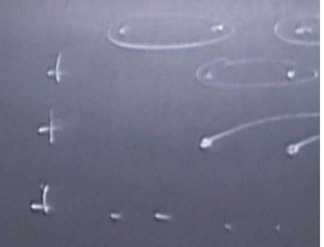
Title: Explorations in Condensed Matter-10

Date: Mar 27, 2015 10:15 AM

URL: http://pirsa.org/15030045

Abstract:

$s = 1/2$ Heisenberg chain $\rightarrow$ gapless, spinon fractionalization

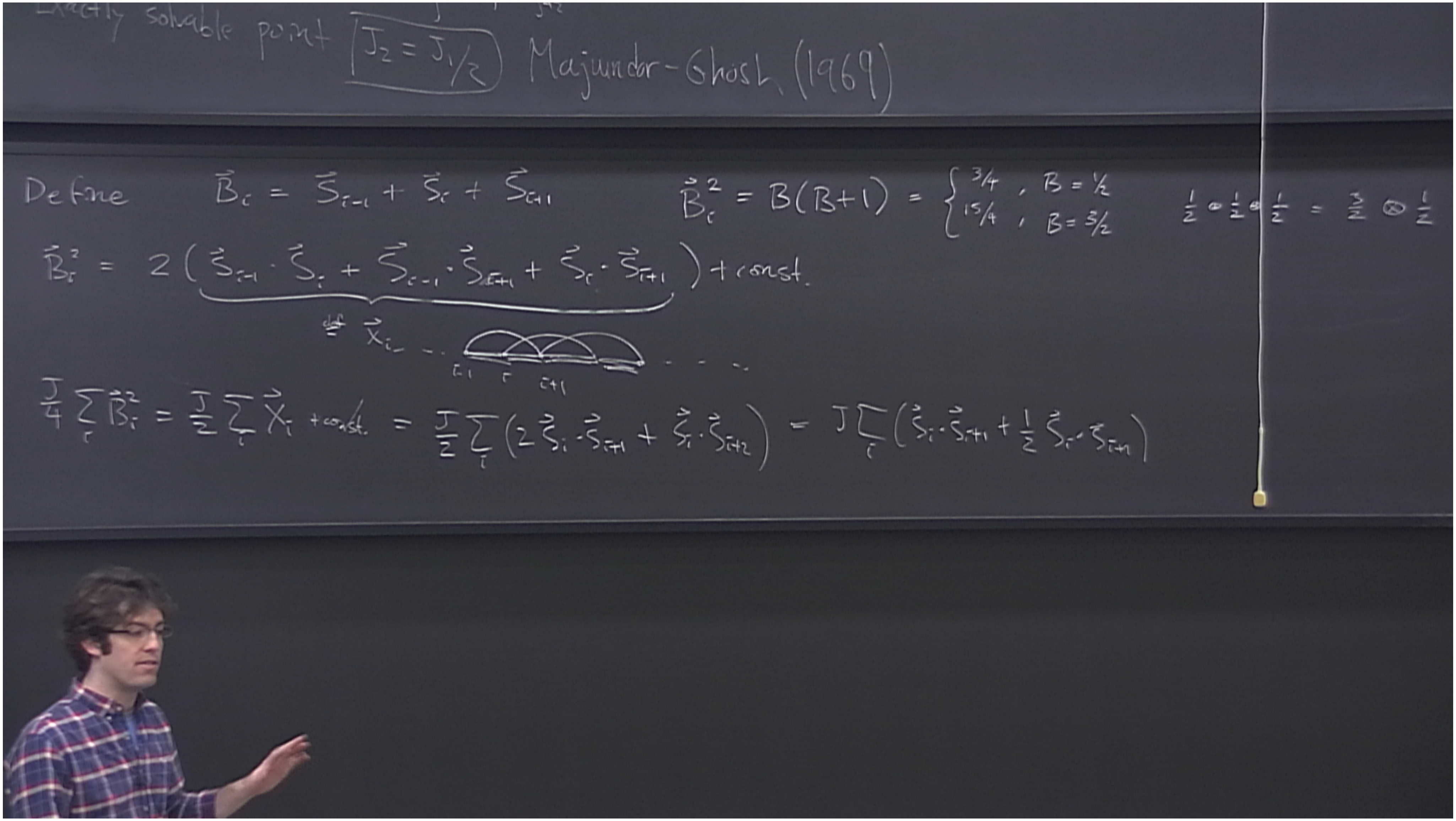


"Frustrate" singlets by extending H?

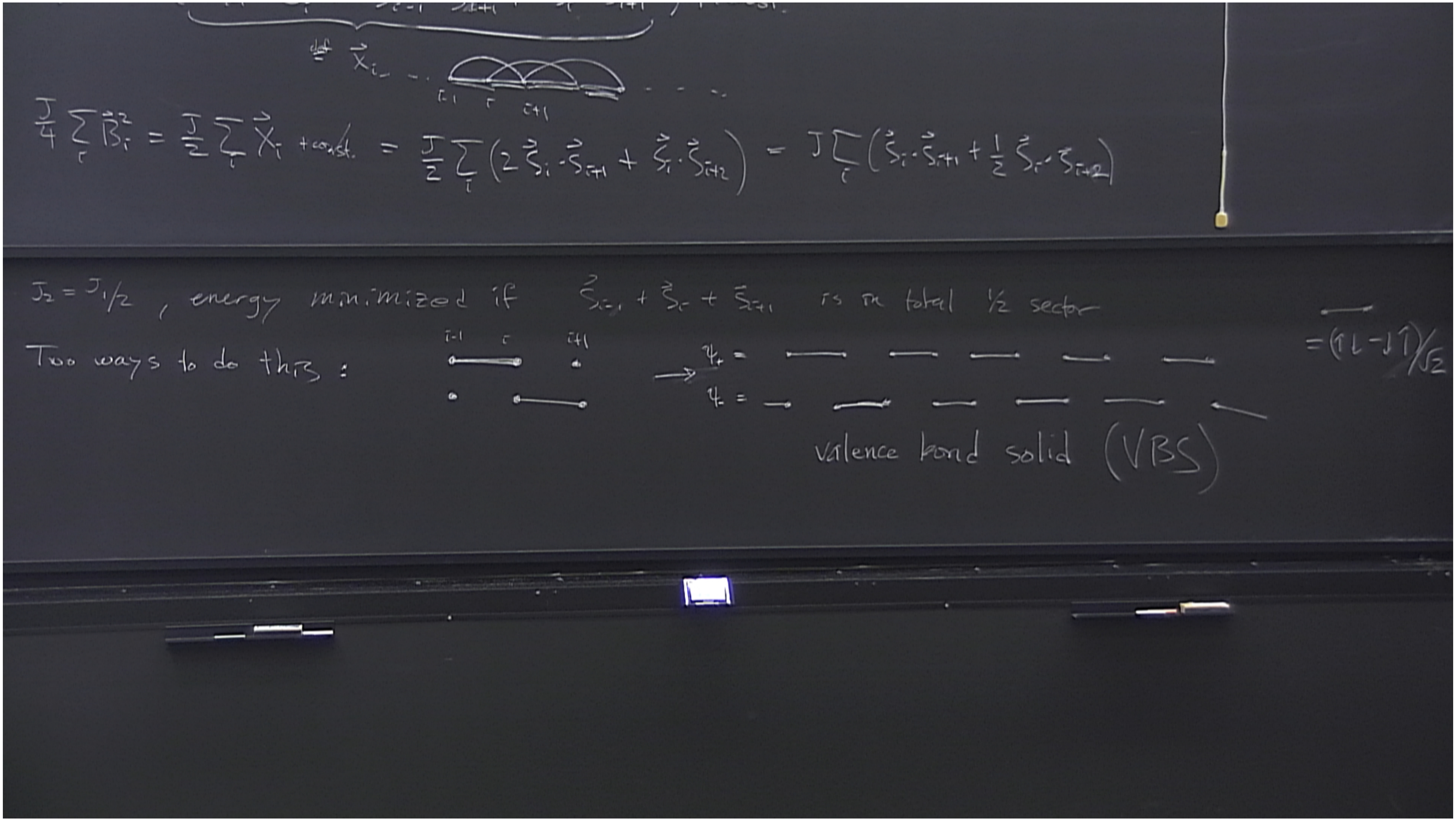
Second-neighbor:

$$H = J_1 \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + J_2 \sum_j \vec{S}_j \cdot \vec{S}_{j+2}$$

Exactly solvable point $\boxed{J_2 = J_1/2}$ Majumdar-Ghosh (1969)

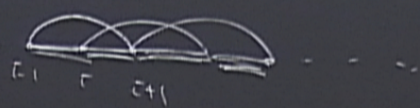


Exactly solvable point $\boxed{J_2 = J_1/2}$ Majumdar-Ghosh (1969)
Define $\vec{B}_i = \vec{S}_{i-1} + \vec{S}_i + \vec{S}_{i+1}$
$\vec{B}_i^2 = B(B+1) = \begin{cases} 3/4 , & B = 1/2 \\ 15/4 , & B = 3/2 \end{cases}$
$\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} = \frac{3}{2} \otimes \frac{1}{2}$
$\vec{B}_i^2 = 2 \big(\underbrace{\vec{S}_{i-1} \cdot \vec{S}_i + \vec{S}_{i-1} \cdot \vec{S}_{i+1} + \vec{S}_i \cdot \vec{S}_{i+1}}_{\stackrel{\text{def}}{=} \vec{X}_i} \big) + \text{const.}$
$i-1$ i $i+1$
$\frac{J}{4} \sum_i \vec{B}_i^2 = \frac{J}{2} \sum_i \vec{X}_i + \text{const.} = \frac{J}{2} \sum_i \left(2 \vec{S}_i \cdot \vec{S}_{i+1} + \vec{S}_i \cdot \vec{S}_{i+2} \right) = J \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{2} \vec{S}_i \cdot \vec{S}_{i+2} \right)$



$\stackrel{\text{def}}{=} \vec{X}_i$
$i-1 \quad i \quad i+1$
$\frac{J}{4}\sum_i \vec{B}_i^2 = \frac{J}{2}\sum_i \vec{X}_i + \text{const.} = \frac{J}{2}\sum_i \left(2\vec{S}_i\cdot\vec{S}_{i+1} + \vec{S}_i\cdot\vec{S}_{i+2}\right) = J\sum_i\left(\vec{S}_i\cdot\vec{S}_{i+1} + \frac{1}{2}\vec{S}_i\cdot\vec{S}_{i+2}\right)$
$J_2 = J_1/2$, energy minimized if $\vec{S}_{i-1} + \vec{S}_i + \vec{S}_{i+1}$ is in total 1/2 sector
Two ways to do this:
$i-1 \quad i \quad i+1$
$\psi_+ =$
$\psi_- =$
$= (\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2}$
valence bond solid (VBS)

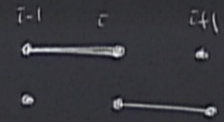
$\underbrace{\qquad\qquad\qquad\qquad}_{\overset{\text{def}}{=}\ \vec{X}_i}$



$$\frac{J}{4}\sum_i \vec{B}_i^2 = \frac{J}{2}\sum_i \vec{X}_i + \text{const.} = \frac{J}{2}\sum_i \left(2\vec{S}_i\cdot\vec{S}_{i+1} + \vec{S}_i\cdot\vec{S}_{i+2}\right) = J\sum_i\left(\vec{S}_i\cdot\vec{S}_{i+1} + \frac{1}{2}\vec{S}_i\cdot\vec{S}_{i+2}\right)$$

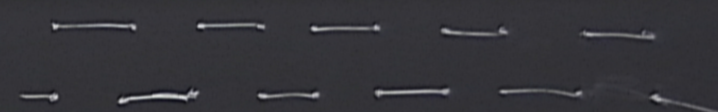
$J_2 = J_1/2$, energy minimized if $\vec{S}_{i-1} + \vec{S}_i + \vec{S}_{i+1}$ is in total $1/2$ sector

Two ways to do this:



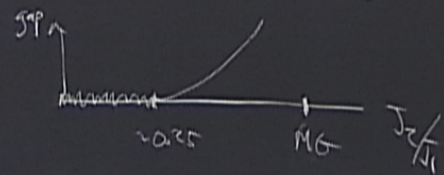
$\psi_+ =$

$\psi_- =$

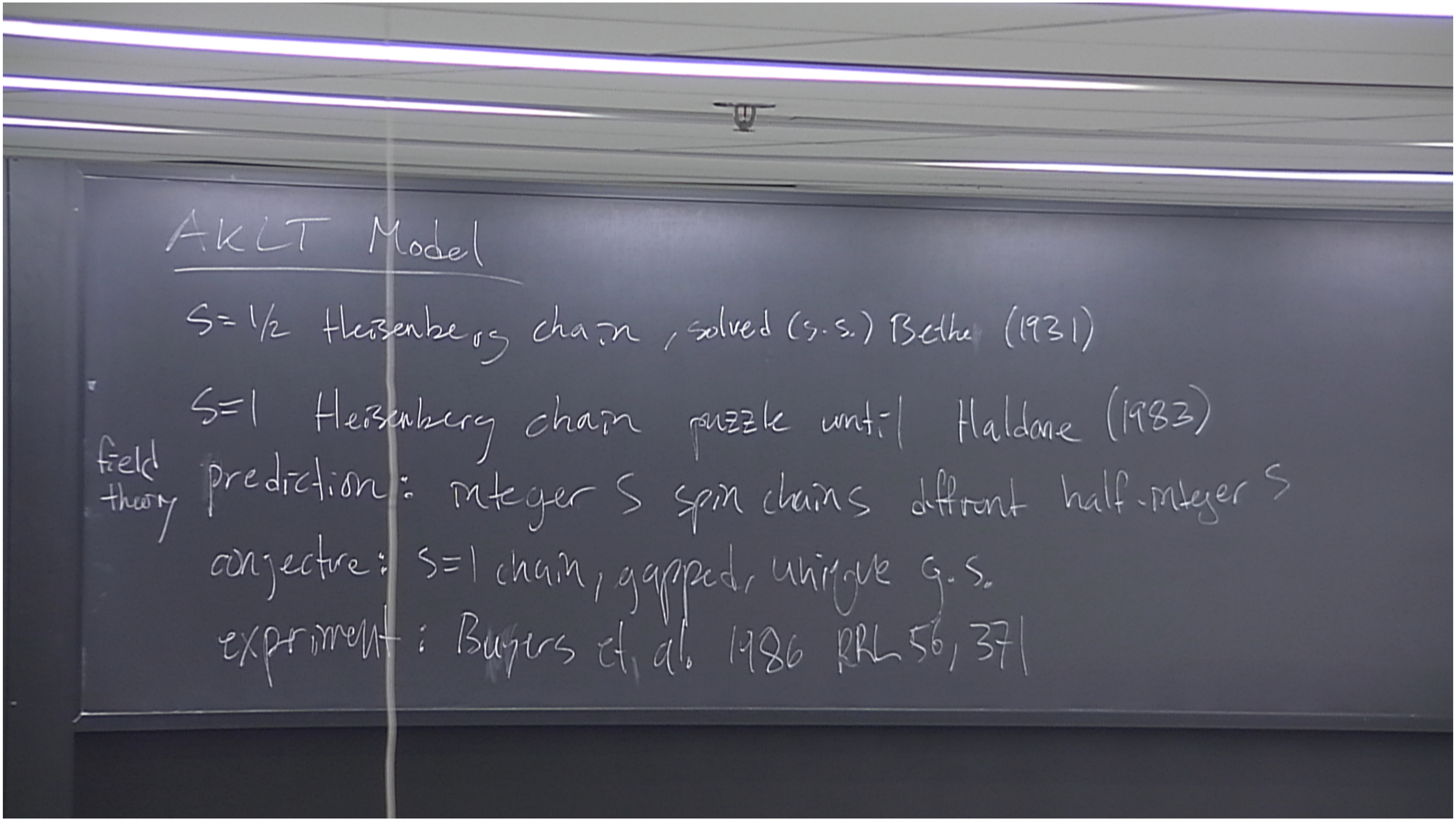


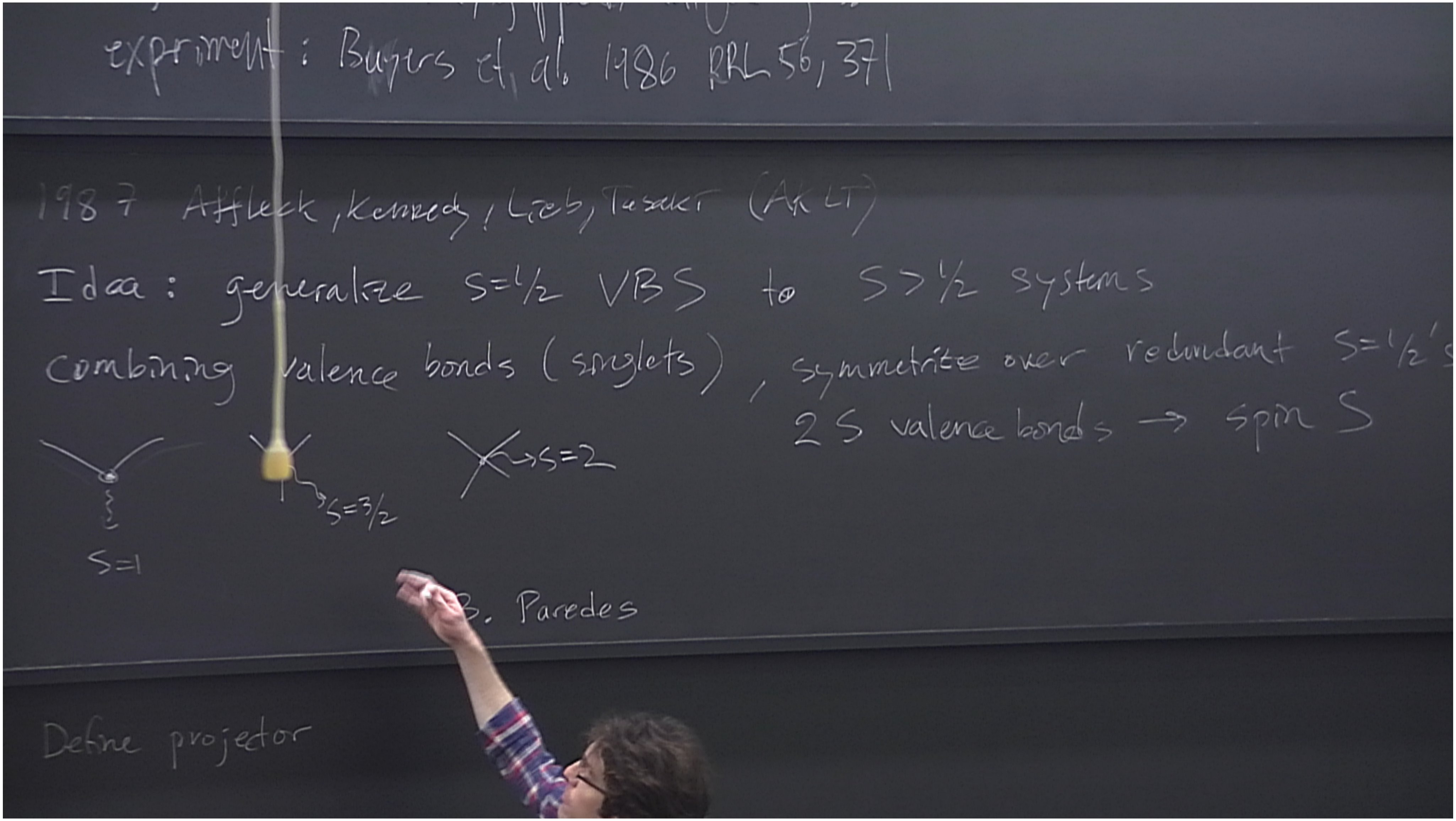
$\text{—} = (\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2}$

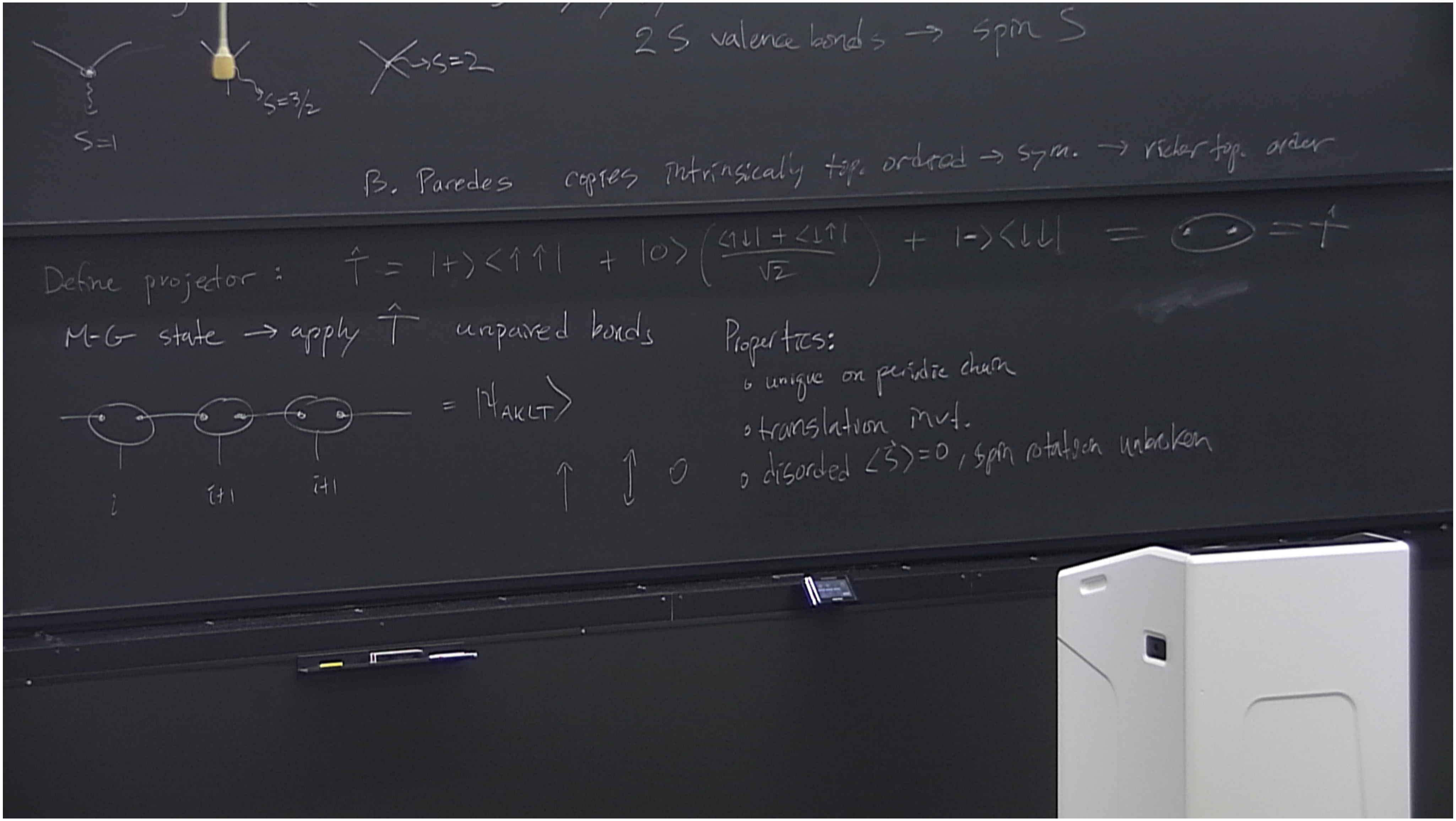
Gapped, stable

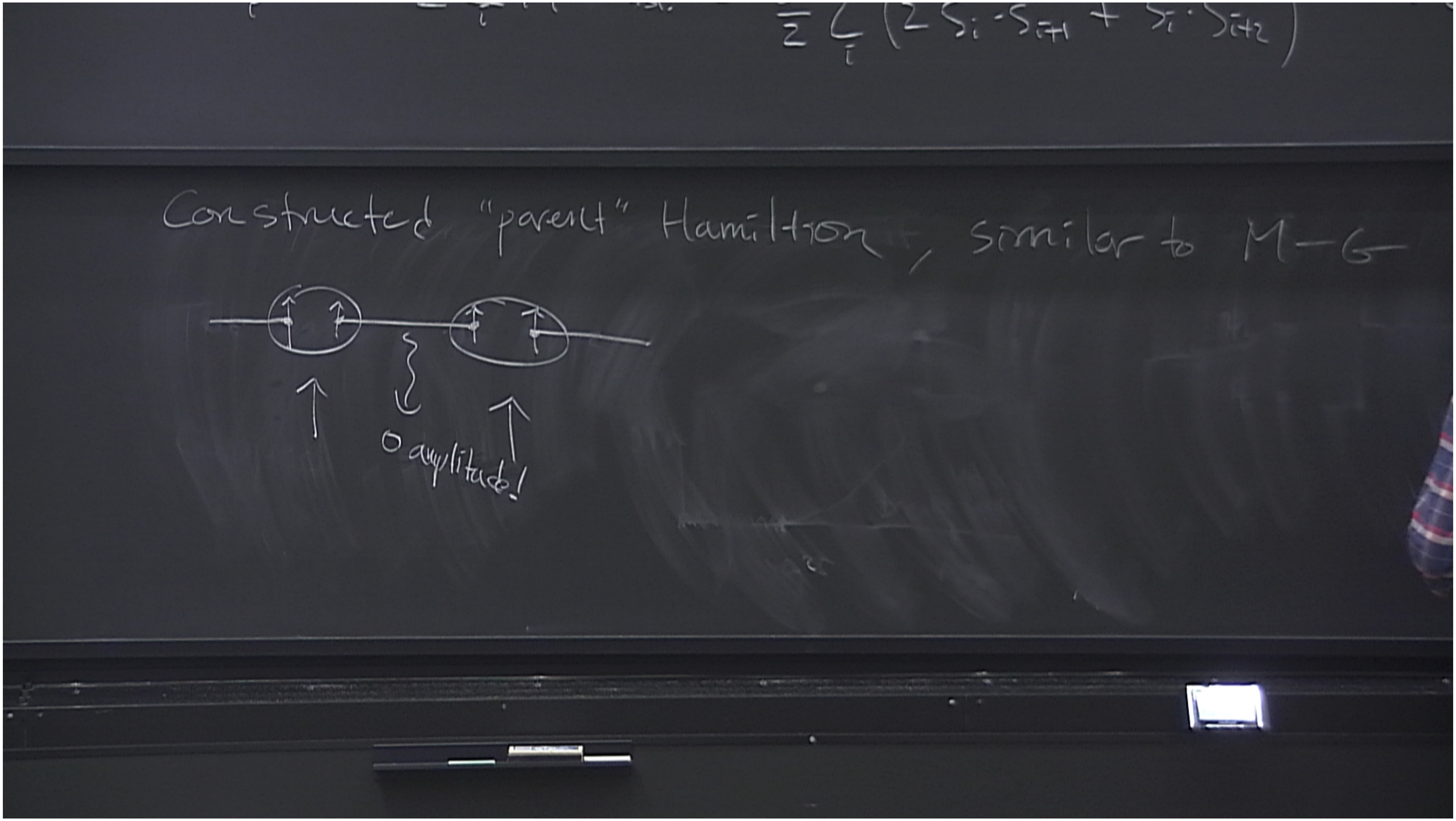


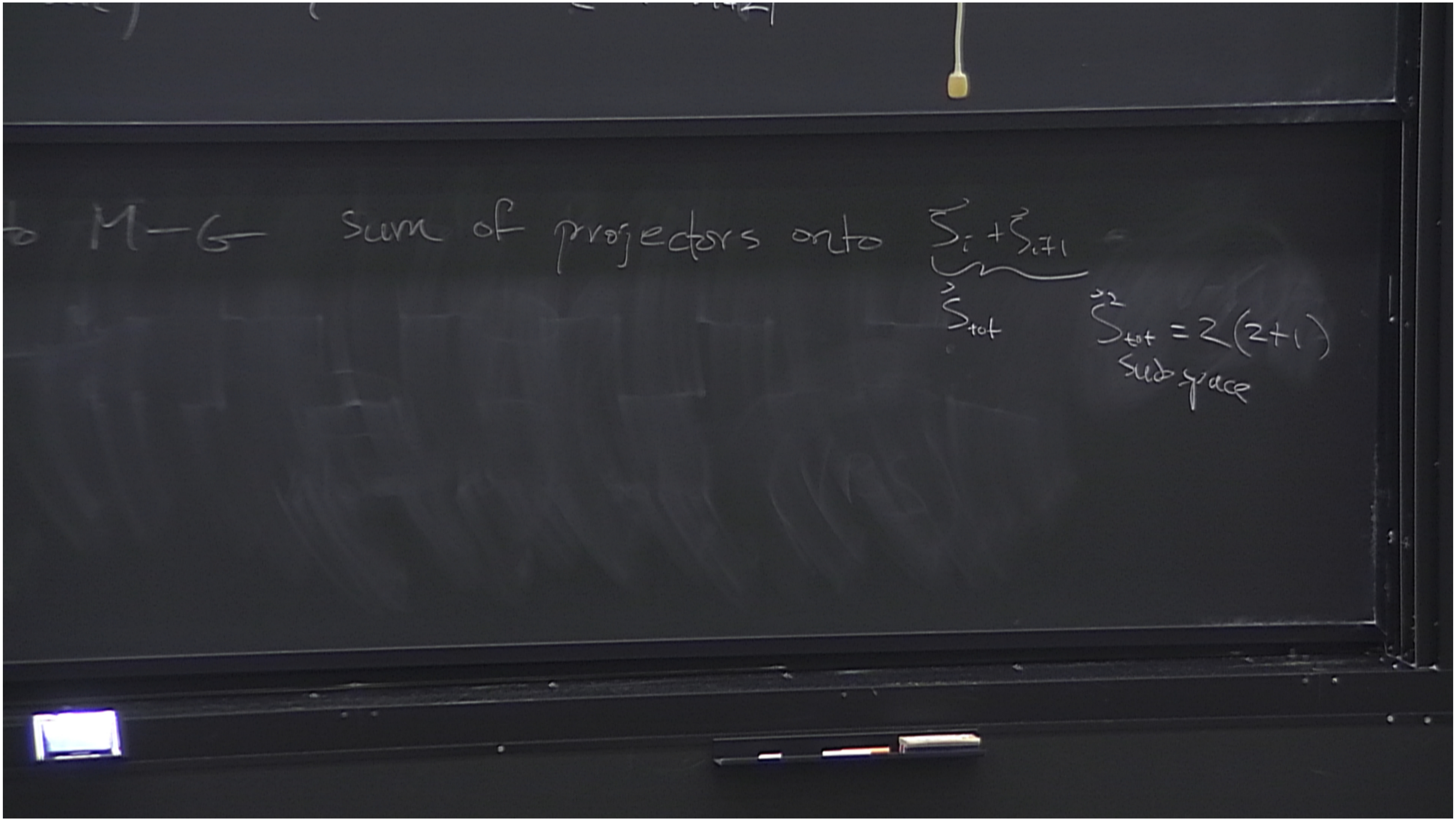
valence bond solid (VBS)


AKLT Model
S=1/2 Heisenberg chain, solved (g.s.) Bethe (1931)
S=1 Heisenberg chain puzzle until Haldane (1983)
field theory
prediction: integer S spin chains different half-integer S
conjecture: S=1 chain, gapped, unique g.s.
experiment: Buyers et al. 1986 PRL 56, 371

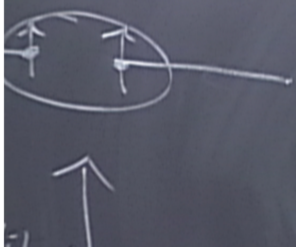

expriment : Buyers et al. 1986 PRL 56, 371
1987 Affleck, Kennedy, Lieb, Tasaki (AKLT)
Idea: generalize S=1/2 VBS to S>1/2 systems
combining valence bonds (singlets), symmetrize over redundant S=1/2's
2S valence bonds → spin S
S=1
S=3/2
S=2
B. Paredes
Define projector


2S valence bonds → spin S
S=1
S=3/2
→S=2
B. Paredes copies intrinsically top. ordered → sym. → richer top. order
Define projector : $\hat{T} = |+\rangle\langle\uparrow\uparrow| + |0\rangle\left(\frac{\langle\uparrow\downarrow| + \langle\downarrow\uparrow|}{\sqrt{2}}\right) + |-\rangle\langle\downarrow\downarrow|$
M-G state → apply $\hat{T}$ unpaired bonds
$= |\Psi_{AKLT}\rangle$
i
i+1
i+1
Properties:
◦ unique on periodic chain
◦ translation invt.
◦ disorded $\langle\vec{S}\rangle = 0$, spin rotation unbroken


$\sum_i J_i \left(2 S_i \cdot S_{i+1} + S_i \cdot S_{i+2}\right)$
Constructed "parent" Hamiltonian, similar to M-G
0 amplitude!


M–G sum of projectors onto $\underbrace{\vec{S}_i + \vec{S}_{i+1}}_{\vec{S}_{tot}}$
$\vec{S}_{tot}^2 = 2(2+1)$
subspace

"parent" Hamiltonian, similar to M–G sum of projectors

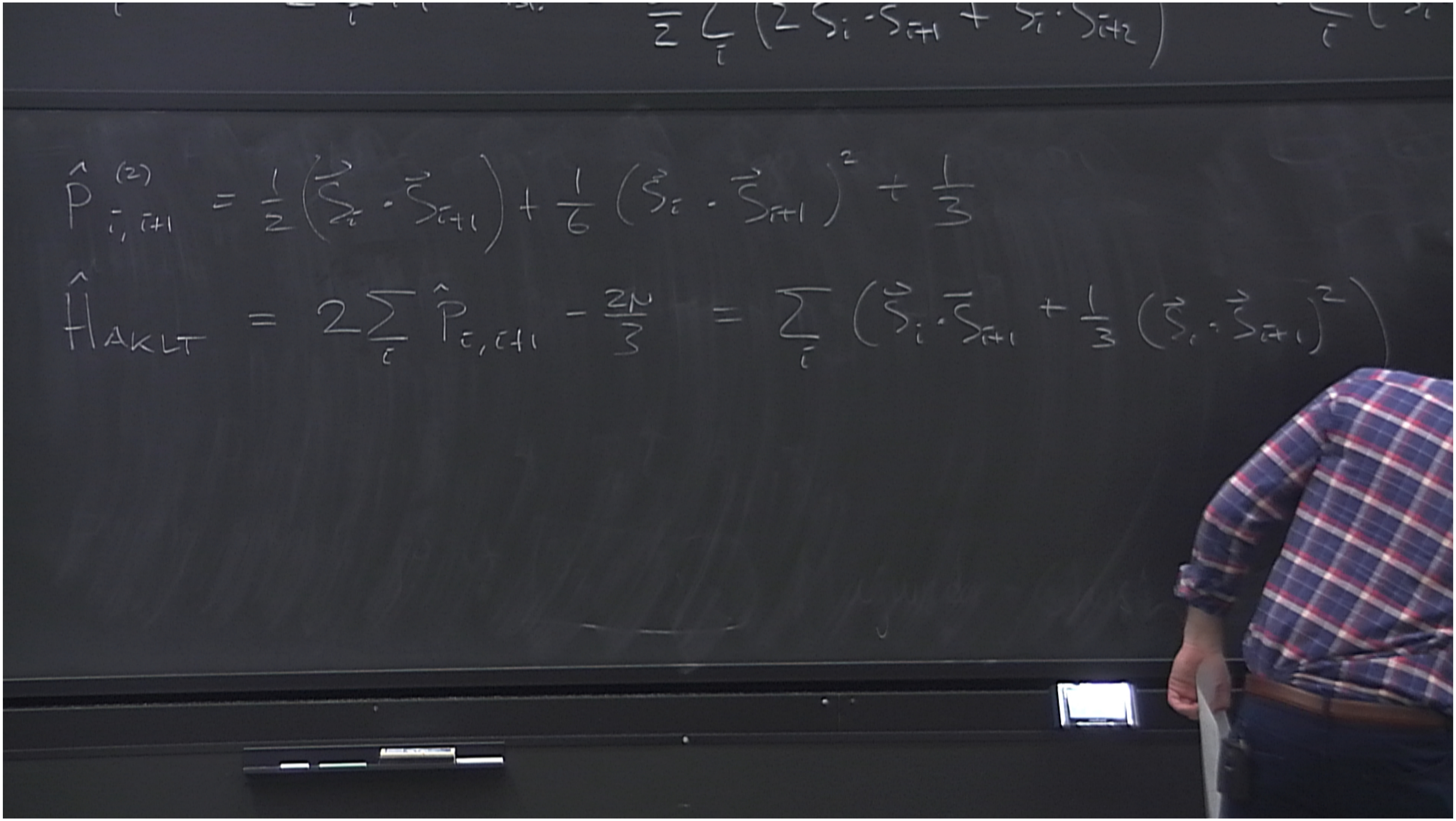


itude!

$$\hat{P}^{(2)}_{i,i+1} \,|\, S^{i,i+1}_{tot}, m^z_{tot} \rangle = \begin{cases} 1 & S_{tot} = 2 \\ 0 & S_{tot} = 0, 1 \end{cases}$$

$$\vec{B}_i = \vec{S}_{i-1} + \vec{S}_i + \vec{S}_{i+1}$$

$$\vec{B}_i^{\,2} = B(B+1) = \begin{cases} 3/4 \,, & B = 1/2 \\ 15/4 \,, & B = 3/2 \end{cases}$$


$\hat{P}^{(2)}_{i,i+1} = \frac{1}{2}\left(\vec{S}_i \cdot \vec{S}_{i+1}\right) + \frac{1}{6}\left(\vec{S}_i \cdot \vec{S}_{i+1}\right)^2 + \frac{1}{3}$
$\hat{H}_{AKLT} = 2\sum_i \hat{P}_{i,i+1} - \frac{2N}{3} = \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3}\left(\vec{S}_i \cdot \vec{S}_{i+1}\right)^2\right)$

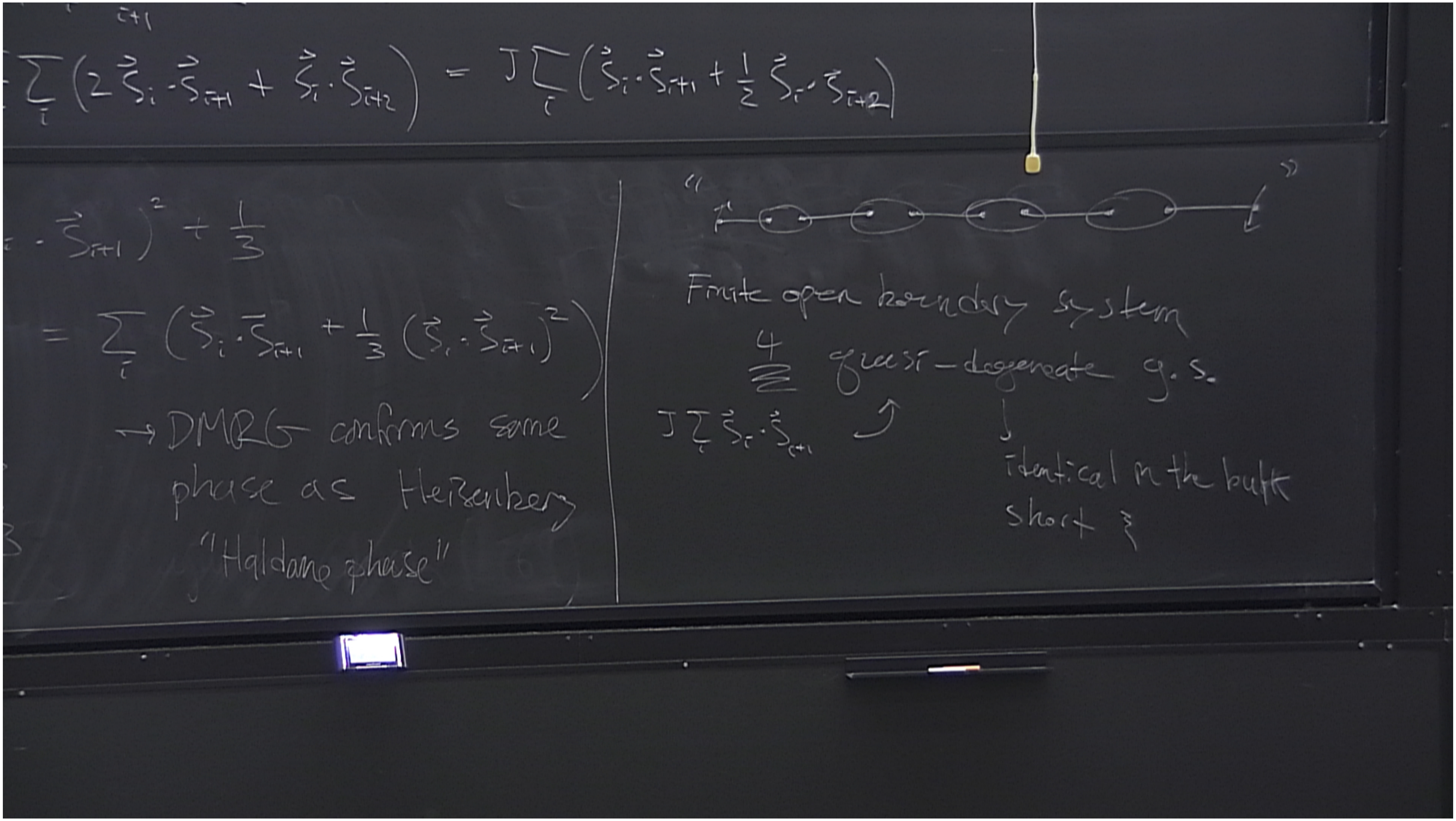
$$\frac{\ }{2}\sum_i \left(2\, S_i \cdot S_{i+1} + S_i \cdot S_{i+2}\right)$$

$$\hat{P}^{(2)}_{i,i+1} = \frac{1}{2}\left(\vec{S}_i \cdot \vec{S}_{i+1}\right) + \frac{1}{6}\left(\vec{S}_i \cdot \vec{S}_{i+1}\right)^2 + \frac{1}{3}$$

$$\hat{H}_{AKLT} = 2\sum_i \hat{P}_{i,i+1} - \frac{2N}{3} = \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3}\left(\vec{S}_i \cdot \vec{S}_{i+1}\right)^2\right)$$

$$\langle \psi_{AKLT} | \vec{S}_i \cdot \vec{S}_{i+1} | \psi_{AKLT} \rangle = -4/3$$

$$\langle \psi_{AKLT} | \frac{1}{3}\left(\vec{S}_i \cdot \vec{S}_{i+1}\right)^2 | \psi_{AKLT} \rangle = 2/3$$


$\sum_i \left(2\vec S_i\cdot\vec S_{i+1} + \vec S_i\cdot\vec S_{i+2}\right) = J\sum_i\left(\vec S_i\cdot\vec S_{i+1} + \frac{1}{2}\vec S_i\cdot\vec S_{i+2}\right)$
$\cdot\vec S_{i+1})^2 + \frac{1}{3}$
$= \sum_i \left(\vec S_i\cdot\vec S_{i+1} + \frac{1}{3}(\vec S_i\cdot\vec S_{i+1})^2\right)$
→ DMRG confirms same phase as Heisenberg
"Haldane phase"
Finite open boundary system
4 quasi-degenerate g.s.
$J\sum_i \vec S_i\cdot\vec S_{i+1}$
identical in the bulk
short ξ