Title: Explorations in Condensed Matter-9

Date: Mar 26, 2015 10:15 AM

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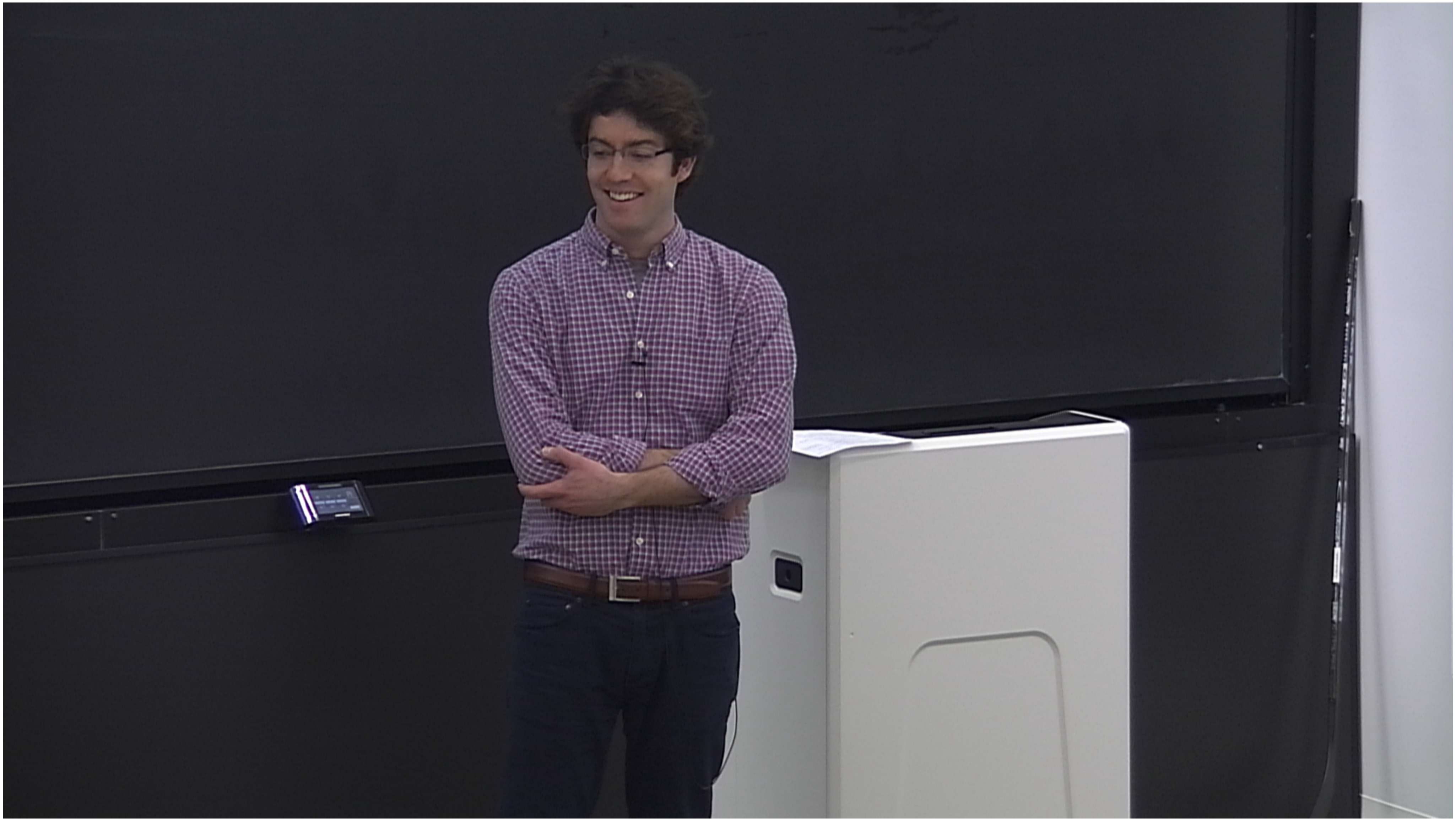
Abstract:



Heisenberg model

Hubbard, $t \ll U$

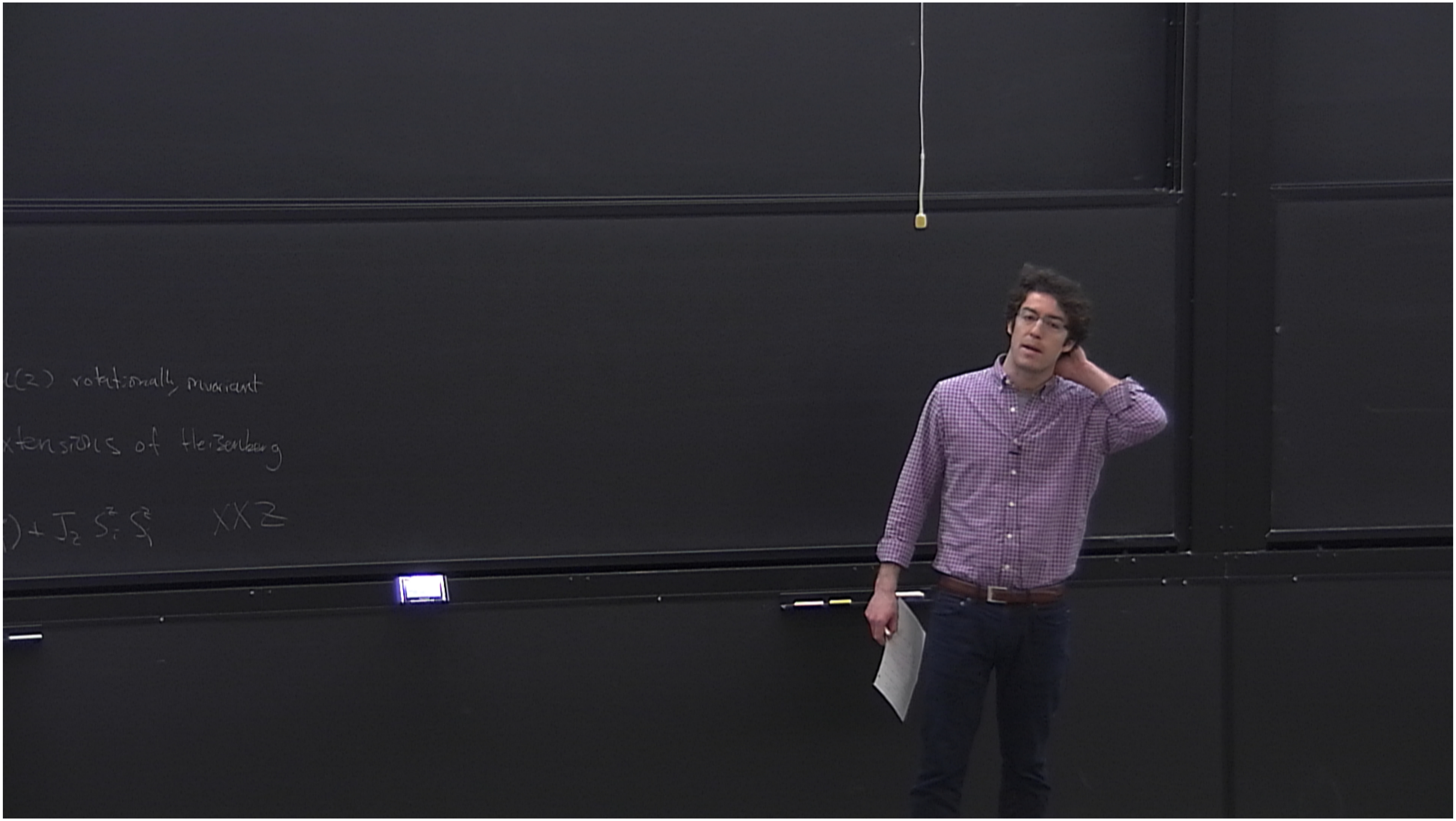
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

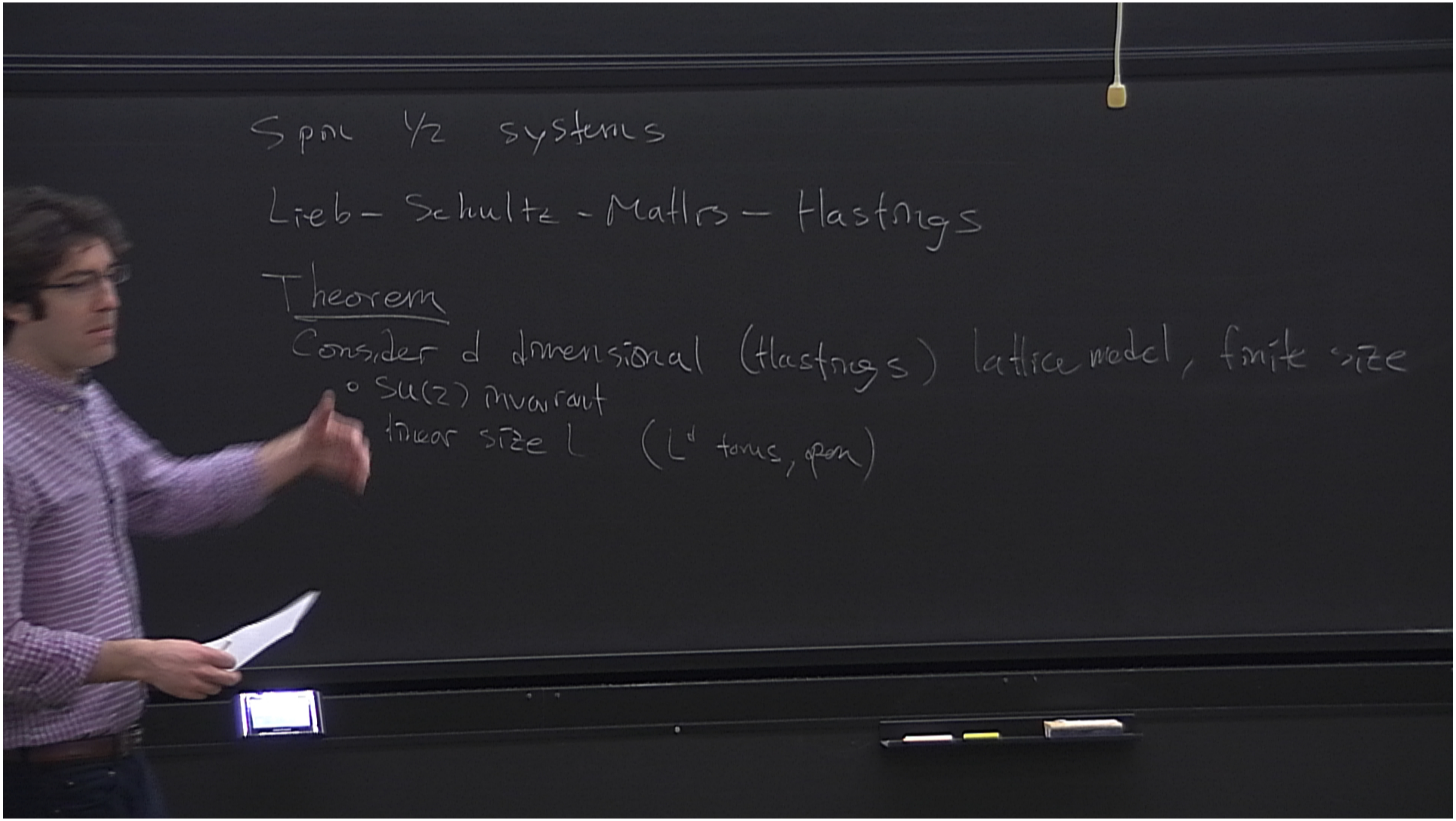
SU(2) rotationally invariant

Real magnetic materials, extensions of Heisenberg

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$H = \sum_{\langle ij \rangle} J_{xy} \left(S_i^x S_j^x + S_i^y S_j^y \right) + J_z S_i^z S_j^z$$


(2) rotationally invariant
xtensions of Heisenberg
XXZ


Spin 1/2 systems
Lieb-Schultz-Mattis-Hastings
Theorem
Consider d dimensional (Hastings) lattice model, finite size
∘ SU(2) invariant
linear size L (L^d torus, open)

Spin 1/2 systems

Lieb-Schultz-Mattis-Hastings

Theorem

Consider d dimensional (Hastings) lattice model, finite size

- SU(2) invariant
- linear size L (L^d torus, open)
- odd number of $S=1/2$'s per unit cell

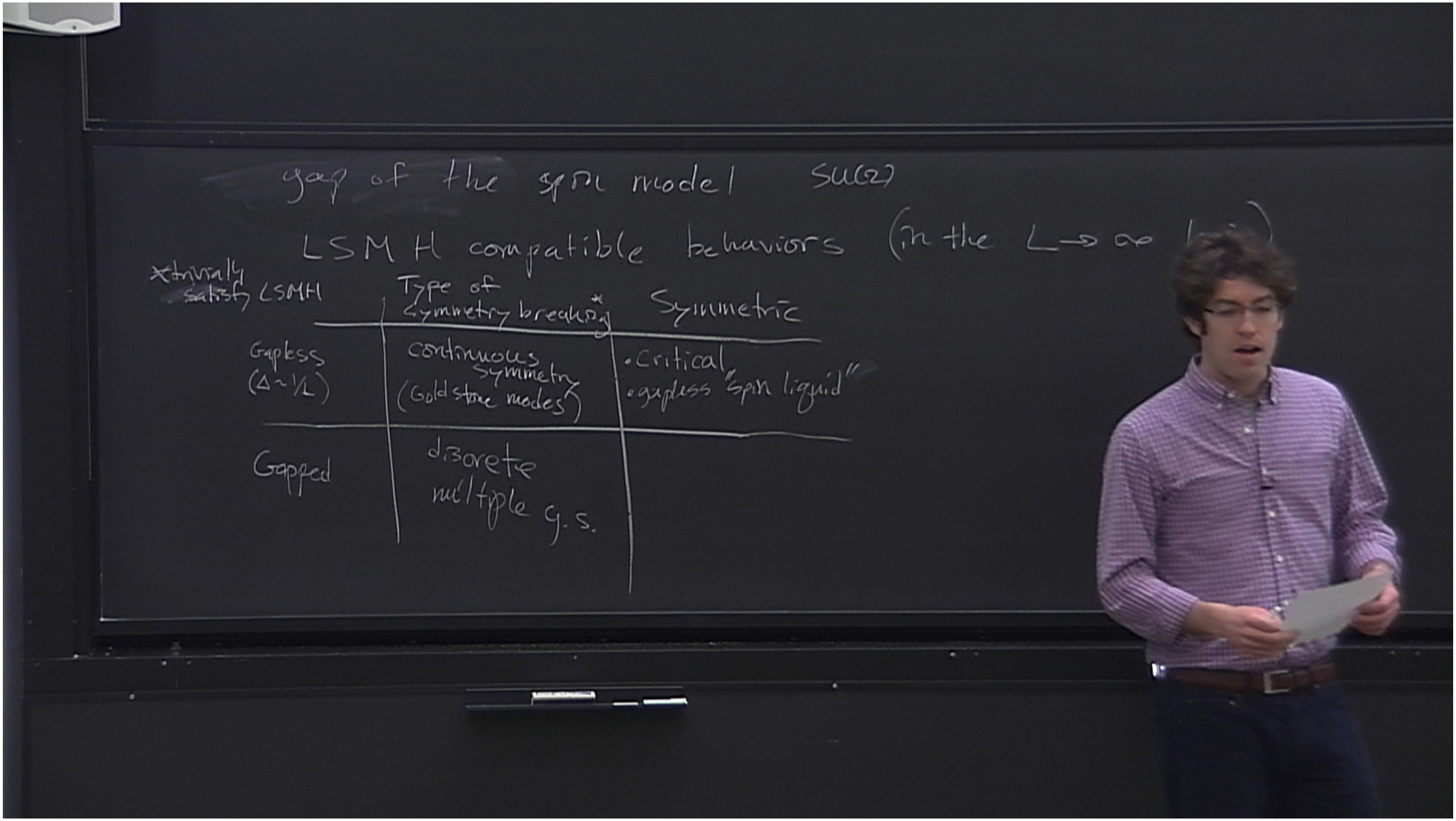
Then the gap to the first excited state $\Delta E = E_1 - E_0 \leq c \ln(L)/L$

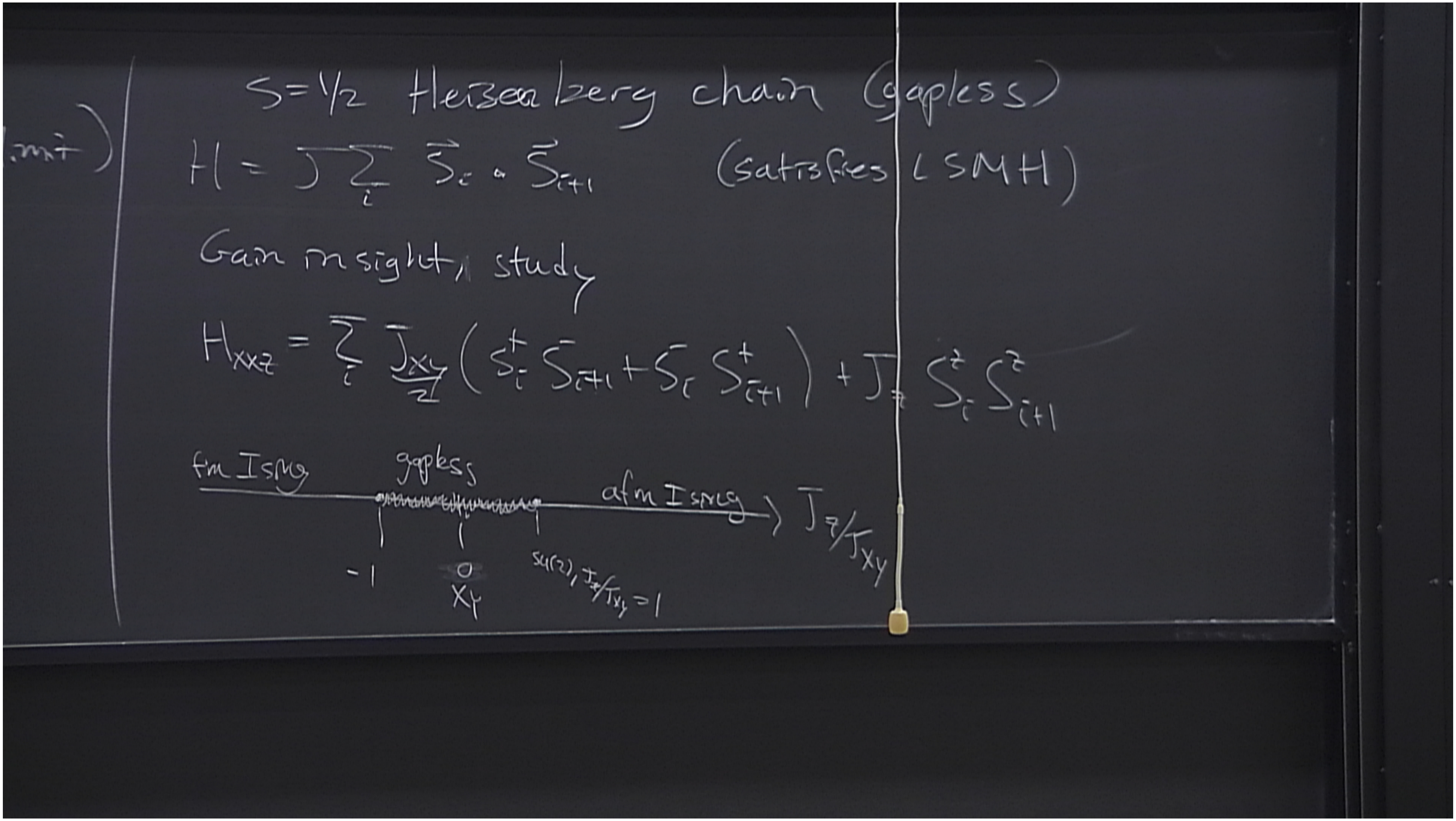
(c indep of L)

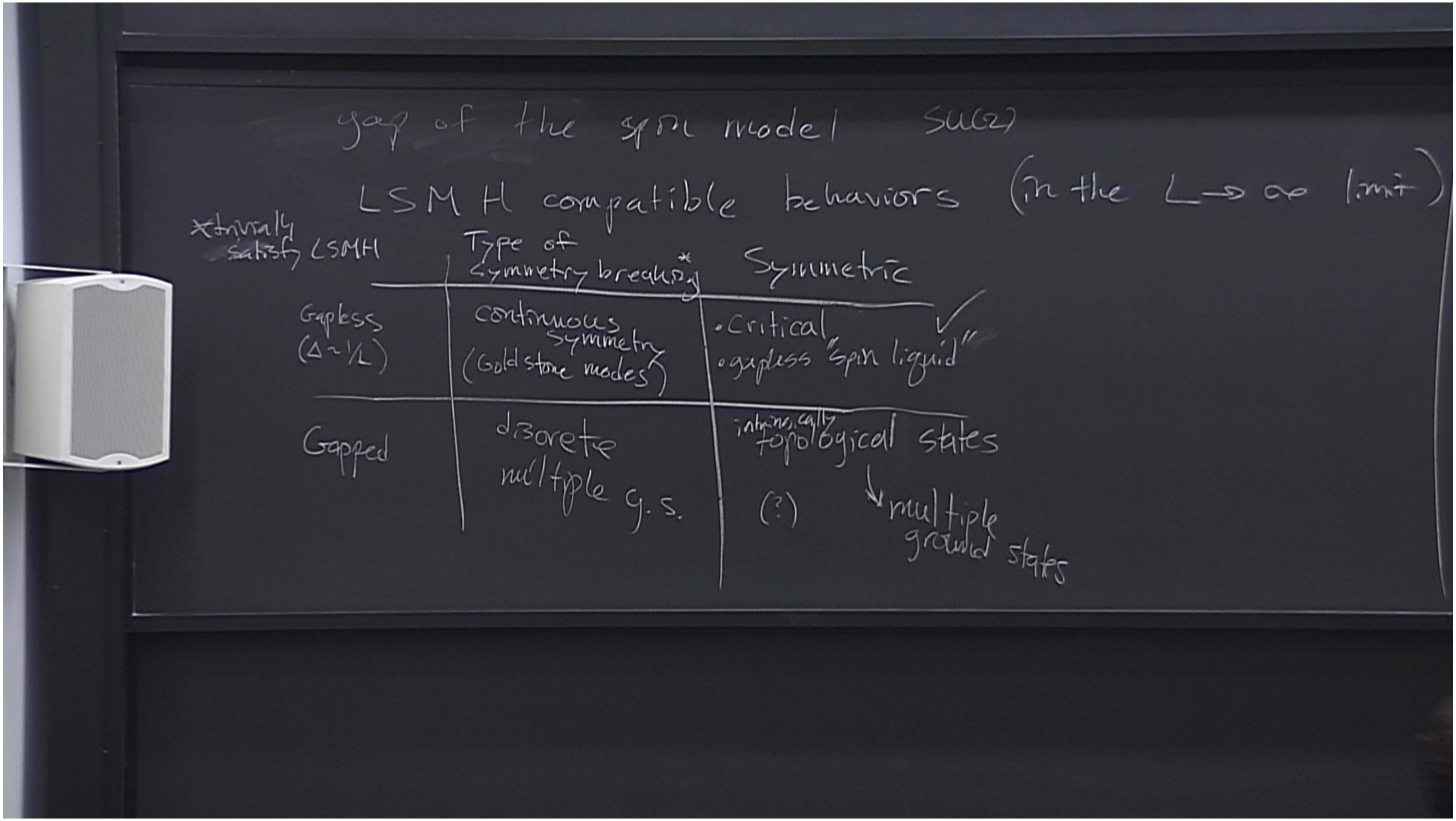
gap of the spin model

LSMH compatible behaviors (in the $L \to \infty$ limit)

	Type of symmetry breaking	Symmetric
Gapless ($\Delta \sim 1/L$)	continuous symmetry	
Gapped		


gap of the spin model SU(2)
LSMH compatible behaviors (in the L → ∞
*trivially satisfy LSMH
Type of symmetry breaking*
Symmetric
Gapless (Δ ~ 1/L)
continuous symmetry (Goldstone modes)
• Critical
• gapless "spin liquid"
Gapped
discrete multiple g.s.


S=1/2 Heisenberg chain (gapless)
$H = J \sum_i \vec{S}_i \cdot \vec{S}_{i+1}$ (satisfies LSMH)
Gain insight, study
$H_{XXZ} = \sum_i \frac{J_{xy}}{2} \left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ \right) + J_z S_i^z S_{i+1}^z$
fm Ising
gapless
afm Ising
J_z / J_{xy}
-1
0
XY
SU(2), $J_z/J_{xy} = 1$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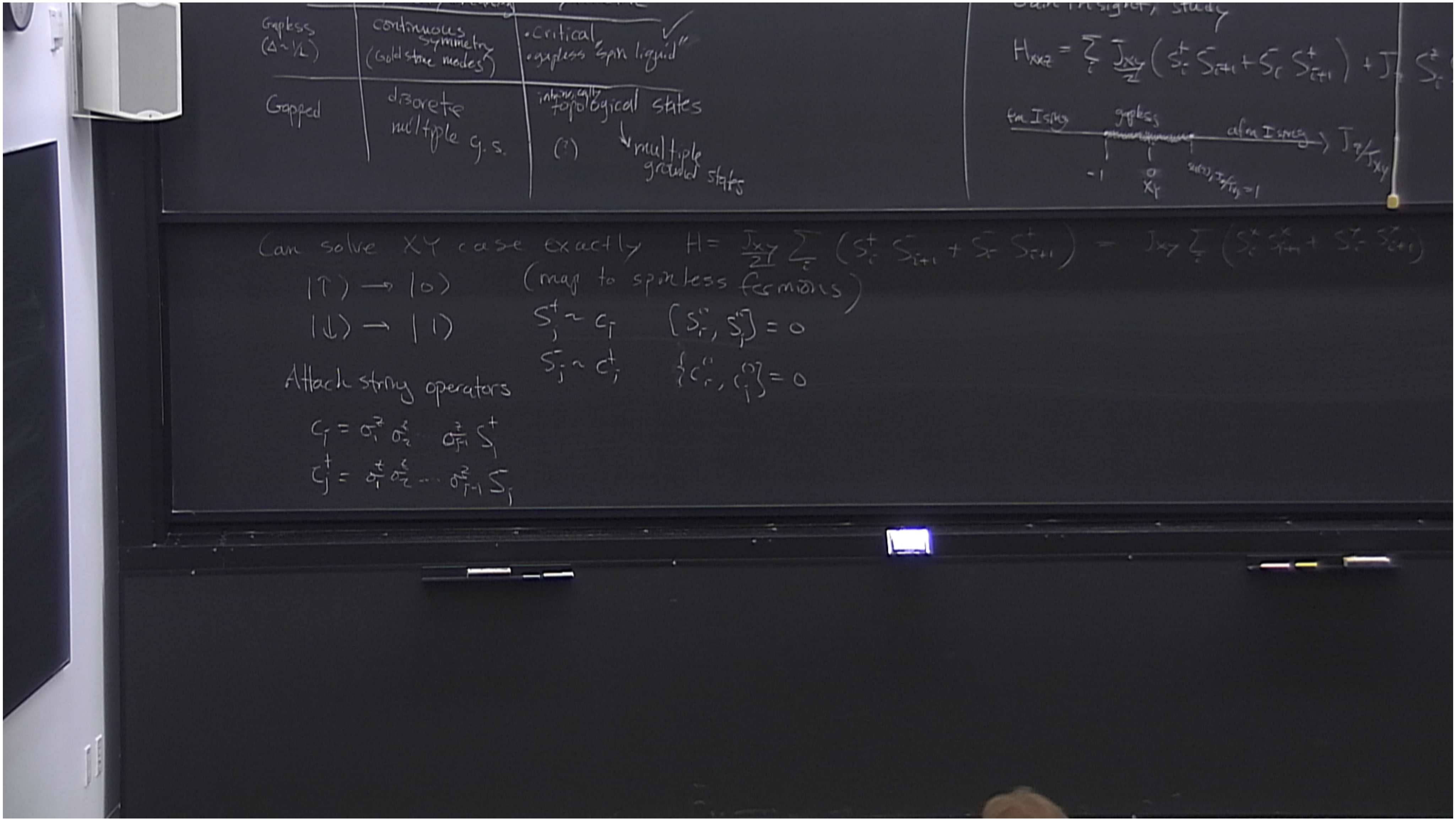
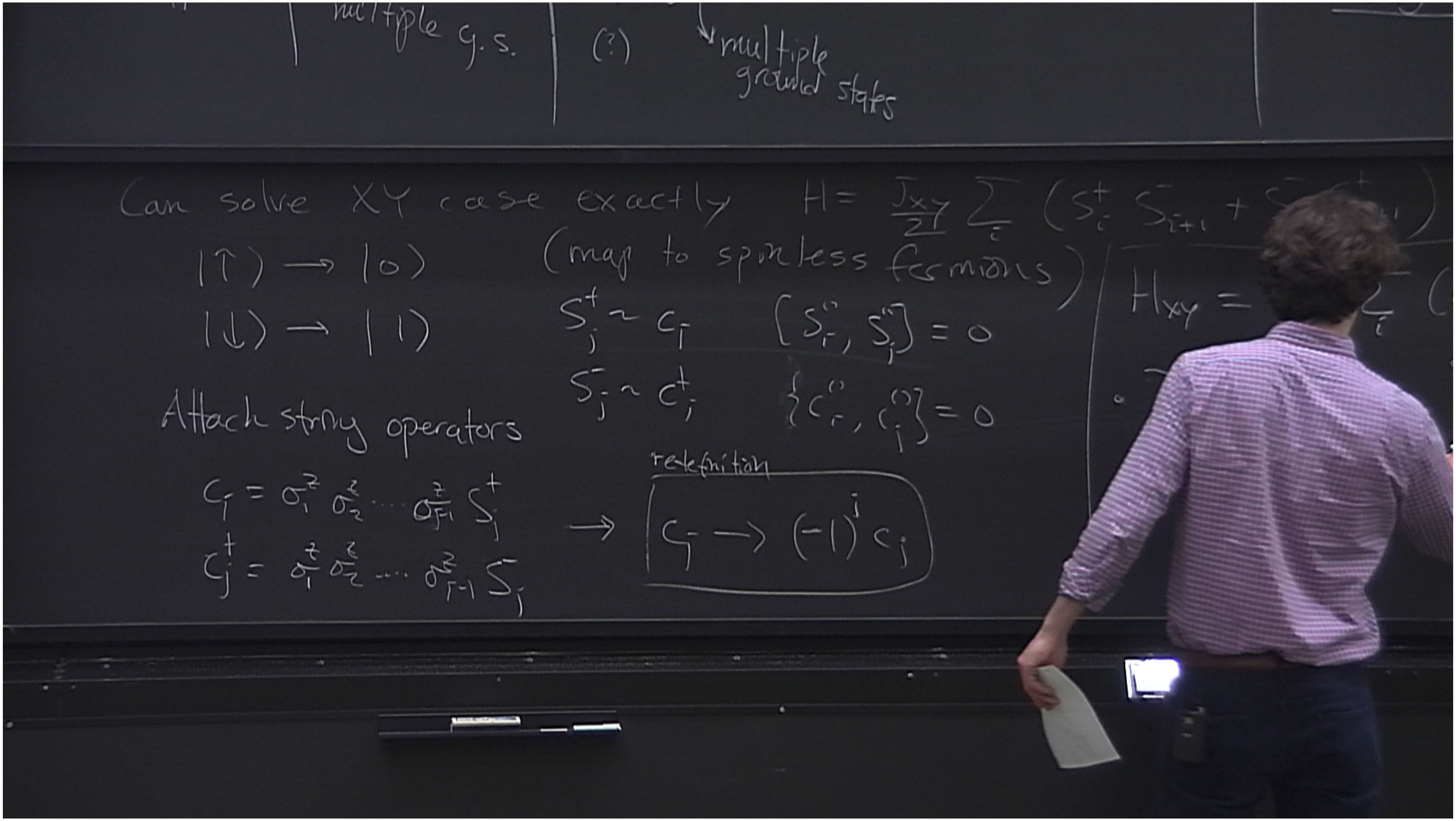
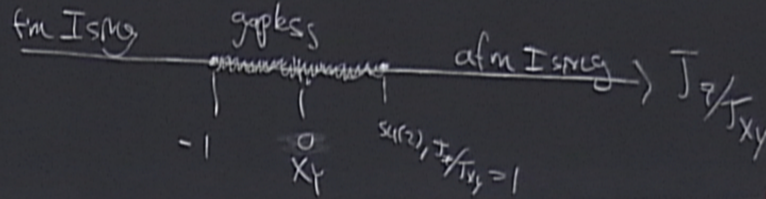
intrinsically topological states
(?)
multiple ground states


Gapless (Δ~1/L) | continuous symmetry (Goldstone modes?) | • critical • gapless "spin liquid"
Gapped | discrete multiple g.s. | intrinsically topological states (?) → multiple ground states
$H_{XXZ} = \sum_i \frac{J_{xy}}{2}(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+) + J_z S_i^z S_{i+1}^z$
fm Ising | gapless | afm Ising → J_z/J_{xy}
-1 | 0 XY | SU(2), $J_z/J_{xy}=1$
Can solve XY case exactly $H = \frac{J_{xy}}{2}\sum_i (S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+) = J_{xy}\sum_i (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y)$
$|\uparrow\rangle \to |0\rangle$ (map to spinless fermions)
$|\downarrow\rangle \to |1\rangle$
$S_j^+ \sim c_j$ $[S_i^\alpha, S_j^\beta] = 0$
$S_j^- \sim c_j^\dagger$ $\{c_i^\alpha, c_j^\beta\} = 0$
Attach string operators
$c_j = \sigma_1^z \sigma_2^z \cdots \sigma_{j-1}^z S_j^+$
$c_j^\dagger = \sigma_1^z \sigma_2^z \cdots \sigma_{j-1}^z S_j^-$


multiple g.s.
(?)
multiple ground states
Can solve XY case exactly H = Jxy/2 Σi (S+i S−i+1 + S−i S+i+1)
|↑⟩ → |0⟩
|↓⟩ → |1⟩
(map to spinless fermions)
S+j ~ cj
S−j ~ c†j
[S(i), S(j)] = 0
{c(i), c(j)} = 0
Hxy =
Attach string operators
cj = σz1 σz2 ⋯ σzj−1 S+j
c†j = σz1 σz2 ⋯ σzj−1 S−j
→
redefinition
cj → (−1)j cj



states

multiple ground states

$$H = \frac{J_{xy}}{2}\sum_i \left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+\right) = J_{xy}\sum_i \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y\right) \; ; \quad J_{xy} > 0$$

...nless fermions)

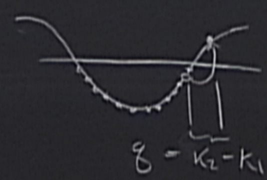
$[S_i^{(-)}, S_j^{(-)}] = 0$

$\{c_i^{()}, c_j^{()}\} = 0$

$\to (-1)^j c_j$

$$H_{xy} = -\frac{J_{xy}}{2}\sum_i \left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i\right) + \text{const}$$

- free fermions
- spectrum gapless (1/2 filled band in g.s.)
- no symmetry breaking!

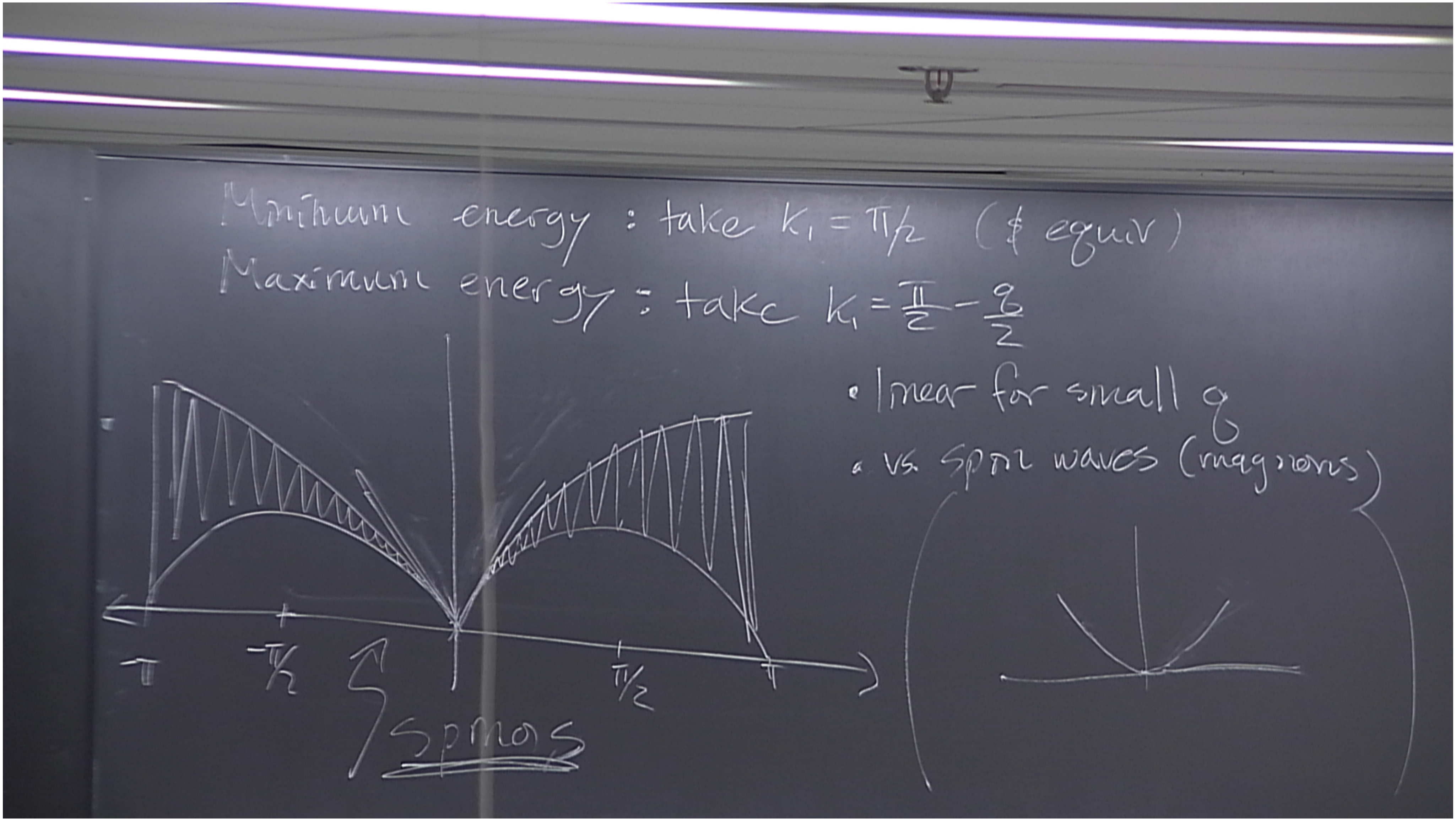


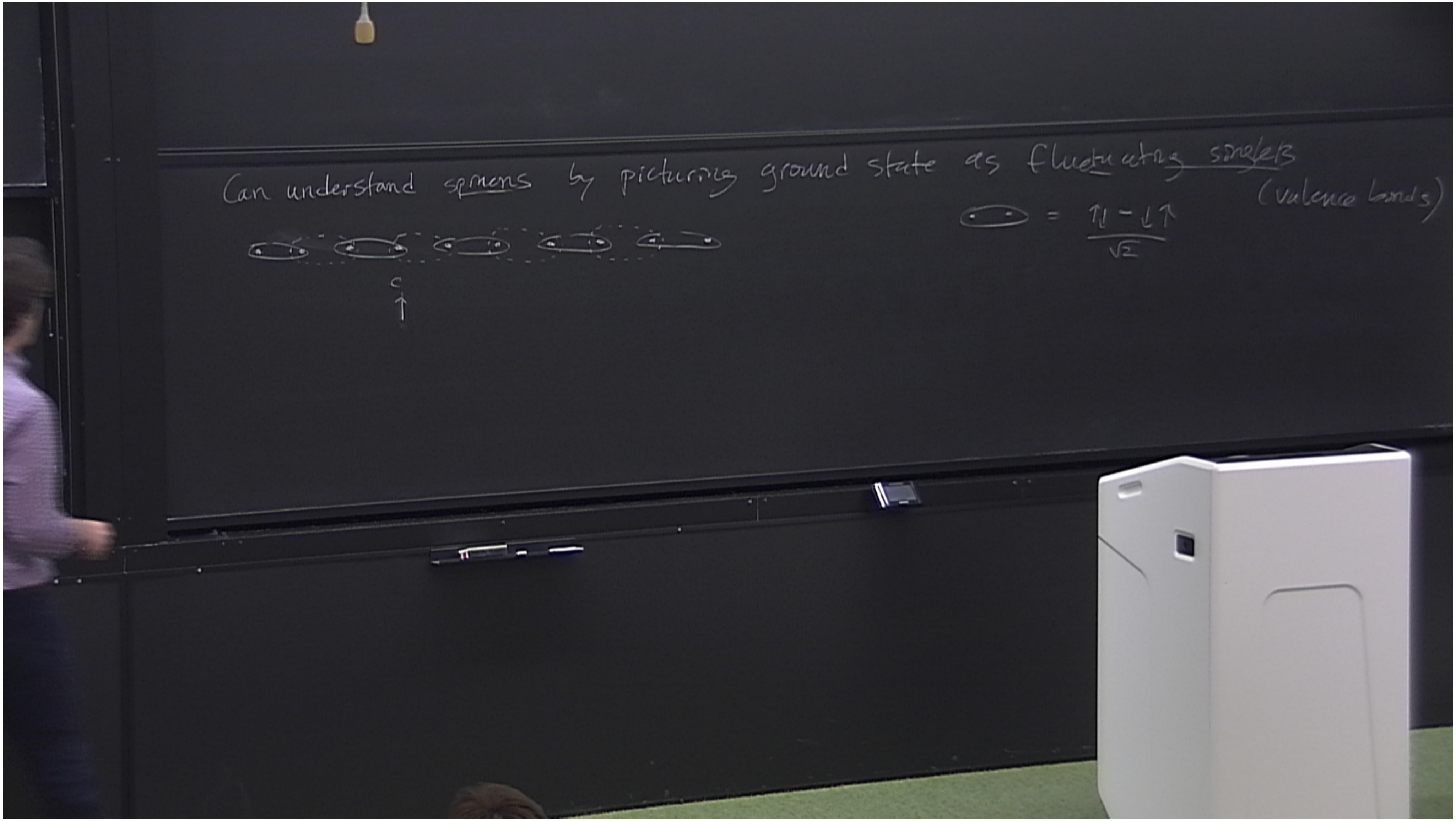
$S_z = 0$ excitations $\hat{\omega}(k_1, k_2) = c_{k_2}^\dagger c_{k_1} = \frac{1}{L}\sum_{j_1 j_2} e^{i(k_2 j_2 - k_1 j_1)} c_{j_2}^\dagger c_{j_1}$

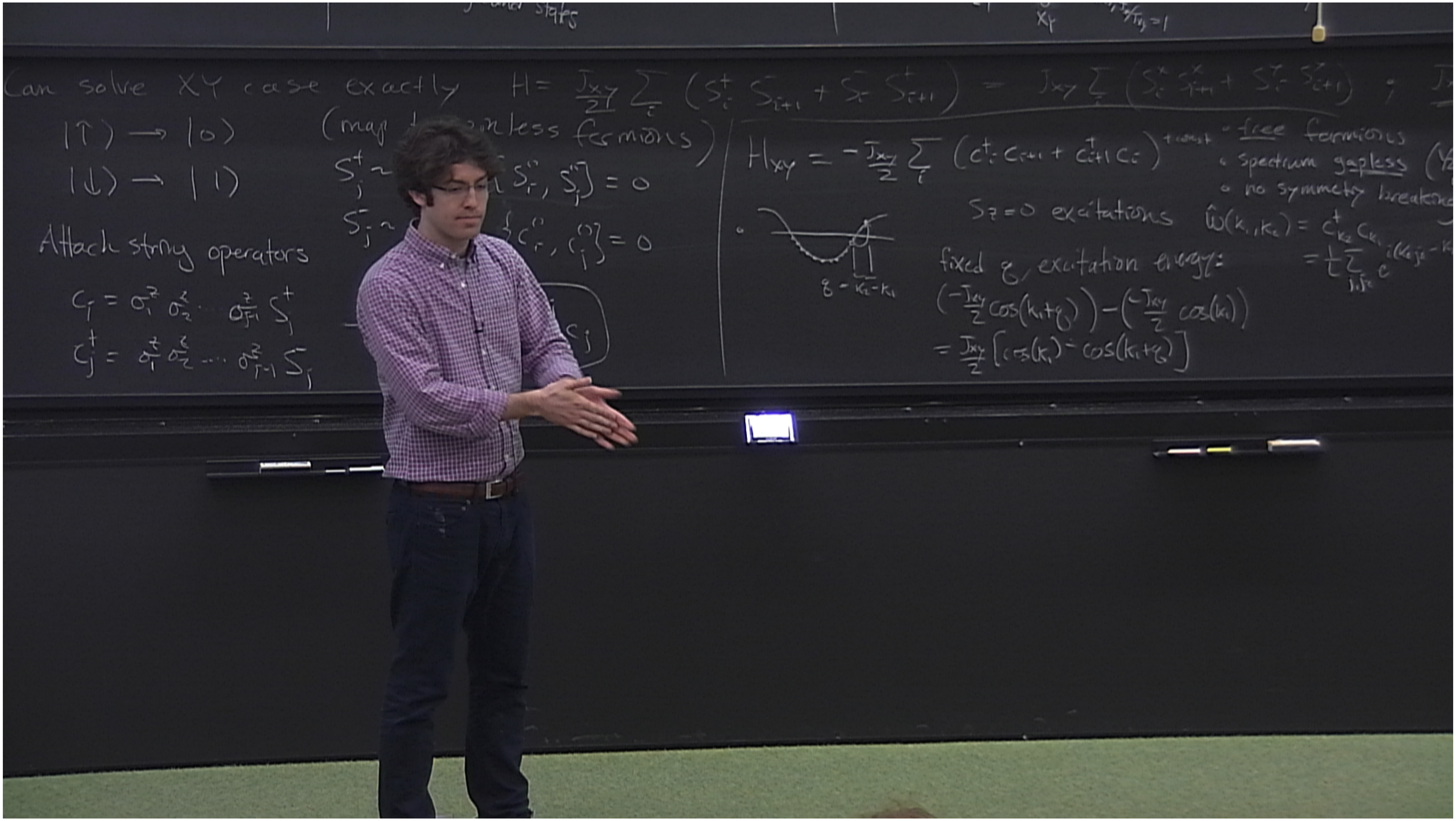
fixed q, excitation energy:

$$\left(-\frac{J_{xy}}{2}\cos(k_1+q)\right) - \left(-\frac{J_{xy}}{2}\cos(k_1)\right)$$

$$= \frac{J_{xy}}{2}\left[\cos(k_1) - \cos(k_1+q)\right]$$


Minimum energy : take $k_1 = \pi/2$ (& equiv)
Maximum energy : take $k_1 = \frac{\pi}{2} - \frac{q}{2}$
• linear for small q
• vs. spin waves (magnons)
$-\pi$
$-\pi/2$
$\pi/2$
Spinons


Can understand spinons by picturing ground state as fluctuating singlets
(valence bonds)
= ↑↓ − ↓↑ / √2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