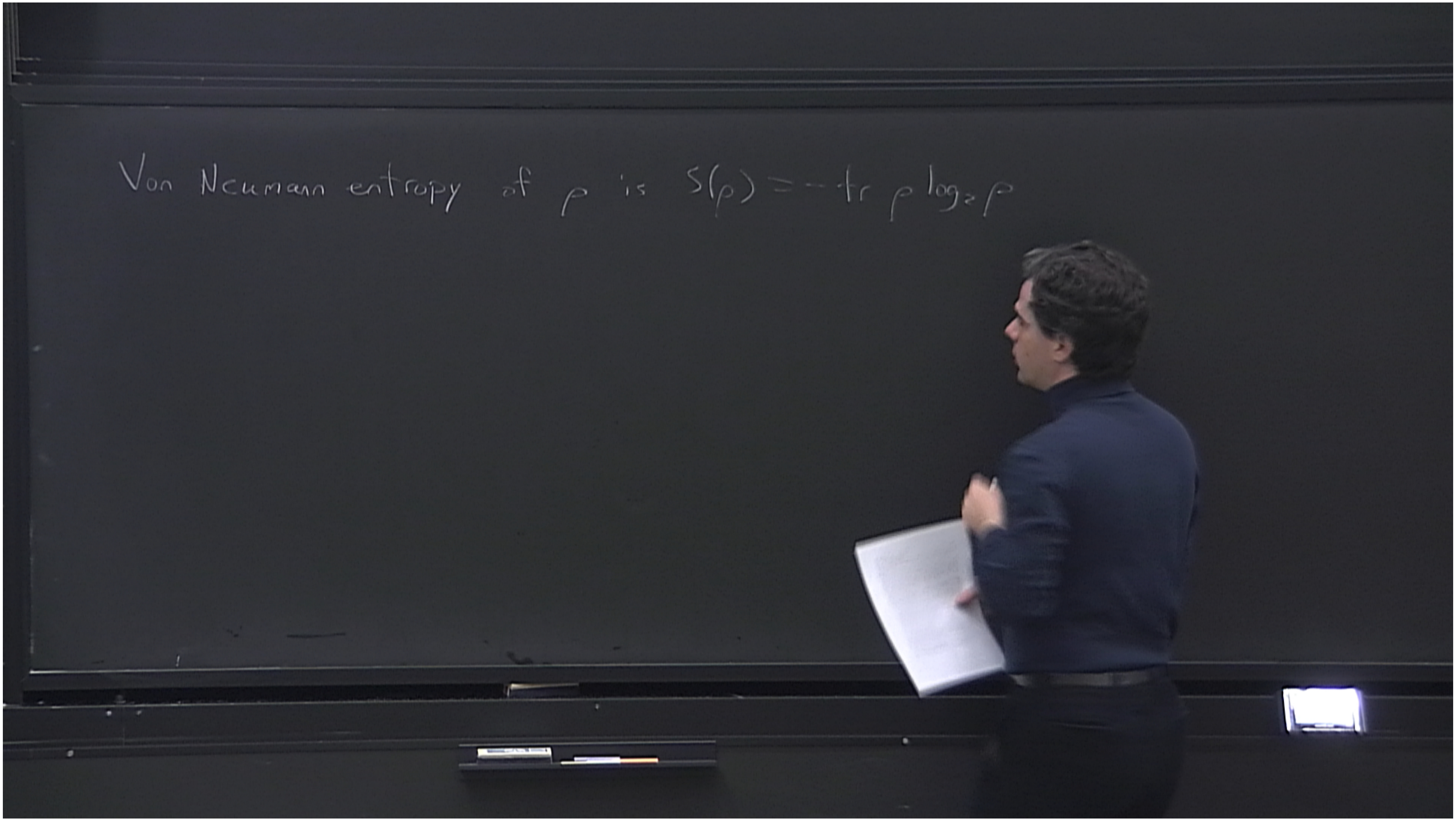
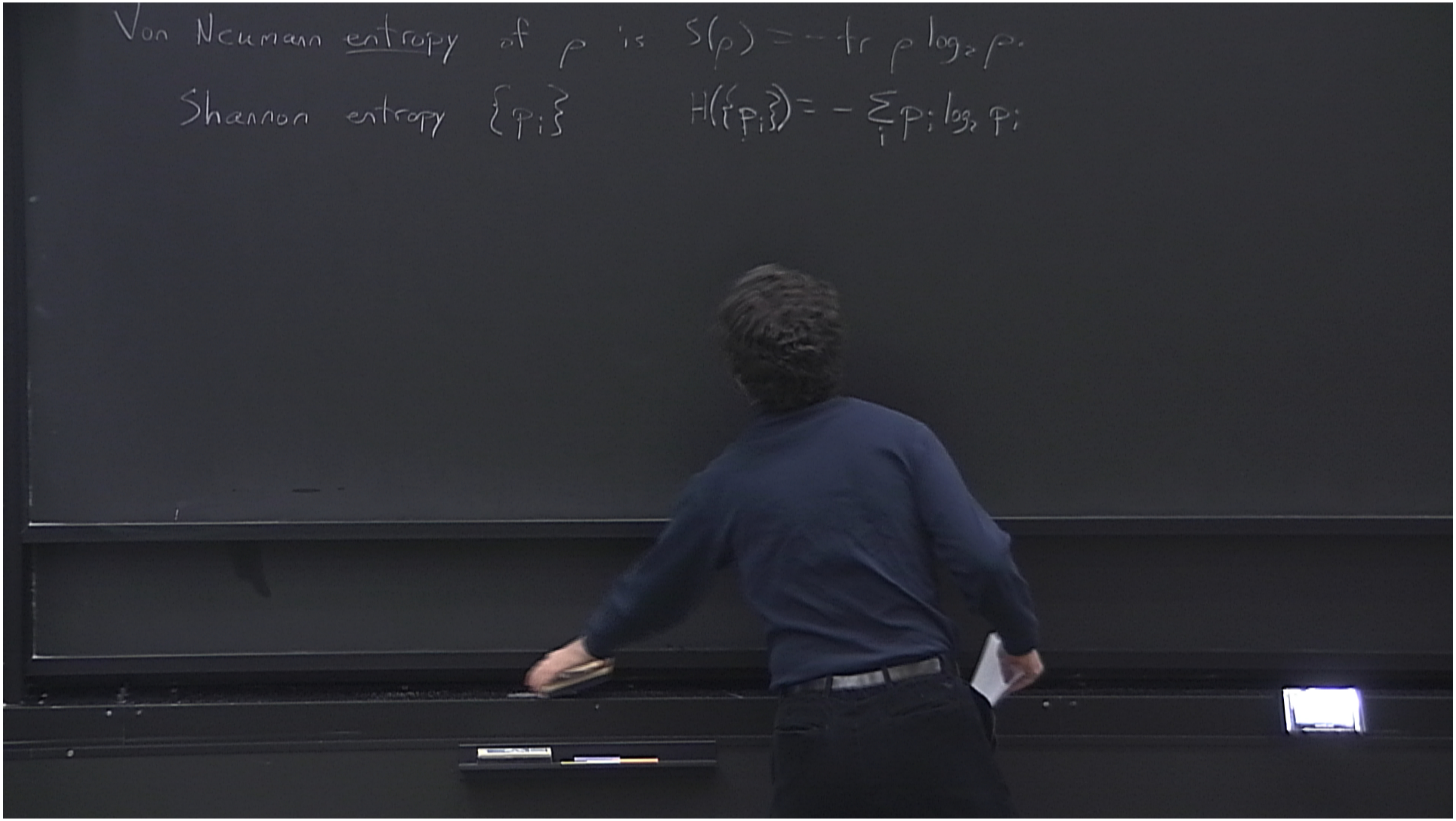
Title: Quantum Information Review-13

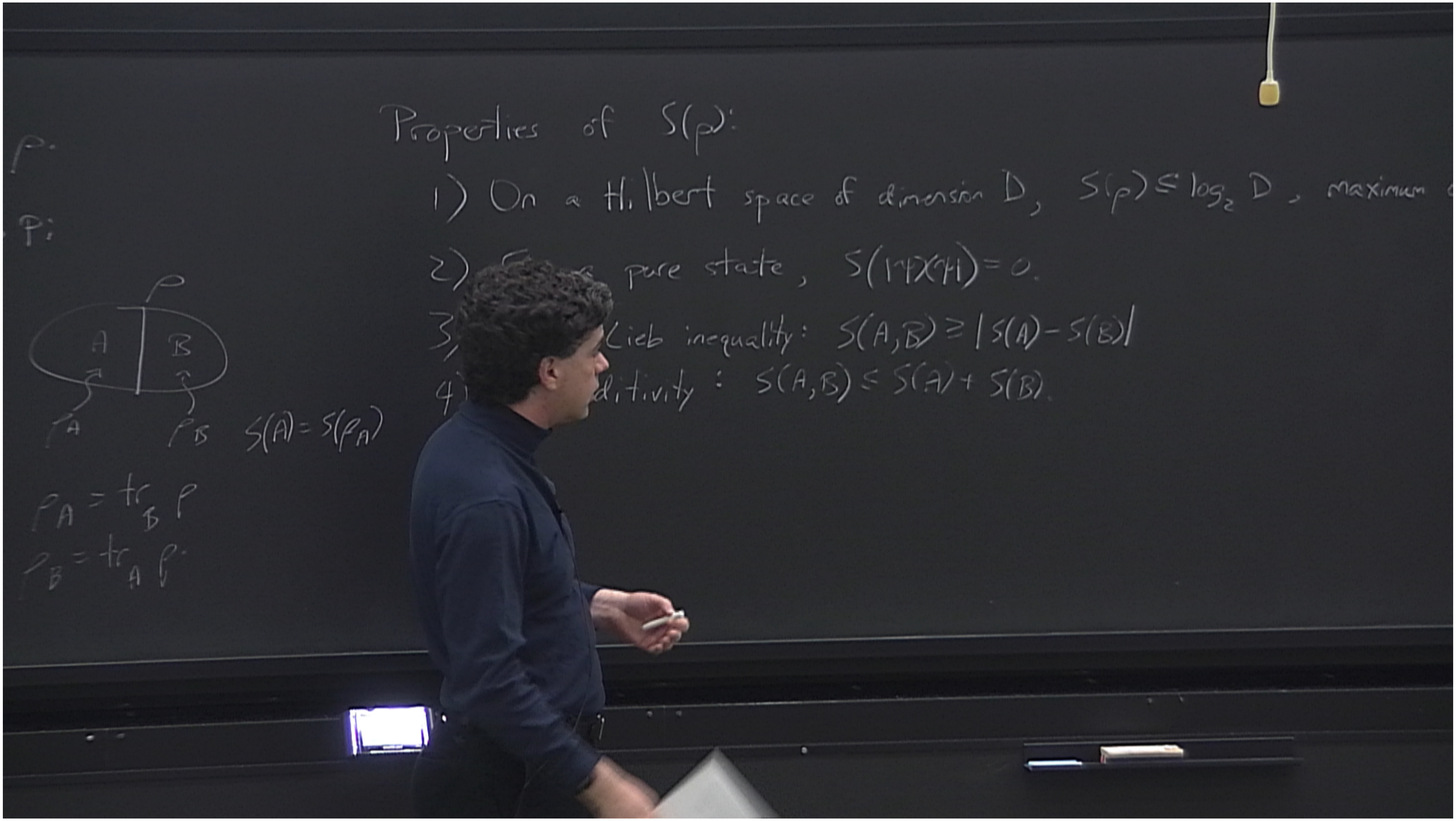
Date: Mar 05, 2015 11:30 AM

URL: http://pirsa.org/15030016

Abstract:


Von Neumann entropy of ρ is $S(\rho) = -\operatorname{tr} \rho \log_2 \rho$


Von Neumann entropy of ρ is $S(\rho) = -\mathrm{tr}\, \rho \log_2 \rho$.
Shannon entropy $\{p_i\}$ $H(\{p_i\}) = -\sum_i p_i \log_2 p_i$



Properties of $S(\rho)$:
1) On a Hilbert space of dimension D, $S(\rho) \leq \log_2 D$, maximum
pure state, $S(|\psi\rangle\langle\psi|) = 0$.
Lieb inequality: $S(A,B) \geq |S(A) - S(B)|$
ditivity: $S(A,B) \leq S(A) + S(B)$.
A
B
ρ_A
ρ_B
$S(A) = S(\rho_A)$
$\rho_A = tr_B\, \rho$
$\rho_B = tr_A\, \rho$

Properties of $S(\rho)$:

1) On a Hilbert space of dimension D, $S(\rho) \leq \log_2 D$, maximum occurs for $\rho = \frac{I}{D}$.

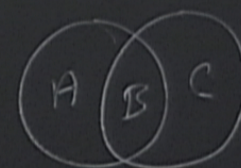
2) For a pure state, $S(|\psi\rangle\langle\psi|) = 0$.

3) Araki-Lieb inequality: $S(A,B) \geq |S(A) - S(B)|$

4) Subadditivity: $S(A,B) \leq S(A) + S(B)$.

5) 2) & 3) $\Rightarrow$ For a global pure state of A&B, $S(A) = S(B)$

6) Strong subadditivity: $S(A,B,C) \leq S(A,B) + S(B,C) - S(B)$



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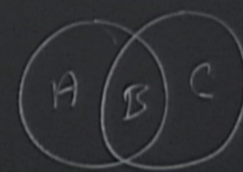

Data compression:

$$A \longrightarrow B$$

Alice has 4 possible messages:

"A" w/ prob. $\frac{1}{2}$, "B" with prob. $\frac{1}{4}$,

"C" w/ prob. $\frac{1}{8}$, "D" with prob. $\frac{1}{8}$.

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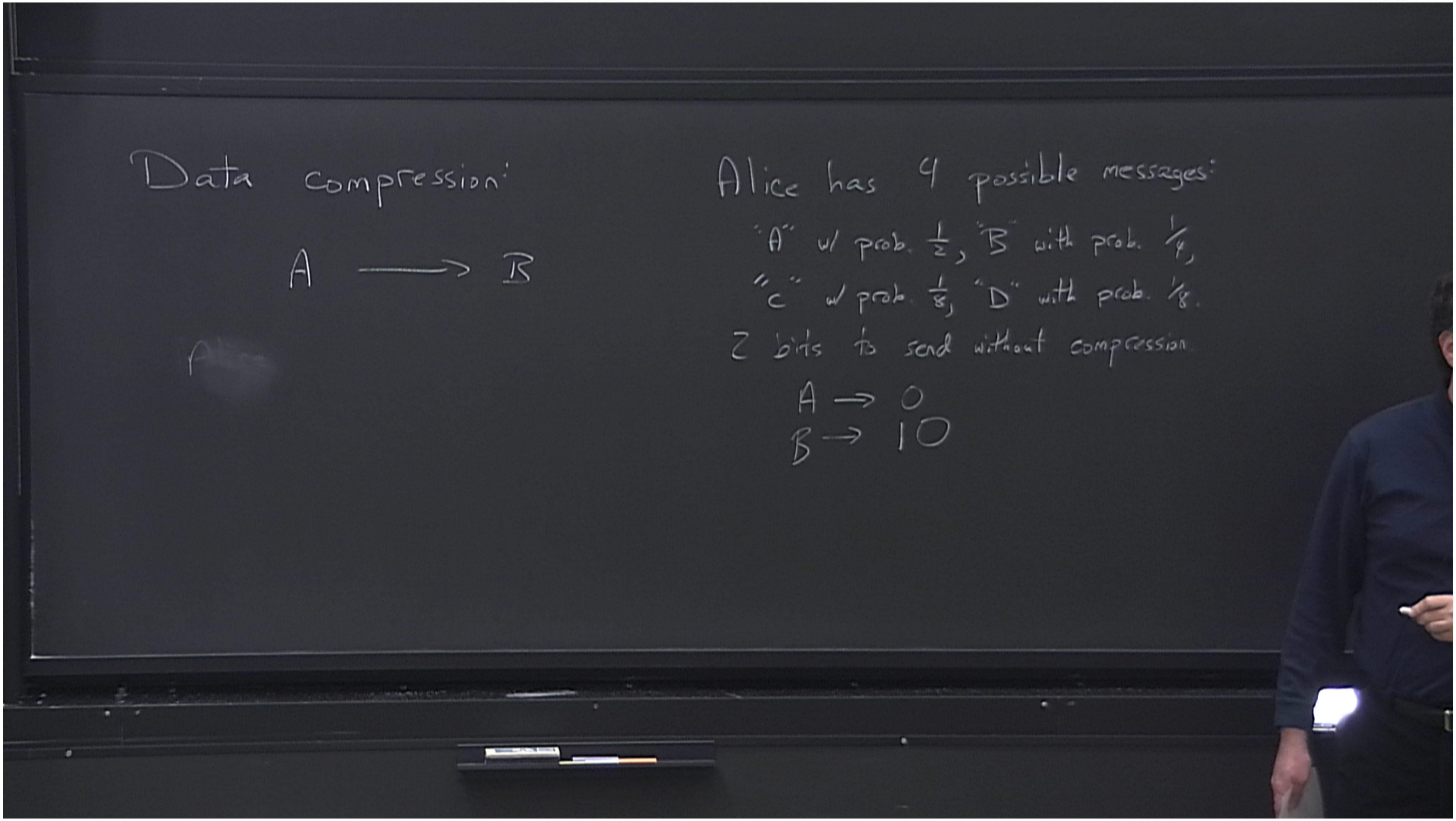

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2 bits to send without compression
A → 0
B → 10

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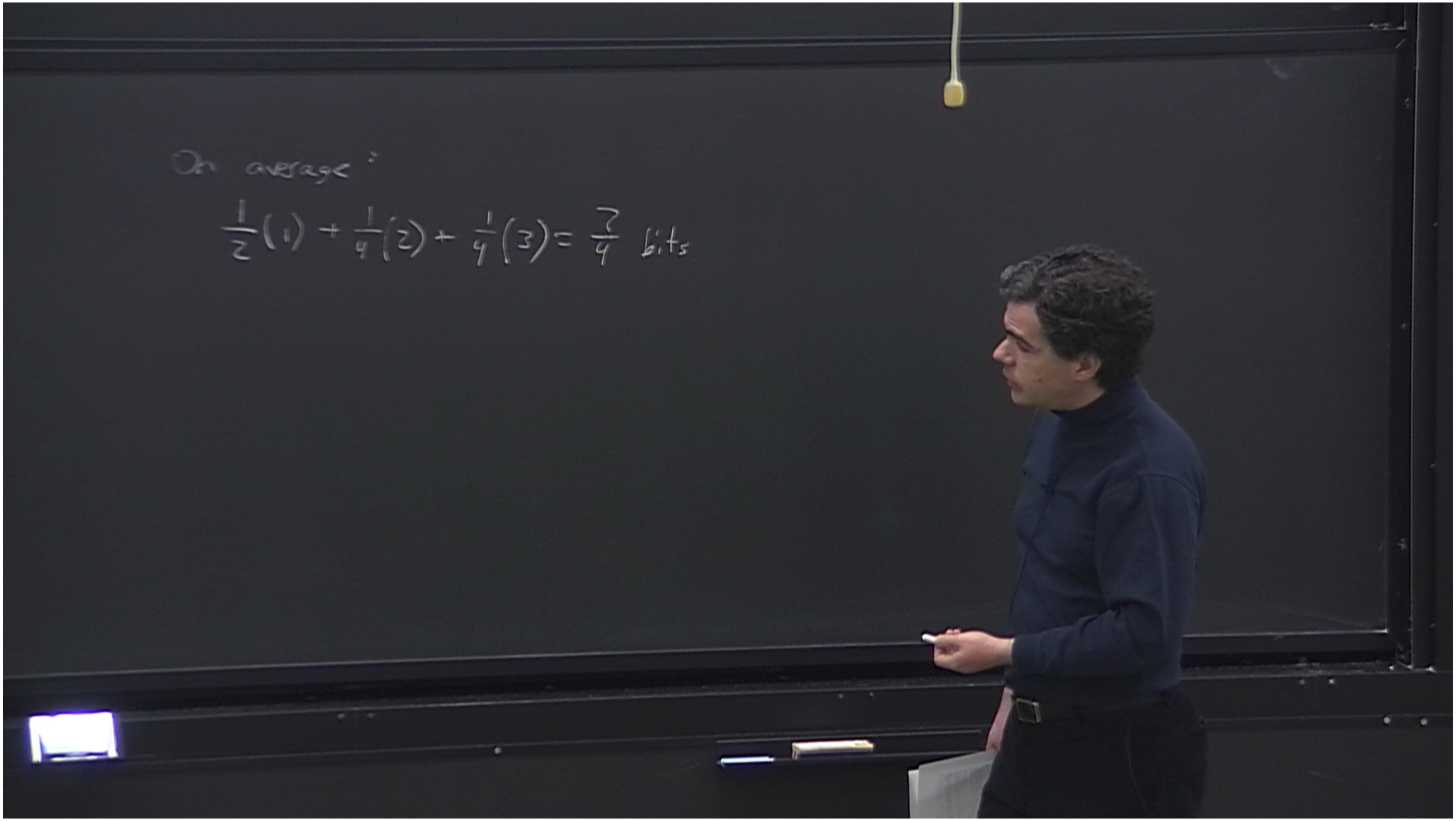
2 bits to send without compression.

$A \rightarrow 0$

$B \rightarrow 10$

$C \rightarrow 110$

$D \rightarrow 111$


On average :
$\frac{1}{2}(1) + \frac{1}{4}(2) + \frac{1}{4}(3) = \frac{7}{4}$ bits

Prob. p_i of message i

Out of n total messages,

$\approx p_i n$ i messages

"typical" set of messages.

Block compression:

List typical messages, assign bit strings to them.

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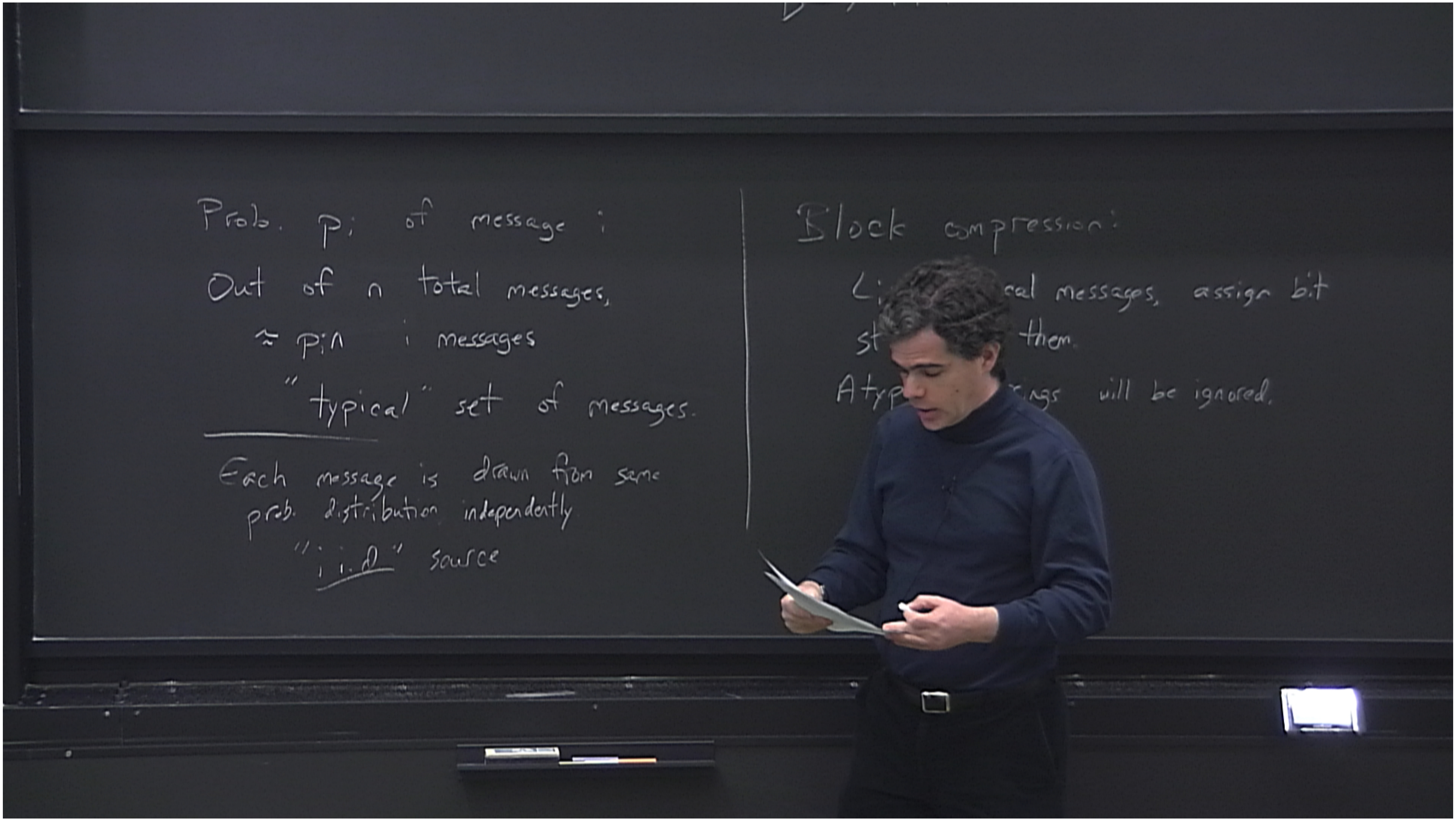
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≈ $p_i n$ i messages
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Each message is drawn from same prob. distribution independently
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Prob. n_i messages of type $i = \prod_i p_i^{n_i}$

bit

- bit

Prob. n_i messages of type i = $\prod_i p_i^{n_i}$ $\qquad |n_i - p_i n| = o(n)$

How many typical messages? $\qquad 2^{nH(\{p_i\}) + o(n)}$

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$\Rightarrow$ We need $nH(\{p_i\}) + o(n)$ bits

Shannon's Source Coding Thm: To compress n messages from an i.i.d source with entropy H with failure rate $\to 0$ as $n \to \infty$, we need at least $nH + o(n)$ bits, and there $\exists$ compression schemes using $nH + o(n)$ bits.

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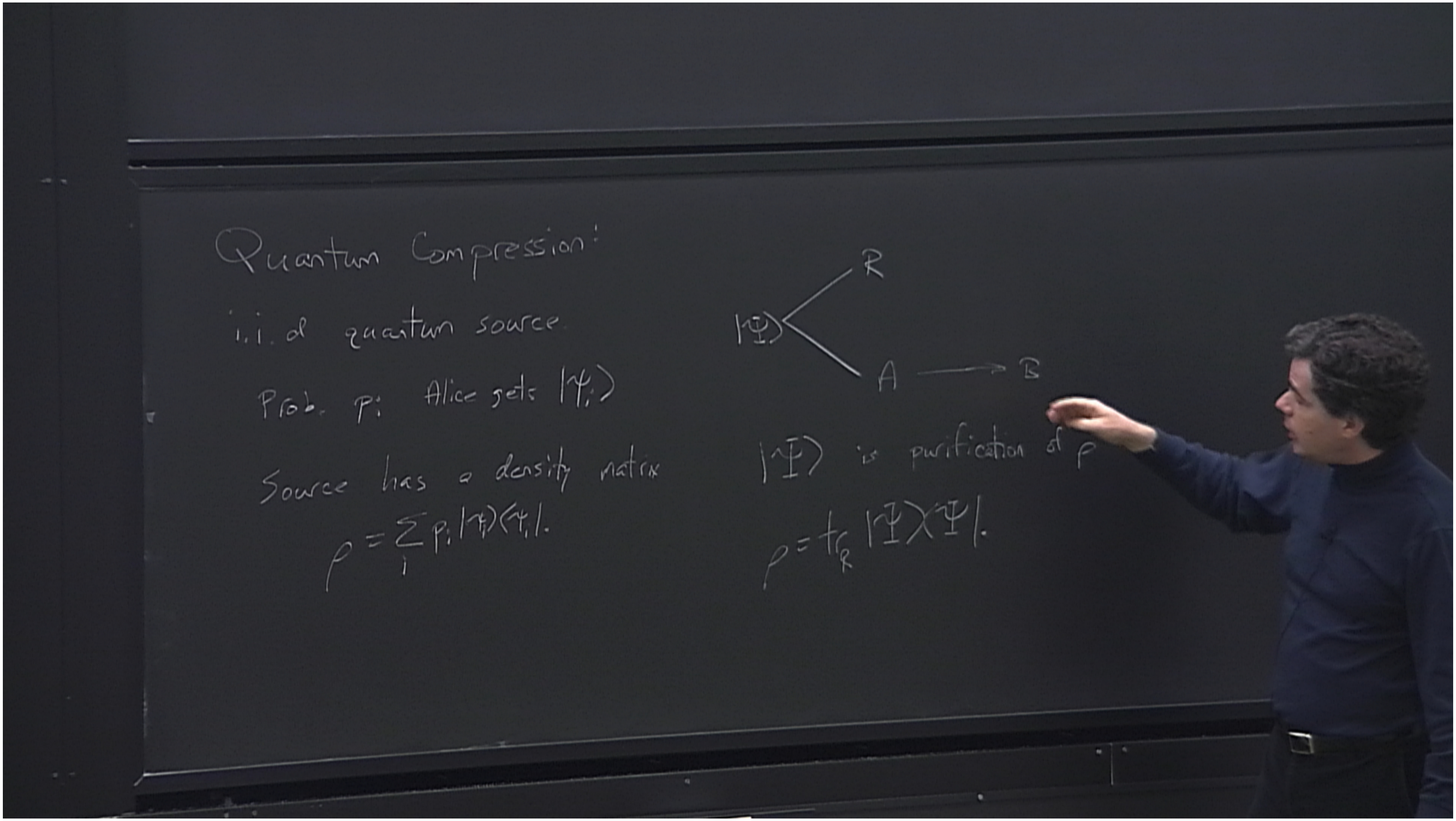
Quantum Compression:

i.i.d quantum source

Prob. p_i Alice gets $|\psi_i\rangle$

Source has a density matrix

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|.$$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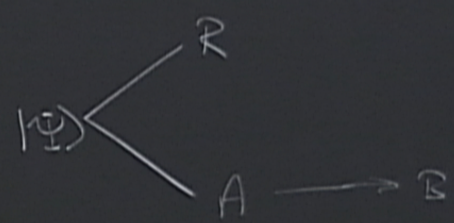

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Want high

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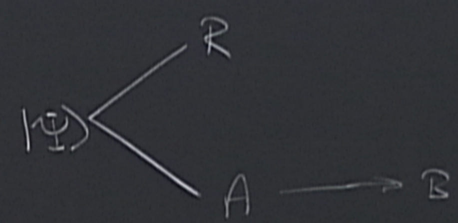

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Want high entanglement fidelity $F(\rho$

$$\rho_{RB}^{(n)} \approx \rho_{RA}^{\otimes n} = |\Psi\rangle\langle\Psi|^{\otimes n}$$