Title: The (macro)-reality of superpositions

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Abstract: <p>This talk touches on three questions regarding the ontological status of quantum states using the ontological models</p>

<p>framework: it is assumed that a physical system has some underlying ontic state and that quantum states correspond to probability distributions over these ontic states.</p>

<p>The first question is whether or not quantum states are necessarily real---that is, whether or not the distributions for different quantum states must be disjoint. The PBR theorem proves the reality of quantum states by making assumptions about the ontic structure of bipartite systems, assumptions that have been challenged. Recent work has therefore concentrated on single systems, producing theorems proving the existence of pairs of quantum states whose overlap region on the ontic state space is very small.</p>

<p>The second question is whether the ontology of a quantum system can be macro-realist---that is, can there be "macroscopic" quantities which always have determinate values? The Leggett-Garg inequalities claim to rule out this possibility, but this conclusion has been disputed.</p>

<p>The third question is less familiar: Must quantum superpositions be ontic? That is, for some superposition with respect to some orthonormal basis, must ontic states exist which can be obtained by preparing the superposition, but not by preparing any of the basis states? In other words, can SchrÃ¶dinger's cat always be either alive xor dead?</p>

The (macro)-reality of superpositions

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PI Foundations Seminar, 2015/3/24

Outline

Real vs. *epistemic*

This work—like PBR and others—considers the **epistemic realist** position.

Realist – there exists some underlying **ontic** *state* $\lambda \in \Lambda$ of a physical system.

Epistemic – preparing a system in quantum state $|\psi\rangle$ results in a (generally) unknown ontic state obtaining according to some probability measure: a **preparation measure**.

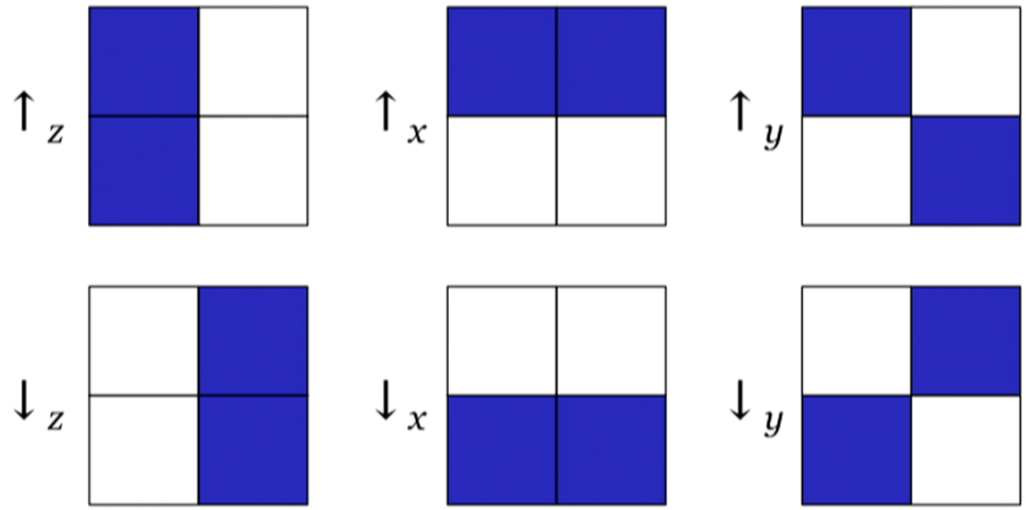
These preparation measures can, in principle, overlap.

This is attractive because perhaps certain quantum features can be explained in terms of overlapping preparation measures:

- Indistinguishability?
- No-cloning?
- Exponential complexity with increasing size?

N.B. here, all quantum states are assumed to be pure.

Example: Spekkens' toy model



What this talk is not about

The natural framework for discussing epistemic realism is that of **ontological models**.

Every* realist approach to quantum theory can be cast as an ontological model.

So Bohmian theories and collapse theories are included, but Copenhagen-esque or QBist theories are exempt from this analysis.

Ontological models (1)

d-dimensional quantum system. Measurable space of ontic states $\Lambda \ni \lambda$.

Preparation of $|\psi\rangle \Rightarrow$ some **preparation measure*** $\mu_{|\psi\rangle}$ gives prob. of each $\lambda \in \Lambda$ obtaining.

$\mu_{|\psi\rangle}$ has support* $\Lambda_\psi \subset \Lambda$.

Measurement of M (POVM or PVM) $\Rightarrow$ Conditional probability* $\mathbb{P}_M(E|\lambda) \in [0,1]$ for each outcome $E \in M$ for ontic state λ.

Therefore, preparation of $|\psi\rangle$ via $\mu_{|\psi\rangle}$ followed by measurement of M produces outcome $E \in M$ with probability

$$\int_\Lambda \mathrm{d}\mu_{|\psi\rangle}(\lambda)\,\mathbb{P}_M(E\,|\,\lambda).$$

Ontological models (2)

Unitary transformation $U \Rightarrow$ stochastic map* γ_U probabilistically transforms the λ.

If some preparation $\mu_{|\psi\rangle}$ has been made, then the action of this map can be understood to transform $\mu_{|\psi\rangle}$ to some other preparation.

Since preparing $|\psi\rangle$ then applying U is a way to prepare $U|\psi\rangle$, then we have

$$\boxed{\mu_{|\psi\rangle} \overset{\gamma_U}{\rightsquigarrow} \mu_{U|\psi\rangle}}$$

Assume quantum statistics

Prepare $|\psi\rangle$, apply U, and measure M to get outcome E with probability

$$\int_\Lambda \mathrm{d}\mu_{U|\psi\rangle}(\lambda)\mathbb{P}_M(E \mid \lambda) = \langle\psi|U^\dagger E U|\psi\rangle.$$

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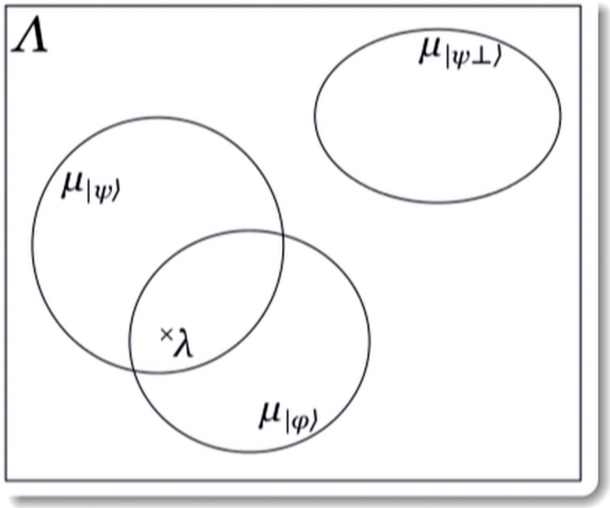
Reminder: Ontological models (3)



Prepare $|\psi\rangle$ and measure $|\phi\rangle$ gives

$$\int_\Lambda \mathrm{d}\mu_{|\psi\rangle}(\lambda)\mathbb{P}_M(|\phi\rangle \mid \lambda) = |\langle\phi|\psi\rangle|^2.$$

Consider the probability of $\mu_{|\psi\rangle}$ producing a λ in the overlap

$$\mu_{|\psi\rangle}(\Lambda_\phi).$$

Clearly, if $\lambda \in \Lambda_\phi$ then $\mathbb{P}_M(|\phi\rangle \mid \lambda) = 1$, so

$$\mu_{|\psi\rangle}(\Lambda_\phi) \leq |\langle\phi|\psi\rangle|^2.$$

Quantifying overlaps: Asymmetric overlap

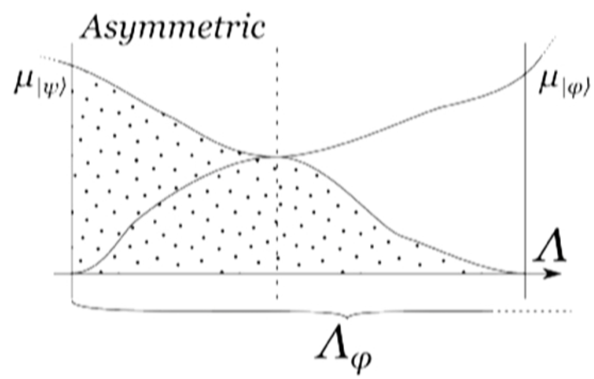
This can be used as way of quantifying overlaps. The **asymmetric overlap** is the probability of getting a $\lambda \in \Lambda_\phi$ given that we prepared $|\psi\rangle$:

$$\varpi(|\phi\rangle \,|\, |\psi\rangle) \stackrel{\text{def}}{=} \mu_{|\psi\rangle}(\Lambda_\phi).$$

Which must therefore satisfy

$$\varpi(|\phi\rangle \,|\, |\psi\rangle) \leq |\langle\phi|\psi\rangle|^2$$

and $|\psi\rangle = |\phi\rangle \Rightarrow \varpi(|\phi\rangle \,|\, |\psi\rangle) = 1$ and $|\psi\rangle \perp |\phi\rangle \Rightarrow \varpi(|\phi\rangle \,|\, |\psi\rangle) = 0$.



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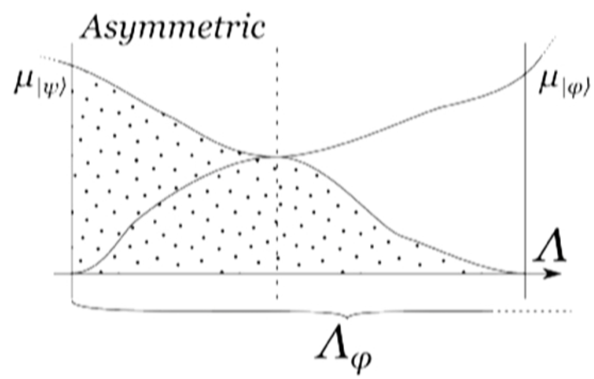

What does it mean for quantum states to be real?

Quantum states are **real** (ontic, state of reality) if they are ontological properties:

- every λ uniquely identifies a quantum state.
- preparation measures for different quantum states do not overlap.
- Λ is partitioned into disjoint regions, each the preparation support of exactly one quantum state.

The model is said to be ψ-**ontic**.

If some preparation measure $\mu_{|\psi\rangle}$ overlaps with another $\mu_{|\phi\rangle}$ for $|\psi\rangle \neq |\phi\rangle$, then quantum state $|\psi\rangle$ is **epistemic** (a state of knowledge)*.

If a model is not ψ-ontic (it has at least one epistemic quantum state) then it is ψ-**epistemic**.

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So, are they real?

The PBR theorem [arXiv:1111.3328] proves that quantum theory is ψ-ontic, but must make additional assumptions about the ontic structure of bipartite systems, which are open to challenge.

So, try to re-create this result with **single-system** arguments and minimal assumptions (just the ontological models framework).

Unfortunately, there exist ψ-epistemic ontological models for quantum system of every finite d [arXiv:1303.2834].

PBR proves that (given assumptions) the **ontic overlaps vanish** between different quantum states.

Next best thing: upper-bound ontic overlaps for preparations of different quantum states can overlap and get close to ψ-ontic.

Bounding overlaps

Several authors have proved single-system theorems of the form: "There exists a pair of non-orthogonal quantum states whose overlap must satisfy this bound..." [arXivs:1310.8302; 1401.7996; 1407.3005].

Some also prove limits in which the bounds approach zero.

There are two common shortcomings:

- The proofs are *existential*: they leave open the possibility that almost all pairs of quantum states have large ontic overlap.
- Where overlap* $\to 0$ is proved, it is in a limit where the quantum states $\to$ orthogonality anyway.

What is macro-realism?

Macro-realism, introduced to quantum theory by Leggett & Garg with their inequalities, is typically defined somewhat informally (contributing to confusion over what it means).

Maroney & Timpson [arXiv:1412.6139] come up with the following:

Macro-realism

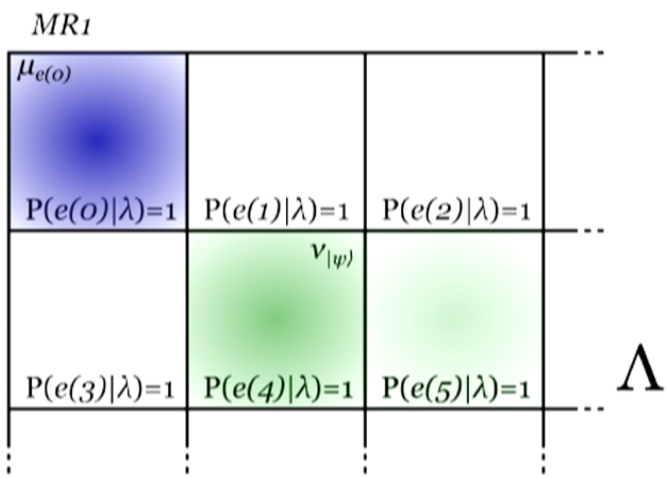
"A macroscopically observable property* [M] with two or more distinguishable values [$\{e(n)\}_n$] available to it will at all times determinately possess one or other of those values."

They then formalise this within the ontological models framework and define three sub-varieties: MR1, MR2, and MR3

First, define **operational eigenstates** $\{\mu_{e(n)}\}$ of M to be preparations which are value-definite for some value $e(n)$ of M.

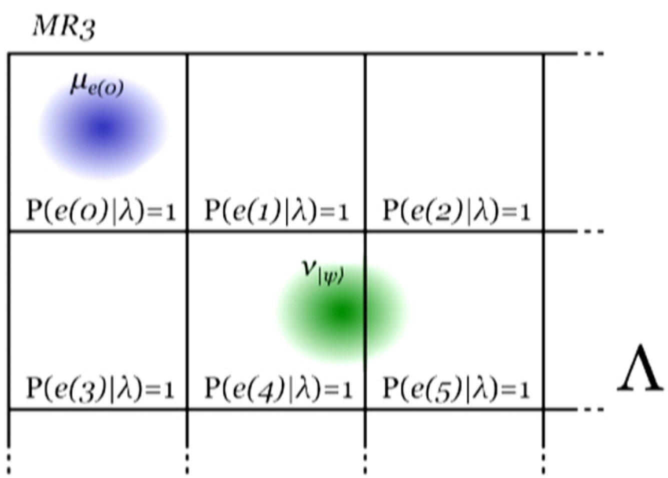
MR1*

The only possible preparations are *convex combinations of operational eigenstate preparations*: $\mu_{|\psi\rangle} = \sum_n p_n \mu_{e(n)}$.
This means that Λ is effectively partitioned according to M, with each $\lambda \in \Lambda$ preparable by exactly one of its operational eigenstates.



MR3*

The ontic state space Λ is again partitioned, but slightly differently.
Every $\lambda \in \Lambda$ will (on measurement of M) return exactly one $e(n)$ with certainty.
In other words, each λ still corresponds to one of these values, but need not be *preparable* by an operational eigenstate.
All MR2 & MR1 models are also MR3.



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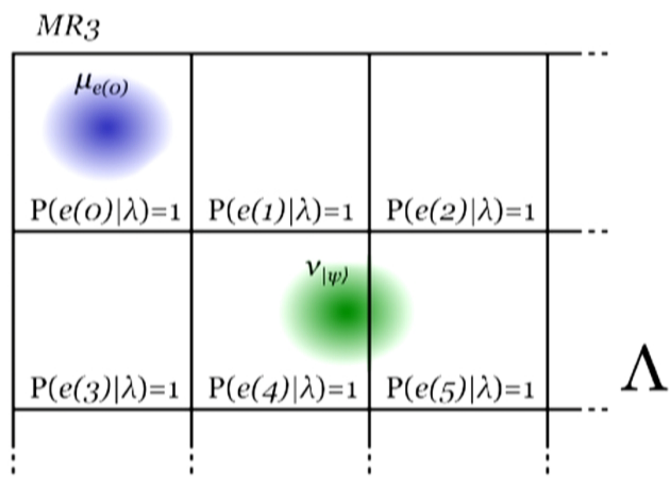
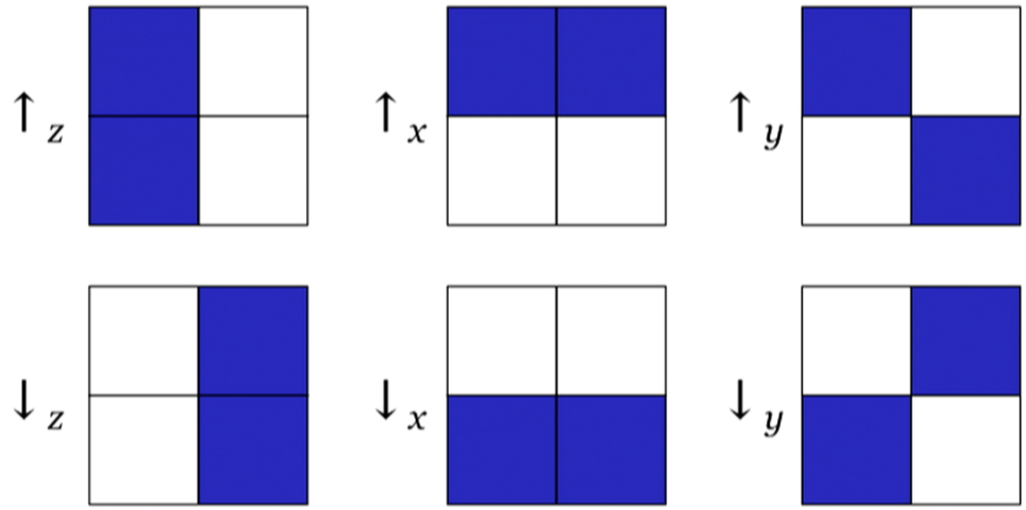


Example: Spekkens' toy model (again)



So this model is MR2 (and therefore MR3), but not MR1.

The Leggett-Garg inequalities

With formal definitions for macro-realism, the Legget-Garg inequalities (LGIs) can be re-evaluated.

These are inequalities designed to hold for some measurement statistics assuming "macro-realism".

The LGIs are violated by quantum experiments, the claim would be that the LGIs therefore prove quantum theory cannot be macro-realist

However, careful analysis by Maroney & Timpson shows that **only MR1** is strong enough to derive the LGIs.

Therefore, without further assumptions, the LGIs say nothing about MR2 and MR3.

Section 4

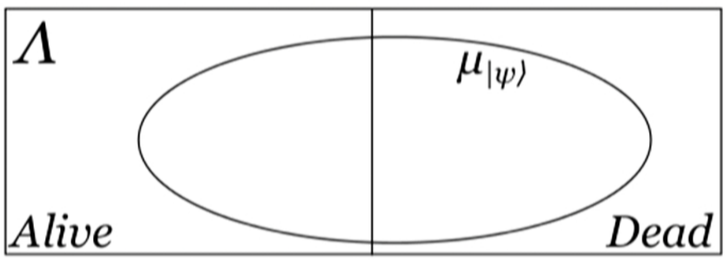
Question 3: Are quantum superpositions real?

Everyone's favourite superposition: Schrödinger's cat

The cat can be engineered to be,
for example, in the superposition state

$$|\psi\rangle = \sqrt{\frac{1}{3}}|\text{dead}\rangle + \sqrt{\frac{2}{3}}|\text{alive}\rangle.$$

It is natural* for epistemic realists to want the cat either alive *xor* dead on the ontological level, *i.e.*



The alternative would be for some ontic states to not correspond t
alive or dead cats.

Real vs. epistemic superpositions

Superpositions are quantum states $|\psi\rangle \notin \mathcal{B}$ defined with respect to some orthonormal basis $\mathcal{B} = \{|i\rangle\}_i$.

A superposition $|\psi\rangle$ is **epistemic** if, for every $\lambda \in \Lambda_{|\psi\rangle}$, $\exists$ some $|i\rangle \in \mathcal{B}$ such that $\lambda \in \Lambda_{|i\rangle}$ too.

Otherwise, $|\psi\rangle$ is **ontic** or **real**: there are novel ontic states for $|\psi\rangle$ which are not required for $\mathcal{B}$ itself.

An epistemic superposition must satisfy

$$\boxed{\varpi(|i\rangle \mid |\psi\rangle) = |\langle i|\psi\rangle|^2, \quad \forall |i\rangle \in \mathcal{B}.}$$

This is exactly what happens, for example, in Spekkens' toy model.

Asymmetric overlap: Monotonicity

Prepare $|\psi\rangle$ (via $\mu_{|\psi\rangle}$) and then apply U.

Each λ is (probabilistically)
mapped into some target set $\Omega_U(\lambda) \subset \Lambda$.

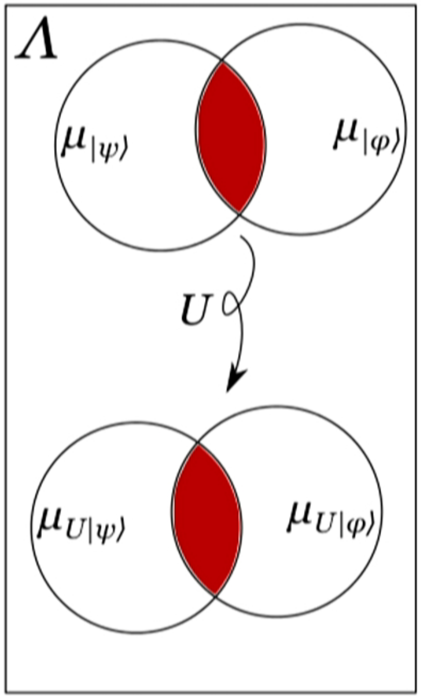
$\lambda \in \Lambda_\psi \Rightarrow \Omega_U(\lambda) \subseteq \Lambda_{U|\psi\rangle}$,

$\lambda \in \Lambda_\phi \Rightarrow \Omega_U(\lambda) \subseteq \Lambda_{U|\phi\rangle}$.

Therefore $\lambda \in \Lambda_\psi \cap \Lambda_\phi \Rightarrow \Omega_U(\lambda) \subseteq \Lambda_{U|\psi\rangle} \cap \Lambda_{U|\phi\rangle}$.

So **overlap** λ**s only map to overlap** λs and thus

$$\boxed{\varpi(U|\phi\rangle \mid U|\psi\rangle) \geq \varpi(|\phi\rangle \mid |\psi\rangle).}$$



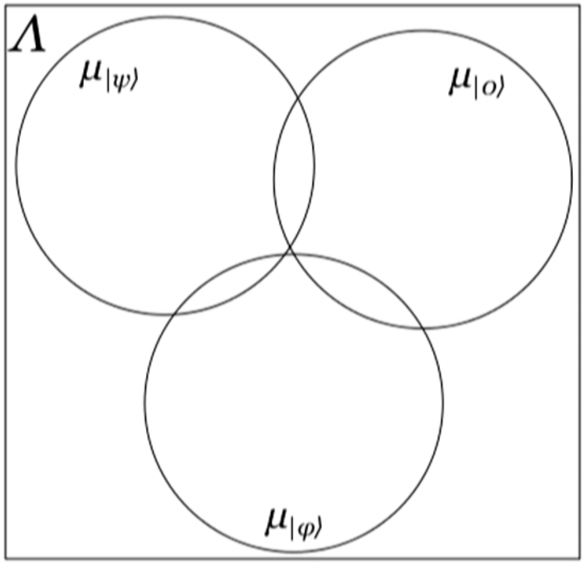
Anti-distinguishable states

A triple of quantum states is **anti-distinguishable** if there is a measurement that, with certainty, identifies one that was **not** prepared.

i.e. $\{|\psi\rangle, |\phi\rangle, |0\rangle\}$ is anti-distinguishable iffe $\exists$ some measurement $M = \{E_{\neg\psi}, E_{\neg\phi}, E_{\neg 0}\}$ for which

$$\langle\psi|E_{\neg\psi}|\psi\rangle = \langle\phi|E_{\neg\phi}|\phi\rangle = \langle 0|E_{\neg 0}|0\rangle = 0.$$

This means that there can be no tri-partite ontic overlap.
arXiv:quant-ph/0206110 derives sufficient conditions for a triple to be anti-distinguishable.



The construction

Suppose $d > 3$ and that there exists some $|0\rangle \in \mathcal{B}$ such that $|\langle 0|\psi\rangle|^2 \stackrel{\text{def}}{=} \alpha^2 < \frac{1}{2}$ (almost always the case).

Define another basis $\mathcal{B}' \ni |0\rangle$ such that

$$\begin{aligned} |\psi\rangle &= \alpha|0\rangle + \beta|1'\rangle + \gamma|2'\rangle, \\ |\phi\rangle &\stackrel{\text{def}}{=} \delta|0\rangle + \eta|1'\rangle + \kappa|3'\rangle. \end{aligned}$$

Where: $\alpha \in (0, \frac{1}{\sqrt{2}})$, $\beta \stackrel{\text{def}}{=} \sqrt{2}\alpha^2$, $\delta \stackrel{\text{def}}{=} 1 - 2\alpha^2$, $\eta \stackrel{\text{def}}{=} \sqrt{2}\alpha^2$.

So that (for any $\alpha < \frac{1}{\sqrt{2}}$):

- $|\langle 0|\psi\rangle| = |\langle\phi|\psi\rangle|$,
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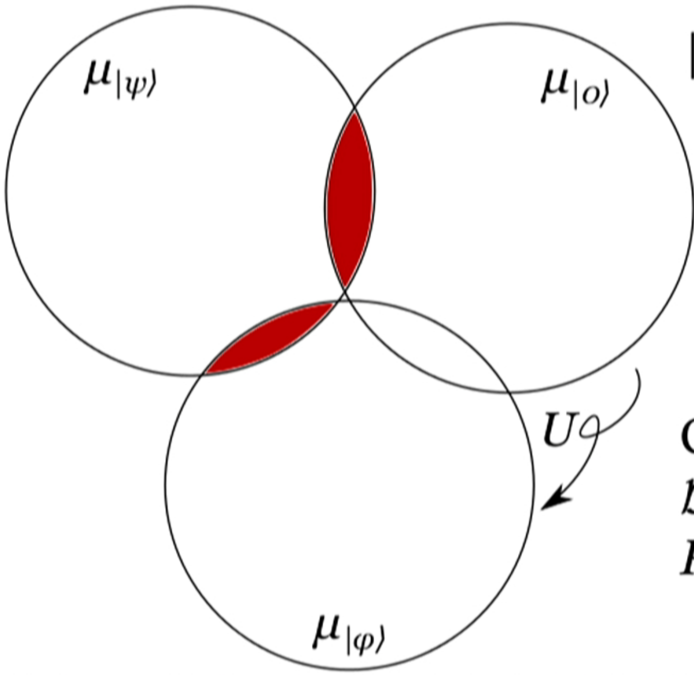
The argument

Λ



$$|\langle\varphi|\psi\rangle| = |\langle o|\psi\rangle|$$

$$\Rightarrow \left.\begin{matrix} U|\psi\rangle = |\psi\rangle \\ U|o\rangle = |\varphi\rangle \end{matrix}\right\}$$

$$\Rightarrow \varpi(o|\psi) \leq \varpi(\varphi|\psi)$$

Consider measuring in the $\mathcal{B}'$ basis:

$$P_{\mathcal{B}'}(o \text{ or } 1' \mid \psi) \geq \varpi(o|\psi) + \varpi(\varphi|\psi)$$
$$\geq 2\varpi(o|\psi)$$

But quantum theory tells us that

$$\mathbb{P}_{\mathcal{B}'}(0 \text{ or } 1' | \, |\psi\rangle) = |\alpha|^2 + |\beta|^2$$

The conclusion

From above

$$\begin{aligned}\mathbb{P}_{\mathcal{B}'}(0 \,\mathrm{or}\, 1' | \, |\psi\rangle) &= |\alpha|^2 + |\beta|^2 \\ \text{and } \mathbb{P}_{\mathcal{B}'}(0 \,\mathrm{or}\, 1' | \, |\psi\rangle) &\geq 2\varpi(|0\rangle \,|\, |\psi\rangle).\end{aligned}$$

together imply

$$\varpi(|0\rangle \,|\, |\psi\rangle) \leq |\alpha|^2 \left(\frac{1}{2} + |\alpha|^2\right) < |\alpha|^2 = |\langle 0|\psi\rangle|^2$$

and, in particular

$$\boxed{\varpi(|0\rangle \,|\, |\psi\rangle) < |\langle 0|\psi\rangle|^2}$$

which rules out $|\psi\rangle$ being epistemic wrt $\mathcal{B}$ (which requires $\varpi(|0\rangle \,|\, |\psi\rangle) = |\langle 0|\psi\rangle|^2$).

Therefore, almost all superpositions are real for $d > 3$.

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Macro-realism and real superpositions

You may already have noticed the similarity between the definitions of macro-realism and ontic superpositions.

Whilst there are important differences, an MR2 model for a quantum system must **necessarily have some epistemic superpositions**.

Specifically, if the "macroscopically observable property" has N distinguishable values* then there exist orthonormal sets $\tilde{\mathcal{B}}$ of at least N quantum states such that every superposition over $\tilde{\mathcal{B}}$ is epistemic.

Epistemic superpositions imply no-MR2

Since almost all superpositions for any basis of $d > 3$ elements must be ontic, it follows that no quantum system can be MR2 macro-realist for a macroscopic quantity with $N > 3$ distinguishable values*.

What about general macro-realism?

We have that MR2 is impossible for quantum systems and "macro" quantities with $N > 3$ distinguishable values.

This immediately implies the impossibility of MR1.

So, for $N > 3$, this supersedes the Leggett-Garg inequalities.

What about MR3? Well, Bohmian mechanics is an example of an MR3 theory so it is impossible to rule out MR3 wholesale.

However, Bohmian mechanics is also ψ-ontic. It may still be possible to rule out all ψ-epistemic MR3 theories.

Bounding overlaps with single-system arguments

Proof of real superpositions works by upper-bounding an asymmetric overlap $\varpi(|0\rangle \,|\, |\psi\rangle)$.

This is exactly the sort of thing that single-system "ψ-ontology" arguments do.

Recall that it is impossible to completely rule out ψ-ontic models without further assumptions, but it is possible to bound ontic overlaps.

These are not strictly "ψ-ontology" arguments, but proving small overlaps is almost as good as proving empty overlaps.

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A single-system ψ-ontology result

By slightly modifying the above proof, we get a single-system ψ-ontology result.

Bounding overlaps

For any $d > 3$ quantum system and any pair of quantum states $|\psi\rangle, |\phi\rangle$ satisfying $|\langle\psi|\phi\rangle|^2 \stackrel{\text{def}}{=} \alpha^2 < \frac{1}{4}$ the asymmetric overlap must satisfy*

$$\begin{aligned} \varpi(|\phi\rangle \,|\, |\psi\rangle) &\leq \alpha^2 \left(\frac{1+2\alpha}{d-2} \right) \\ \Rightarrow \lim_{d\to\infty} \varpi(|\phi\rangle \,|\, |\psi\rangle) &= 0. \end{aligned}$$

So in the limit of large dimension, this bound approaches zero independent from the inner product α.

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Comparison with previous results

Recall the shortcomings of previous single-system results noted earlier.

This theorem addresses both shortcomings:

- This result is **not existential**. The bound holds for **arbitrary pairs** of quantum states satisfying an inequality.
- A bound is proved $\to 0$ in a limit where quantum states **remain non-orthogonal**. In fact the states themselves are not affected by the limiting procedure.

This means that for large-dimensional quantum systems there are **large numbers** of pairs of states which **overlap negligibly**.

On the other hand, the bound itself is weaker than some of the previous bounds.

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Section 8

Robustness

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But what about error?

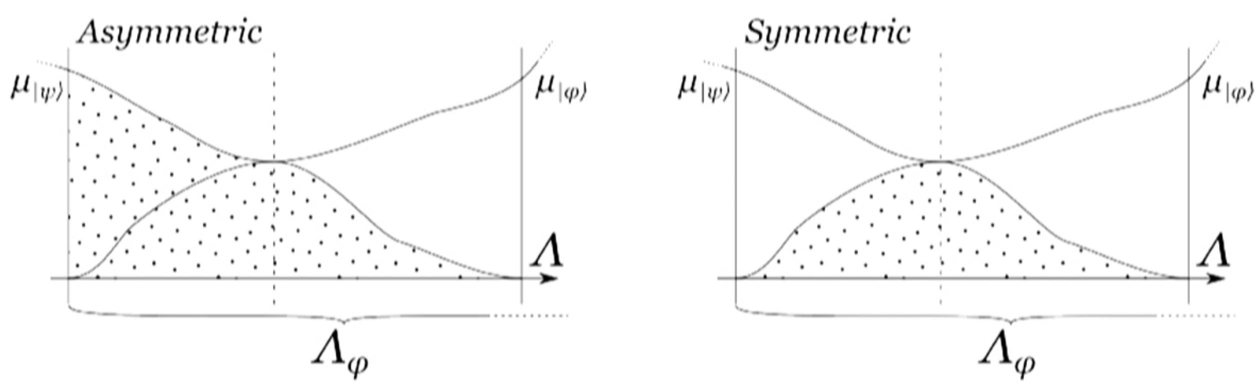
I'm sure you've noticed: I've been assuming *exact* quantum statistics.

What if the ontological model only reproduces quantum probabilities to within $\pm\epsilon \in (0, 1)$?

Unfortunately, the asymmetric overlap is not error-tolerant. At all.

Fortunately, the alternative **symmetric overlap** $\omega(|\psi\rangle, |\phi\rangle)$ is error-tolerant.

$\omega(|\psi\rangle, |\phi\rangle) \in [0, 1]$. $\quad \omega(|\psi\rangle, |\phi\rangle) \leq 1 - \sqrt{1 - |\langle\psi|\phi\rangle|^2}$



Error-tolerant result

The above result can also be carefully re-derived to use the symmetric overlap.

Error-tolerant ψ-ontology result

For any $d > 3$ quantum system and any pair of quantum states $|\psi\rangle, |\phi\rangle$ satisfying $|\langle\psi|\phi\rangle|^2 \stackrel{\text{def}}{=} \alpha^2 < \frac{1}{4}$ the asymmetric overlap must satisfy*

$$\begin{aligned} \omega(|\phi\rangle, |\psi\rangle) &\leq \alpha^2 \left(\frac{1+2\alpha}{d-2}\right) + \frac{(d-3)(d-4)}{2(d-2)}\epsilon, \\ \lim_{d\to\infty} \omega(|\phi\rangle, |\psi\rangle) &\leq \mathcal{O}(d\epsilon), \end{aligned}$$

if quantum probabilities are reproduced to within $\pm\epsilon$.

What does this prove?

This generalises the previous bound to apply in the presence of finite error.

However, it is only non-trivial for $d > 5$ (since we're now using the symmetric overlap).

Also, this result does not imply an error-tolerant no-go for epistemic superpositions or MR2.

It does, however, suggest that these results can, in principle, be made error-tolerant.

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Section 9

Conclusions

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Almost all superpositions (wrt any given $\mathcal{B}$, for $d > 3$) must be real: there must be novel ontic states that are superposition states.

As a result, all MR2 models for "macro" observables with $N > 3$ values are ruled out*.

For large-dimensional quantum systems, many quantum state pairs cannot have significant overlap*.

This third result can be made tolerant to small error.

These three results are three perspectives on essentially the same proof.

An operational characterisation would be nice. It would help to devise possible experimental set-ups and perhaps suggest information-theoretic implications.

Thank you for listening

Any questions?

Thanks also to Jon Barrett, Owen Maroney, and Stefano Gogioso

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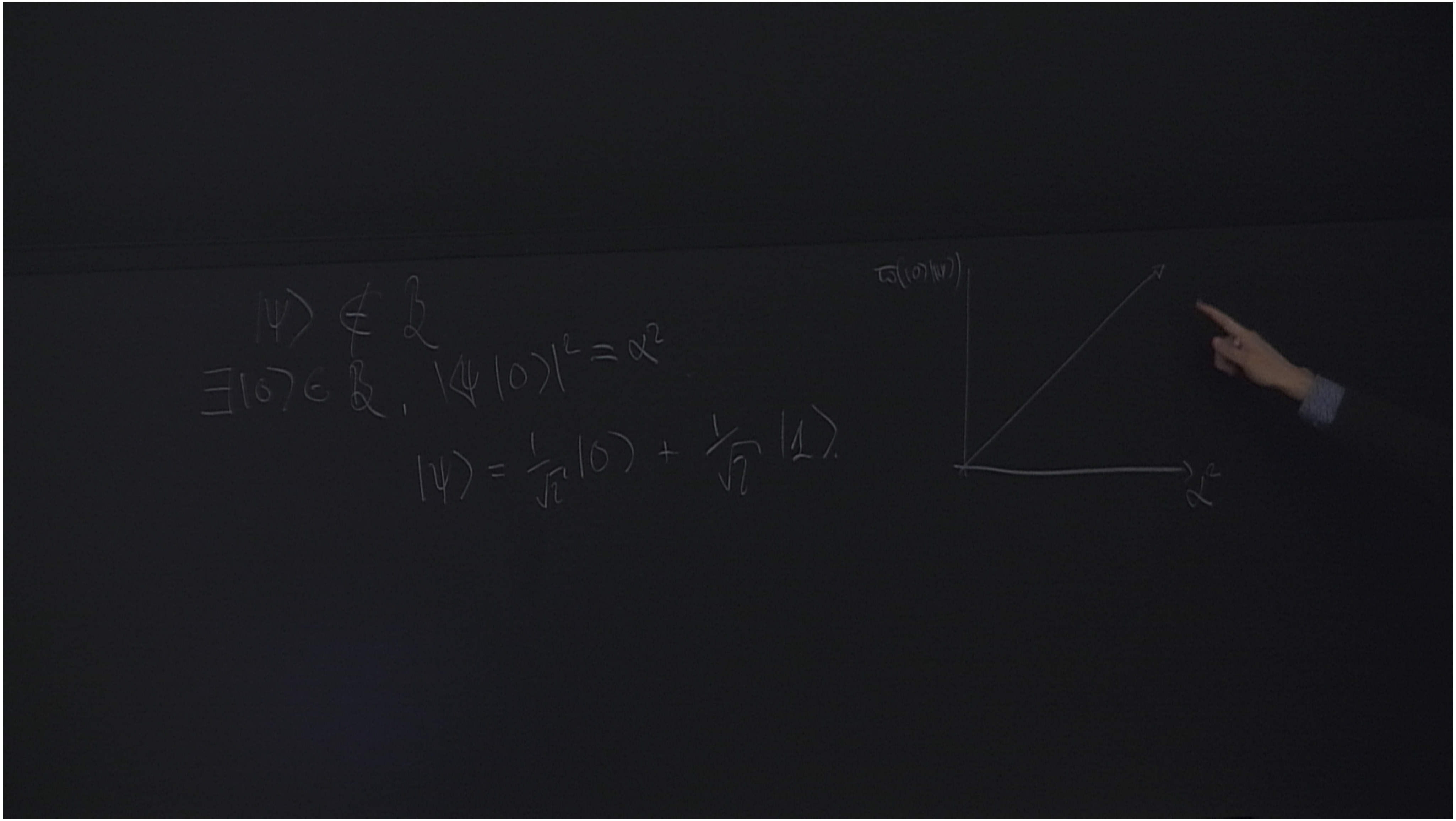
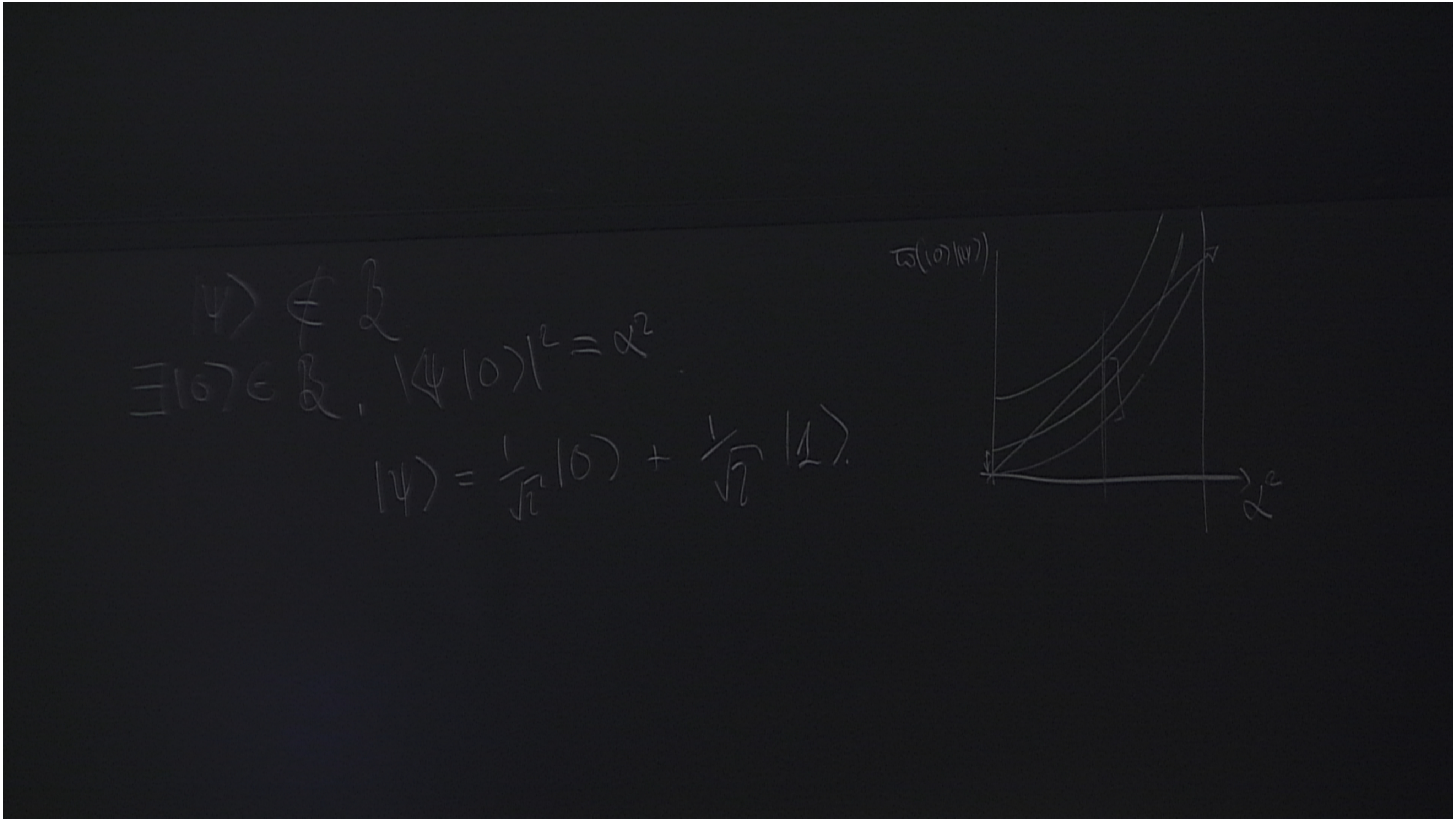

|ψ⟩ ∉ B
∃|0⟩ ∈ B, |⟨ψ|0⟩|² = α²
|ψ⟩ = 1/√2 |0⟩ + 1/√2 |1⟩.
ϖ(|0⟩|ψ⟩)
α²



$|\psi\rangle \notin \mathcal{B}$
$\exists |0\rangle \in \mathcal{B},\ |\langle\psi|0\rangle|^2 = \alpha^2$
$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$
α^2

Questions summary

Question 1

What are the limits on ontic overlaps for single-system ontological models of quantum systems? How close to ψ-ontic can we get without further assumptions?

Question 2

Bearing in mind that MR1 models are ruled out by the Legget-Garg inequalities and that Bohmian mechanics is MR3, what limits are there for macro-realist models for quantum theory?

Question 3

Given any $|\psi\rangle \notin \mathcal{B}$ of a d-dimensional quantum system, can $|\psi\rangle$ be epistemic wrt $\mathcal{B}$, or must it be ontic? Must it be real?