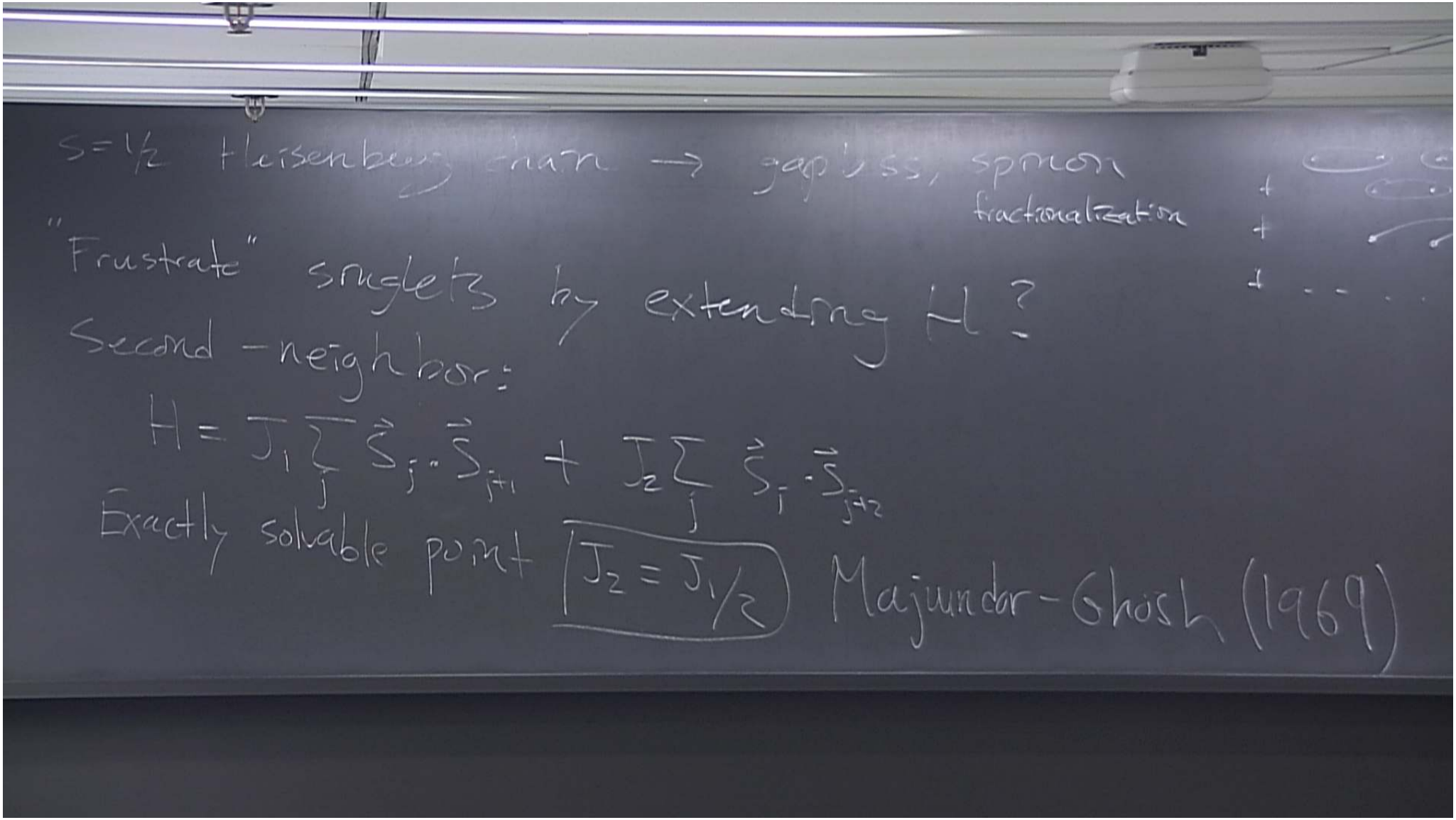
Title: Explorations in Condensed Matter-10

Date: Mar 27, 2015 10:15 AM

URL: http://pirsa.org/15030045

Abstract:


S=1/2 Heisenberg chain → gapless, spinon fractionalization
"Frustrate" singlets by extending NL?
Second-neighbor:
$H = J_1 \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + J_2 \sum_j \vec{S}_j \cdot \vec{S}_{j+2}$
Exactly solvable point $J_2 = J_1/2$ Majumdar-Ghosh (1969)

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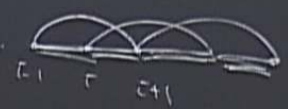
Define $\vec{B}_i = \vec{S}_{i-1} + \vec{S}_i + \vec{S}_{i+1}$ $\quad \vec{B}_i^2 = B(B+1) = \begin{cases} 3/4, & B = 1/2 \\ 15/4, & B = 3/2 \end{cases}$ $\quad \frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} = \frac{3}{2} \oplus \frac{1}{2}$

$$\vec{B}_i^2 = 2\underbrace{\left(\vec{S}_{i-1}\cdot\vec{S}_i + \vec{S}_{i-1}\cdot\vec{S}_{i+1} + \vec{S}_i\cdot\vec{S}_{i+1}\right)}_{\stackrel{\text{def}}{=} \vec{X}_i} + \text{const.}$$

$$\frac{J}{4}\sum_i \vec{B}_i^2 = \frac{J}{2}\sum_i \vec{X}_i + \text{const.} = \frac{J}{2}\sum_i \left(2\vec{S}_i\cdot\vec{S}_{i+1} + \vec{S}_i\cdot\vec{S}_{i+2}\right) = J\sum_i \left(\vec{S}_i\cdot\vec{S}_{i+1} + \frac{1}{2}\vec{S}_i\cdot\vec{S}_{i+2}\right)$$

$\overset{\text{def}}{=} \vec{X}_i$



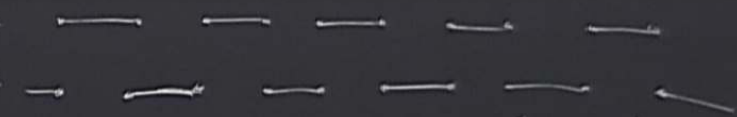
$$\frac{J}{4}\sum_i \vec{B}_i^2 = \frac{J}{2}\sum_i \vec{X}_i + \text{const.} = \frac{J}{2}\sum_i \left(2\,\vec{S}_i\cdot\vec{S}_{i+1} + \vec{S}_i\cdot\vec{S}_{i+2}\right) = J\sum_i\left(\vec{S}_i\cdot\vec{S}_{i+1} + \frac{1}{2}\vec{S}_i\cdot\vec{S}_{i+2}\right)$$

$J_2 = J_1/2$, energy minimized if $\vec{S}_{i-1} + \vec{S}_i + \vec{S}_{i+1}$ is in total ½ sector

Two ways to do this:

$\longrightarrow$ $\psi_+ =$

$\psi_- =$



$= (\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2}$

valence bond solid (VBS)

$\frac{J}{4}\sum_i \vec{B}_i^2 = \frac{J}{2}\sum_i \vec{X}_i + \text{const.} = \frac{J}{2}\sum_i \left(2\vec{S}_i\cdot\vec{S}_{i+1} + \vec{S}_i\cdot\vec{S}_{i+2}\right) = J\sum_i \left(\vec{S}_i\cdot\vec{S}_{i+1} + \frac{1}{2}\vec{S}_i\cdot\vec{S}_{i+2}\right)$

$J_2 = J_1/2$, energy minimized if $\vec{S}_{i-1} + \vec{S}_i + \vec{S}_{i+1}$ is in total $\frac{1}{2}$ sector

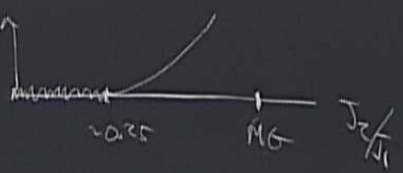
Two ways to do this:

$\rightarrow \psi_+ =$

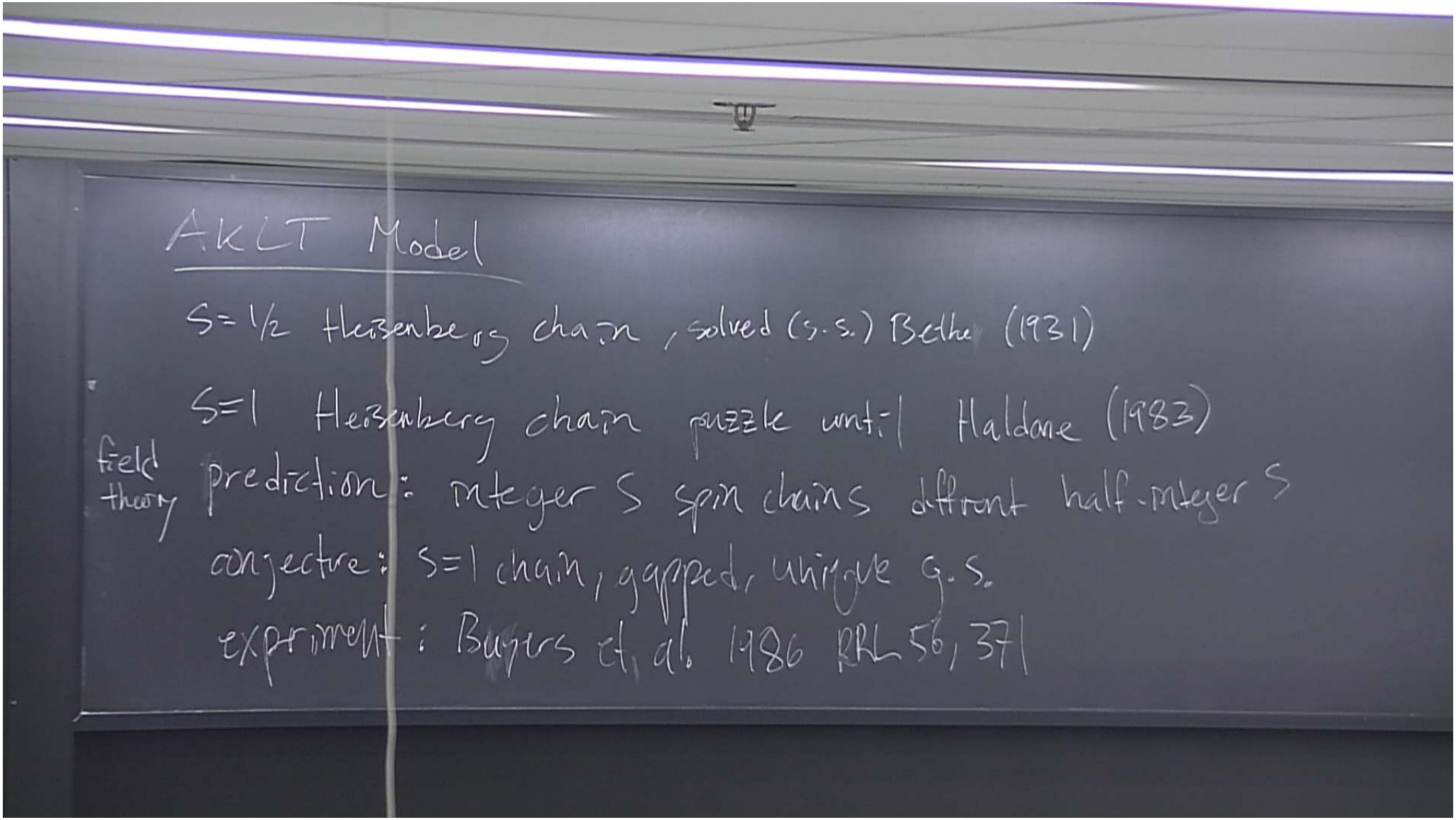
$\psi_- =$

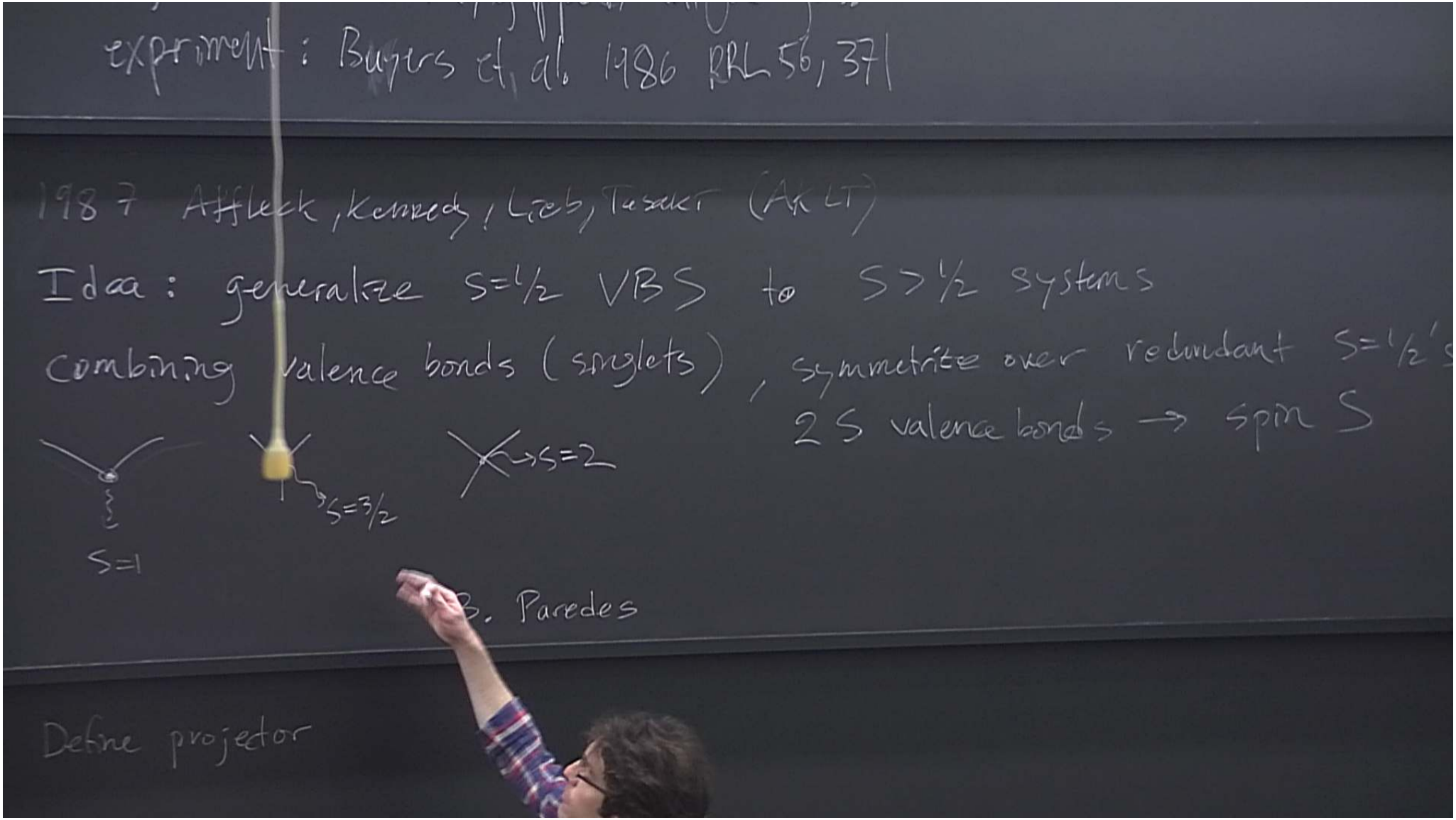
$= (\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2}$

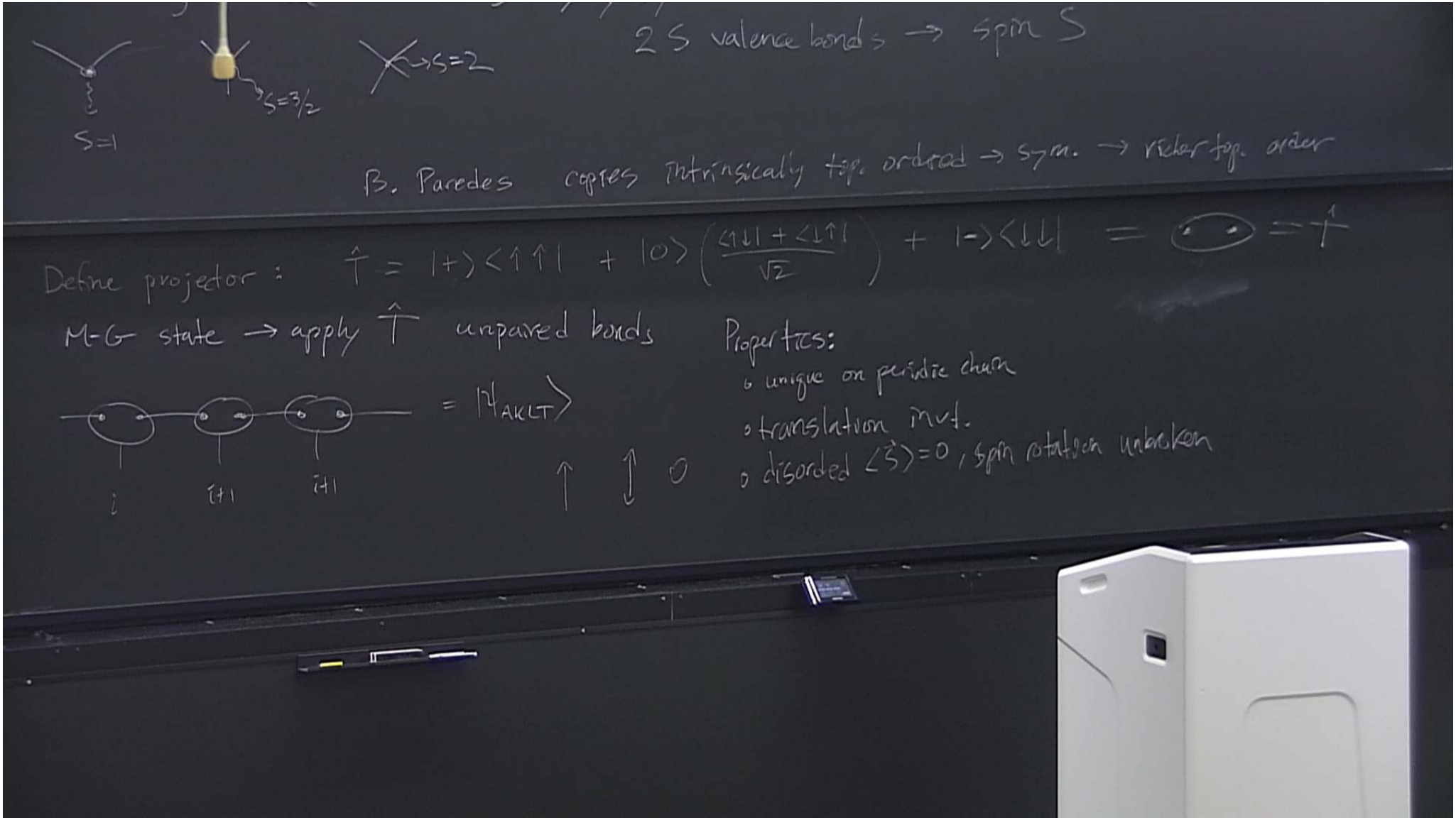
Gapped, stable



valence bond solid (VBS)

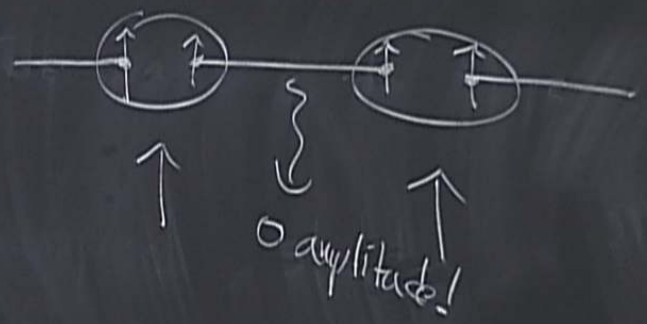

AKLT Model
S=1/2 Heisenberg chain, solved (s.s.) Bethe (1931)
S=1 Heisenberg chain puzzle until Haldane (1983)
field theory prediction: integer S spin chains diffrent half-integer S
conjecture: S=1 chain, gapped, unique g.s.
experiment: Buyers et al. 1986 RRL 56, 371

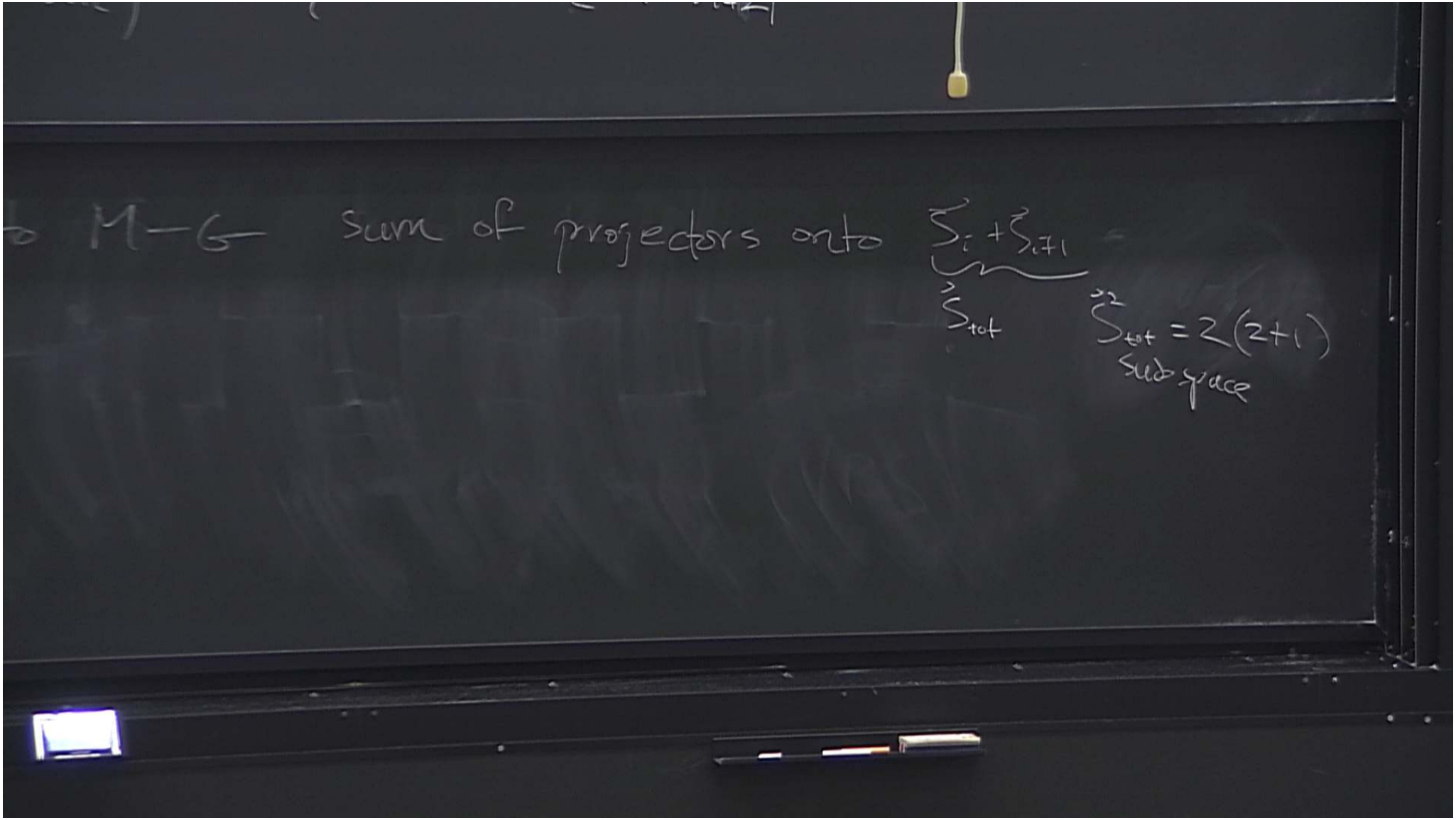

experiment: Buyers et al. 1986 PRL 56, 371
1987 Affleck, Kennedy, Lieb, Tasaki (AKLT)
Idea: generalize S=1/2 VBS to S>1/2 systems
combining valence bonds (singlets), symmetrize over redundant S=1/2's
2S valence bonds → spin S
S=1
S=3/2
S=2
B. Paredes
Define projector


S=1
S=3/2
S=2
2 S valence bonds → spin S
B. Paredes copies Intrinsically top. ordered → sym. → richer top. order
Define projector : $\hat{T} = |+\rangle\langle\uparrow\uparrow| + |0\rangle\left(\frac{\langle\uparrow\downarrow| + \langle\downarrow\uparrow|}{\sqrt{2}}\right) + |-\rangle\langle\downarrow\downarrow|$ = $\hat{T}$
M-G state → apply $\hat{T}$ unpaired bonds
$= |\Psi_{AKLT}\rangle$
i
i+1
i+1
Properties:
• unique on periodic chain
• translation inv.
• disorded $\langle\vec{S}\rangle = 0$, spin rotation unbroken

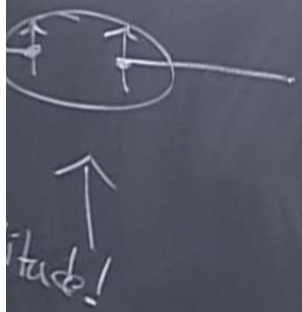
$\frac{1}{2}\sum_i \left(2\, S_i \cdot S_{i+1} + S_i \cdot S_{i+2}\right)$

Constructed "parent" Hamiltonian, similar to M-G




M−G sum of projectors onto $\vec{S}_i + \vec{S}_{i+1}$
$\vec{S}_{tot}$
$\vec{S}^2_{tot} = 2(2+1)$
subspace

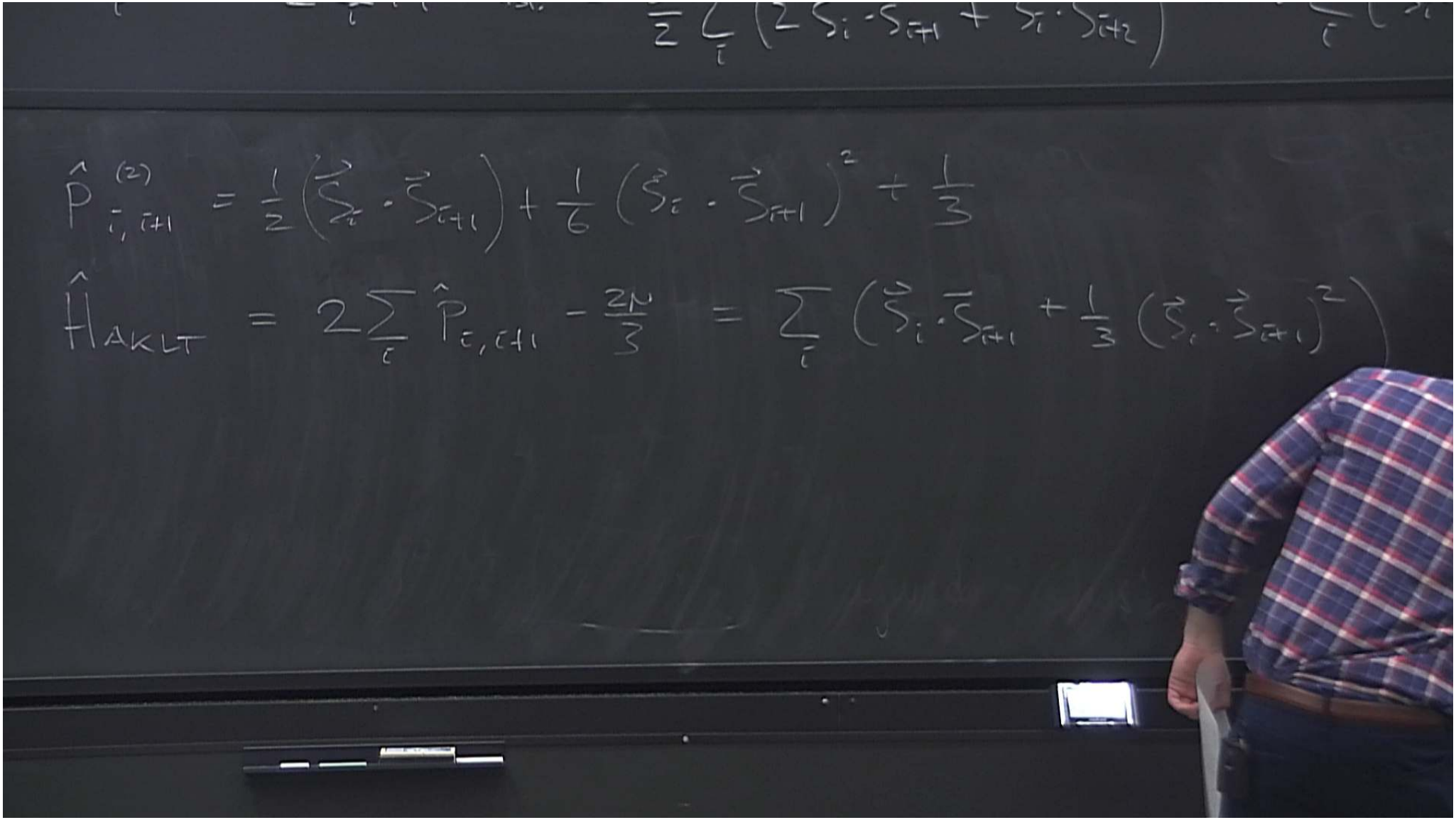
"parent" Hamiltonian, similar to M–G sum of projectors



itude!

$$\hat{P}^{(2)}_{i,i+1}\left(S^{i,i+1}_{tot}, m^{z}_{tot}\right) = \begin{cases} 1 & S_{tot}=2 \\ 0 & S_{tot}=0,1 \end{cases}$$

$$\vec{B}_i = \vec{S}_{i-1} + \vec{S}_i + \vec{S}_{i+1} \qquad \vec{B}_i^{\,2} = B(B+1) = \begin{cases} 3/4, & B=1/2 \\ 15/4, & B=3/2 \end{cases}$$


$\hat{P}^{(2)}_{i,i+1} = \frac{1}{2}\left(\vec{S}_i \cdot \vec{S}_{i+1}\right) + \frac{1}{6}\left(\vec{S}_i \cdot \vec{S}_{i+1}\right)^2 + \frac{1}{3}$
$\hat{H}_{AKLT} = 2\sum_i \hat{P}_{i,i+1} - \frac{2N}{3} = \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3}\left(\vec{S}_i \cdot \vec{S}_{i+1}\right)^2\right)$

$$\frac{1}{2}\sum_i \left(2\,\vec{S}_i\cdot\vec{S}_{i+1} + \vec{S}_i\cdot\vec{S}_{i+2}\right)$$

$$\hat{P}^{(2)}_{i,i+1} = \frac{1}{2}\left(\vec{S}_i\cdot\vec{S}_{i+1}\right) + \frac{1}{6}\left(\vec{S}_i\cdot\vec{S}_{i+1}\right)^2 + \frac{1}{3}$$

$$\hat{H}_{AKLT} = 2\sum_i \hat{P}_{i,i+1} - \frac{2N}{3} = \sum_i \left(\vec{S}_i\cdot\vec{S}_{i+1} + \frac{1}{3}\left(\vec{S}_i\cdot\vec{S}_{i+1}\right)^2\right)$$

$$\langle \psi_{AKLT} |\ \vec{S}_i\cdot\vec{S}_{i+1}\ | \psi_{AKLT}\rangle = -4/3$$

$$\langle \psi_{AKLT} |\ \tfrac{1}{3}\left(\vec{S}_i\cdot\vec{S}_{i+1}\right)^2 | \psi_{AKLT}\rangle = 2/3$$

$$\sum_i \left(2\vec{S}_i \cdot \vec{S}_{i+1} + \vec{S}_i \cdot \vec{S}_{i+2} \right) = J \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{2} \vec{S}_i \cdot \vec{S}_{i+2} \right)$$

$$\cdot \vec{S}_{i+1} \Big)^2 + \frac{1}{3}$$

$$= \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3} \left(\vec{S}_i \cdot \vec{S}_{i+1} \right)^2 \right)$$

→ DMRG confirms same phase as Heisenberg "Haldane phase"

Finite open boundary system

$\equiv$ 4 quasi-degenerate g.s.

$J \sum_i \vec{S}_i \cdot \vec{S}_{i+1}$

↓ identical in the bulk short }