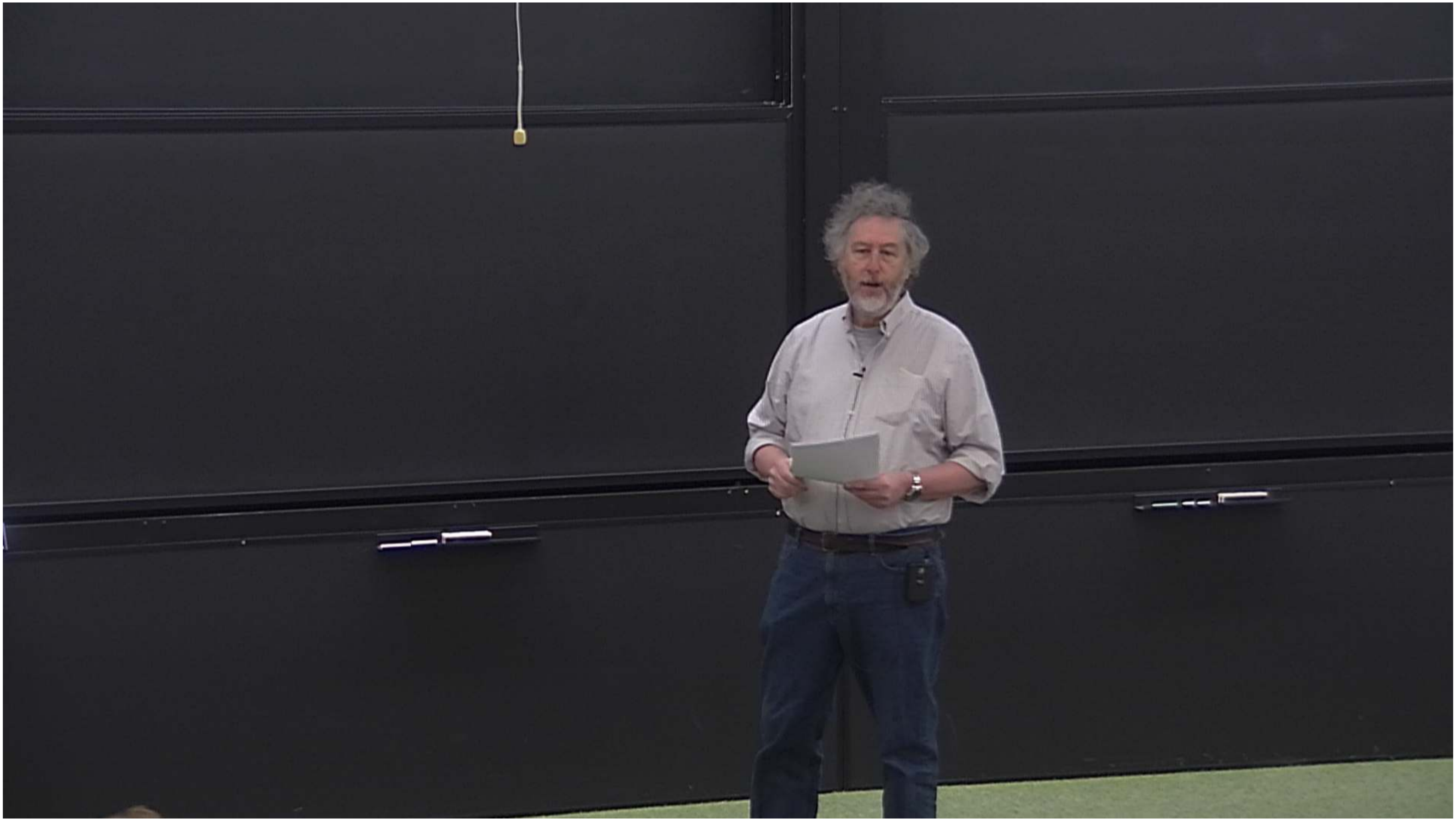
Title: Explorations in Quantum Information-7

Date: Mar 24, 2015 09:00 AM

URL: http://pirsa.org/15030026

Abstract:



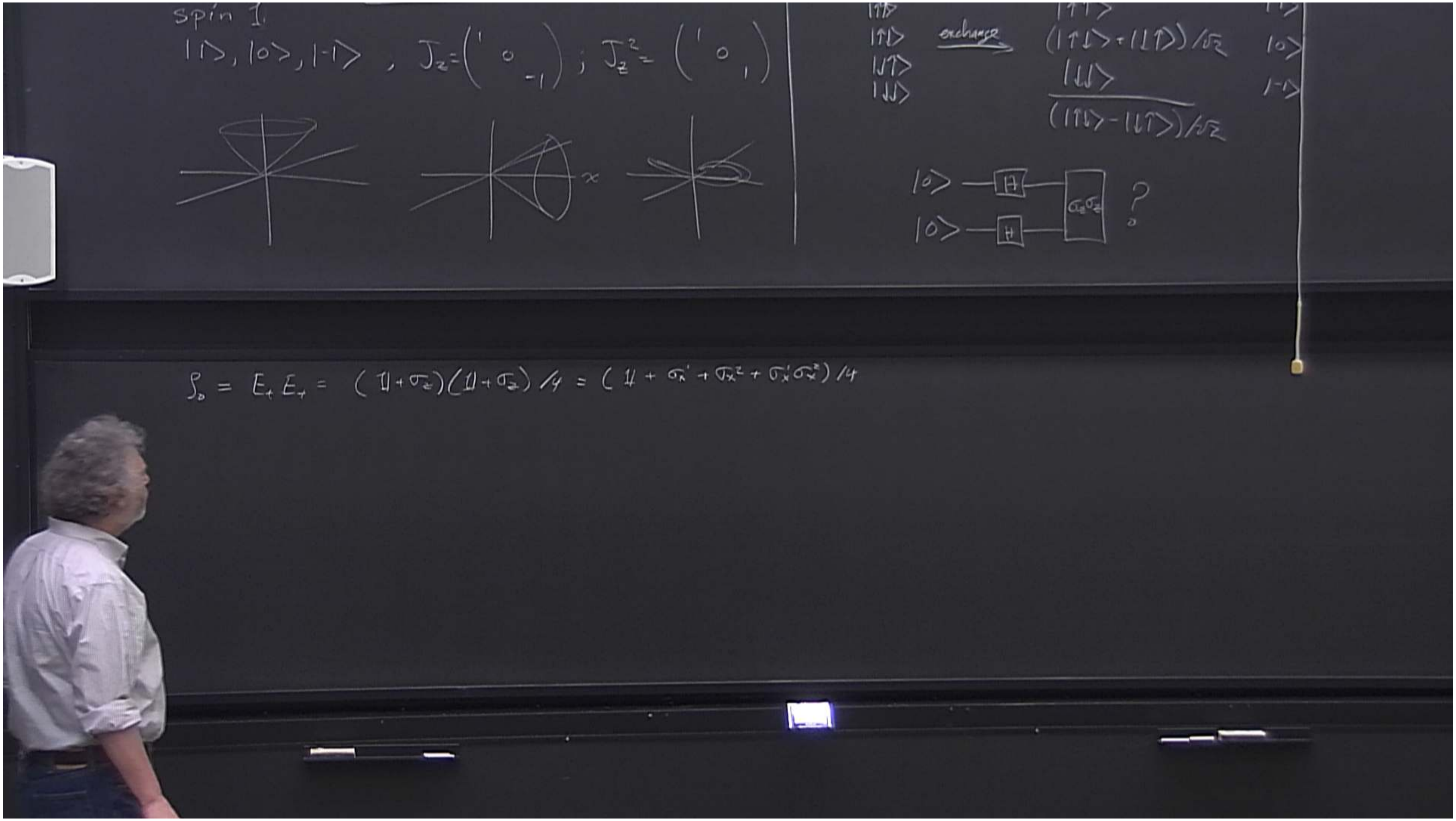
Non-linear operator

spin 1:

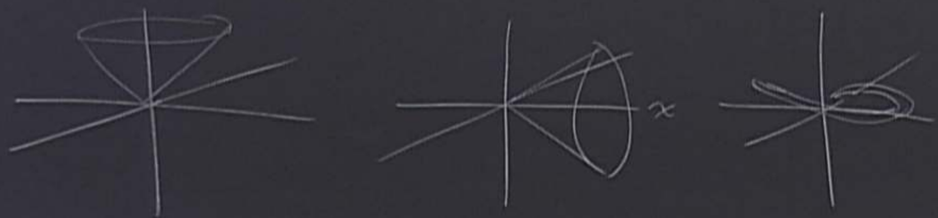
$|1\rangle, |0\rangle, |-1\rangle$; $J_z = \begin{pmatrix} 1 & & \\ & 0 & \\ & & -1 \end{pmatrix}$; $J_z^2 = \begin{pmatrix} 1 & & \\ & 0 & \\ & & 1 \end{pmatrix}$

spin 1 subspace of 2 spin 1/2.

$$\begin{array}{lcll}
|\uparrow\uparrow\rangle & & |\uparrow\uparrow\rangle & |1\rangle \\
|\uparrow\downarrow\rangle & \xrightarrow{\text{exchange}} & (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2} & |0\rangle \\
|\downarrow\uparrow\rangle & & |\downarrow\downarrow\rangle & |-1\rangle \\
\cline{3-3}
|\downarrow\downarrow\rangle & & (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2} &
\end{array}$$


spin 1
|1>, |0>, |-1> , J_z = (1 0 -1) ; J_z^2 = (1 0 1)
x
|↑↑>
|↑↓>
|↓↑>
|↓↓>
exchange
|↑↑>
(|↑↓> + |↓↑>)/√2
|↓↓>
(|↑↓> - |↓↑>)/√2
|1>
|0>
|-1>
|0> — H —
|0> — H —
σ_z σ_z
?
ρ_0 = E_+ E_+ = (𝟙 + σ_z)(𝟙 + σ_z)/4 = (𝟙 + σ_x^1 + σ_x^2 + σ_x^1 σ_x^2)/4

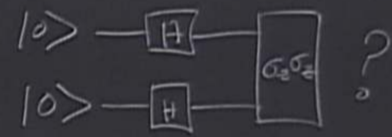
$|1\rangle, |0\rangle, |-1\rangle$; $J_z = \begin{pmatrix} 0 & \\ & -1 \end{pmatrix}$; $J_z^2 = \begin{pmatrix} 0 & \\ & 1 \end{pmatrix}$



$|\downarrow\uparrow\rangle$
$|\downarrow\downarrow\rangle$

$|\downarrow\downarrow\rangle$ $\quad |-1\rangle$

$(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



$$\rho_0 = E_+ E_+ = (\mathbb{1} + \sigma_z)(\mathbb{1} + \sigma_z)/4 = (\mathbb{1} + \sigma_z^1 + \sigma_z^2 + \sigma_z^1 \sigma_z^2)/4$$

$$e^{-i\frac{\theta}{2}\sigma_z\sigma_z}$$

$$\longrightarrow \frac{1}{4}(\mathbb{1} + \sigma_x^1 + \sigma_x^2 + \sigma_x^1\sigma_x^2) \quad ; \quad [\sigma_x^1\sigma_x^2, \sigma_z^1\sigma_z^2] = 0 \quad ; \quad \sigma_x \mathbb{1} \xrightarrow{\sigma_z\sigma_z} \sigma_x \mathbb{1}\cos(\theta) + \sigma_y\sigma_z\sin(\theta)$$

$$\xrightarrow{\sigma_z\sigma_z} \frac{1}{4}\left(\cos(\theta)(\sigma_x^1 - \sigma_x^2) + \sin(\theta)(\sigma_y^1\sigma_z^2 + \sigma_z^1\sigma_y^2) + \mathbb{1} + \sigma_x\sigma_x\right)$$

Operators for Spin 1

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

9 operators

total spin $\quad J_k = \left(\sigma_k^1 + \sigma_k^2\right), \quad \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = J_x$

total spin squared $\quad J_k^2 = \left(\sigma_k \mathbb{1} + \mathbb{1}\sigma_k\right)\left(\sigma_k \mathbb{1} + \mathbb{1}\sigma_k\right) = 2\mathbb{1} + 2\sigma_k\sigma_k \quad , \quad J_x^2 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

sym bi-linear $\quad J_{kl} = \left(\sigma_k^1 \sigma_l^2 + \sigma_l^1 \sigma_k^2\right) = J_{lk}$

Operators for Spin 1

$$\begin{pmatrix}0&1&1&0\\1&0&0&1\\1&0&0&1\\0&1&1&0\end{pmatrix}$$

9 operators

total spin $J_k = (\sigma^1_k + \sigma^2_k)$ $\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix} = J_x$

total spin squared $J_k^2 = (\sigma_k \mathbb{1} + \mathbb{1}\sigma_k)(\sigma_k \mathbb{1} + \mathbb{1}\sigma_k) = 2\mathbb{1} + 2\sigma_k\sigma_k$, $J_x^2 = \begin{pmatrix}1&0&1\\0&2&0\\1&0&1\end{pmatrix}$

sym bi-linear $J_{kl} = (\sigma^1_k\sigma^2_l + \sigma^1_l\sigma^2_k) = J_{lk}$, $J_{yz} = \begin{pmatrix}0&1&0\\-1&0&-1\\0&1&0\end{pmatrix}$

$J_{yz} \propto [J_x, J_z^2]$

$|0\rangle$ — H —

$$\rho_0 = E_+ E_+ = (\mathbb{1}+\sigma_z)(\mathbb{1}+\sigma_z)/4 = (\mathbb{1} + \sigma_z^1 + \sigma_z^2 + \sigma_z^1\sigma_z^2)/4$$

$$e^{-i\frac{\theta}{2}\sigma_z\sigma_z}$$

$$\longrightarrow \frac{1}{4}\left(\mathbb{1} + \sigma_x^1 + \sigma_x^2 + \sigma_x^1\sigma_x^2\right) \quad ; \quad [\sigma_x^1\sigma_x^2, \sigma_z^1\sigma_z^2] = 0 \quad ; \quad \sigma_x\mathbb{1} \xrightarrow{\sigma_z\sigma_z} \sigma_x\mathbb{1}\cos$$

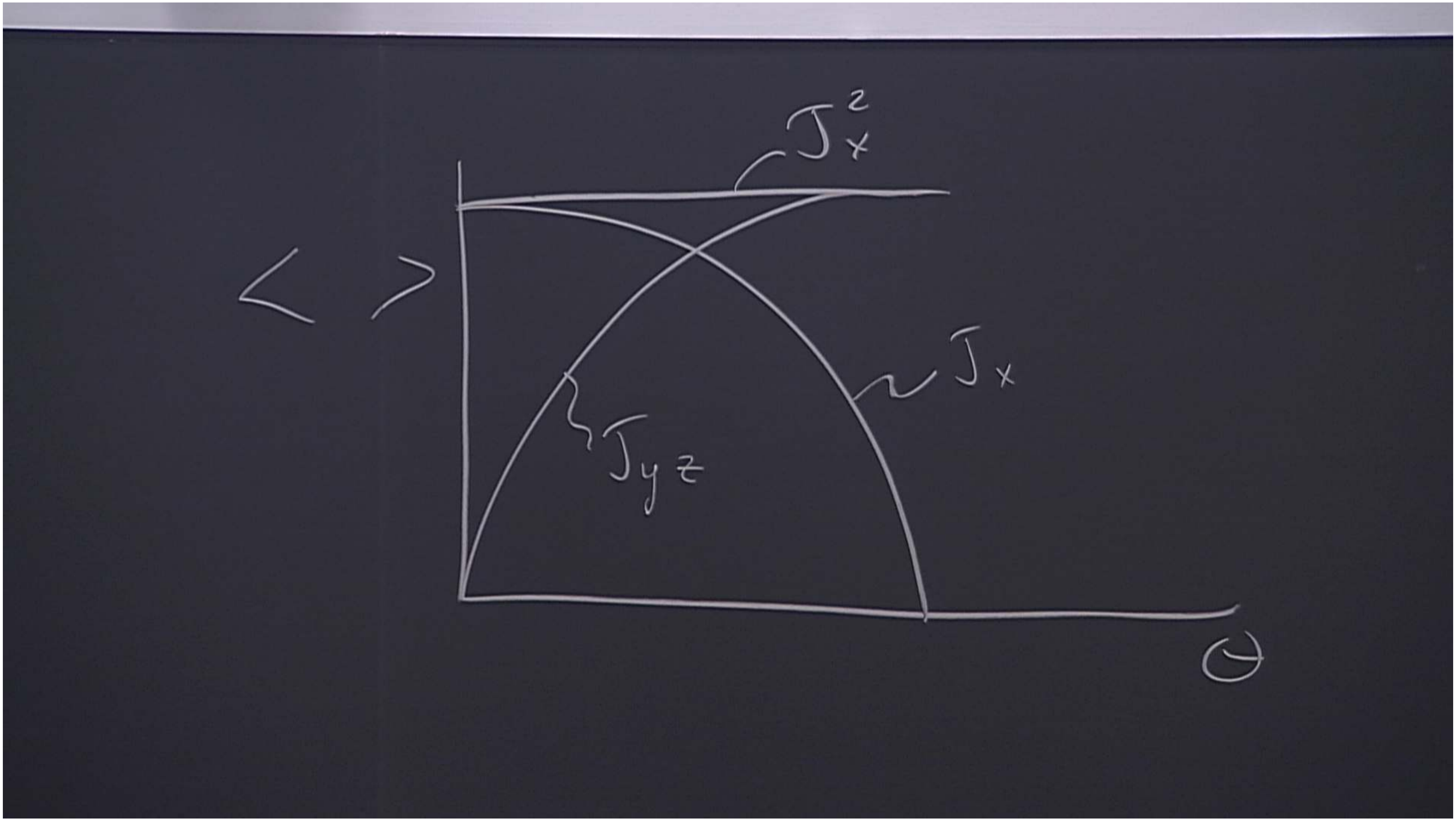
$$\xrightarrow{\sigma_z\sigma_z} \frac{1}{4}\left(\cos(\theta)(\sigma_x^1+\sigma_x^2) + \sin(\theta)(\sigma_y^1\sigma_z^2 + \sigma_z^1\sigma_y^2) + \mathbb{1} + \sigma_x\sigma_x\right) = \rho(\theta)$$

$$\rho(\theta) = \frac{1}{4}\left[\cos(\theta) J_x + \sin(\theta) J_{yz} + \frac{J_x^2}{2}\right]$$

$$\ldots \Big[\cos(\theta)\, J_x + \sin(\theta)\, J_{yz} + \frac{J_x}{2}\Big]$$

$$\langle J_x(\theta)\rangle = \mathrm{Tr}\left[J_x\, \rho(\theta)\right] = \cos(\theta) \quad ; \quad \langle J_y\rangle = \langle J_z\rangle = 0$$

$$\langle J_{yz}(\theta)\rangle = \overset{?}{+}\sin(\theta)$$


J_x^2
$\langle\ \rangle$
J_x
J_{yz}
Θ

$$\left[\cos(\theta)\, J_x + \sin(\theta)\, J_{yz} + \frac{J_x}{2}\right]$$

$$\langle J_x(\theta) \rangle = \mathrm{Tr}\left[J_x\, \rho(\theta) \right] = \cos(\theta) \quad ; \quad \langle J_y \rangle = \langle J_z \rangle = 0$$

$$\langle J_{yz}(\theta) \rangle = \left(\overset{?}{+}\right) \sin(\theta)$$

$$\langle \Delta J_x(\theta) \rangle = \sqrt{\underbrace{\langle J_x^2(\theta) \rangle}_{1} - \langle J_x(\theta) \rangle^2}$$

$$\rho(\theta) = \tfrac{1}{4}\left[\cos(\theta)\, J_x + \sin(\theta)\, J_{yz} + \frac{J_x^2}{2}\right]$$

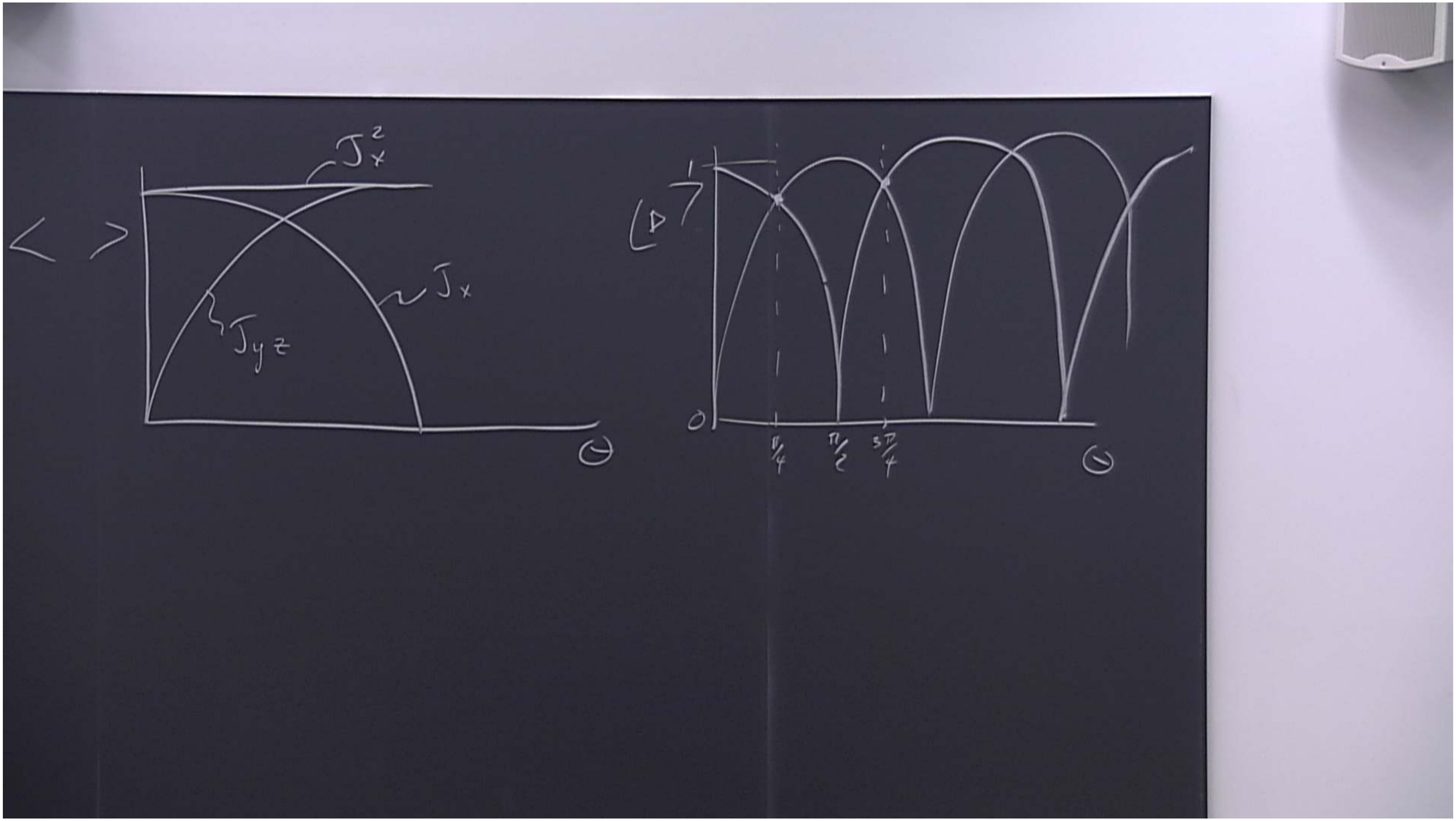
$$\langle J_x(\theta)\rangle = \mathrm{Tr}\left[J_x\, \rho(\theta)\right] = \cos(\theta) \quad ; \quad \langle J_y\rangle = \langle J_z\rangle = 0$$

$$\langle J_{yz}(\theta)\rangle = \overset{?}{+} \sin(\theta)$$

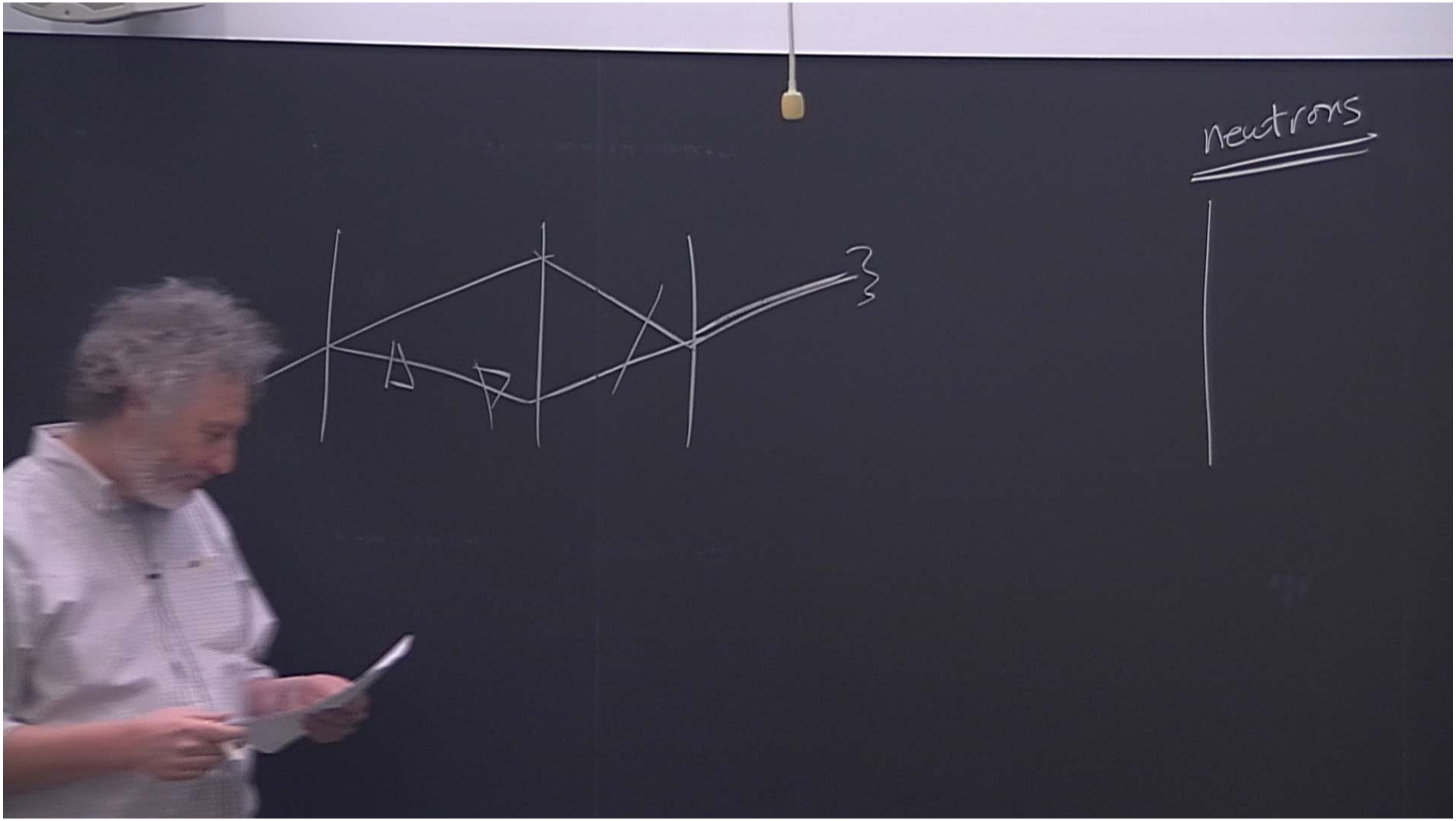
$$\langle \Delta J_x(\theta)\rangle = \sqrt{\underbrace{\langle J_x^2(\theta)\rangle}_{1} - \underbrace{\langle J_x(\theta)\rangle^2}_{\cos\theta}}$$

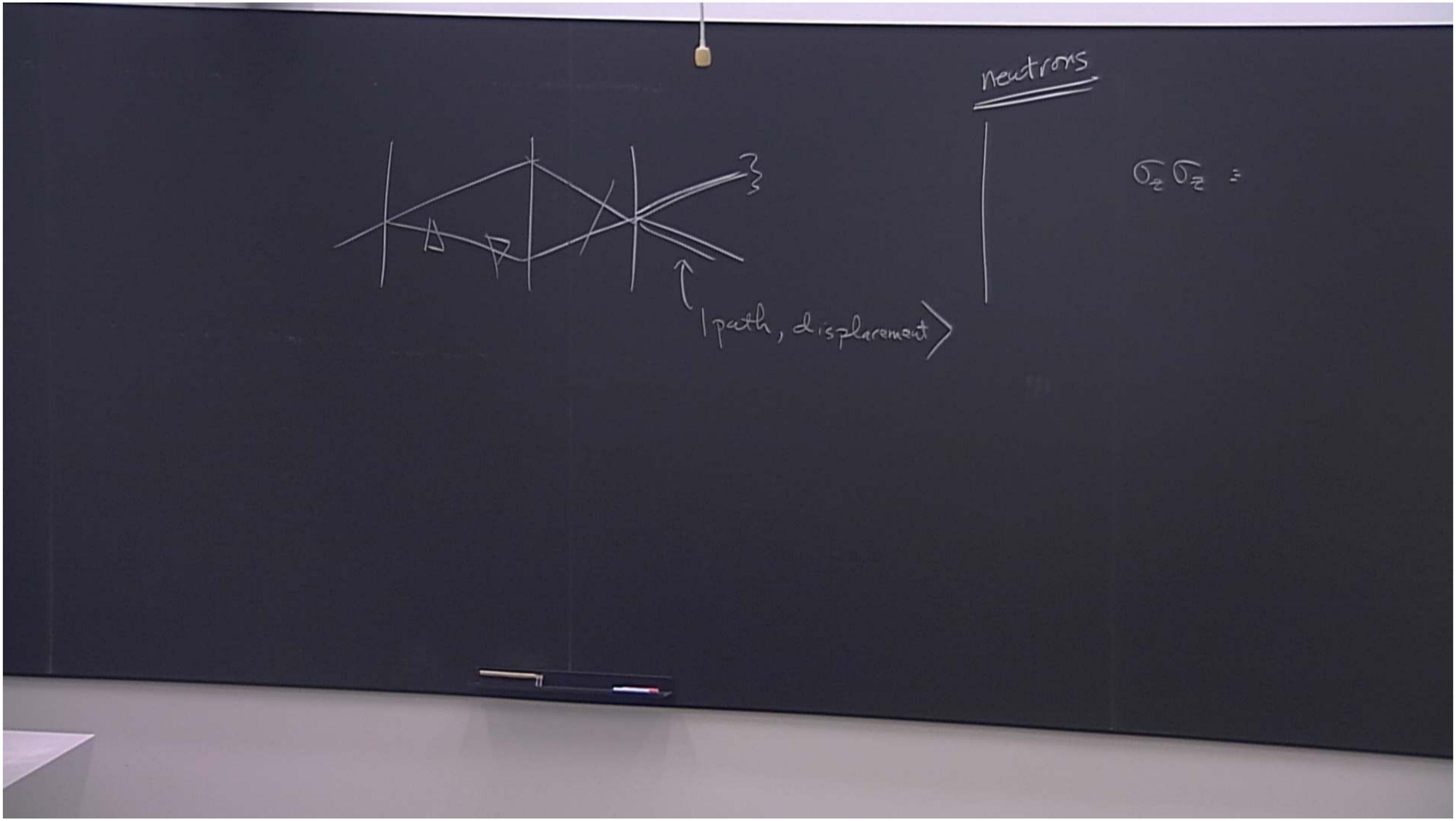
$$\langle \Delta J_x(\theta)\rangle = \sqrt{1 - \cos^2(\theta)}$$

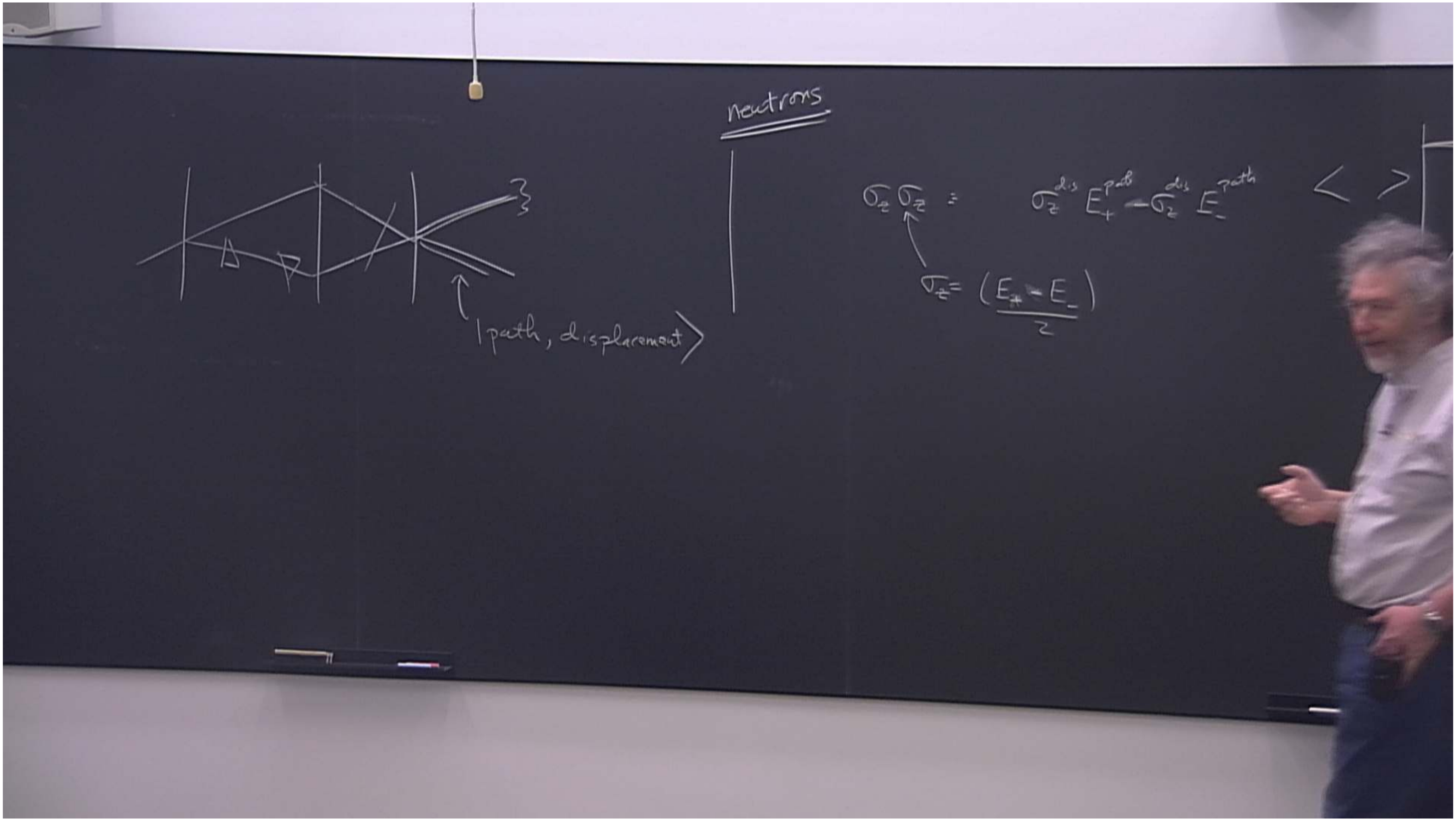
$$\langle \Delta J_{yz}(\theta)\rangle = \sqrt{1 - \sin^2(\theta)}$$



J_x^2
$\langle\ \rangle$
J_x
J_{yz}
Θ
$\langle D \rangle$
1
0
$\pi/4$
$\pi/2$
$3\pi/4$
Θ


neutrons


neutrons
|path, displacement⟩
σz σz =


neutrons
|path, displacement>
σz σz = σz^dis E+^path − σz^dis E−^path
σz = (E+ − E−)/2
< >

$J_{yz} \propto [J_x, J_z]$

4-qubits

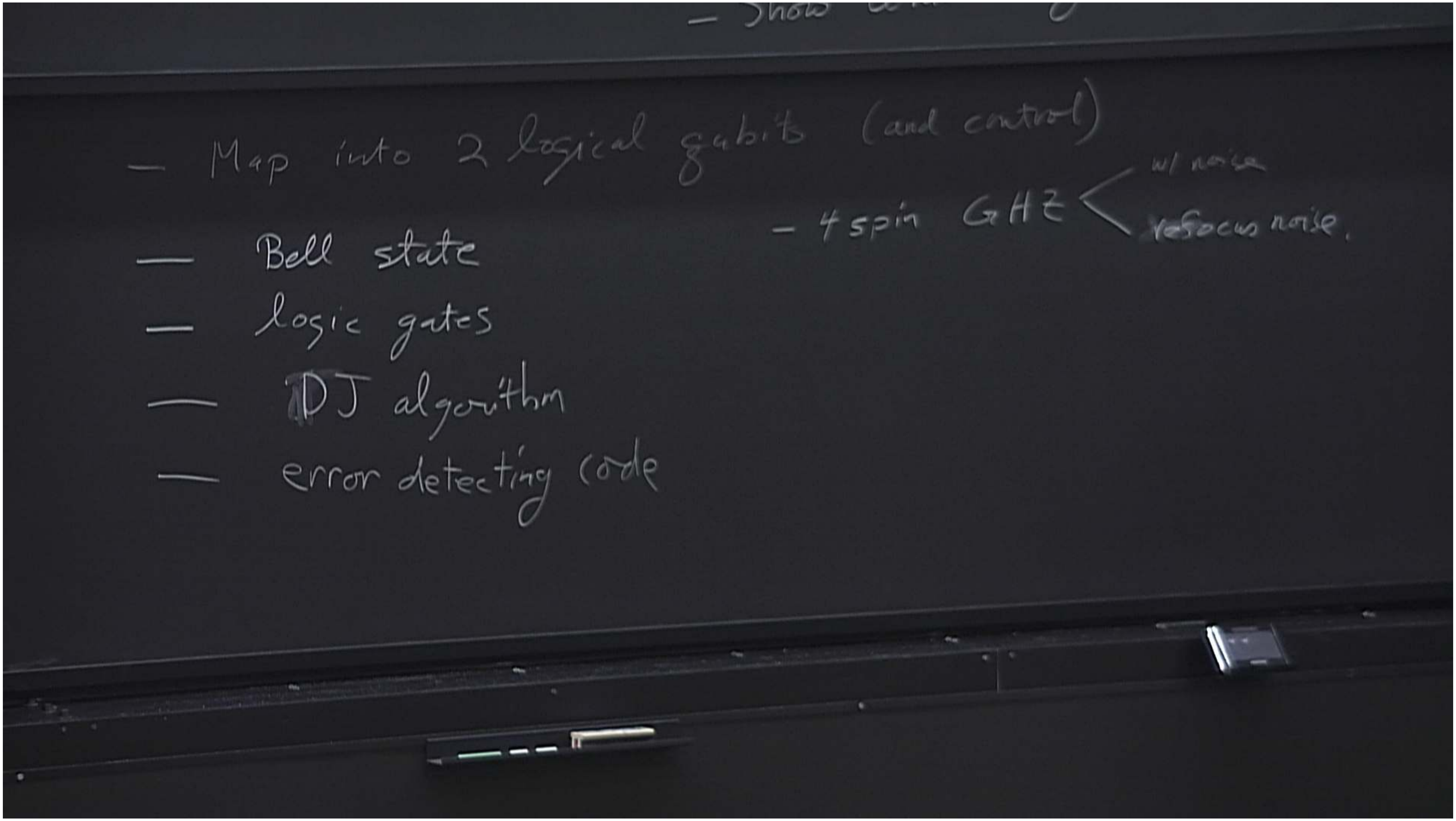
$$\mathcal{H}_{int} = \sum_k \omega_k \sigma_z^k + \omega_c \sum_k \sigma_z^k \sigma_z^{k+1} \quad ; \quad \omega_k = \omega_t + \Delta\omega_k, \quad \frac{\Delta\omega_k}{\omega_t} < 10^{-3}$$

$$\mathcal{H}_{noise}(t) = \omega_n^A(t)\left(\sigma_z^1 + \sigma_z^2\right) + \omega_n^B(t)\left(\sigma_z^3 + \sigma_z^4\right)$$

slow

$$\mathcal{H}_{cont}(t) = \omega_{RF}(t)\cos(\omega_t t + \phi(t)) \sum_k \sigma_x^k$$

— Map into 2 log


– Map into 2 logical qubits (and control)
– 4 spin GHZ
w/ noise
refocus noise.
– Bell state
– logic gates
– DJ algorithm
– error detecting code