

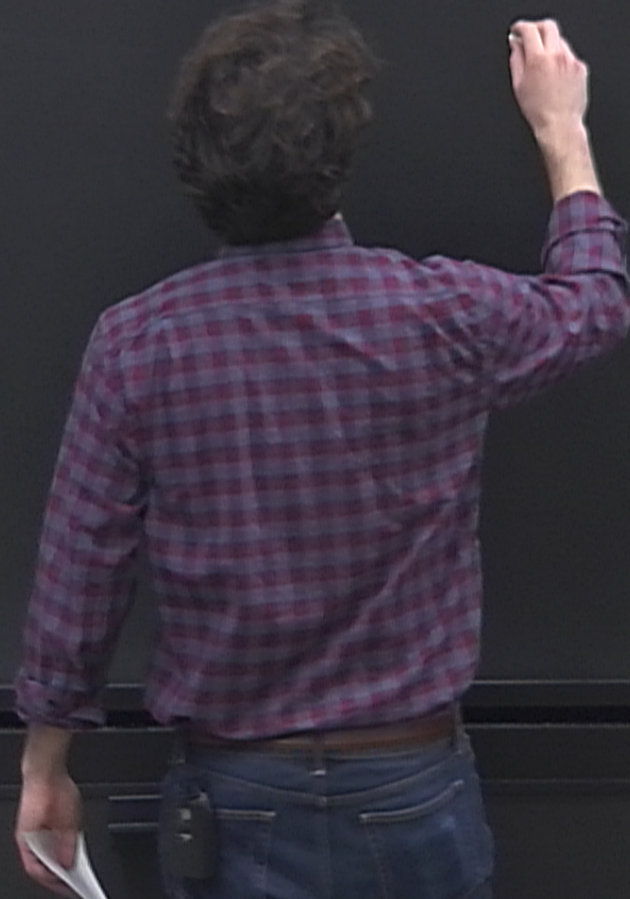
Title: Explorations in Condensed Matter-11

Date: Mar 30, 2015 10:15 AM

URL: <http://pirsa.org/15030048>

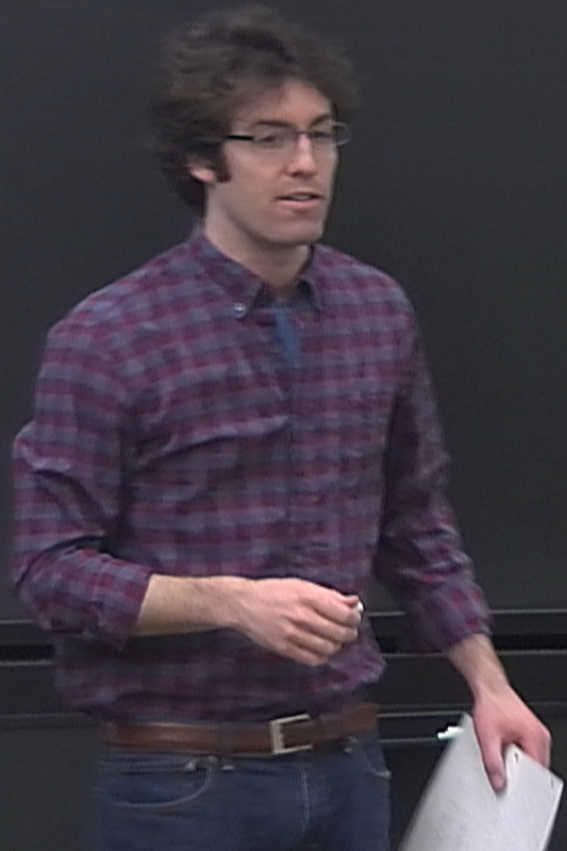
Abstract:

AKLT state

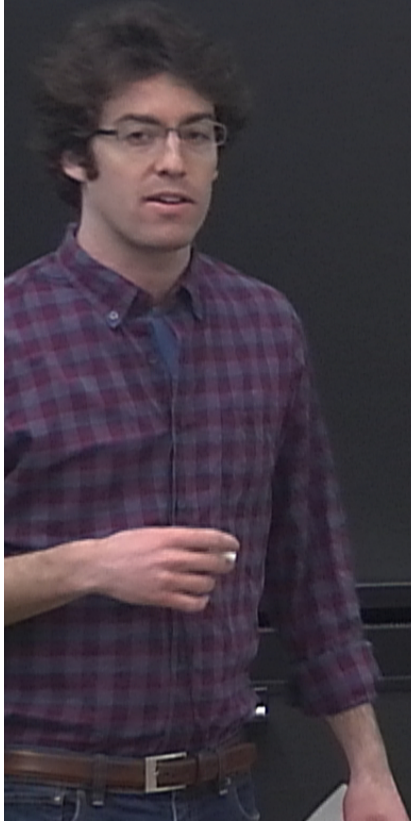


AKLT state

$$\hat{H} = J \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3} (\vec{S}_i - \vec{S}_{i+1})^2 \right)$$



$$H = J \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3} (\vec{S}_i - \vec{S}_{i+1})^2 \right) \quad S=1$$



$$(\vec{S}_i - \vec{S}_{i+1})^2$$

$$S=1$$



emergent $S=1/2$ like edge states

$$(\vec{S}_i - \vec{S}_{i+1})^2$$

$$S=1$$



emergent $S=1/2$ like edge states
• 4 ground states
 $\langle S \rangle$

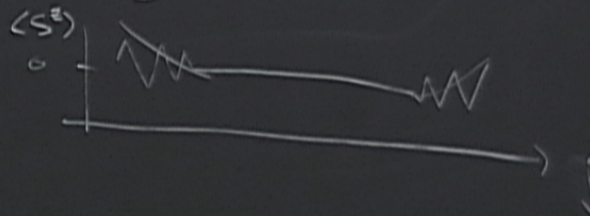
$$(\vec{S}_i - \vec{S}_{i+1})^2$$

$$S=1$$



emergent $S=1/2$ like edge states

- 4 ground states



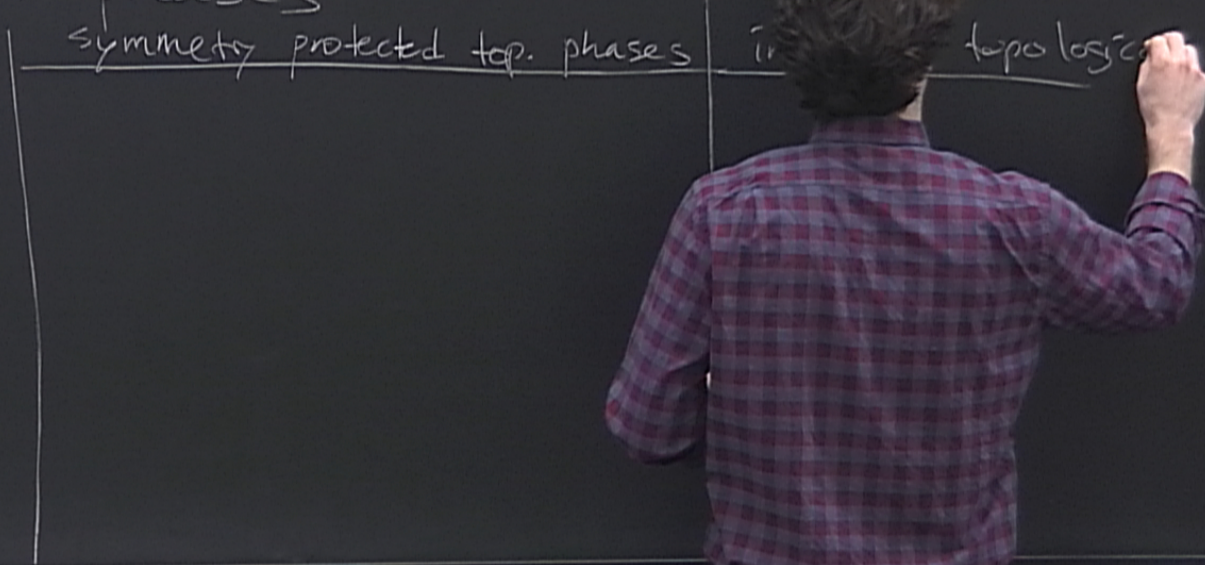
AKLT state
"topological" phases

$$\hat{H} = J \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3} (\vec{S}_i - \vec{S}_{i+1})^2 \right)$$

AKLT state $\hat{H} = J \sum_i (\vec{S}_i \vec{S}_{i+1} + \frac{1}{3} (\vec{S}_i - \vec{S}_{i+1})^2)$
"topological" phases
symmetry protected top.

AKLT state $\hat{H} = J \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3} (\vec{S}_i - \vec{S}_{i+1})^2 \right)$

"topological" phases



AKLT state

$$\hat{H} = J \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3} (\vec{S}_i - \vec{S}_{i+1})^2 \right)$$

"topological" states

protected top. phases

intrinsic topological phases FQHE

AKLT state

$$\hat{H} = J \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3} (\vec{S}_i - \vec{S}_{i+1})^2 \right)$$

"topological" phases

| # ground states on closed manifold of genus g | symmetry protected top. phases | intrinsic topological phases <u>FQHE</u> |
|---|--------------------------------|--|
| | 1 | 1, depends on g |

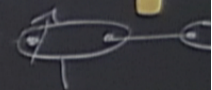
$$\hat{H} = J \sum_{\vec{r}} \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3} (\vec{S}_i - \vec{S}_{i+1})^2 \right) \quad S=1$$

S
protected top. phases

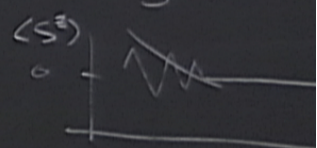
Intrinsic topological phases FQHE

1

> 1 , depends on g ; torus, finite cylinder
quasiparticle types



emergent S
4 ground



state $\hat{H} = J \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3} (\vec{S}_i - \vec{S}_{i+1})^2 \right) \quad S=1$

1" phases

| symmetry protected top. phases | intrinsic topological phases <u>FQHE</u> |
|--------------------------------|--|
| 1 | > 1, depends on g; torus, finite cylinder # quasiparticle types |
| no | yes |
| yes | sometimes, may require symmetries |

AKLT state

$$\hat{H} = J \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3} (\vec{S}_i - \vec{S}_{i+1})^2 \right)$$

topological" phases

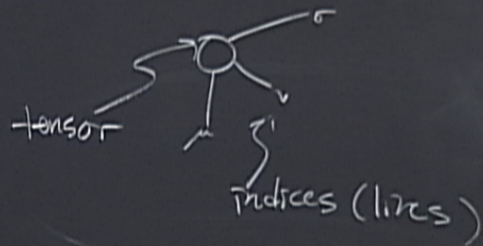
| (symmetry protected) top. phases | (intrinsic) topological phases <u>FQHE</u> |
|----------------------------------|---|
| 1 | > 1, depends on g ; torus = $2\pi i g$; # quasip |
| no | yes |
| yes | sometimes, may require symmetries |
| yes | no |
| yes | no for bosonic degrees of freedom |

$AKLT$: some example of matrix product

AkLT : some example of matrix product state (M)

le of matrix product state (MPS)

A kLT : some example of matrix product



$$T_{\mu\nu\sigma}$$

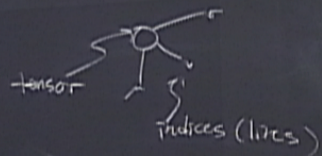
$$T_{111} \quad T_{222}$$

$$\text{O} - i = v_i \quad (\text{vector})$$

$$i - \text{O} - j = M_{ij} \quad (\text{matrix})$$

$$\begin{matrix} j \\ | \\ i - \text{O} - k \\ | \\ m \end{matrix} = T_{ijk} \quad (\text{rank 3 tensor})$$

AKLT : simple example of matrix product state (MPS)



$$T_{\mu\nu\sigma}$$

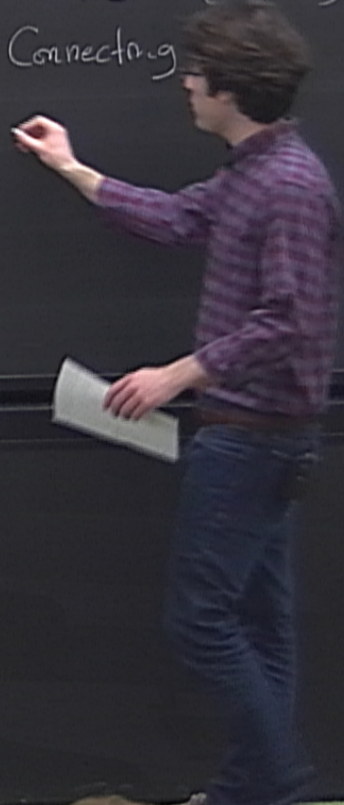
$$T_{111} \quad T_{222}$$

$$\bigcirc - i = v_i \quad (\text{vector})$$

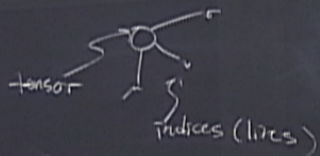
$$i - \bigcirc - j = M_{ij} \quad (\text{matrix})$$

$$\begin{matrix} j \\ | \\ i - \bigcirc - k \\ | \\ \sum \end{matrix} = T_{ijk} \quad \begin{matrix} (\text{rank 3} \\ \text{tensor}) \end{matrix}$$

Connecting means contraction (sum over indices)



AKLT : simple example of matrix product state (MPS)



$$T_{\mu\nu\sigma}$$

$$T_{111} \quad T_{222}$$

$$O-i = v_i \quad (\text{vector})$$

$$i-O-j = M_{ij} \quad (\text{matrix})$$

$$\begin{matrix} j \\ | \\ i-O-k \\ | \\ \sum \end{matrix} = T_{ijk} \quad (\text{rank 3 tensor})$$

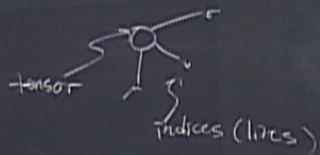
Connecting lines means contraction (sum over indices)

$$\begin{matrix} O & O \\ A & B \end{matrix} = \sum_i O-i-i-O = \sum_i A_i B_i = \vec{A} \cdot \vec{B}$$

$$i-O-O = \sum_j M_{ij} v_j = M_i^j v_j$$

$$O-O-O =$$

AKLT : simple example of matrix product state (MPS)



$$T_{\mu\nu\sigma}$$

$$T_{111} \quad T_{222}$$

$$\text{---}i = v_i \quad (\text{vector})$$

$$i\text{---}j = M_{ij} \quad (\text{matrix})$$

$$\begin{matrix} & j \\ & | \\ i\text{---} & \text{---} k \\ & | \\ & \mu \end{matrix} = T_{ijk} \quad (\text{rank 3 tensor})$$

Connecting lines means contraction (sum over indices)

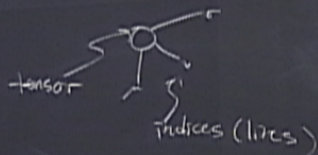
$$\text{---}i\text{---}j = \sum_c \text{---}i\text{---}c\text{---}j = \sum_c A_c \cdot B_c = \vec{A} \cdot \vec{B}$$

$$i\text{---}j\text{---}k = \sum_j M_{ij} v_j = M \vec{v}$$

$$\text{---}i\text{---}j\text{---}k = \vec{a}^T M \vec{b}$$

$$\text{---}i\text{---}j = \text{Tr}[A B]$$

AKLT : simple example of matrix product state (MPS)



$$T_{\mu\nu\sigma}$$

$$T_{111} \quad T_{222}$$

$$O \text{---} i = v_i \quad (\text{vector})$$

$$i \text{---} O \text{---} j = M_{ij} \quad (\text{matrix})$$

$$\begin{matrix} j \\ | \\ i \text{---} O \text{---} k \\ | \\ \mu \end{matrix} = T_{ijk} \quad \begin{matrix} (\text{rank 3} \\ \text{tensor}) \end{matrix}$$

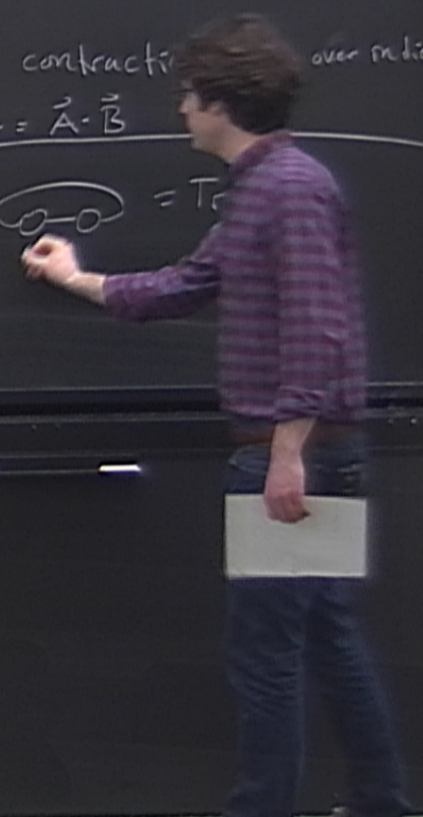
Connecting lines means contraction (over indices)

$$\begin{matrix} O & O \\ A & B \end{matrix} = \sum_i O \text{---} i \text{---} O = \sum_i A_i B_i = \vec{A} \cdot \vec{B}$$

$$i \text{---} O \text{---} O = \sum_j M_{ij} v_j = M \vec{v}$$

$$O \text{---} O \text{---} O = \vec{a}^T M \vec{b}$$

$$\text{O---O} = T_{\mu\nu\sigma}$$



matrix product state (MPS)

(vector)

(matrix)

(rank 3 tensor)

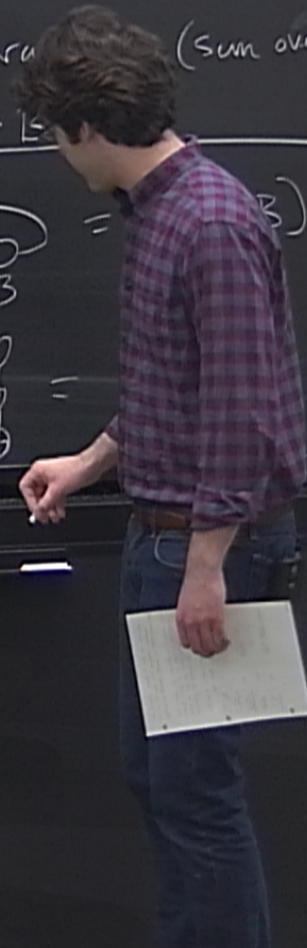
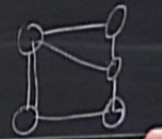
Connecting lines means contraction (sum over indices)

$$\begin{array}{c} \text{O} \text{---} \text{O} \\ \text{A} \quad \text{B} \end{array} = \sum_i \begin{array}{c} \text{O} \text{---} i \text{---} \text{O} \\ \text{A}_i \quad \text{B}_i \end{array} = \sum_i A_i B_i = \vec{A} \cdot \vec{B}$$

$$i \text{---} \text{O} \text{---} \text{O} = \sum_j M_{ij} v_j = M \vec{v}$$

$$\text{O} \text{---} \text{O} \text{---} \text{O} = \vec{a}^T M \vec{b}$$

$$\begin{array}{c} \text{O} \text{---} \text{O} \\ \text{A} \quad \text{B} \end{array} = \text{tr}(A B)$$



matrix product state (MPS)

(vector)

(matrix)

(rank 3 tensor)

Connecting lines means contraction (sum over indices)

$$\begin{array}{c} \bigcirc - \bigcirc \\ A \quad B \end{array} = \sum_i \begin{array}{c} \bigcirc - i - \bigcirc \\ A_i \quad B_i \end{array} = \sum_i A_i B_i = \vec{A} \cdot \vec{B}$$

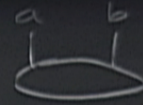
$$i - \bigcirc - \bigcirc = \sum_j M_{ij} v_j = M \vec{v}$$

$$\bigcirc - \bigcirc - \bigcirc = \vec{a}^T M \vec{b}$$

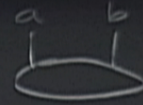
$$\begin{array}{c} \bigcirc - \bigcirc \\ A \quad B \end{array} = \text{Tr}[AB]$$

$$\begin{array}{c} \bigcirc - \bigcirc \\ \bigcirc - \bigcirc \end{array} = \# \text{ number scalar}$$

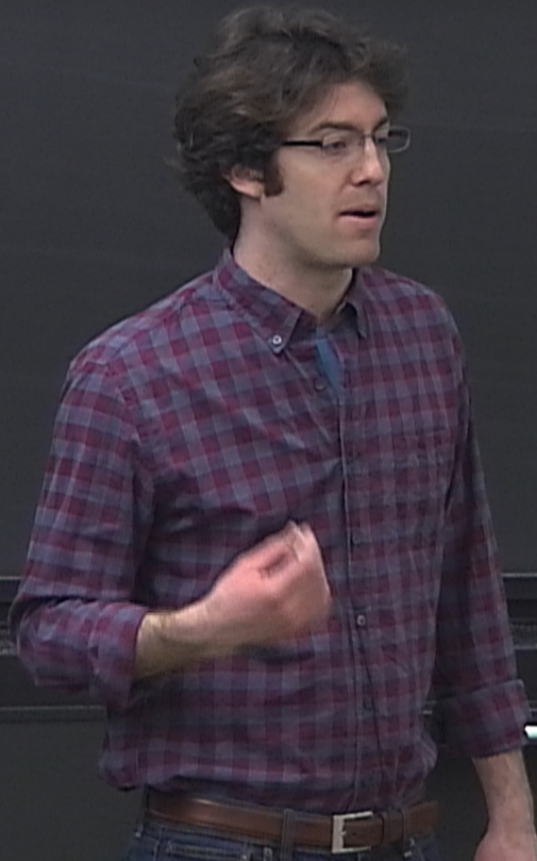
"valence bond" tensor



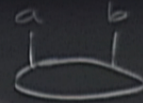
"valence bond" tensor



| a | b | value |
|---|---|-----------------------|
| ↑ | ↓ | $\frac{1}{\sqrt{2}}$ |
| ↓ | ↑ | $-\frac{1}{\sqrt{2}}$ |
| ↑ | ↑ | 0 |
| ↓ | ↓ | 0 |



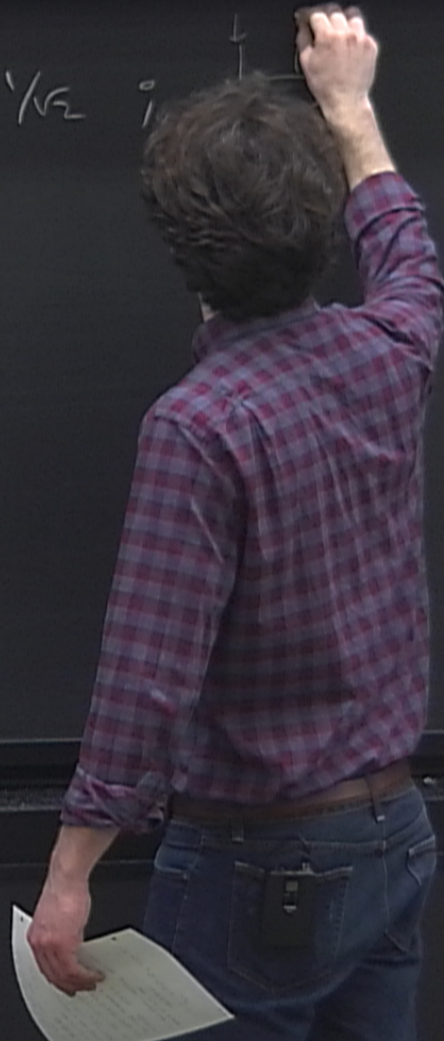
"valence bond" tensor



| a | b | value |
|---|---|-----------------------|
| ↑ | ↓ | $\frac{1}{\sqrt{2}}$ |
| ↓ | ↑ | $-\frac{1}{\sqrt{2}}$ |
| ↑ | ↑ | 0 |
| ↓ | ↓ | 0 |

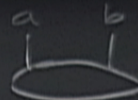
| a | b | value |
|---|---|---------------|
| ↑ | ↓ | $1/\sqrt{2}$ |
| ↓ | ↑ | $-1/\sqrt{2}$ |
| ↑ | ↑ | 0 |
| ↓ | ↓ | 0 |

$$\begin{matrix} \uparrow & \downarrow \\ | & | \\ \text{---} & \text{---} \\ | & | \\ \downarrow & \downarrow \end{matrix} = 1/\sqrt{2}$$

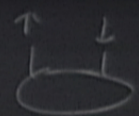


| a | b | value |
|---|---|---------------|
| ↑ | ↓ | $1/\sqrt{2}$ |
| ↓ | ↑ | $-1/\sqrt{2}$ |
| ↑ | ↑ | 0 |
| ↓ | ↓ | 0 |

$$= \begin{matrix} \uparrow & \downarrow \\ | & | \\ \circlearrowleft & \circlearrowright \end{matrix} = 1/\sqrt{2} ; \begin{matrix} \downarrow & \uparrow \\ | & | \\ \circlearrowleft & \circlearrowright \end{matrix} = -1/\sqrt{2}$$

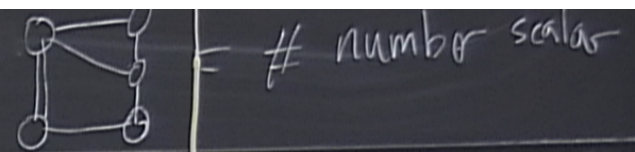
"valence bond" tensor: 

| a | b | value |
|---|---|-----------------------|
| ↑ | ↓ | $\frac{1}{\sqrt{2}}$ |
| ↓ | ↑ | $-\frac{1}{\sqrt{2}}$ |
| ↑ | ↑ | 0 |
| ↓ | ↓ | 0 |

=  =

μ (tensor)

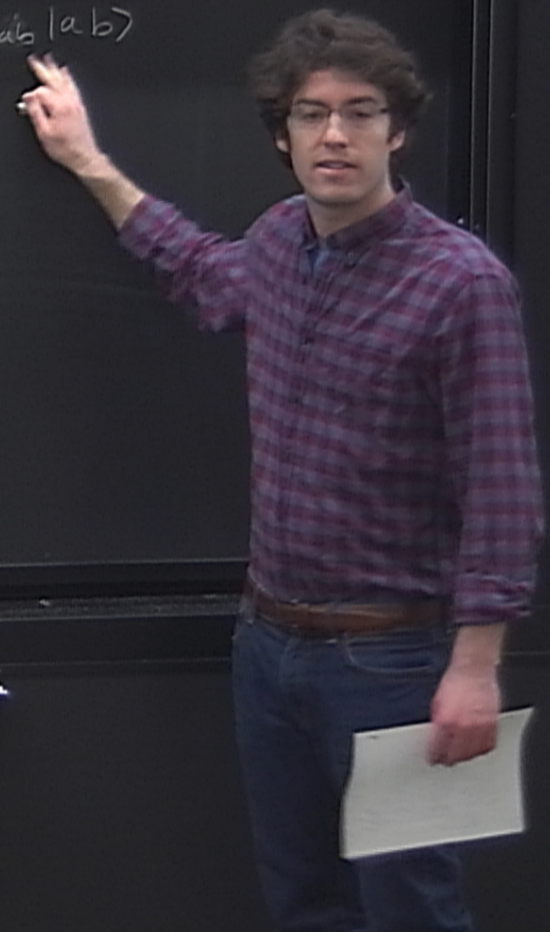
$$o-o-o = \vec{a}^T M b$$



| a | b | value |
|---|---|---------------|
| ↑ | ↓ | $1/\sqrt{2}$ |
| ↓ | ↑ | $-1/\sqrt{2}$ |
| ↑ | ↑ | 0 |
| ↓ | ↓ | 0 |

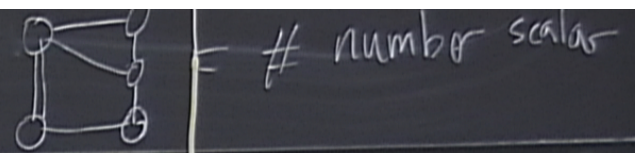
$$= \begin{matrix} \uparrow & \downarrow \\ | & | \\ \hline \end{matrix} = 1/\sqrt{2} ; \begin{matrix} \downarrow & \uparrow \\ | & | \\ \hline \end{matrix} = -1/\sqrt{2}$$

$$|\psi_{singlet}\rangle = \sum_{ab} \psi_{ab} |ab\rangle$$



μ (tensor)

$$\bigcirc - \bigcirc - \bigcirc = \vec{a}^T M \vec{b}$$



| a | b | value |
|--------------|--------------|---------------|
| \uparrow | \downarrow | $1/\sqrt{2}$ |
| \downarrow | \uparrow | $-1/\sqrt{2}$ |
| \uparrow | \uparrow | 0 |
| \downarrow | \downarrow | 0 |

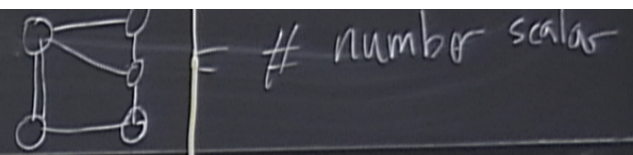
$$= \begin{array}{c} \uparrow \quad \downarrow \\ \bigcirc \end{array} = 1/\sqrt{2} ; \quad \begin{array}{c} \downarrow \quad \uparrow \\ \bigcirc \end{array} = -1/\sqrt{2}$$

$$|\psi_{\text{singlet}}\rangle = \sum_{ab} \psi_{ab} |ab\rangle$$

$$\langle ab | \psi_{\text{singlet}} \rangle = \psi_{ab} = \begin{array}{c} a \quad b \\ \bigcirc \end{array}$$

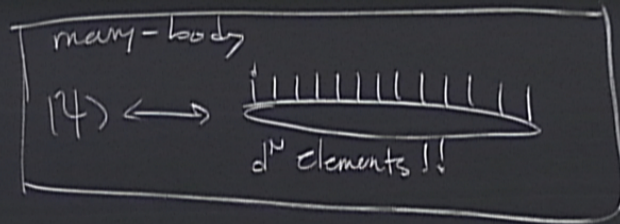
μ (tensor)

$$\bigcirc - \bigcirc - \bigcirc = \vec{a}^T M \vec{b}$$



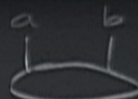
| a | b | value |
|---|---|---------------|
| ↑ | ↓ | $1/\sqrt{2}$ |
| ↓ | ↑ | $-1/\sqrt{2}$ |
| ↑ | ↑ | 0 |
| ↓ | ↓ | 0 |

$$= \begin{matrix} \uparrow & \downarrow \\ \bigcirc & \bigcirc \end{matrix} = 1/\sqrt{2} ; \begin{matrix} \downarrow & \uparrow \\ \bigcirc & \bigcirc \end{matrix} = -1/\sqrt{2}$$



$$|\psi_{\text{singlet}}\rangle = \sum_{ab} \psi_{ab} |ab\rangle$$


$$\langle ab | \psi_{\text{singlet}} \rangle = \psi_{ab} = \begin{matrix} a & b \\ \bigcirc \end{matrix}$$

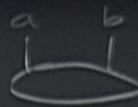
"valence bond" tensor: 

| a | b | value |
|---|---|---------------|
| ↑ | ↓ | $1/\sqrt{2}$ |
| ↓ | ↑ | $-1/\sqrt{2}$ |
| ↑ | ↑ | 0 |
| ↓ | ↓ | 0 |

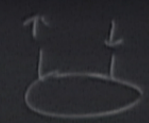
$=$  $=$

projector $\hat{T} = |+\rangle\langle\uparrow\uparrow| + |0\rangle\left(\frac{\langle\uparrow\downarrow| + \langle\downarrow\uparrow|}{\sqrt{2}}\right) + |-\rangle\langle\downarrow\downarrow|$


many-body
 $|4\rangle \leftrightarrow$ 

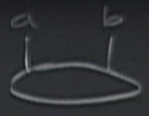
"valence bond" tensor: 

| a | b | value |
|---|---|---------------|
| ↑ | ↓ | $1/\sqrt{2}$ |
| ↓ | ↑ | $-1/\sqrt{2}$ |
| ↑ | ↑ | 0 |
| ↓ | ↓ | 0 |

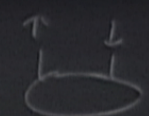
$=$  $=$


projector $\hat{T} = |+\rangle\langle\uparrow\uparrow| + |0\rangle\left(\frac{\langle\uparrow\downarrow| + \langle\downarrow\uparrow|}{\sqrt{2}}\right) + |-\rangle\langle\downarrow\downarrow|$

many-body
 $|4\rangle \leftrightarrow$ 

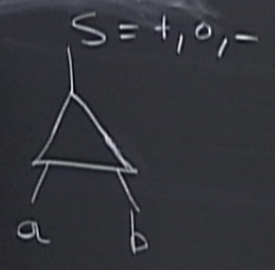
"valence bond" tensor: 

| a | b | value |
|---|---|-----------------------|
| ↑ | ↓ | $\frac{1}{\sqrt{2}}$ |
| ↓ | ↑ | $-\frac{1}{\sqrt{2}}$ |
| ↑ | ↑ | 0 |
| ↓ | ↓ | 0 |

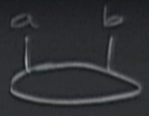
$=$  $=$

many-body $|4\rangle \leftrightarrow$ 

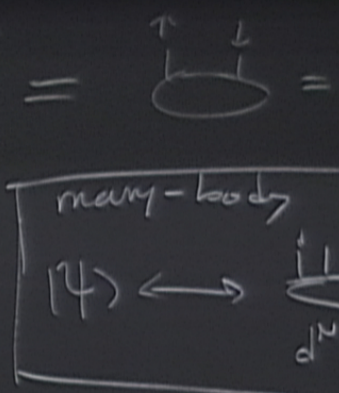
projector $\hat{T} = |+\rangle\langle ++| + |0\rangle\left(\frac{\langle \uparrow\downarrow| + \langle \downarrow\uparrow|}{\sqrt{2}}\right) + |-\rangle\langle --|$



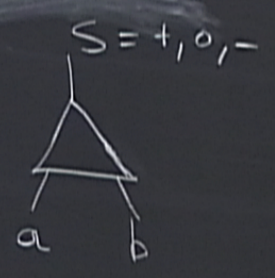
| a | b | S | Value |
|---|---|---|----------------------|
| ↑ | ↑ | + | 1 |
| ↑ | ↓ | 0 | $\frac{1}{\sqrt{2}}$ |
| ↓ | ↑ | 0 | $\frac{1}{\sqrt{2}}$ |
| ↓ | ↓ | - | 1 |

"valence bond" tensor: 

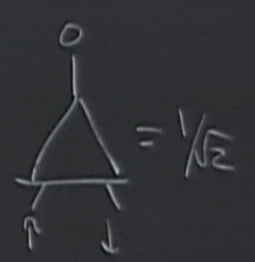
| a | b | value |
|---|---|---------------|
| ↑ | ↓ | $1/\sqrt{2}$ |
| ↓ | ↑ | $-1/\sqrt{2}$ |
| ↑ | ↑ | 0 |
| ↓ | ↓ | 0 |



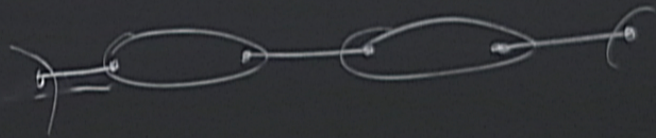
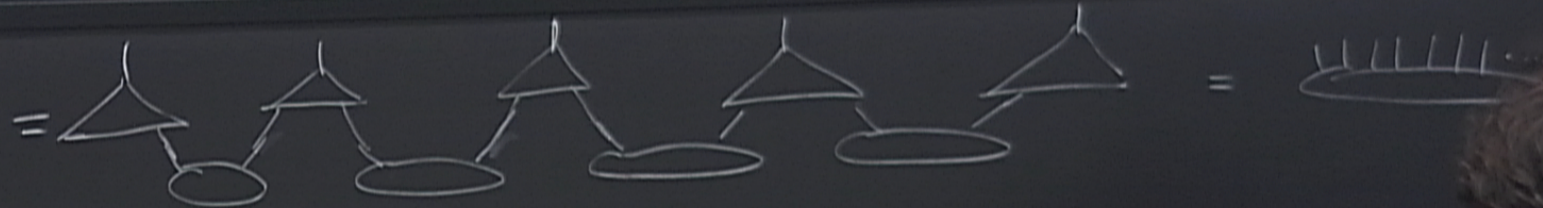
projector $\hat{T} = |+\rangle\langle ++| + |0\rangle\left(\frac{\langle \uparrow\downarrow| + \langle \downarrow\uparrow|}{\sqrt{2}}\right) + |-\rangle\langle \downarrow\downarrow|$

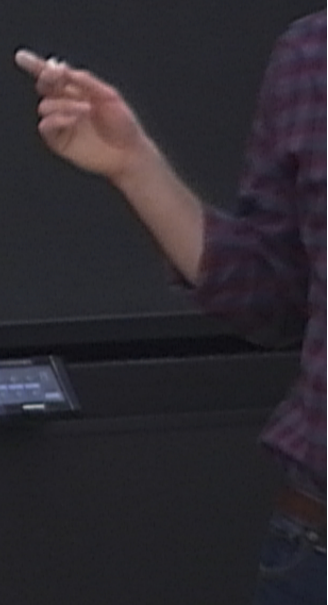
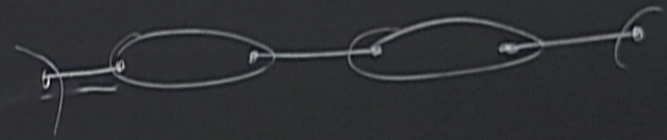


| a | b | S | Value |
|---|---|---|--------------|
| ↑ | ↑ | + | 1 |
| ↑ | ↓ | 0 | $1/\sqrt{2}$ |
| ↓ | ↑ | 0 | $1/\sqrt{2}$ |
| ↓ | ↓ | - | 1 |



$|4_{AKLT}\rangle$





$$|4_{AKLT}\rangle = \text{[Diagram: 5 triangles with indices } i_1^+, i_2^0, i_3^0, i_4^-, i_5^+ \text{]} = \text{[Diagram: 5 vertical bars in a row]}$$

In traditional notation

$$= \text{[Diagram: A chain of three ovals connected by lines]}$$

$$= \sum_{i_1, i_2, i_3} \text{[Diagram: Summed triangles with indices } i_1, i_2, i_3 \text{]}$$

$$= \text{[Diagram: MPS tensor chain with three tensors]}$$

↑ MPS

$$A_{i_1} = \text{[Diagram: Single tensor with index } i_1 \text{]}$$

$$|4_{AKLT}\rangle = \text{[Diagram: 4 sites with spin indices } \uparrow, 0, 0, -, \uparrow \text{ and circles below]} = \text{[Diagram: 4 vertical bars in a circle]}$$

In traditional notation

$$= \text{[Diagram: 4 sites with circles and lines connecting them horizontally]}$$

$$= \sum_{s_1 s_2 \dots s_N} \text{Tr}[A^{s_1} A^{s_2} \dots A^{s_N}] |s_1 \dots s_N\rangle$$

$$= \sum_{i_1 i_2 i_3} \text{[Diagram: 3 sites with circles and lines, some crossed out]} = \text{[Diagram: 3 sites with circles and lines, labeled MPS]}$$

$$= \text{[Diagram: 3 sites with circles and lines, labeled MPS]} = \text{[Diagram: 3 sites with circles and lines, labeled MPS]}$$

$$|\psi_{AKLT}\rangle = \text{Diagram with 5 tensors} = \text{Diagram with 5 vertical lines}$$

In traditional notation

$$= \text{Diagram with 3 tensors and 2 loops}$$

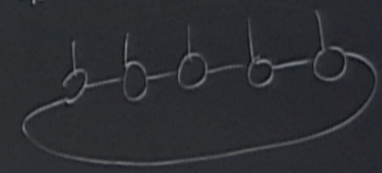
$$= \sum_{i_1, i_2, i_3} \text{Diagram with 3 tensors and 3 loops}$$

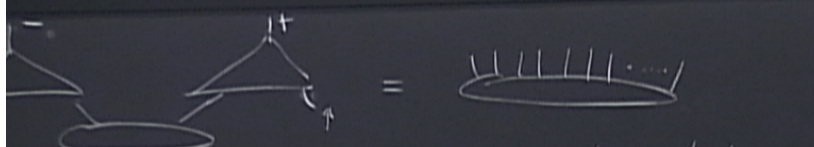
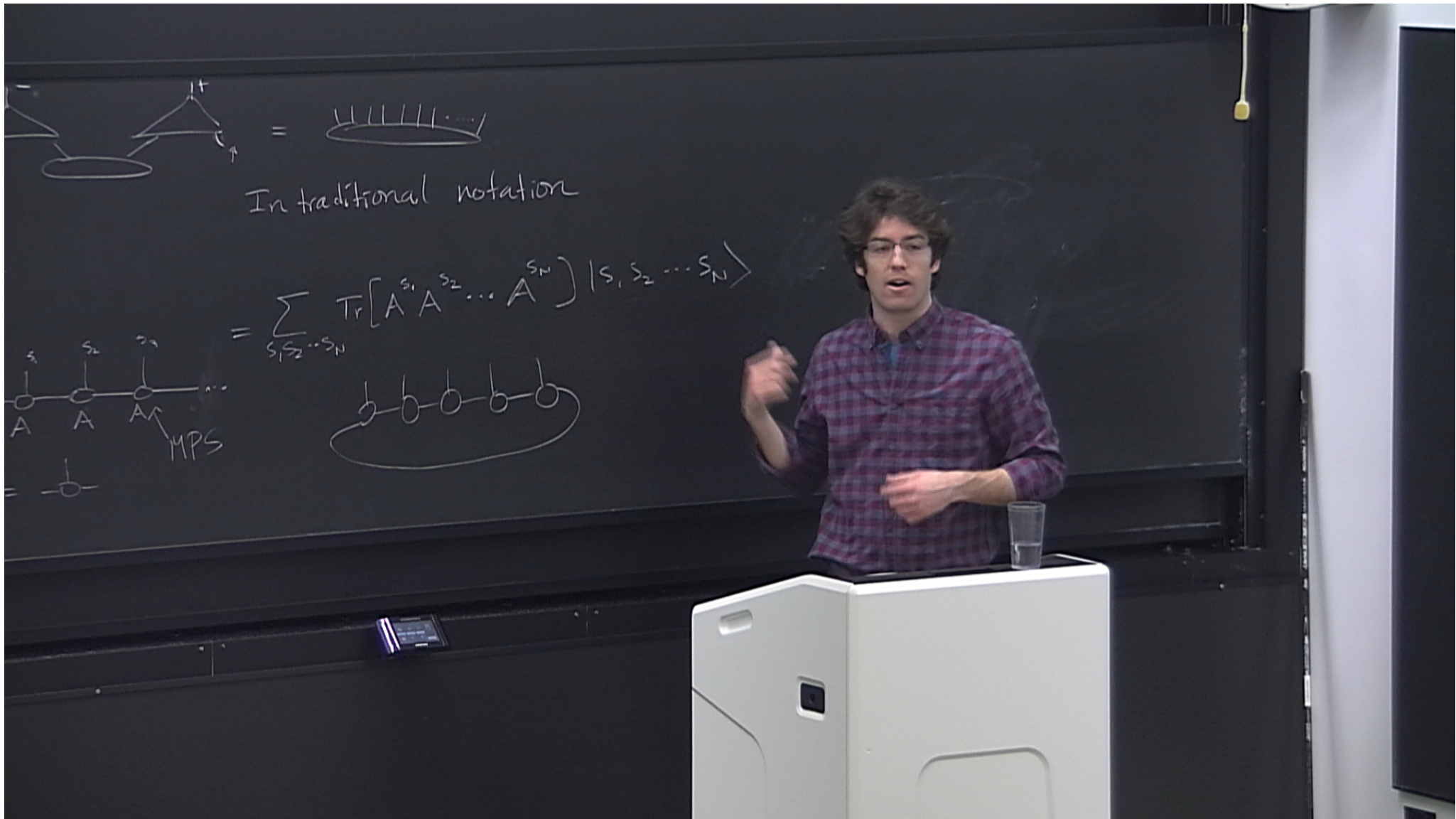
$$= \dots \overset{s_1}{\underset{A}{\circ}} \overset{s_2}{\underset{A}{\circ}} \overset{s_3}{\underset{A}{\circ}} \dots$$

MPS

$$A_{i_1 i_2} = \text{Diagram with 1 tensor and 1 loop}$$

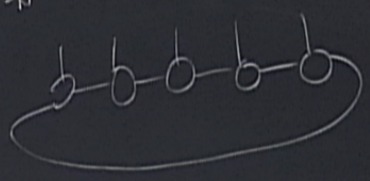
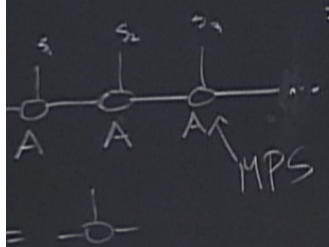
$$= \sum_{s_1, s_2, \dots, s_N} \text{Tr}[A^{s_1} A^{s_2} \dots A^{s_N}] |s_1, s_2, \dots, s_N\rangle$$

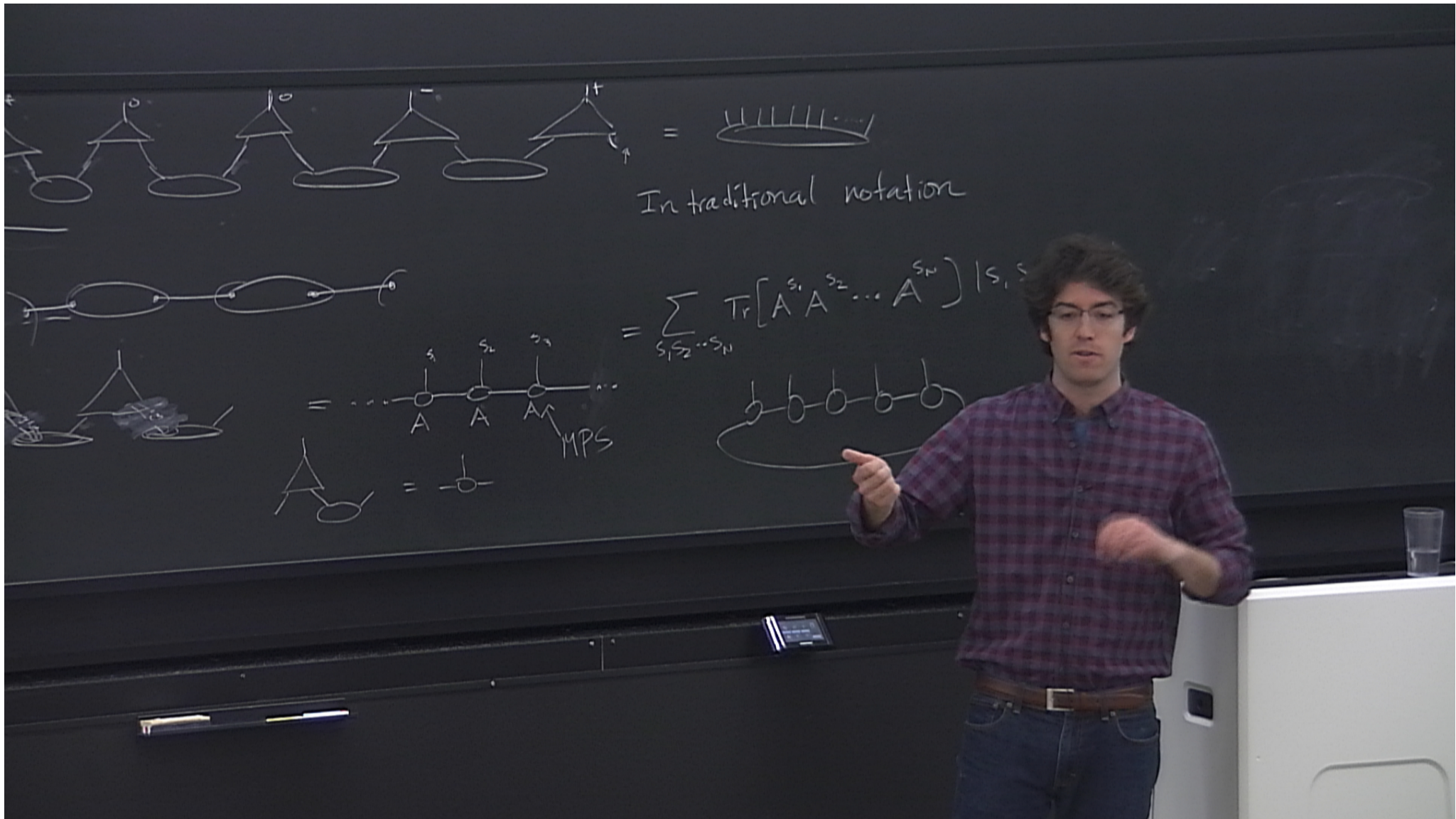


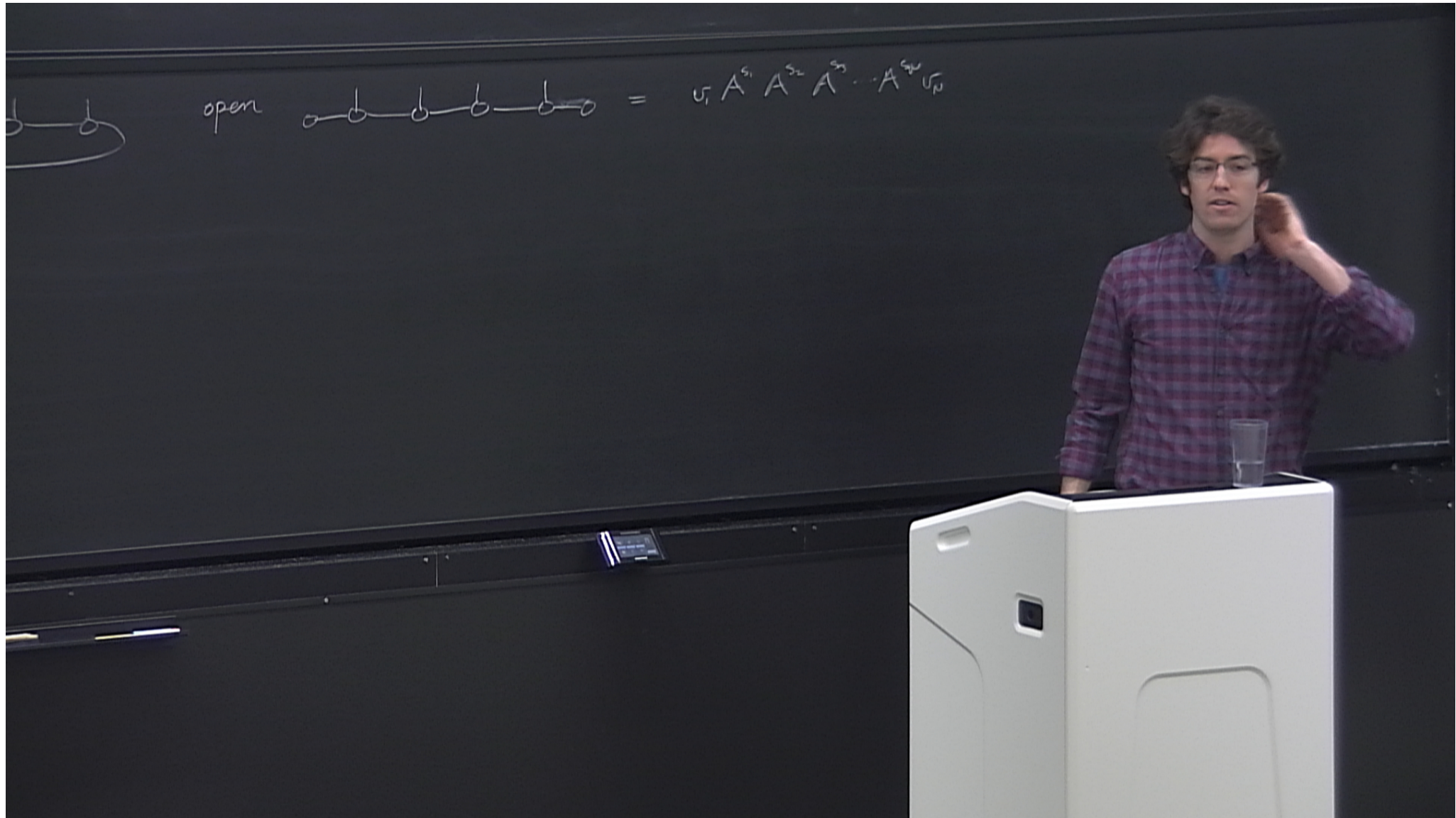


In traditional notation

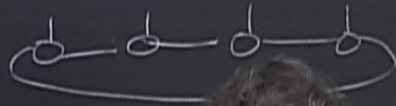
$$= \sum_{s_1 s_2 \dots s_N} \text{Tr} [A^{s_1} A^{s_2} \dots A^{s_N}] |s_1 s_2 \dots s_N\rangle$$



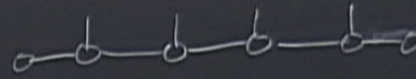




periodic



open



$$= U_1 A^{s_1} A^{s_2} A^{s_3} \dots A^{s_p} U_p$$

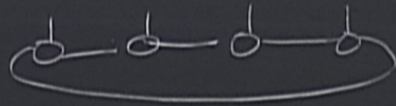
AKLT

$$A^{\sigma_1} \sigma_1$$

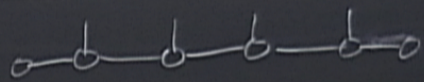
$$A^{\sigma_2} \sigma_2$$

$$A^{\sigma_3} \sigma_3$$

periodic



open



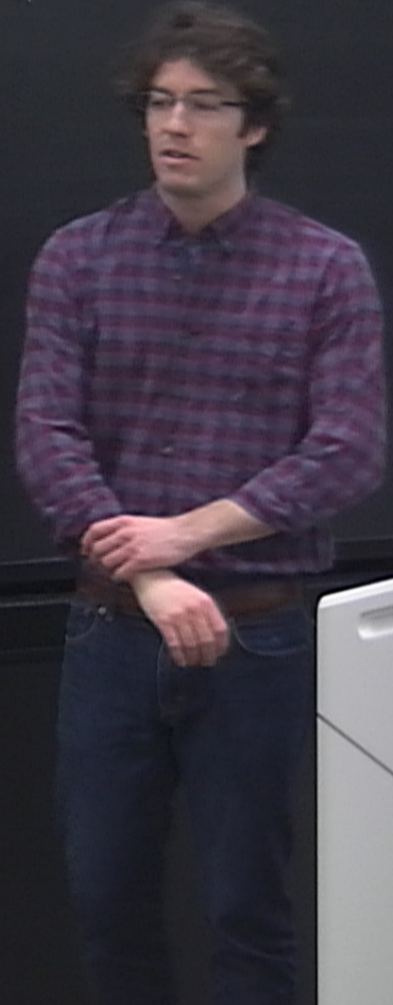
$$U_1 A^{s_1} A^{s_2} A^{s_3} \dots A^{s_N} U_2$$

AKLT

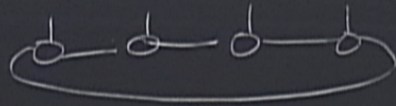
$$A^+ = \sqrt{\frac{2}{3}} \sigma_T$$

$$A^0 = -\sqrt{\frac{1}{3}} \sigma_z$$

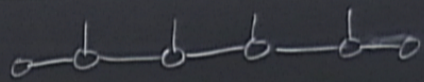
$$A^- = -\sqrt{\frac{2}{3}} \sigma_x$$



periodic



open



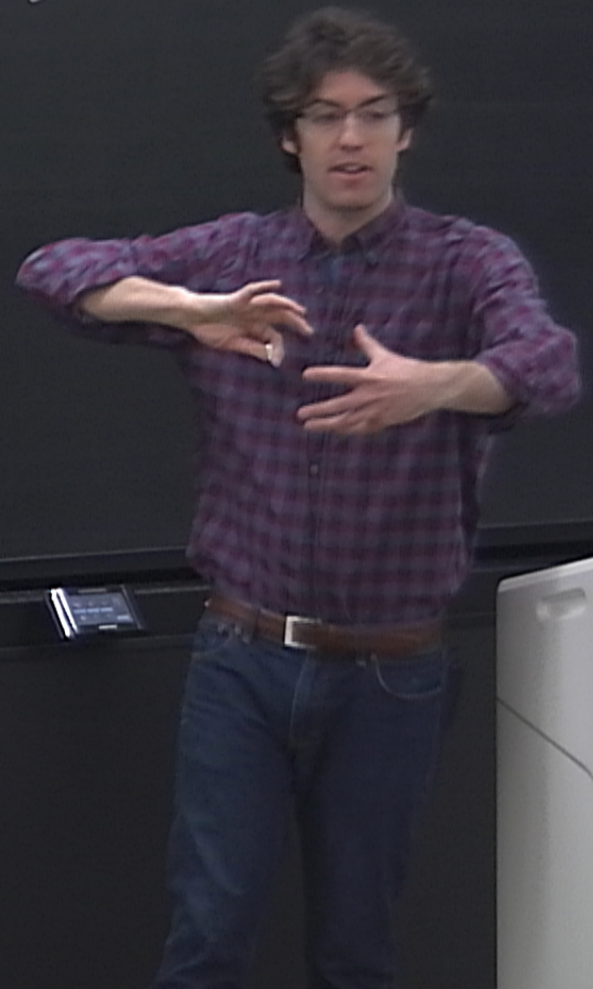
$$= U_1 A^{s_1} A^{s_2} A^{s_3} \dots A^{s_N} U_2$$

AKLT

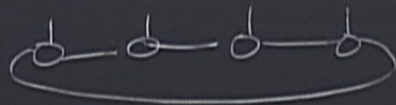
$$A^+ = \sqrt{\frac{2}{3}} \sigma^+ = \sqrt{\frac{3}{3}} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$A^0 = -\sqrt{\frac{1}{3}} \sigma^z \sim \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

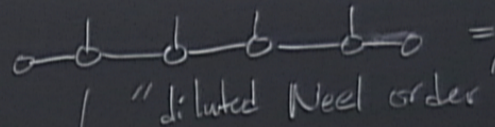
$$A^- = -\sqrt{\frac{2}{3}} \sigma^- \sim \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$



periodic



open



"diluted Neel order"

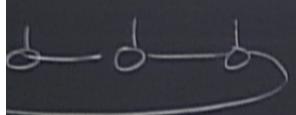
$\sigma_1^z \sigma_2^z \sigma_3^z \dots \sigma_{2n}^z \sigma_{2n+1}^z$
 Neel $\uparrow \downarrow \uparrow \downarrow \uparrow$

AKLT

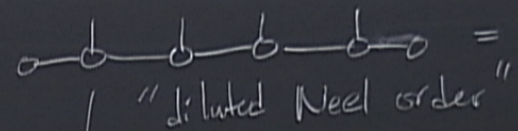
$$A^+ = \sqrt{\frac{2}{3}} \sigma^+ \quad \sigma^+ = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$A^0 = -\sqrt{\frac{1}{3}} \sigma^z \quad \sim \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$A^- = -\sqrt{\frac{2}{3}} \sigma^- \quad \sim \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$



open



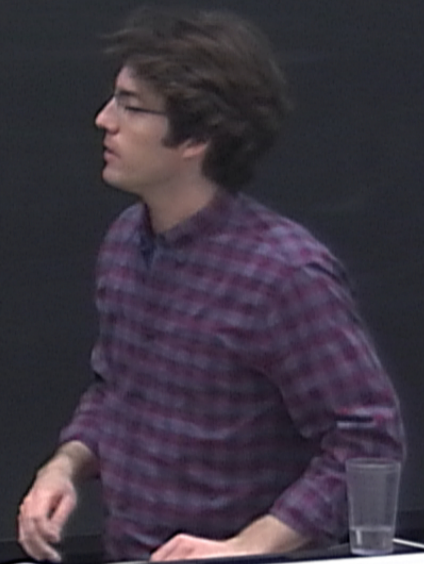
$$v_1 A^{s_1} A^{s_2} A^{s_3} \dots A^{s_N} v_2$$

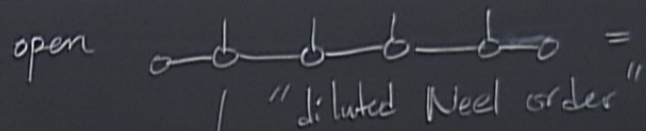
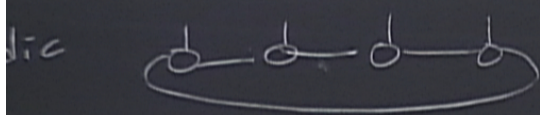
Neel $\uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$

$$A^+ = \sqrt{\frac{2}{3}} \sigma^+ = \sqrt{\frac{2}{3}} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$A^0 = -\sqrt{\frac{1}{3}} \sigma^z \sim \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$A^- = -\sqrt{\frac{2}{3}} \sigma^- \sim \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$





$$u_i A^{s_1} A^{s_2} A^{s_3} \dots A^{s_N} u_N$$

Neel $\uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$

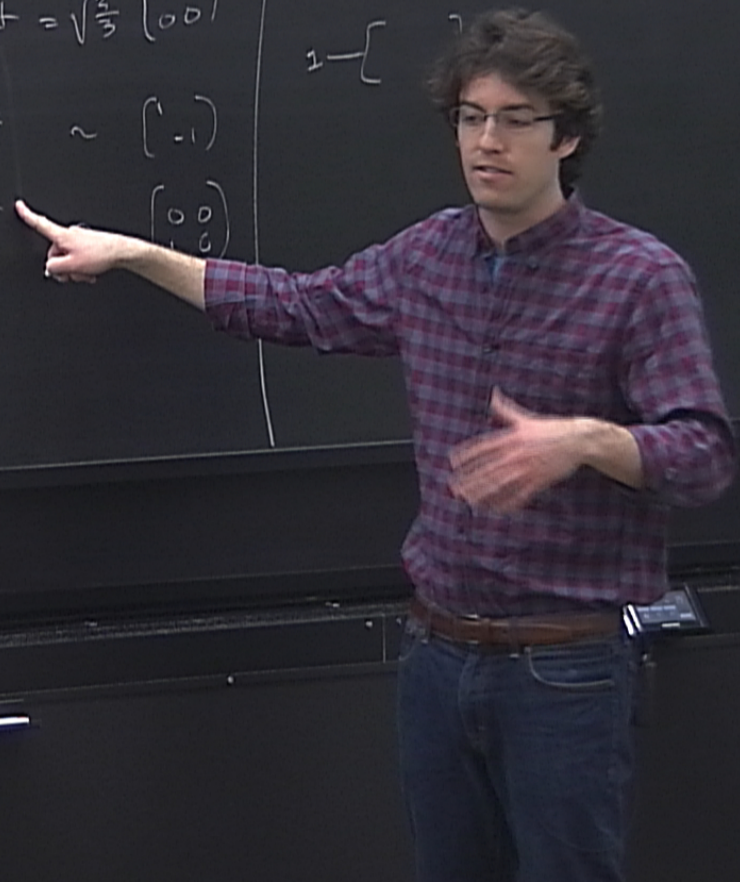
LT

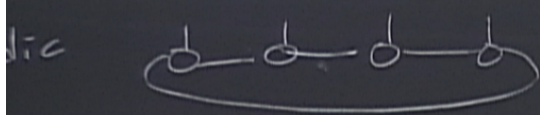
$$A^+ = \sqrt{\frac{v_1 v_2}{v_1 v_2}} \sigma^+ = \sqrt{\frac{2}{3}} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$A^0 = -\sqrt{\frac{v_1 v_2}{v_1 v_2}} \sigma^z \sim \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

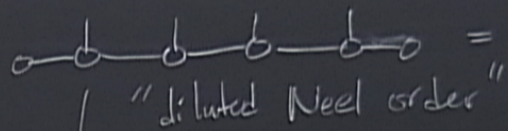
$$A^- = -\sqrt{\frac{v_1 v_2}{v_1 v_2}} \sigma^- \sim \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$1 - [\dots]$





open



$$U_i A^{s_1} A^{s_2} A^{s_3} \dots A^{s_N} U_N$$

Neel ↑ ↓ ↑ ↓ ↑ ↓

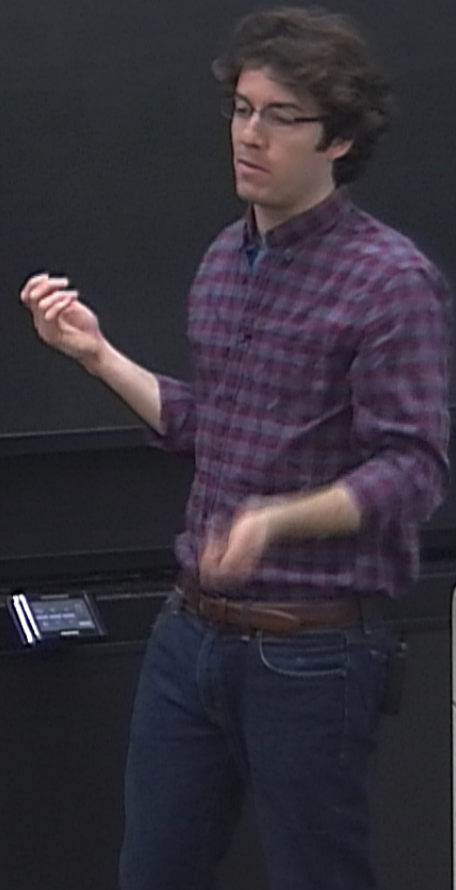
LT

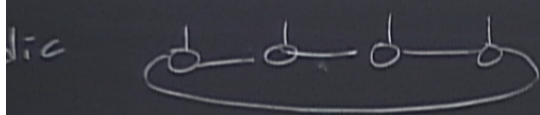
$$A^+ = \sqrt{\frac{2}{3}} \sigma_T = \sqrt{\frac{2}{3}} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$A^0 = -\sqrt{\frac{1}{3}} \sigma_{T^2} \sim \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

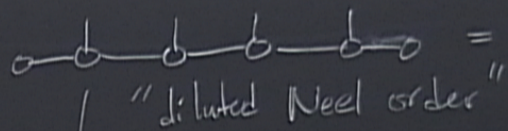
$$A^- = -\sqrt{\frac{2}{3}} \sigma_{T^2} \sim \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$1 \Rightarrow \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}^+$$





open



$$U_1 A^{s_1} A^{s_2} A^{s_3} \dots A^{s_N} U_N$$

Neel $\uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$

LT

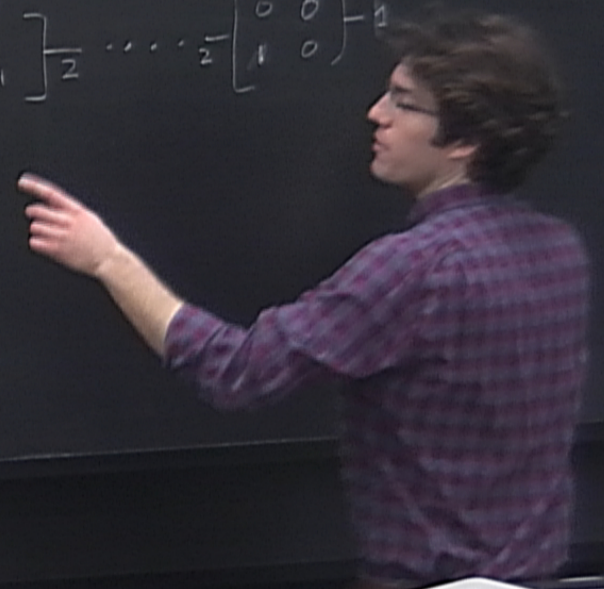
$$A^+ = \sqrt{\frac{w}{w'}} \sigma_T = \sqrt{\frac{2}{3}} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

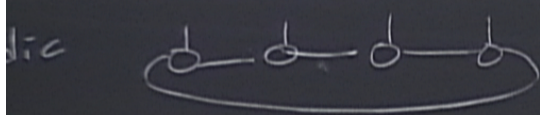
$$A^0 = -\sqrt{\frac{w}{w'}} \sigma_{T^2} \quad ? \quad \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

$$A^- = -\sqrt{\frac{w}{w'}} \sigma_{T^1} \quad ? \quad \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

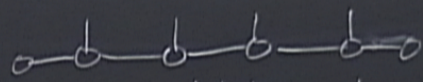
"diluted Neel order"

$$1 \rightarrow \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & \\ & -1 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & \\ & -1 \end{bmatrix} \dots \frac{1}{2} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \frac{1}{2}$$





open



$$U_1 A^{s_1} A^{s_2} A^{s_3} \dots A^{s_N} U_N$$

Neel $\uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$

"diluted Neel order"

LT

$$A^+ = \sqrt{\frac{2}{3}} \sigma_T = \sqrt{\frac{2}{3}} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$A^0 = -\sqrt{\frac{1}{3}} \sigma_{T^2} \sim \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$A^- = -\sqrt{\frac{2}{3}} \sigma_{T^2} \sim \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$1 \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \dots \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} - 1$$

$$|\Psi_{AKLT}\rangle = |+000 \dots 0-000 \dots + \dots$$

$$+ |-+-+ \dots -0+0-+00 \dots$$

$$+ |-+-00+0-+0 \dots$$

$$+ \dots$$

