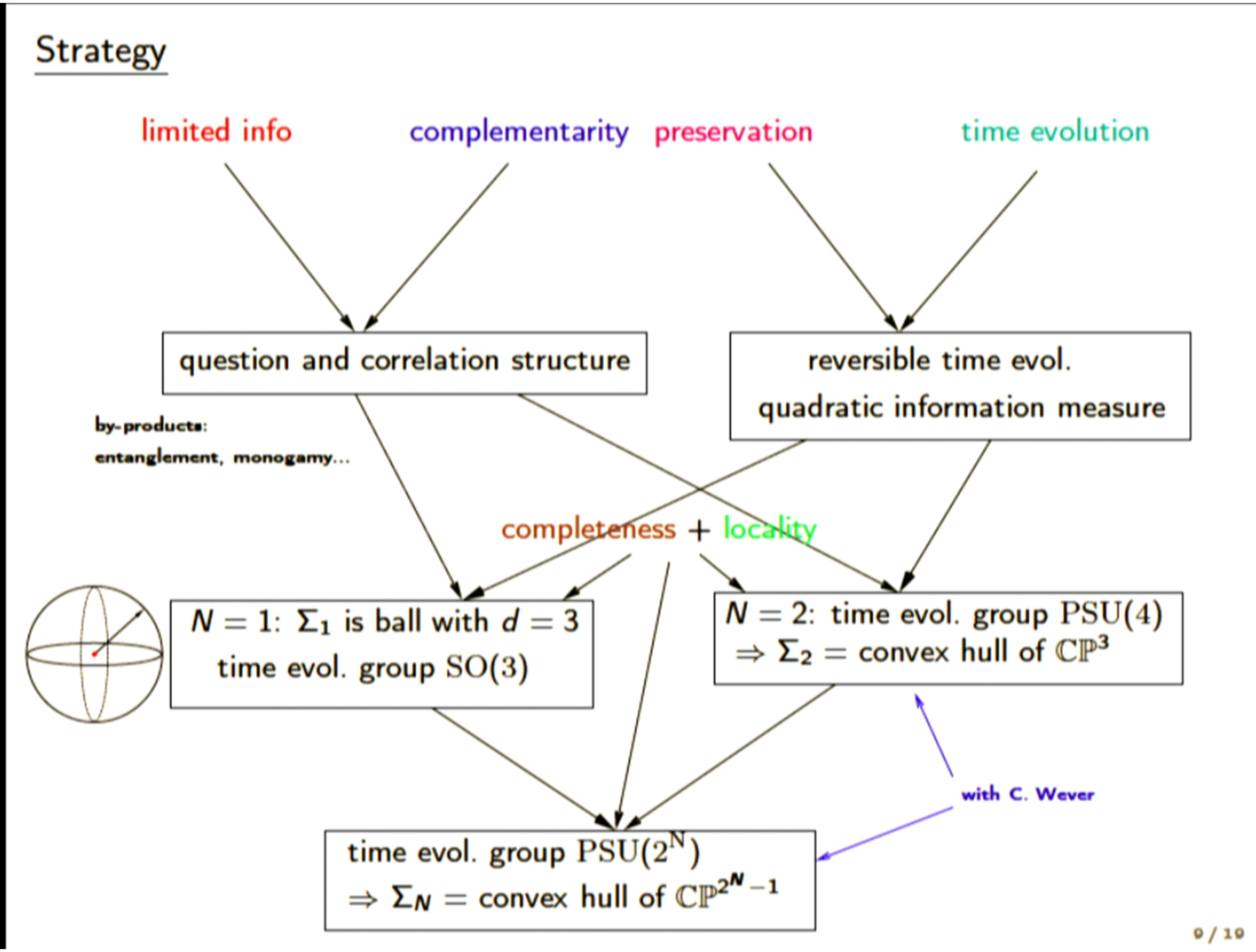
Claim: Σ_N is space of $2^N \times 2^N$ density matrices over $(\mathbb{C}^2)^{\otimes N}$, states evolve unitarily and $\mathcal{Q}_N \simeq \{\text{Pauli operators}\}$


Strategy
limited info
complementarity
preservation
time evolution
question and correlation structure
reversible time evol.
quadratic information measure
by-products:
entanglement, monogamy...
completeness + locality
$N = 1$: Σ_1 is ball with $d = 3$
time evol. group $\mathrm{SO}(3)$
$N = 2$: time evol. group $\mathrm{PSU}(4)$
$\Rightarrow \Sigma_2 =$ convex hull of $\mathbb{CP}^3$
with C. Wever
time evol. group $\mathrm{PSU}(2^N)$
$\Rightarrow \Sigma_N =$ convex hull of $\mathbb{CP}^{2^N-1}$
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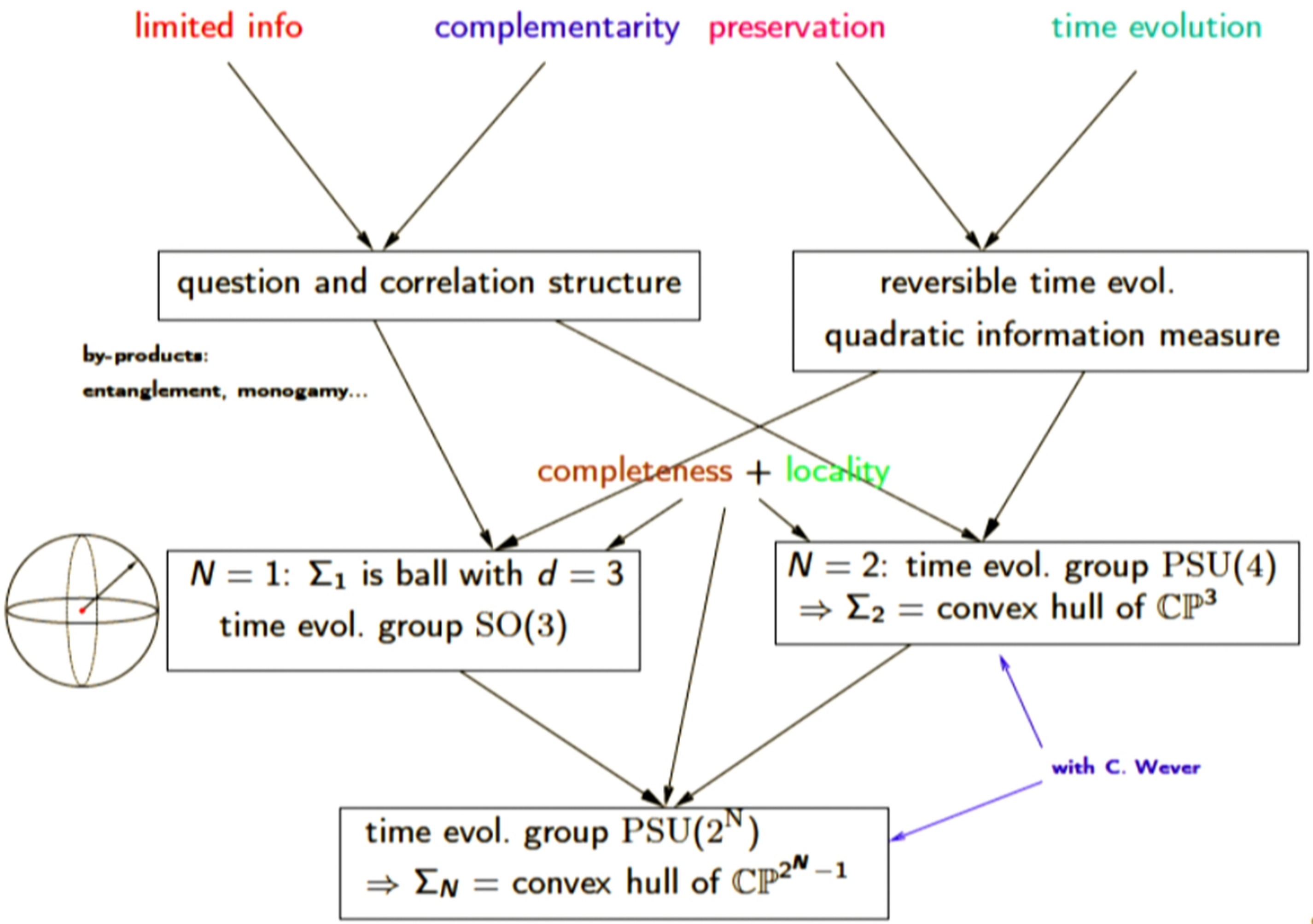
Strategy



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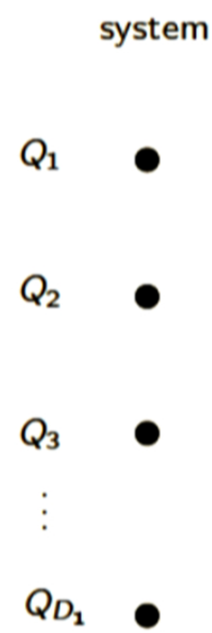
Compatibility and independence structure of questions [PH '14]

$N = 1$: only individual Q_i, $i = 1, \ldots, D_1 \Rightarrow D_1 =$? (know $D_1 \geq 2$)

$N = 2$: $2D_1$ individual Q_i

<u>vertex:</u> individual question Q_i



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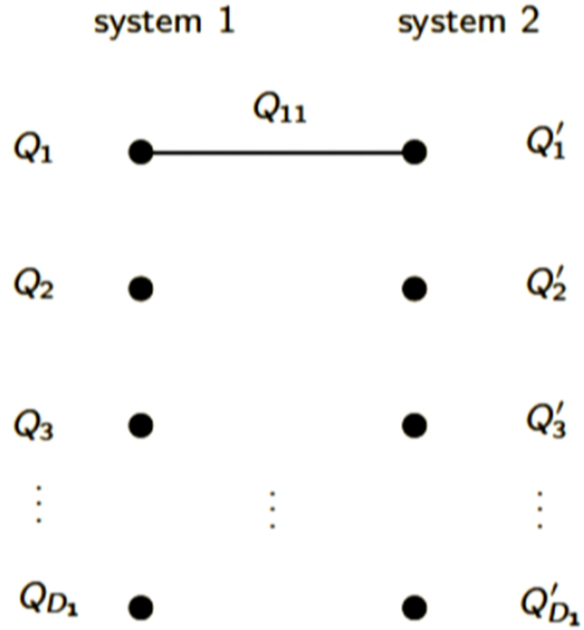
$N = 2$: $2D_1$ individual Q_i + D_1^2 composite questions:
$Q_{ij} := Q_i \leftrightarrow Q'_j$ "Are answers to Q_i and Q'_j the same?"
+ ???

<u>vertex:</u> individual question Q_i, Q'_j

<u>edge:</u> composite question Q_{ij}



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+ ???

vertex: individual question Q_i, Q'_j

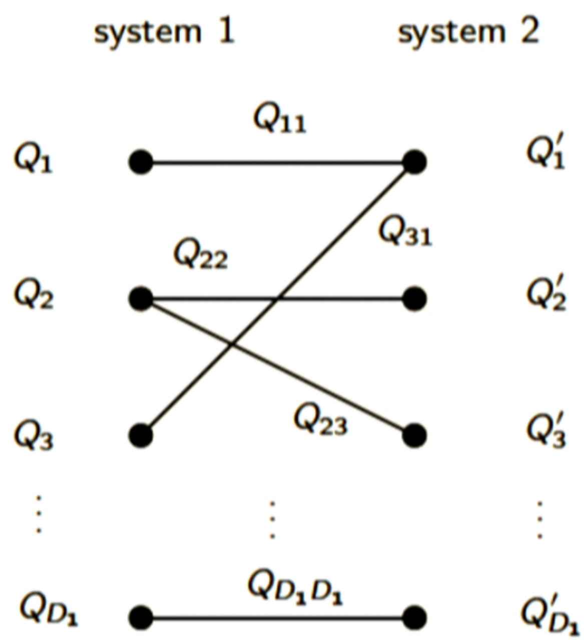
edge: composite question Q_{ij}

can show: Q_{ij}

1. pairwise indep.
2. complementary if corresp. edges intersect (e.g., Q_{11}, Q_{31})
3. compatible if corresp. edges non-intersecting (e.g., Q_{11}, Q_{22})

$\Rightarrow$ entanglement: > 1 bit in Q_{ij}

[see also Brukner, Zeilinger]

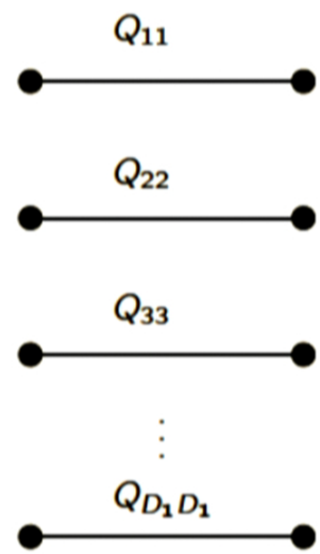


What is the dimension of the Bloch sphere? [PH '14]

Logical argument from $N = 2$ case:

- Q_{ii}, $i =, \ldots, D_1$ pairwise independent, compatible
- O can acquire answers to all D_1 composites Q_{ii} simultaneously (Specker)
- limited info: O cannot know more than $N = 2$ indep. bits about S

⇒ answers to any two Q_{ii} determine answers to all other Q_{jj}



- e.g., truth table for any three Q_{ii} ($a \neq b$): $\Rightarrow Q_{33} = Q_{11} \leftrightarrow Q_{22}$ or $\neg(Q_{11} \leftrightarrow Q_{22})$

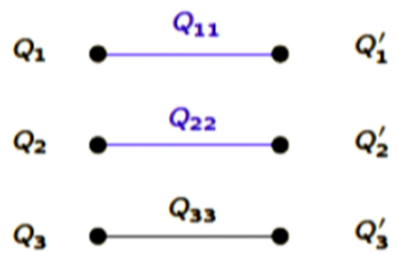
Q_{11}	Q_{22}	Q_{33}
0	1	a
1	0	a
1	1	b
0	0	b

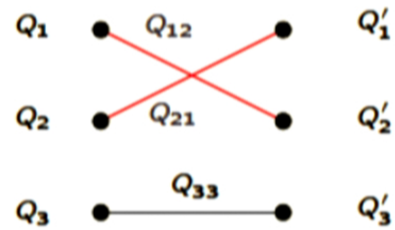
⇒ holds for all compatible sets of Q_{ij}: $2 \leq D_1 \leq 3$

⇒ # DoFs: 15 if $D_1 = 3$; 9 if $D_1 = 2$

Odd and even correlations [PH '14]

- know: $Q_{33} = \overset{?}{\neg}(Q_{11} \leftrightarrow Q_{22}) = \overset{?}{\neg}(Q_{12} \leftrightarrow Q_{21})$





Hence,
(a) $Q_{11} \leftrightarrow Q_{22} = Q_{12} \leftrightarrow Q_{21}$, or (b) $Q_{11} \leftrightarrow Q_{22} = \neg(Q_{12} \leftrightarrow Q_{21})$

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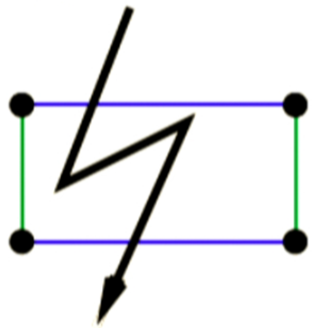
case (a):
e.g., suppose $Q_{11} = Q_{22} = 1$ $\Rightarrow$ $Q_{12} \leftrightarrow Q_{21} = 1$
$\Rightarrow$ diagrams can be consistently joined



$\Rightarrow$ $Q_1 \leftrightarrow Q_2 = Q_1' \leftrightarrow Q_2'$
$\Rightarrow$ illegal complementary info
$\Rightarrow$ would obtain same diagram in local 'hidden variable' model

Odd and even correlations [PH '14]

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case (b):
e.g., suppose $Q_{11} = Q_{22} = 1$ $\Rightarrow$ $Q_{12} \leftrightarrow Q_{21} = 0$

Odd and even correlations [PH '14]

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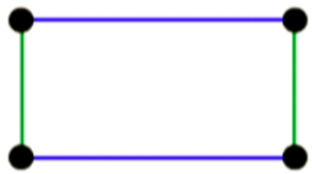
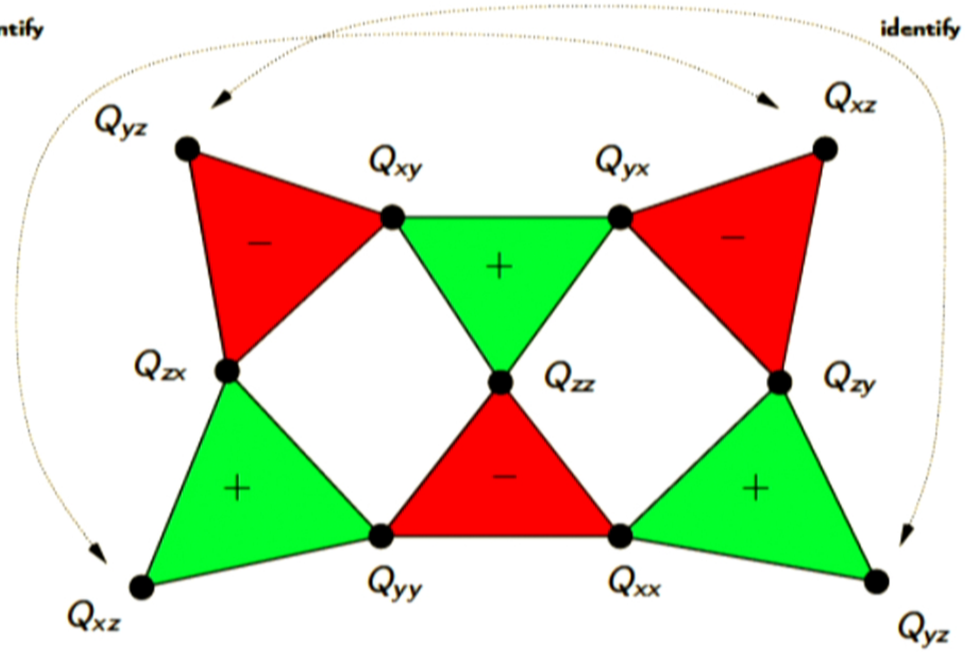


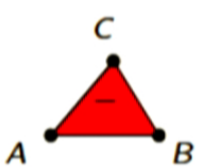
$\Rightarrow Q_1 \leftrightarrow Q_2 = Q_1' \leftrightarrow Q_2'$

Correlation structure for qubits ($N = 2$ and $D_1 = 3$) [PH '14]

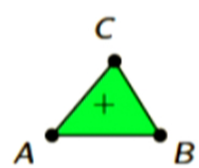
Compatibility structure of Qs $\Rightarrow$ correlation structure for 2 qubits in QT

Q, Q' compatible if connected by edge, otherwise complementary





$\Leftrightarrow$ odd correlation
$A = \neg(B \leftrightarrow C)$,
etc...



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case (b):
e.g., suppose $Q_{11} = Q_{22} = 1$ $\Rightarrow$ $Q_{12} \leftrightarrow Q_{21} = 0$
$\Rightarrow$ diagrams cannot be joined consistently (while assigning values simultaneously to compl. Qs)

$\Rightarrow$ no illegal info can be extracted

Information measure [PH '14]

recall: state of S relative to O:

$$\vec{P}_{O\to S} = \begin{pmatrix} p_1 \\ \vdots \\ p_{D_N} \end{pmatrix}, \quad p_i \text{ prob. that } Q_i = 1, Q_i \text{ indep.}$$

preservation and time evolution imply:

1. reversible time evolution $T \in$ some 1-param. group

$$\vec{r}(t) = T(t) \cdot \vec{r}(0)$$

with 'Bloch' vector $\vec{r} = 2\,\vec{P}_{O\to S} - \vec{1}$

2. O's info about Q_i $\alpha_i = (2p_i - 1)^2 \Rightarrow O$'s total info about S:

$$I_{O\to S} = ||2\,\vec{P}_{O\to S} - \vec{1}||^2 = \sum_{i=1}^{D_N}(2\,p_i - 1)^2 = \sum_{i=1}^{D_N} r_i^2$$

[from different perspective also proposed by Brukner, Zeilinger]

3. $\{\text{all possible time evolutions}\} \subset \mathrm{SO}(\mathrm{D_N})$

$\Rightarrow$ info $I_{O\to S}$ 'conserved charge' of time evol.

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$N = 1$ and the Bloch ball [PH '14]

argued before: $D_1 = 3 \Rightarrow$ have: $\vec{P}_{O\to S} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$

- **pure states:**

$$I_{O\to S} = (2\,p_1 - 1)^2 + (2p_2 - 1)^2 + (2p_3 - 1)^2 = 1\,\texttt{bit} \qquad (1)$$

- **mixed states:**

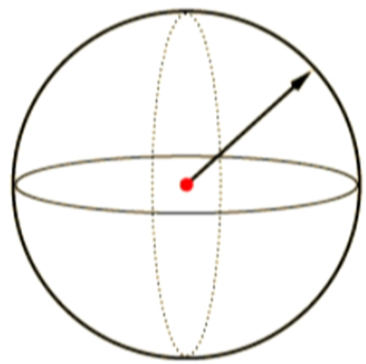
$$0\,\texttt{bit} < (2\,p_1 - 1)^2 + (2p_2 - 1)^2 + (2p_3 - 1)^2 < 1\,\texttt{bit} \qquad (2)$$

- **completely mixed state:**

$$(2\,p_1 - 1)^2 + (2p_2 - 1)^2 + (2p_3 - 1)^2 = 0\,\texttt{bit} \qquad (3)$$

using completeness axiom:

1. $\Sigma_1 =$ Bloch ball ✓
2. $\{$all time evolutions $T\} = \mathrm{SO}(3)$ ✓
3. $\mathcal{Q}_1 = S^2$ (require every pure state corresponds to answered Q)



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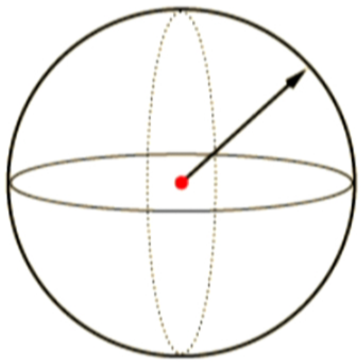
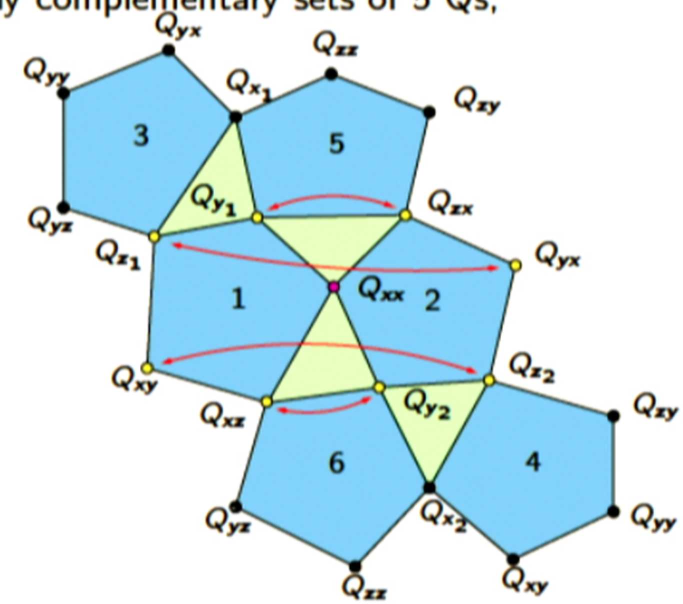


$N = 2$: complementarity, unitarity and pure states [with C. Wever]

from before: $D_2 = 15$

- $\exists$ 6 max. mutually complementary sets of 5 Qs,



- pure states have "conserved charges": Info(Pent$_i$) $= 1$ bit, $i = 1, \ldots, 6$,

  e.g., $\quad \text{Info}(\text{Pent}_1) = r_{y_1}^2 + r_{z_1}^2 + r_{xx}^2 + r_{xy}^2 + r_{xz}^2 = 1, \qquad \forall$ pures states

- which transformations allowed?

  e.g., 'information swap' between Pent$_1$ and Pent$_2$ preserves "charges"

$$r_{y_1}^2 \longleftrightarrow r_{zx}^2,\ r_{z_1}^2 \longleftrightarrow r_{yx}^2,\ r_{xy}^2 \longleftrightarrow r_{z_2}^2,\ r_{xz}^2 \longleftrightarrow r_{y_2}^2$$

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$N = 2$: From complementarity to unitarity and state spaces [with C. Wever]

- info swap defines generator at Bloch vector level, $\vec{r} \to e^{t\,G}\,\vec{r}$
  using correlation structure can show:

$$G_{ij}^{\text{Pent}_1,\text{Pent}_2} = \delta_{iy_1}\delta_{jzx} - \delta_{iz_1}\delta_{jyx} + \delta_{ixy}\delta_{jz_2} - \delta_{ixz}\delta_{jy_2} - (i \longleftrightarrow j).$$

- similarly for any pair of pentagons $\Rightarrow \exists$ 15 such swap generators

$\Rightarrow$ form $\mathfrak{su}(4) \simeq \mathfrak{so}(6) \simeq \mathfrak{psu}(4)$

$\Rightarrow$ exponentiation yields $\mathrm{PSU}(4)$ which is max. subgroup of $\mathrm{SO}(15)$

$\Rightarrow$ {all time evolutions} $= \mathrm{PSU}(4)$ as in QT ✓

pure states $\vec{r}_{\text{pure}}$ defined by 21 (part. dep.) equalities

1. 6 'pentagon equalities'

$$\text{Info}(\text{Pent}_k) = 1\,\text{bit}, \; k = 1, \ldots, 6$$

2. 15 'pentagon preservation conditions'

$$\vec{r} \cdot A^{\text{Pent}_k,\text{Pent}_l} \cdot G^{\text{Pent}_k,\text{Pent}_l} \cdot \vec{r} = 0, \qquad A_{ij}^{\text{Pent}_k,\text{Pent}_l} = \delta_{ij \in \text{Pent}_k}$$

$\Rightarrow$ yields $\mathbb{CP}^3$ for pure states

$\Rightarrow$ Σ_2 = convex hull of $\mathbb{CP}^3$ ✓

$\Rightarrow$ unitary group and states from complementarity relations and 'conserved informational charges'

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The case for $N > 2$

limited info and complementarity yield

Informationally complete set [PH '14]

$4^N - 1$ questions

$$Q_{\mu_1\mu_2\cdots\mu_N} := Q_{\mu_1} \leftrightarrow Q_{\mu_2} \leftrightarrow \cdots \leftrightarrow Q_{\mu_N}, \qquad \mu = 0, 1, 2, 3,$$

$\leftrightarrow =$ XNOR and $Q_{0_i} = 1$
$\Rightarrow$ correct number of DoFs ✓

permit: group of time evol. contains pairwise qubit unitaries

get: [with C. Wever]

1. pairwise unitaries generate all unitaries (universality [Harrow]) $\Rightarrow$ time evol. group $\mathrm{PSU}(2^{\mathrm{N}})$ as in QT ✓
2. $\Sigma_N =$ convex hull of $\mathbb{CP}^{2^N-1}$ ✓
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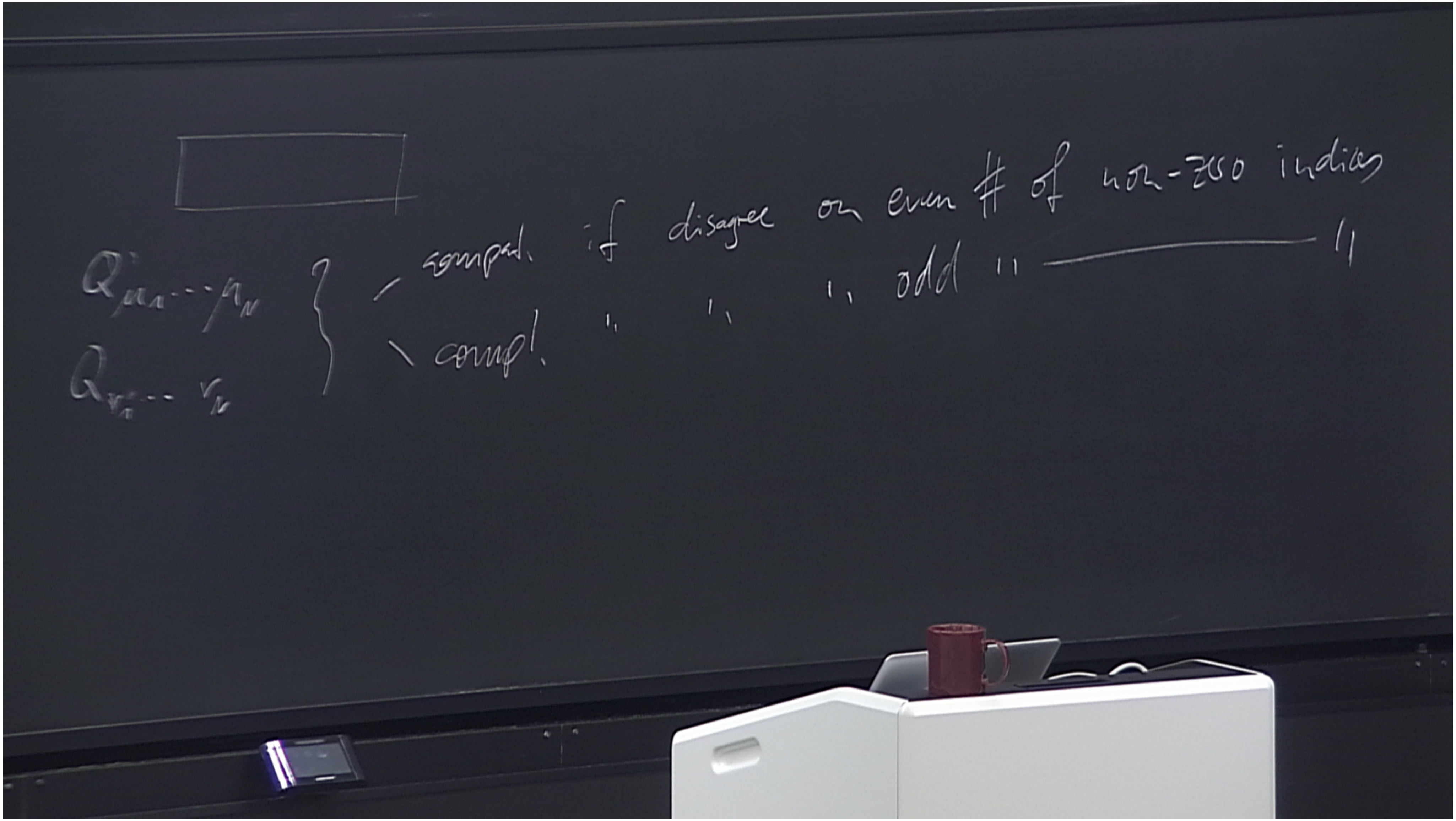
Conclusions

Rules on O's acquisition of information about S yield formalism of QT

novel constructive perspective on:

- dimensionality of state spaces
- entanglement and correlation structure
- monogamy
- quantifying O's info
- origin of unitary group from complementarity and 'conserved info charges'

further reading: PH arXiv:1412.8323 (minor revision coming), PH and C. Wever (forthcoming)



compat. if disagree on even # of non-zero indices
odd
compl.