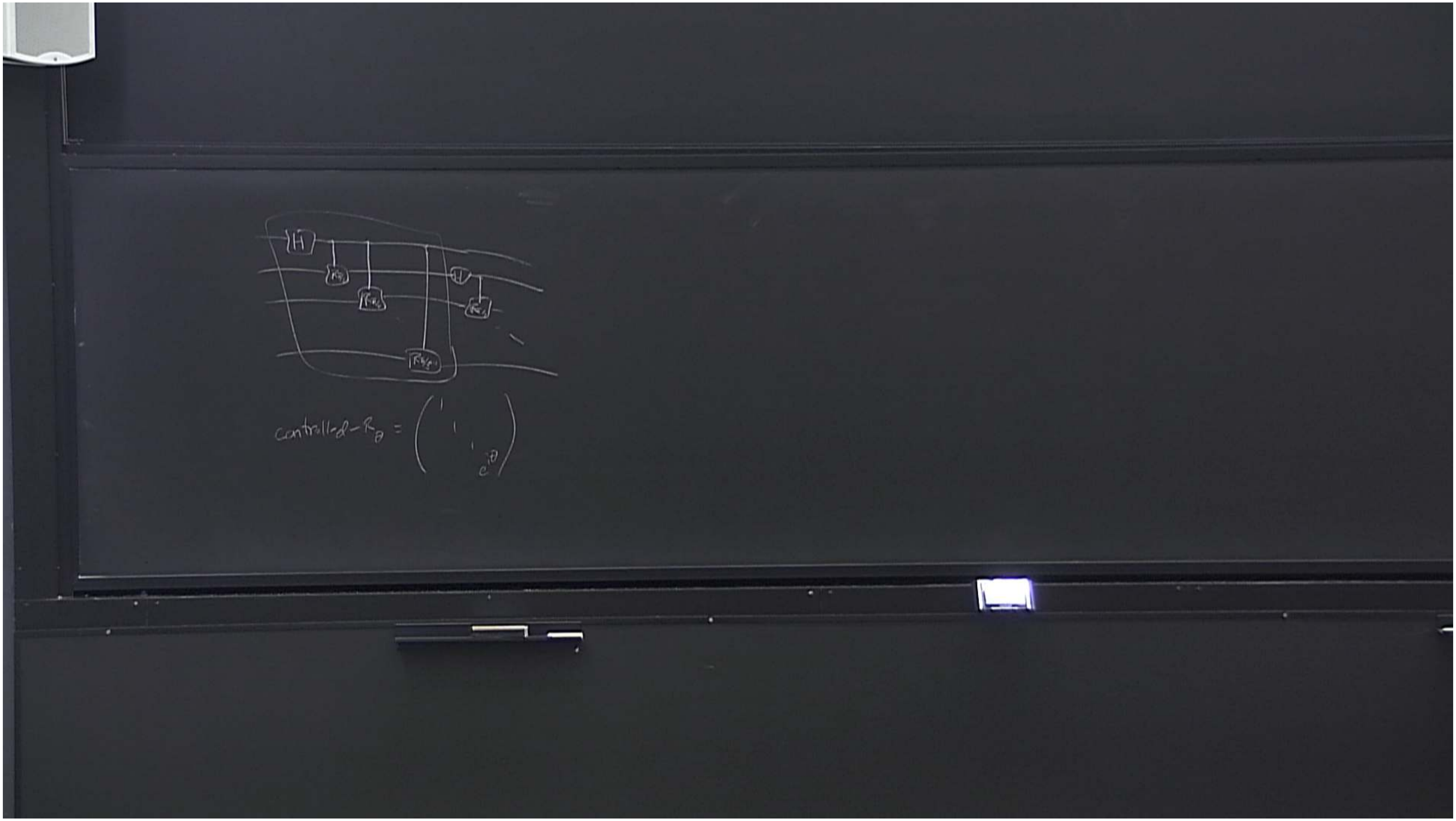
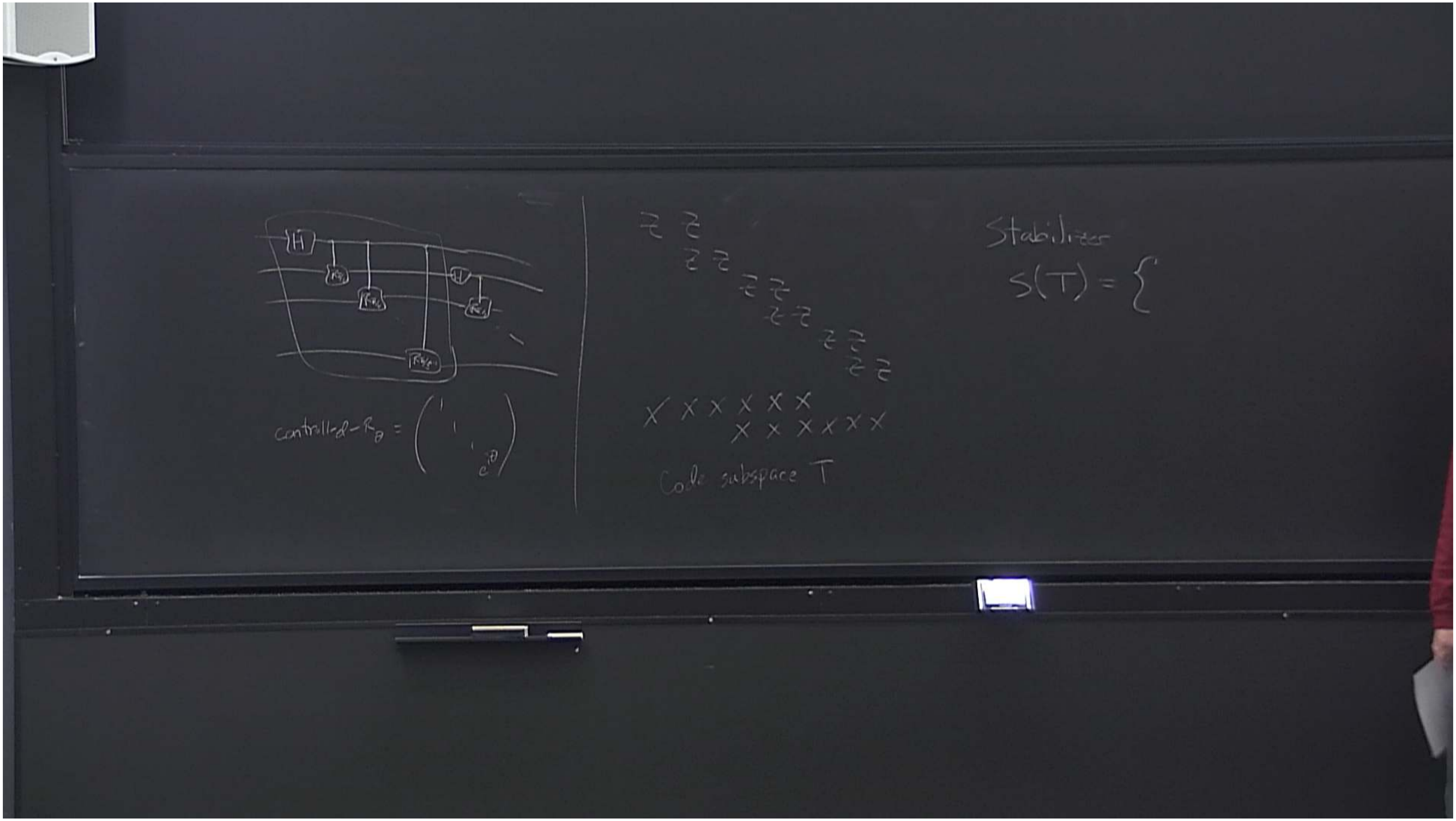
Title: Quantum Information Review-11

Date: Mar 03, 2015 11:30 AM

URL: http://pirsa.org/15030014

Abstract:


H
controlled-R_θ =


H
controlled-R_θ =
Z Z
Z Z
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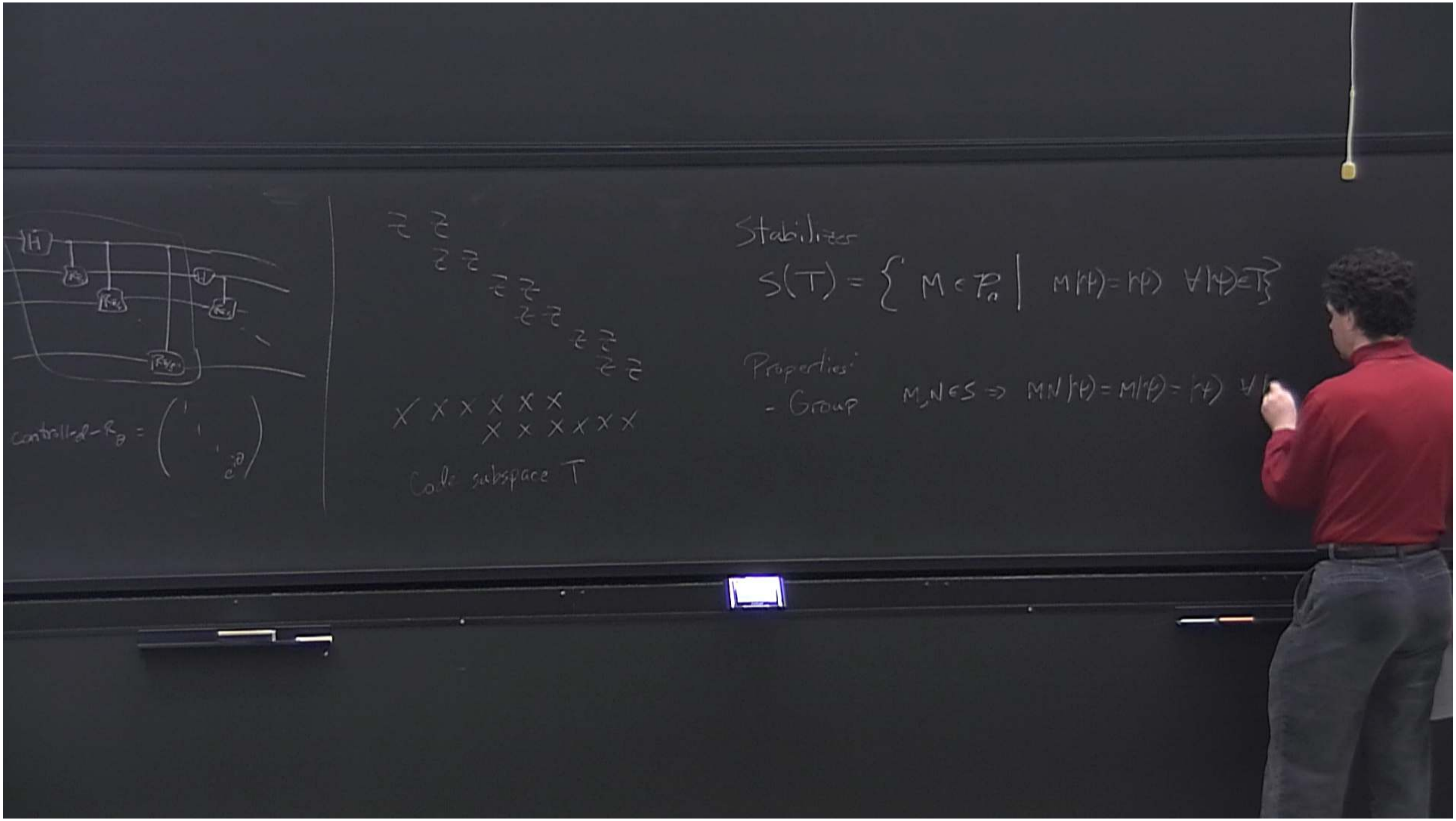
Z Z
X X X X X X
X X X X X X
Code subspace T
Stabilizer
S(T) = {


Stabilizer
S(T) = { M ∈ P_n | M|ψ⟩ = |ψ⟩ ∀|ψ⟩ ∈ T }
Properties:
- Group
M, N ∈ S ⇒ MN|ψ⟩ = M|ψ⟩ = |ψ⟩ ∀
Code subspace T
controlled-R_θ =

Let S be an Abelian subgroup of P_n, with $-I \notin S$. Let

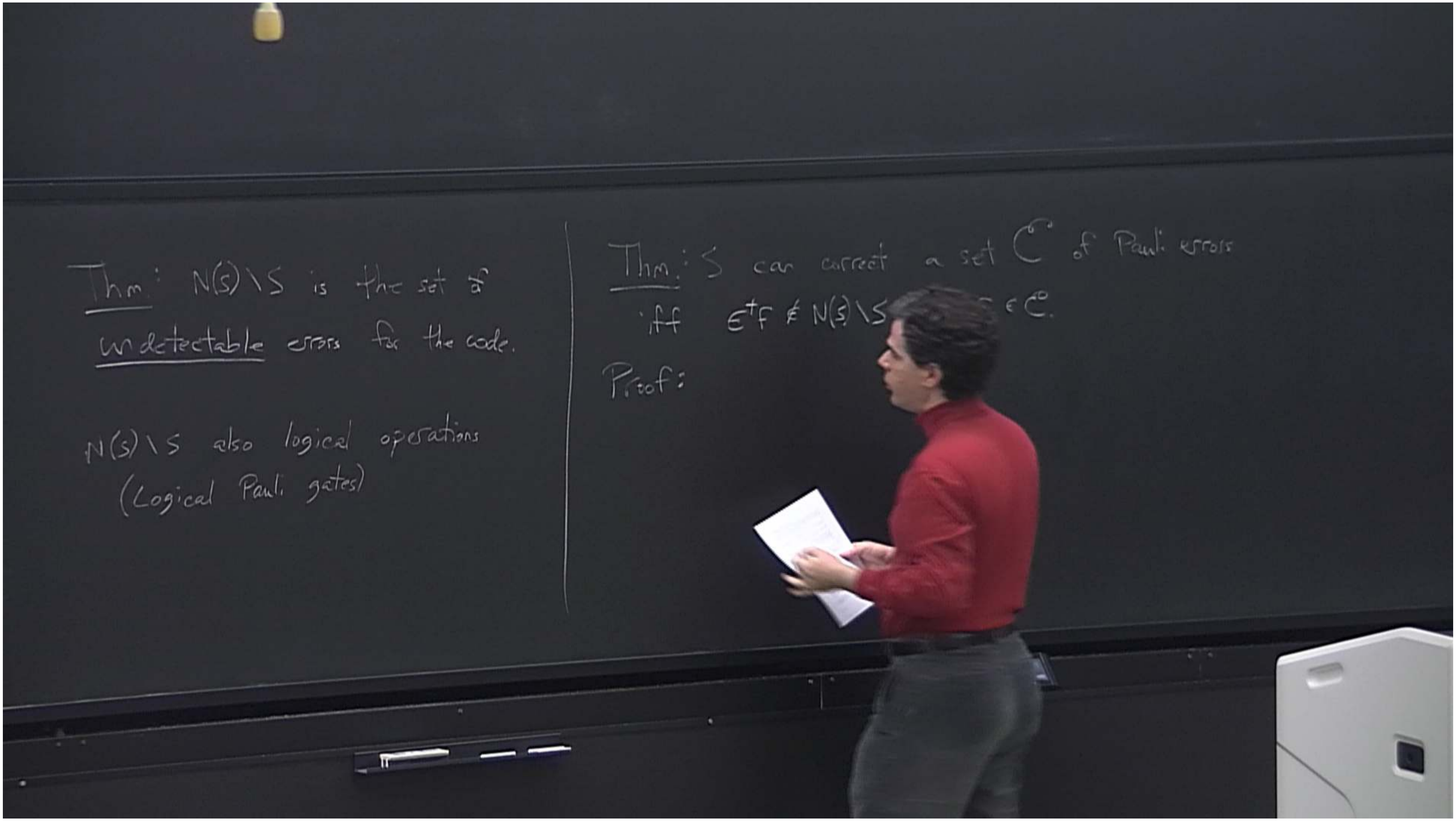
$$T(S) = \{ |\psi\rangle \mid M|\psi\rangle = |\psi\rangle \ \forall M \in S \}.$$

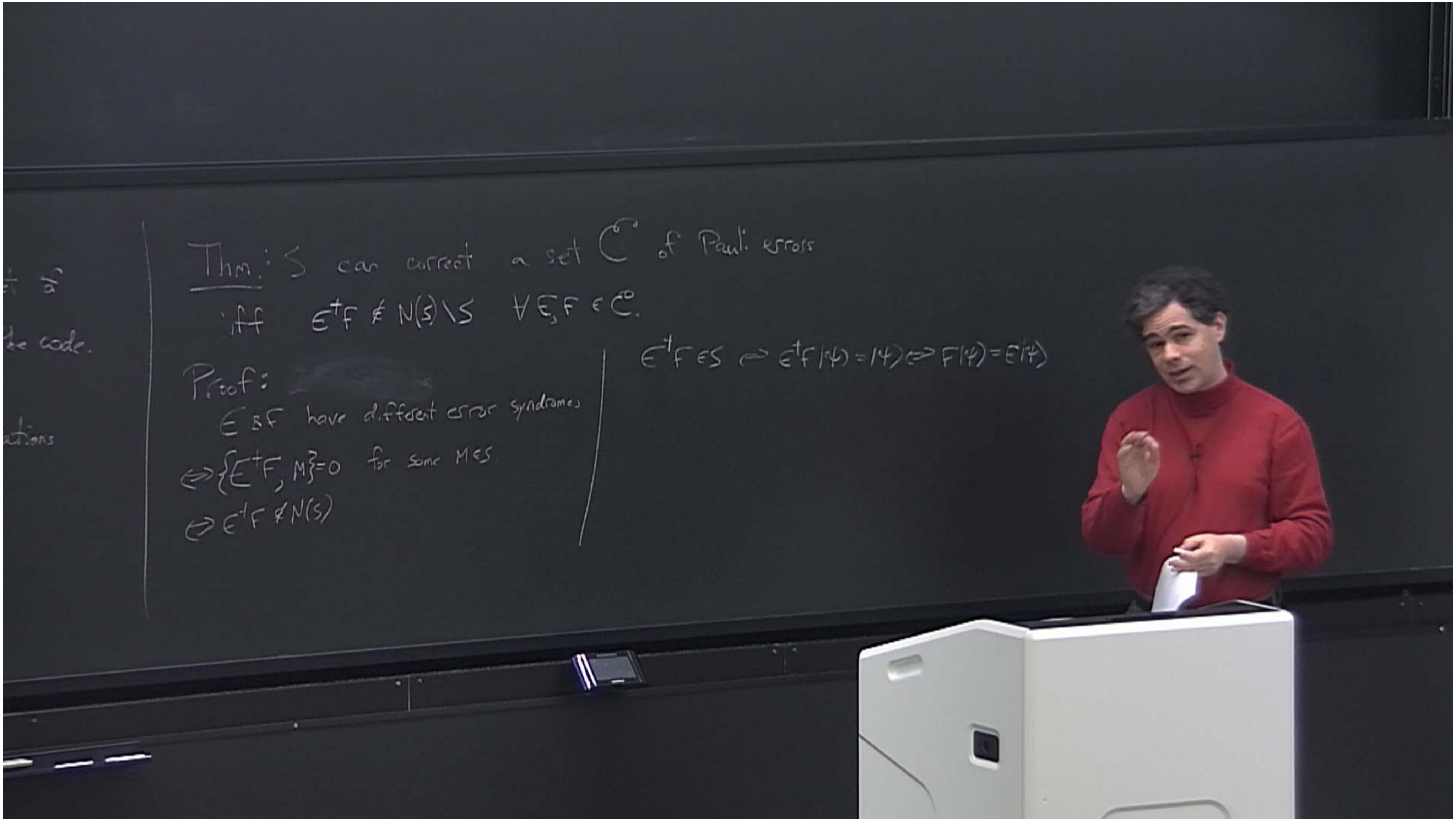
Stabilizer code

Thm: If a stabilizer has r generators, $|S| = 2^r$, $k = n - r$. Then $\dim T(S) = 2^k$.

Thm: $N(S)\setminus S$ is the set of undetectable errors for the code.

$N(S)\setminus S$ also logical operations
(Logical Pauli gates)


Thm: N(S)\S is the set of
undetectable errors for the code.
N(S)\S also logical operations
(Logical Pauli gates)
Thm: S can correct a set of Pauli errors
iff
Proof:



Thm: S can correct a set ℰ of Pauli errors
iff E†F ∉ N(S)\S ∀ E,F ∈ ℰ.
Proof:
E & F have different error syndromes
⇔ {E†F, M} = 0 for some M ∈ S
⇔ E†F ∉ N(S)
E†F ∈ S ⇔ E†F|ψ⟩ = |ψ⟩ ⇔ F|ψ⟩ = E|ψ⟩
the code.
ations

$|\psi\rangle \in T(S)$, $E \in \mathcal{P}_n$

$M \in S$ s.t. $\{M, E\} = 0$: $M(E|\psi\rangle) = -EM|\psi\rangle = -(E|\psi\rangle)$

$M \in S$ s.t. $[M, E] = 0$: $M(E|\psi\rangle) = +EM|\psi\rangle = E|\psi\rangle$

Error syndrome of E = r-bit vector

bit i is 0 if $[E, M_i] = 0$, 1 if $\{E, M_i\} = 0$

M_i ith generator of S.

Let

If $E \in$

S s

$|\psi\rangle \in T(S)$, $E \in \mathcal{P}_n$

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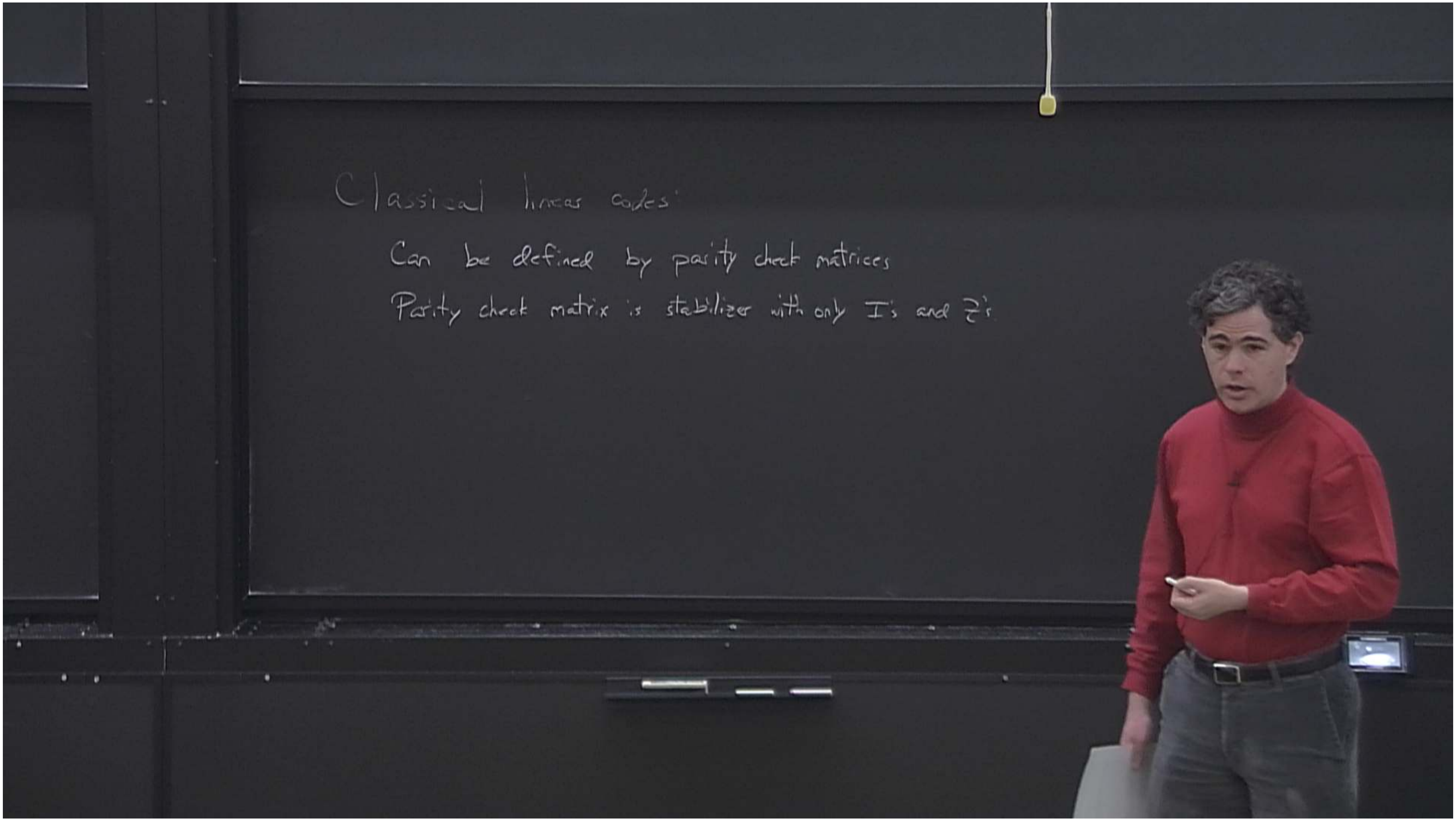
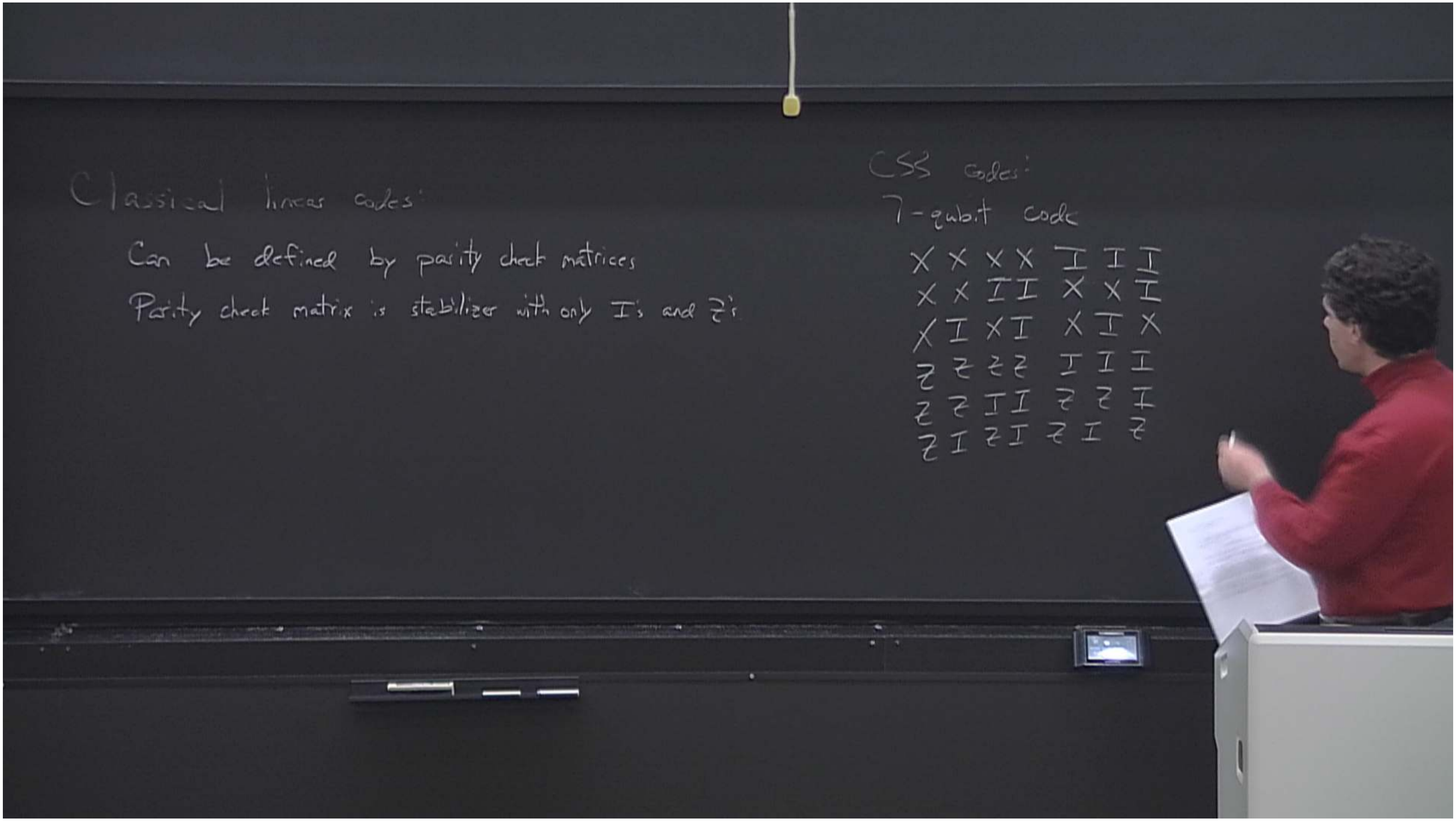
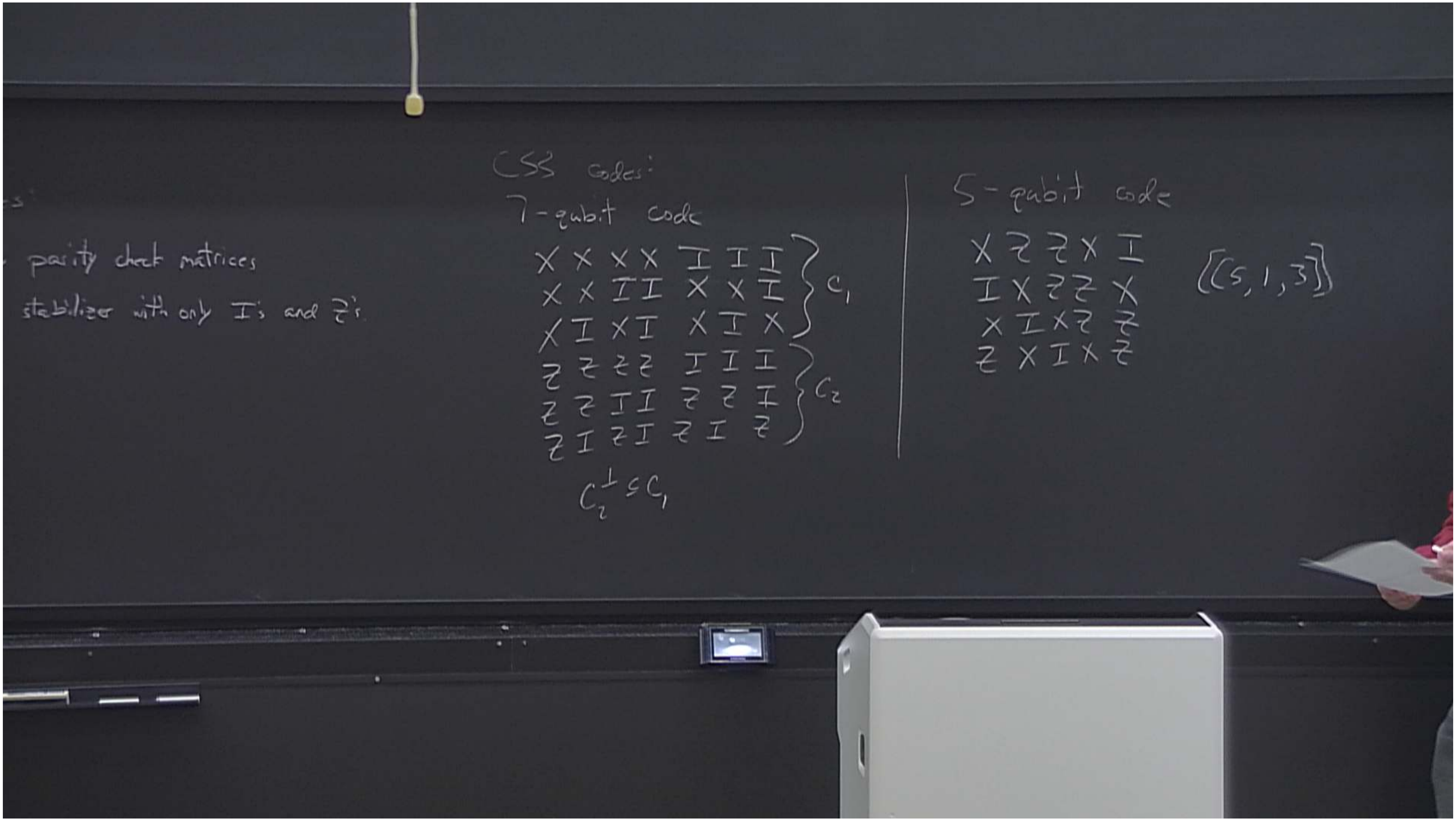

Classical linear codes:
Can be defined by parity check matrices
Parity check matrix is stabilizer with only I's and Z's


Classical linear codes:
Can be defined by parity check matrices
Parity check matrix is stabilizer with only I's and Z's
CSS codes:
7-qubit code
X X X X I I I
X X I I X X I
X I X I X I X
Z Z Z Z I I I
Z Z I I Z Z I
Z I Z I Z I Z


parity check matrices
stabilizer with only I's and Z's
CSS codes:
7-qubit code
X X X X I I I
X X I I X X I
X I X I X I X
C_1
Z Z Z Z I I I
Z Z I I Z Z I
Z I Z I Z I Z
C_2
$C_2^{\perp} \subseteq C_1$
5-qubit code
X Z Z X I
I X Z Z X
X I X Z Z
Z X I X Z
[[5, 1, 3]]

with $-I \notin S$. Let

$$T(S) = \{ |\psi\rangle \mid M|\psi\rangle = |\psi\rangle \ \forall M \in S \}$$

Stabilizer code

generators, $|S| = 2^r$

$k = n - r$. Then $\dim T(S) = 2^k$.

Fault-Tolerance:

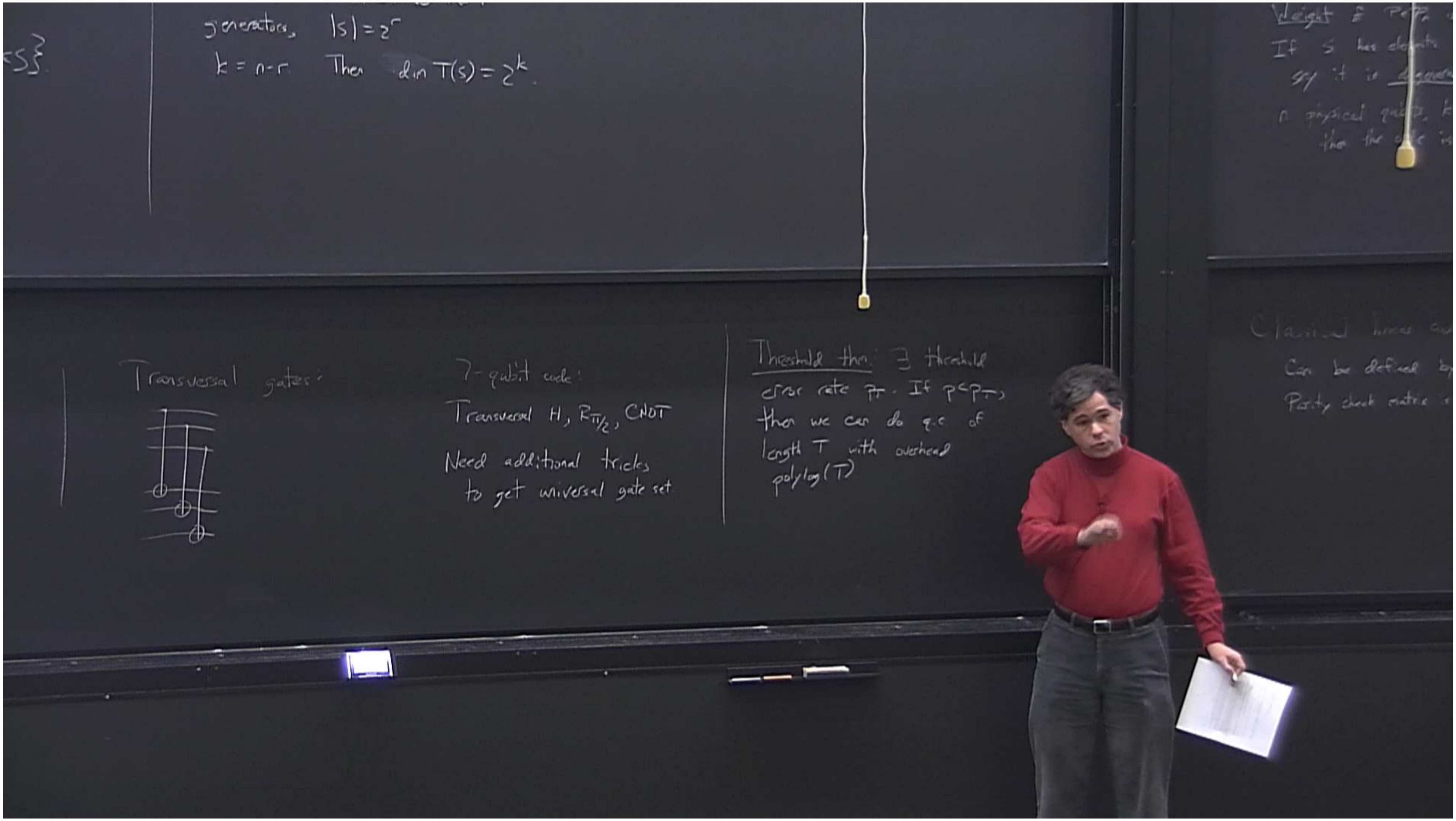
Avoid error propagation

Transversal gates:

7-qubit code:

Transversal H, $R_{\pi/2}$, CNOT

Need additional tricks to get universal gate set


generators, |S| = 2^r
k = n-r Then dim T(S) = 2^k.
Transversal gates:
7-qubit code:
Transversal H, R_{π/2}, CNOT
Need additional tricks
to get universal gate set
Threshold thm: ∃ threshold
error rate p_T. If p<p_T,
then we can do q.c of
length T with overhead
polylog(T)
Can be defined by
Parity check matrix