Title: Even a tiny cosmological constant casts a long shadow

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Abstract: <p>Surprisingly, several basic questions in classical and quantum gravity, which were resolved some 40-50 years ago for zero Λ, still remain open in the $\Lambda >0$ case. In particular, for $\Lambda >0$, we still do not have a satisfactory notion of gravitational radiation or Bondi 4-momentum in exact general relativity, nor a positive energy theorem. Similarly, the standard constructions of `in' and `out' Hilbert spaces that we routinely use (e.g. in the analysis of black hole evaporation) do not extend to the $\Lambda >0$ case. In this talk I will present some illustrative examples of these quandaries and introduce a systematic approach to resolve the open issues.

</p>

Even a tiny positive Λ casts a long shadow

Abhay Ashtekar
Institute for Gravitation & the Cosmos, Penn State

First three parts: Joint work with Beatrice Bonga & Aruna Kesavan;
The first part appeared in CQG: 32, 025004-46

Discussions/correspondence with Bianchi, Bicak, Blanchet, Chrusciel, Corichi, Costa, Garriga, Goldberg, Robinson & Saulson

Perimeter Institute: Strong Gravity Seminar, March 19th 2015

Isolated Systems, Gravitational Waves & the S-matrix

- Confusion re gravitational waves in full GR till 1960s (Exs: Einstein 1916 vs 1936; Eddington)

- The Bondi-Penrose et al Framework

  Notion of null infinity $\mathcal{I}^{\pm}$; (1960s to 1980s)

  Bondi, Metzner, Sachs (BMS) group $\mathcal{B} = \mathcal{S} \ltimes \mathcal{L}$

- Gravitational radiation:

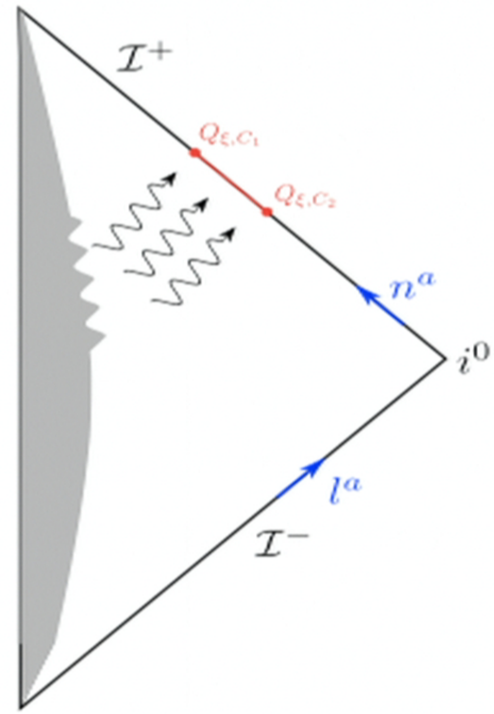
  Gauge invariant Bondi News N_{ab} at $\mathcal{I}^{\pm}$;

  No incoming radiation at $\mathcal{I}^{-}$: $N_{ab} = 0$.

  Balance law for Bondi-energy:

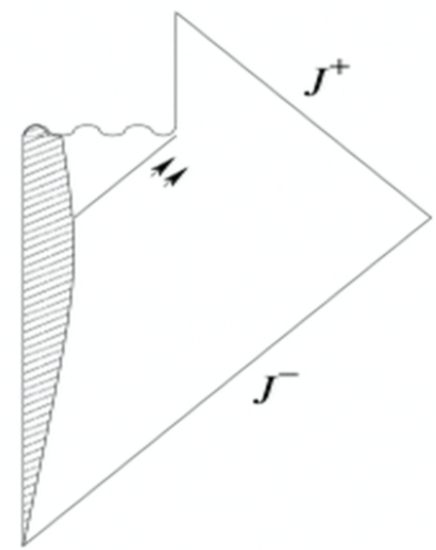
$$Q_\xi[C_2] - Q_\xi[C_1] = \int_{\Delta\mathcal{I}} \xi\, |N_{ab}|^2$$

  Positive $Q_\xi[C]$ and Flux positive (`Gravitational waves are real; you can boil water with them' ...Bondi)



- The BMS group $\mathcal{B}$ admits a unique 4-d Abelian normal subgroup of translations $\mathcal{T}$. Also, if $N_{ab} = 0$, then $\mathcal{B}$ reduces to the Poincare group.

- In quantum theory, one routinely uses $\mathcal{T}$ and $\mathcal{B}$ to define spin and mass of zero rest mass fields, and introduce asymptotic Hilbert spaces for the S-matrix theory, in particular to analyze the issue if information loss during black hole evaporation.



- None of this rich structure discussed so far goes over to the positive Λ case. We do not have even basic notions: Bondi news; Balance law; positive energy or flux; the `no incoming radiation' condition. Don't know what gravitational waves mean in full, non-linear GR for positive Λ, however small !
- Don't have the positive and negative frequency decomposition needed for asymptotic Hilbert spaces in quantum theory.

Organization of the Rest of the Talk

1. Asymptotically de Sitter space-times & difficulties
2. New Strategy: gravitational physics with positive Λ
3. Discussion

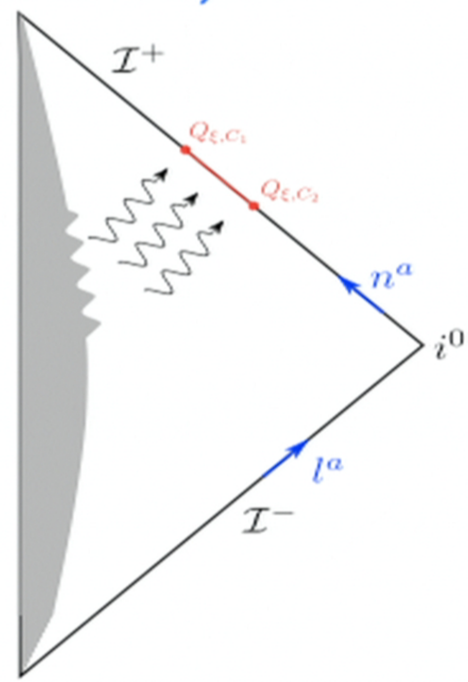
1. Asymptotically de Sitter space-times

- Recall the notion of asymptotic flatness: A physical space-time $(\tilde{M}, \tilde{g})$ is said to be asymptotically Minkowski if it admits a conformal completion **(M,g)**, where M = $\tilde{M} \cup \mathcal{I}$ is a manifold with boundary $\mathcal{I}$, & **g =** $\Omega^2 \tilde{g}$ on M , s.t.

  i) At the boundary $\mathcal{I}$, we have $\Omega = 0$ and $\nabla\Omega \neq 0$;

  ii) $\tilde{g}$ satisfies Einstein Eqs $\tilde{G}_{ab} = 8\pi\, G_N\, \tilde{T}_{ab}$ with $\tilde{T}_{ab}$ falling off sufficiently fast as $\Omega \to 0$;

  iii) and $\mathcal{I}$ is topologically S^2 X R and complete in an appropriate sense.



Asymptotically flat case: summary (contd)

- Field equations imply that $\mathcal{I}$ is null, hence ruled by the integral curves of its null normal, $n^a = \nabla^a \Omega$. Hence, the asymptotic symmetry group is reduced from Diff ($\mathcal{I}$) to the BMS group $\mathcal{B}$, which admits a preferred 4-d (Abelian, normal) sub-group of BMS-translations $\mathcal{T}$, that is then used to define energy-momentum, positive and negative frequency decomposition, etc.

Asymptotically de Sitter space-times

- A physical space-time $(\tilde{M}, \tilde{g})$ is said to be asymptotically de Sitter if it admits a conformal completion (M,g), where M = $\tilde{M} \cup \mathcal{I}$ is a manifold with boundary $\mathcal{I}$, & g = $\Omega^2 \tilde{g}$ on M , such that :

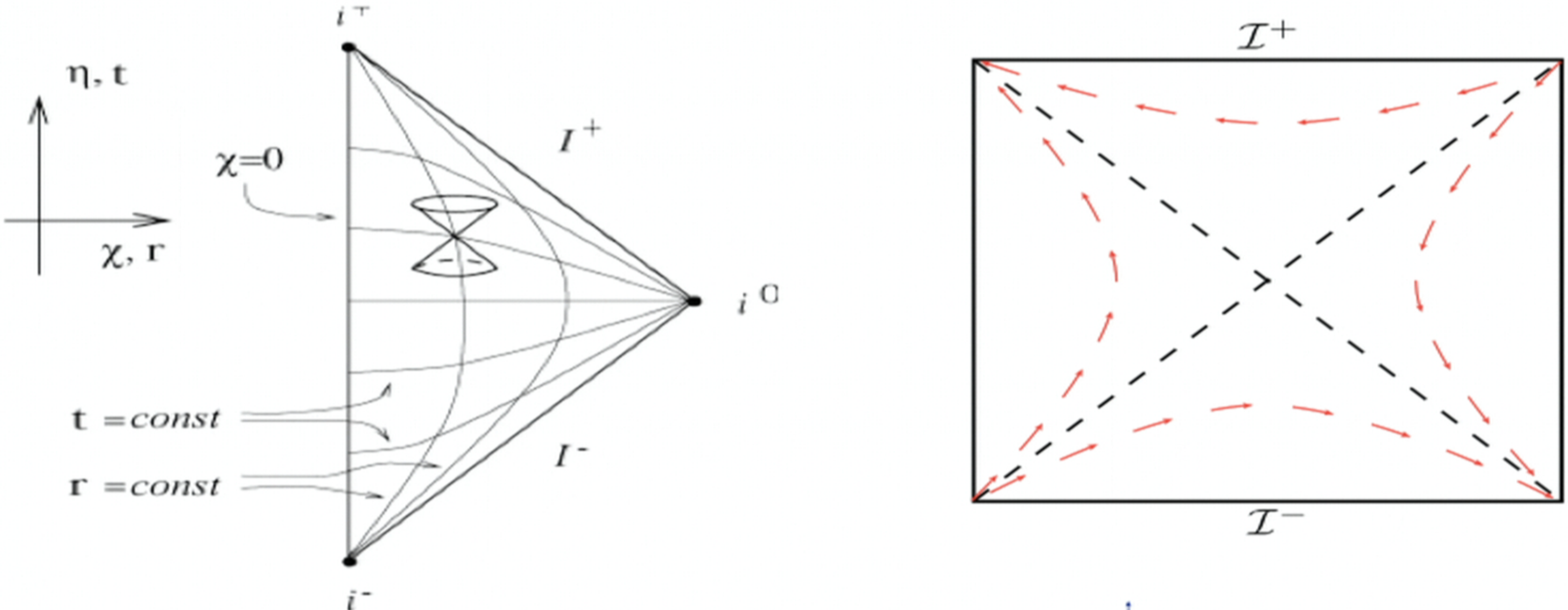
  i) At the boundary $\mathcal{I}$, we have $\Omega = 0 \text{ and } \nabla\Omega \neq 0$;

  ii) $\tilde{g}$ satisfies Einstein Eqs $\tilde{G}_{ab} = 8\pi G_N \tilde{T}_{ab} - \Lambda \tilde{g}_{ab}$ with $\tilde{T}_{ab}$ falling off sufficiently fast as $\Omega \longrightarrow 0$; and,

  iii) $\mathcal{I}$ is topologically S^3 (minus punctures, e.g. $S^2 \times R$) and complete in an appropriate sense.

Field equations now imply that $\mathcal{I}$ is space-like rather than null. Hence, no extra structure like a preferred ruling. Hence the asymptotic symmetry group is just Diff ($\mathcal{I}$) ! Not clear how to define Energy & linear (or, angular) momentum.

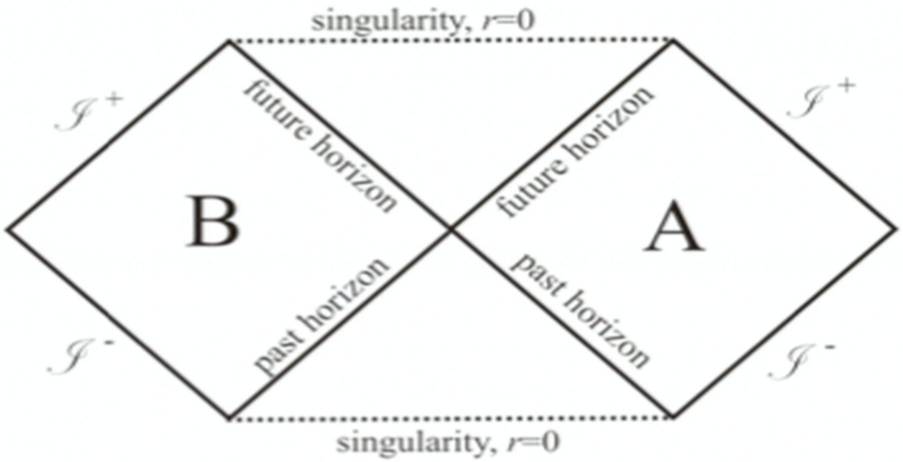
Contrasting Minkowski and deSitter



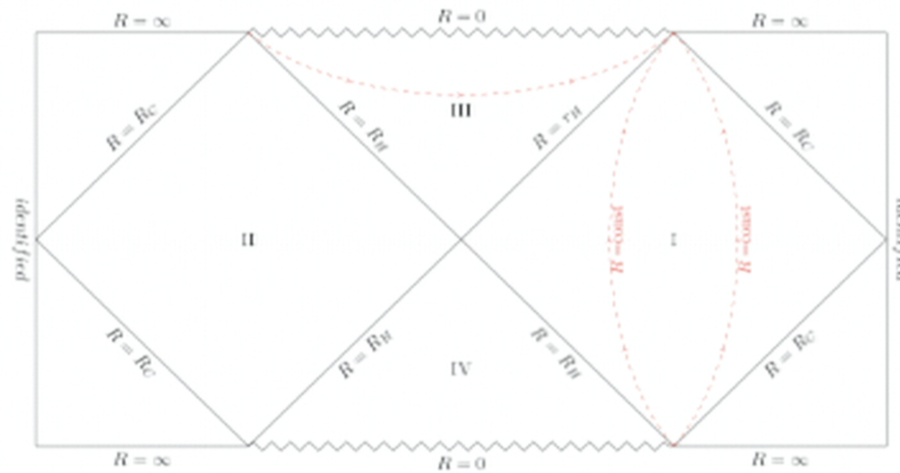
Time translation Killing fields are time-like near Minkowskian $\mathcal{I}$, whence energy fluxes of test fields across $\mathcal{I}$ are positive. In de Sitter, **all** Killing fields are space-like near $\mathcal{I}$. So fluxes associated with them, including the `energy flux' can be arbitrarily negative in de Sitter space-time irrespective of how small Λ is.

Asymptotic flatness vs Asymptotically deSitter

Eternal BH with $\Lambda = 0$



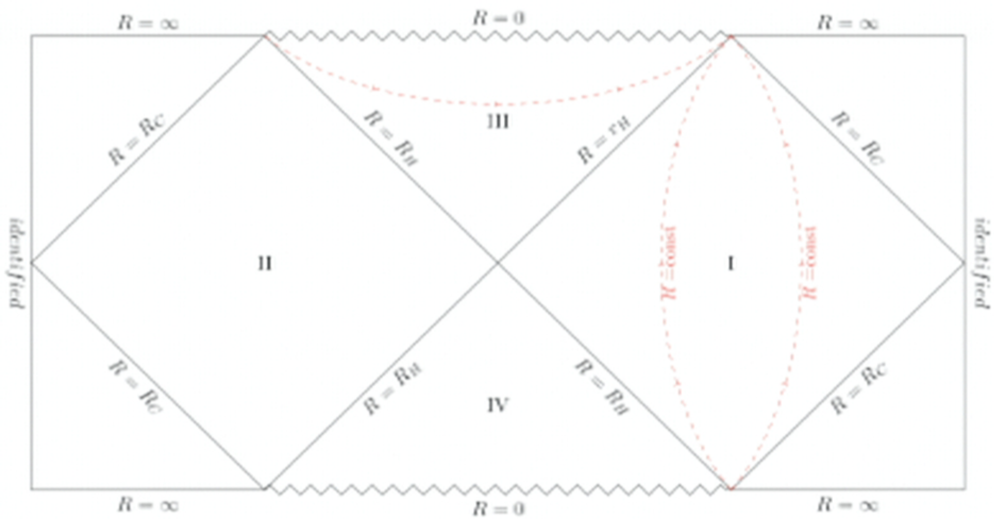
Eternal BH with positive Λ



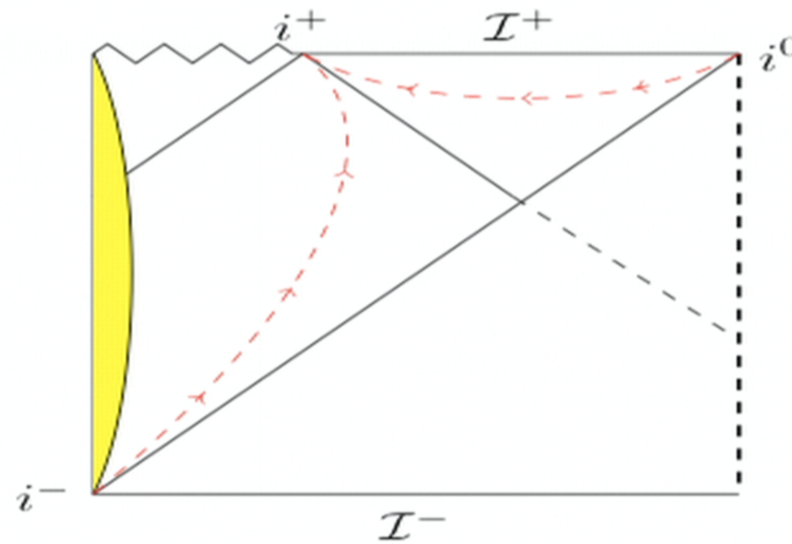
With positive Λ , the maximal extension has an infinite number of Asymptotic regions. One generally terminates the sequence via identification. Spatial topology is then $S^2 \times S^1$ rather than $S^2 \times R$.

Asymptotic flatness vs Asymptotically deSitter

Eternal Asym. dS BH



Asym. ds BH resulting from a Spherical collapse



In the collapsing case, one cannot identify. So space-time has a time-like boundary on the right. Cannot specify incoming states just on $\mathcal{I}^-$!

Restricted symmetries and Bondi-like Charges

- Can we strengthen the boundary conditions at $\mathcal{I}$ to reduce the Diff ($\mathcal{I}$) to a manageable size? A natural strategy; commonly used in the literature: Demand that, q_{ab}, the intrinsic +,+,+ metric at $\mathcal{I}$ be conformally flat, as in de Sitter.

- Not only is the group reduced; but it is reduced to the de Sitter group! Following Bondi, One can an now define charges $Q_\xi[C]$ at $\mathcal{I}$ in full GR as in asymptotically flat space-times: $Q_\xi[C] = \oint_C E_{ab}\xi^a dS^b$. Expected answers in Kerr-deSitter .

Gravitational radiation considerations: Problem

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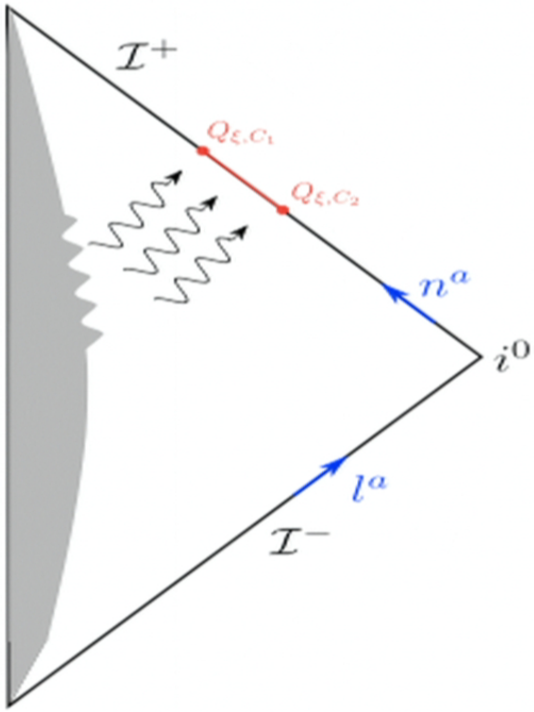
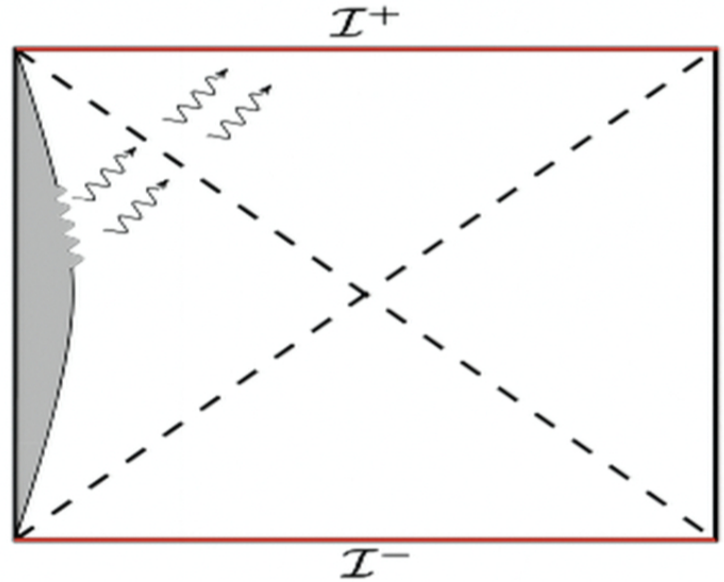
- However, there are two serous problems:
  i) Conformal flatness of $\mathcal{I}$ $\Leftrightarrow B_{ab} = 0$ at $\mathcal{I}$.
  Since $\mathcal{I}$ is space-like, half the solutions simply thrown out!
  ii) Secondly, $Q_\xi[C]$ well-defined, but absolutely conserved
  no flux of energy, momentum, etc through $\mathcal{I}$!

Contrasting Minkowski and deSitter





In asymptotically flat space-times, non-trivial flux of Bondi-energy through $\mathcal{I}$. If Λ is positive and $B_{ab} = 0$, fluxes across $\mathcal{I}$ vanish identically irrespective of how tiny Λ is!

Why do all fluxes vanish?

- In retrospect, however, this is not surprising. In the asymptotically flat case, if impose the condition: $B^{ab} = {}^\star C^{abcd} n_a n_b = 0$ at $\mathcal{I}$, we find (AA, 1980s) :

i) The BMS group reduces to the Poincare, just as Diff ($\mathcal{I}$) reduced to the de Sitter group here; and,

ii) The Bondi-news N_{ab} vanishes identically on $\mathcal{I}$; there is no flux of gravitational radiation across $\mathcal{I}$!

- Thus, the condition is too restrictive. But, if we remove it, we lose the entire machinery we routinely use in the asymptotically flat case: No `charges' representing de Sitter energy, momentum etc, let alone the positive energy theorem; no analog of the gauge invariant Bondi news $N_{ab}$; no access to the structure needed in simulations of BH mergers to calculate the `kicks' via emission of 3-momentum; no obvious Hilbert spaces of asymptotic states to for quantum theory!

2. Linear Gravitational Waves on de Sitter

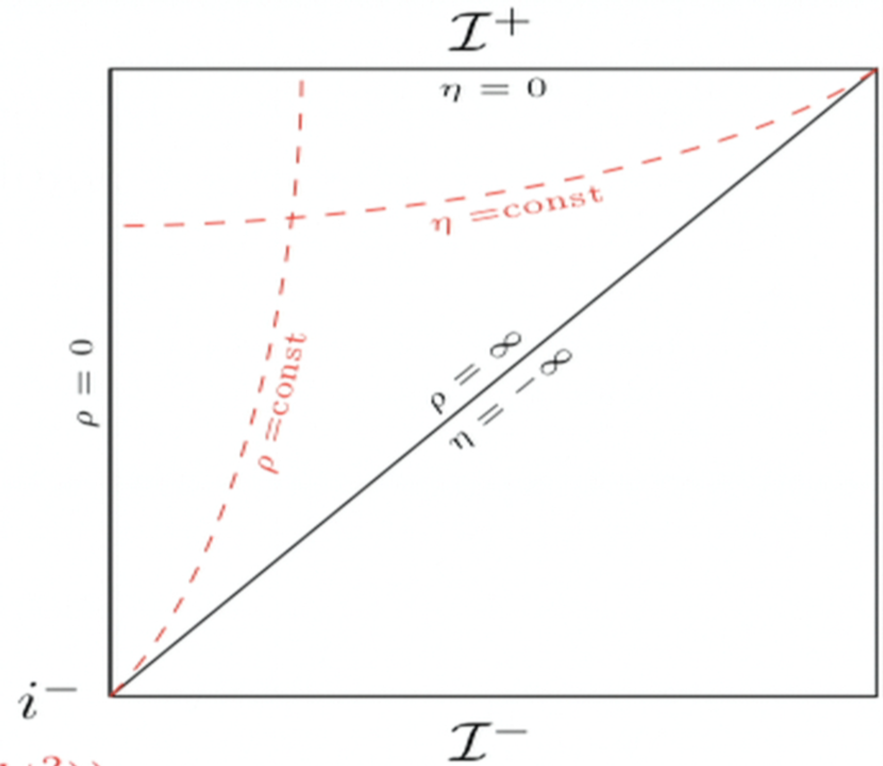
- Can seek some guidance from linearized gravitational waves as in the $\Lambda = 0$ case (where non -linear effects fall-off rapidly as one approaches $\mathcal{I}^+$.
- In the $\Lambda > 0$ case, one can obtain:
  * Explicit consequences of the $B_{ab} = 0$ condition.
  * Expressions of energy, momentum and angular-momentum fluxes carried by gravitational waves;
  * Positive energy-flux in physically relevant situations;
  * A generalization of the quadrupole formula (subtleties)

$B_{ab} = 0$ Condition in linear theory

Linearized gravitational waves:

Perturbation theory used in cosmology.

Explicit calculations show that the condition $B_{ab} = 0$ at $\mathcal{I}$ requires B= 0, leaving only the `decaying' modes (for which h_1 vanishes at $\mathcal{I}$), where



$$h_1(\vec{x}, \eta) = E(k, H)(\sin k\eta - k\eta \cos k\eta)\, e^{i\vec{k}\cdot\vec{x}}$$

$$h_2(\vec{x}, \eta) = B(k, H)(k\eta \sin k\eta - \cos k\eta)\, e^{i\vec{k}\cdot\vec{x}}$$

de Sitter metric:

$$ds^2 = (1/H\eta)^2\,(-d\eta^2 + d\rho^2 + \rho^2(d\theta^2 + \sin^2\theta\, d\phi^2))$$

Note: Limit $\Lambda \to 0$ corresponds to $H \to \infty$

de Sitter-momentum fluxes

- Energy-momentum and angular momentum carried by electromagnetic waves: $F_\xi = \int_M T_{ab}\xi^a \, dS^b = \int_M \epsilon_{abc} E^a B^b \xi^c \, dV$ if ξ^a is tangential to M (as at de-Sitter $\mathcal{I}$).

- No stress-energy tensor for gravitational waves. But we can use symplectic/Hamiltonian methods. For Maxwell theory: Covariant phase space = space of solutions. Symplectic form:

$$\omega(A, A') = \int_M (A_a F'^{ab} - A'_a F^{ab}) \, dS_b$$ (conserved, gauge invariant).

- Infinitesimal canonical transformation: $A_a \to A_a + \epsilon \mathcal{L}_\xi \, A_a$ is generated by the Hamiltonian $H_\xi = \int_M \epsilon_{abc} E^a B^b \xi^c \, dV$!
- Thus the covariant phase space provides a natural avenue to calculate fluxes associated with Killing fields. If ξ^a is a translation, that component of energy-momentum flux, etc.

de Sitter-momentum fluxes

- Energy-momentum and angular momentum carried by gravitational waves: Start with the covariant phase space of linear gravitational perturbations h_{ab} . Symplectic structure (derived from the action)

$$\omega(h, h') = (\ell/8\pi G)\int_M (h_{ab}E'^{ab} - h'_{ab}E^{ab})\, dV.$$

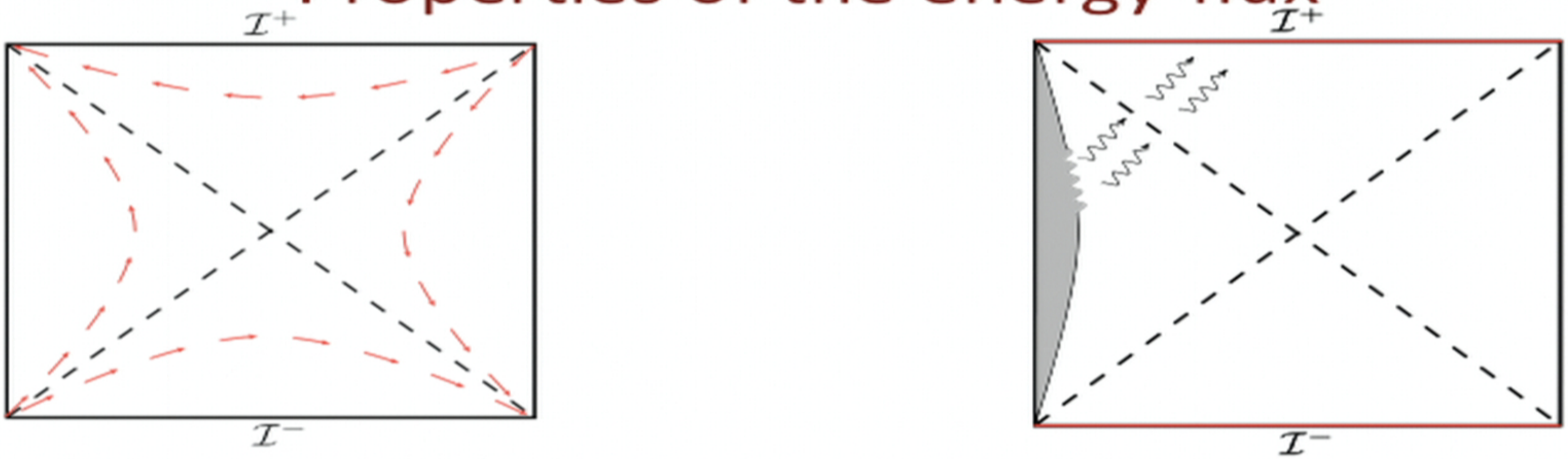
- We can compute Hamiltonians corresponding to de Sitter symmetries. Energy defined by a de Sitter time translation T^a:

$$H_T \equiv \frac{1}{2}\,\omega(h,\, \mathcal{L}_T h) = \frac{\ell}{8\pi G}\int_M E^{ab}(\mathcal{L}_T h_{ab} - \frac{2}{3}(D_c T^c)h_{ab})\, dV$$

Now, $h_{ab} = 0$ at $\mathcal{I}$ if $B_{ab} = 0$ there, and the flux vanishes. Same is true for fluxes of linear and angular momentum. (Correct/standard answer the $\Lambda \to 0$ limit but the limit is subtle!)

- Thus, if $B_{ab} = 0$, not only we rule out by fiat ½ the DOF but the remaining DOF do not carry any de Sitter fluxes!

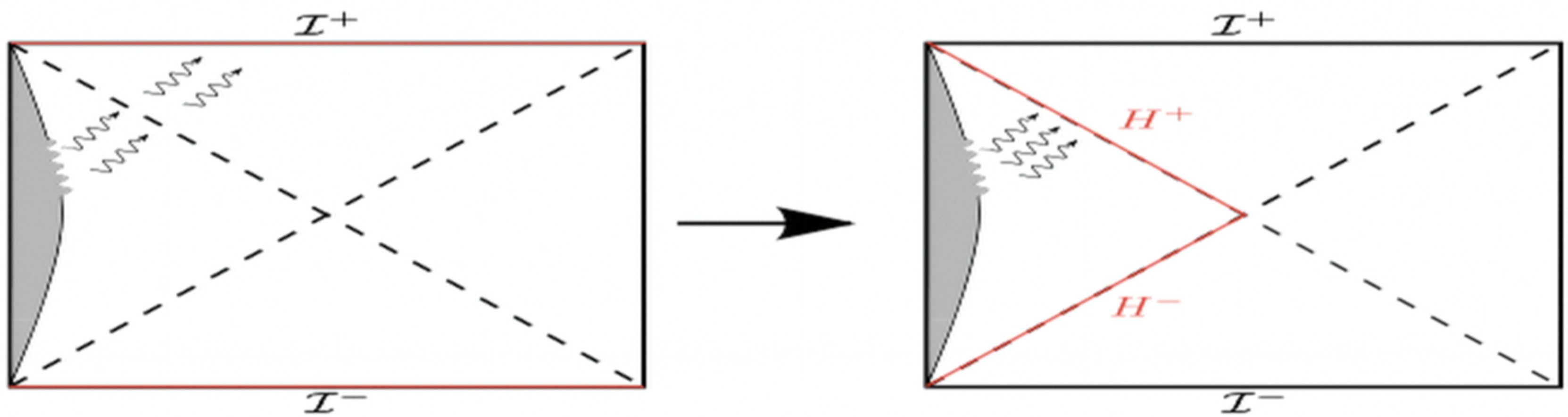
Properties of the energy-flux



- Positivity: The Killing field T^a is time-like in the left quadrant. This implies that H_T is positive in all physically interesting situations shown in the two figures
- The explicit expression of H_T : Agrees with the (2nd order) linearization of the flux one would get in the exact theory if we used $Q_\xi[C] = \oint E_{ab}\xi^a dS^b$ for charge integrals in the exact theory! Powerful hint for the Exact theory.

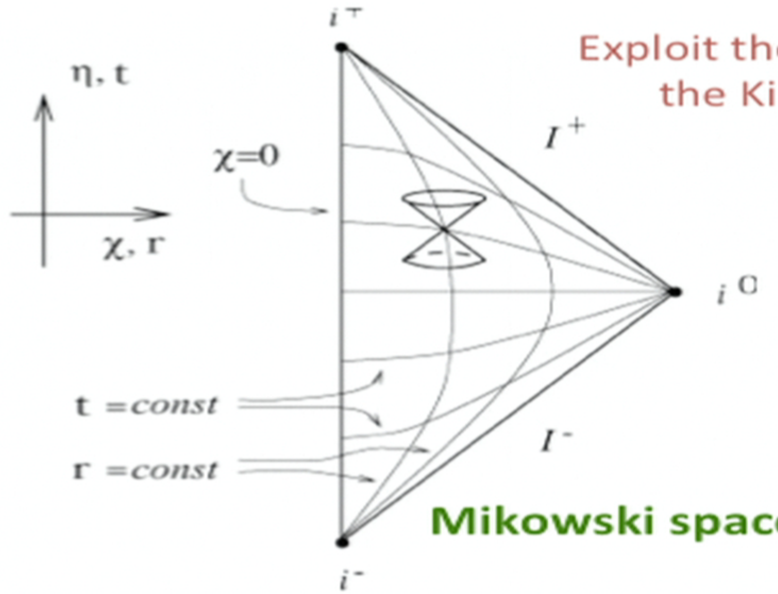
3. Full non-linear theory: One Strategy

- The strategy is to construct the theory using cosmological horizons in place of $\mathcal{I}$.



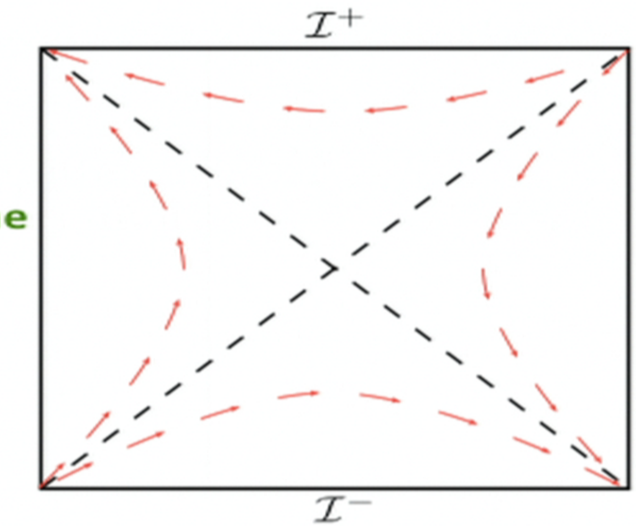
An oscillating star emitting gravitational waves. These are registered at the future horizon. Requiring that the past horizon be a Weakly Isolated Horizon (WIH) naturally incorporates the no incoming radiation boundary condition.

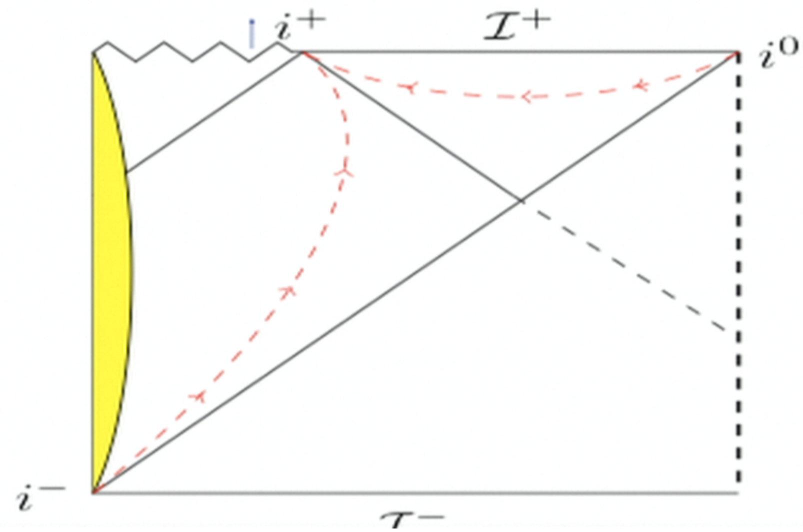
Exploit the properties of the Killing field



Mikowski space-time

de Sitter Space-time

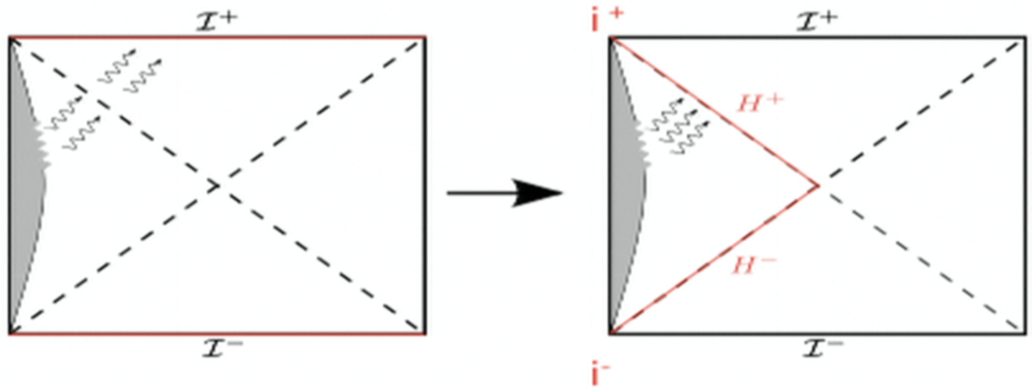




For the region bounded by the two horizons $H^{\pm}$, information coming from the right time-like boundary (world-tune of i° is irrelevant!

Collapse To a BH

Full, non-linear GR: The Setup

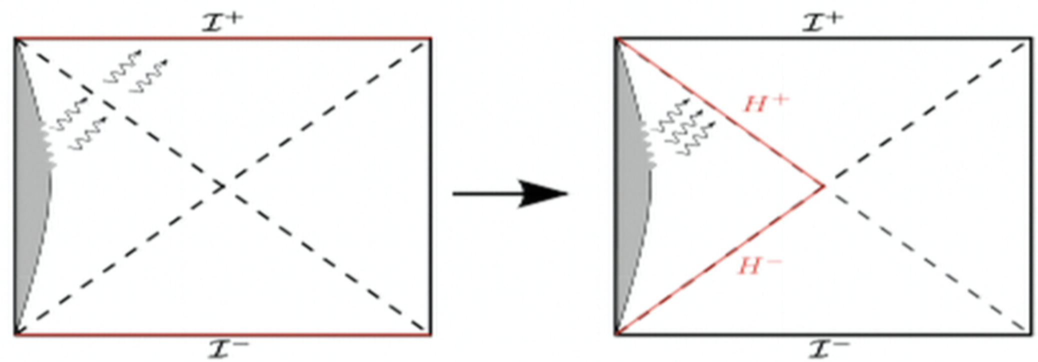


Focus on gravitating systems that remain in a spatially bounded region. Then we obtain a point i^- on $\mathcal{I}^-$ and a point i^+ on $\mathcal{I}^+$.

Assume that, H^-, the future event horizon of i^- is a weakly isolated horizon (WIH): A null non-expanding submanifold, $S^2 \times R$, equipped with a null normal l^a which is a symmetry of the intrinsic metric and `exitrinsic curvature' of $H^-$. Implements the `no incoming radiation' condition. (AA, Beetle, Fairhurst,Lewandowski,...) . Area constant. H^- is the local $\mathcal{I}^-$. H^+, the past event horizon of i^+, serves as the local $\mathcal{I}^+$
This will be our notion of an isolated system in presence of positive Λ .

4. Discussion

- For positive Λ, literature has focused primarily on $\mathcal{I}$, assuming conformally flat intrinsic geometry. But this is too restrictive because it halves the number of modes and, furthermore, ill suited to study gravitational radiation in full GR and for quantum considerations.
- Inclusion of Λ, however small, introduces qualitatively new, conceptual issues. Exs: $\mathcal{I}$ and hence all symmetry vector fields there are space-like; energy can be arbitrarily negative; an extra time-like boundary in gravitational collapse changing the S-matrix theory paradigm; no `peeling' at $\mathcal{I}$, making it difficult to isolate radiative modes there, ...
- New framework: Focus instead on H^- and H^+ adapted to the isolated system of interest.



- Can find the required symmetries; construct `charges' and fluxes associated with them; introduce the $\pm$ frequency decomposition and construct the Hilbert spaces necessary for S-matrix theory, infra-structure needed for the

- But several new elements from the familiar asymptotically Minkowskian context: Boundary conditions used in asymptotic matching; frameworks for testing no hair theorems; infrastructure underlying calculations of energy loss, black hole kicks, wave form extraction; ... Even in the TT linear approximation, we have a mass term: $(\Box - (2\Lambda/3)\, h_{ab} = 0$, affecting the propagation.

- Viewpoint: In the end, in almost all calculations, corrections will be very small (R_c ~ 5 Gpc!). But we need controlled approximations. For example LIGO will see sources from 1 Gpc. Do small effects accumulate over this distance? Surprises (like the memory effect and super-translation ambiguity in angular momentum)?

Experimenters and numerical GR experts have done a fantastic job. Theorists need to provide a solid infrastructure to match that!