

Title: Explorations in Condensed Matter-7

Date: Mar 24, 2015 10:15 AM

URL: <http://pirsa.org/15030042>

Abstract:



$$H = \int \psi_{z\sigma}^\dagger \left[-\frac{1}{2} \nabla^2 + U(\vec{r}) \right] \psi_{z\sigma} + \frac{1}{2} \int U_{cc}(r-r') \hat{n}(r) \hat{n}(r') \rightarrow H = -\sum_{ij\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_{\substack{ij\sigma\sigma' \\ \sigma\sigma'}} V_{ij\sigma\sigma'} c_{i\sigma}^\dagger c_{j\sigma} c_{i\sigma'}^\dagger c_{j\sigma'}$$

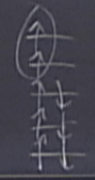
Coulomb exchange

$$\sum_{\sigma\sigma'} V_{ij\sigma\sigma'} c_{i\sigma}^\dagger c_{j\sigma} c_{j\sigma'}^\dagger c_{i\sigma'} = - \sum_{\sigma\sigma'} V_{ij\sigma\sigma'} c_{i\sigma}^\dagger c_{i\sigma'} c_{j\sigma'}^\dagger c_{j\sigma} = -2V_{ij\sigma\sigma'} \left(\vec{S}_i \cdot \vec{S}_j + \frac{1}{4} \hat{n}_i \cdot \hat{n}_j \right)$$

$\leadsto -J_H \vec{S}_i \cdot \vec{S}_j$

$J_H = V_{ij\sigma\sigma'} = \frac{1}{2} \int_{r,r'} U_{cc}(r-r') (\phi_i^\dagger(r) \phi_j(r)) (\phi_j^\dagger(r') \phi_i(r')) \Big|_{S_i^z \uparrow \uparrow} \Big|_{S_j^z \uparrow \uparrow} = \frac{1}{2} \sum_{\sigma\sigma'} c_{i\sigma}^\dagger c_{j\sigma} c_{j\sigma'}^\dagger c_{i\sigma'}$

\leadsto ferromagnetic



$N_i^z \uparrow$

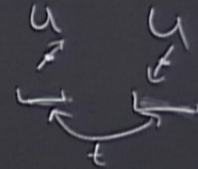
\leadsto generally positive

$N_i = G_a z S_4$

\leadsto Hund's first rule

Kinetic exchange

interplay between kinetic and interactions
(+potential)



minimal model $\hat{H} = -t(c_1^\dagger c_2 + c_2^\dagger c_1) + U \hat{n}_{1\uparrow} \hat{n}_{1\downarrow} + U \hat{n}_{2\uparrow} \hat{n}_{2\downarrow}$, limit $t \ll U$

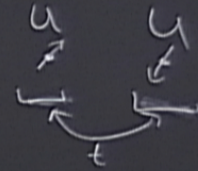
$$\Psi_{\pm}(\vec{r}_1, \vec{r}_2) = \left(\phi_1(\vec{r}_1) \phi_2(\vec{r}_2) \pm \phi_2(\vec{r}_1) \phi_1(\vec{r}_2) \right) \times \text{spin part}$$

\rightarrow singlet
 \rightarrow $m^z=0$ triplet

Compute matrix elements

Kinetic exchange

interplay between kinetic and interactions
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Compute matrix elements

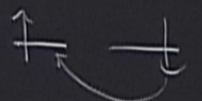
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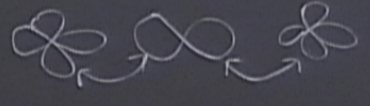
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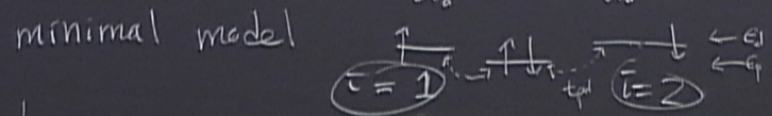
Compute matrix elements $\langle \Psi_a | \hat{H} | \Psi_b \rangle$ (overlaps $\langle \Psi_a | \Psi_b \rangle$) $a, b = \pm$

\hookrightarrow diagonalize: lower energy is Ψ_{\pm} , spatially symmetric, spin anti-symmetric
 perturb in t/U

 \rightarrow only possible if spins anti-aligned, Pauli exclusion

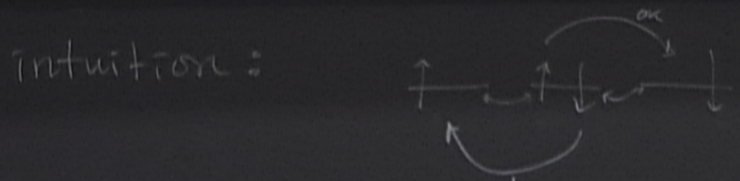
Superexchange → notes by Erik Koch <http://www.cond-mat.de/events/correl>

many materials → active orbitals connected by "spacer" 



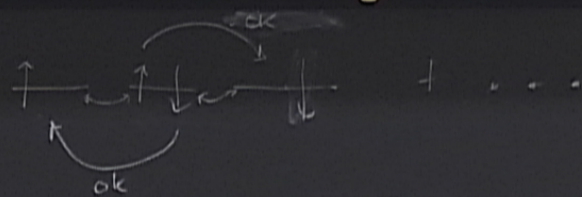
low energy subspace $\langle n_{z1} n_{z2} \rangle = 0$ pair, no U_d

$\hat{H} \rightarrow \hat{P} \hat{H} \hat{P} \sim J \hat{S}_1 \cdot \hat{S}_2$ $J \sim \frac{t^4}{U_d^3} > 0$ antiferromagnet



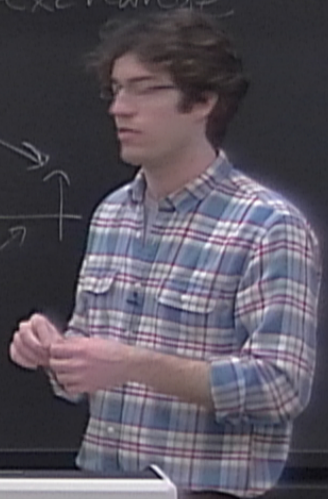
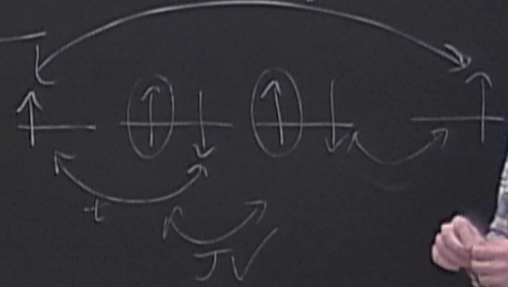
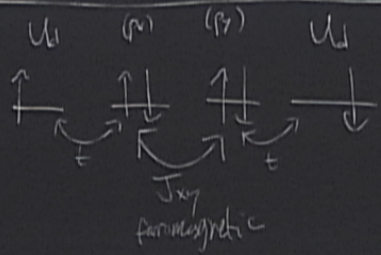
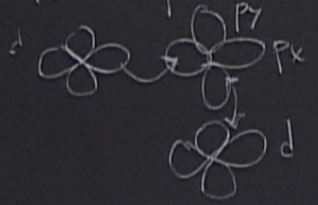
low energy subspace $\langle n_{i\uparrow} n_{i\downarrow} \rangle = 0$ pair no U_d
 $\hat{H} \rightarrow \hat{P} \hat{H} \hat{P} \sim J \vec{S}_1 \cdot \vec{S}_2$ $J \sim \frac{t^4}{U_d^3} > 0$ antiferromagnet

Intuition:



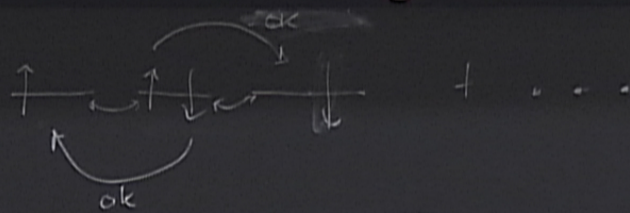
180° superexchange
ferromagnetic

90° superexchange



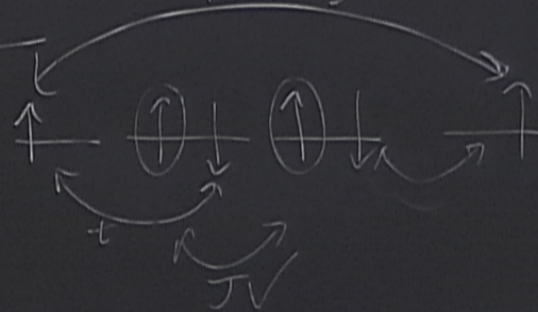
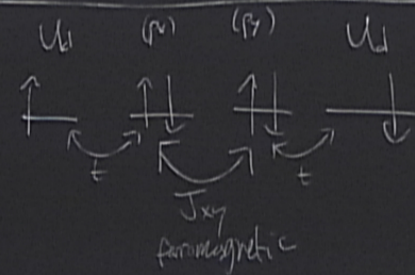
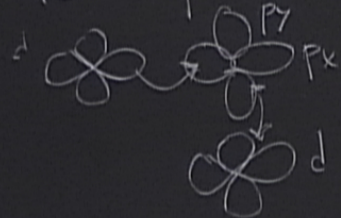
$\hat{H} \rightarrow \hat{P}\hat{H}\hat{P} \sim J\vec{S}_1 \cdot \vec{S}_2$
 $\langle n_{\uparrow} n_{\downarrow} \rangle = 0$
 $J \sim \frac{t^4}{U^3} > 0$ antiferromagnet

Intuition:

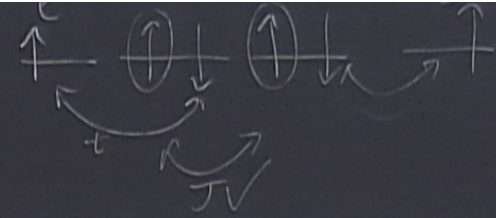
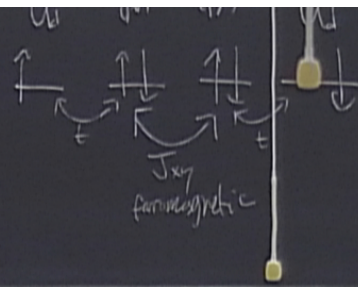
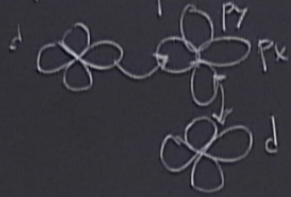


180° superexchange
 ferromagnetic ~ weaker than afm

90° superexchange



90° superexchange

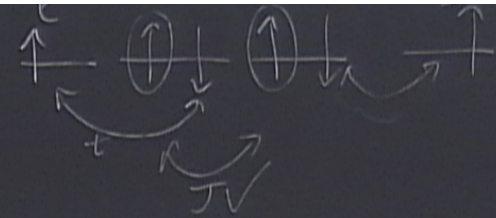
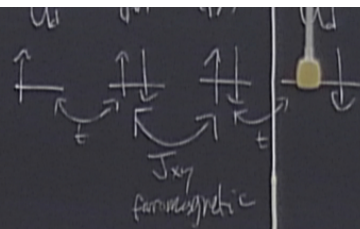
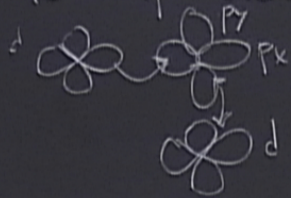


Hubbard model → simplification

$$\hat{H} = -t \sum_{\langle ij \rangle \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_j n_{j\uparrow} n_{j\downarrow}$$

difficult, rich, (too special?), behavior depends on $\begin{cases} \checkmark \text{ dimensionality} \\ \checkmark \text{ lattice geometry, connectivity} \end{cases}$

90° superexchange



Hubbard model → simplification

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difficult, rich, (too special?), behavior depends on

"half filling" ⇒ # electrons = # sites $\uparrow \downarrow \uparrow \downarrow$

gapping, more away from half filling

{ dimensionality
lattice geometry, connectivity