Title: Explorations in Condensed Matter-9

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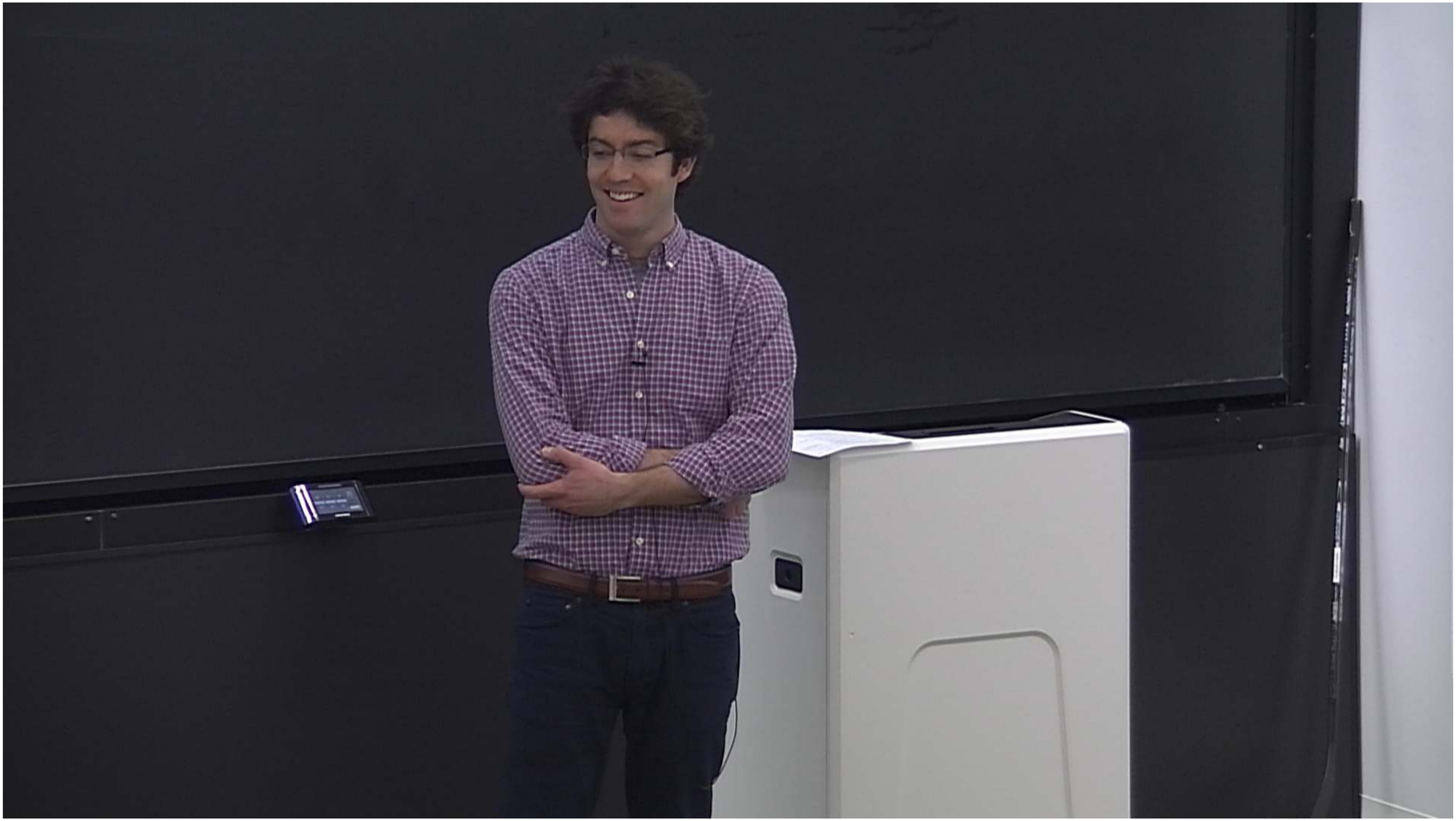
Abstract:



# Heisenberg model

Hubbard, $t \ll U$

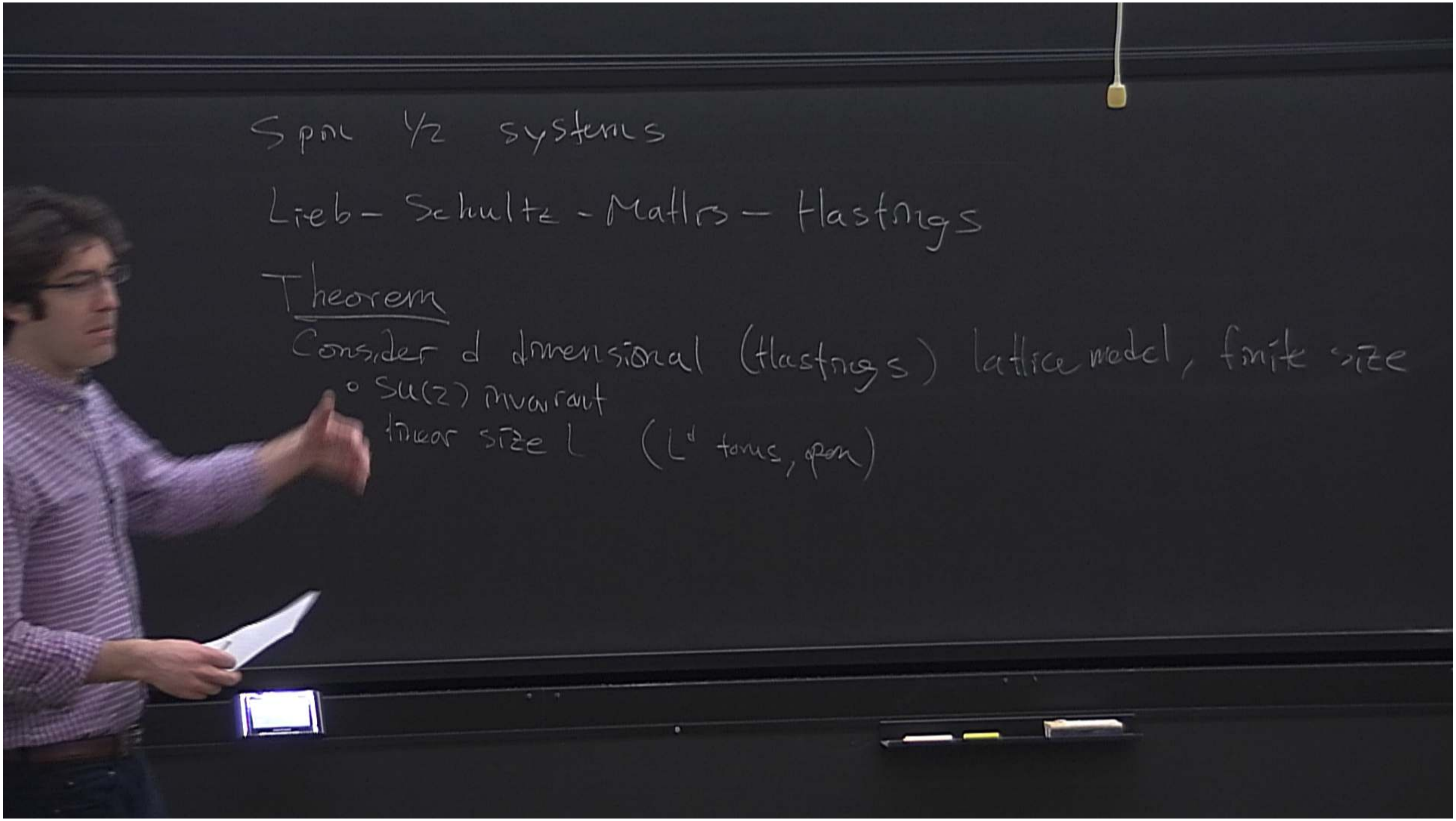
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \qquad SU(2) \text{ rotationally invariant}$$

Real magnetic materials, extensions of Heisenberg

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$H = \sum_{\langle ij \rangle} J_{xy} (S_i^x S_j^x + S_i^y S_j^y) + J_z S_i^z S_j^z$$


rotationally invariant
xtensions of Heisenberg
XXZ


Spin 1/2 systems
Lieb-Schultz-Mattis-Hastings
Theorem
Consider d dimensional (Hastings) lattice model, finite size
∘ SU(2) invariant
linear size L (L^d torus, open)

Spin 1/2 systems

Lieb – Schultz – Mattis – Hastings

Theorem

Consider d dimensional (Hastings) lattice model, finite size

- SU(2) invariant
- linear size L ($L^d$ torus, open)
- odd number of $S = 1/2$'s per unit cell

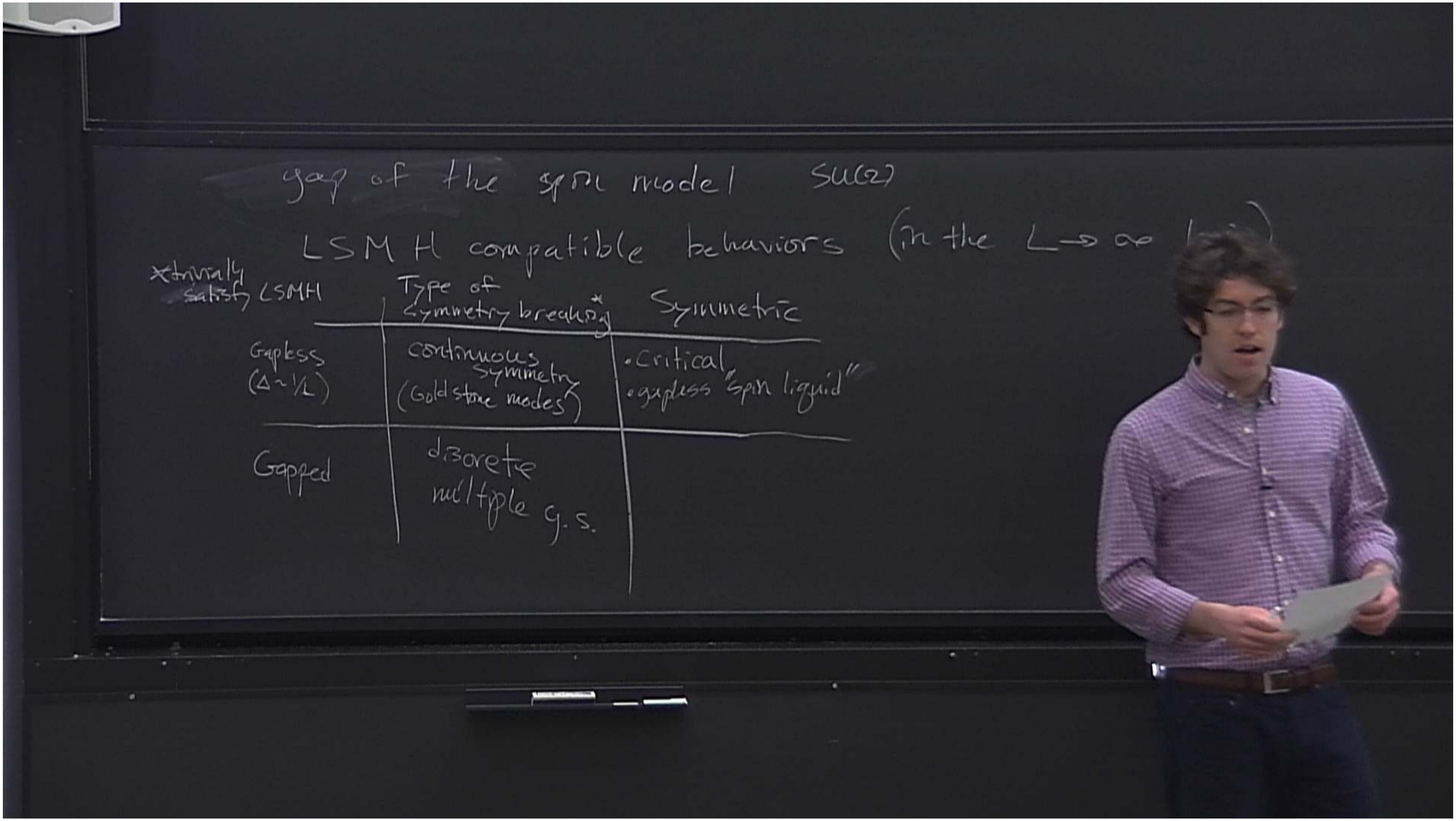
(c indep of L)

Then the gap to the first excited state $\Delta E = E_1 - E_0 \leq c \ln(L)/L$

gap of the spin model

LSMH compatible behaviors (in the $L \to \infty$ limit)

| | Type of symmetry breaking | Symmetric |
|---|---|---|
| Gapless ($\Delta \sim 1/L$) | continuous symmetry | |
| Gapped | | |


gap of the spin model SU(2)
LSMH compatible behaviors (in the L → ∞
*trivially satisfy LSMH
Type of symmetry breaking*
Symmetric
Gapless (Δ ~ 1/L)
continuous symmetry (Goldstone modes)
• Critical
• gapless "spin liquid"
Gapped
discrete multiple g.s.

$S=1/2$ Heisenberg chain (gapless)

$$H = J\sum_i \vec{S}_i \cdot \vec{S}_{i+1}$$ (satisfies LSMH)

Gain insight, study

$$H_{XXZ} = \sum_i \frac{J_{xy}}{2}\left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+\right) + J_z S_i^z S_{i+1}^z$$

fm Ising — gapless — afm Ising $\rightarrow J_z/J_{xy}$

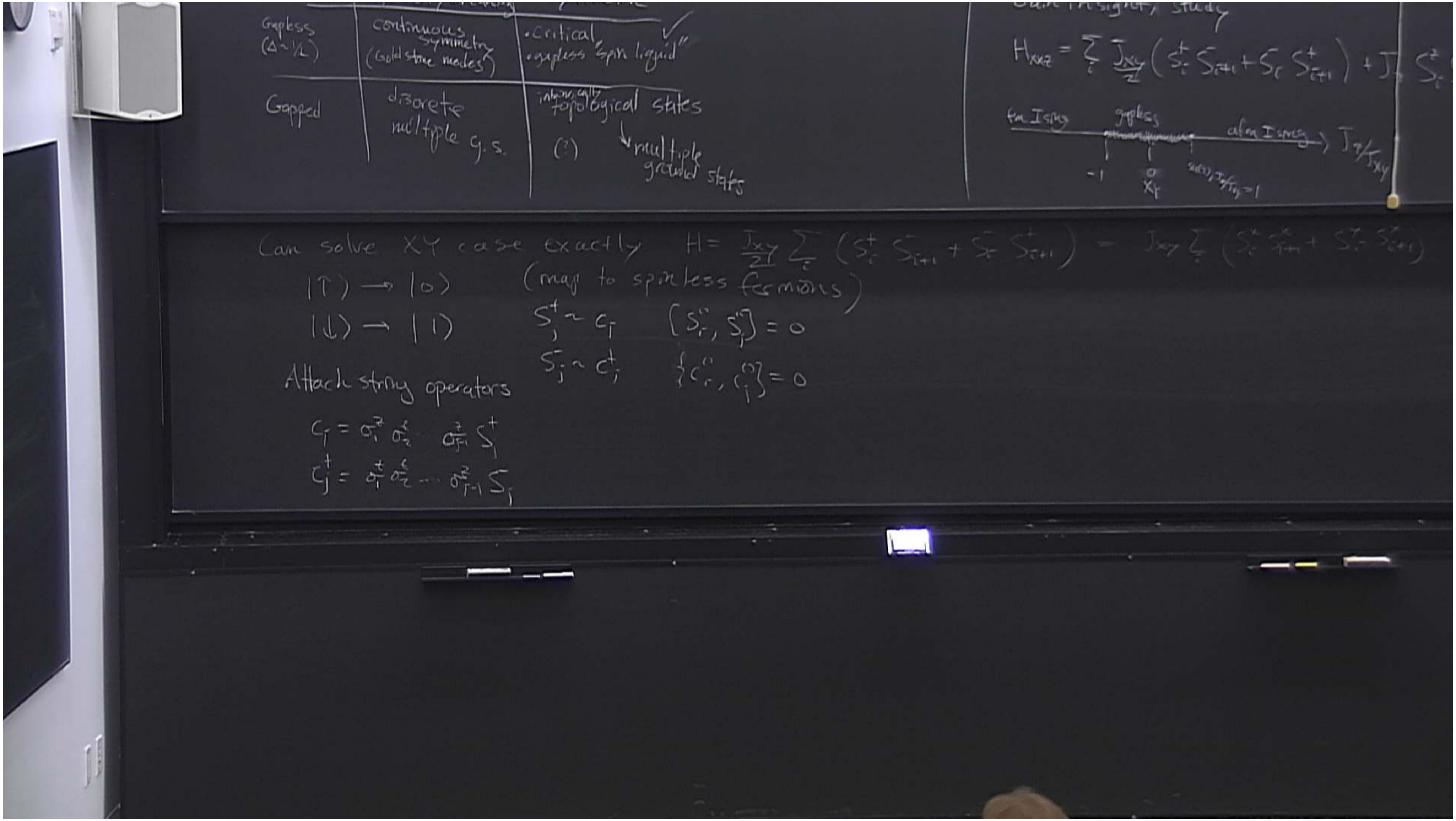
$-1$ — $0$ (XY) — $SU(2)$, $J_z/J_{xy}=1$

gap of the spin model   SU(2)

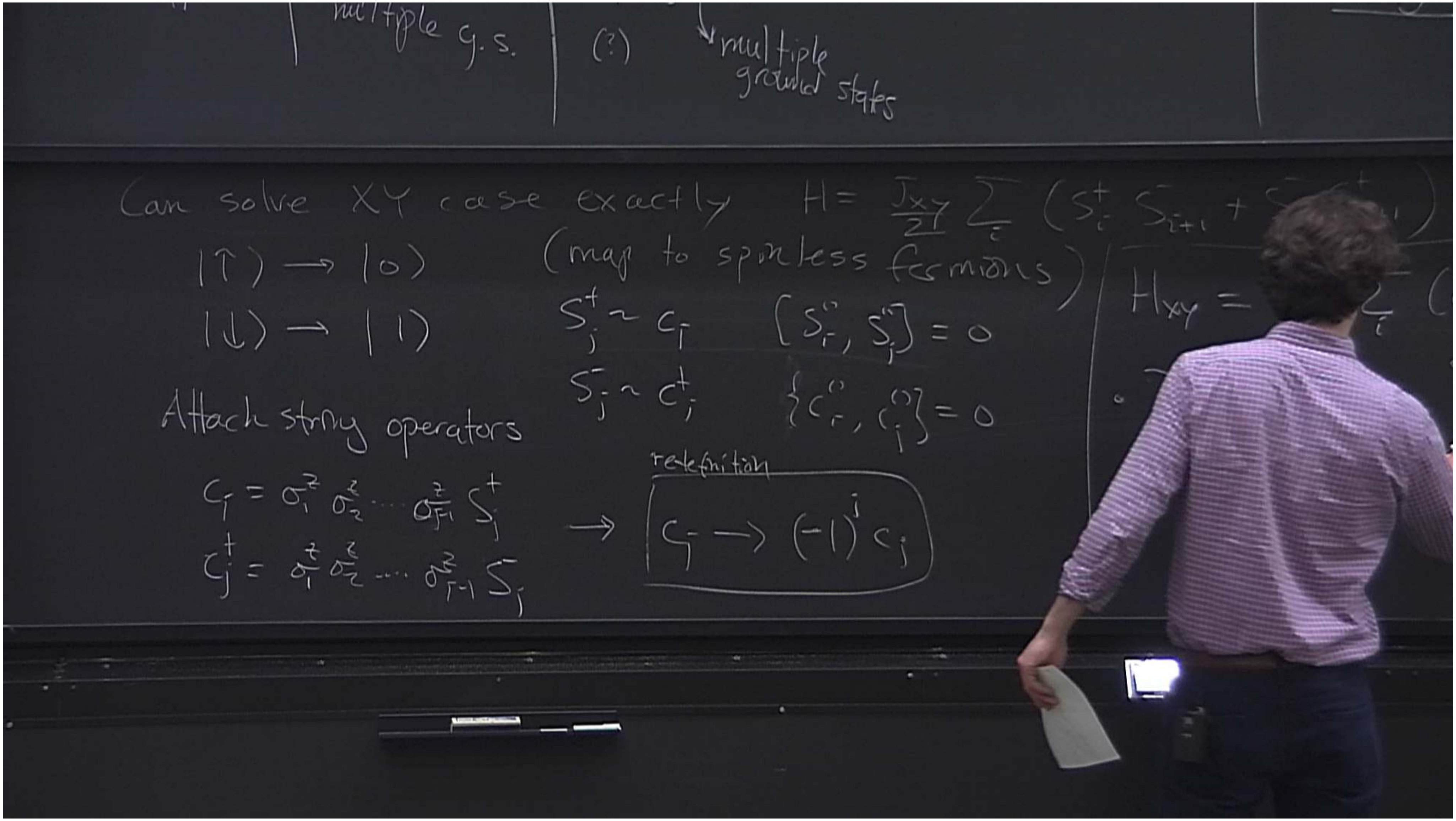
LSMH compatible behaviors (in the $L \to \infty$ limit)

*trivially satisfy LSMH

| | Type of symmetry breaking* | Symmetric |
|---|---|---|
| Gapless ($\Delta \sim 1/L$) | continuous symmetry (Goldstone modes) | • Critical<br>• gapless "spin liquid" |
| Gapped | discrete multiple g.s. | intrinsically topological states<br>(?) ↓ multiple ground states |



Gapless (Δ~½)
continuous symmetry (Goldstone modes?)
• critical
• gapless "spin liquid"
Gapped
discrete multiple g.s.
intrinsically topological states
(?)
multiple ground states
H_XXZ = Σ_i J_xy/2 (S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+) + J_z S_i^z
fm Ising
gapless
afm Ising
J_z/J_xy
-1
0
XY
SU(2), J_z/J_xy = 1
Can solve XY case exactly H = J_xy/2 Σ_i (S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+) = J_xy Σ_i (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y)
|↑⟩ → |0⟩
|↓⟩ → |1⟩
(map to spinless fermions)
S_j^+ ~ c_j
S_j^- ~ c_j^†
[S_i^α, S_j^β] = 0
{c_i^α, c_j^β} = 0
Attach string operators
c_j = σ_1^z σ_2^z ... σ_{j-1}^z S_j^+
c_j^† = σ_1^z σ_2^z ... σ_{j-1}^z S_j^-


multiple g.s.
(?)
multiple ground states
Can solve XY case exactly $H = \frac{J_{xy}}{2} \sum_i (S_i^+ S_{i+1}^- + \ldots)$
(map to spinless fermions)
$|\uparrow\rangle \rightarrow |0\rangle$
$|\downarrow\rangle \rightarrow |1\rangle$
$S_j^+ \sim c_j$
$S_j^- \sim c_j^\dagger$
$[S_i^{(\cdot)}, S_j^{(\cdot)}] = 0$
$\{c_i^{(\cdot)}, c_j^{(\cdot)}\} = 0$
$H_{xy} =$
Attach string operators
$c_j = \sigma_1^z \sigma_2^z \cdots \sigma_{j-1}^z S_j^+$
$c_j^\dagger = \sigma_1^z \sigma_2^z \cdots \sigma_{j-1}^z S_j^-$
redefinition
$c_j \rightarrow (-1)^j c_j$

states

multiple ground states

fm Ising — gapless — afm Ising $\rightarrow J_z/J_{xy}$

$-1$ $\quad 0$ (XY) $\quad SU(2), J_z/J_{xy}=1$

$$H = \frac{J_{xy}}{2}\sum_i \left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+\right) = J_{xy}\sum_i \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y\right) \; ; \; J_{xy} > 0$$

...nless fermions)

$$[S_i^{(\cdot)}, S_j^{(\cdot)}] = 0$$

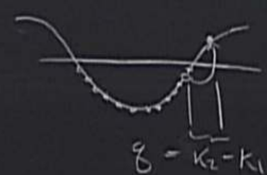
$$\{c_i^{(\cdot)}, c_j^{(\cdot)}\} = 0$$

$$\boxed{\Rightarrow (-1)^j c_j}$$

$$H_{xy} = -\frac{J_{xy}}{2}\sum_i \left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i\right) + \text{const}$$

- free fermions
- spectrum gapless (½ filled band in g.s.)
- no symmetry breaking!

$S_z = 0$ excitations $\quad \hat{\omega}(k_1,k_2) = c_{k_2}^\dagger c_{k_1} = \frac{1}{L}\sum_{j_1 j_2} e^{i(k_2 j_2 - k_1 j_1)} c_{j_2}^\dagger c_{j_1}$

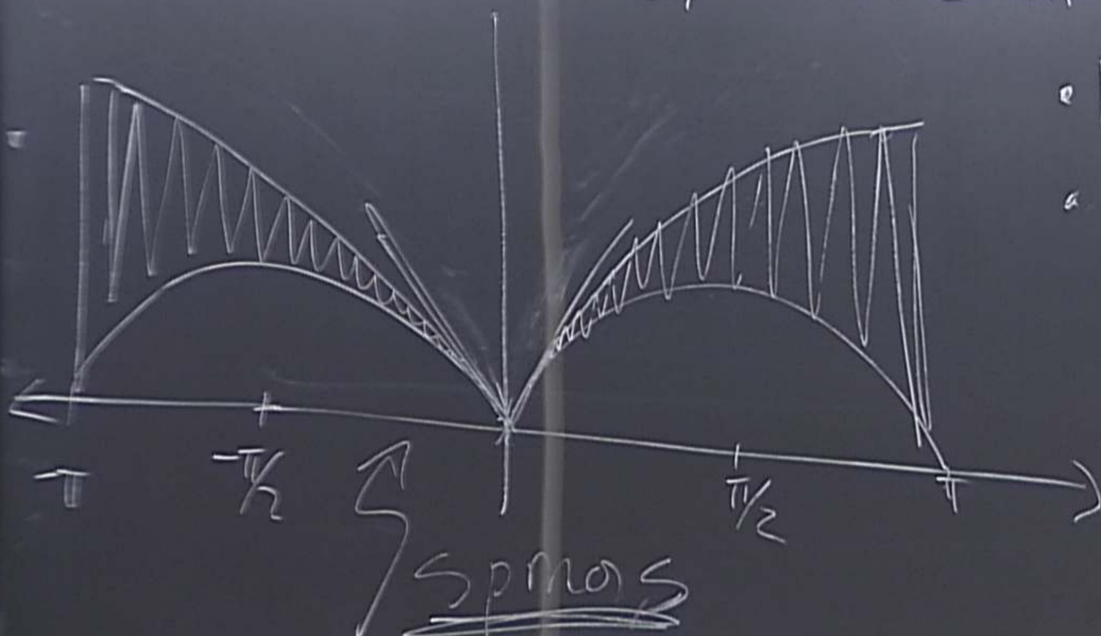



fixed $q$, excitation energy:

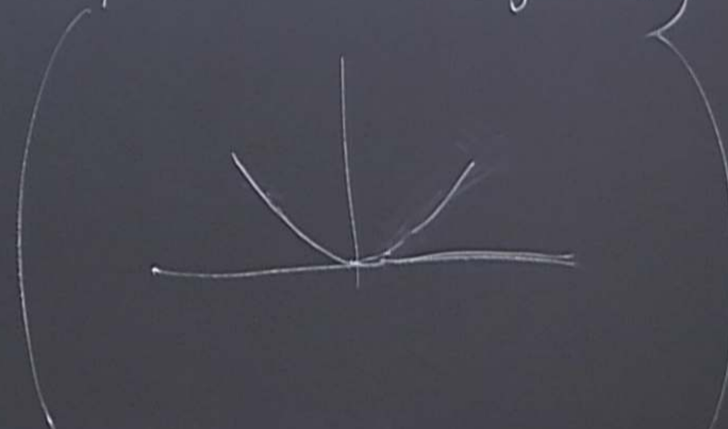
$$\left(-\frac{J_{xy}}{2}\cos(k_1+q)\right) - \left(-\frac{J_{xy}}{2}\cos(k_1)\right)$$
$$= \frac{J_{xy}}{2}\left[\cos(k_1) - \cos(k_1+q)\right]$$

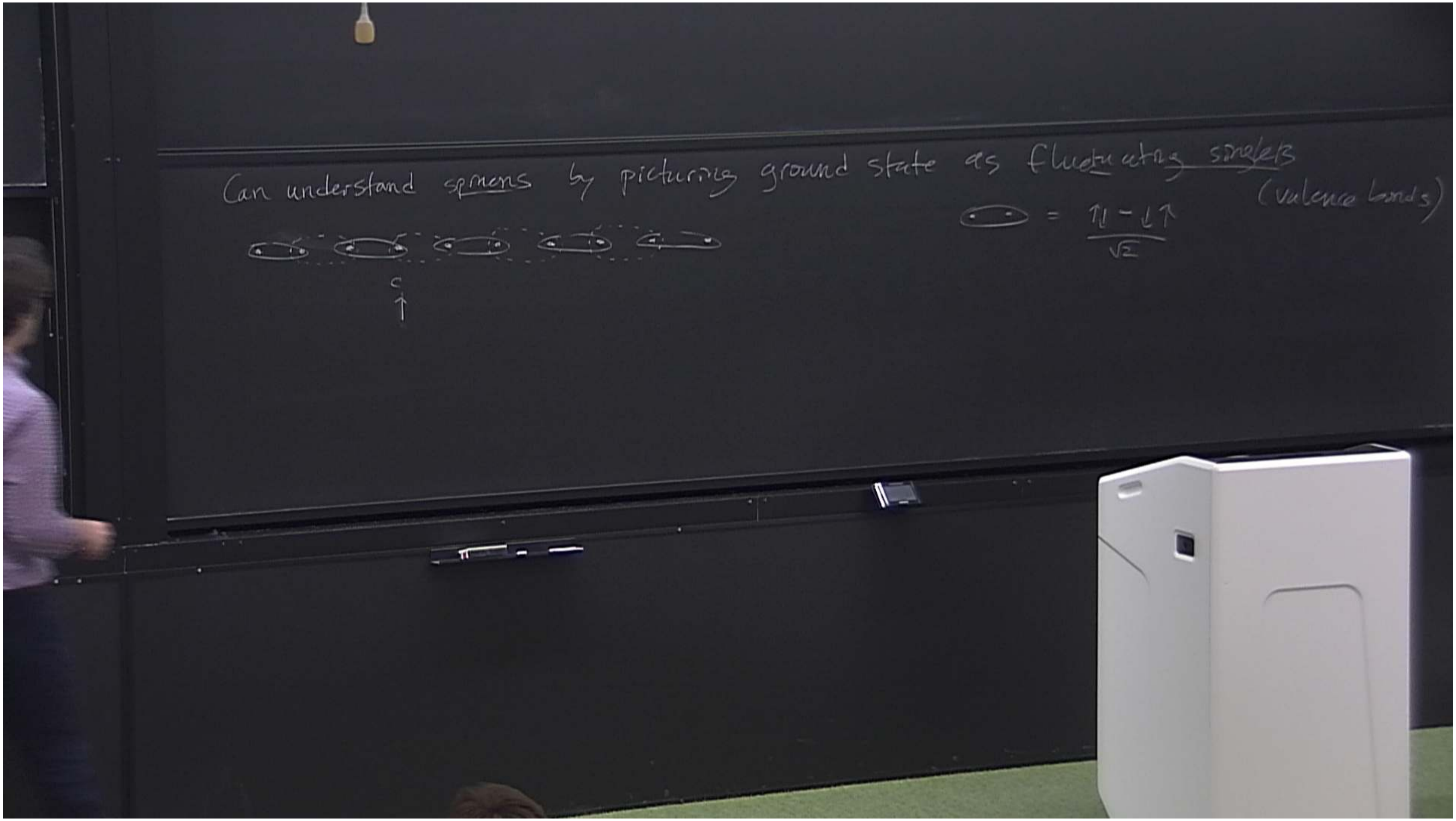
Minimum energy : take $k_1 = \pi/2$ (& equiv)

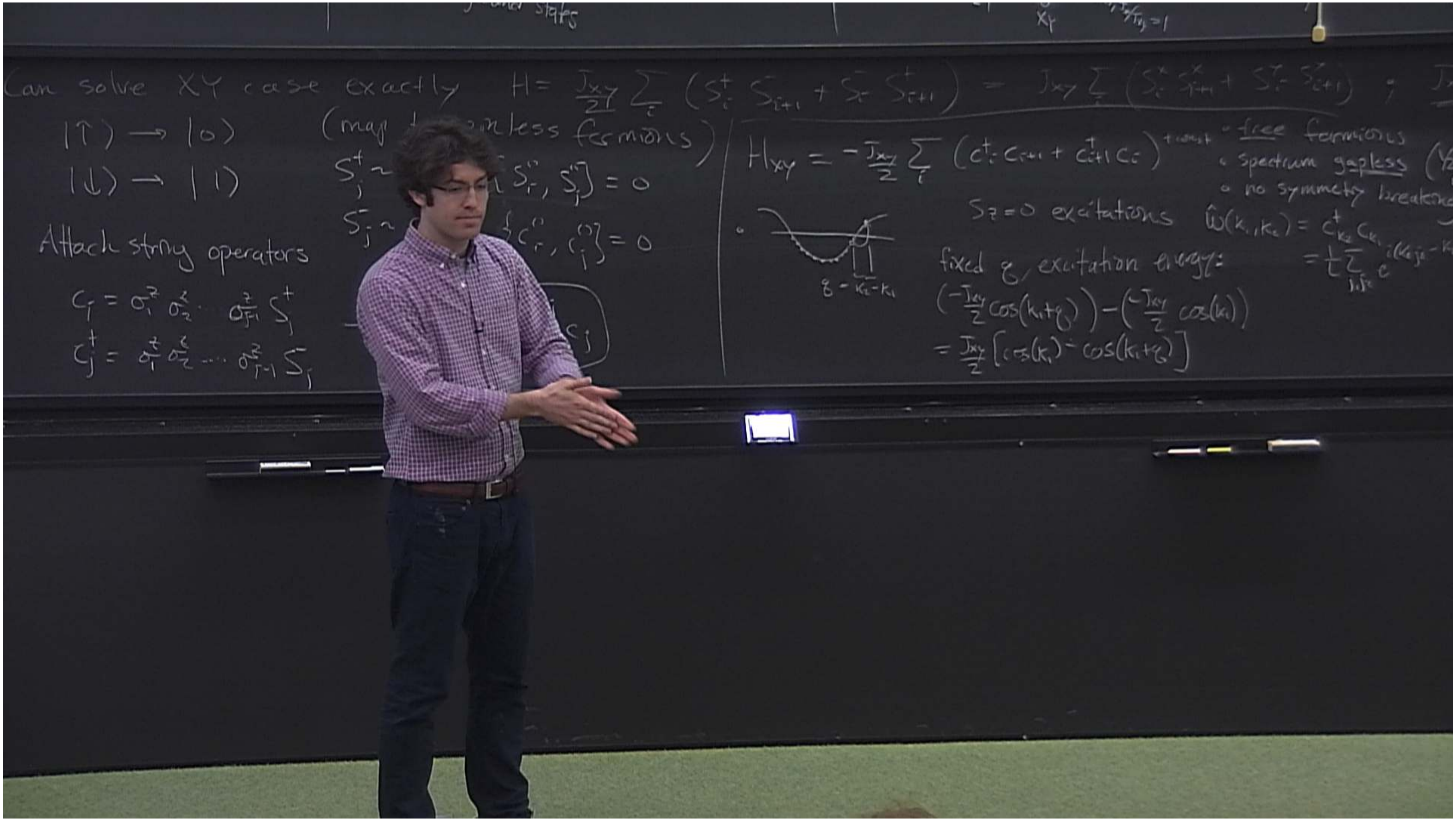
Maximum energy : take $k_1 = \frac{\pi}{2} - \frac{q}{2}$




- linear for small $q$
- vs. spin waves (magnons)




Can understand spinons by picturing ground state as fluctuating singlets
(valence bonds)
= $\frac{\uparrow\downarrow - \downarrow\uparrow}{\sqrt{2}}$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