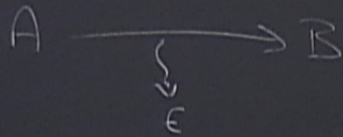


Title: Quantum Information Review-12

Date: Mar 04, 2015 11:30 AM

URL: <http://pirsa.org/15030015>

Abstract:



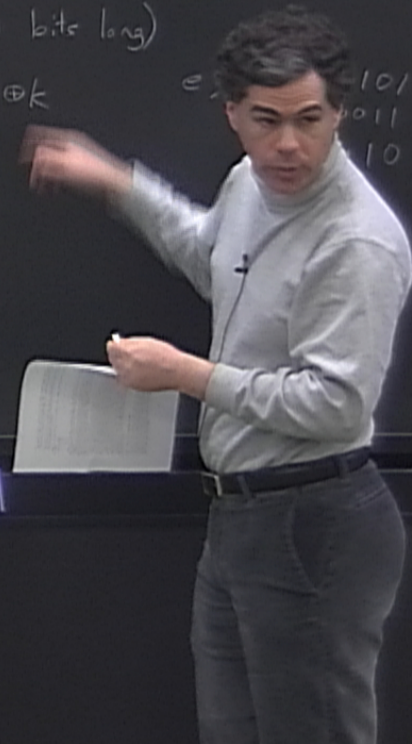
One-time pad:

Alice & Bob share secret key  $k$  ( $n$  bits long)

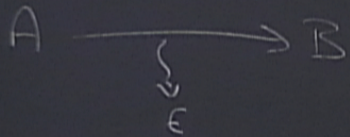
Encryption:  $n$ -bit message  $m \rightarrow e = m \oplus k$

Decrypt: ciphertext  $c \rightarrow c \oplus k =$

$e =$  101  
011  
10







One-time pad:

Alice & Bob share secret key  $k$  ( $n$  bits long)

Encryption:  $n$ -bit message  $m \rightarrow e = m \oplus k$

Decrypt: ciphertext  $c \rightarrow c \oplus k = (m \oplus k) \oplus k = m$

$e = 0110$   
 $m = 0101$   
 $k = 0011$   
 $c = 0110$

One-time pad:

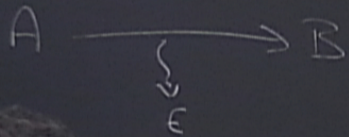
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Encryption:  $n$ -bit message  $m \rightarrow e = m \oplus k$

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e.g.  $m = 0101$   
 $k = 0011$   
 $e = 0110$





One-time pad:

Alice & Bob share secret key  $k$  ( $n$  bits long)

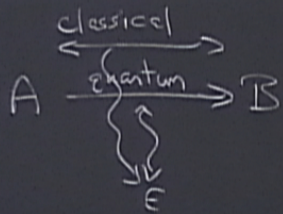
Encryption:  $n$ -bit message  $m \rightarrow e = m \oplus k$

Decrypt: ciphertext  $e \rightarrow e \oplus k = (m \oplus k) \oplus k = m$

Information-theoretic security

e.g.  $m = 0101$   
 $k = 0011$   
 $e = 0110$





Quantum key distribution (QKD)

A & B set up secret key  $k$

BB84:



key distribution (QKD)

up secret key  $k$

BB84:

1. Alice generates  $N$  random bit pairs  $(a_i, r_i)$
- 2.

$a_i$	$r_i$	qubit
0	0	
0	1	
1	0	
1	1	

key distribution (QKD)

up secret key  $k$

BB84:

1. Alice generates  $N$  random bit pairs  $(a_i, r_i)$

2.

$a_i$	$r_i$	qubit
0	0	$ 0\rangle$
0	1	$ 1\rangle$
1	0	$ +\rangle$
1	1	$ -\rangle$



key distribution (QKD)

up secret key  $k$

BB84:

1. Alice generates  $N$  random bits  $a_i$  and  $r_i$
2. Alice sends corresponding qubits
3. Bob chooses  $N$  random bits  $b_i$

$a_i$	$r_i$	qubit
0	0	$ 0\rangle$
0	1	$ 1\rangle$
1	0	$ +\rangle$
1	1	$ -\rangle$



distribution (QKD)

secret key  $k$

BB84:

1. Alice generates  $N$  random bit pairs  $(a_i, r_i)$
2. Alice sends corresponding qubits to Bob.
3. Bob choose  $N$  random bits  $b_i$ , measures qubit  $i$  in base

$a_i$	$r_i$	qubit
0	0	$ 0\rangle$
1	0	$ 1\rangle$
0	1	$ +\rangle$
1	1	$ -\rangle$



distribution (QKD)

secret key  $k$

BB84:

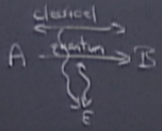
1. Alice generates  $N$  random bit pairs  $(a_i, r_i)$
2. Alice sends corresponding qubits to Bob.
3. Bob chooses  $N$  random bits  $b_i$ , measures qubit  $i$  in basis  $b_i$ , getting result  $s_i$ .
4. Alice & Bob classically announce  $a_i$  &  $b_i$ . Discard any bits for which  $a_i \neq b_i$ . Keep raw key.

$a_i$	$r_i$	qubit
0	0	$ 0\rangle$
0	1	$ 1\rangle$
1	0	$ +\rangle$
1	1	$ -\rangle$



Imagine





### Quantum key distribution (QKD)

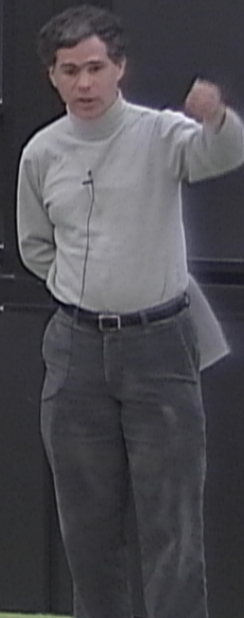
A & B set up secret key  $k$

Let's A & B detect the presence of an eavesdropper

### BB84:

1. Alice generates  $N$  random bit pairs  $(a_i, r_i)$
2. Alice sends corresponding qubits to Bob
3. Bob chooses  $N$  random bits  $b_i$ , measures qubit  $i$  in basis  $b_i$ , getting result  $s_i$ .
4. Alice & Bob classically announce  $a_i, b_i$ . Discard any bits for which  $a_i \neq b_i$ . Keep remaining.
5. Alice & Bob choose 50% of bits  $a_i, s_i$  to announce & compare. Calculate error rate & abort if it is too high.

$a$	$r$	qubit
0	0	$ 0\rangle$
0	1	$ 1\rangle$
1	0	$ +\rangle$
1	1	$ -\rangle$





Quantum key distribution (QKD)

B set up secret key  $k$

A & B detect the presence  
of eavesdropper

BB84:

1. Alice generates  $N$  random bit pairs  $(a_i, r_i)$
2. Alice sends corresponding qubits to Bob.
3. Bob chooses  $N$  random bits  $b_i$ , measures qubit  $i$  in basis  $b_i$ , getting result  $s_i$ .
4. Alice & Bob classically announce  $a_i$  &  $b_i$ . Discard any bits for which  $a_i \neq b_i$ . Keep raw key.
5. Alice & Bob choose 50% of bits  $a_i/s_i$  to announce & compare.  
Calculate error rate & abort if it is too high.
6. Take parities corresponding to a classical ECC & compare

$a_i$	$r_i$	qubit
0	0	$ 0\rangle$
0	1	$ 1\rangle$
1	0	$ +\rangle$
1	1	$ -\rangle$



Quantum key distribution (QKD)

B set up secret key  $k$

A & B detect the presence  
of eavesdropper

BB84:

1. Alice generates  $N$  random bit pairs  $(a_i, r_i)$
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4. Alice & Bob classically announce  $a_i$  &  $b_i$ . Discard any bits for which  $a_i \neq b_i$ . Keep raw key.
5. Alice & Bob choose 50% of bits  $r_i/s_i$  to announce & compare.  
Calculate error rate & abort if it is too high.
6. Take parities corresponding to a classical ECC & compare, correct errors.  
Discard 1 bit for each parity they announce

$a_i$	$r_i$	qubit
0	0	$ 0\rangle$
0	1	$ 1\rangle$
1	0	$ +\rangle$
1	1	$ -\rangle$



## BB84:

1. Alice generates  $N$  random bit pairs  $(a_i, r_i)$
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5. Alice & Bob choose 50% of bits  $a_i/s_i$  to announce & compare.  
Calculate error rate & abort if it is too high.
6. Take parities corresponding to a classical ECC & compare to correct errors.  
Discard 1 bit for each parity they announce

$a_i$	$r_i$	qubit
0	0	$ 0\rangle$
0	1	$ 1\rangle$
1	0	$ +\rangle$
1	1	$ -\rangle$



## BB84:

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5. Alice & Bob choose 50% of bits  $a_i/s_i$  to announce & compare.  
Calculate error rate & abort if it is too high.
6. Take parities corresponding to a classical ECC & compare, correct errors.  
Discard 1 bit for each parity they announce

7. Privacy amplification: They choose random subsets & take the parity. The parities become bits of the final key.



3. Bob choose  $N$  random bits  $b_i$ , measures qubit  $i$  in basis  $b_i$ , getting result  $s_i$ .
4. Alice & Bob classically announce  $a_i$  &  $b_i$ . Discard any bits for which  $a_i \neq b_i$ . Keep raw key.
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Calculate error rate & abort if it is too high.
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Discard 1 bit for each parity they announce

7. Privacy amplification: They choose random subsets & take the parity. The parities become bits of the final key.

Security condition:  $\forall$  attacks by Eve, either she is caught w/  
high probability (exponentially close to 1), or she has (with high probability)  
almost no information (exponentially small,  $\ll 1$  bit) about the final key.





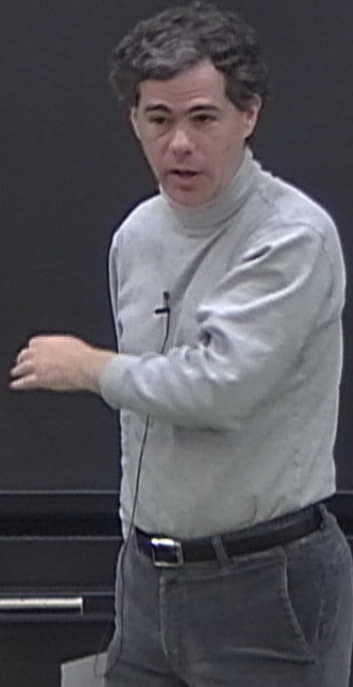
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Discard 1 bit for each parity they announce

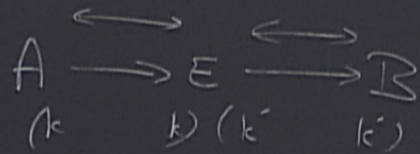
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Security condition:  $\forall$  attacks by Eve, either she is caught w/  
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$A \rightleftharpoons E$      $B$



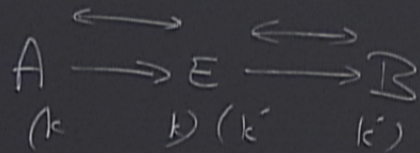


Man-in-the-Middle

Need authenticated classical channel.



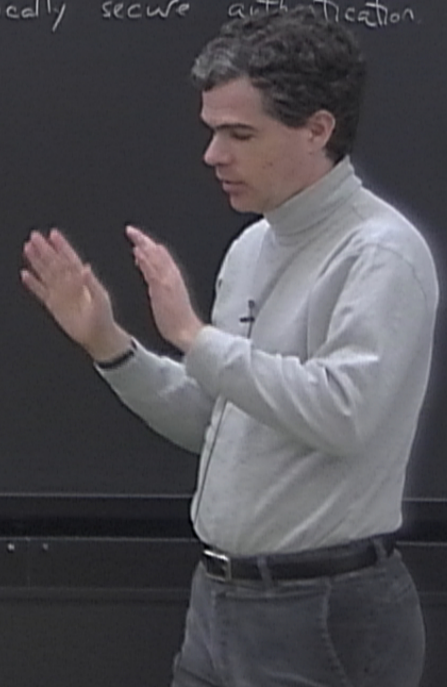


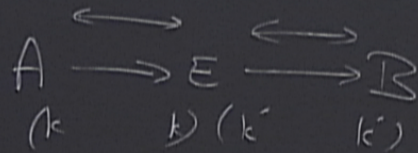


Man-in-the-Middle

Need authenticated classical channel.

Alice & Bob have prior secret key,  
information-theoretically secure authentication





Man-in-the-Middle

Need authenticated classical channel.

Alice & Bob have prior secret key,  
information-theoretically secure authentication

