

Title: Explorations in Condensed Matter-12

Date: Mar 31, 2015 10:15 AM

URL: <http://pirsa.org/15030049>

Abstract:

Calculations / measurements of an MPS

Consider $\hat{O}_j = \hat{I}_2 \otimes \dots \otimes \hat{O}_j \otimes \hat{I}_2 \otimes \dots$; $|\psi\rangle = \sum_{s_1, s_2, \dots, s_n} A_1^{s_1} A_2^{s_2} \dots A_n^{s_n} |s_1, s_2, \dots, s_n\rangle$

open BC \rightarrow $D \times D$ \dots $D \times 1$

Calculations / measurements of an MPS

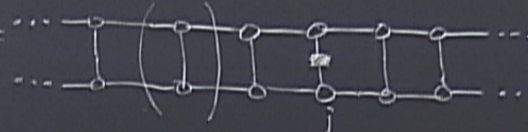
Consider $\hat{O}_j = \hat{I}_1 \otimes \hat{I}_2 \otimes \dots \otimes \hat{O}_j \otimes \hat{I}_{j+1}$

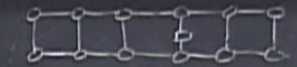
$$\langle 4 | \hat{O}_j | \Psi \rangle = \sum_{s'_1, s'_2} (\dots \bar{A}^{s'_1} \bar{A}^{s'_2} \dots) (A^{s_1} A^{s_2} \dots) \langle s'_1 s'_2 \dots | \hat{O}_j | s_1 s_2 \dots s_l \rangle$$

$$= \dots \left(\sum_{s'_1} \bar{A}^{s'_1} A^{s_1} \right) \left(\sum_{s'_2} \bar{A}^{s'_2} A^{s_2} \right) \dots \left(\sum_{s'_l} \bar{A}^{s'_l} A^{s_l} \right) \dots$$

Infinite
open BC
periodic T \rightarrow 1x1 1x1 ... 1x1

$$|\Psi\rangle = \sum_{s_1, s_2, \dots, s_l} \bar{A}^{s_1} A^{s_2} A^{s_3} \dots A^{s_l} |s_1, s_2, \dots, s_l\rangle$$


$$\langle 4 | \hat{O} | 4 \rangle = \dots \left(\sum_k \bar{A}^k A^k \right) \dots$$




$$dD^2 + dD^3 + \dots \alpha(D^3)$$

$$\sim ND^3 d$$

Cost of computation



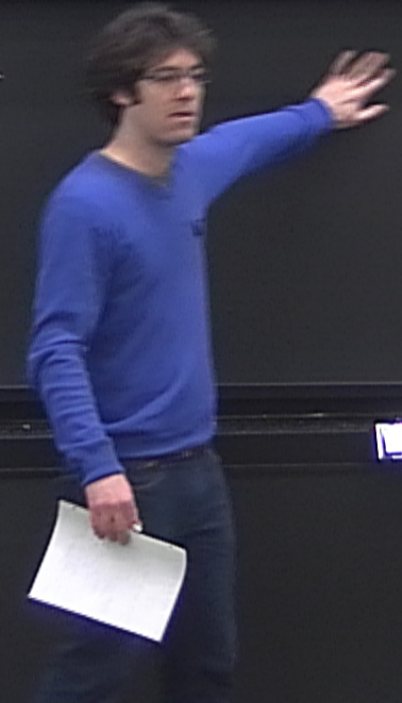
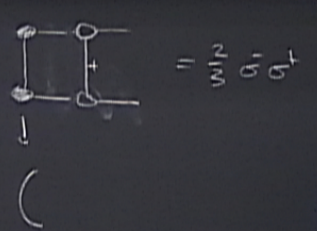
$$= \sum_{s_1, s_2, \dots, s_N} \prod_{i=1}^N \psi_{s_i, s_{i+1}}$$

$$|4_{AKLT}\rangle$$

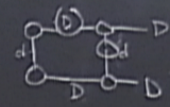
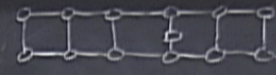
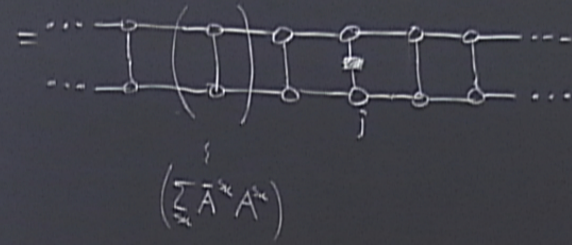
$$A^+ = \sqrt{\frac{2}{3}} \sigma^+ |0\rangle$$

$$A^0 = \frac{1}{\sqrt{3}} |1\rangle$$

$$A^- = -\sqrt{\frac{2}{3}} \sigma^- |0\rangle$$



$\langle 4 | \hat{O} | 4 \rangle$



$dD^2 + dD^3 + \dots \propto (D^3)$
 $\sim ND^3d$

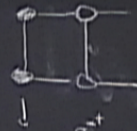


• Cost of computation

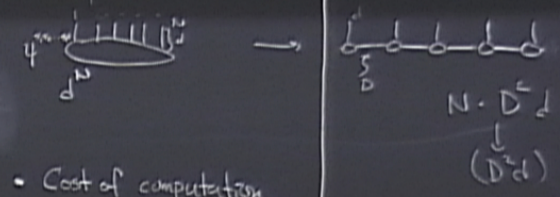
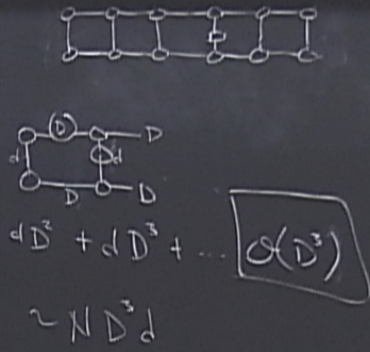
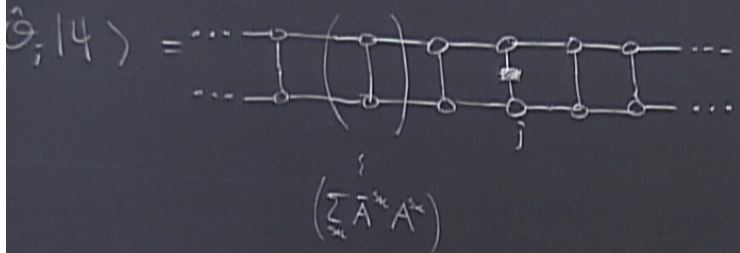
$\sum_{s_1, s_2, \dots, s_N} \prod_{i=1}^N \gamma_{s_i, s_{i-1}, s_{i+1}}$

$|4_{AKLT}\rangle$

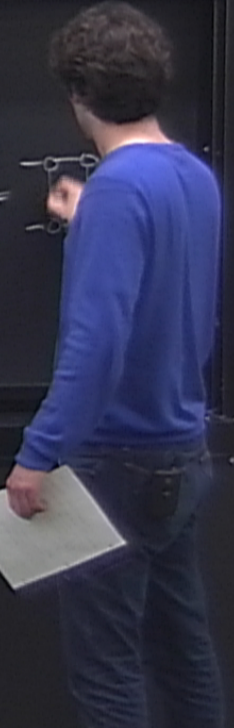
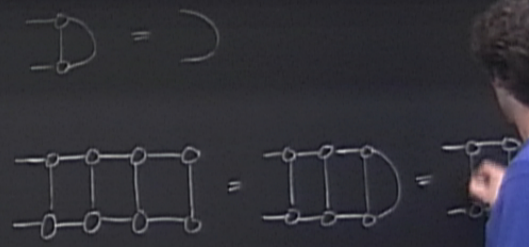
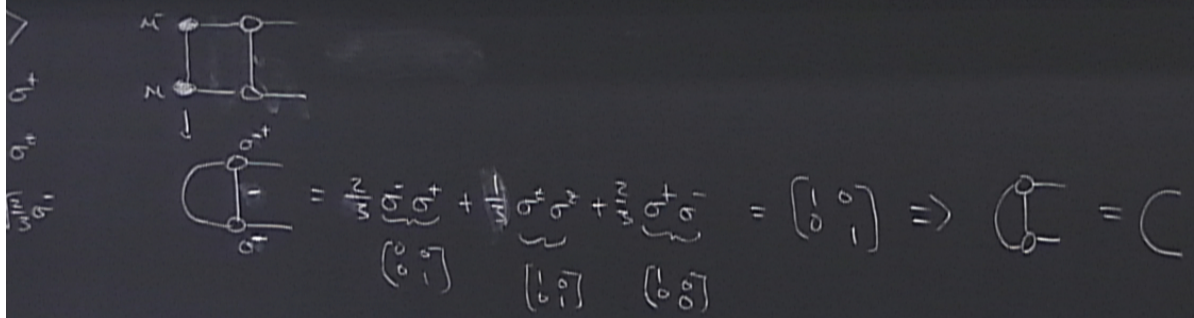
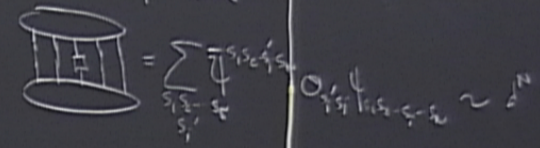
$A^+ = \sqrt{\frac{2}{3}} \sigma^+$
 $A^0 = \frac{1}{\sqrt{3}} \sigma^z$
 $A^- = -\sqrt{\frac{2}{3}} \sigma^-$

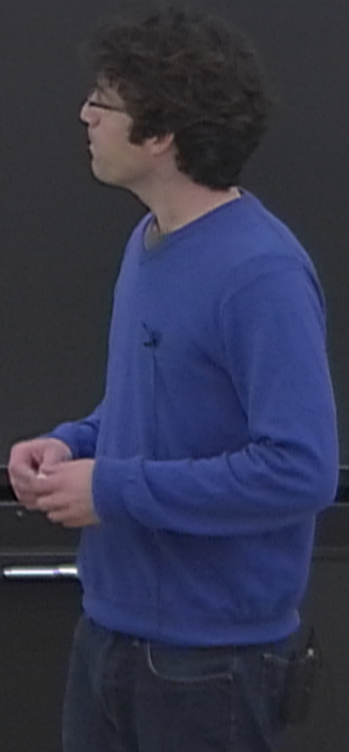
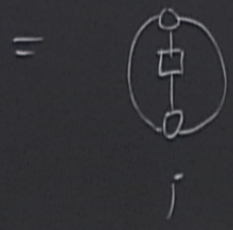
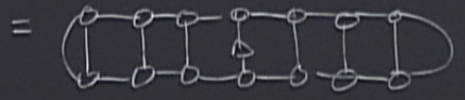
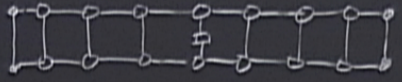


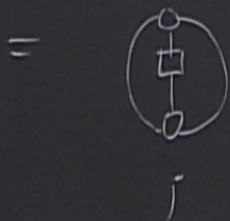
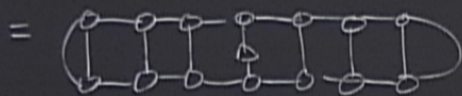
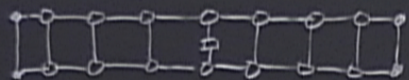
$\begin{matrix} \circ & \circ \\ \circ & \circ \end{matrix} \begin{matrix} \sigma^+ \\ \sigma^+ \end{matrix} = \frac{2}{3} \begin{matrix} \sigma^+ \\ \sigma^+ \end{matrix} \begin{matrix} \sigma^+ \\ \sigma^+ \end{matrix} + \frac{1}{3} \begin{matrix} \sigma^z \\ \sigma^z \end{matrix} \begin{matrix} \sigma^z \\ \sigma^z \end{matrix} + \frac{2}{3} \begin{matrix} \sigma^- \\ \sigma^- \end{matrix} \begin{matrix} \sigma^- \\ \sigma^- \end{matrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \Rightarrow \begin{matrix} \circ & \circ \\ \circ & \circ \end{matrix} = \left(\begin{matrix} \circ & \circ \\ \circ & \circ \end{matrix} \right)$



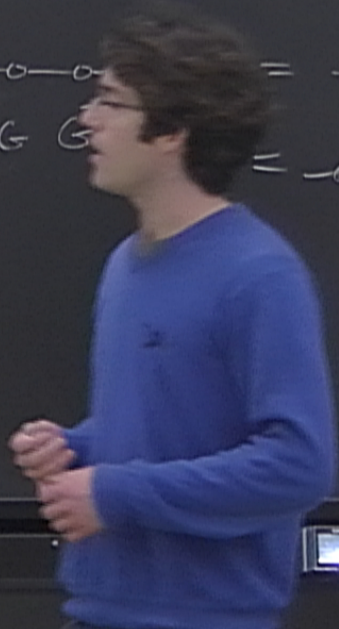
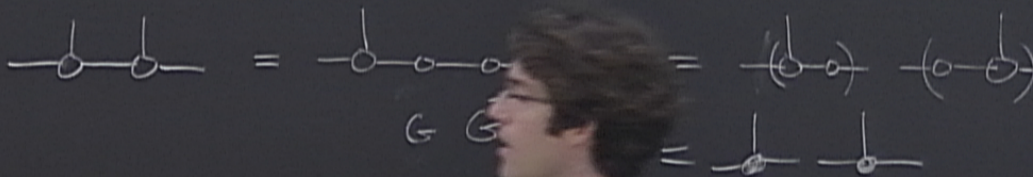
• Cost of computation





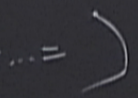
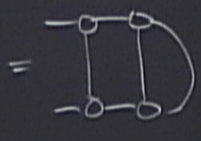
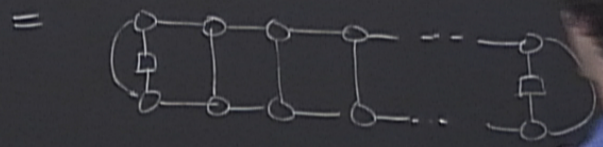
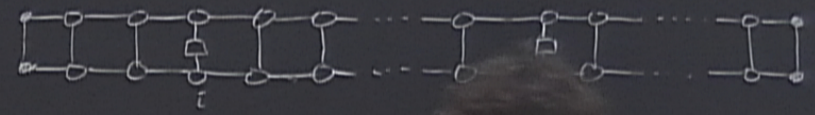


Can demand this of any MPS



Correlations

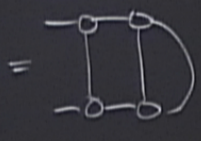
$$\langle \psi | \hat{\sigma}_i \hat{\sigma}_j | \psi \rangle =$$



Correlations

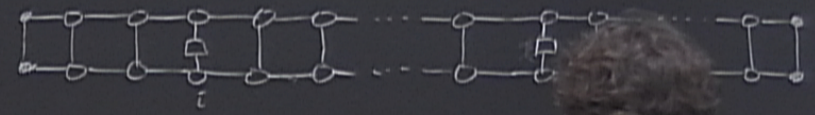
$$\langle \Psi | \hat{\sigma}_i \hat{\sigma}_j | \Psi \rangle =$$

$$=$$

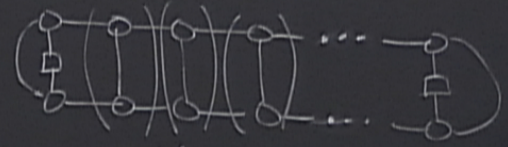


Correlations

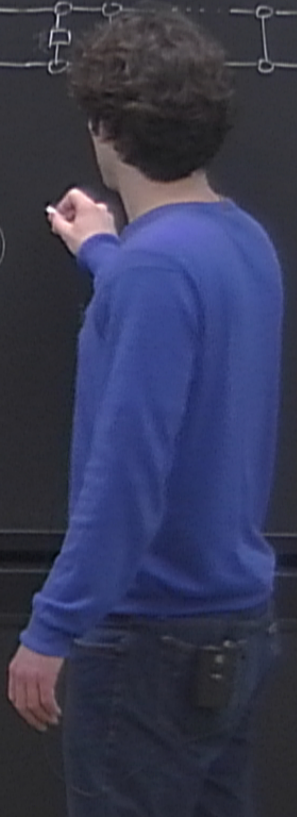
$$\langle \psi | \hat{O}_i \hat{O}_j | \psi \rangle =$$



$$=$$

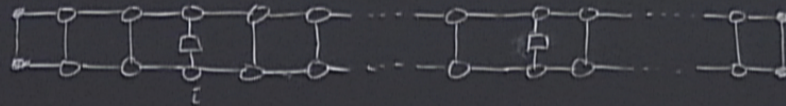


$$T \stackrel{\text{def}}{=} \sum \bar{A}^s \otimes A^s$$



Correlations

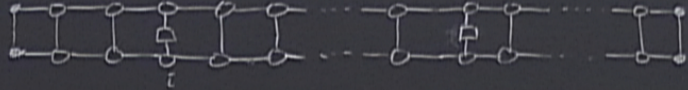
$$\langle \psi | \hat{O}_i \hat{O}_j | \psi \rangle =$$

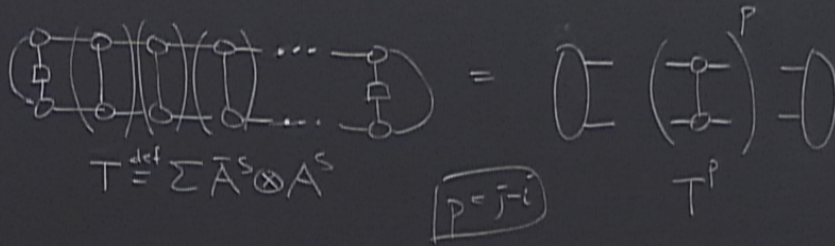


$$=$$

$T \stackrel{\text{def}}{=} \sum A^S \otimes A^S$
 $p = j - i$
 T^p

Correlations

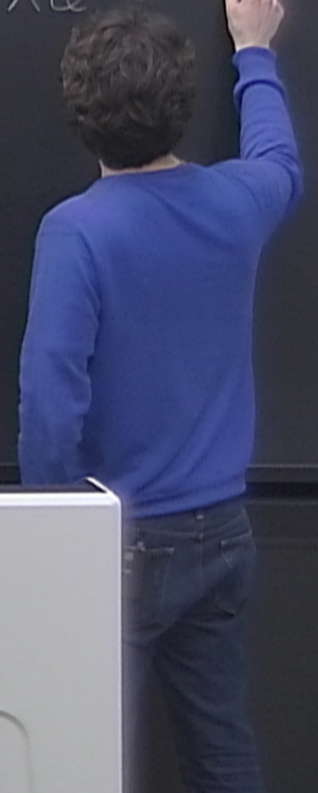
$$\langle 4 | \hat{O}_i \hat{O}_j | 4 \rangle =$$


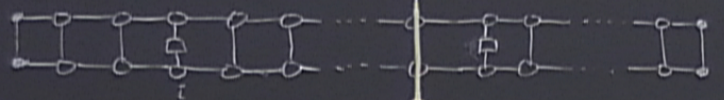
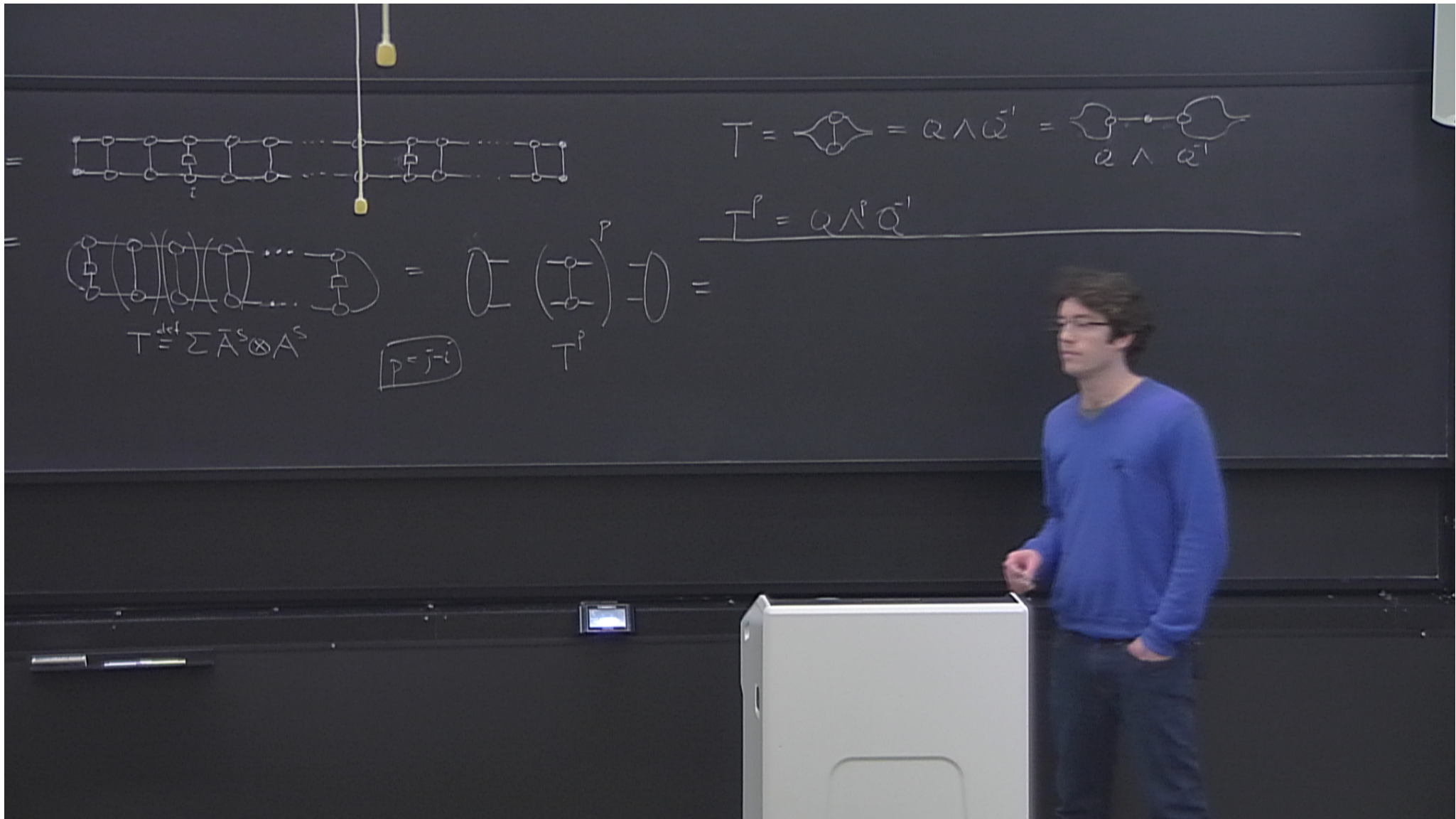
$$=$$


$T \stackrel{\text{def}}{=} \sum \bar{A}^S \otimes A^S$

$p = j - i$

$$T = \langle \text{loop} \rangle = \alpha \wedge \bar{\omega}' =$$



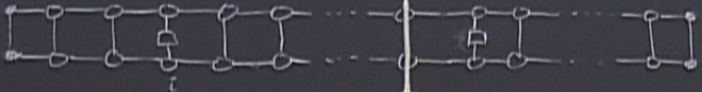


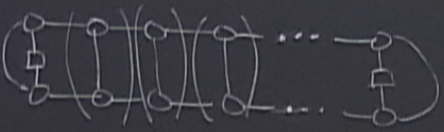
$$T = \text{Diagram} = Q \wedge Q^{-1} = \text{Diagram}$$

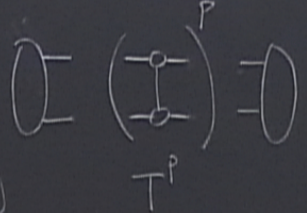
$$\underline{T^p = Q \wedge^p Q^{-1}}$$

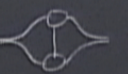
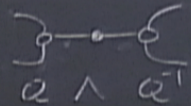
$$\text{Diagram} = \text{Diagram} \begin{pmatrix} \text{Diagram} \\ T^p \end{pmatrix} \text{Diagram} =$$

$$T \stackrel{\text{def}}{=} \sum \bar{A}^S \otimes A^S \quad \boxed{p = j-i}$$

$\rangle =$ 

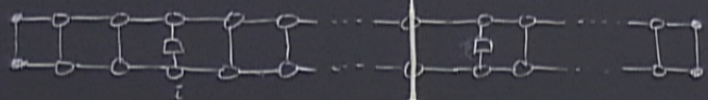
$=$ 
 $T \stackrel{\text{def}}{=} \sum \bar{A}^s \otimes A^s$

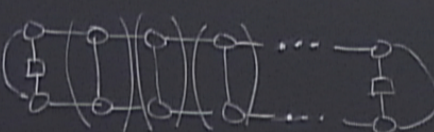
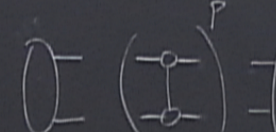
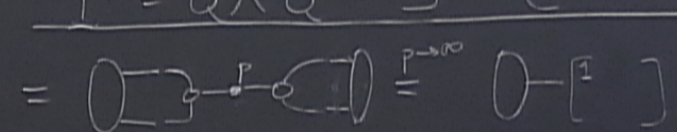
$=$ 
 T^p

$T =$ 
 $= Q \wedge \bar{Q}' =$ 
 $Q \wedge Q'$

$T^p = Q \wedge Q'$

A man in a blue sweater and glasses stands to the right of the chalkboard, gesturing with his hands.

$\rangle =$ 

$=$ 
 $=$ 
 $=$ 

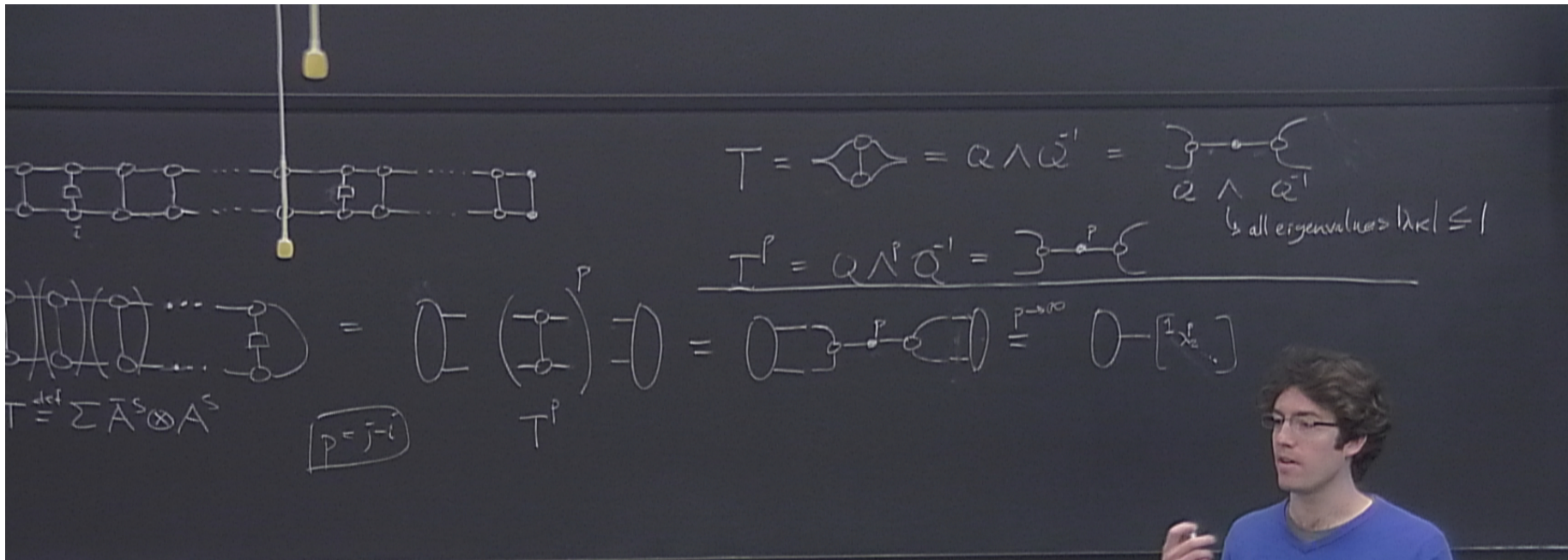
$T \stackrel{\text{def}}{=} \sum \bar{A}^S \otimes A^S$

$T^P = \sum_{p=j-i} \dots$

$T = \langle \dots \rangle = Q \wedge Q^{-1} = \dots$

$T^P = Q \wedge^P Q^{-1} = \dots$

$\lim_{P \rightarrow \infty} \dots = \dots$



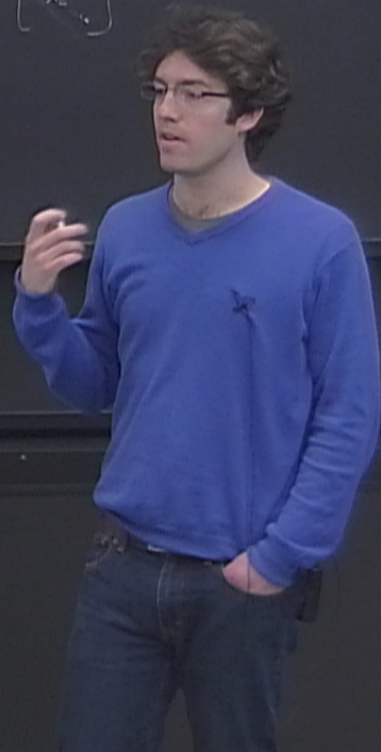
$$T = Q \Lambda Q^{-1} = \sum_{i=1}^n \lambda_i \mathbf{q}_i \mathbf{q}_i^T$$

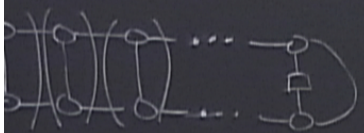
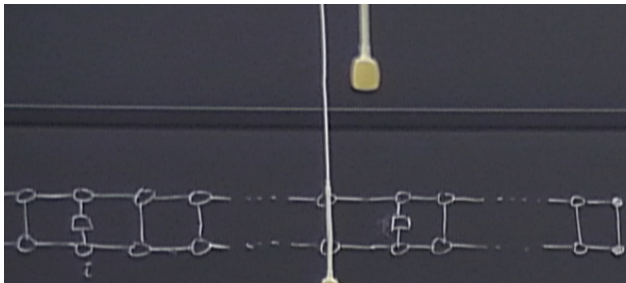
↳ all eigenvalues $|\lambda_i| \leq 1$

$$T^p = Q \Lambda^p Q^{-1} = \sum_{i=1}^n \lambda_i^p \mathbf{q}_i \mathbf{q}_i^T$$

$$T \stackrel{\text{def}}{=} \sum \bar{A}^S \otimes A^S = \left(\sum \bar{A}^S \otimes A^S \right)^p = \sum \bar{A}^S \otimes A^S \stackrel{p \rightarrow \infty}{=} \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix}$$

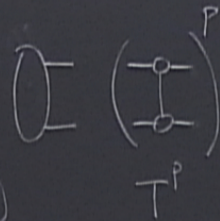
$p = j - i$





$$T \stackrel{\text{def}}{=} \sum \bar{A}^S \otimes A^S$$

$$p = j - i$$



$$T = \text{---} \circ \text{---} = Q \Lambda Q^{-1} = \left. \begin{array}{c} \text{---} \circ \text{---} \\ Q \Lambda Q^{-1} \end{array} \right\} \begin{array}{l} \text{---} \circ \text{---} \\ \text{---} \circ \text{---} \end{array}$$

↳ all eigenvalues $|\lambda_k| \leq 1$

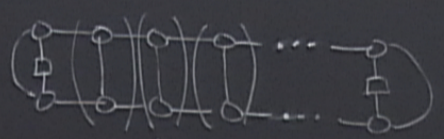
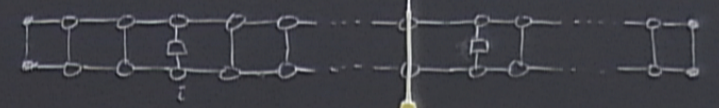
$$T^p = Q \Lambda^p Q^{-1} = \left. \begin{array}{c} \text{---} \circ \text{---} \\ Q \Lambda^p Q^{-1} \end{array} \right\} \text{---} \circ \text{---}$$

$$= \left(\text{---} \right) \left(\text{---} \right)^p \left(\text{---} \right) = \left(\text{---} \right) \left(\text{---} \right)^p \left(\text{---} \right) \stackrel{p \rightarrow \infty}{=} \left(\text{---} \right) \left[\begin{array}{c} \lambda_1 \\ \lambda_2 \end{array} \right] \left(\text{---} \right) = \text{const.} + c \lambda_2^p$$

Correlations

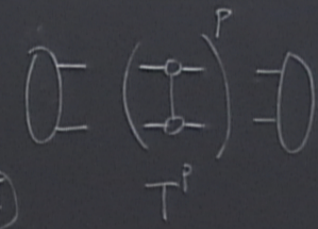
$$\langle \Psi | \hat{O}_i \hat{O}_j | \Psi \rangle =$$

$$- \langle \Psi | \hat{O}_i | \Psi \rangle \langle \Psi | \hat{O}_j | \Psi \rangle$$



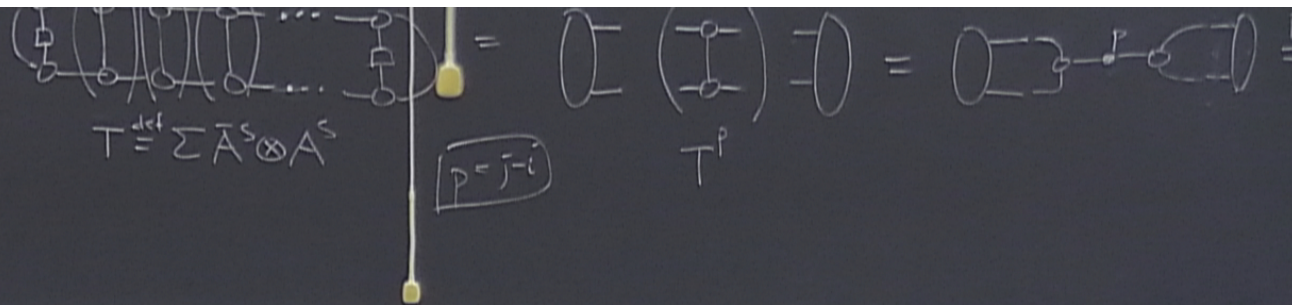
$$T \stackrel{\text{def}}{=} \sum \bar{A}^s \otimes A^s$$

$$p = j - i$$



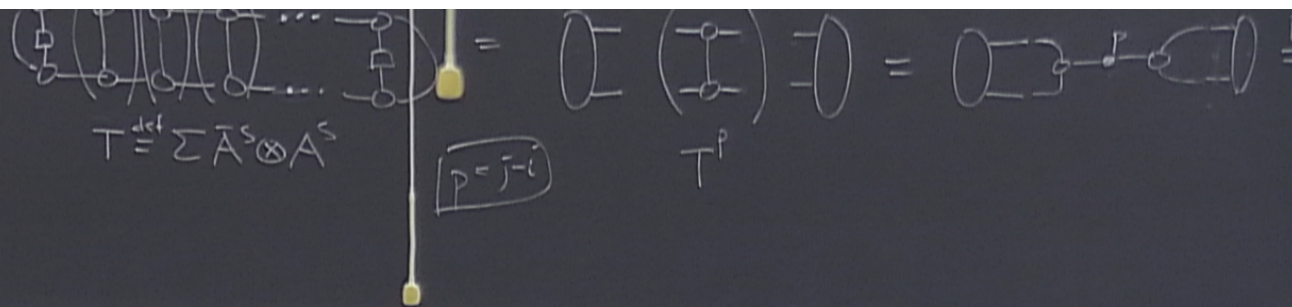
$$T = \text{tr}(\Lambda) = Q \Lambda$$

$$T^p = Q \Lambda^p Q^{-1} =$$



$$\langle \hat{\theta}_i, \hat{\theta}_i \rangle_c \xrightarrow{|j-i| \rightarrow \infty} \lambda_2^{|j-i|} = \left(e^{\ln \lambda_2} \right)^{|j-i|} = e^{(|j-i| \ln \lambda_2)} \stackrel{\text{def}}{=} e^{-|j-i|/\xi}$$

$$\sum_{\text{max}} = \frac{-1}{|\ln \lambda_2|}$$



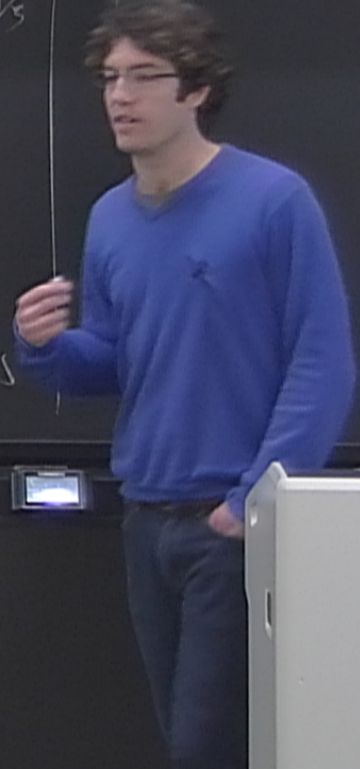
$$\langle \hat{O}_i \hat{O}_j \rangle_c \xrightarrow{|j-i| \rightarrow \infty} \lambda_2^{|j-i|} = \left(e^{\ln \lambda_2} \right)^{|j-i|} = e^{|j-i| \ln \lambda_2} \stackrel{\text{def}}{=} e^{-|j-i|/\xi}$$

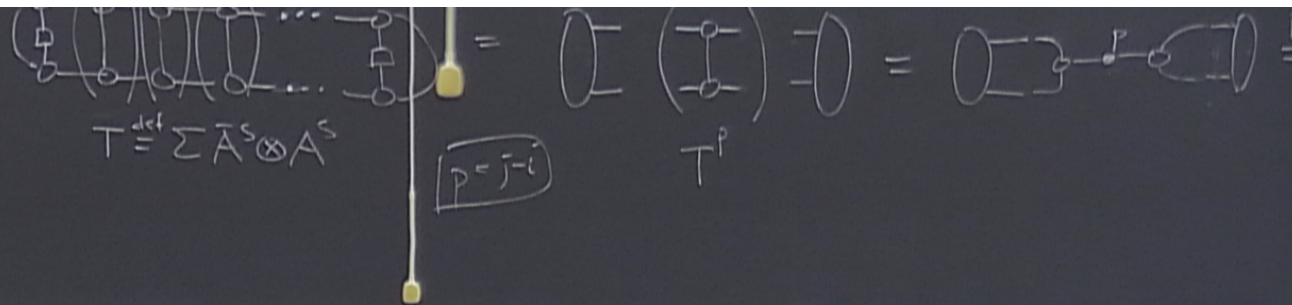
MPS for critical

$$\xi_{\text{max}} = \frac{-1}{\ln |\lambda_2|}$$

correlation length bound
local property of MPS

MPS finite D \leftrightarrow gapped ground states





$$T \stackrel{\text{def}}{=} \sum \bar{A}^s \otimes A^s$$

$$p = j-i$$

$$T^p$$

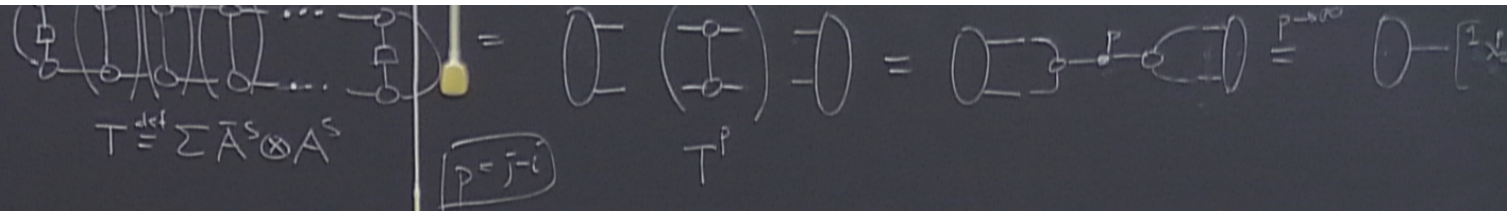
$$\langle \hat{O}_i \hat{O}_j \rangle_c \xrightarrow{|j-i| \rightarrow \infty} \lambda_2^{|j-i|} = \left(e^{\ln \lambda_2} \right)^{|j-i|} = e^{|j-i| \ln \lambda_2} \stackrel{\text{def}}{=} e^{-|j-i|/\xi}$$

MPS for critical

$$\xi = \frac{-1}{\ln |\lambda_2|}$$

correlation length bound
local property of MPS

MPS finite D \leftrightarrow gapped ground states



$$\langle \hat{O}_i \hat{O}_j \rangle_c \xrightarrow{|j-i| \rightarrow \infty} \lambda_2^{|j-i|} = (e^{\ln \lambda_2})^{|j-i|} = e^{(|j-i| \ln \lambda_2)} \stackrel{\text{def}}{=} e^{-|j-i|/\xi}$$

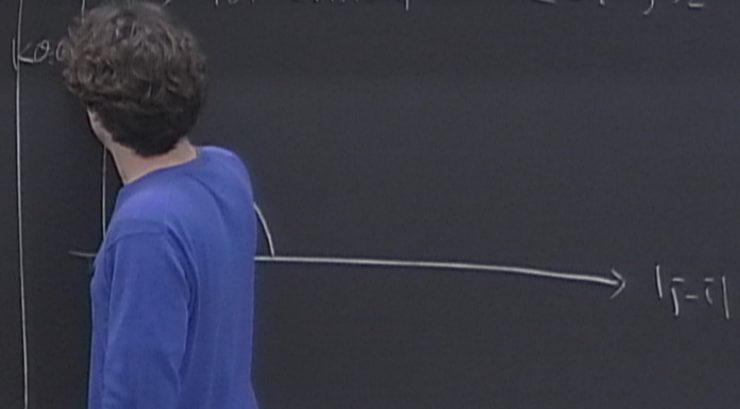
$$\xi = \frac{-1}{\ln |\lambda_2|}$$

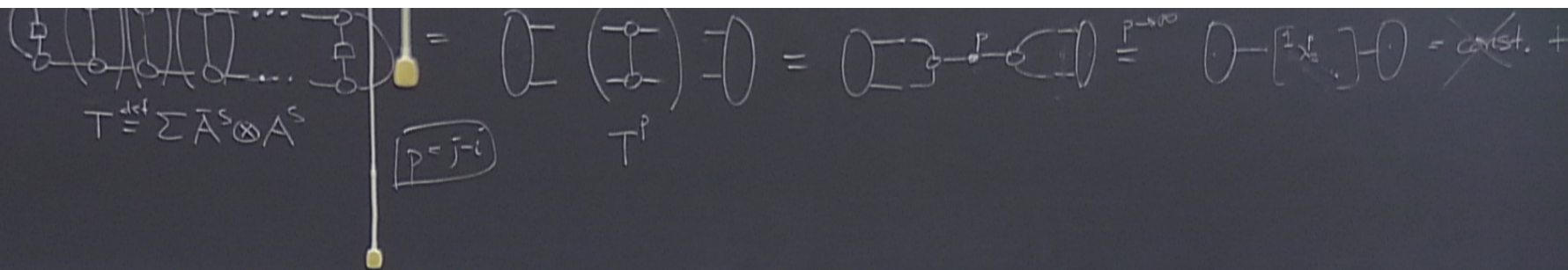
correlation length bound
local property of MPS

MPS finite D \leftrightarrow gapped ground states

MPS for critical

$$\langle \hat{O}_i \hat{O}_j \rangle_c \sim$$





$\langle \hat{O}_i \rangle_c \xrightarrow{|i-j| \rightarrow \infty} \lambda_2 = (e^{\ln \lambda_2})^{|j-i|} = e^{(|j-i| \ln \lambda_2)} \stackrel{\text{def}}{=} e^{-|j-i|/\xi}$

$\xi = \frac{-1}{\ln |\lambda_2|}$ correlation length bound
 local property of MPS
 MPS finite D \leftrightarrow gapped ground states

