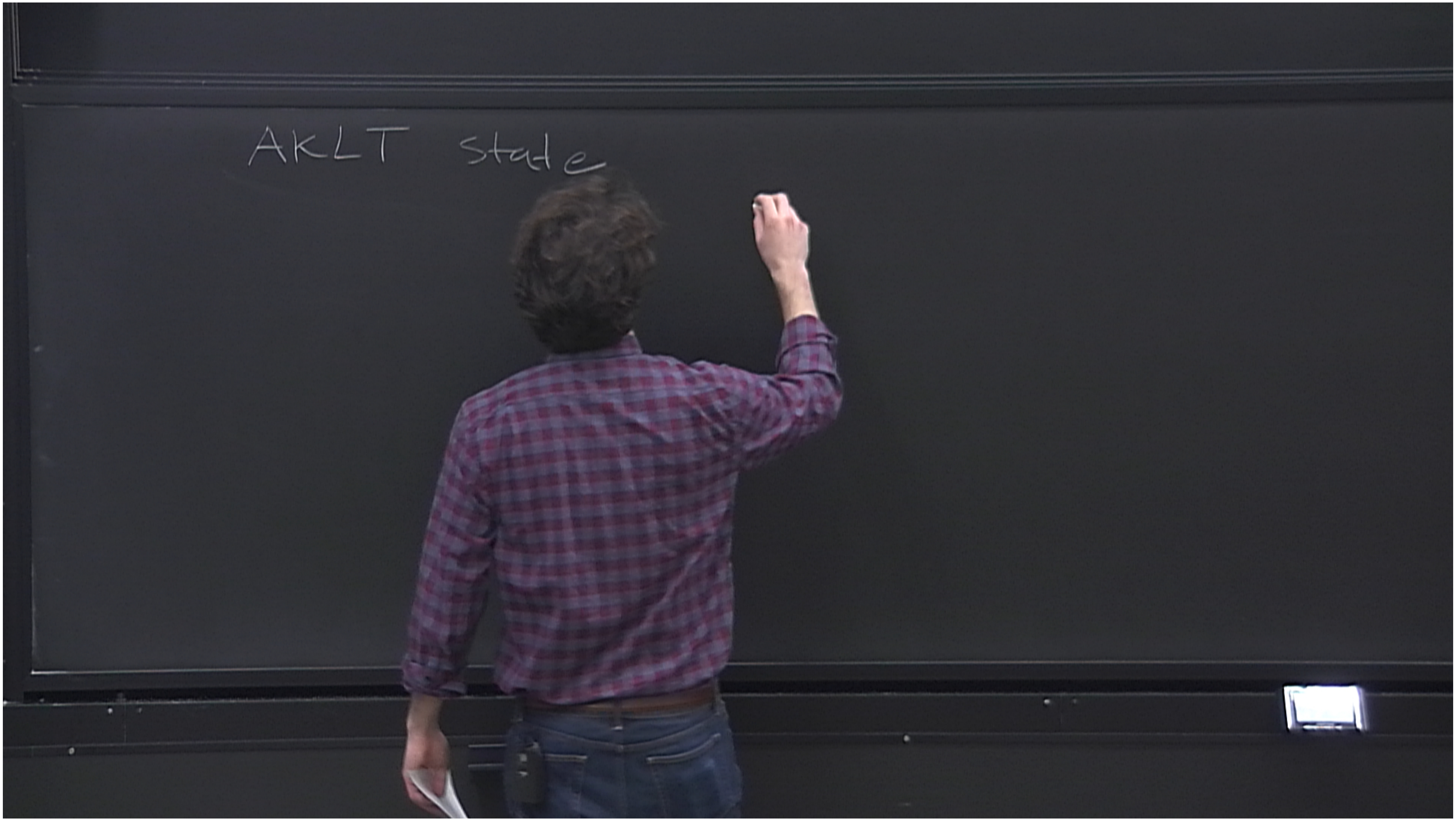
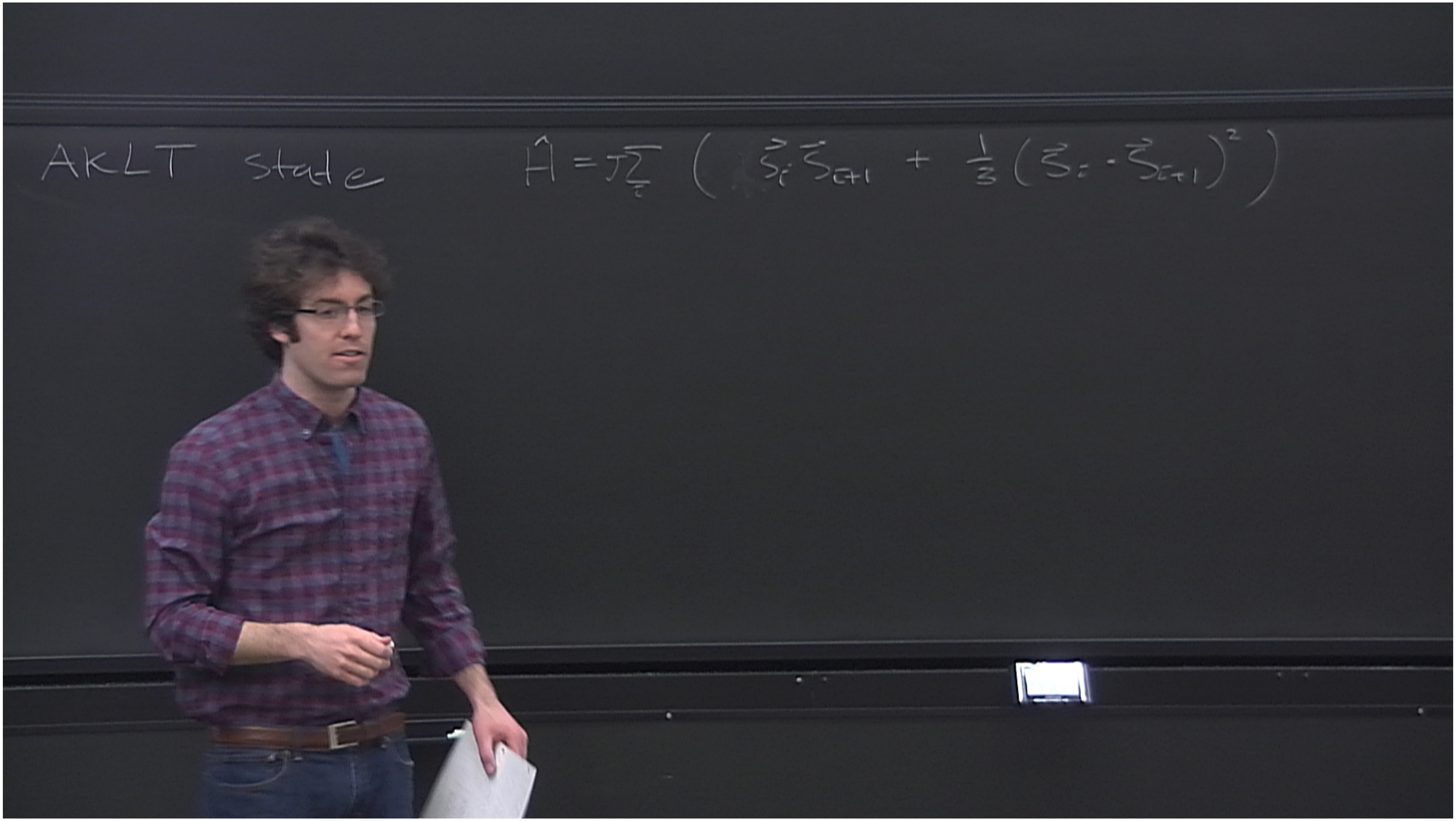
Title: Explorations in Condensed Matter-11

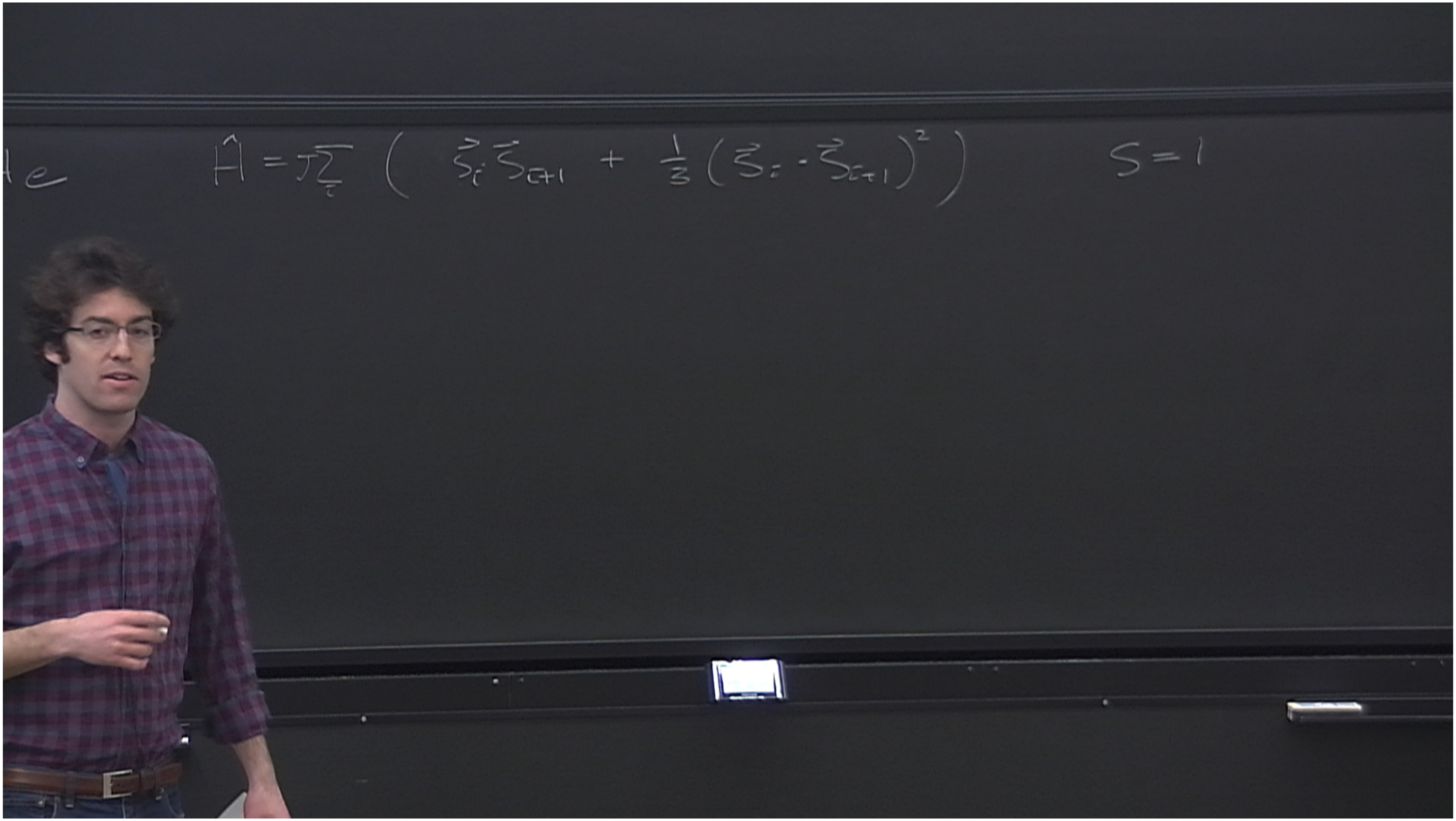
Date: Mar 30, 2015 10:15 AM

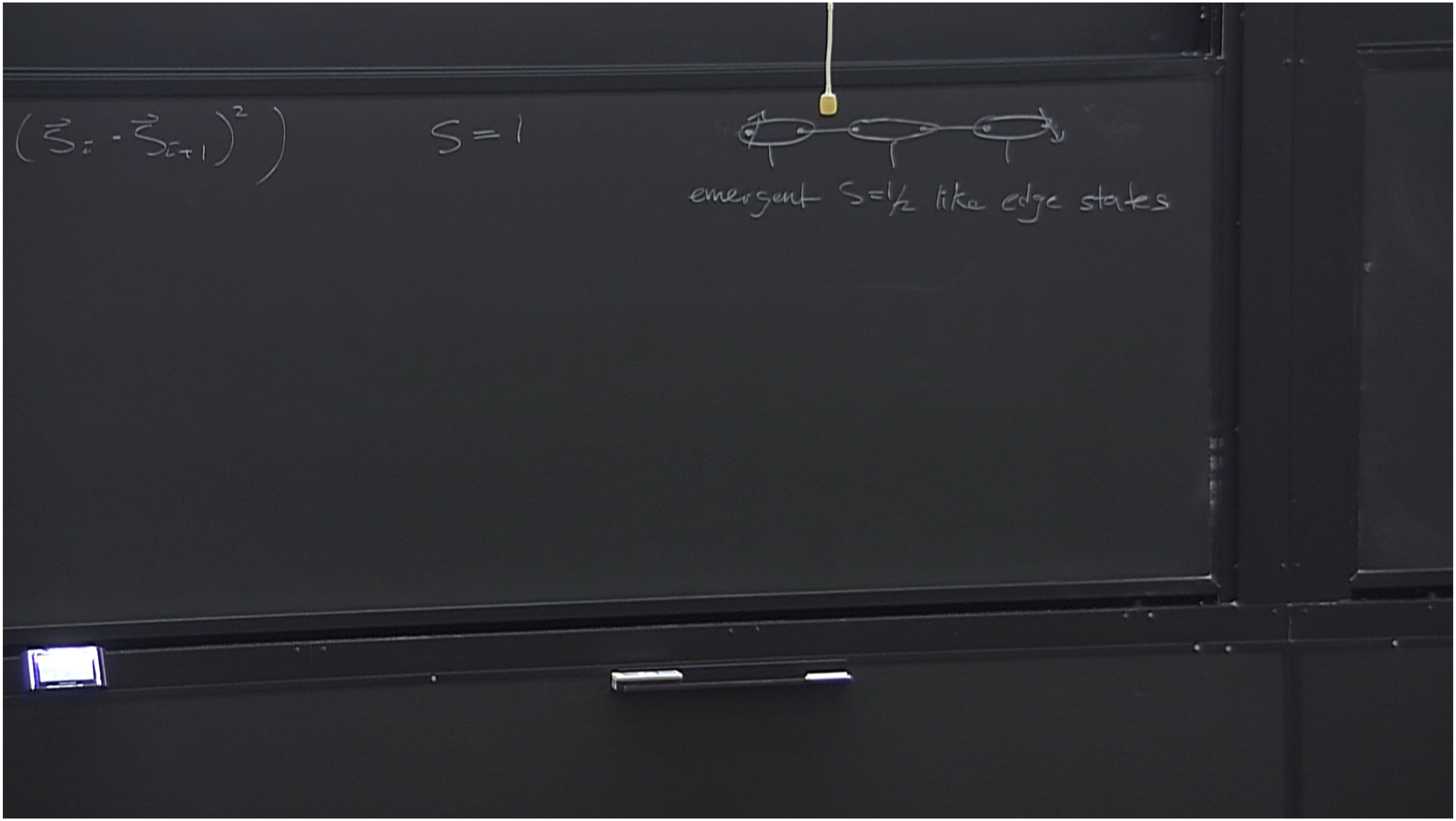
URL: http://pirsa.org/15030048

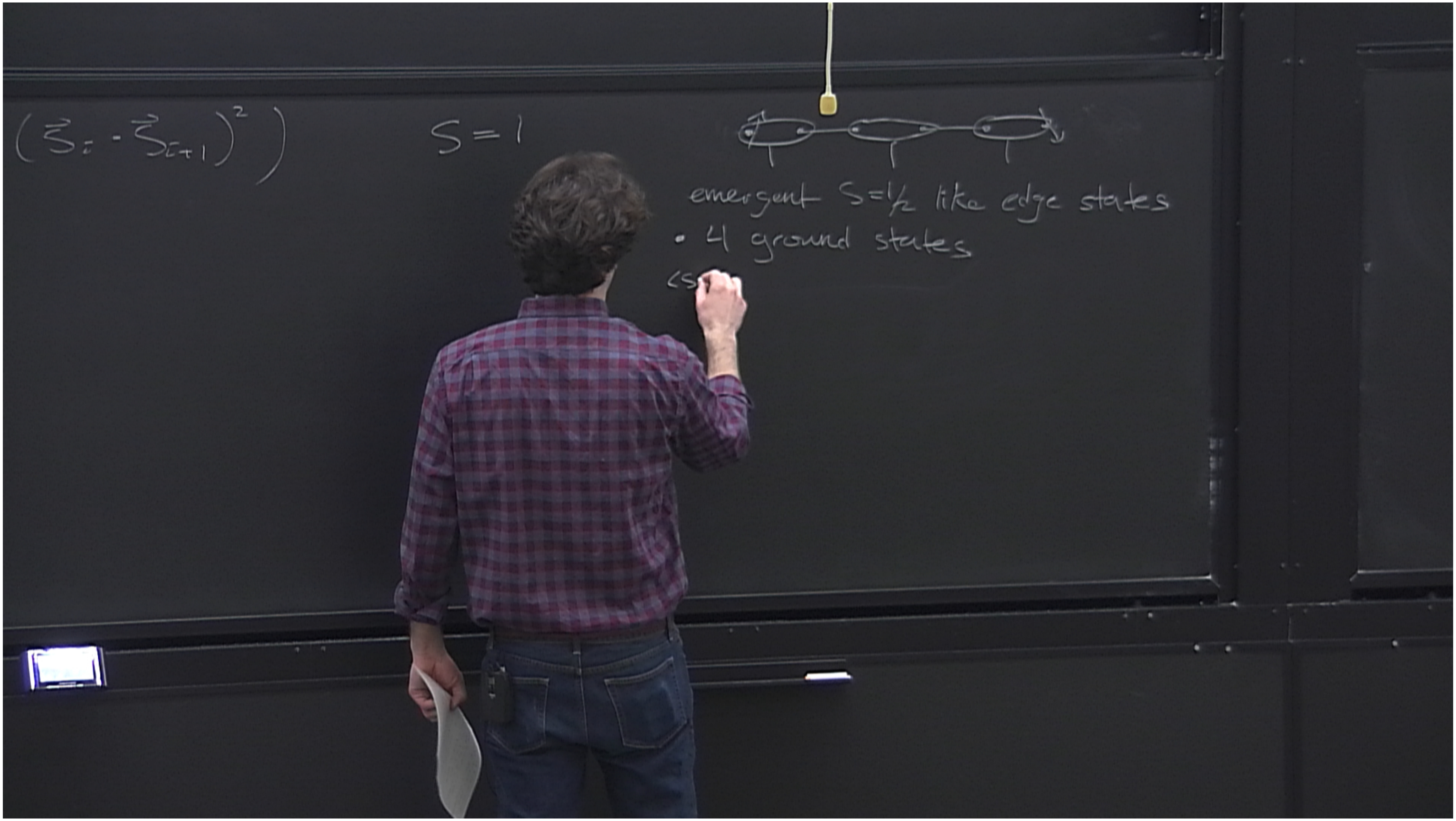
Abstract:

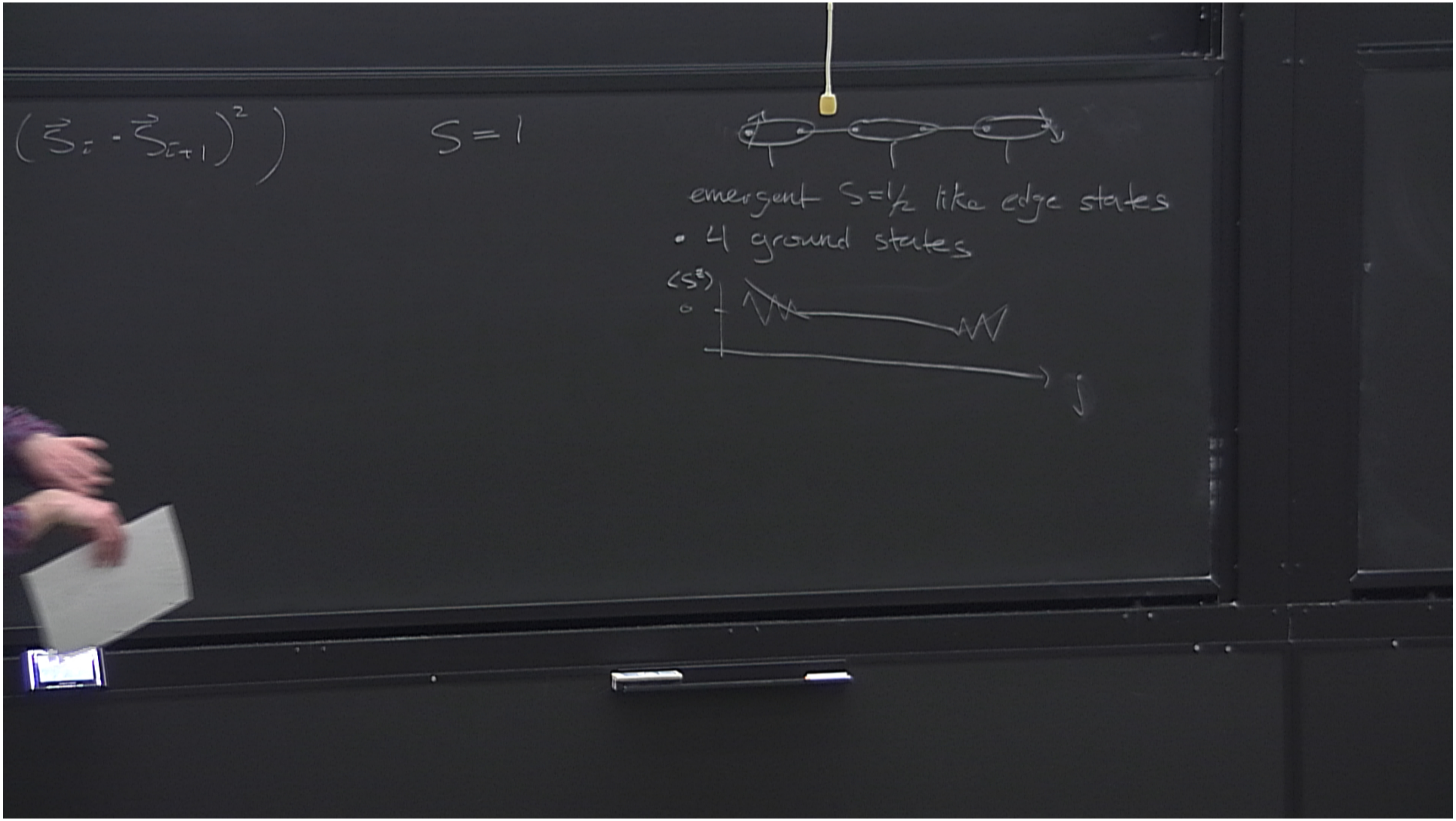

AKLT state

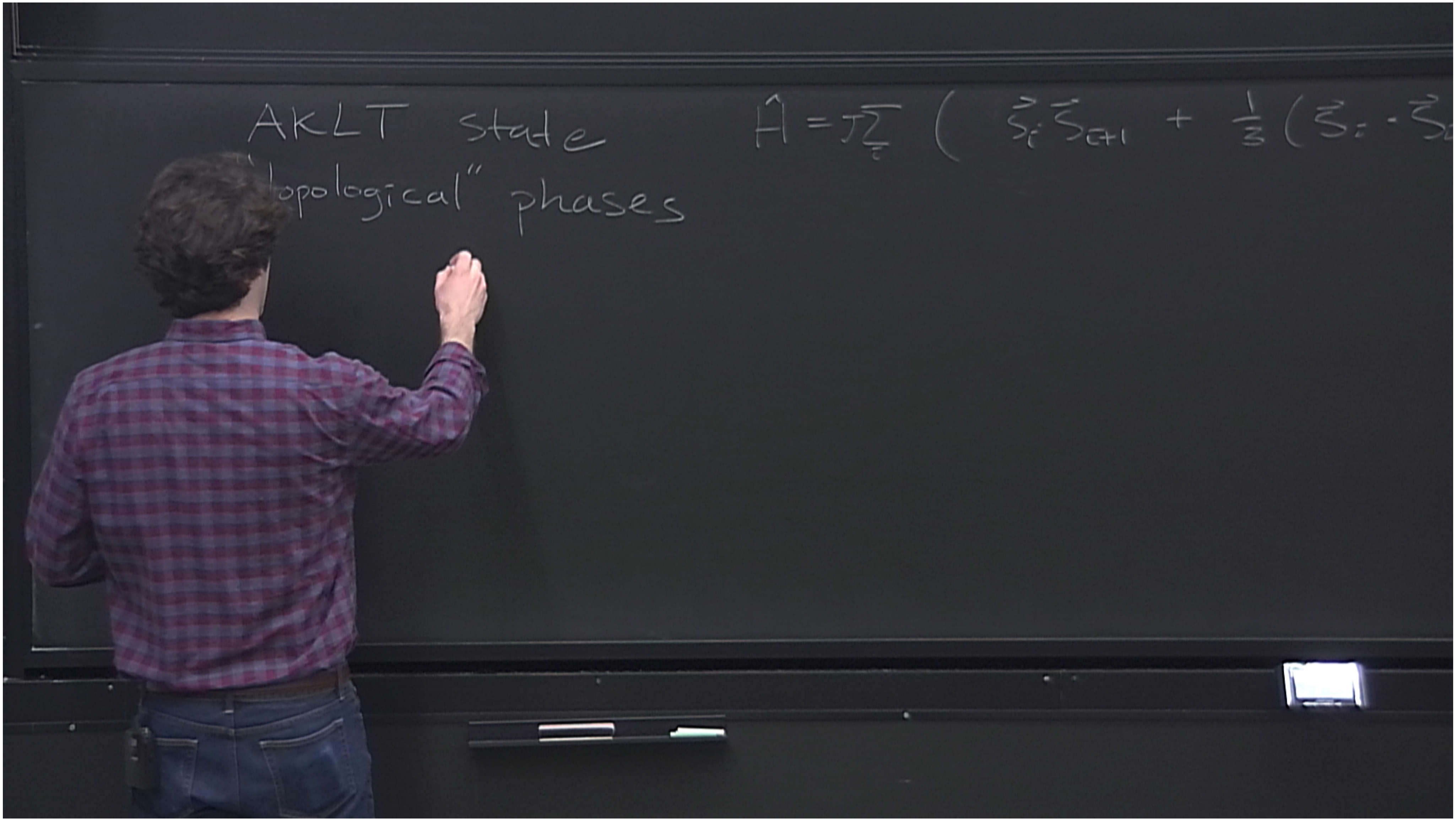

AKLT state
$\hat{H} = J\sum_i \left(\vec{S}_i \vec{S}_{i+1} + \frac{1}{3}\left(\vec{S}_i \cdot \vec{S}_{i+1}\right)^2 \right)$

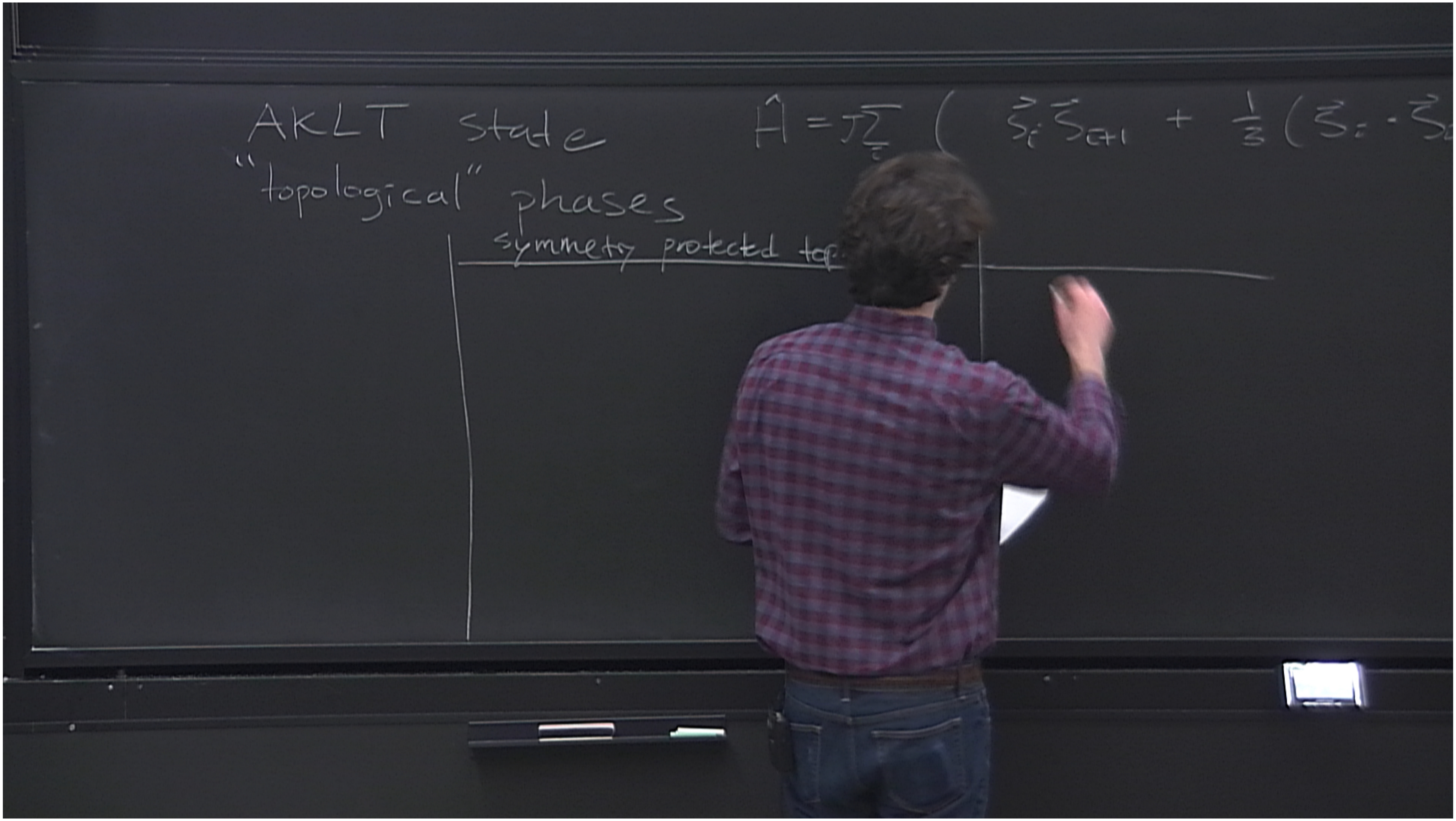

$\hat{H} = J\sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3}\left(\vec{S}_i \cdot \vec{S}_{i+1}\right)^2 \right)$
$S = 1$

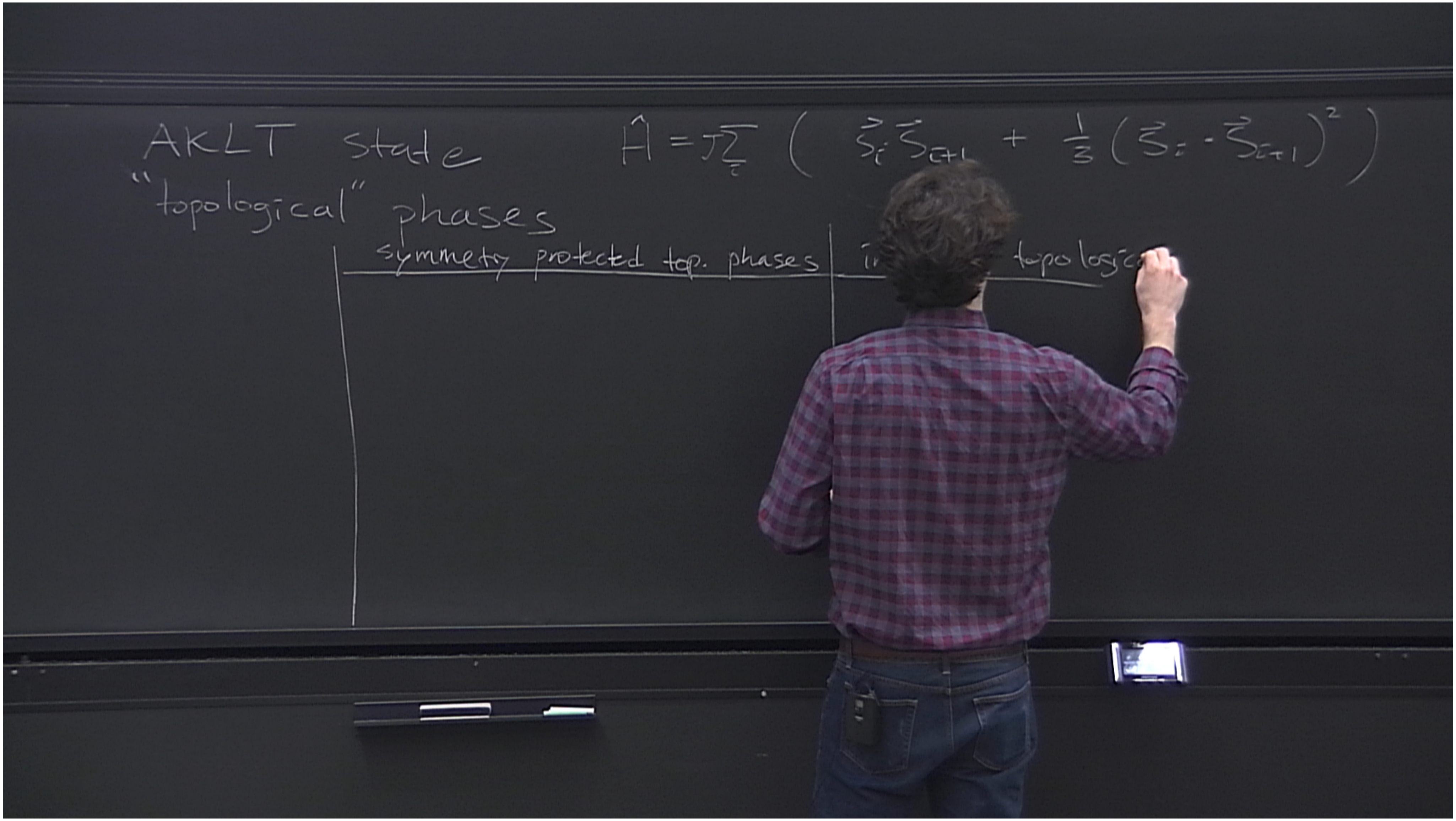

$(\vec{S}_i \cdot \vec{S}_{i+1})^2)$
$S=1$
emergent $S=1/2$ like edge states

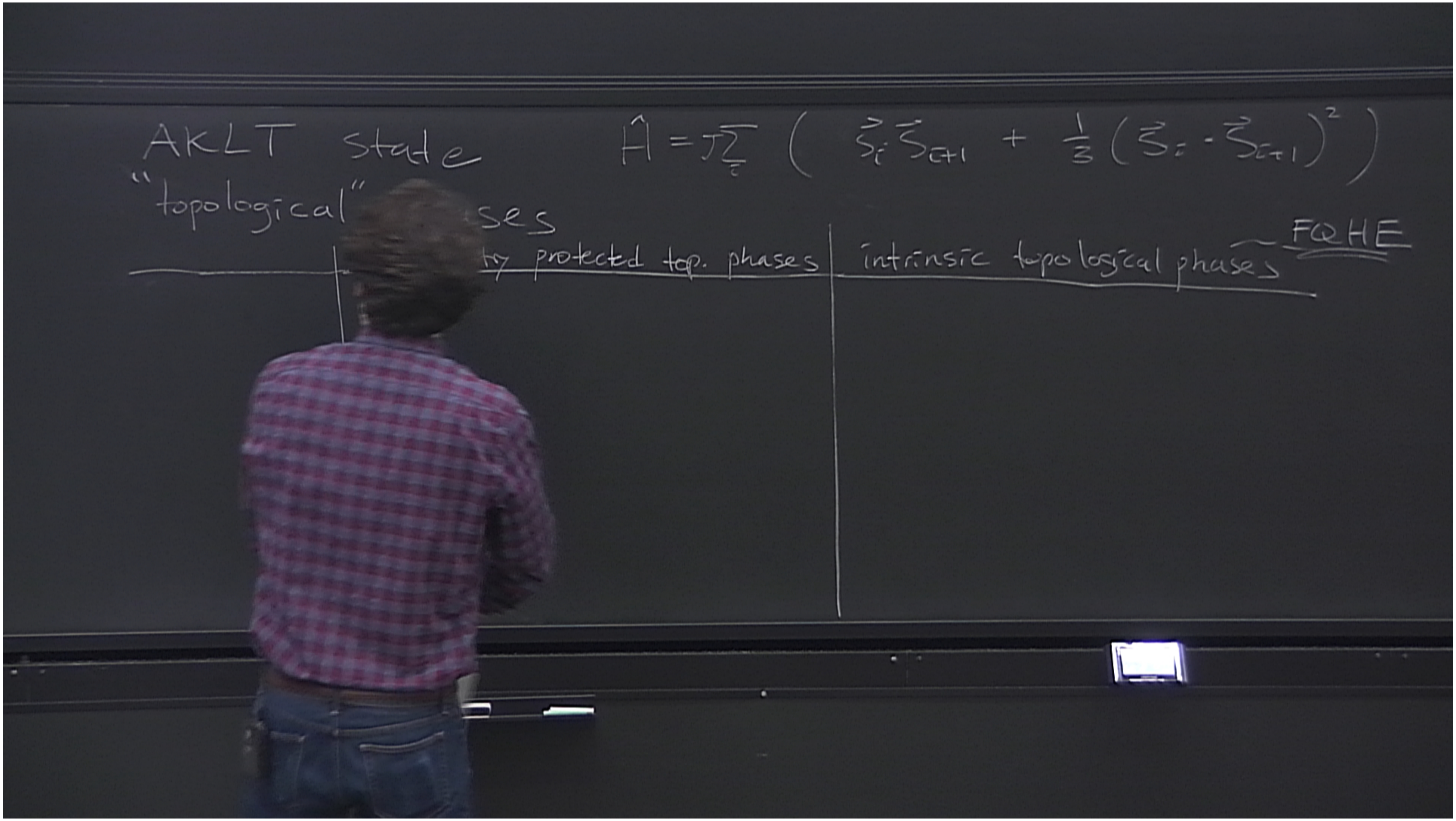

$(\vec{S}_i \cdot \vec{S}_{i+1})^2)$
S = 1
emergent S=1/2 like edge states
• 4 ground states

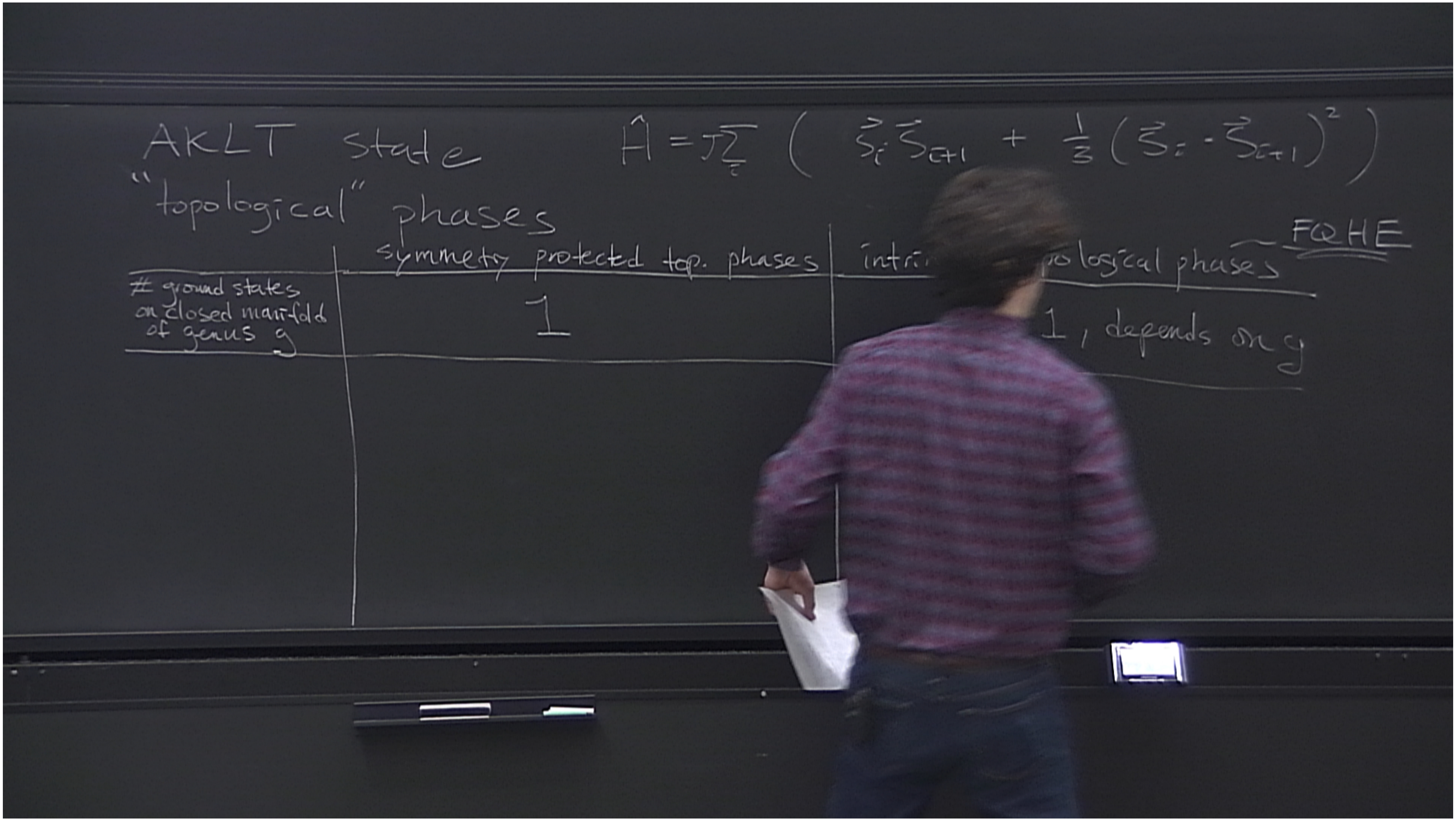

$(\vec{S}_i \cdot \vec{S}_{i+1})^2$)
$S=1$
emergent $S=1/2$ like edge states
• 4 ground states
$\langle S^z \rangle$
0
j

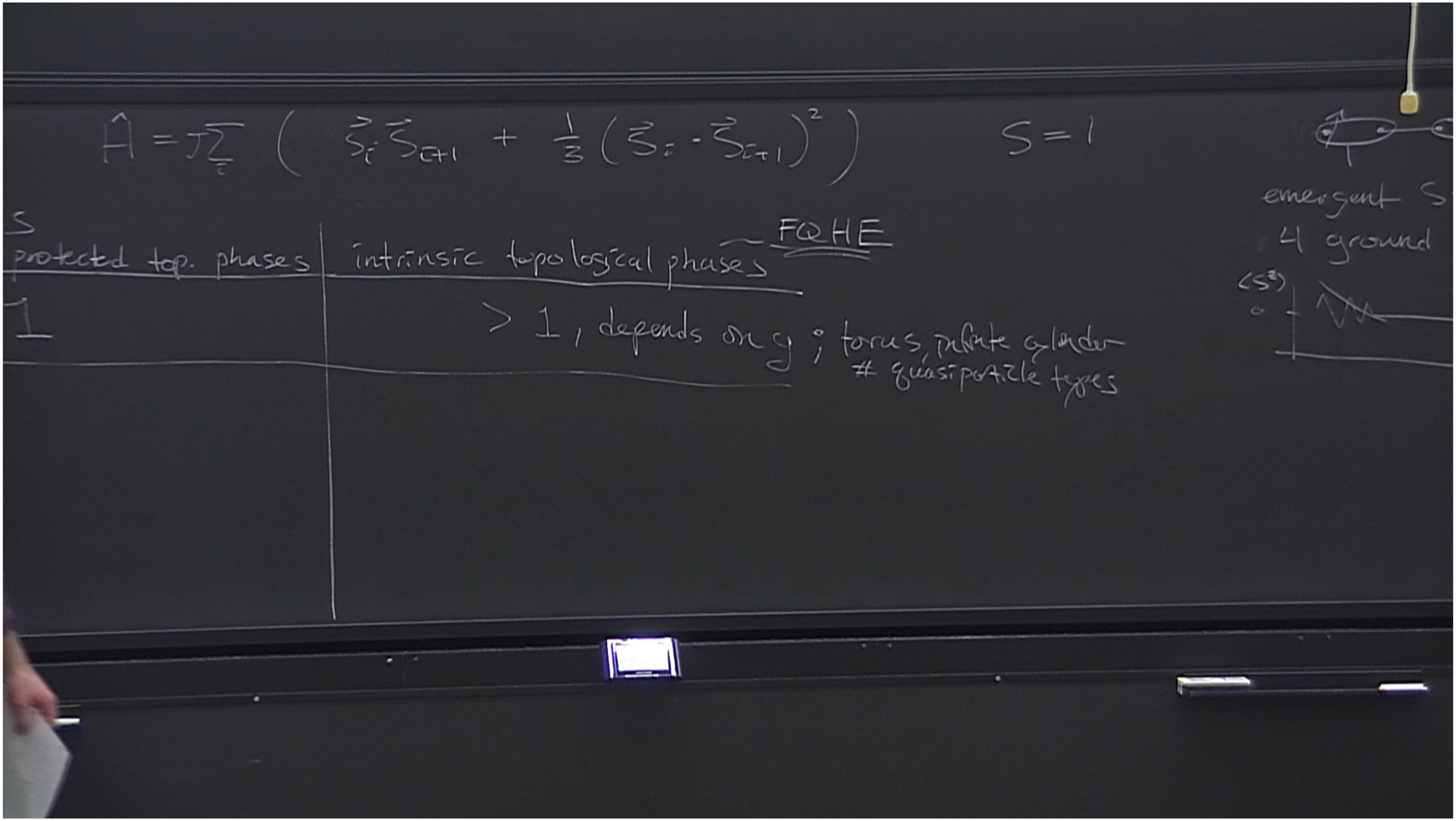

AKLT state
opological" phases
Ĥ = J Σ_i (S_i · S_{i+1} + 1/3 (S_i ·


AKLT state
$\hat{H} = J\sum_i (\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3}(\vec{S}_i \cdot \vec{S}$
"topological" phases
symmetry protected top


AKLT state
$\hat{H} = J \sum_i \left(\vec{S}_i \vec{S}_{i+1} + \frac{1}{3} (\vec{S}_i \cdot \vec{S}_{i+1})^2 \right)$
"topological" phases
symmetry protected top. phases
topological


AKLT state
$\hat{H} = J \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3} \left(\vec{S}_i \cdot \vec{S}_{i+1} \right)^2 \right)$
"topological" ses
ry protected top. phases
intrinsic topological phases
FQHE

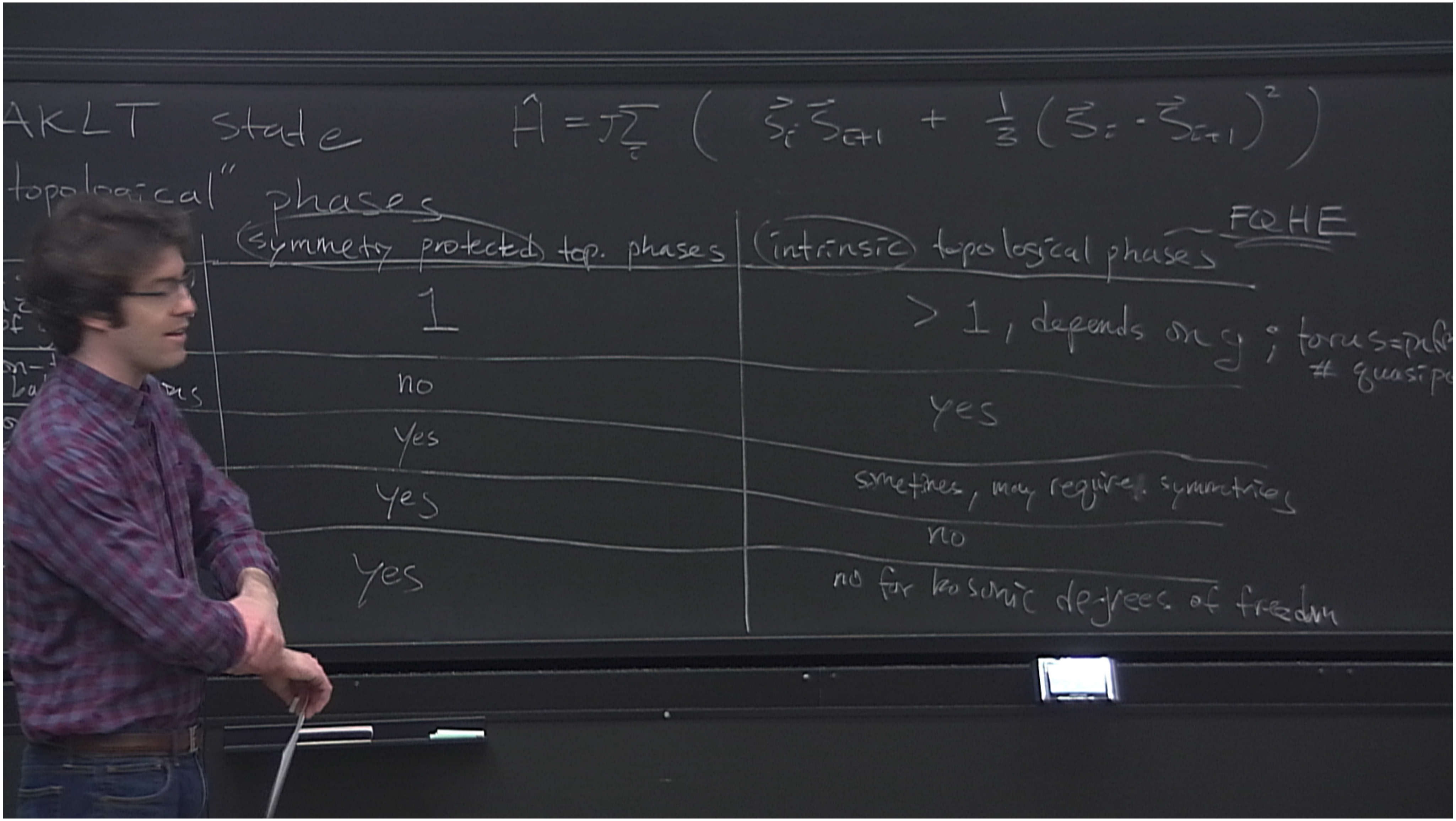

AKLT state
"topological" phases
symmetry protected top. phases
intri... ...pological phases
FQHE
ground states on closed manifold of genus g
1
1, depends on g


$\hat{H} = J \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3} (\vec{S}_i \cdot \vec{S}_{i+1})^2 \right)$
$S = 1$
emergent S
4 ground
protected top. phases
intrinsic topological phases
FQHE
1
> 1, depends on g ; torus, infinite cylinder
quasiparticle types

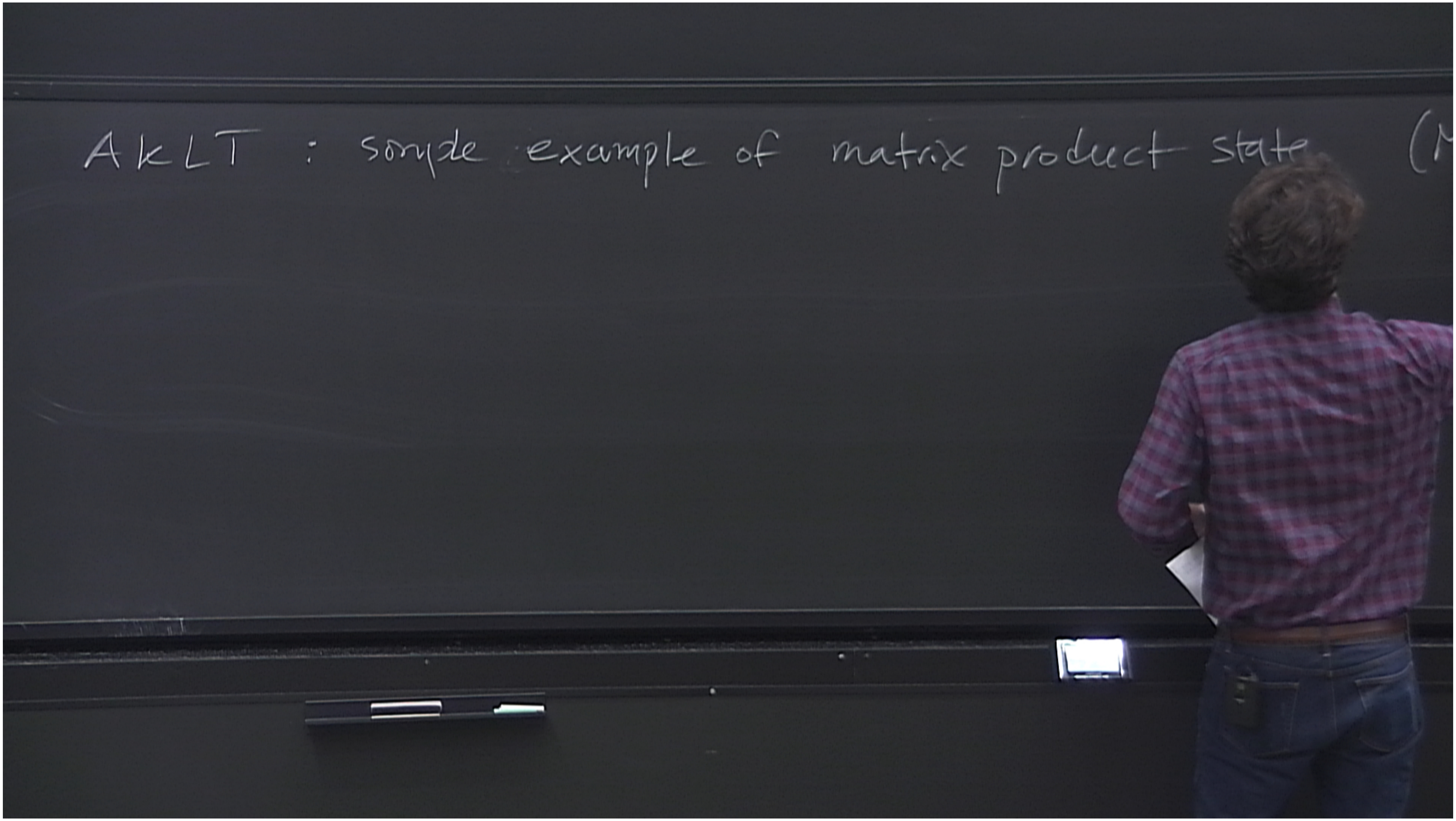
state $\hat{H} = J\sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3}(\vec{S}_i \cdot \vec{S}_{i+1})^2 \right)$ $S=1$

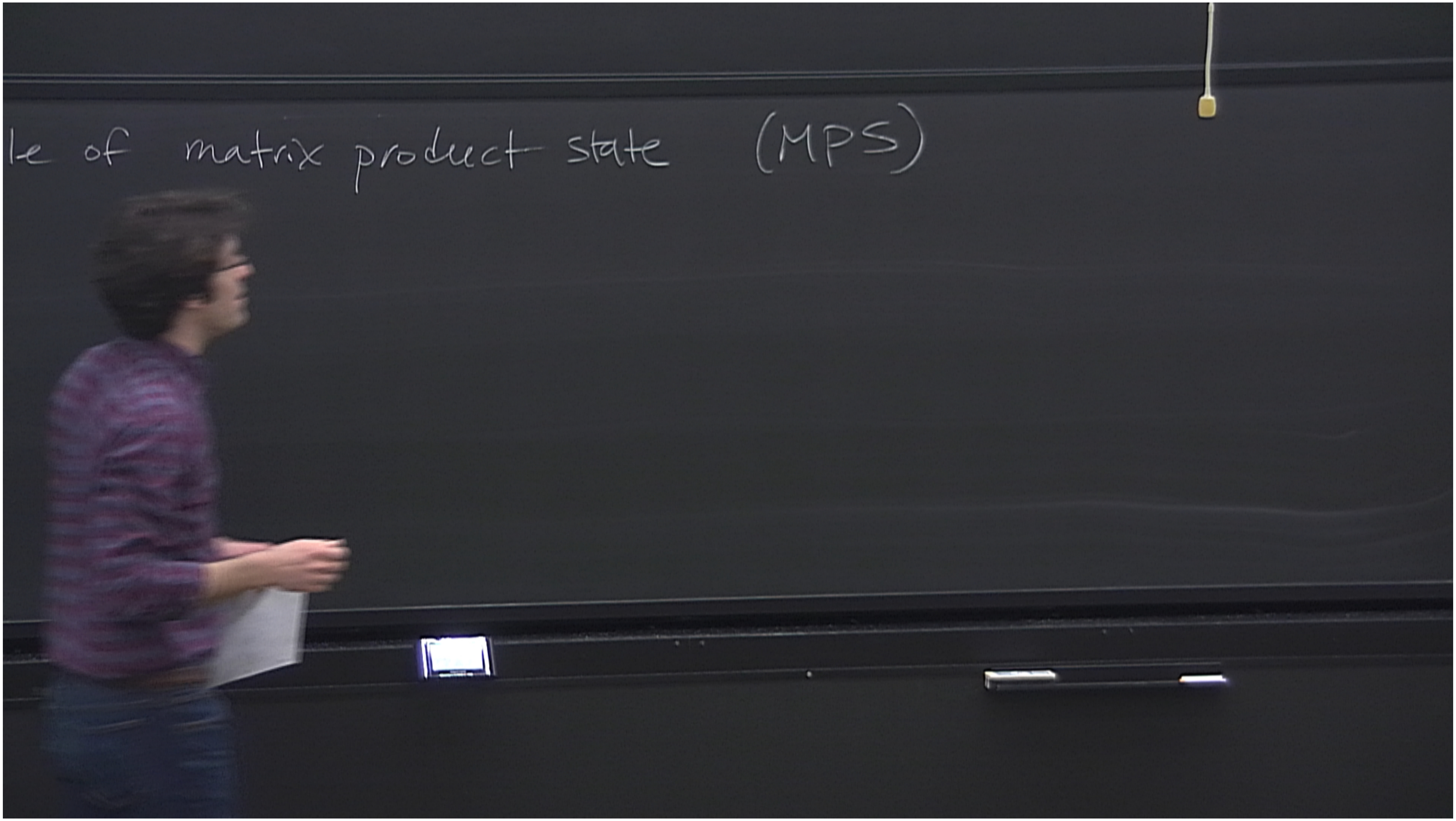
l" phases

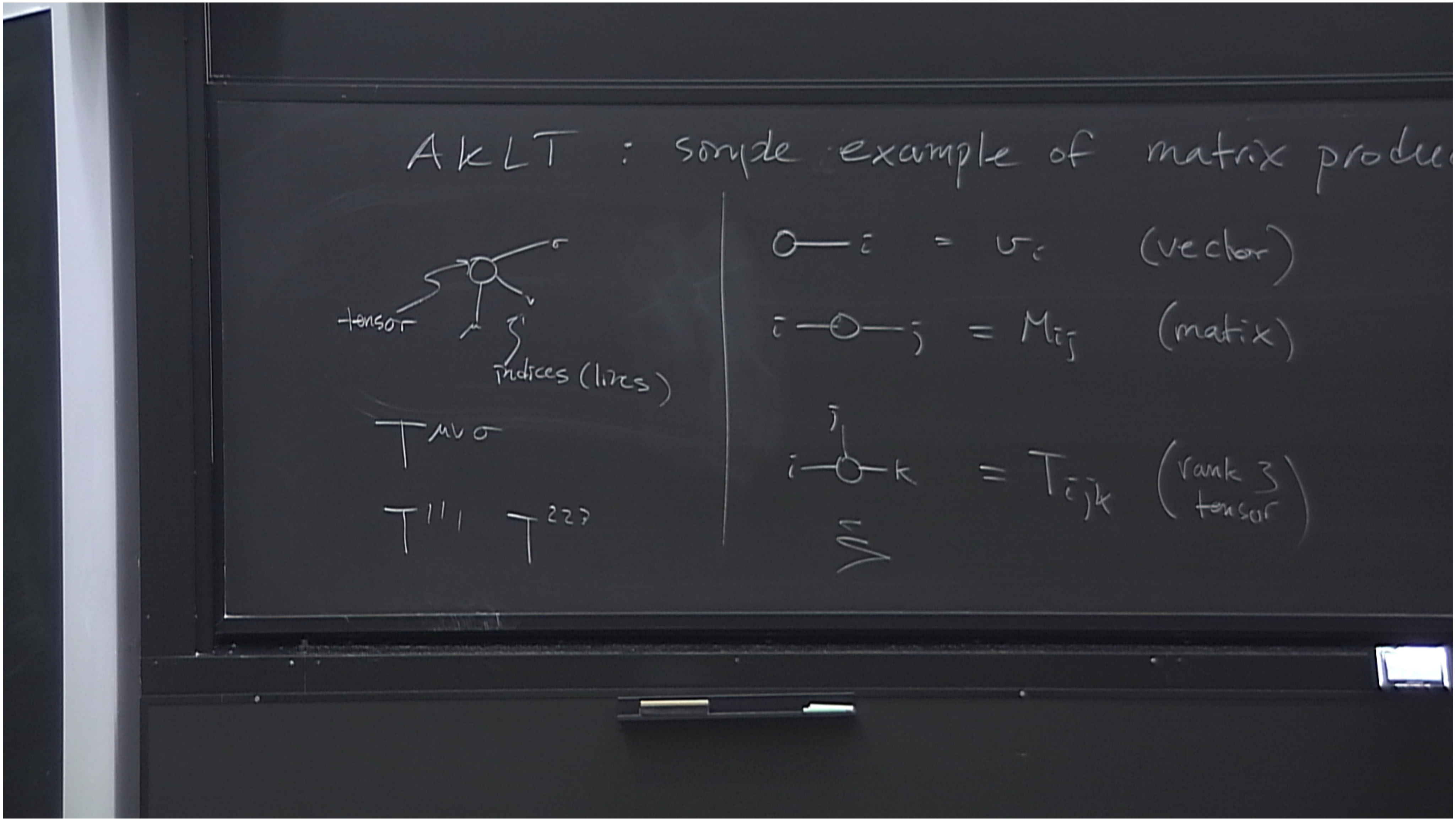
	symmetry protected top. phases	intrinsic topological phases ~ FQHE
	1	> 1, depends on g ; torus, infinite cylinder # quasiparticle types
s	no	yes
	yes	sometimes, may require symmetries

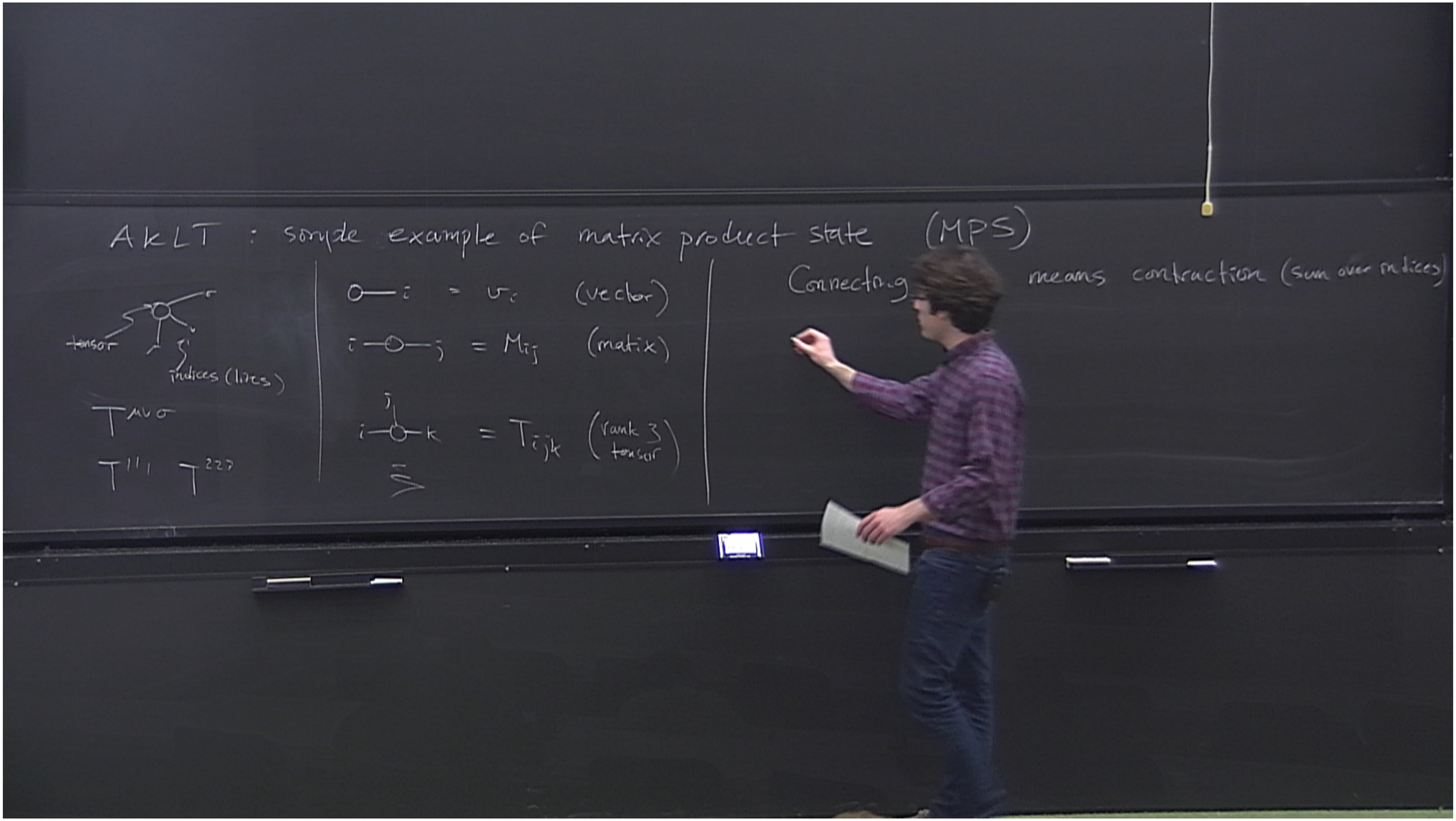

AKLT state
"topological" phases
symmetry protected top. phases
intrinsic topological phases
FQHE
1
> 1, depends on g ; torus
no
yes
yes
sometimes, may require symmetries
yes
no
yes
no for bosonic degrees of freedom

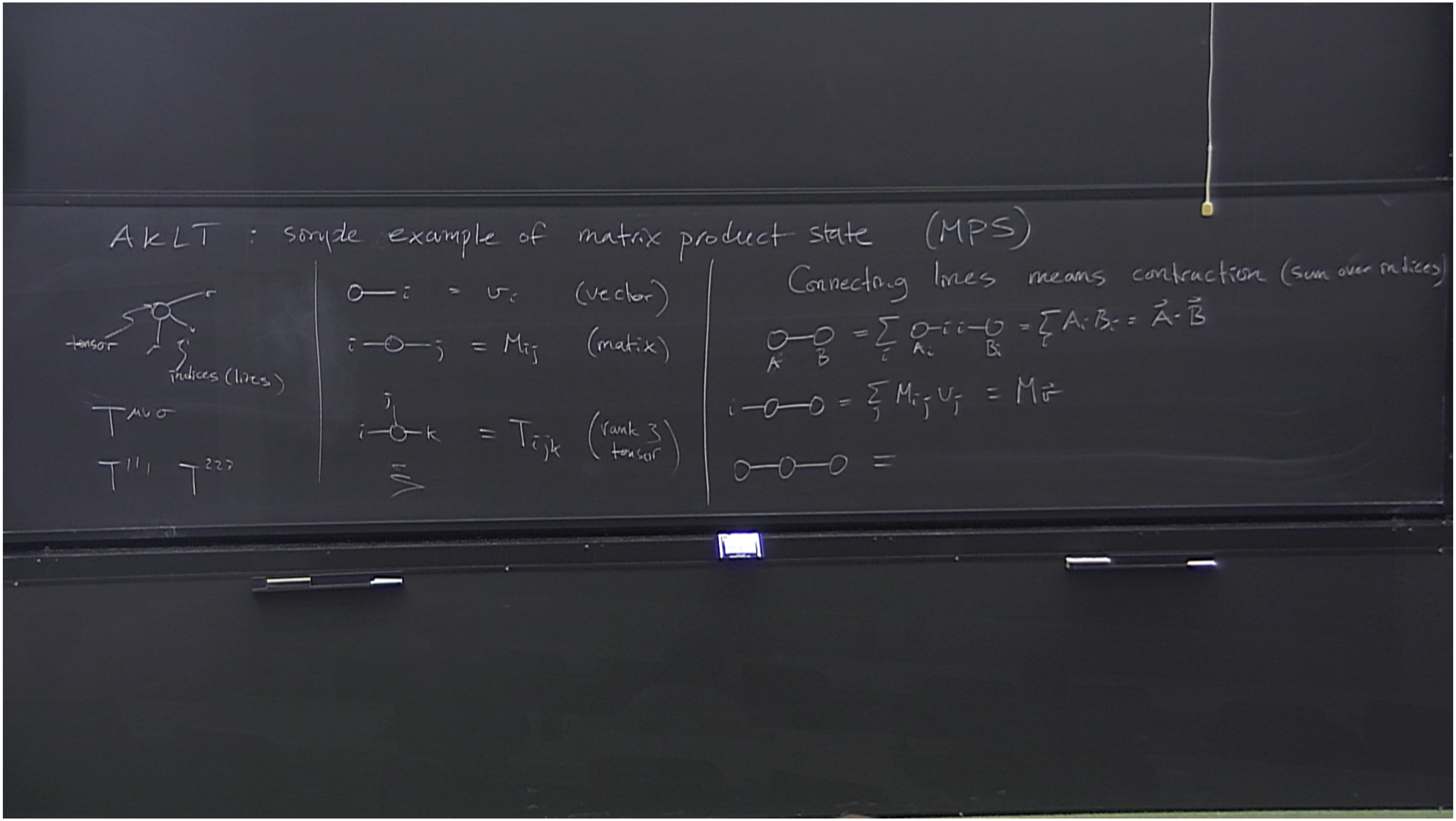

AKLT : simple example of matrix

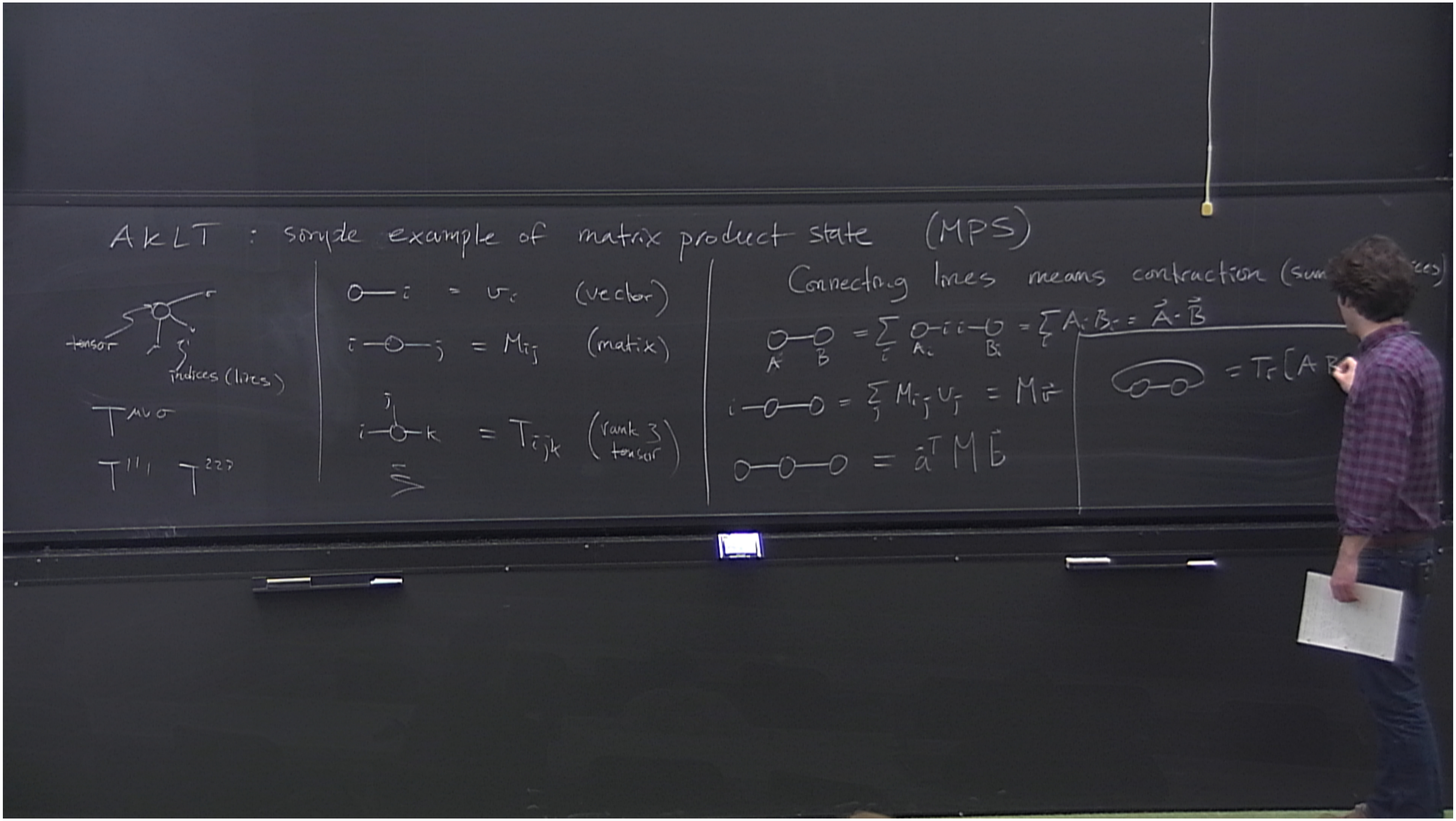

AKLT : simple example of matrix product state

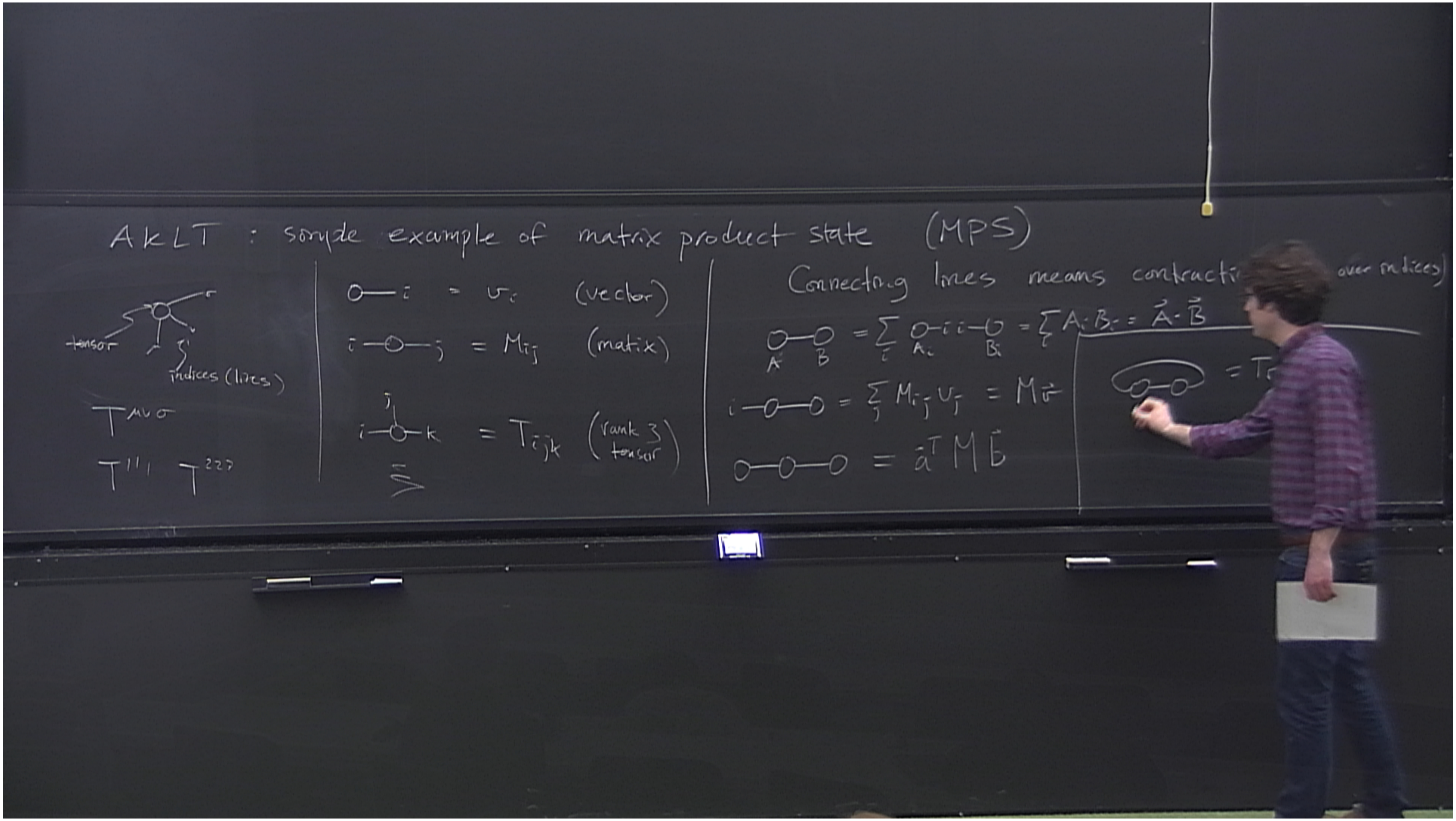

le of matrix product state (MPS)


AKLT : simple example of matrix produc
tensor
indices (lines)
(vector)
(matrix)
(rank 3
tensor)


AKLT : simple example of matrix product state (MPS)
tensor
indices (lines)
(vector)
(matrix)
(rank 3 tensor)
Connecting
means contraction (sum over indices)


AKLT : simple example of matrix product state (MPS)
tensor
indices (lines)
$T^{\mu\nu\sigma}$
T^{111} T^{223}
$= v_i$ (vector)
$= M_{ij}$ (matrix)
$= T_{ijk}$ (rank 3 tensor)
Connecting lines means contraction (sum over indices)
$= \sum_i A_i B_i = \vec{A} \cdot \vec{B}$
$= \sum_j M_{ij} v_j = M\vec{v}$


AKLT : simple example of matrix product state (MPS)
tensor
indices (lines)
(vector)
(matrix)
(rank 3 tensor)
Connecting lines means contraction (sum
= Tr [A B



AKLT : simple example of matrix product state (MPS)
tensor
indices (lines)
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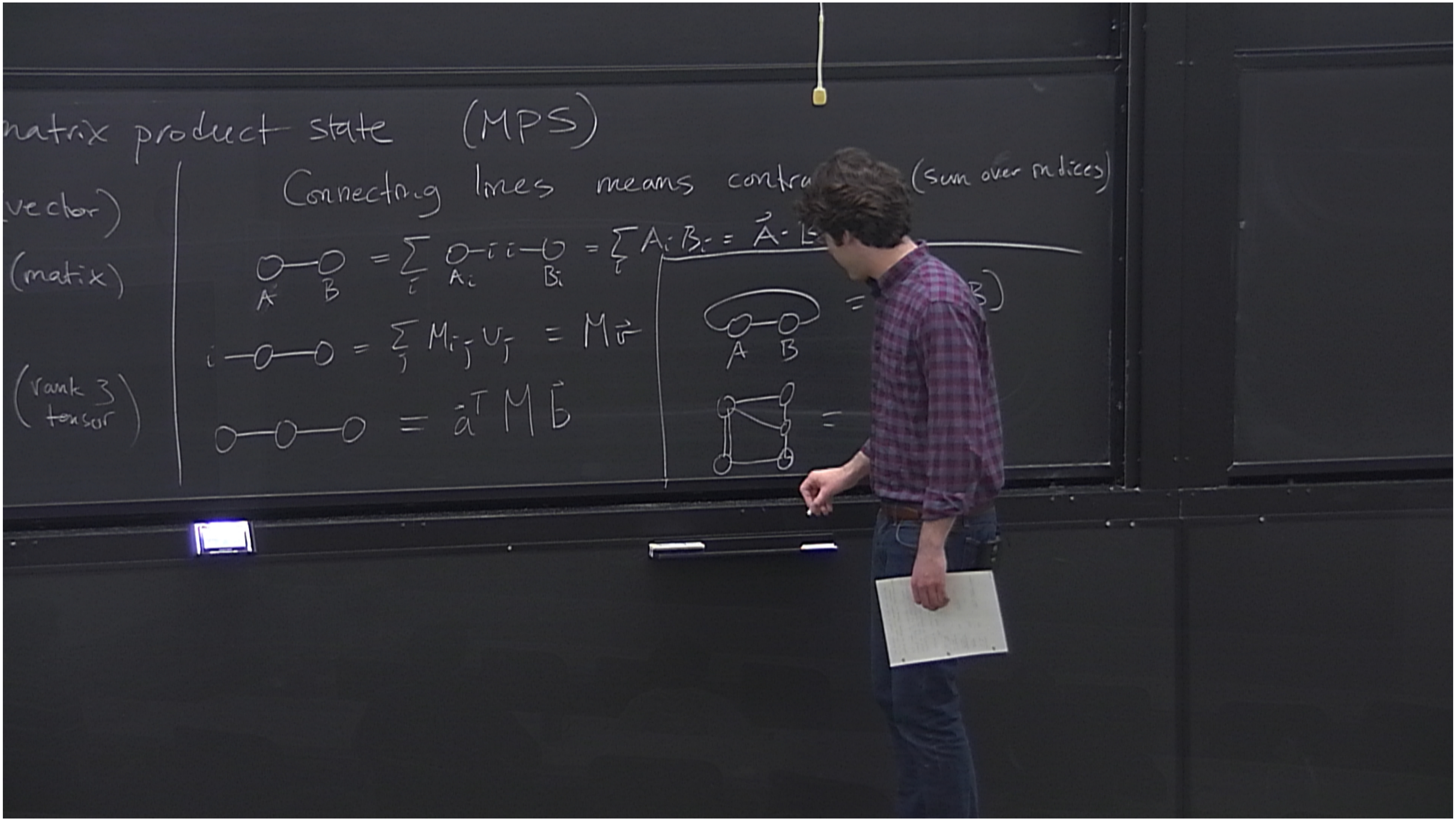
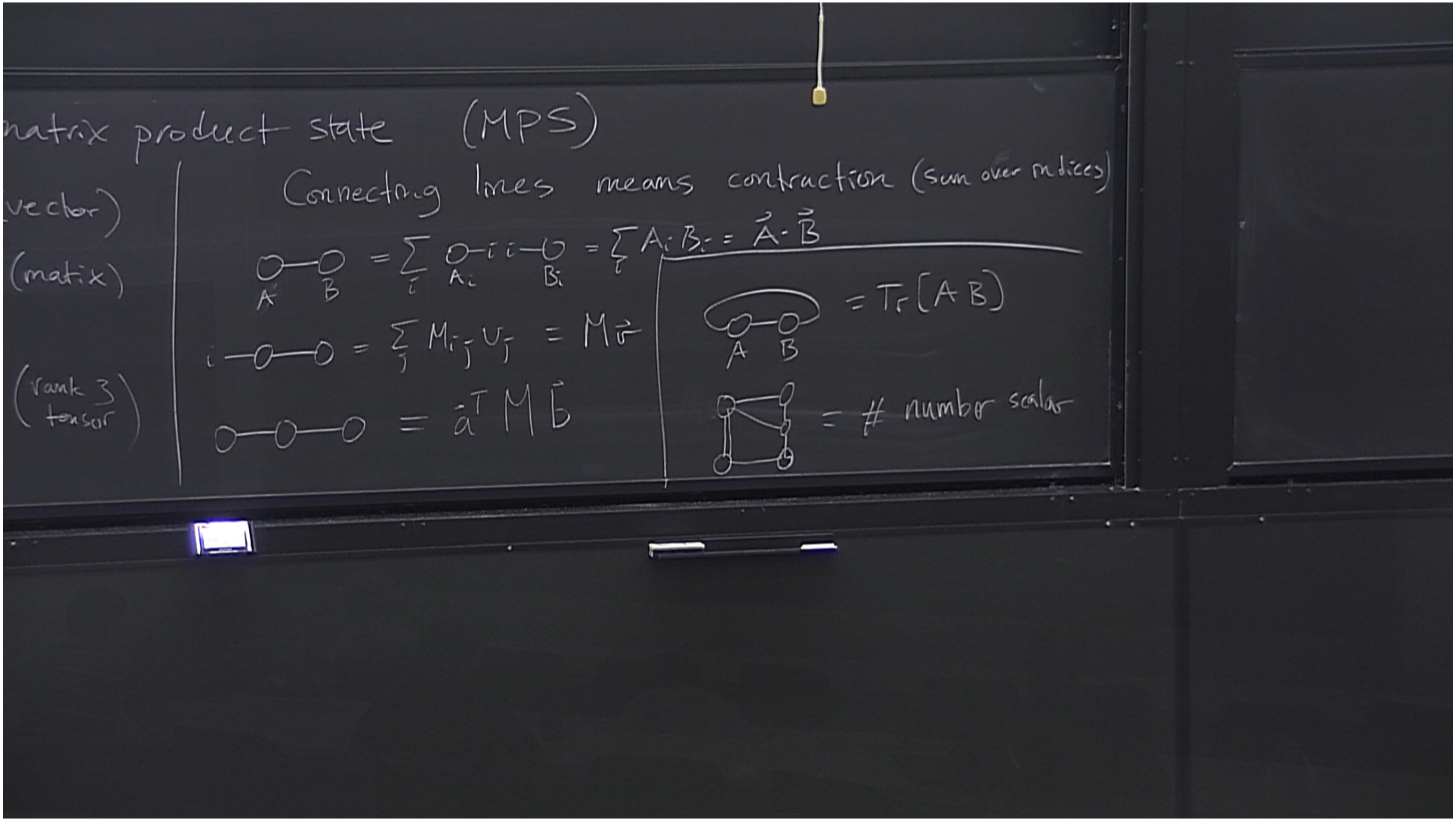
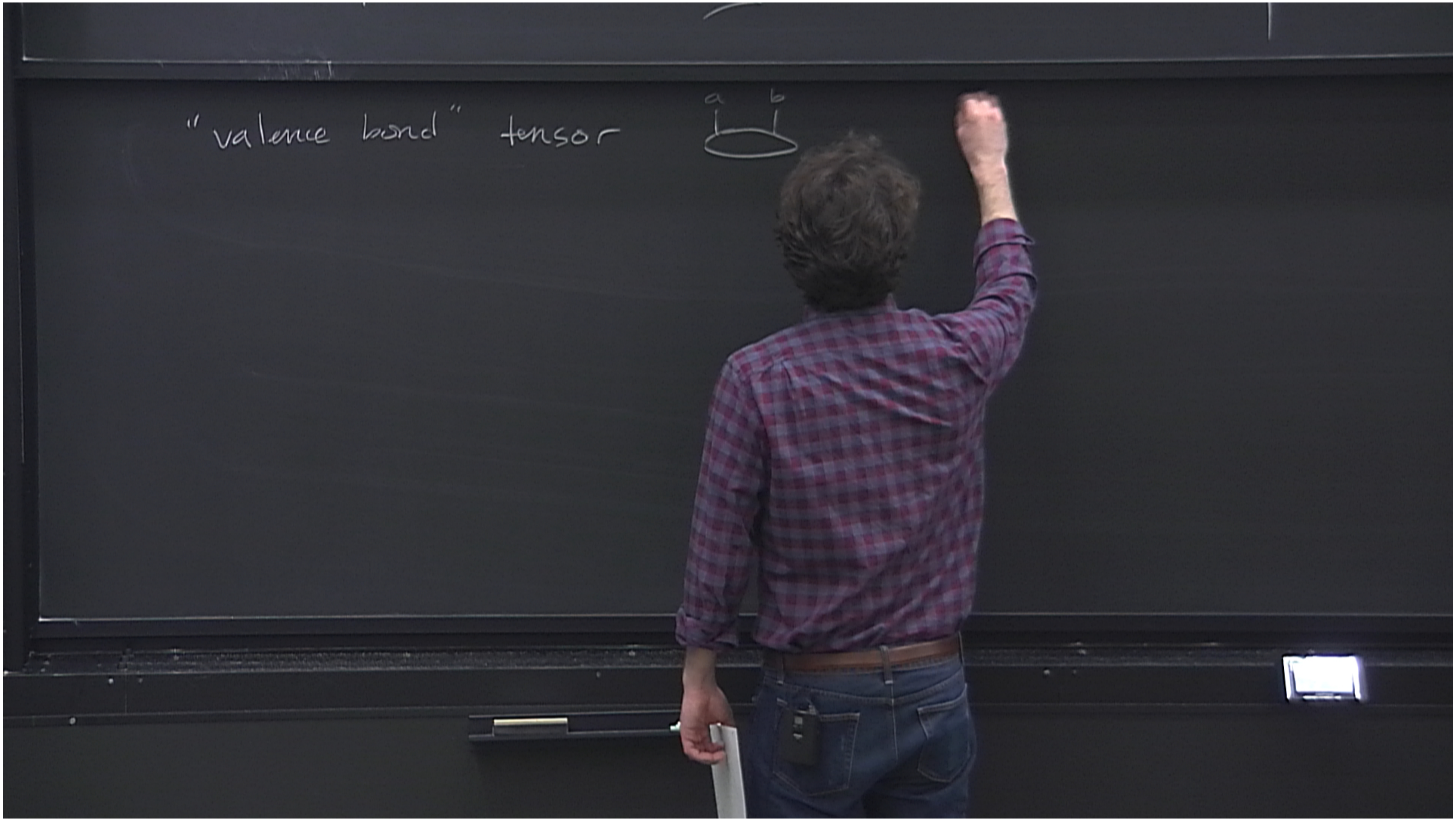
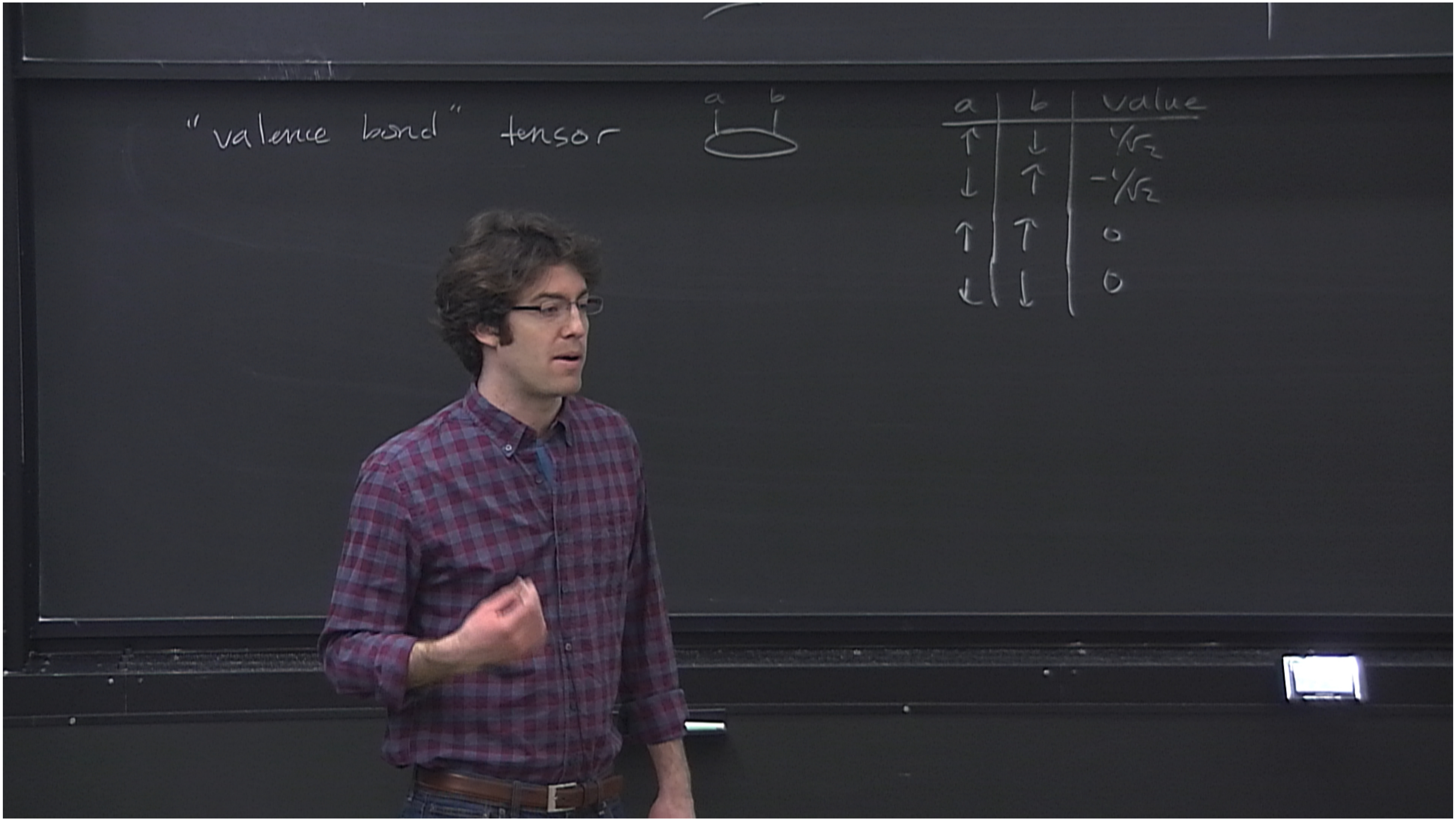
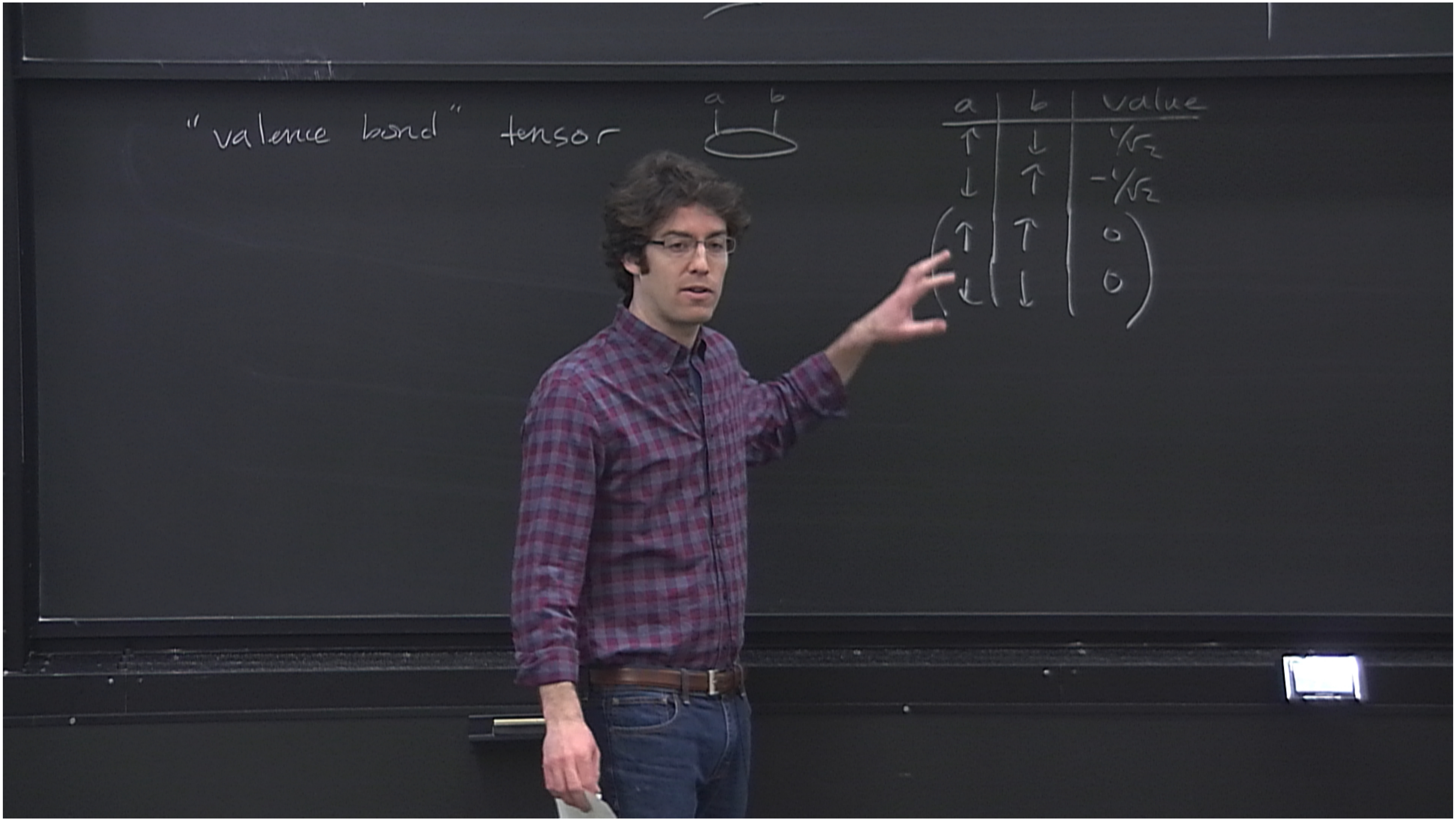
rank 3 tensor
Connecting lines means contracti
over indices)


matrix product state (MPS)
(vector)
(matrix)
(rank 3 tensor)
Connecting lines means contra... (sum over indices)
= Σ A_i B_i = A · B
= Σ_j M_ij v_j = M v
= a^T M b
A B
A_i B_i


matrix product state (MPS)
(vector)
(matrix)
(rank 3 tensor)
Connecting lines means contraction (sum over indices)
A B
A B
= Tr[A B]
= # number scalar


"valence bond" tensor
a b


"valence bond" tensor
a b
a | b | value
↑ ↓ 1/√2
↓ ↑ −1/√2
↑ ↑ 0
↓ ↓ 0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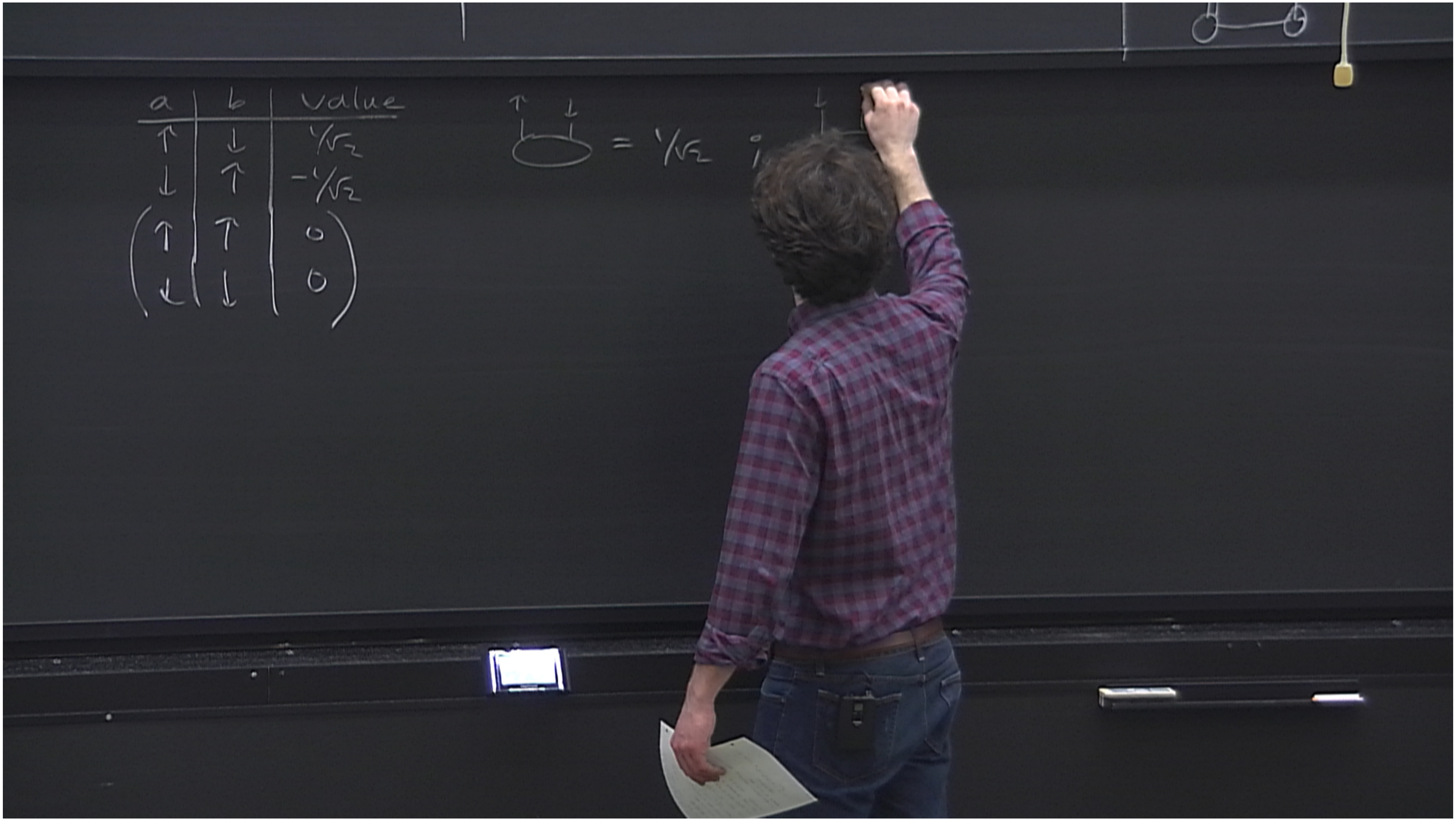
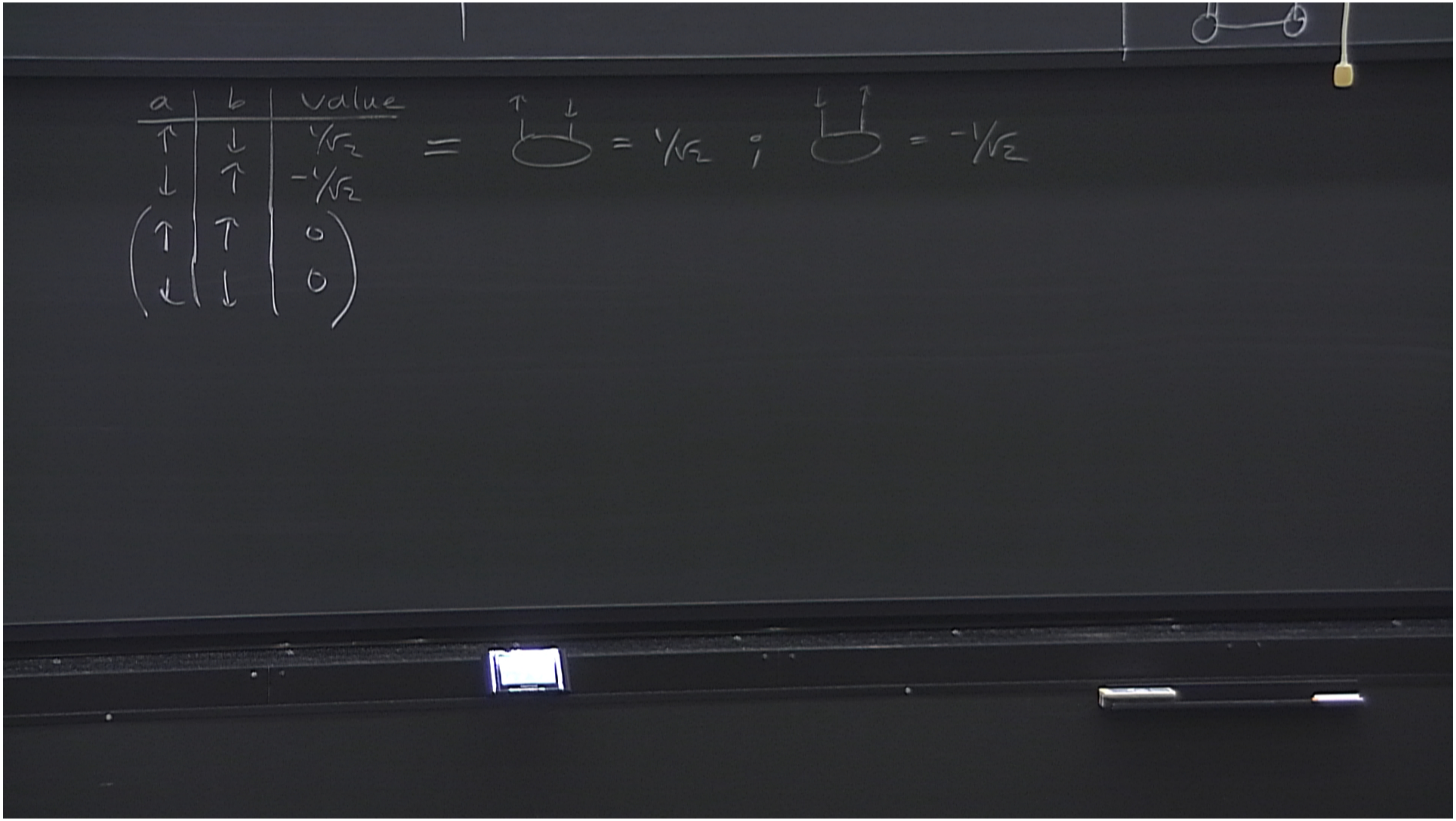
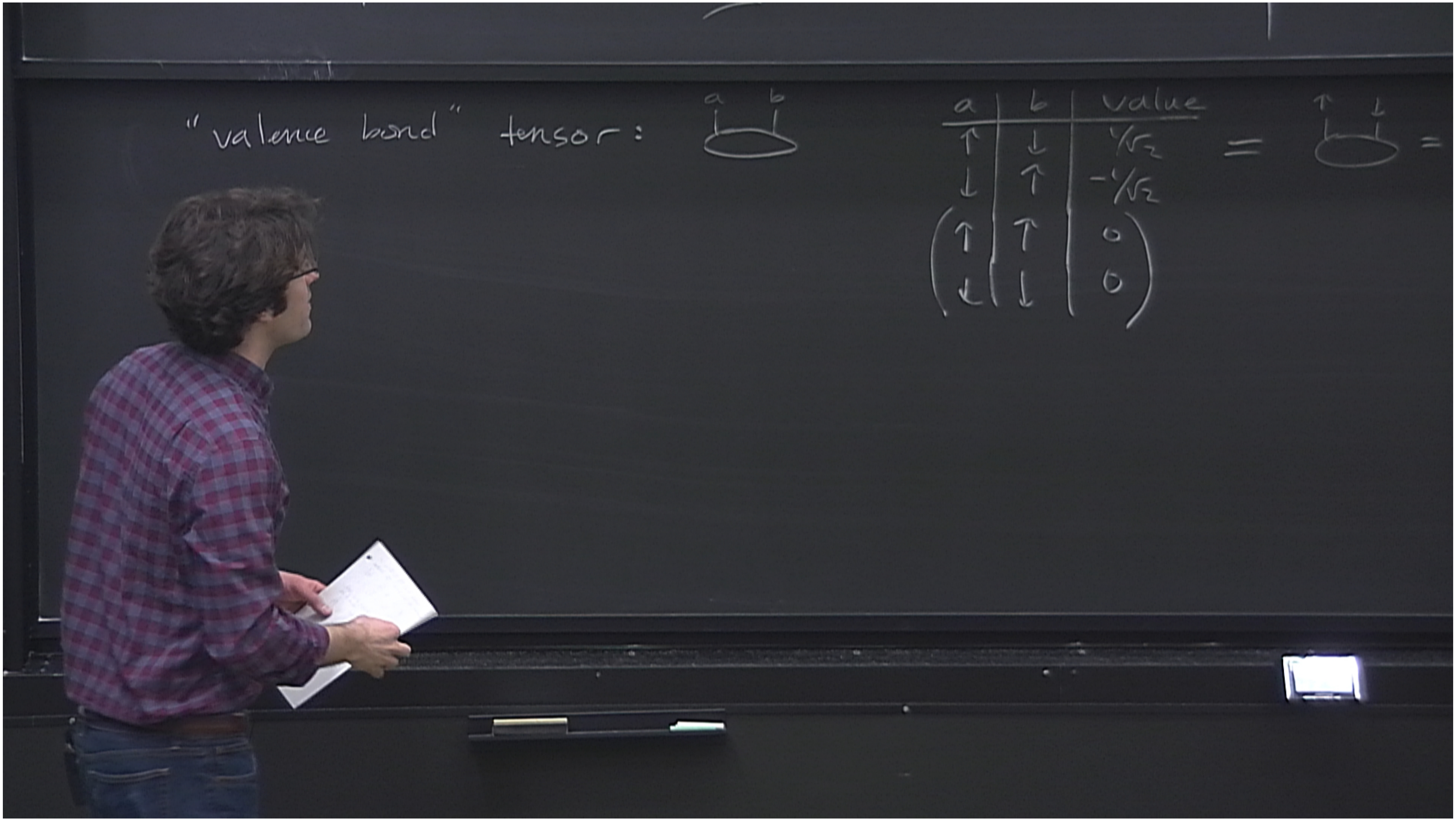
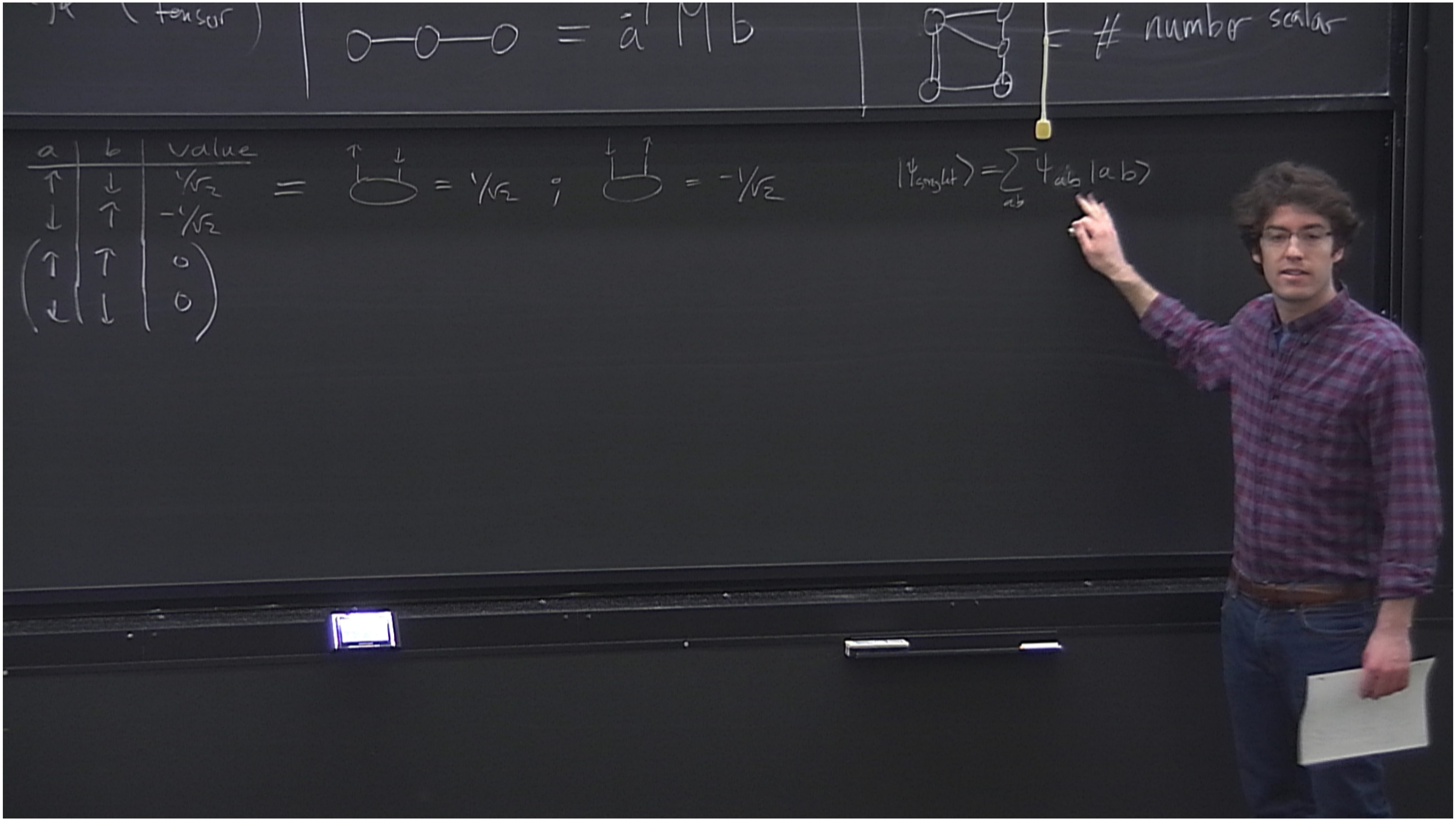
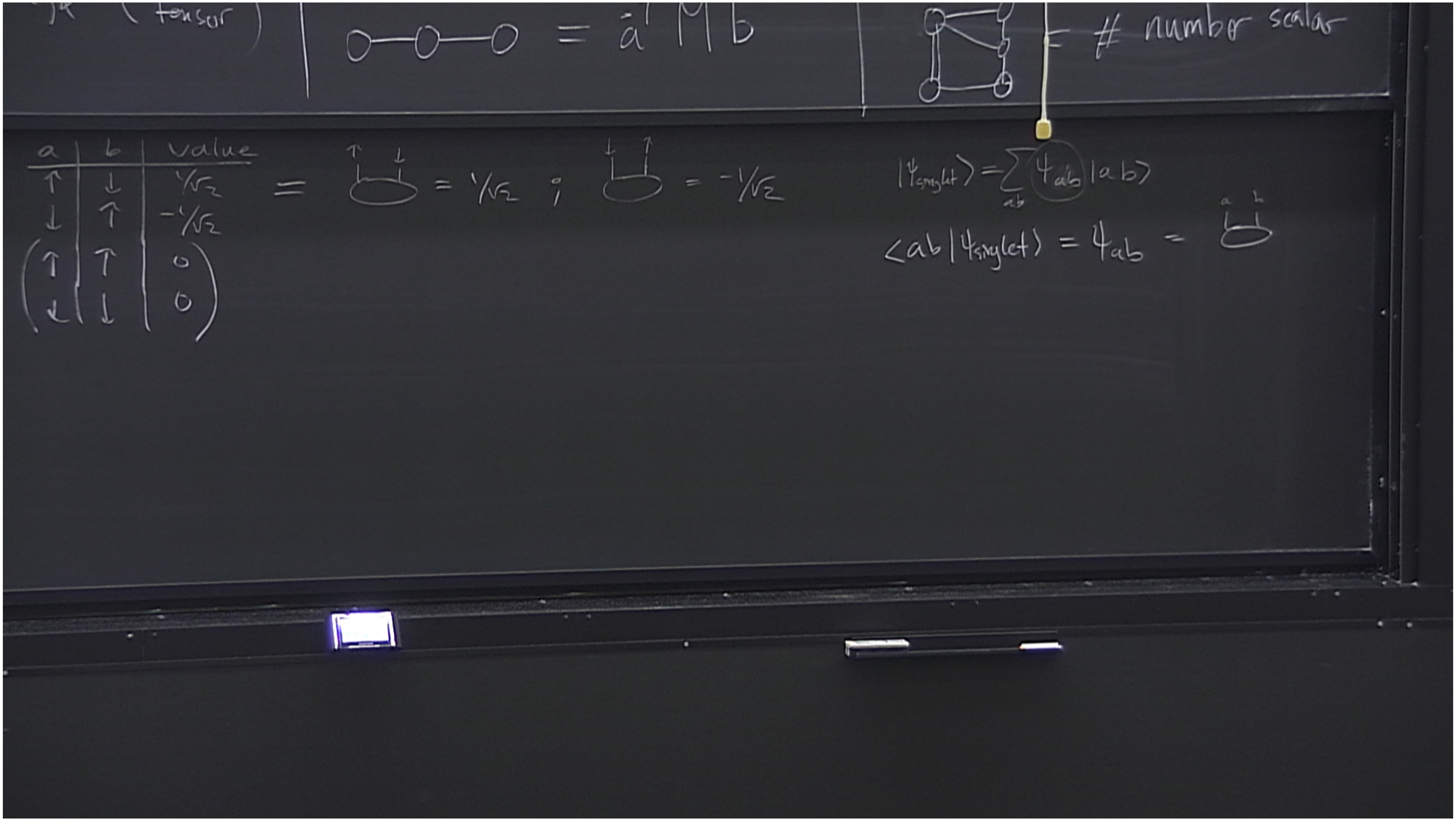

a | b | value
↑ | ↓ | 1/√2
↓ | ↑ | −1/√2
↑ | ↑ | 0
↓ | ↓ | 0
= 1/√2

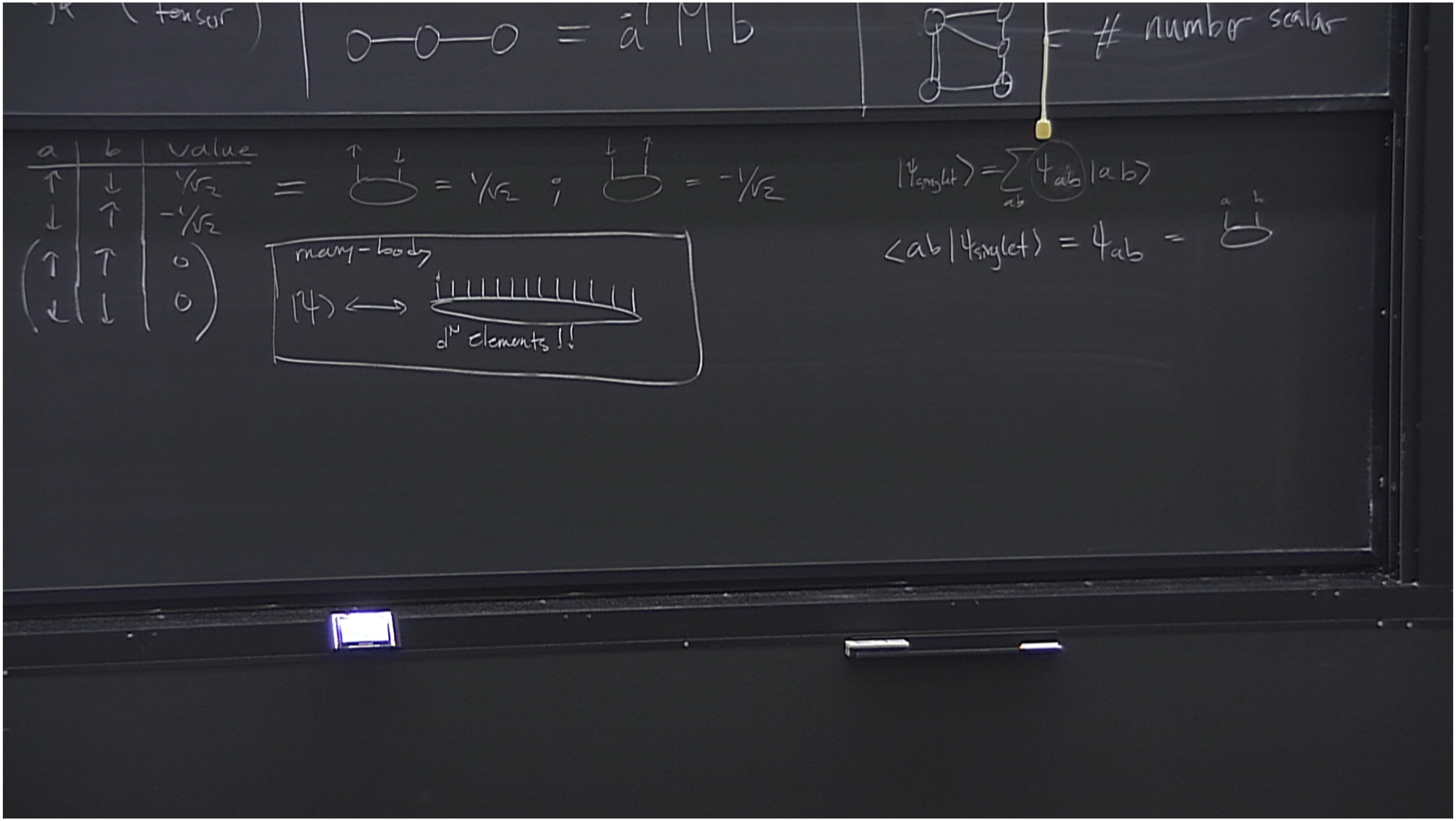

a | b | value
↑ | ↓ | 1/√2
↓ | ↑ | −1/√2
↑ | ↑ | 0
↓ | ↓ | 0
=
↑ ↓
= 1/√2 ;
↓ ↑
= −1/√2

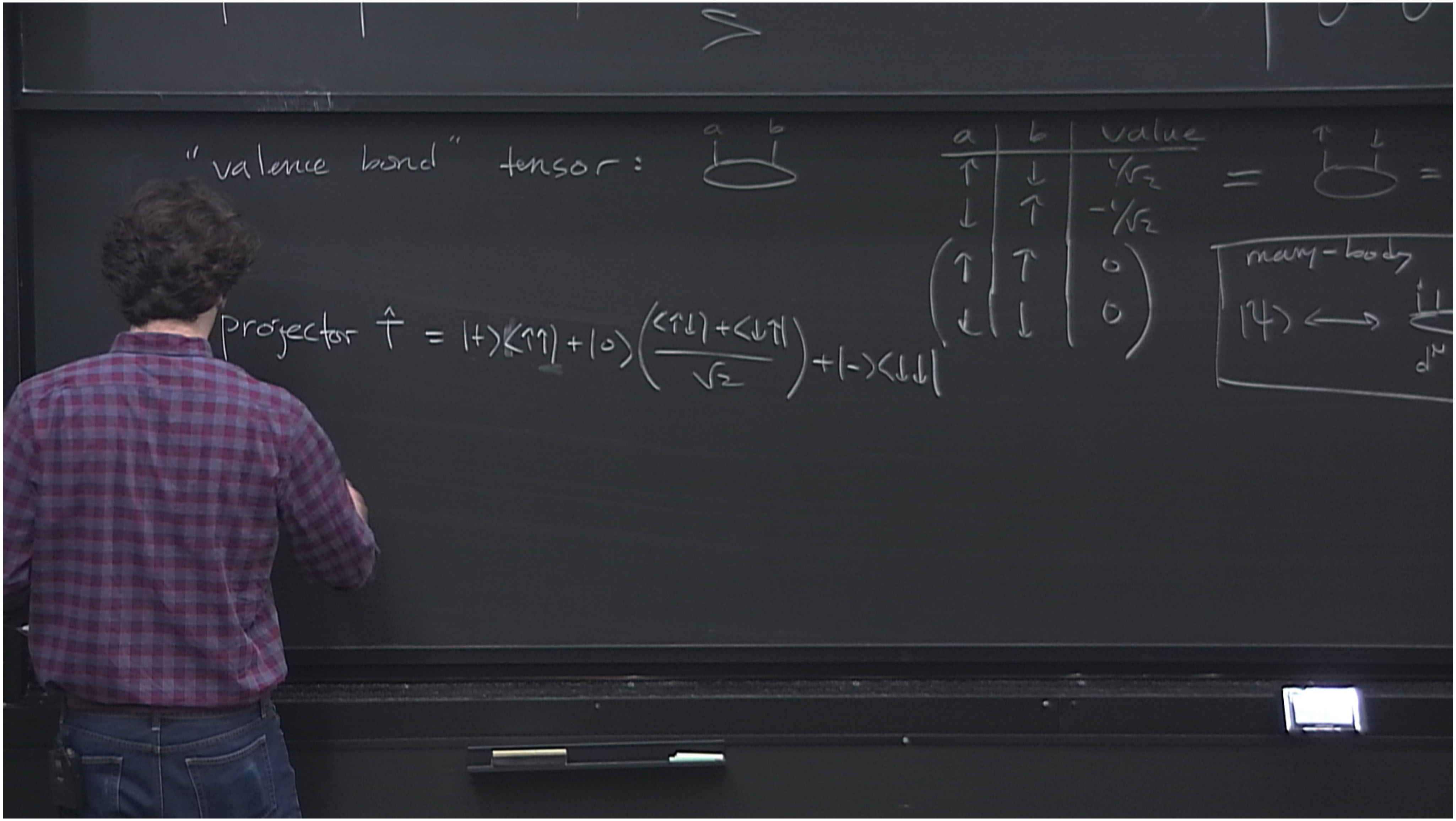

"valence bond" tensor:
a b
a | b | value
1/√2
-1/√2
0
0

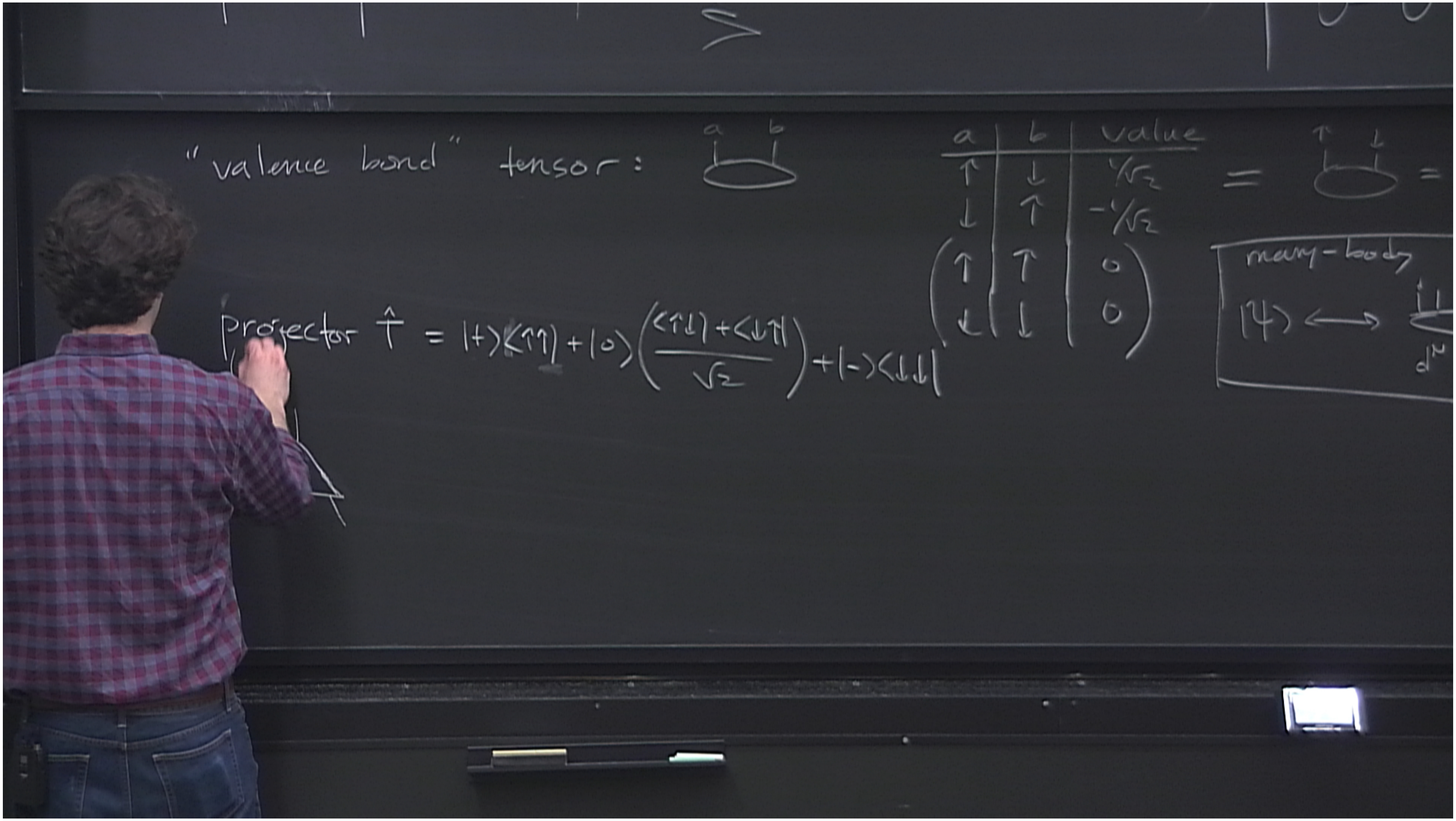

tensor
= a M b
number scalar
a | b | value
↑ | ↓ | 1/√2
↓ | ↑ | -1/√2
↑ | ↑ | 0
↓ | ↓ | 0
= 1/√2 ; = -1/√2
|ψsinglet⟩ = Σab ψab |ab⟩

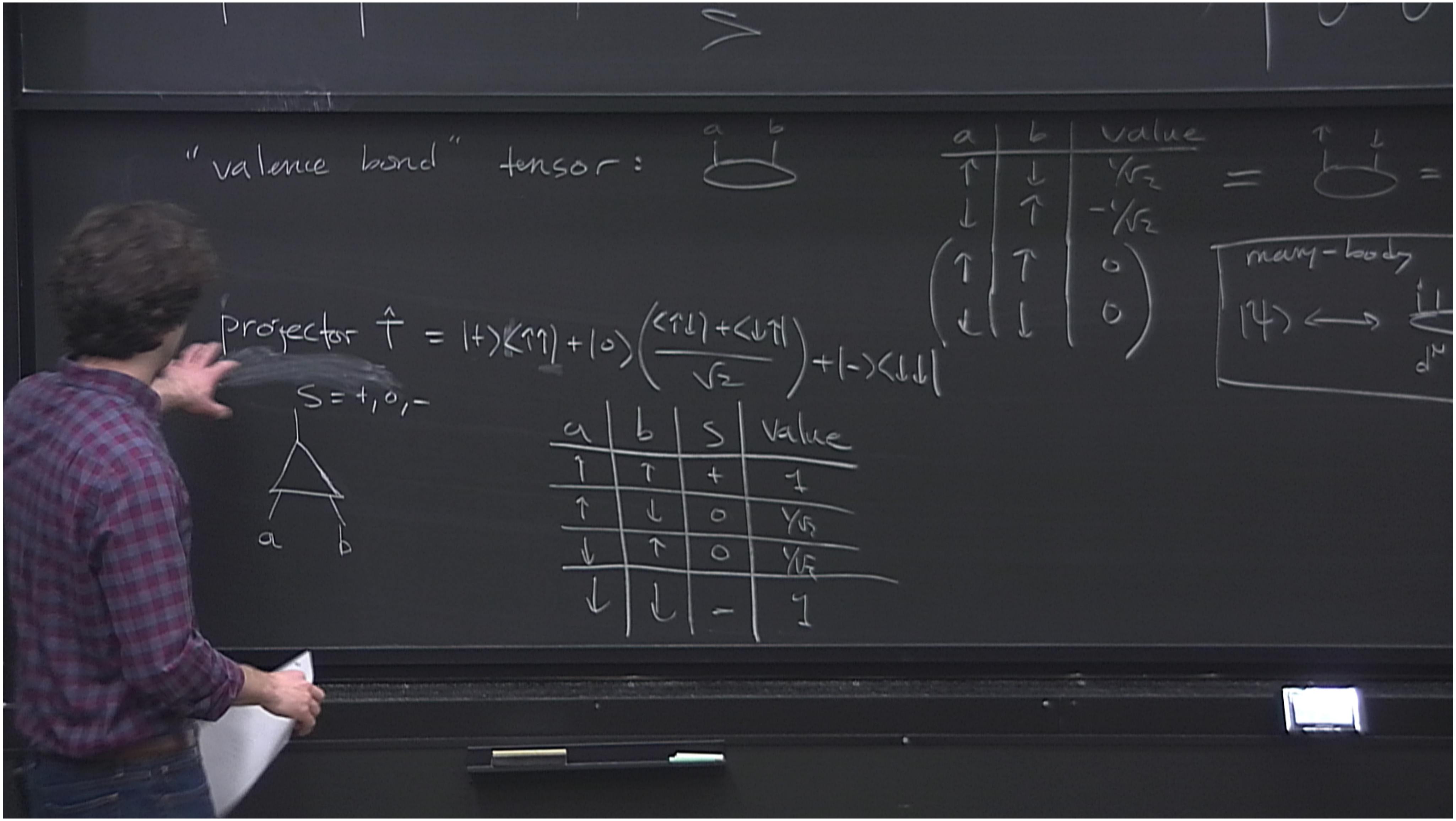


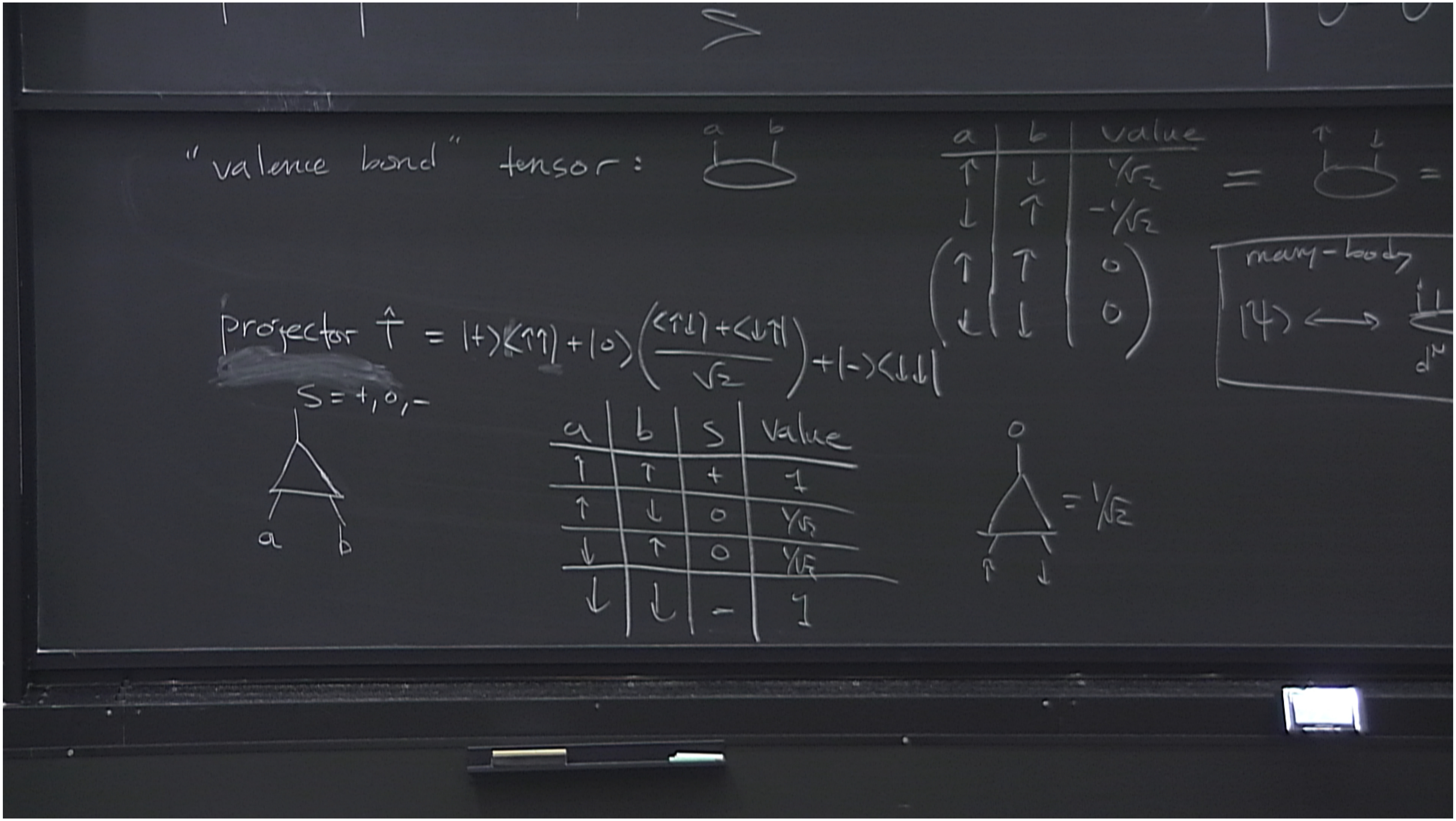
(tensor)
= a M b
number scalar
a | b | value
= 1/√2 ; = -1/√2
|ψsinglet⟩ = ∑ψab |ab⟩
⟨ab|ψsinglet⟩ = ψab =

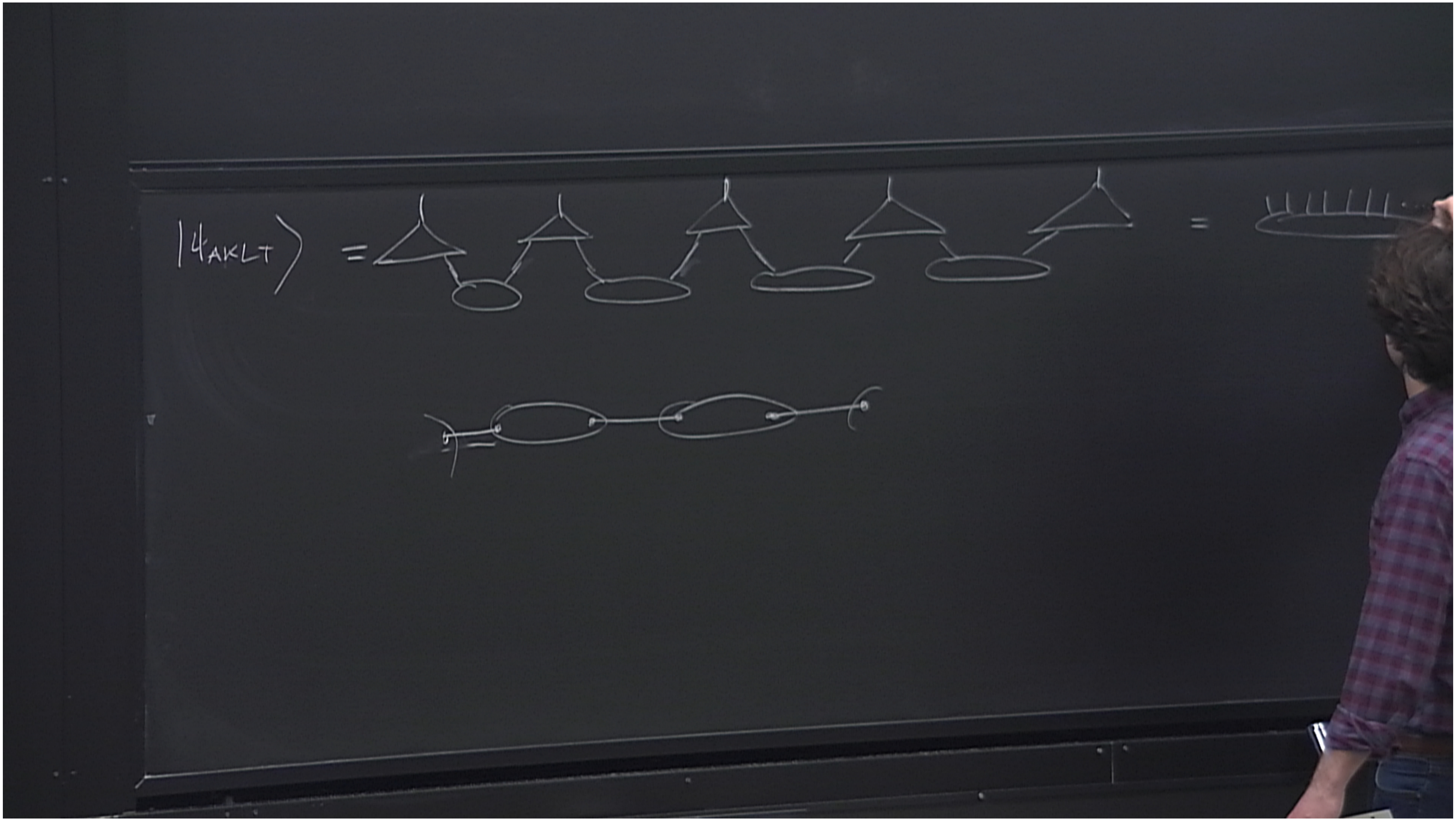

tensor
= a† M b
number scalar
a | b | value
= 1/√2 ; = -1/√2
many-body
d^N elements !!
|ψsinglet> = Σ ψab |ab>
<ab|ψsinglet> = ψab =

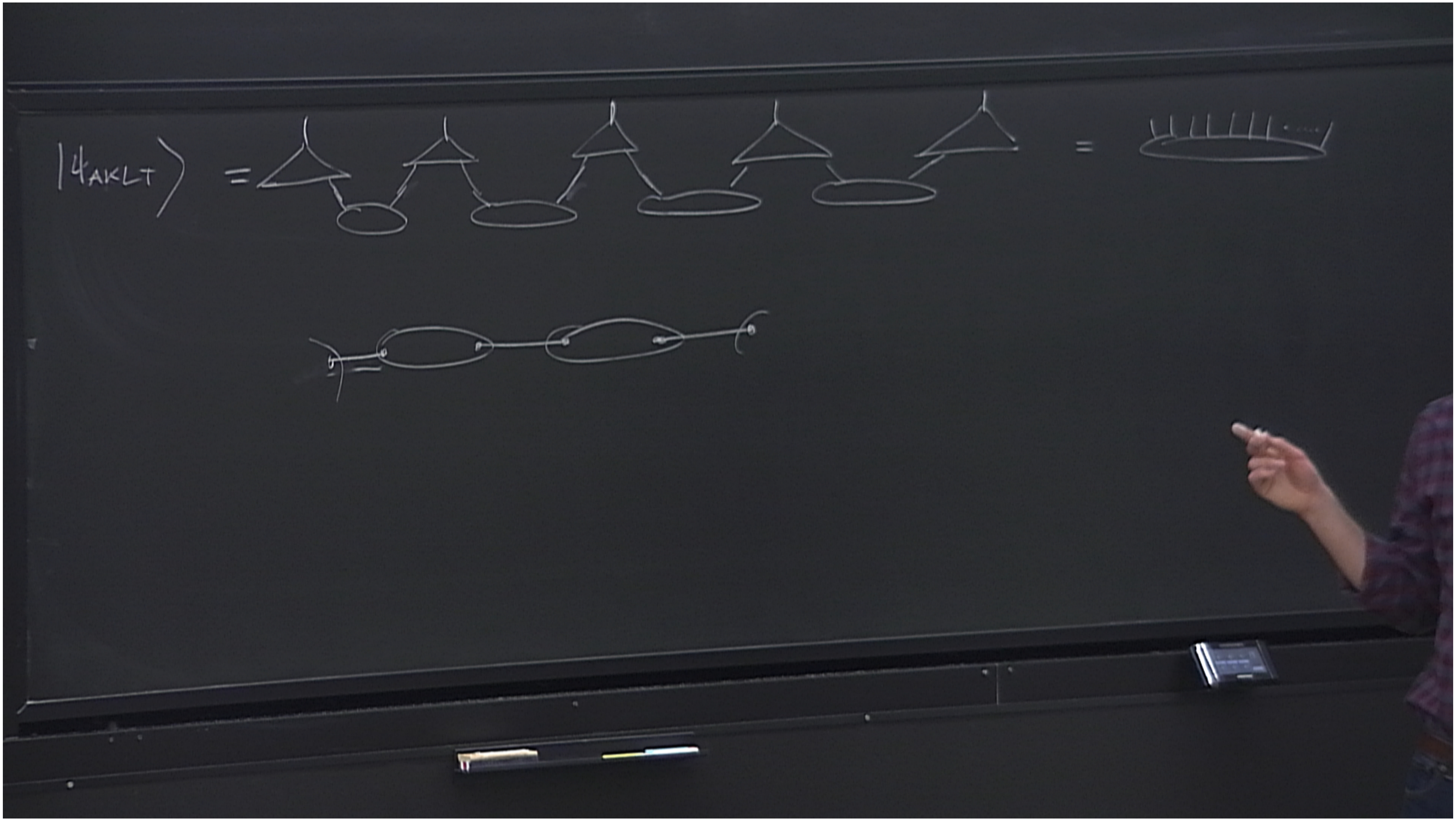

"valence bond" tensor:
a b value
↑ ↓ 1/√2
↓ ↑ -1/√2
↑ ↑ 0
↓ ↓ 0
many-body
|ψ⟩ ⟷
projector T̂ = |+⟩⟨↑↑| + |0⟩(⟨↑↓| + ⟨↓↑| / √2) + |-⟩⟨↓↓|

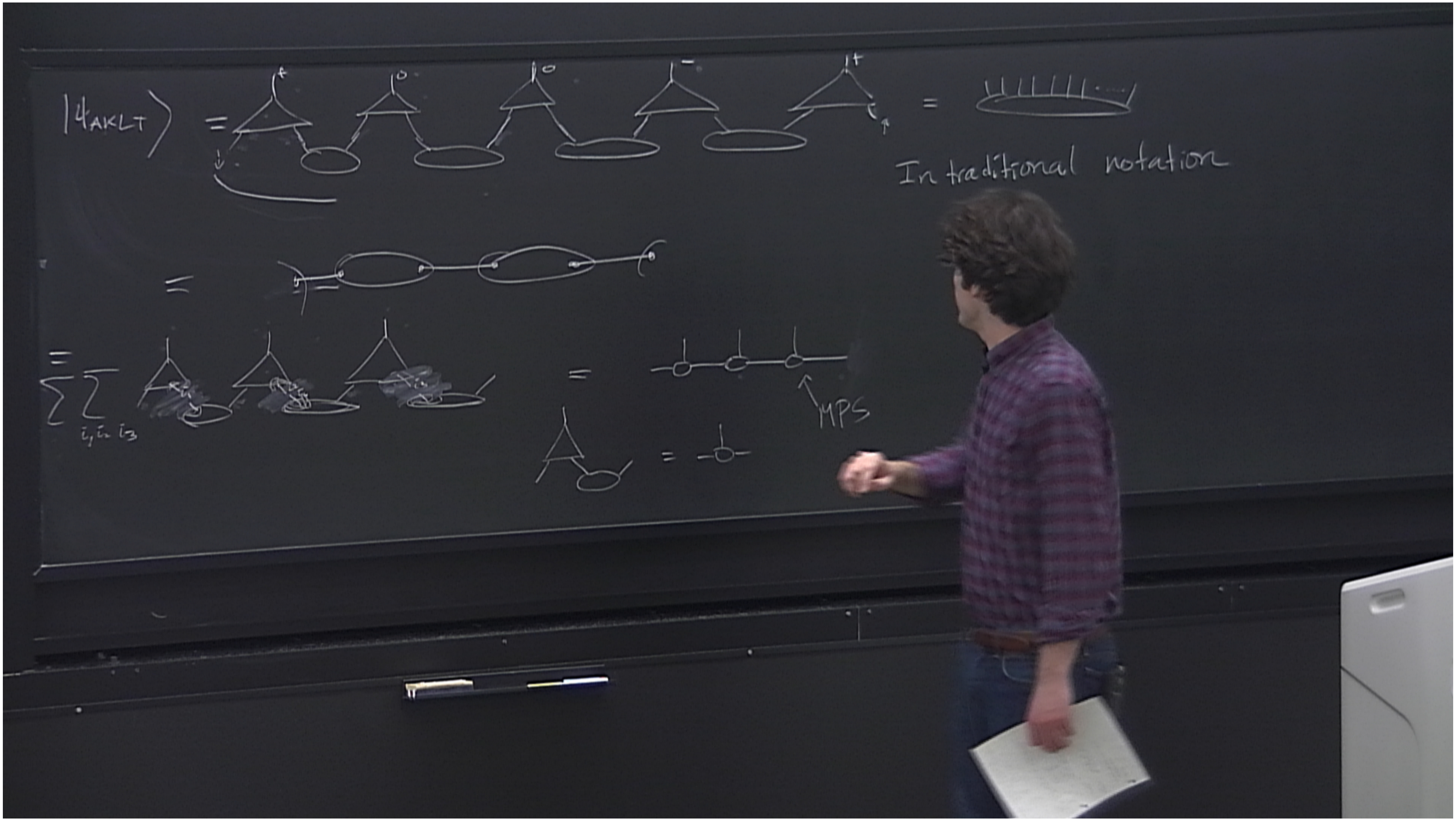

"valence bond" tensor:
a b value
↑ ↓ 1/√2
↓ ↑ -1/√2
↑ ↑ 0
↓ ↓ 0
many-body
|ψ⟩ ⟷
projector T̂ = |+⟩⟨↑↑| + |0⟩(⟨↑↓| + ⟨↓↑| / √2) + |-⟩⟨↓↓|

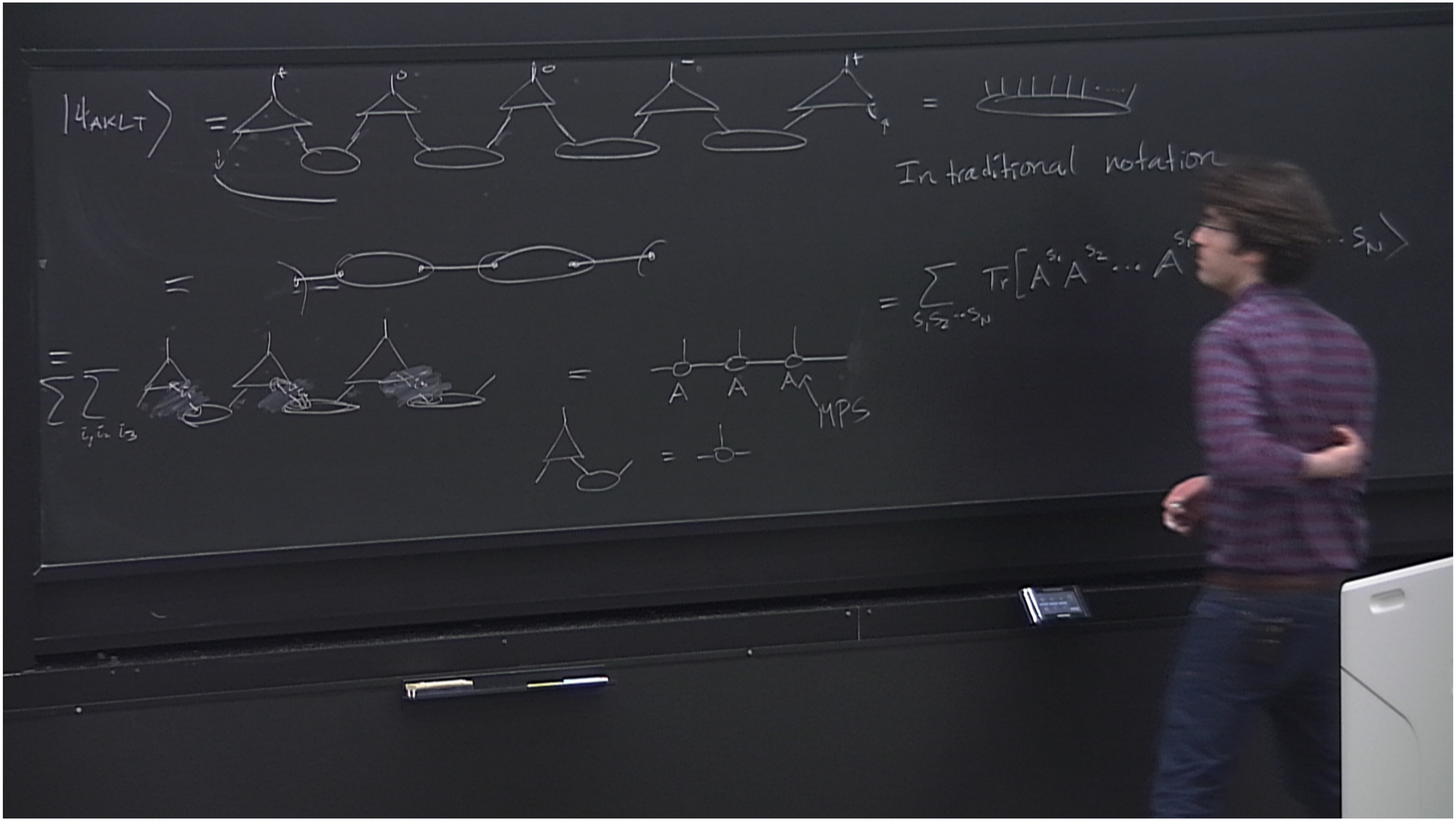

"valence bond" tensor:
a b
a | b | value
↑ | ↓ | 1/√2
↓ | ↑ | −1/√2
↑ | ↑ | 0
↓ | ↓ | 0
many-body
|ψ⟩ ⟷
projector T̂ = |+⟩⟨↑↑| + |0⟩(⟨↑↓| + ⟨↓↑| / √2) + |−⟩⟨↓↓|
s = +, 0, −
a b
a | b | s | value
↑ | ↑ | + | 1
↑ | ↓ | 0 | 1/√2
↓ | ↑ | 0 | 1/√2
↓ | ↓ | − | 1

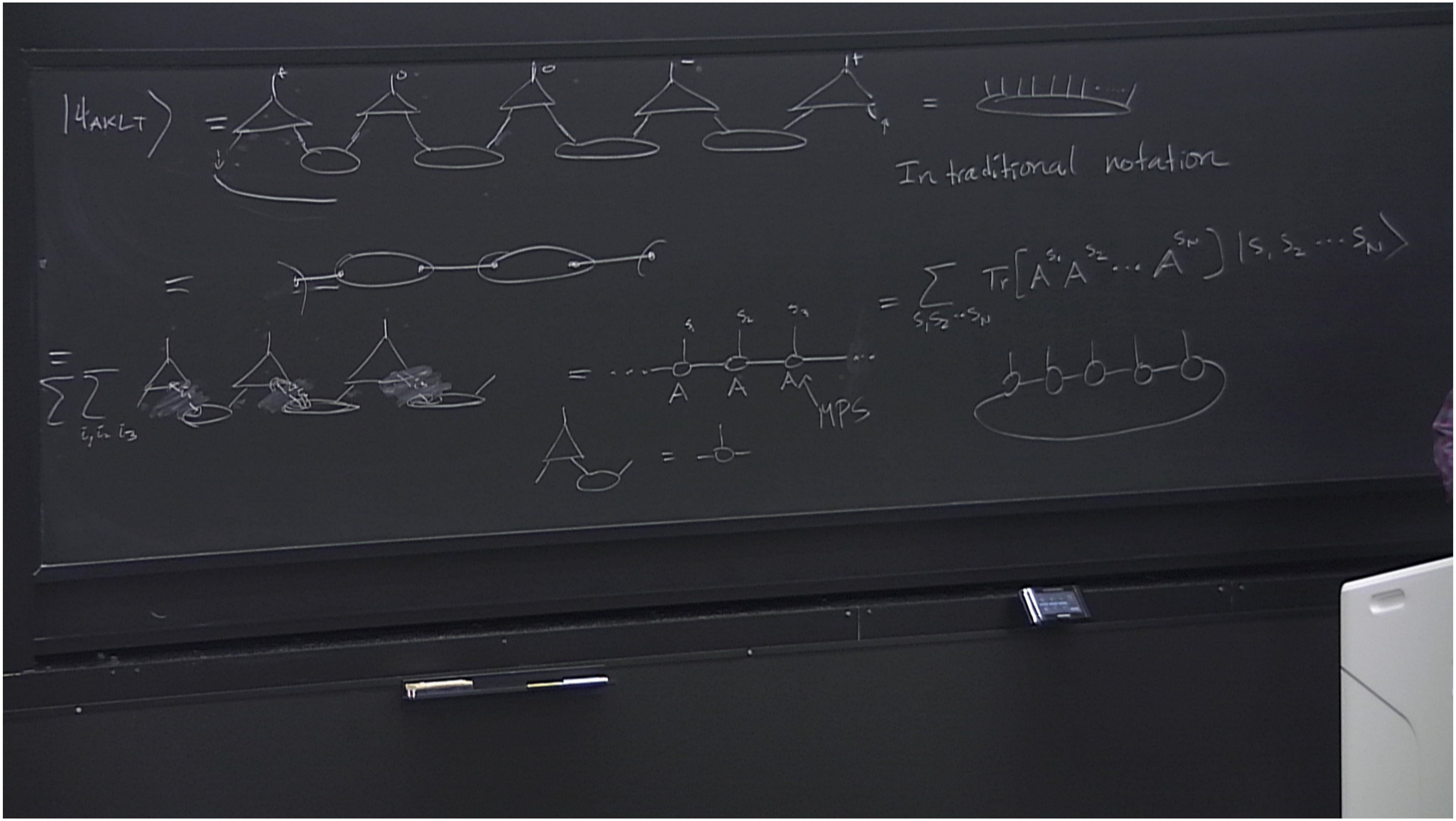

"valence bond" tensor:
projector
S = +, 0, -
value
many-body

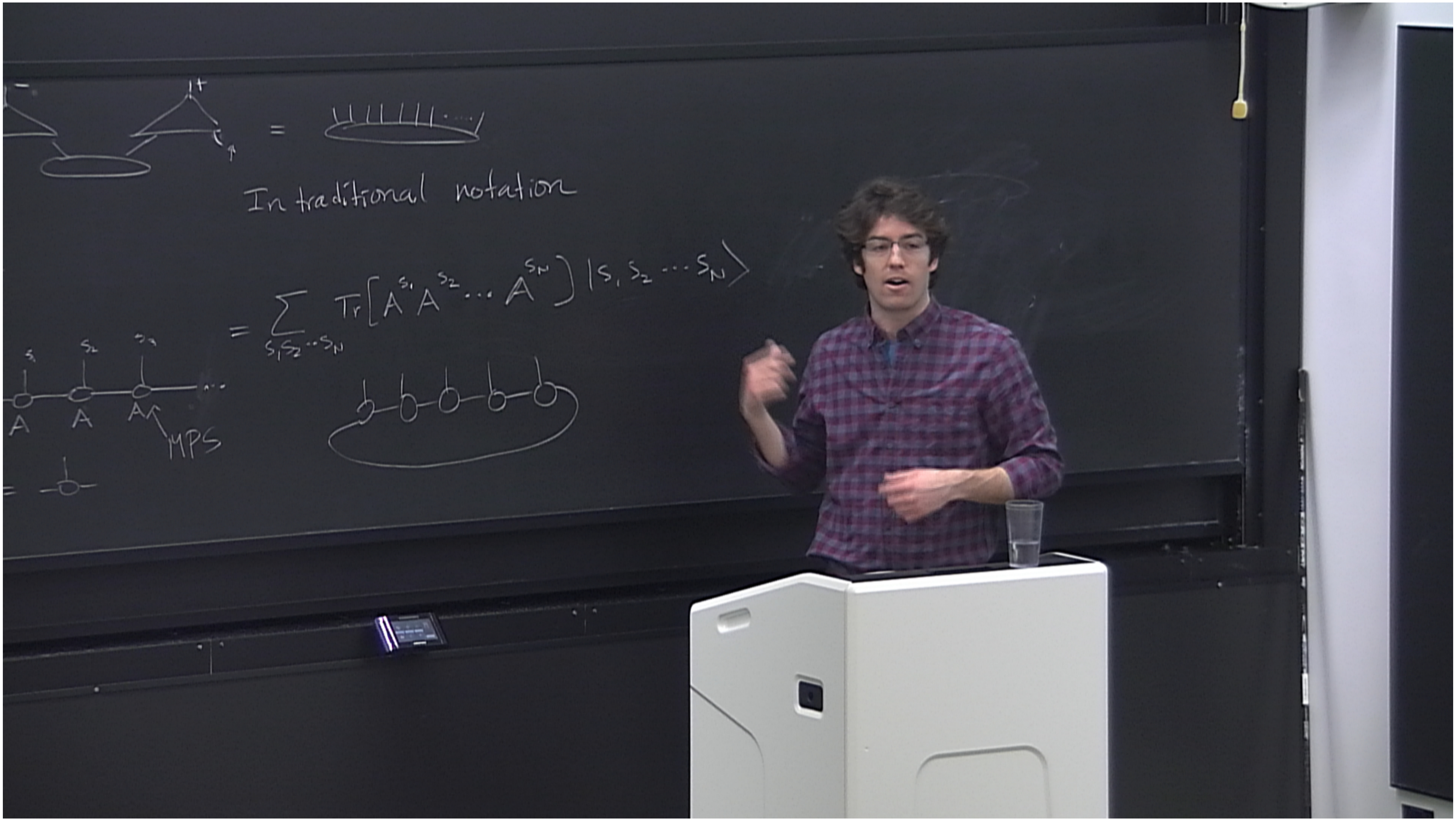

$|\psi_{AKLT}\rangle$ =


$|\psi_{AKLT}\rangle =$
=

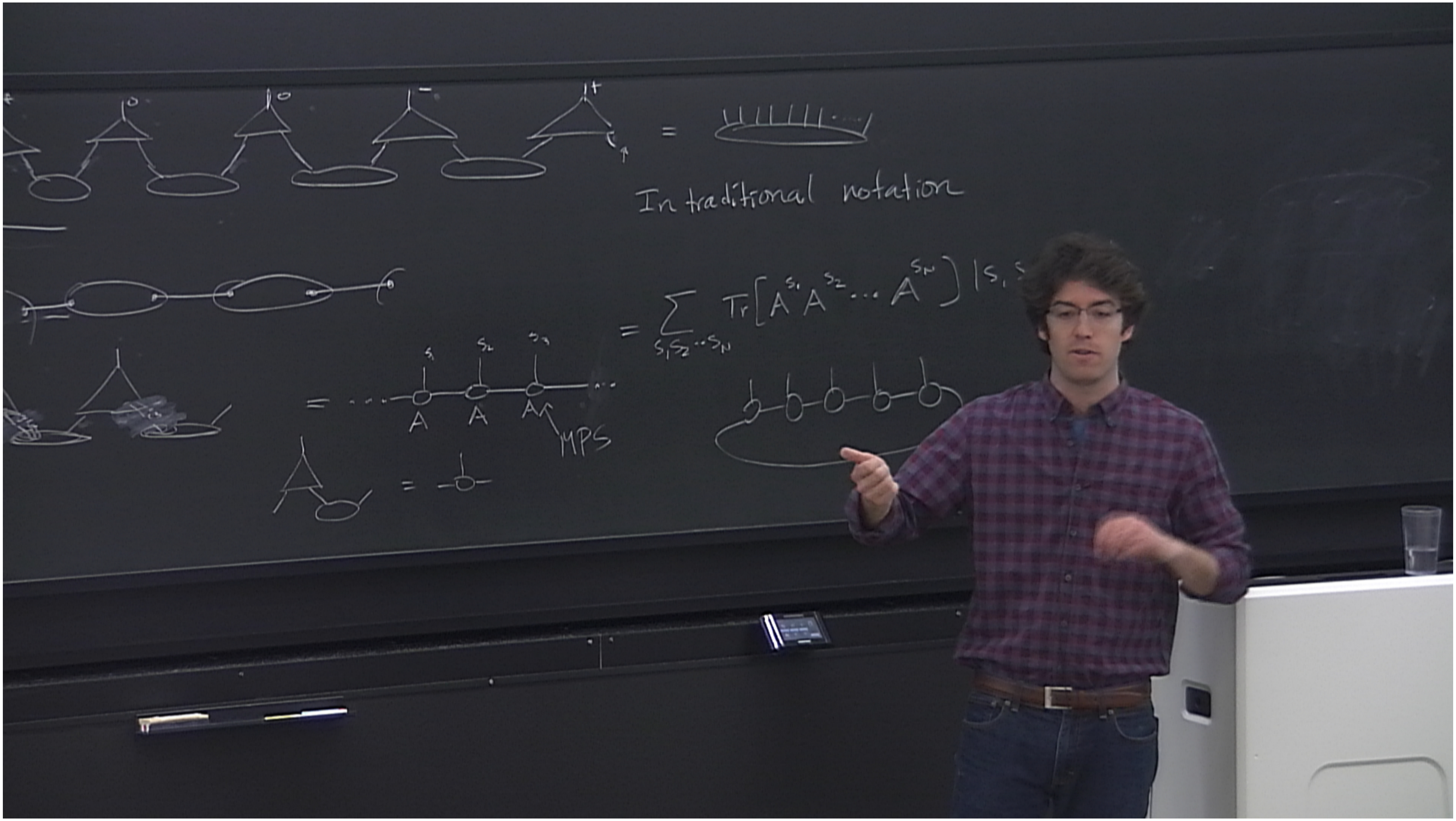

$|\psi_{AKLT}\rangle$ =
In traditional notation
MPS

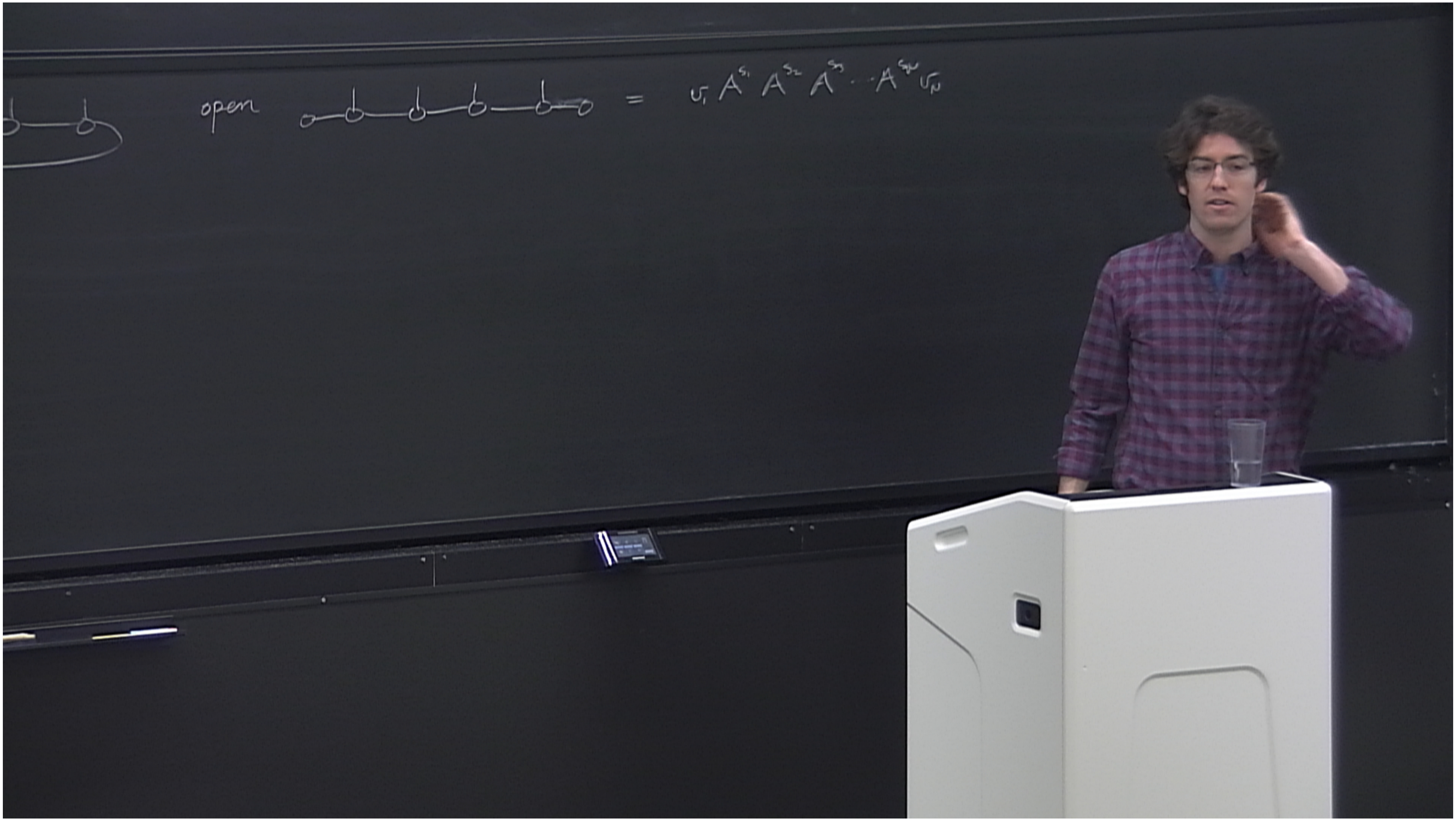

In traditional notation
A A A
MPS

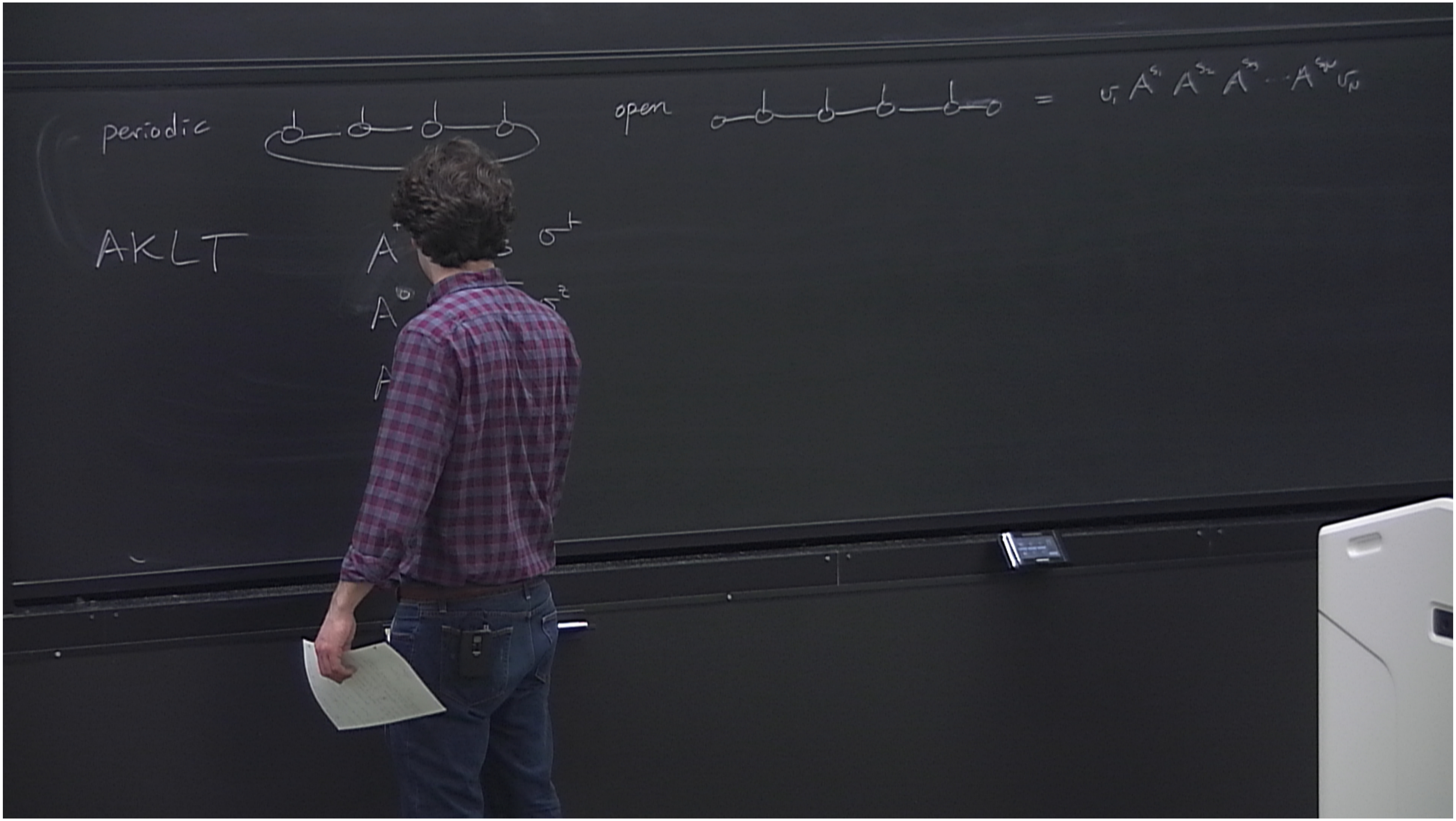

|ψAKLT⟩ =
In traditional notation
= Σ s1 s2 ··· sN Tr[A^s1 A^s2 ··· A^sN] |s1 s2 ··· sN⟩
A A A
MPS

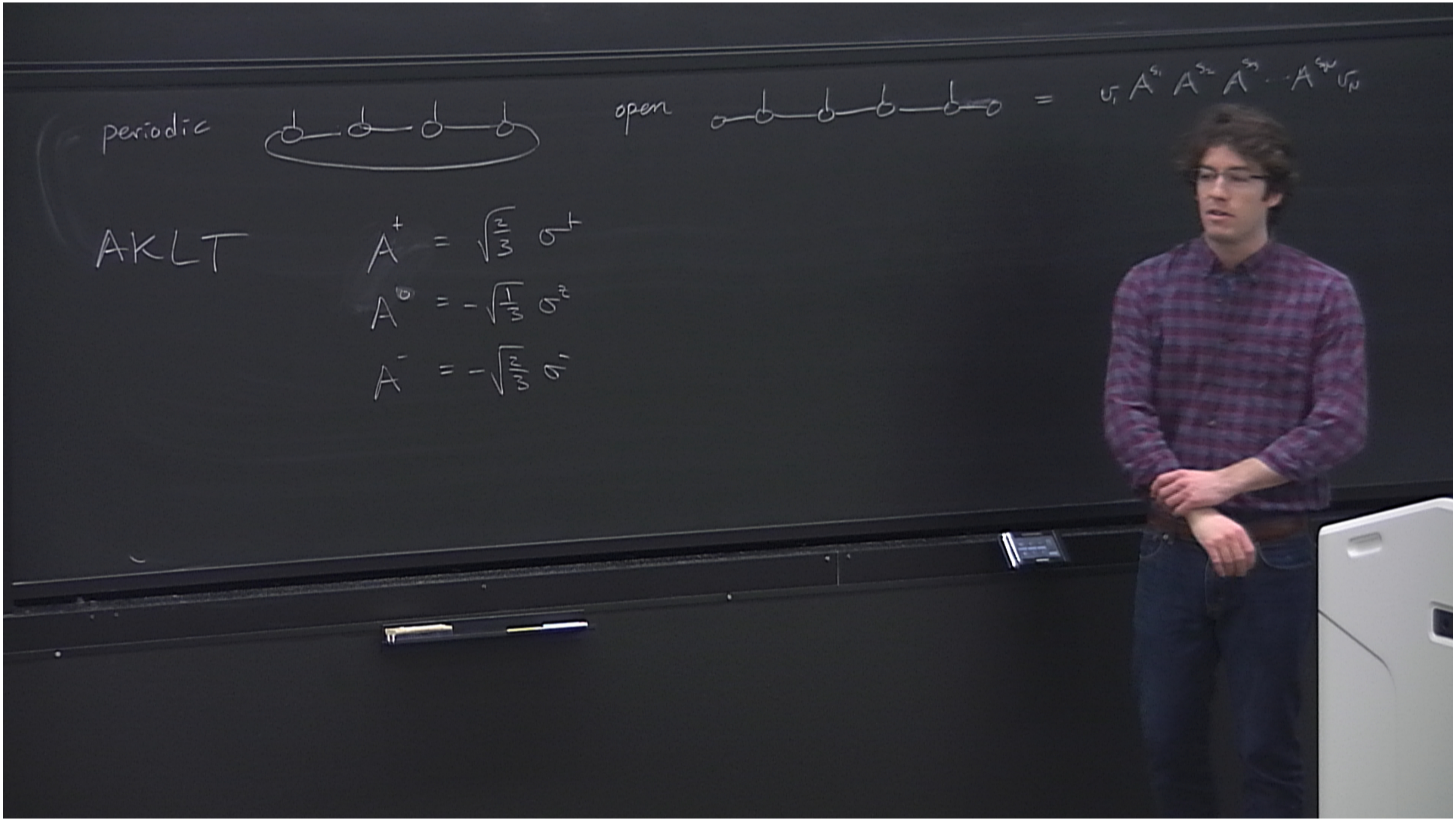


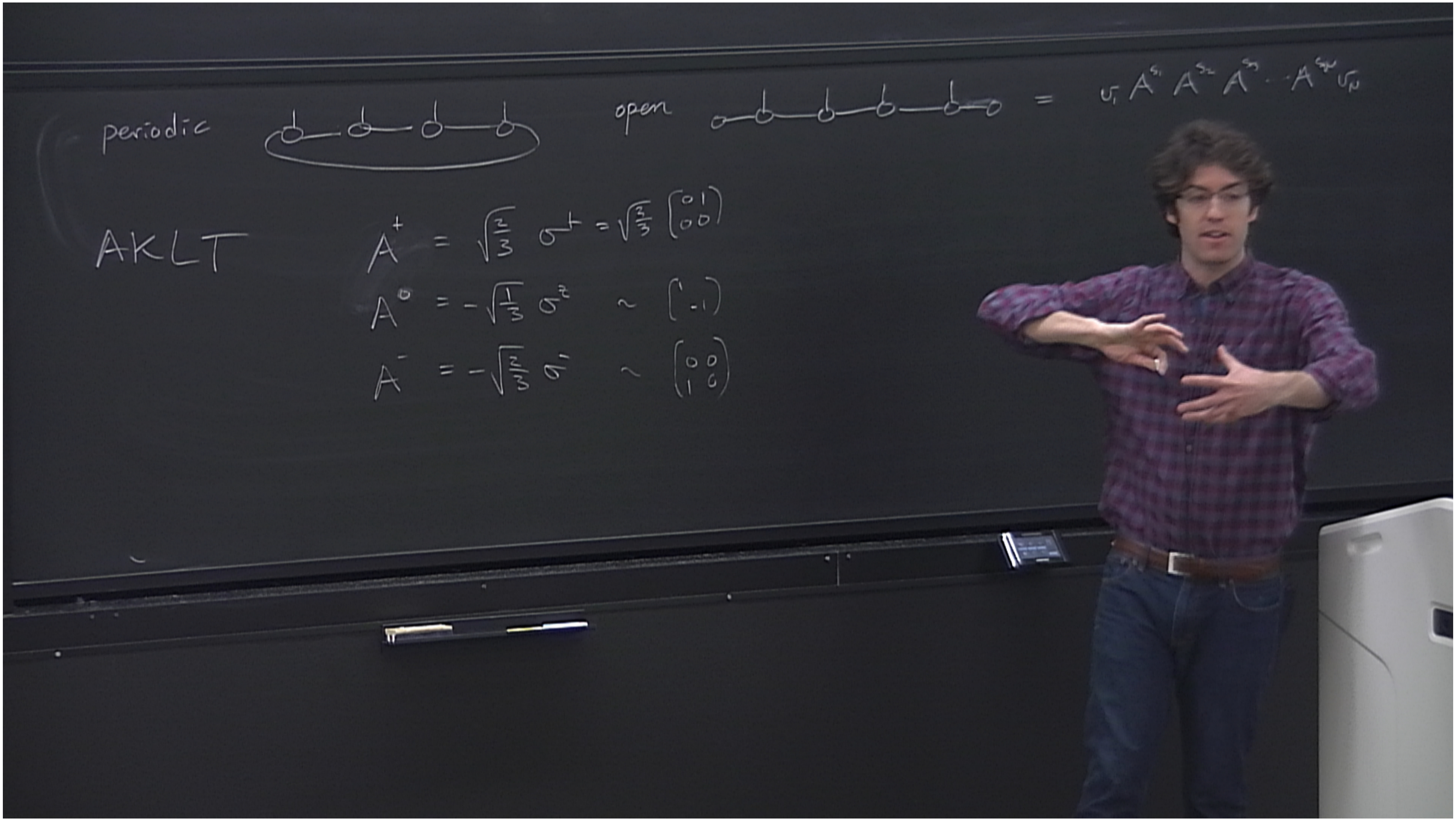
In traditional notation
MPS

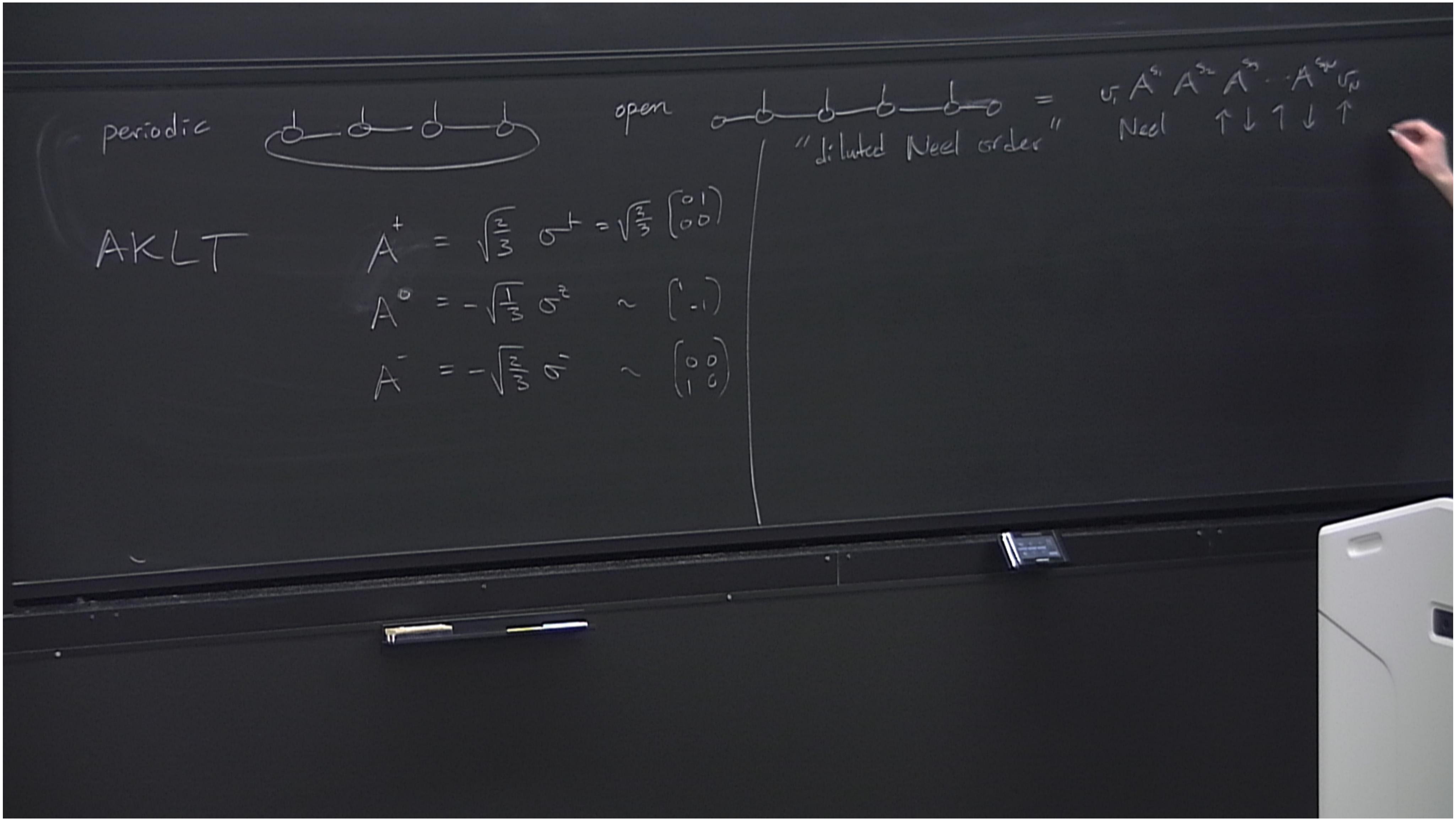

In traditional notation
A A A
MPS

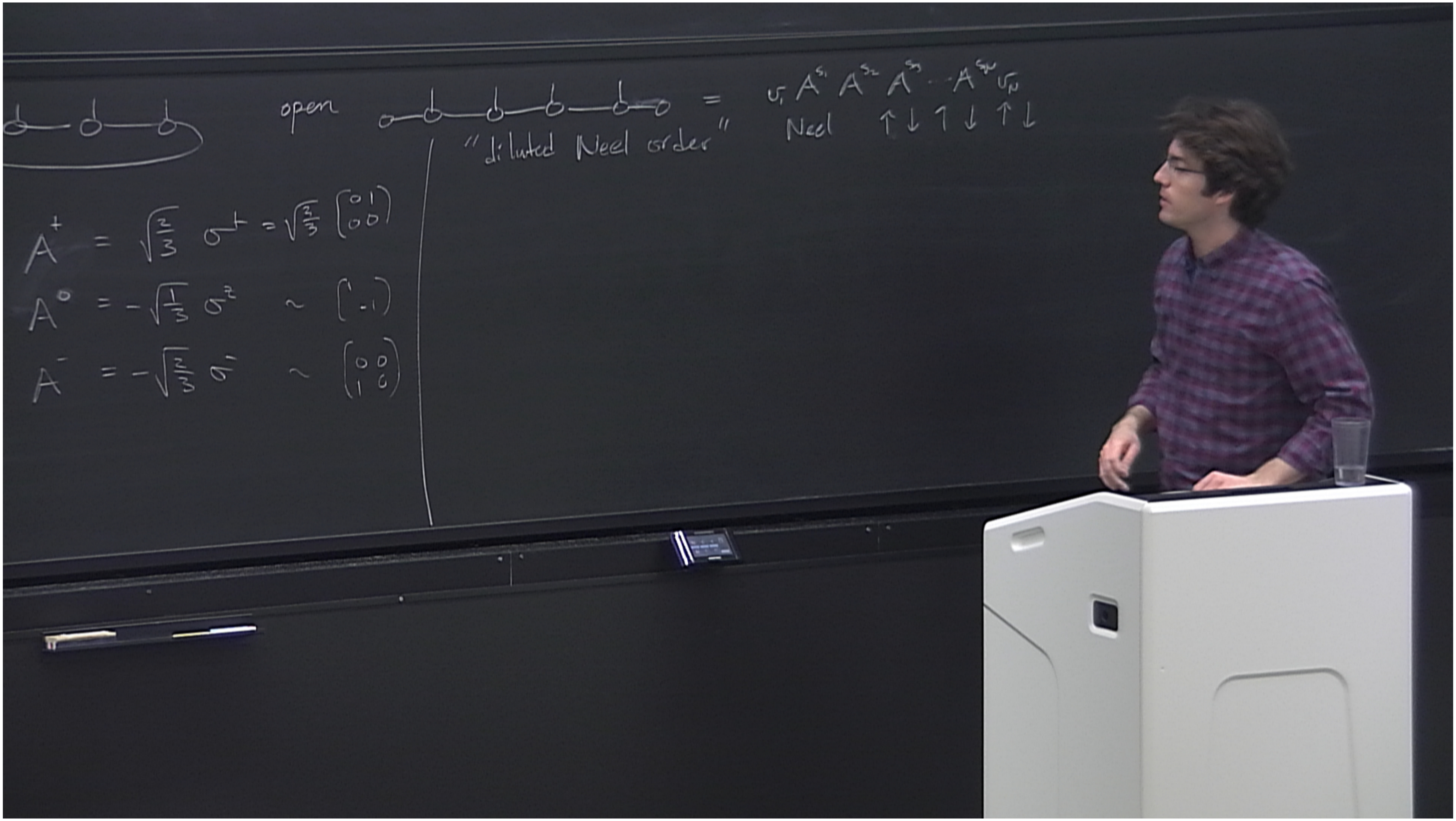

open
= v1 A^s1 A^s2 A^s3 ··· A^sN vN

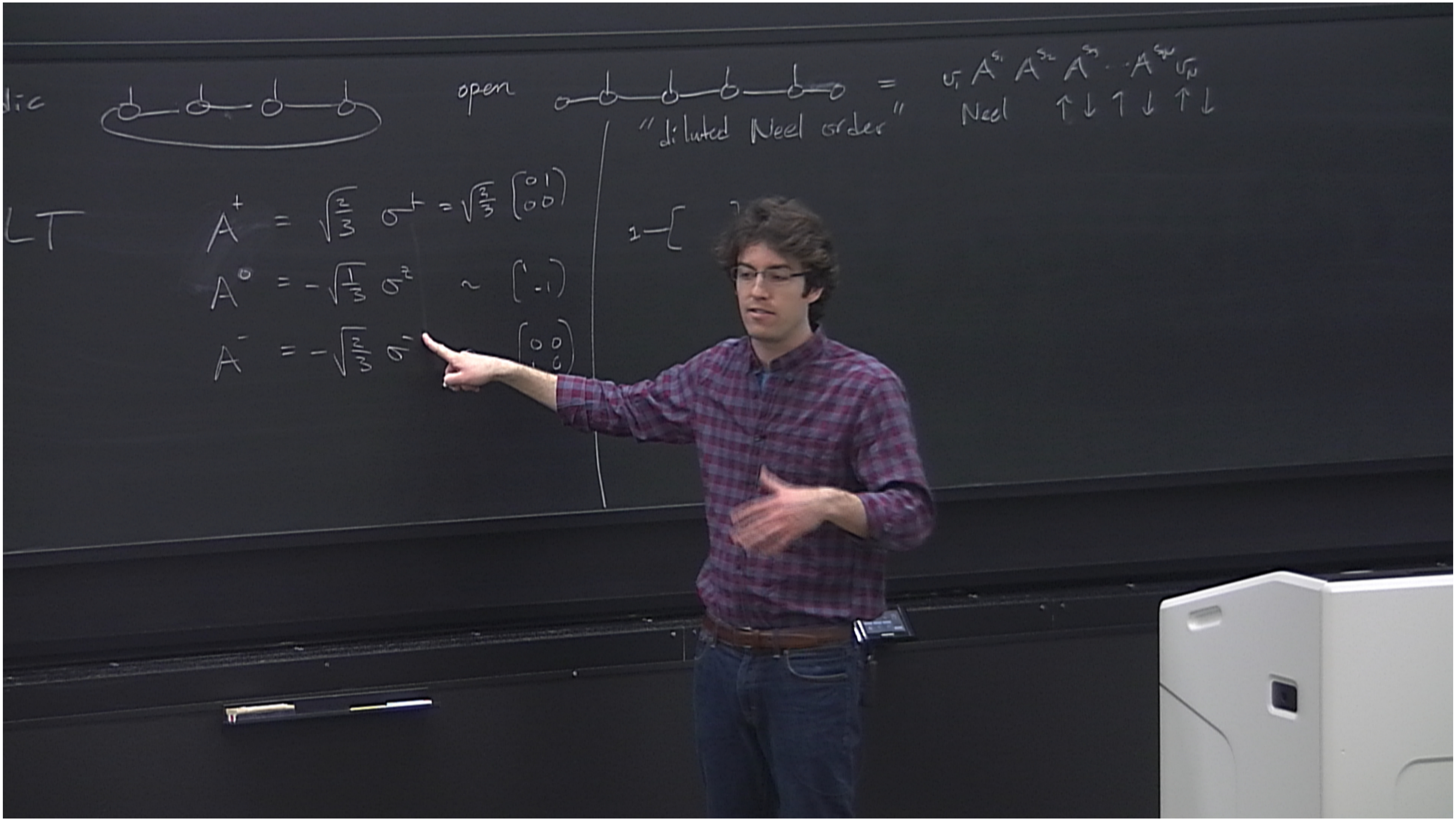

periodic
open
AKLT

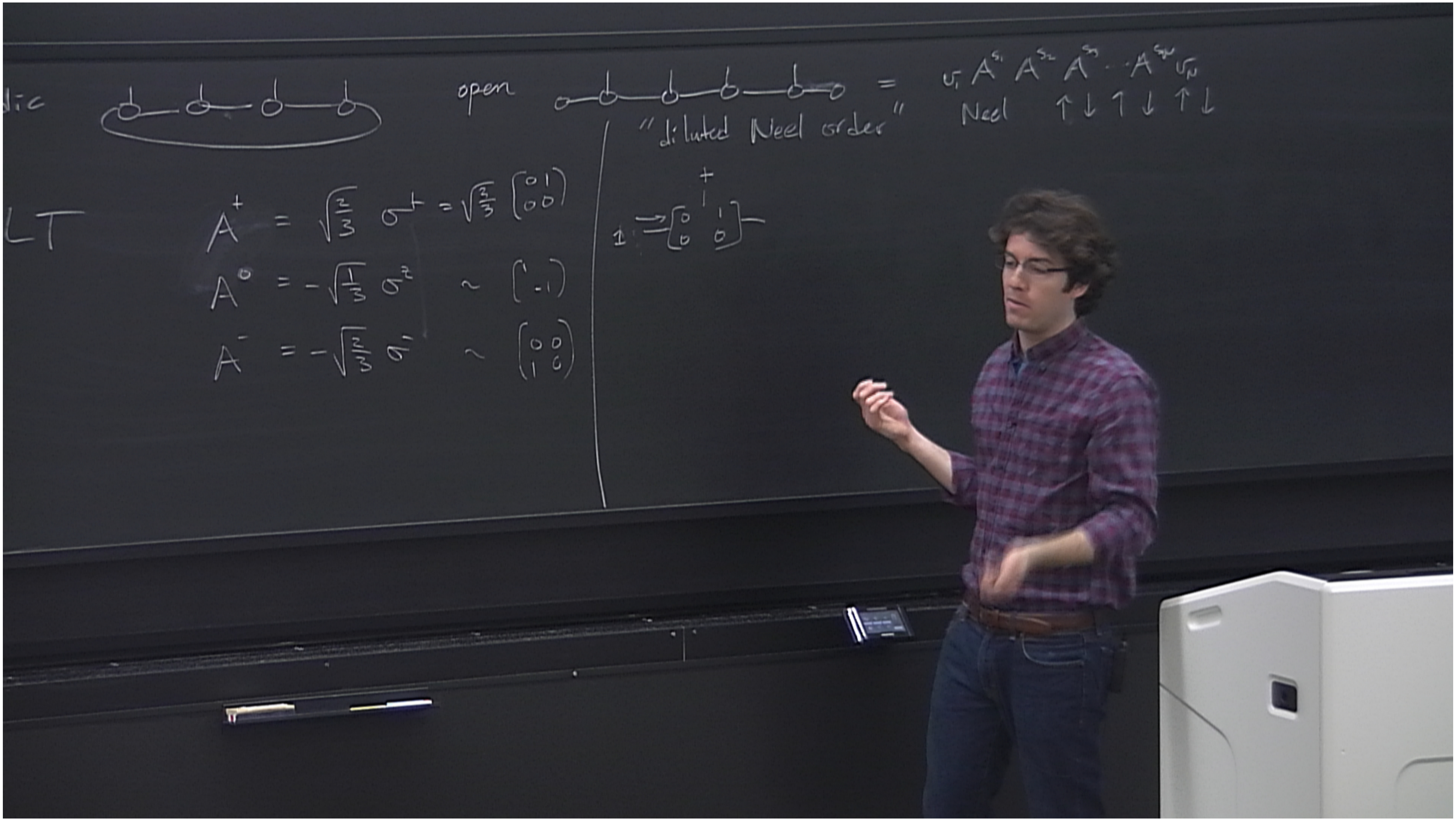

periodic
open
$= v_1 A^{s_1} A^{s_2} A^{s_3} \cdots A^{s_N} v_N$
AKLT
$A^{+} = \sqrt{\frac{2}{3}}\, \sigma^{+}$
$A^{0} = -\sqrt{\frac{1}{3}}\, \sigma^{z}$
$A^{-} = -\sqrt{\frac{2}{3}}\, \sigma^{-}$


periodic
open
= $v_1 A^{s_1} A^{s_2} A^{s_3} \cdots A^{s_N} v_N$
AKLT
$A^{+} = \sqrt{\frac{2}{3}}\,\sigma^{+} = \sqrt{\frac{2}{3}} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$
$A^{0} = -\sqrt{\frac{1}{3}}\,\sigma^{z} \sim \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$
$A^{-} = -\sqrt{\frac{2}{3}}\,\sigma^{-} \sim \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

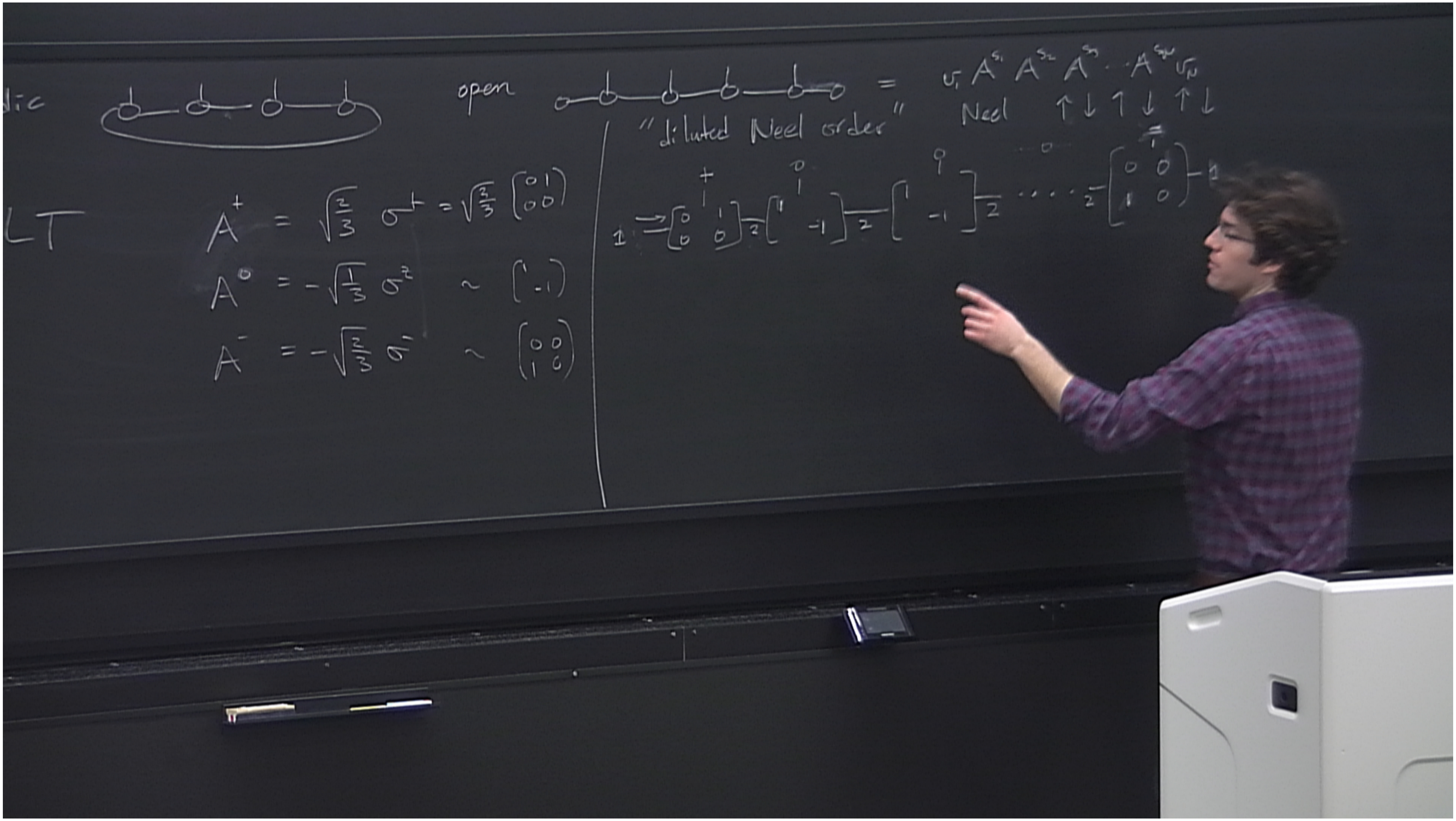

periodic
open
"diluted Neel order"
Neel ↑ ↓ ↑ ↓ ↑
AKLT

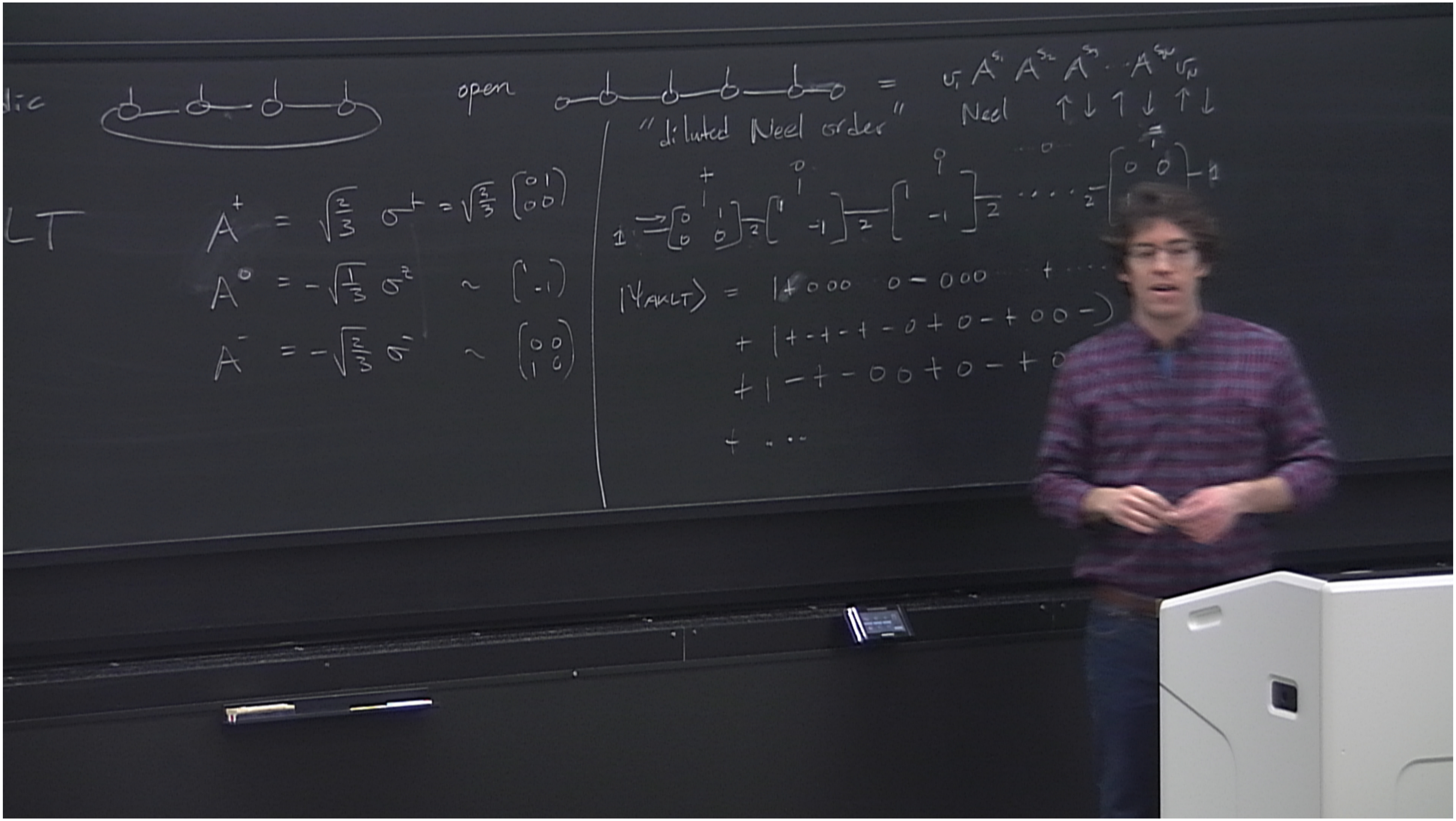

open
"diluted Neel order"
Neel ↑ ↓ ↑ ↓ ↑ ↓


open
"diluted Neel order"
Neel ↑ ↓ ↑ ↓ ↑ ↓



open
"diluted Neel order"
Neel ↑ ↓ ↑ ↓ ↑ ↓


open
"diluted Neel order"
Neel
LT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LT