Title: Quantum Information Review-14

Date: Mar 06, 2015 11:30 AM

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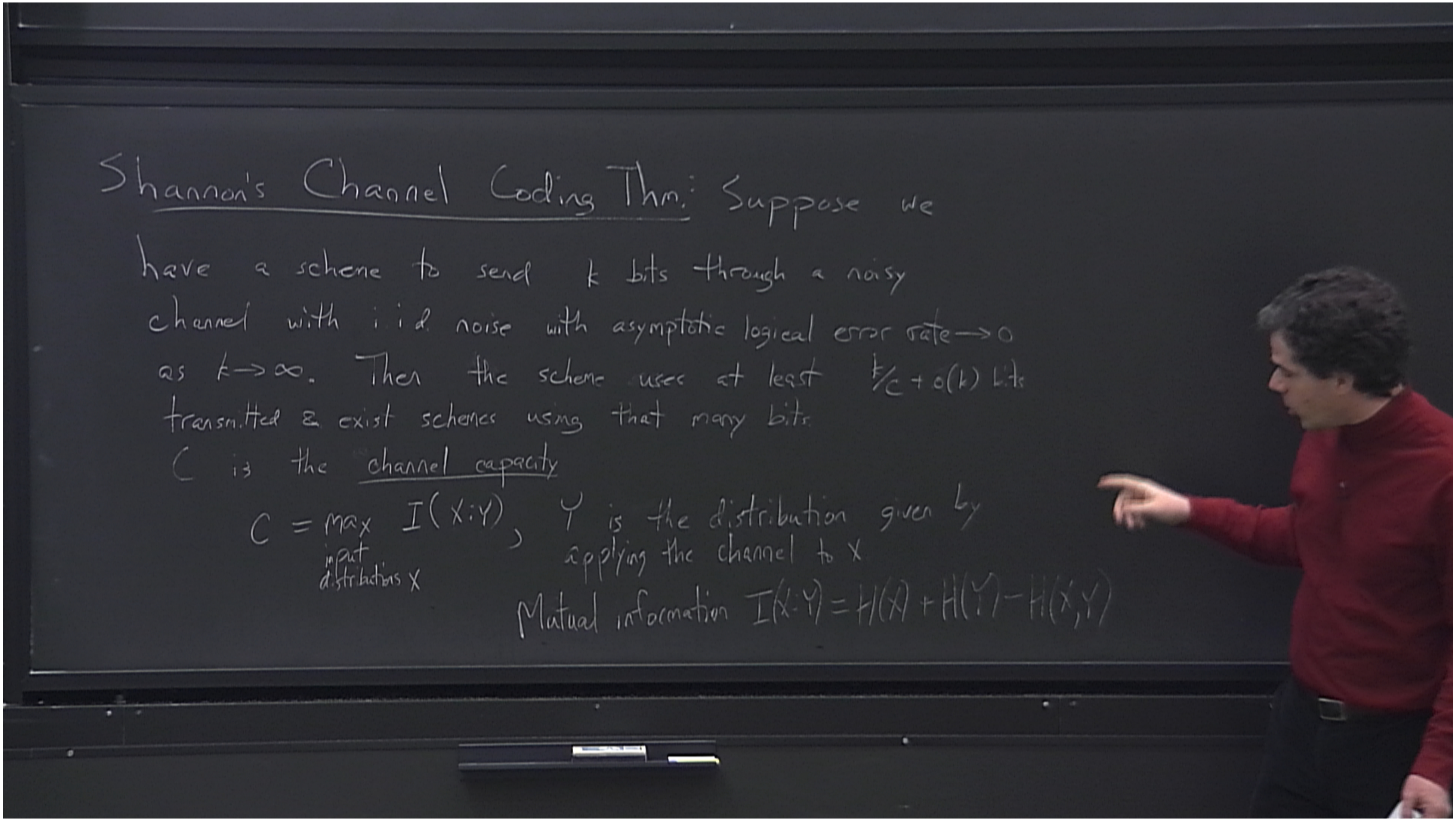
Abstract:

Shannon's Channel Coding Thm: Suppose we have a scheme to send k bits through a noisy channel with i.i.d. noise with asymptotic logical error rate $\to 0$ as $k \to \infty$. Then the scheme uses at least $k/C + o(k)$ bits transmitted & exist schemes using that many bits.

C is the channel capacity

$$C = \max_{\substack{\text{input} \\ \text{distributions } X}} I(X:Y),$$ Y is the distribution given by applying the channel to X

Mutual information $I(X:Y) = H(X) + H(Y) - H(X,Y)$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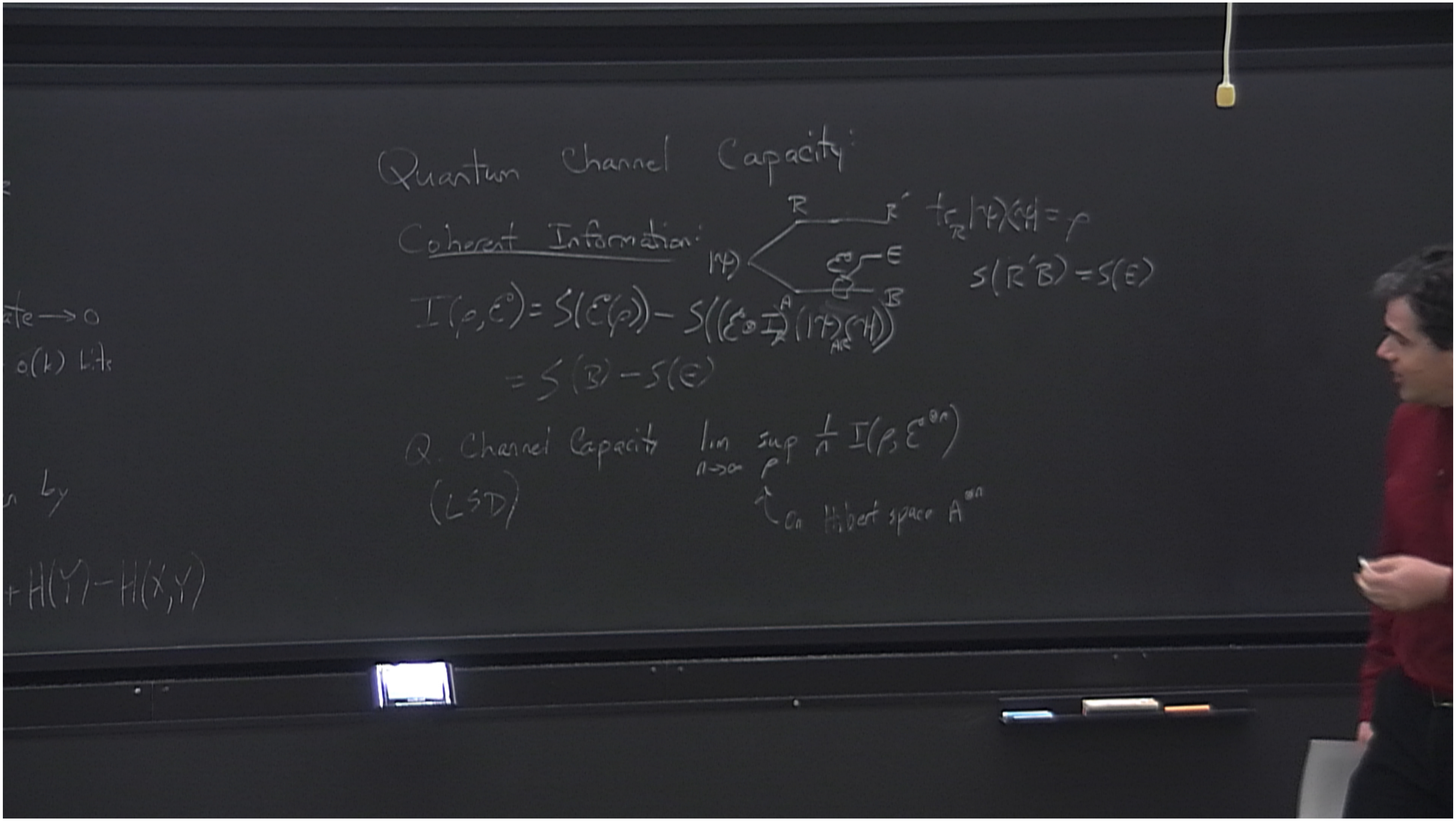
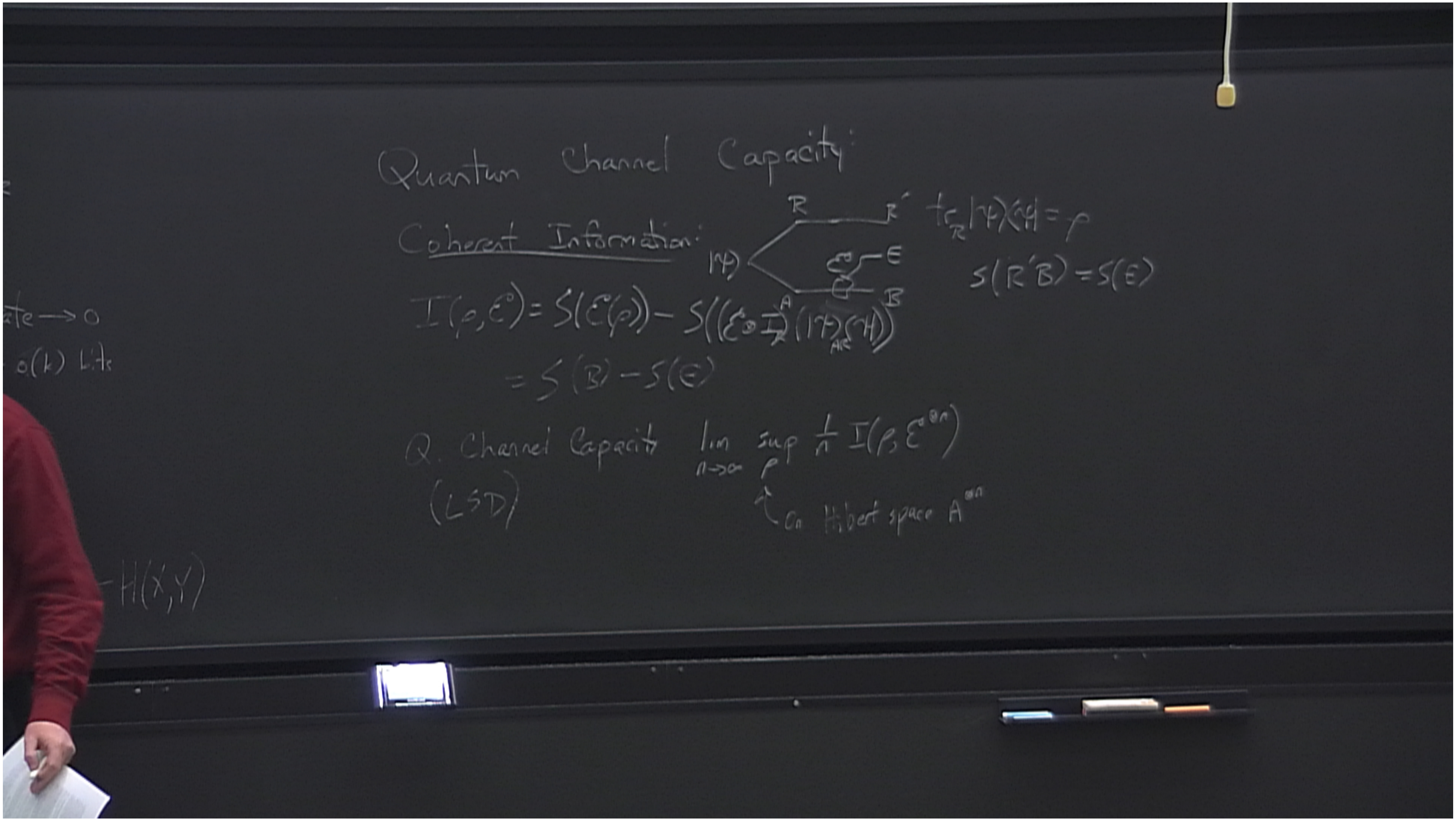

Quantum Channel Capacity:
Coherent Information:
R
R'
$|\psi\rangle$
E
A
B
$tr_R |\psi\rangle\langle\psi| = \rho$
$S(R'B) = S(E)$
$I(\rho, \mathcal{E}) = S(\mathcal{E}(\rho)) - S((\mathcal{E} \otimes I_R)(|\psi\rangle\langle\psi|_{AR}))$
$= S(B) - S(E)$
Q. Channel Capacity $\lim_{n\to\infty} \sup_\rho \frac{1}{n} I(\rho, \mathcal{E}^{\otimes n})$
(LSD)
on Hilbert space $A^{\otimes n}$
ate $\to 0$
$o(k)$ bits
n by
$+H(Y) - H(X,Y)$


Quantum Channel Capacity:
Coherent Information:
tr_R |ψ⟩⟨ψ| = ρ
S(R'B) = S(E)
I(ρ, E) = S(E(ρ)) − S((E ⊗ I_R)(|ψ⟩⟨ψ|_AR))
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Q. Channel Capacity lim_{n→∞} sup_ρ 1/n I(ρ, E^{⊗n})
(LSD)
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ate $\rightarrow 0$

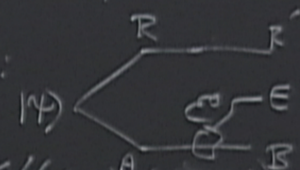
$o(k)$ bits

n by

$+H(Y)-H(X,Y)$

Quantum Channel Capacity:

Coherent Information:



$\mathrm{tr}_R |\psi\rangle\langle\psi| = \rho$

$S(R'B) = S(E)$

$$I(\rho,\mathcal{E}) = S(\mathcal{E}(\rho)) - S\big((\mathcal{E}\otimes I_R)(|\psi\rangle\langle\psi|_{AR})\big)$$

$$= S(B) - S(E)$$

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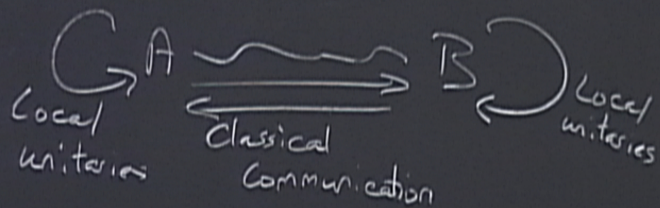


LOCC

<u>Separable state</u> if

$$\rho = \sum_i p_i \, \rho_{A_i} \otimes \rho_{B_i}$$

Local unitaries A → B Local unitaries

Classical Communication

LOCC

<u>Separable state</u> if

$$\rho = \sum_i p_i \, \rho_{A_i} \otimes \rho_{B_i}$$

<u>Entangle</u>

1) For

Entanglement monotone $f(\rho) \in \mathbb{R}$ s.t.

1) For separable state, $f(\rho) = 0$.

$f(\rho) > 0$ for some entangled states

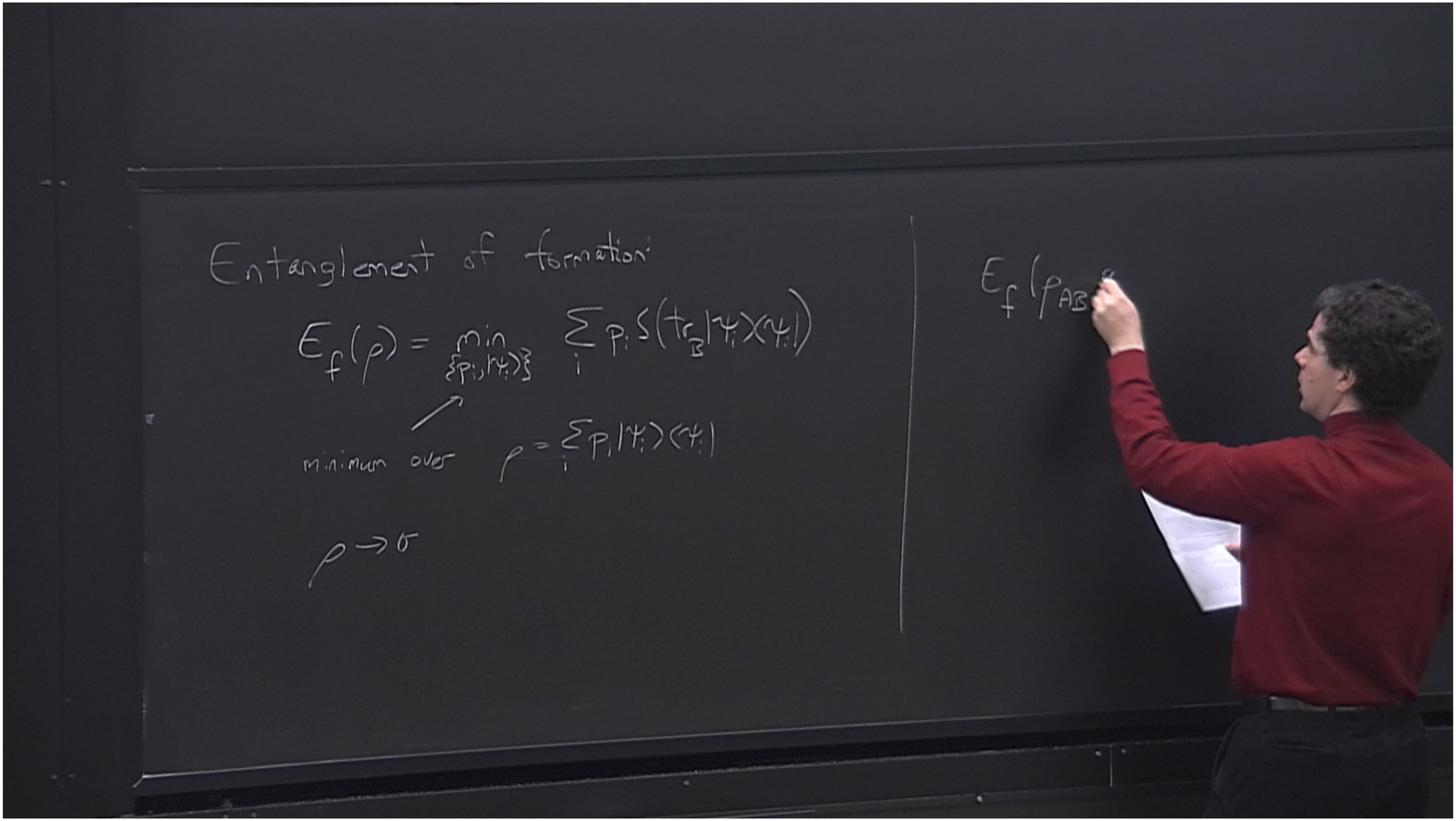
2)

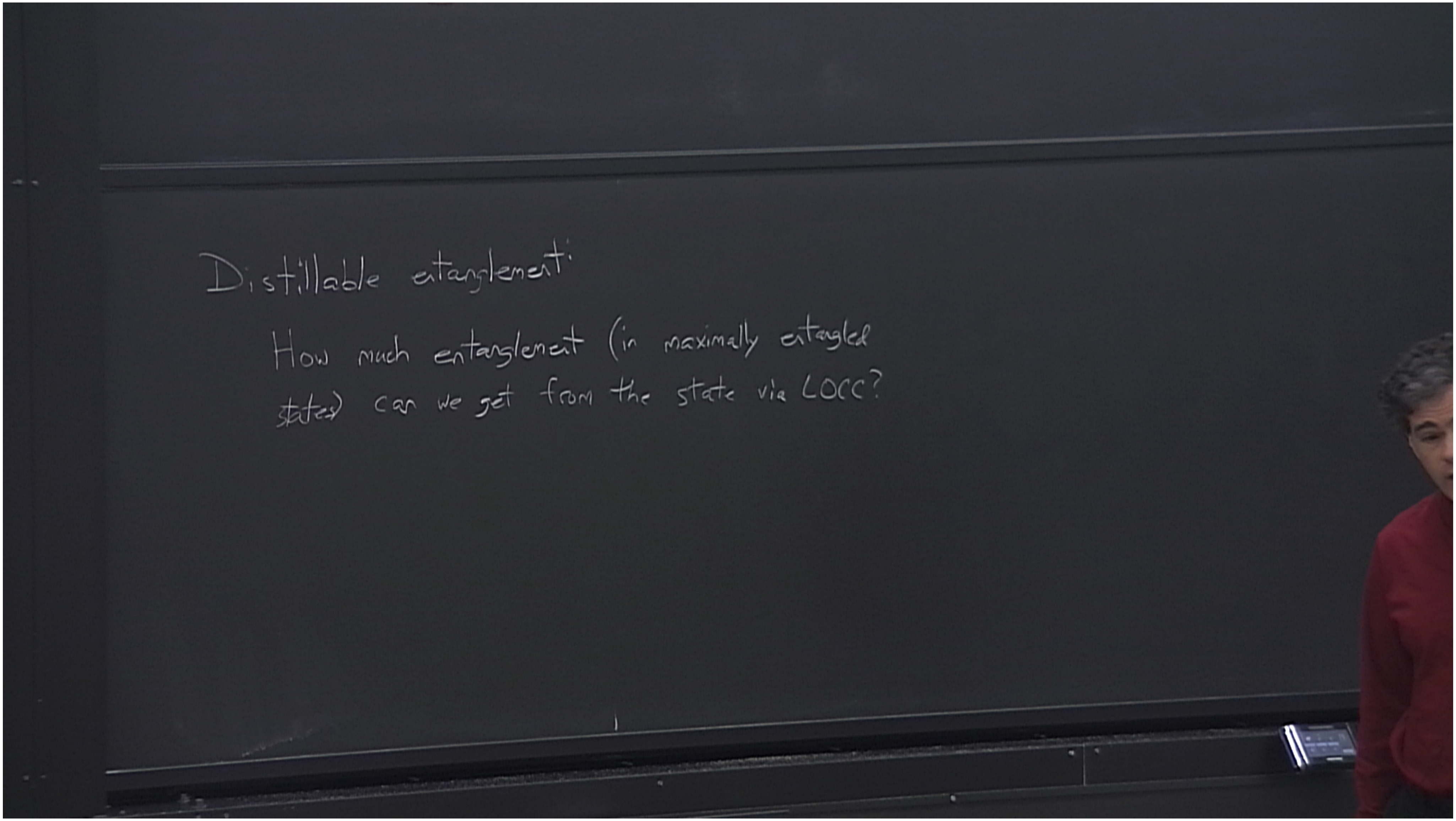
For pure states:

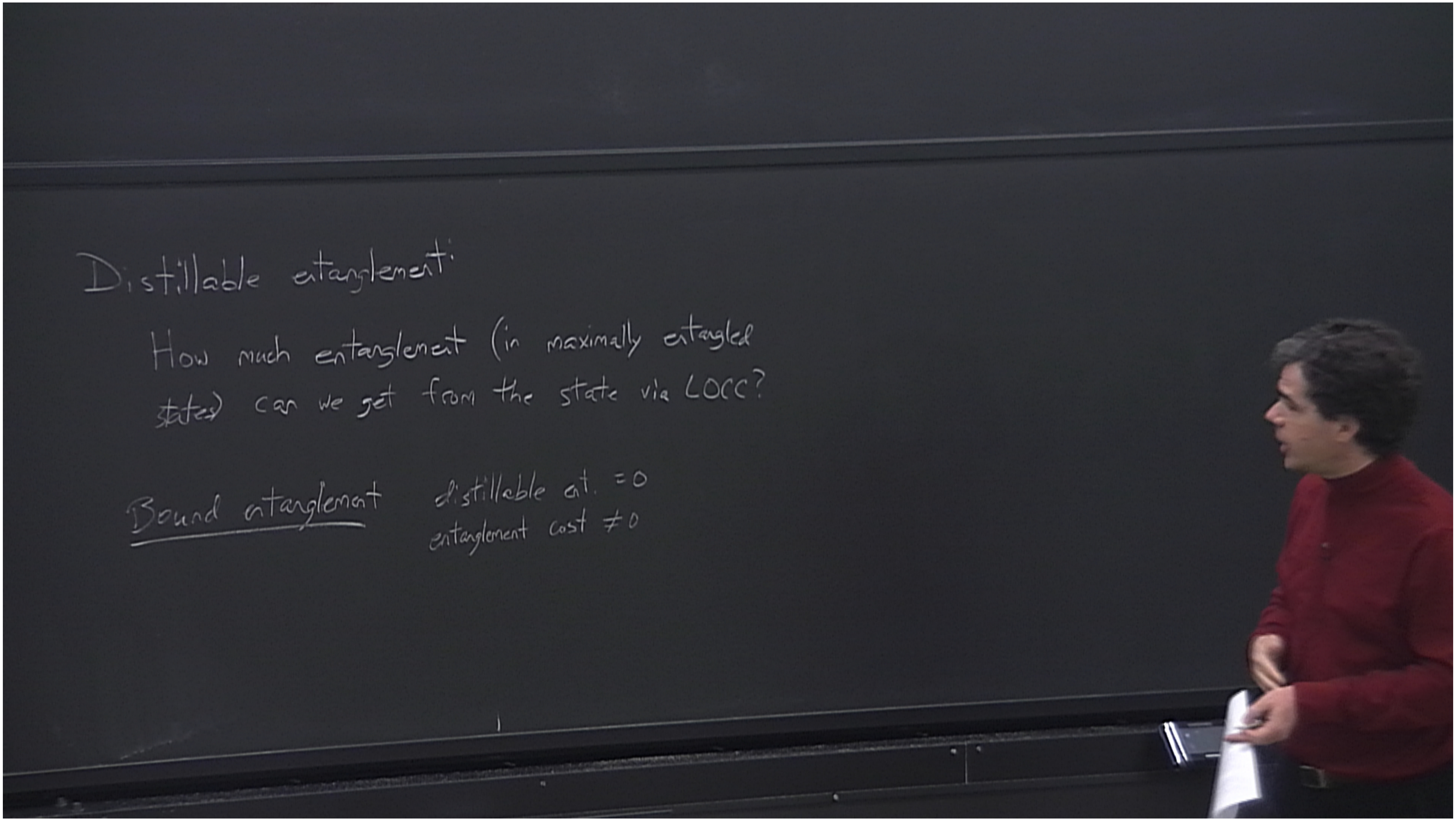
$$S(A) = S(B)$$

<u>entanglement entropy</u>

$$(1-\varepsilon)\,|00\rangle + \varepsilon\,|11\rangle$$


Entanglement of formation:
$E_f(\rho) = \min_{\{p_i, |\psi_i\rangle\}} \sum_i p_i S(\mathrm{tr}_B |\psi_i\rangle\langle\psi_i|)$
minimum over $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$
$\rho \to \sigma$
$E_f(\rho_{AB}$


Distillable entanglement:
How much entanglement (in maximally entangled
states) can we get from the state via LOCC?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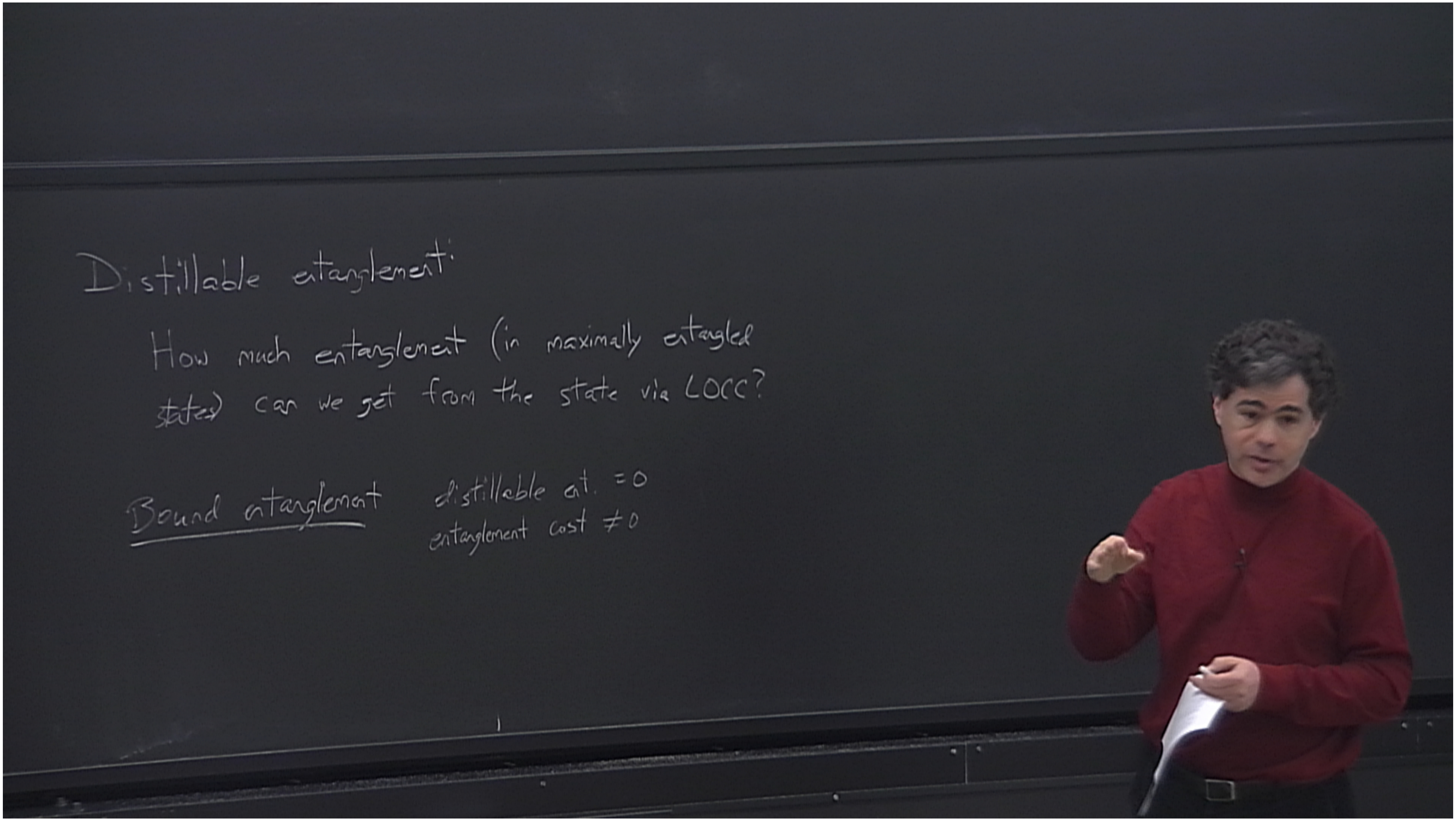
Bound entanglement
distillable ent. = 0
entanglement cost ≠ 0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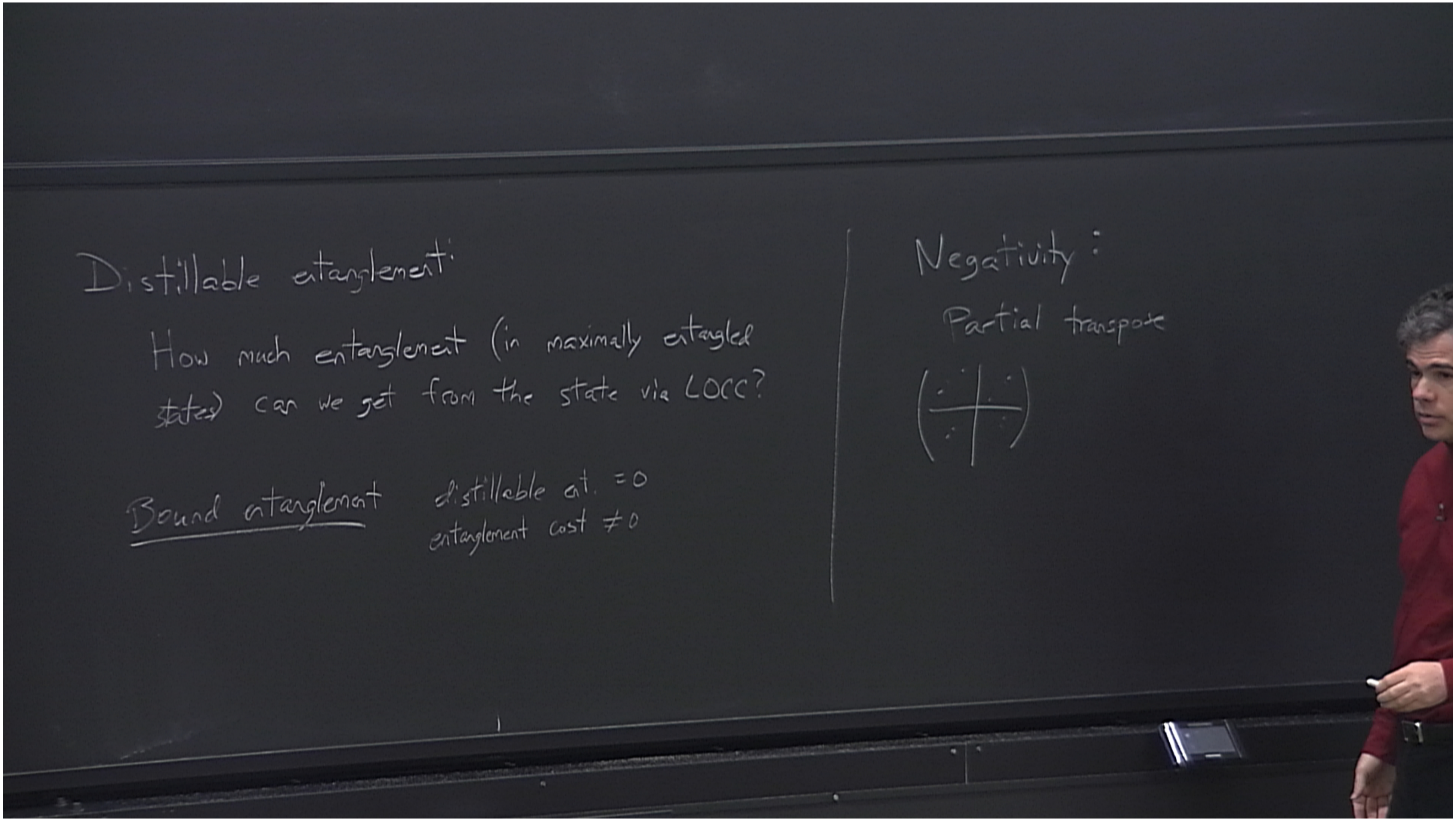
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Negativity:
Partial transpose

Distillable entanglement:

How much entanglement (in maximally entangled states) can we get from the state via LOCC?

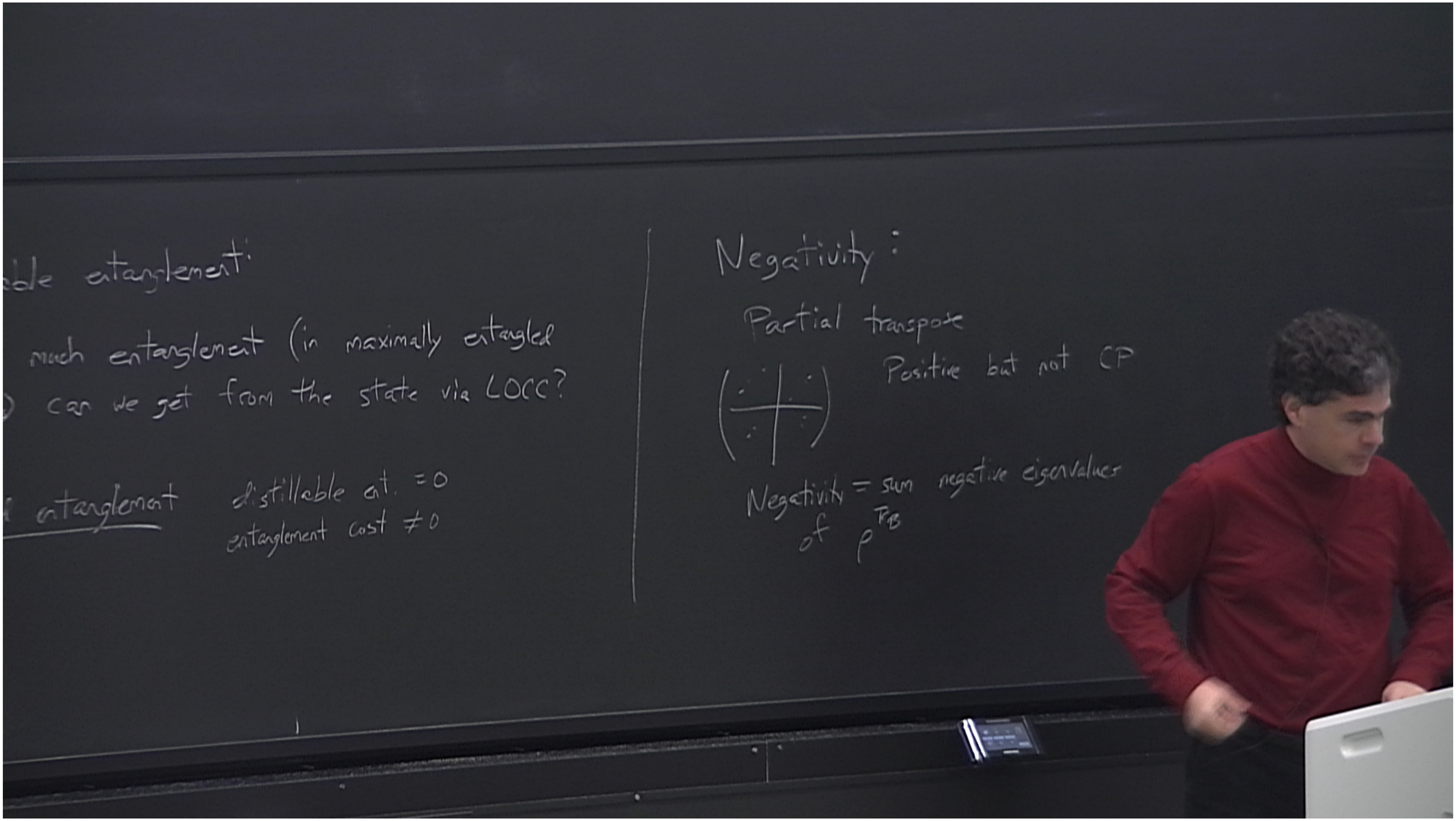
Bound entanglement distillable ent. $= 0$

entanglement cost $\neq 0$

Negativity:

Partial transpose

Positive but not CP


ble entanglement:
much entanglement (in maximally entangled
can we get from the state via LOCC?
entanglement
distillable ent. = 0
entanglement cost ≠ 0
Negativity:
Partial transpose
Positive but not CP
Negativity = sum negative eigenvalues
of ρ^{T_B}