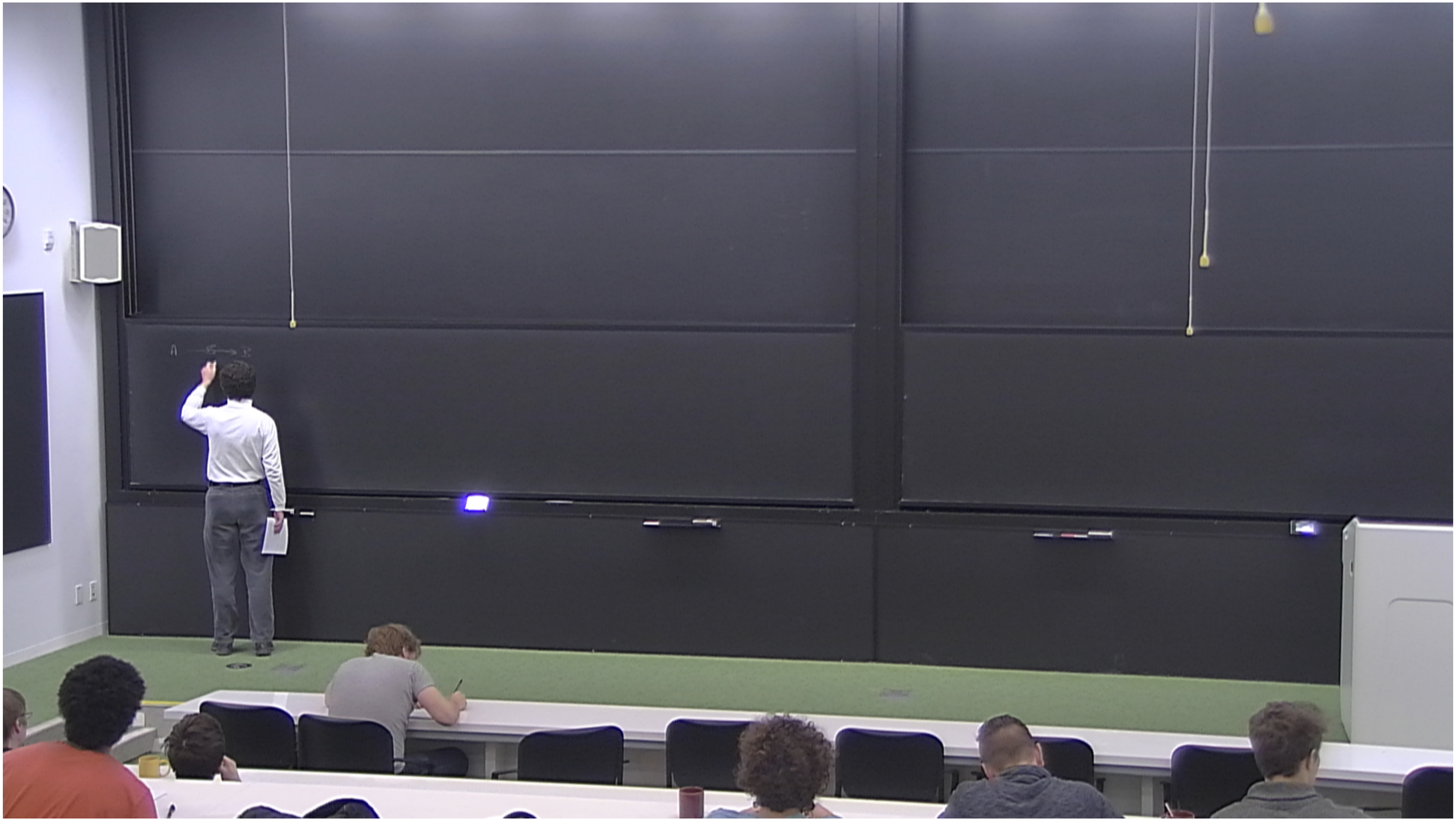
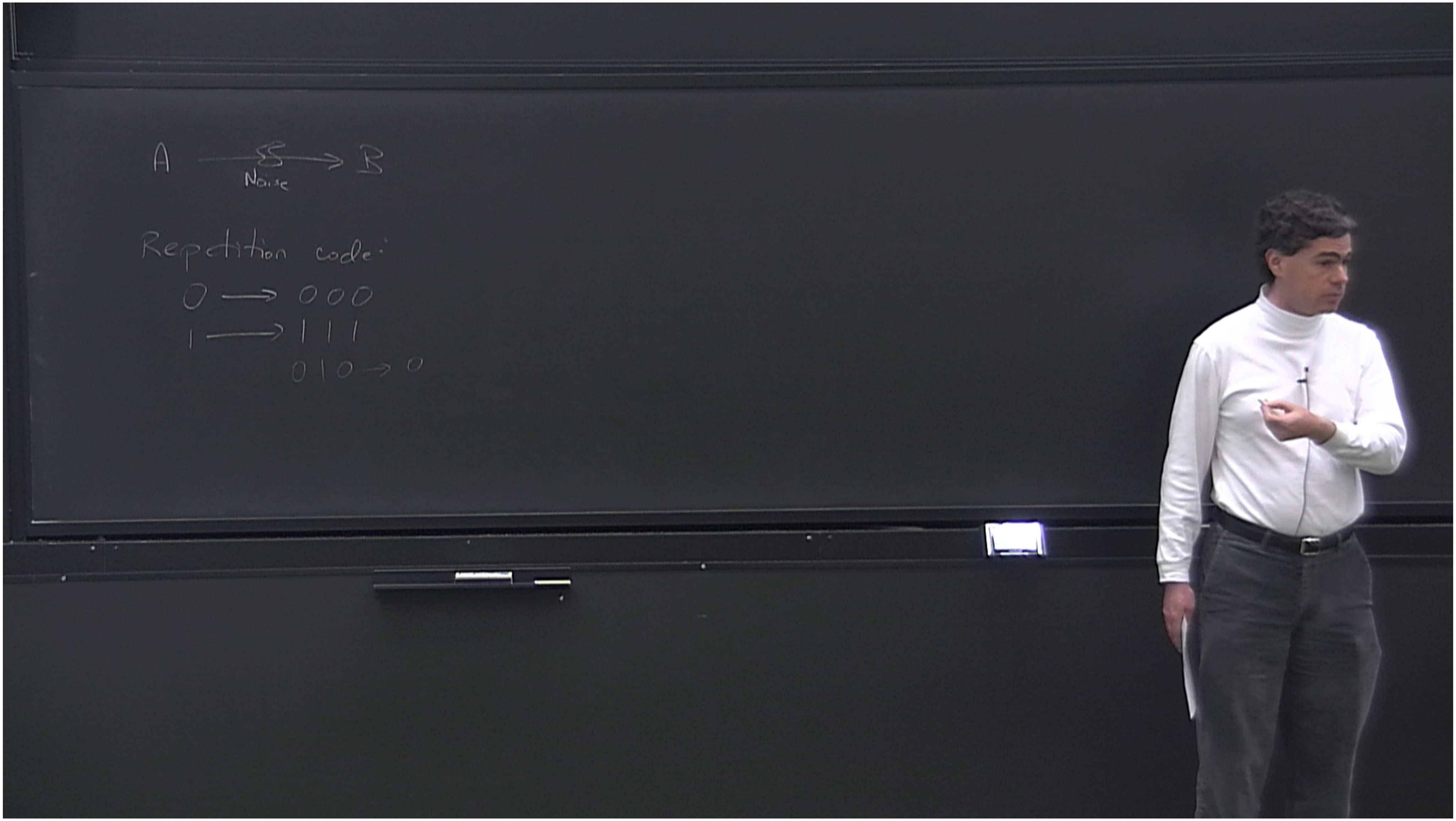
Title: Quantum Information Review-10

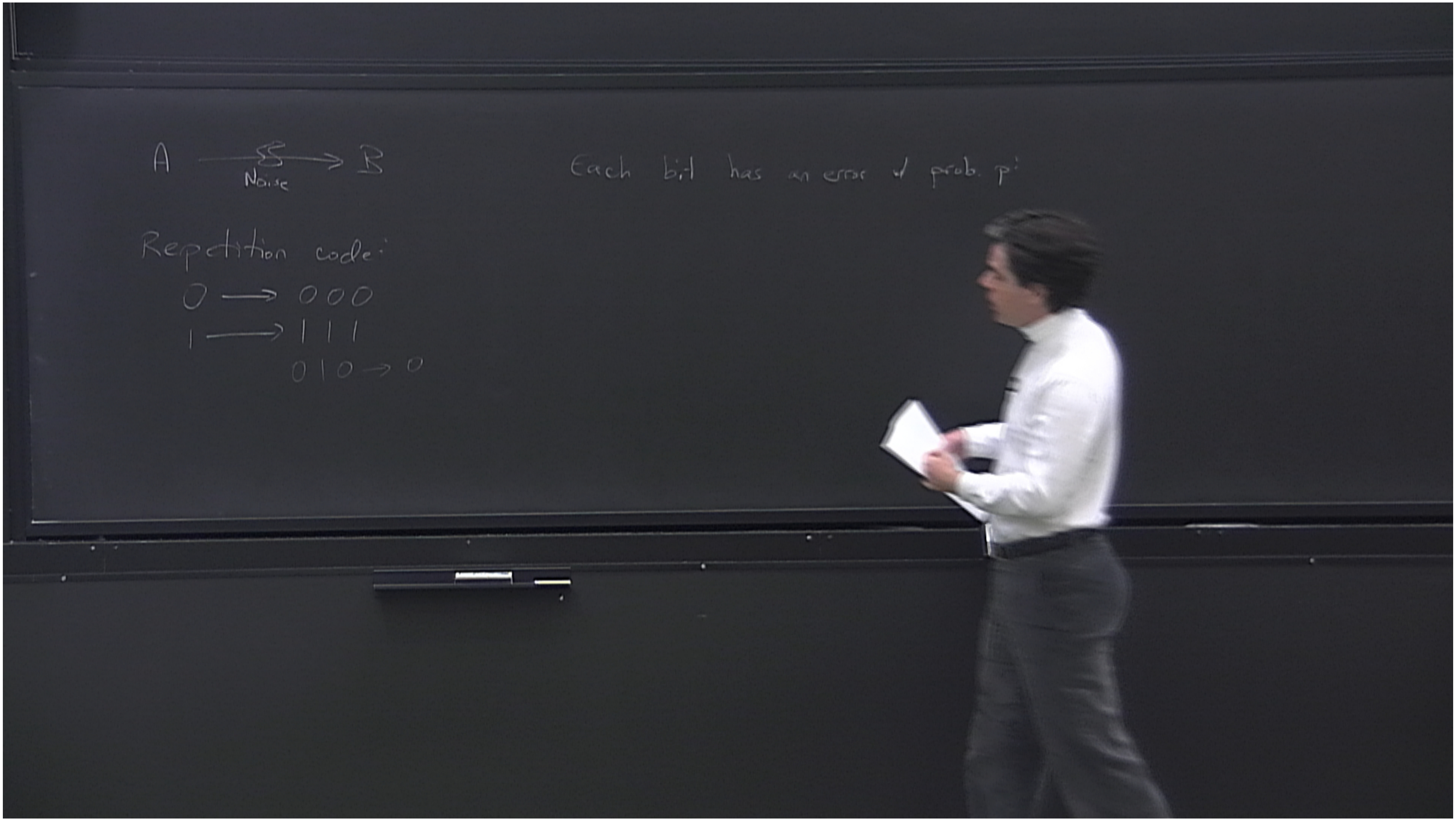
Date: Mar 02, 2015 11:30 AM

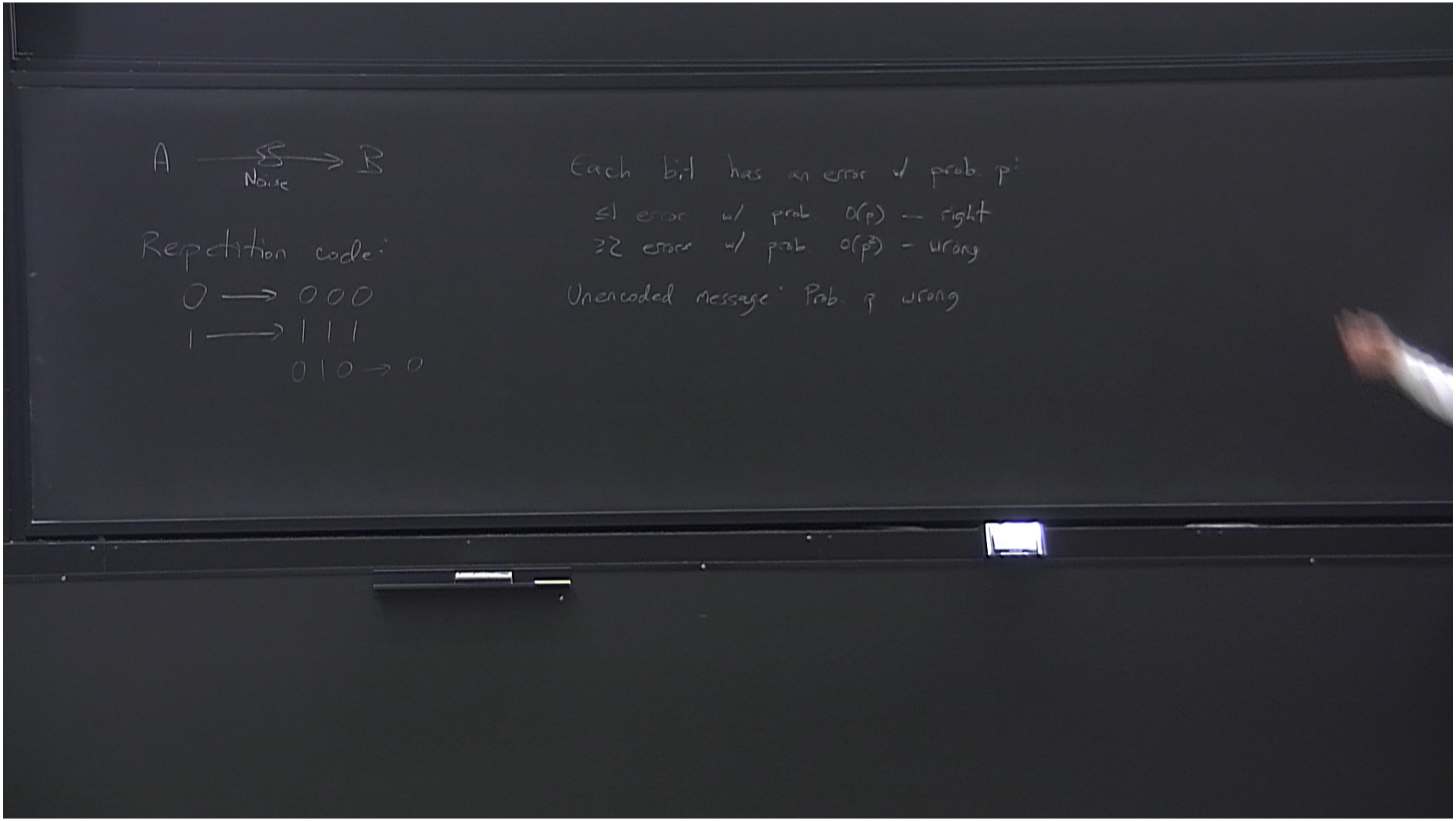
URL: http://pirsa.org/15030013

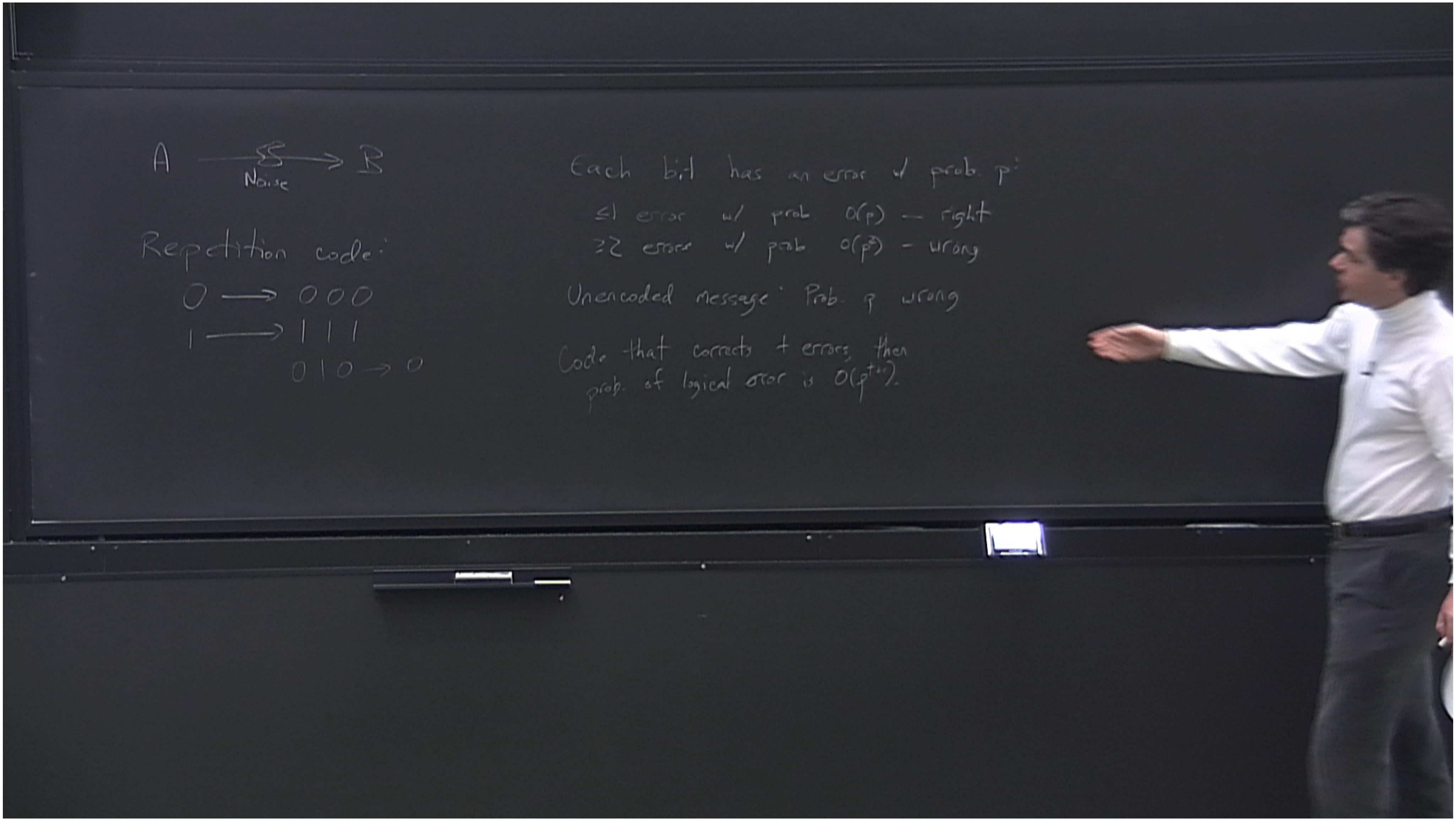
Abstract:

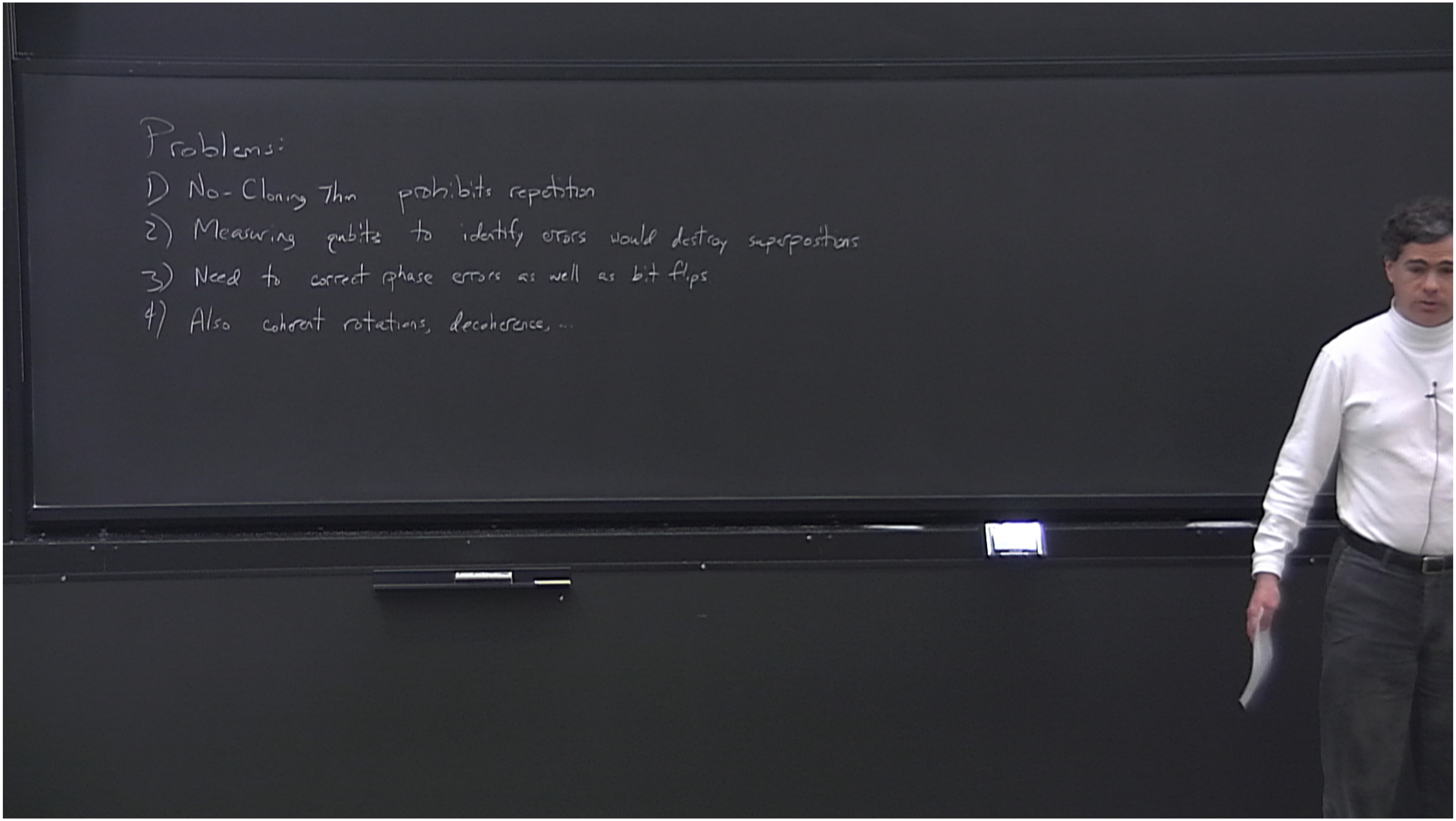


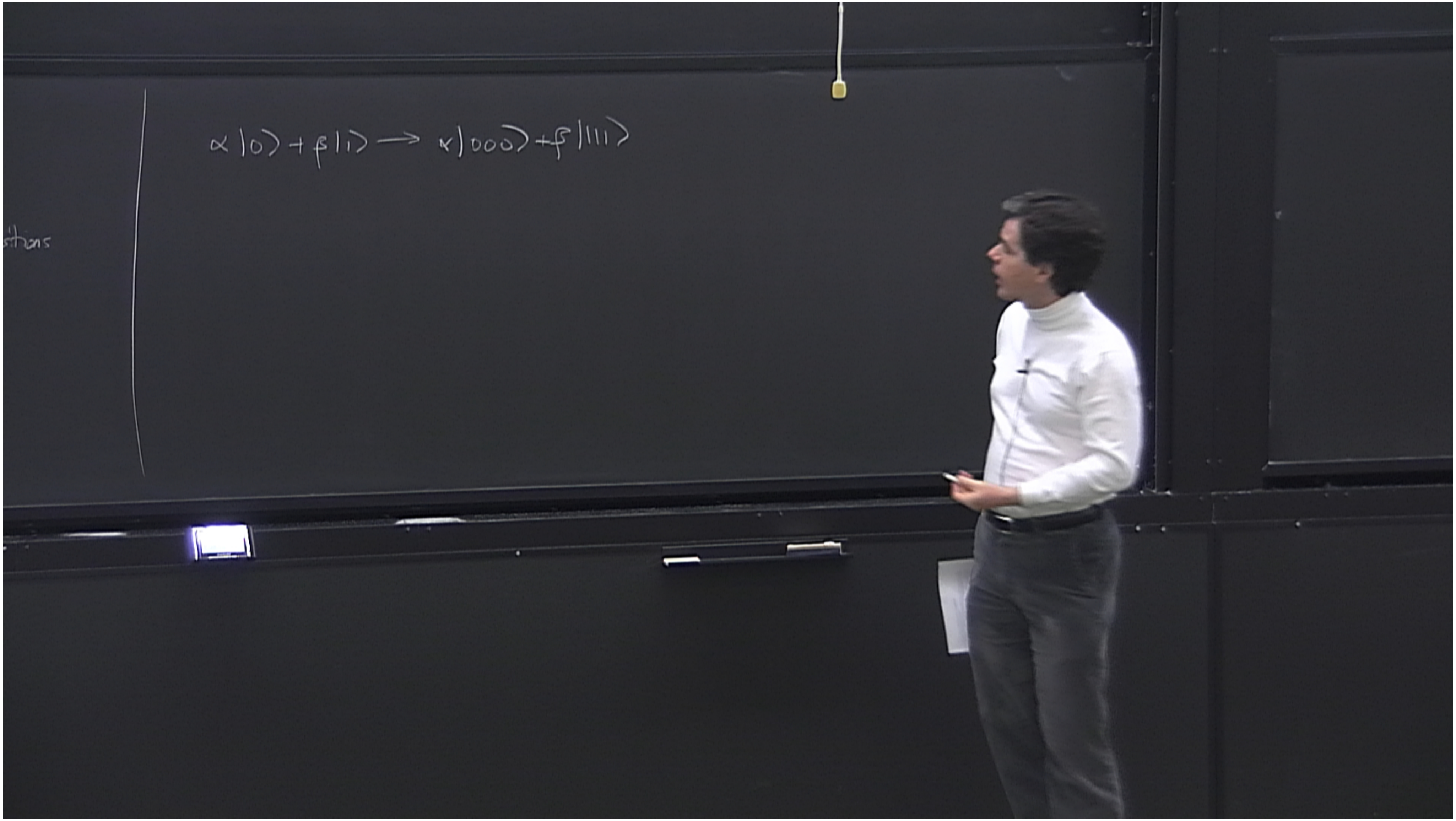

A
Noise
B
Repetition code:
0 → 000
1 → 111
010 → 0

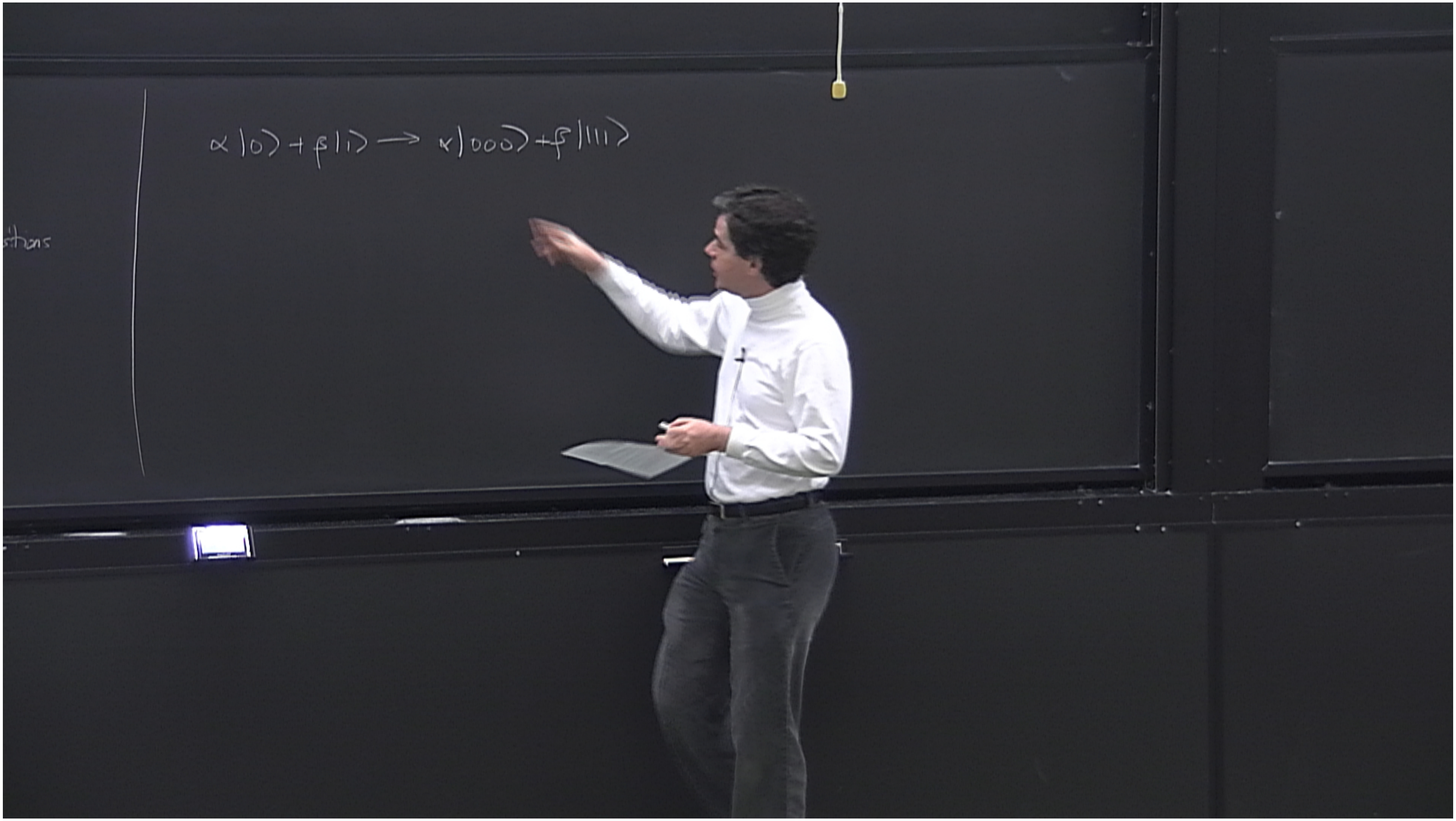

A → B
Noise
Each bit has an error of prob. p:
Repetition code:
0 → 000
1 → 111
010 → 0

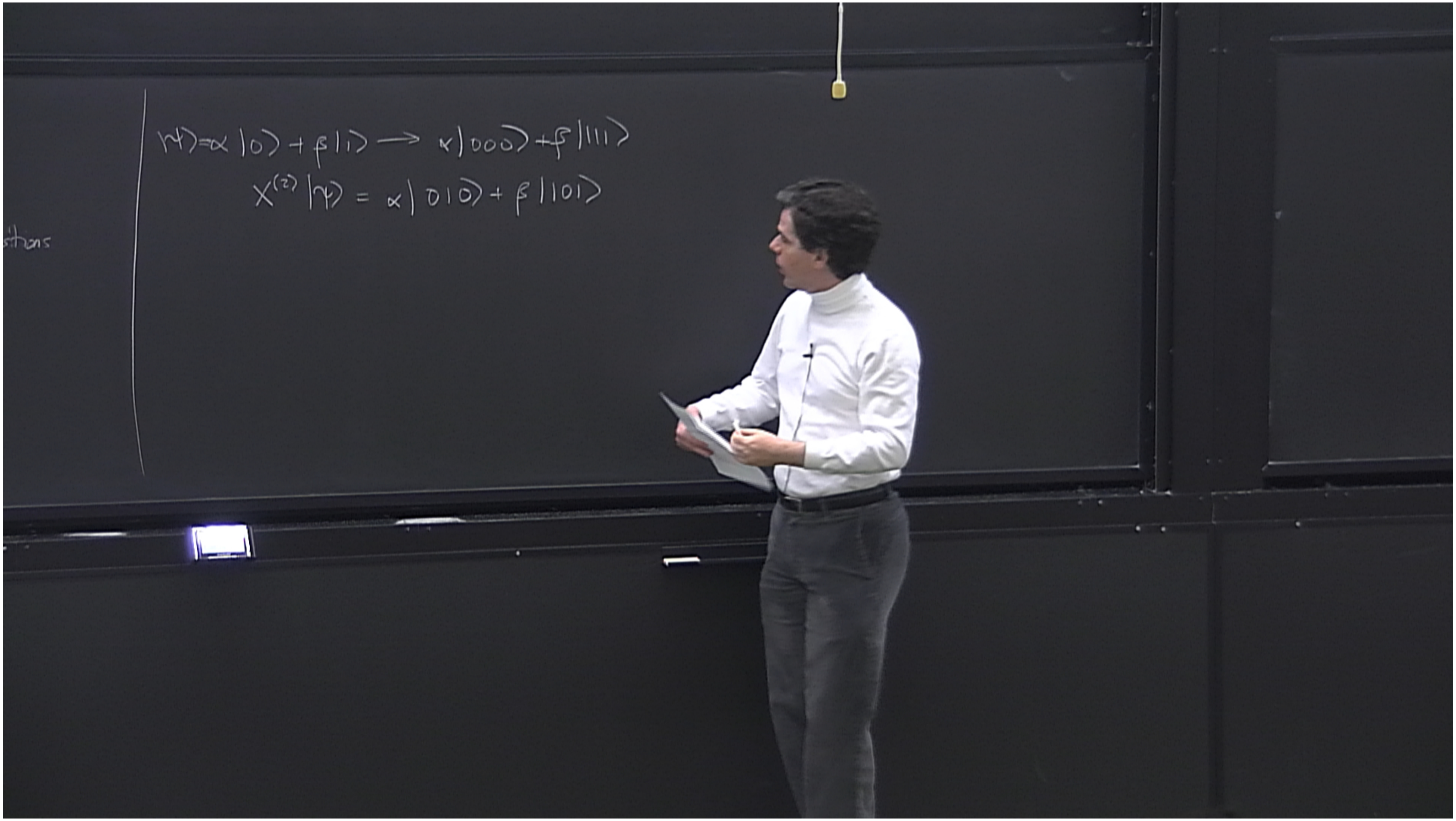

A → B
Noise
Repetition code:
0 → 000
1 → 111
010 → 0
Each bit has an error w/ prob. p:
≤1 error w/ prob. O(p) — right
≥2 errors w/ prob. O(p²) - wrong
Unencoded message: Prob. p wrong

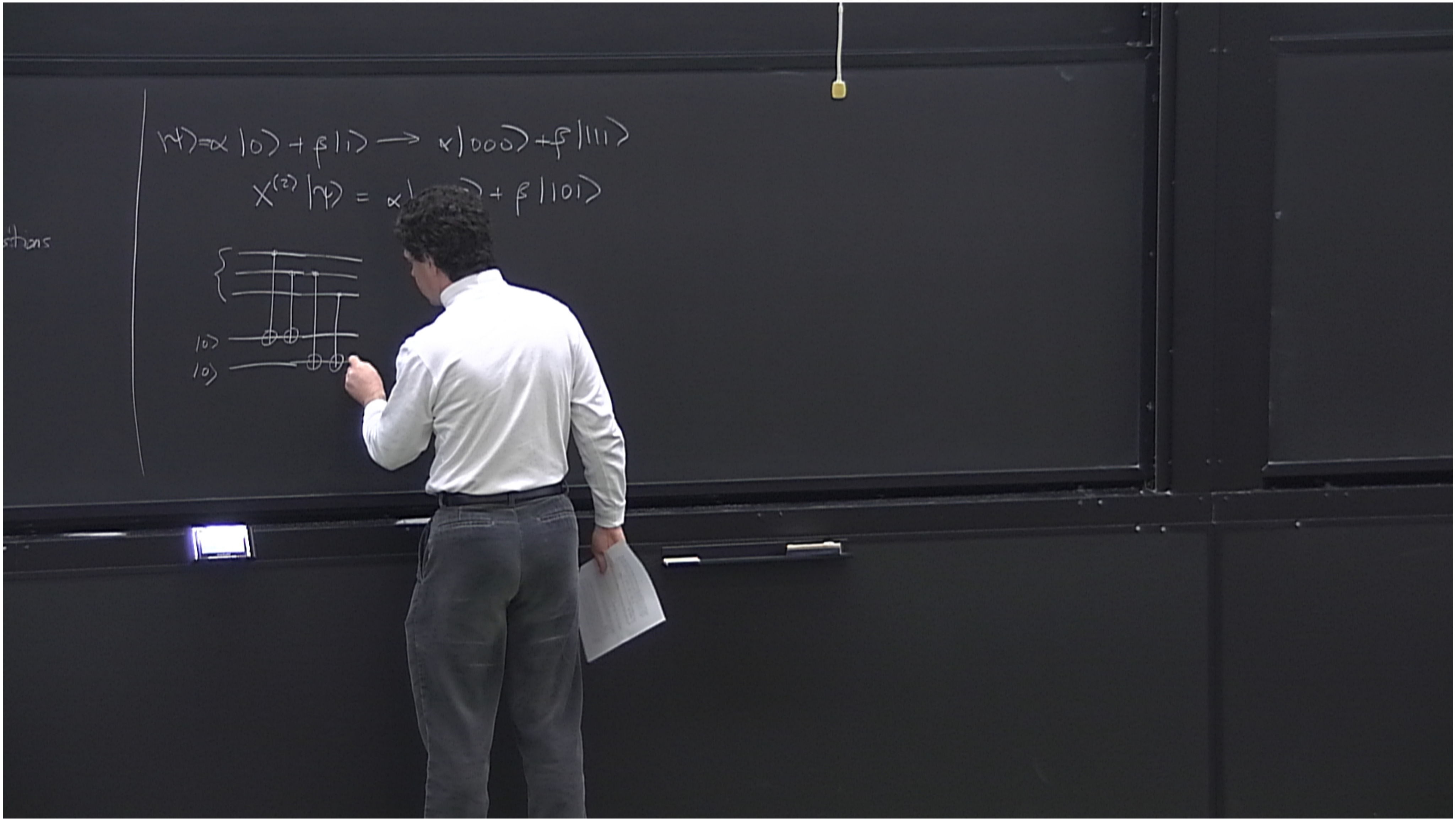

A → B
Noise
Repetition code:
0 → 000
1 → 111
010 → 0
Each bit has an error w/ prob. p:
≤1 error w/ prob. O(p) – right
≥2 errors w/ prob. O(p²) – wrong
Unencoded message: Prob. p wrong
Code that corrects t errors, then
prob. of logical error is O(p^{t+1}).

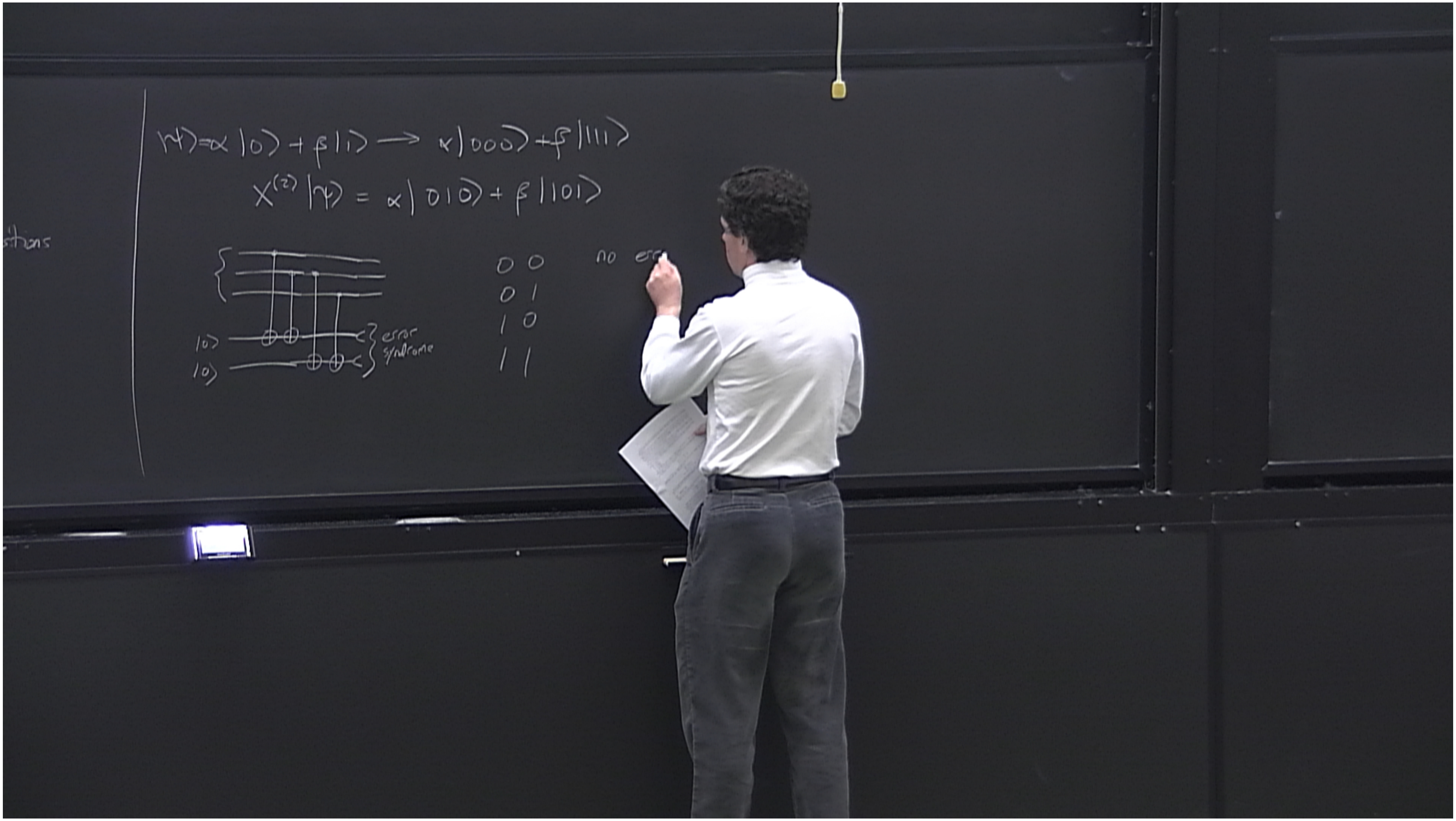

Problems:
1) No-Cloning Thm prohibits repetition
2) Measuring qubits to identify errors would destroy superpositions
3) Need to correct phase errors as well as bit flips
4) Also coherent rotations, decoherence, ...

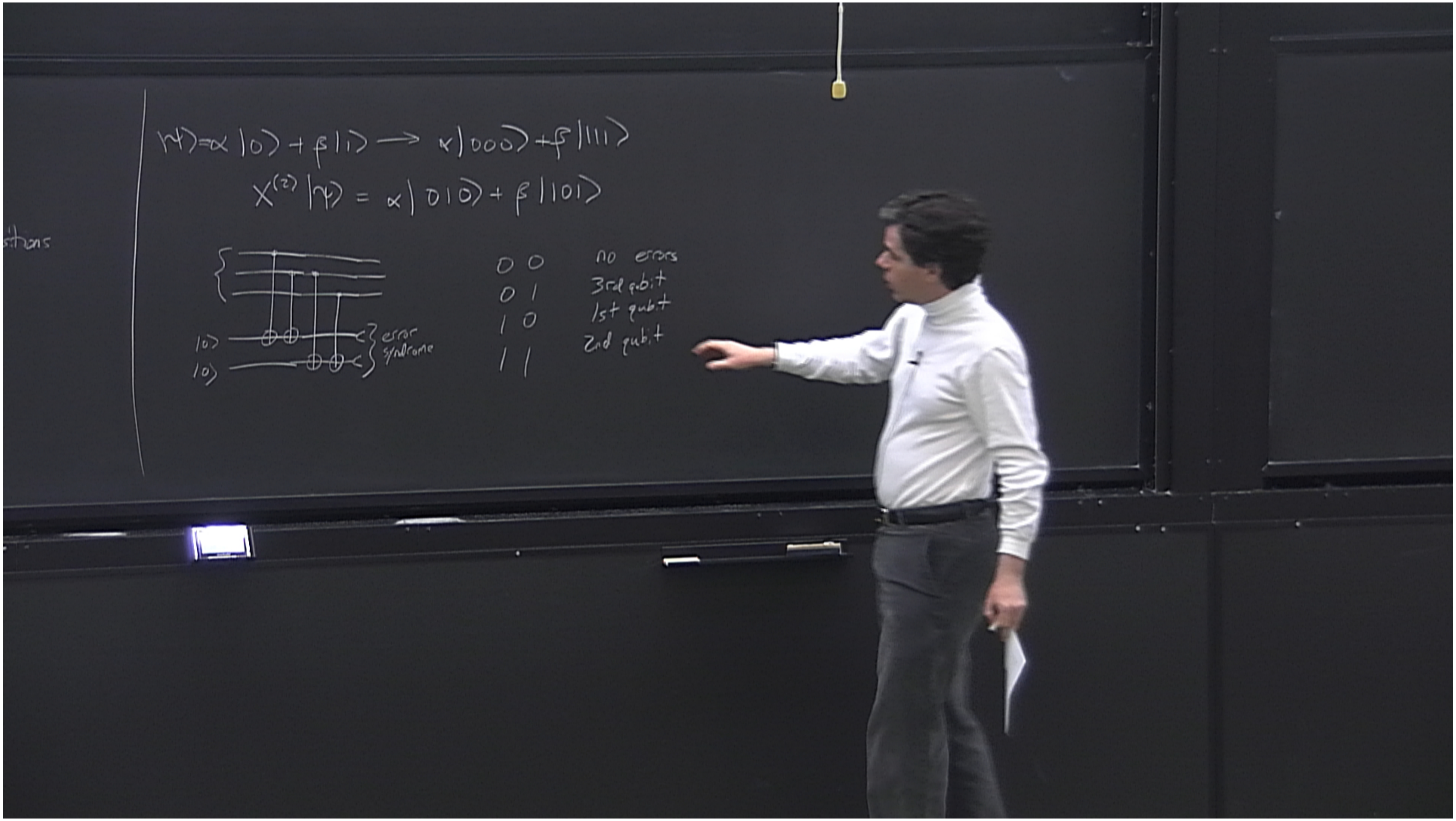

$\alpha|0\rangle + \beta|1\rangle \longrightarrow \alpha|000\rangle + \beta|111\rangle$
sitions

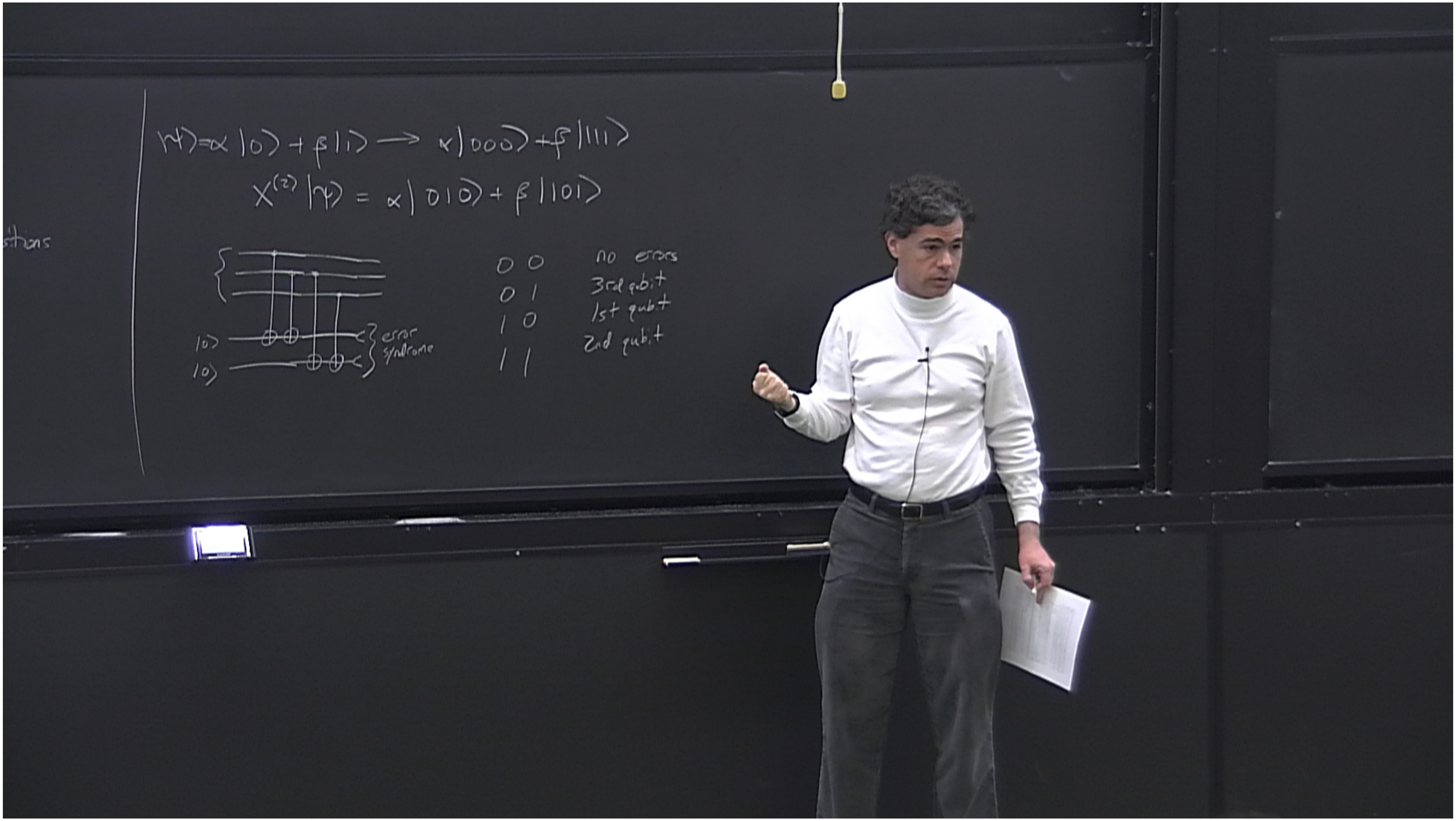

α|0⟩ + β|1⟩ ⟶ α|000⟩ + β|111⟩

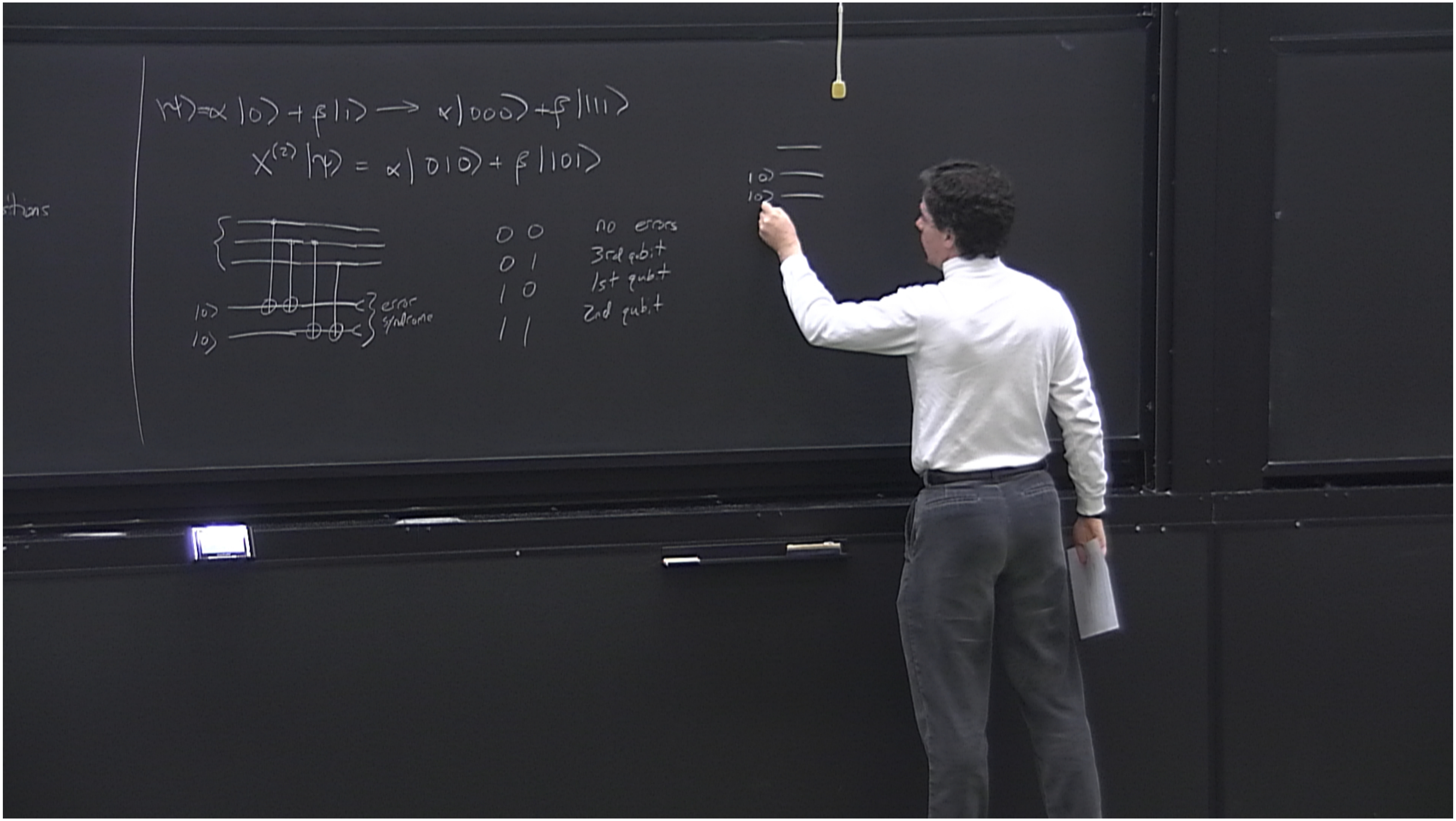

|ψ⟩=α|0⟩+β|1⟩ ⟶ α|000⟩+β|111⟩
X^(2)|ψ⟩ = α|010⟩+β|101⟩


|ψ⟩=α|0⟩+β|1⟩ ⟶ α|000⟩+β|111⟩
+ β|101⟩
|0⟩
|0⟩

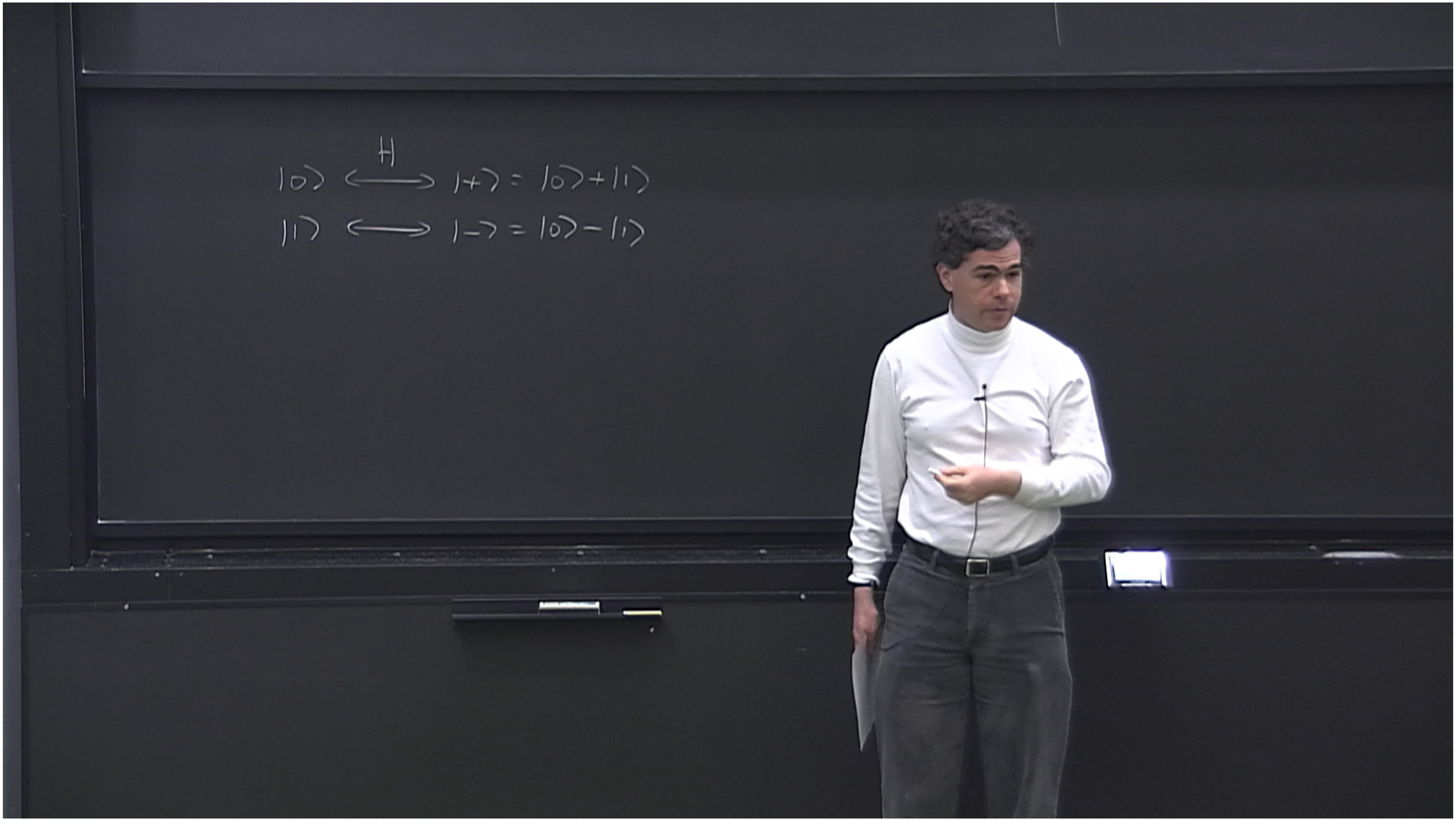

|ψ⟩=α|0⟩+β|1⟩ → α|000⟩+β|111⟩
X^(2)|ψ⟩ = α|010⟩+β|101⟩
|0⟩
|0⟩
error syndrome
0 0 no
0 1
1 0
1 1

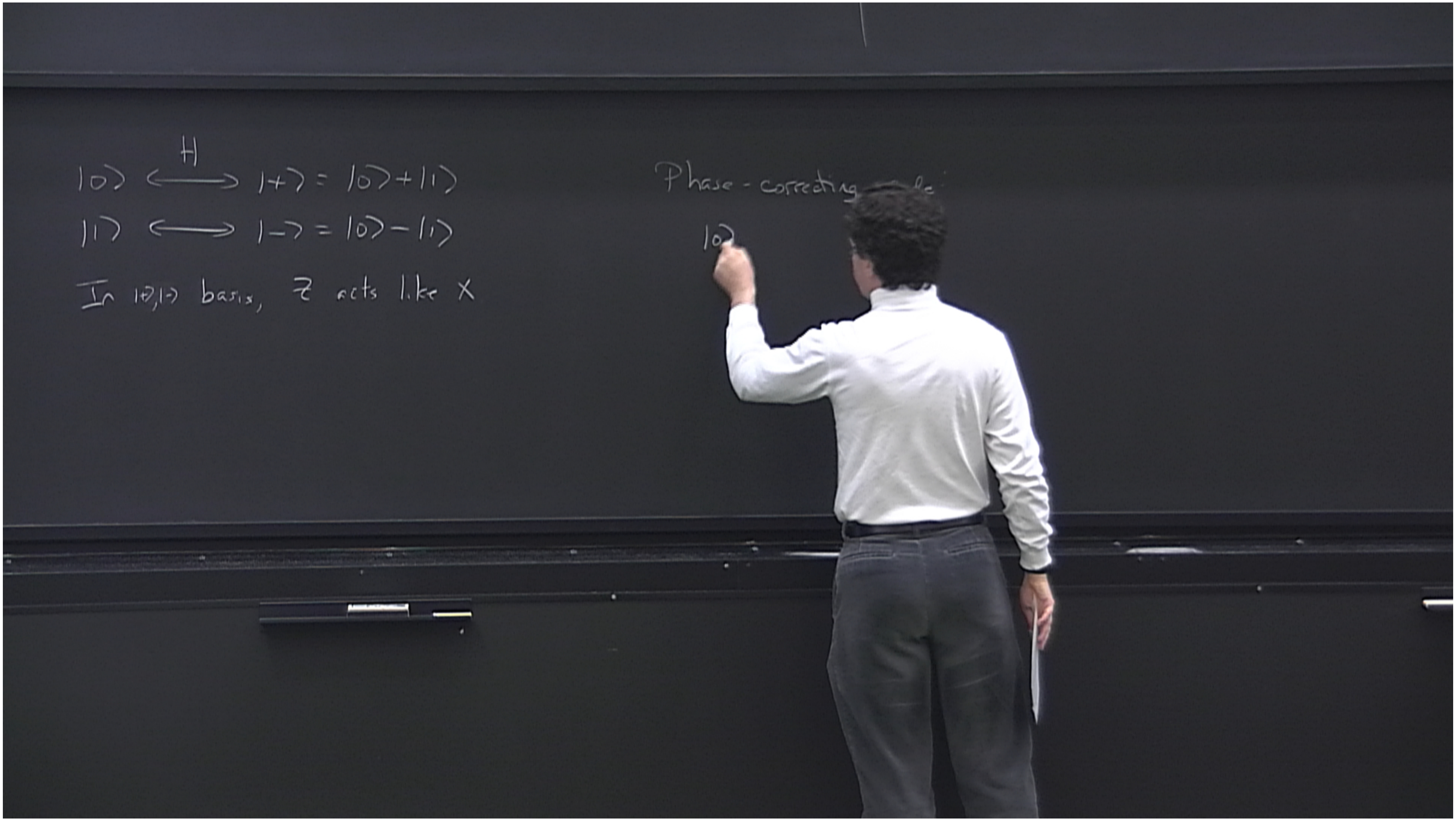

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \longrightarrow \alpha|000\rangle + \beta|111\rangle$
$X^{(2)}|\psi\rangle = \alpha|010\rangle + \beta|101\rangle$
$|0\rangle$
$|0\rangle$
error syndrome
0 0 no errors
0 1 3rd qubit
1 0 1st qubit
1 1 2nd qubit

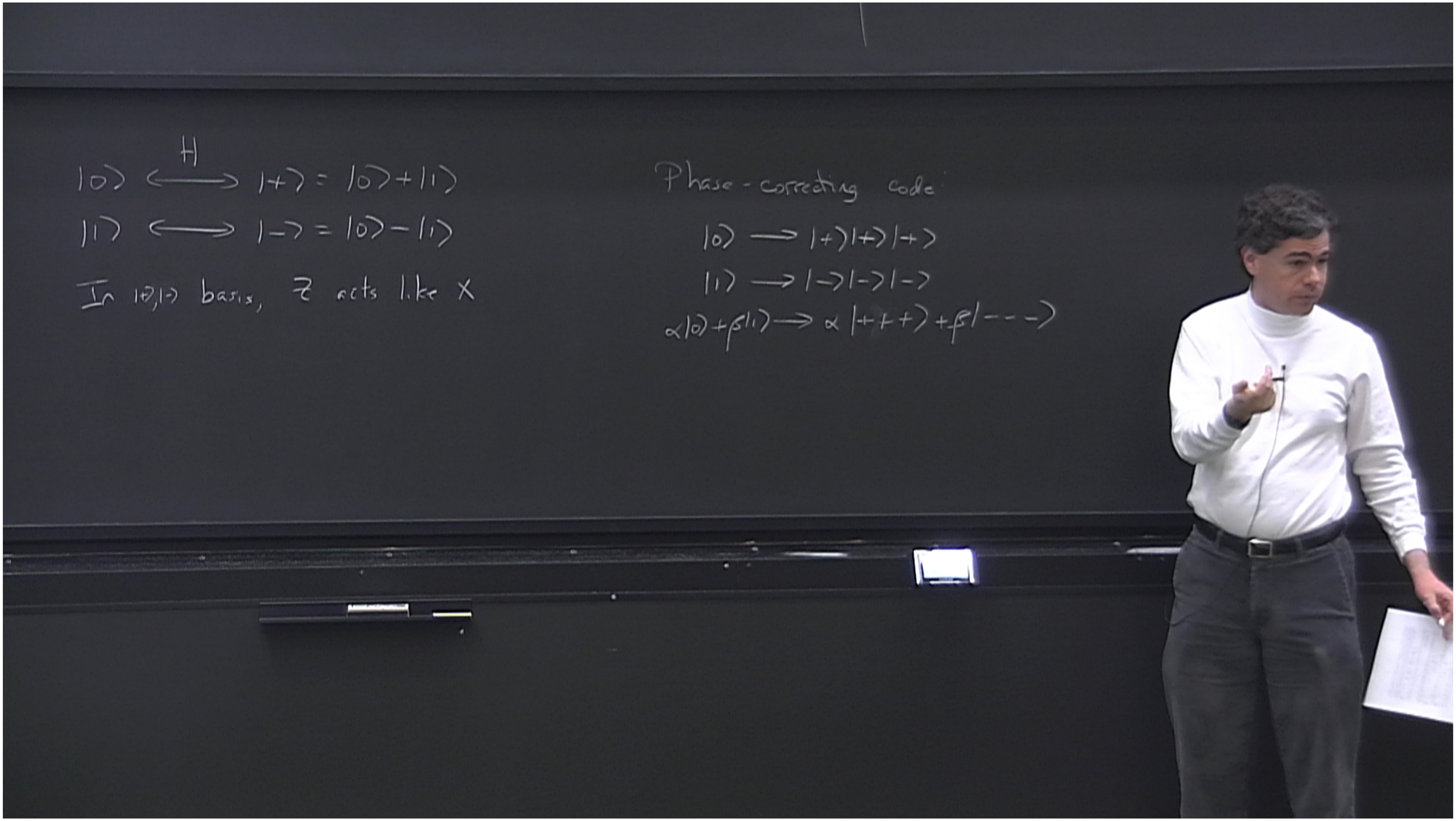

|ψ⟩=α|0⟩+β|1⟩ ⟶ α|000⟩+β|111⟩
X^(2)|ψ⟩ = α|010⟩+β|101⟩
|0⟩
|0⟩
error syndrome
0 0 no errors
0 1 3rd qubit
1 0 1st qubit
1 1 2nd qubit

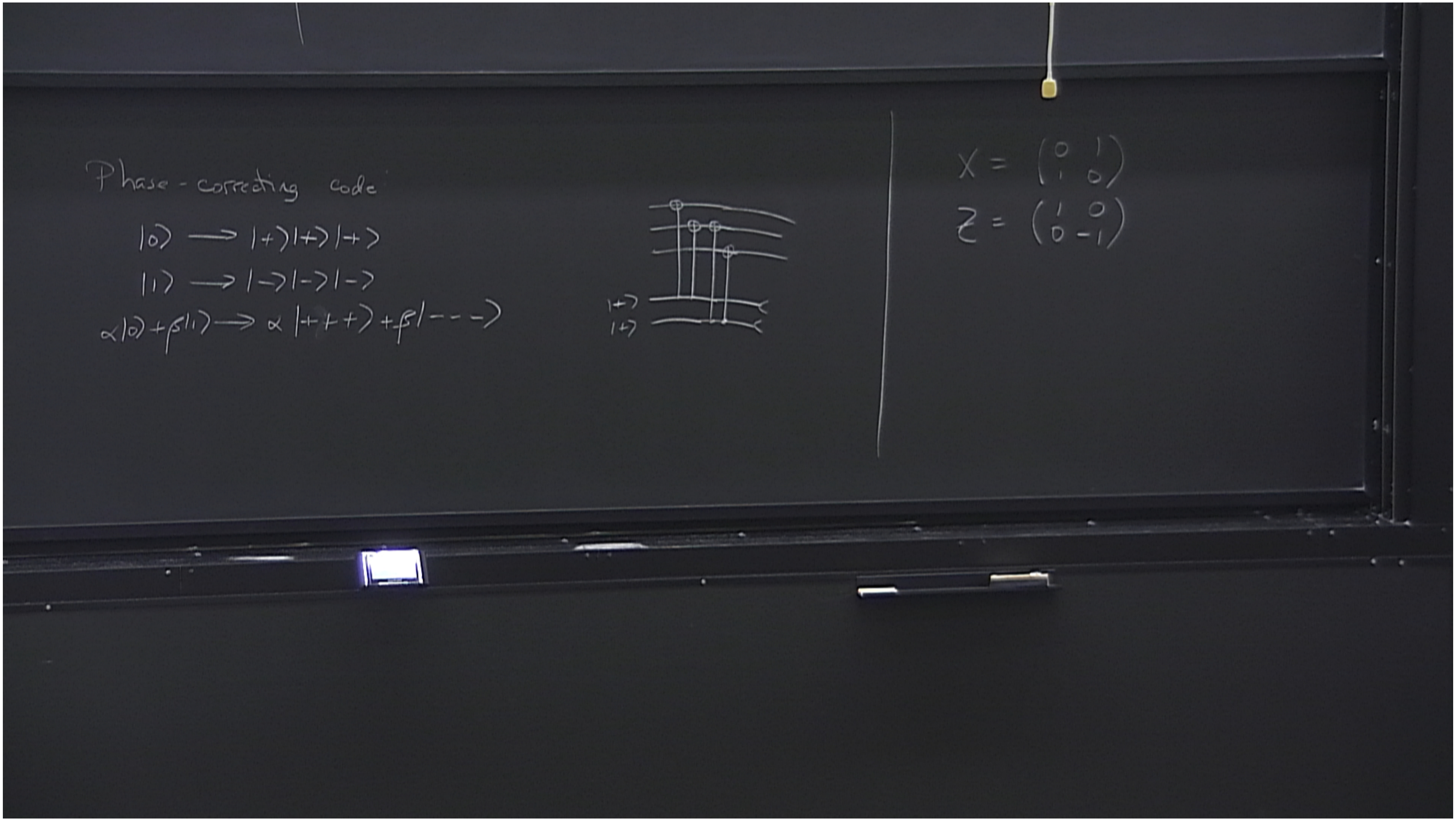


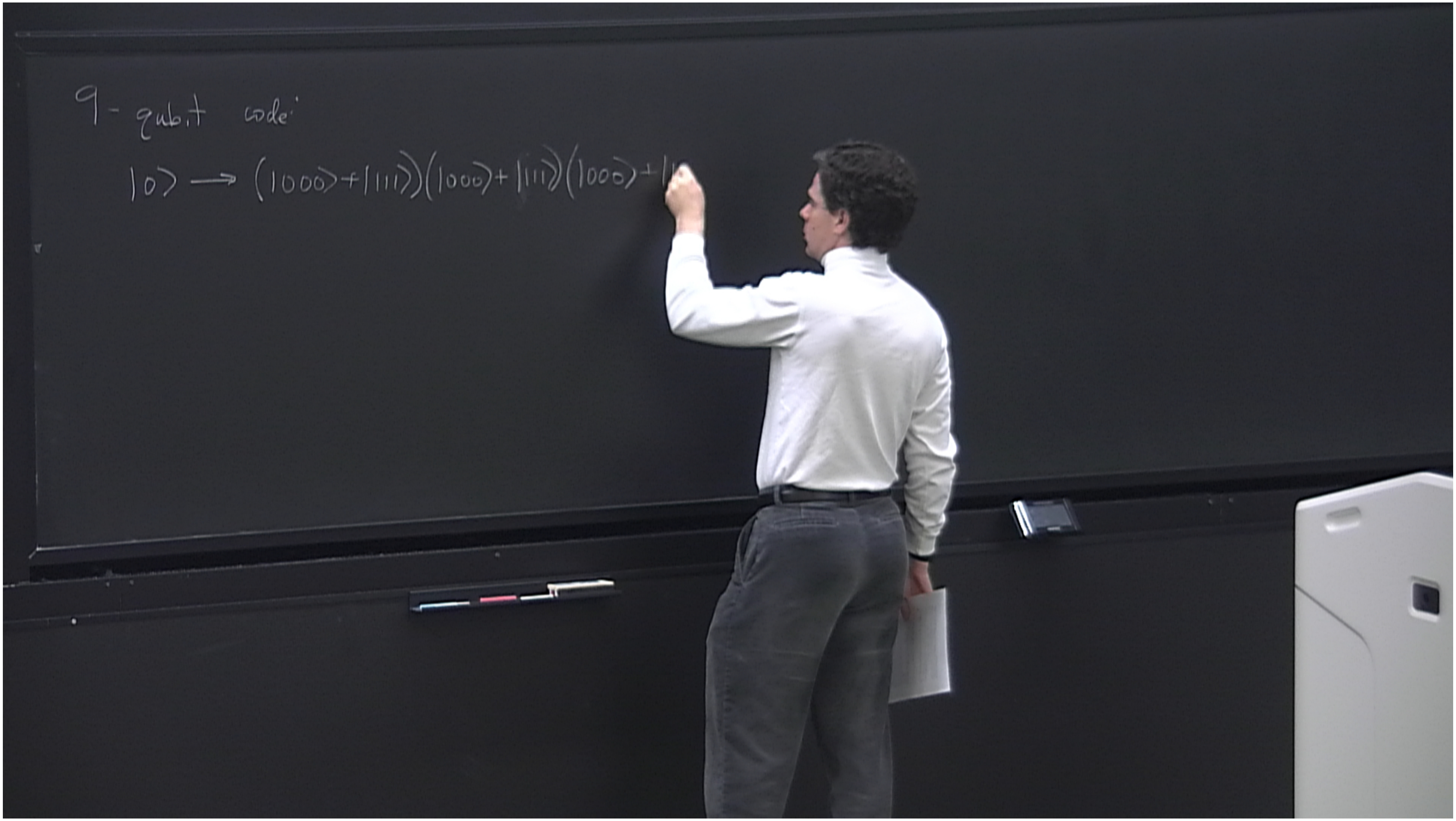
|ψ⟩=α|0⟩+β|1⟩ ⟶ α|000⟩+β|111⟩
X^(2)|ψ⟩ = α|010⟩+β|101⟩
|0⟩
|0⟩
error
syndrome
0 0 no errors
0 1 3rd qubit
1 0 1st qubit
1 1 2nd qubit
|0⟩
|0⟩

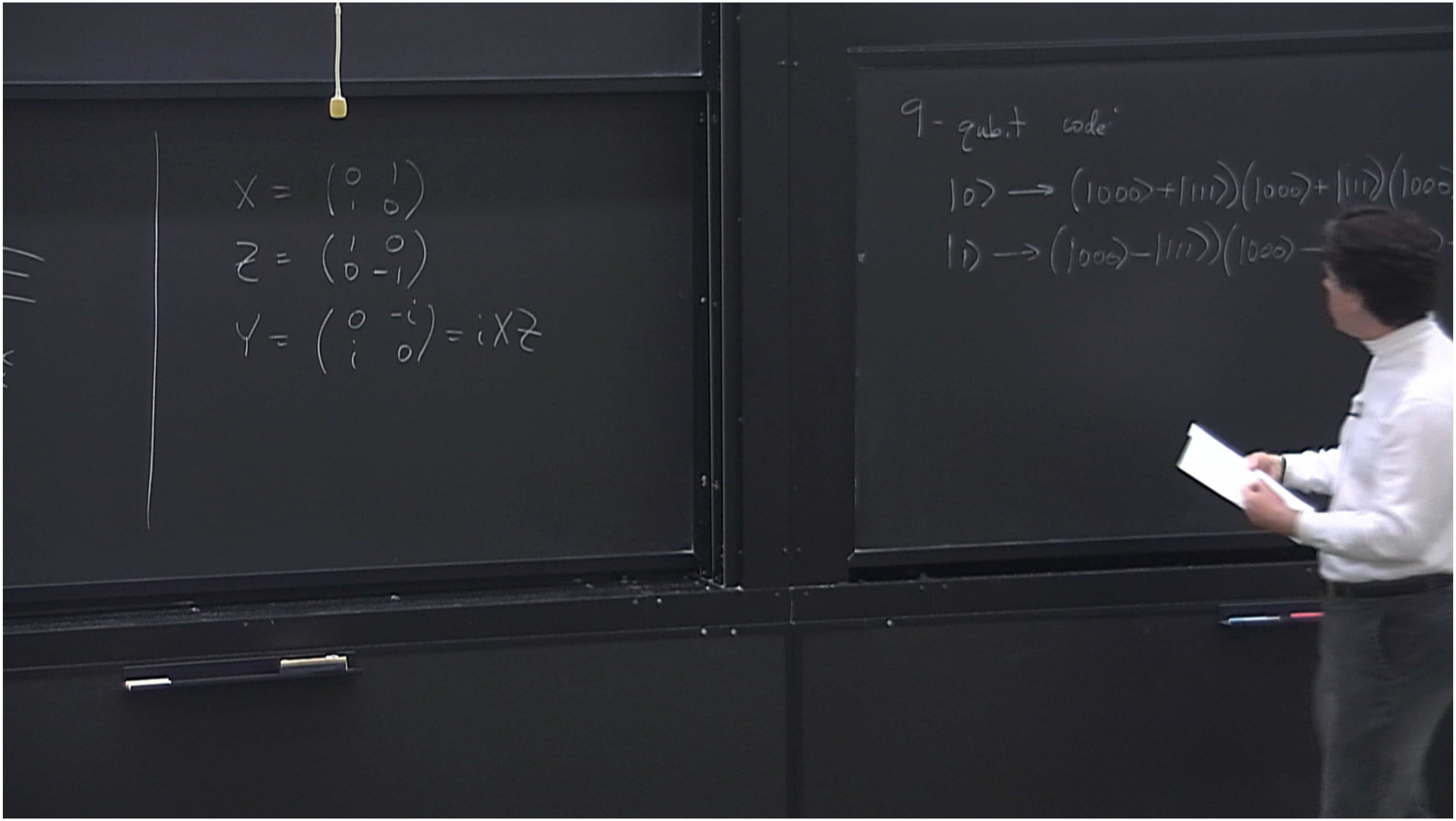

H
|0⟩ ⟷ |+⟩ = |0⟩ + |1⟩
|1⟩ ⟷ |−⟩ = |0⟩ − |1⟩

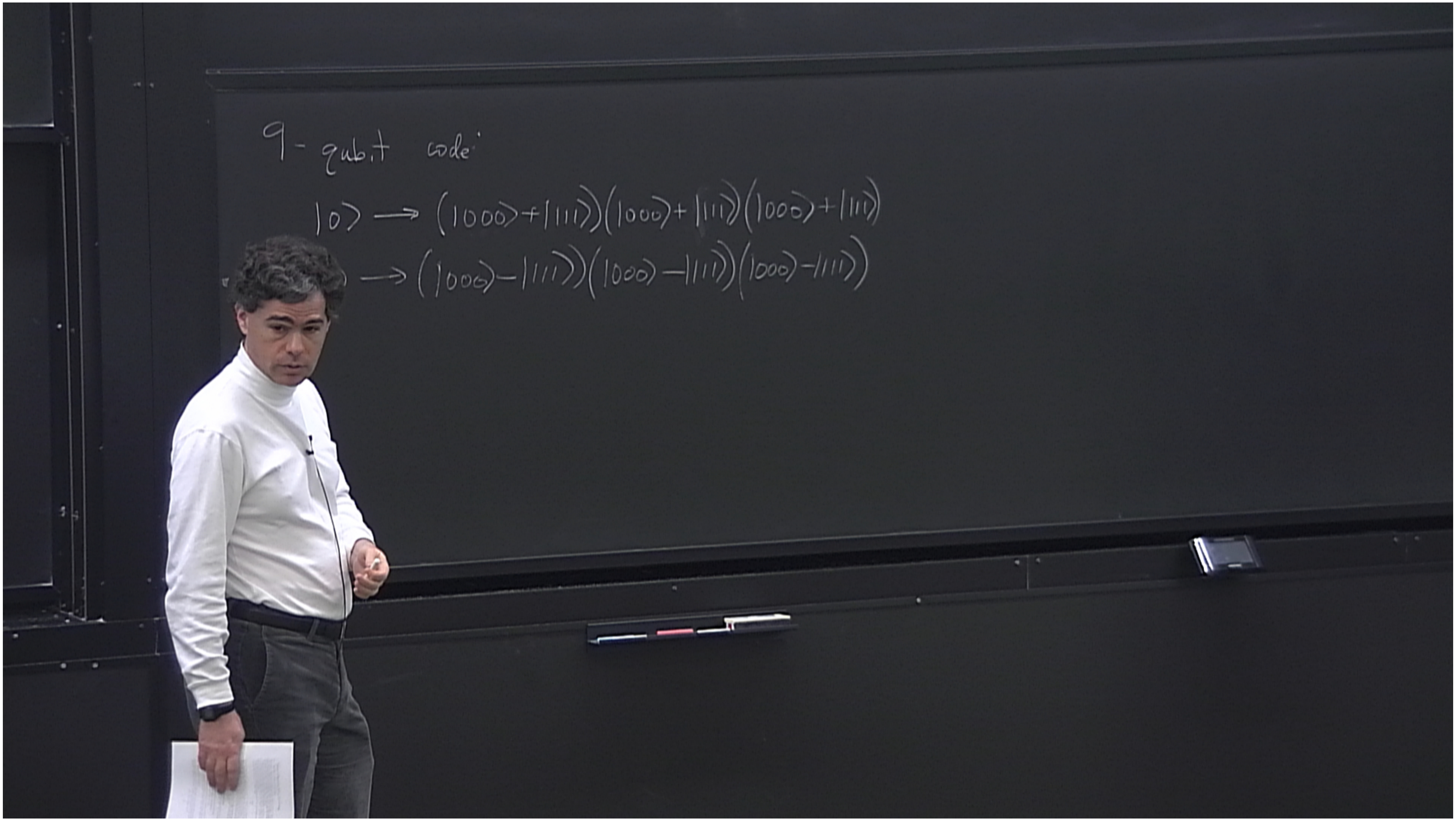

H
|0⟩ ⟷ |+⟩ = |0⟩+|1⟩
|1⟩ ⟷ |−⟩ = |0⟩−|1⟩
In |+⟩,|−⟩ basis, Z acts like X
Phase - correcting
|0⟩

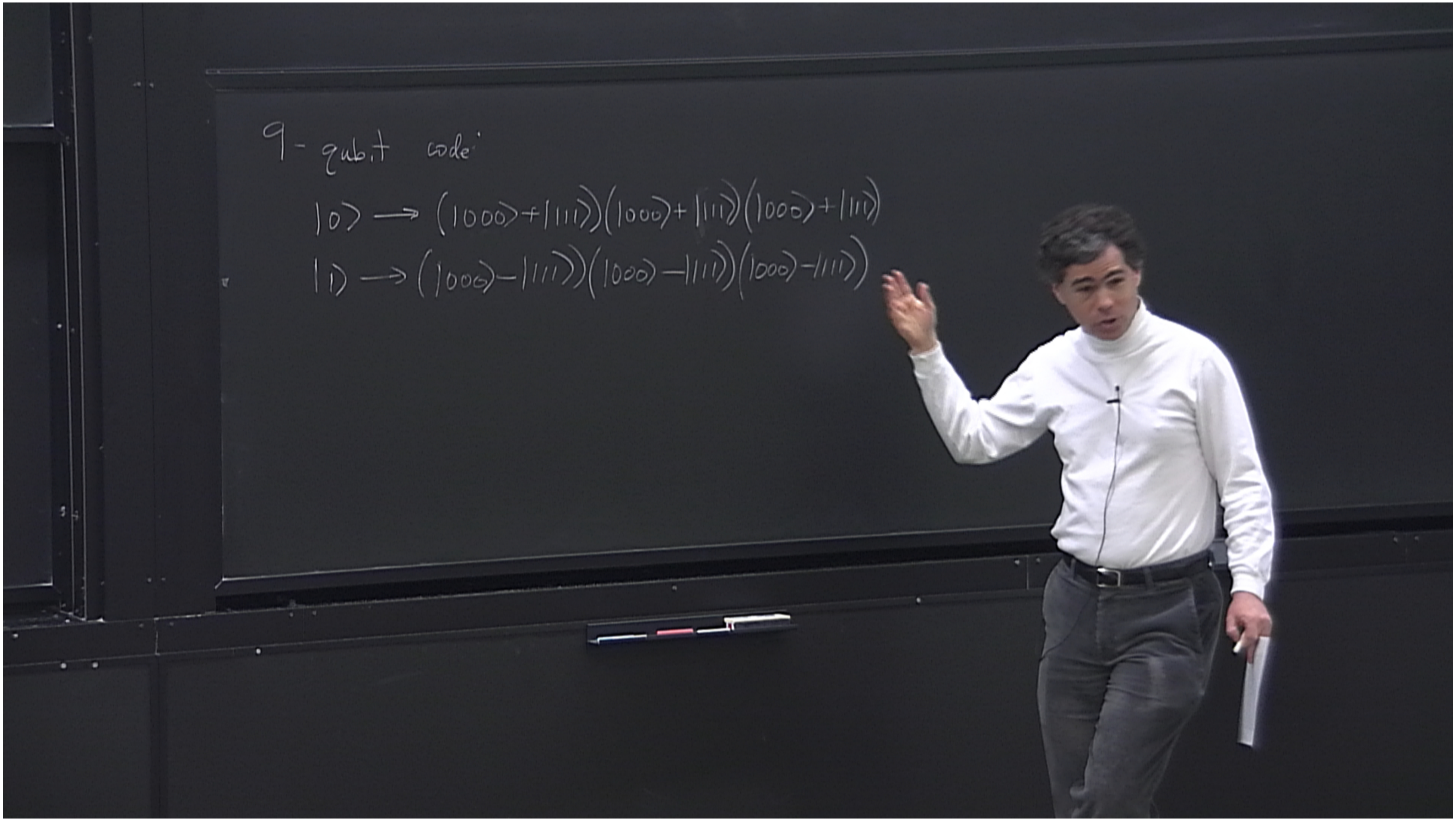

H
|0⟩ ⟷ |+⟩ = |0⟩+|1⟩
|1⟩ ⟷ |−⟩ = |0⟩−|1⟩
In |+⟩,|−⟩ basis, Z acts like X
Phase-correcting code:
|0⟩ → |+⟩|+⟩|+⟩
|1⟩ → |−⟩|−⟩|−⟩
α|0⟩+β|1⟩ → α|+++⟩+β|−−−⟩

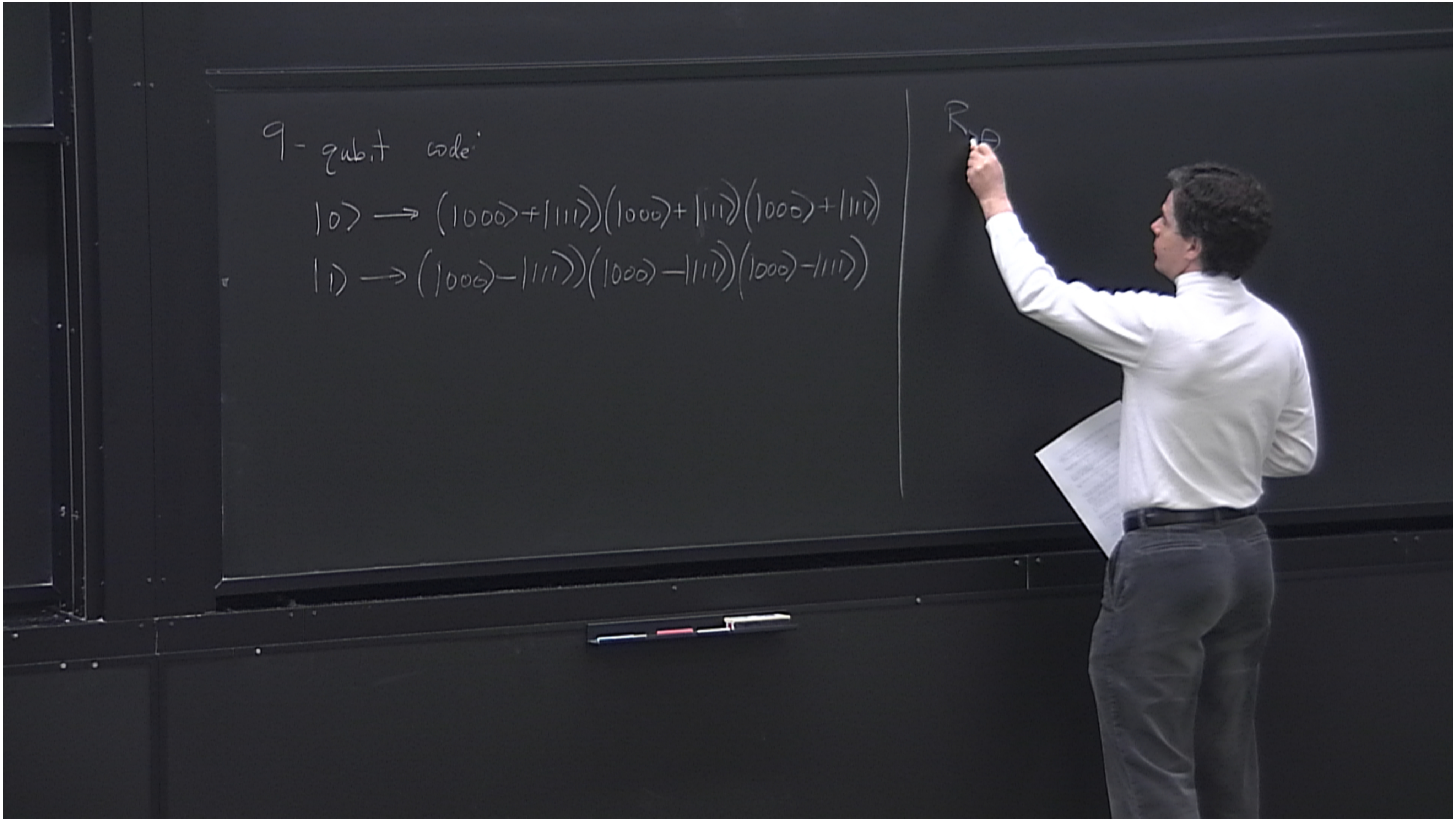

Phase-correcting code
|0⟩ → |+⟩|+⟩|+⟩
|1⟩ → |−⟩|−⟩|−⟩
α|0⟩+β|1⟩ → α|+++⟩+β|−−−⟩
|+⟩
|+⟩
X = (0 1; 1 0)
Z = (1 0; 0 −1)

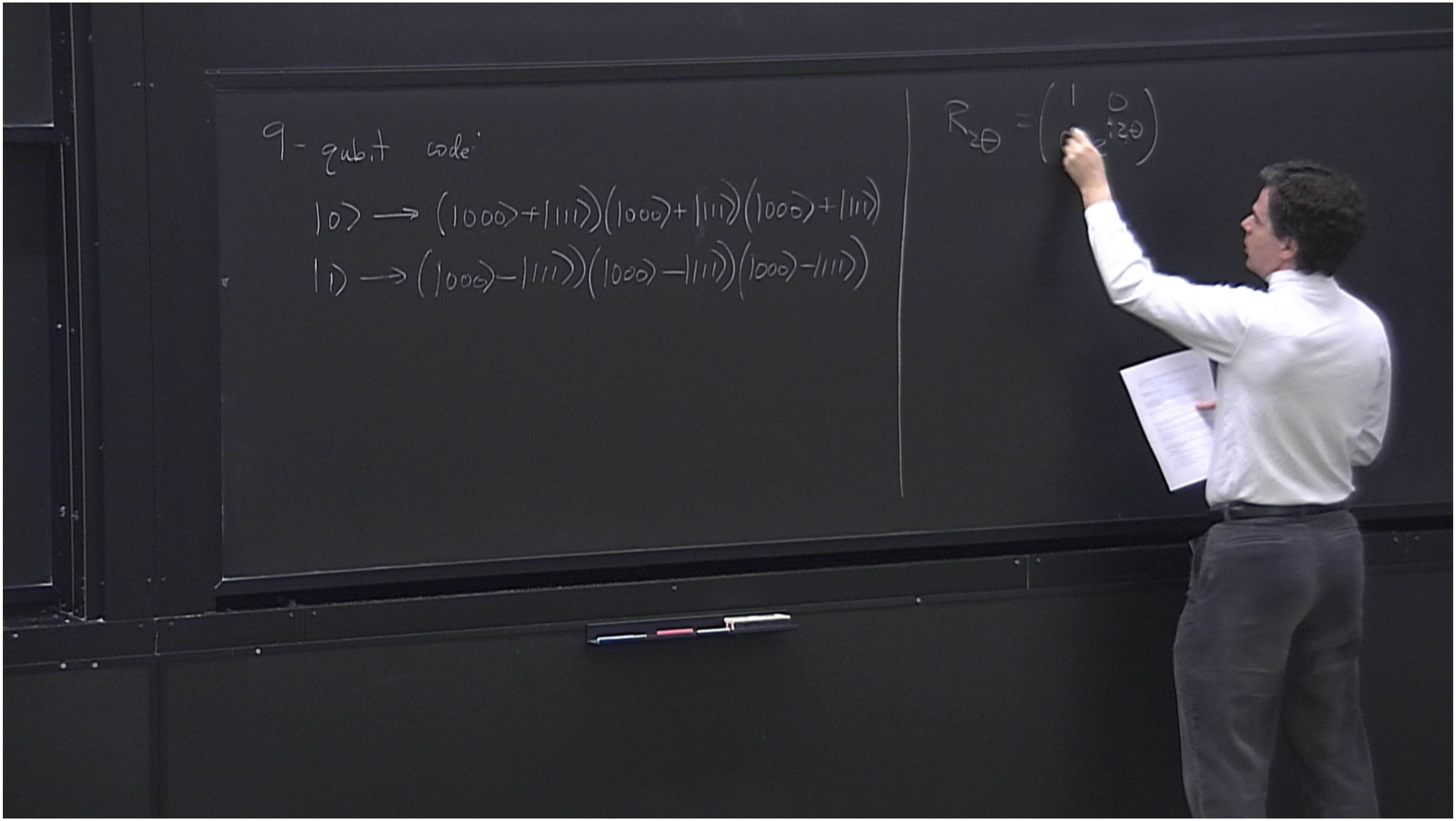

9-qubit code:
|0⟩ → (|000⟩+|111⟩)(|000⟩+|111⟩)(|000⟩+

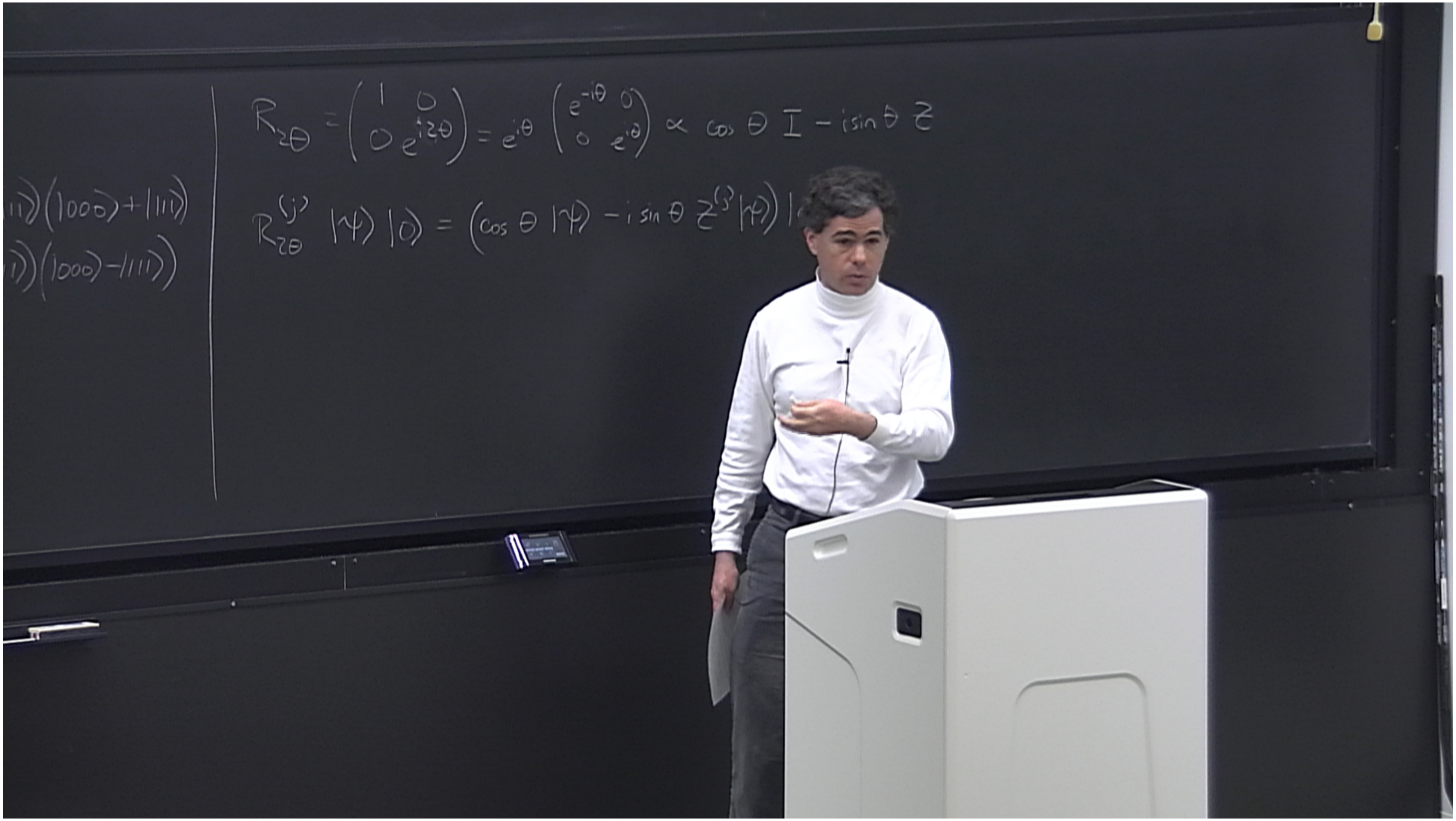

X = (0 1 / 1 0)
Z = (1 0 / 0 -1)
Y = (0 -i / i 0) = iXZ
9-qubit code:
|0> → (|000>+|111>)(|000>+|111>)(|000
|1> → (|000>-|111>)(|000>

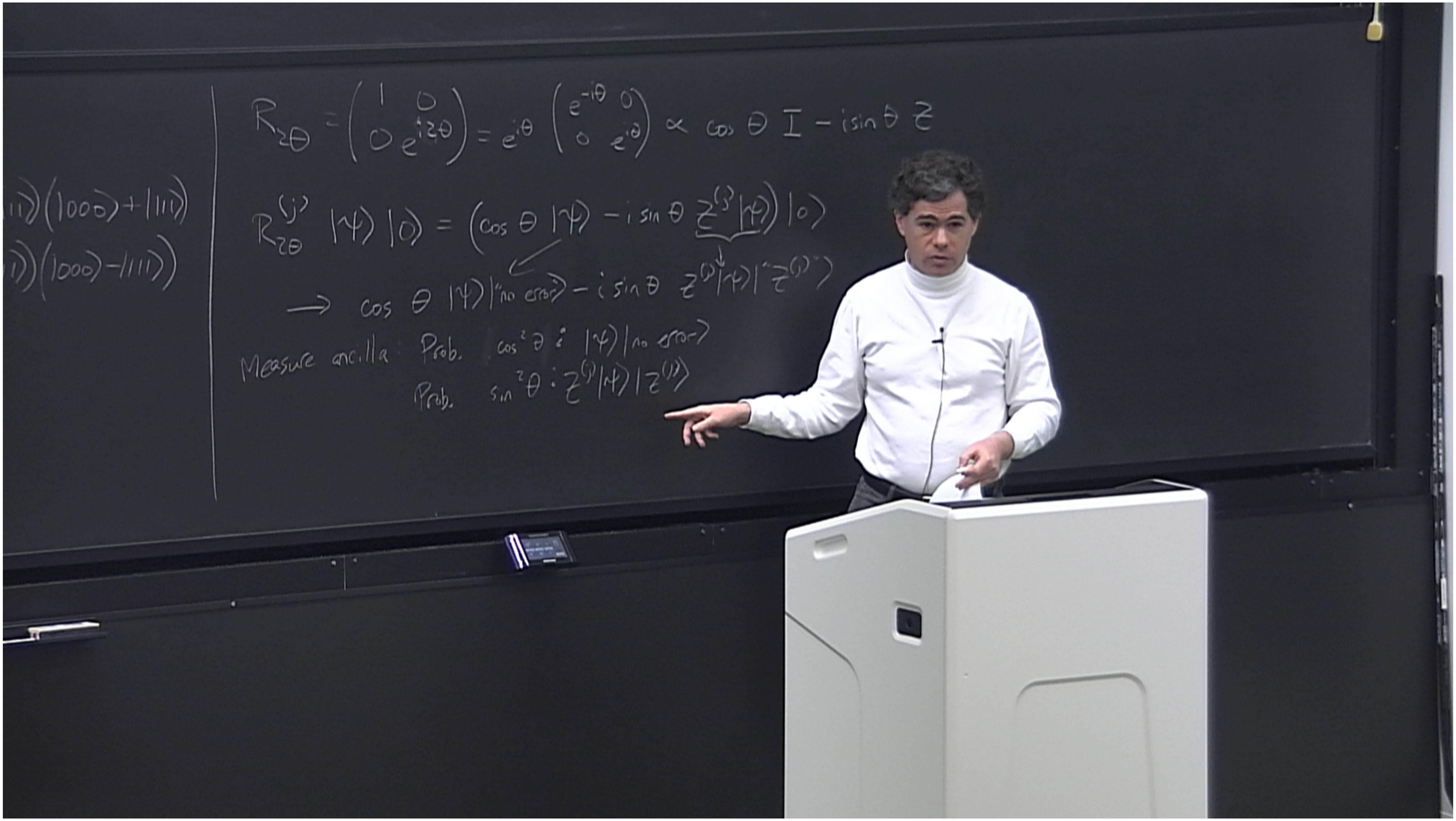

9-qubit code:
|0⟩ → (|000⟩+|111⟩)(|000⟩+|111⟩)(|000⟩+|111⟩)
→ (|000⟩−|111⟩)(|000⟩−|111⟩)(|000⟩−|111⟩)

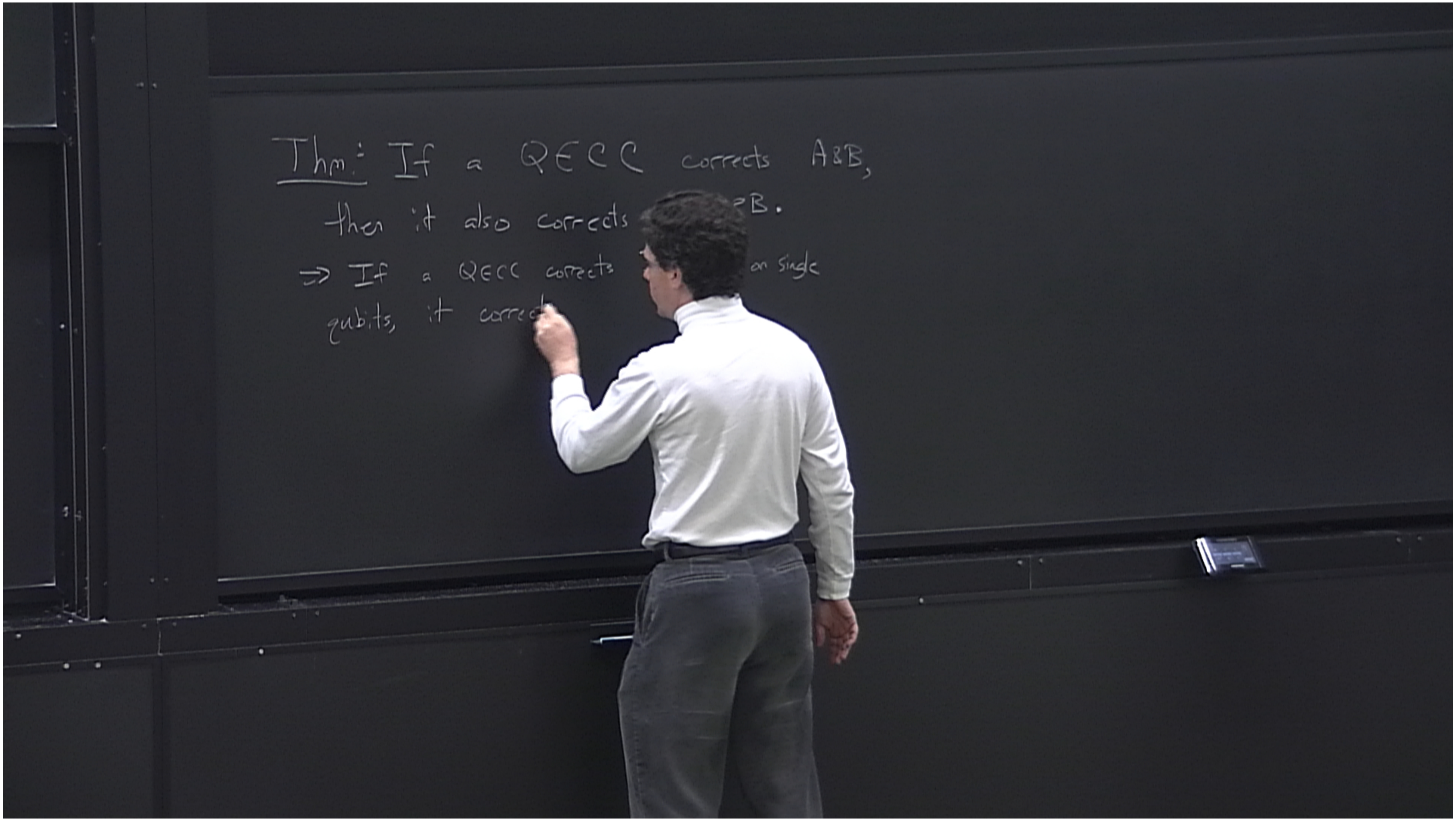

9-qubit code:
|0⟩ → (|000⟩+|111⟩)(|000⟩+|111⟩)(|000⟩+|111⟩)
|1⟩ → (|000⟩−|111⟩)(|000⟩−|111⟩)(|000⟩−|111⟩)

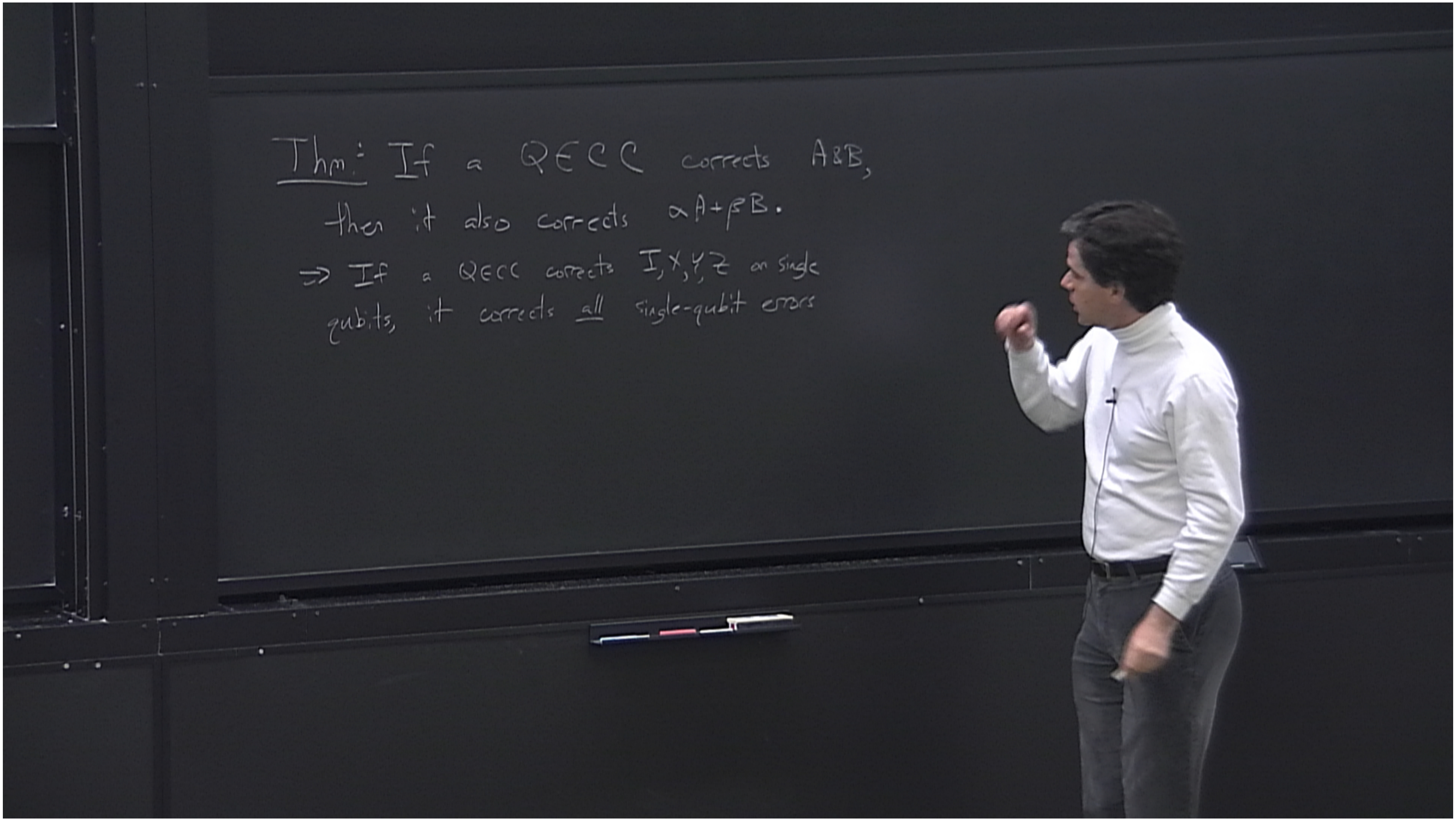

9-qubit code:
|0⟩ → (|000⟩+|111⟩)(|000⟩+|111⟩)(|000⟩+|111⟩)
|1⟩ → (|000⟩−|111⟩)(|000⟩−|111⟩)(|000⟩−|111⟩)

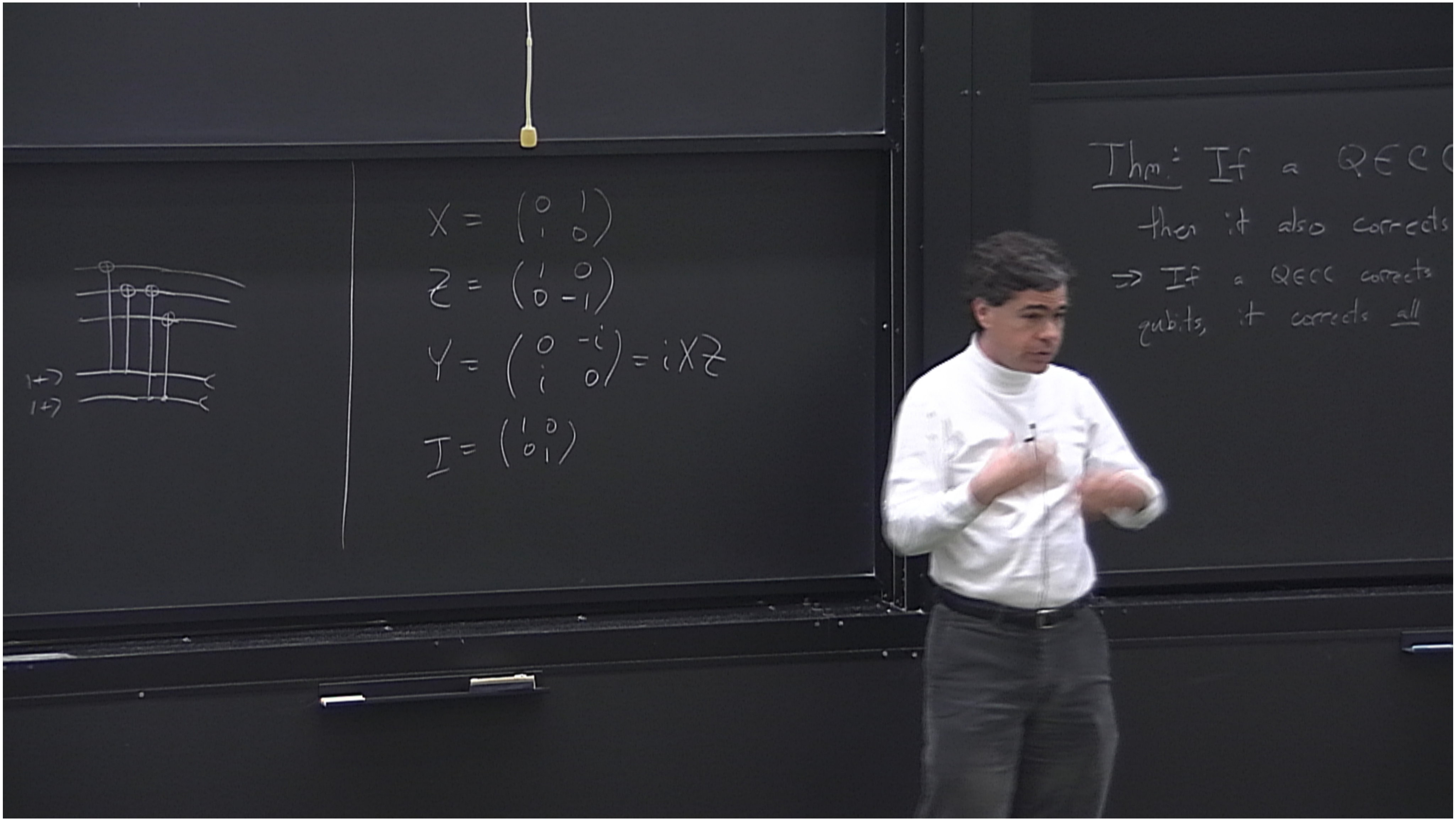

9-qubit code:
|0⟩ → (|000⟩+|111⟩)(|000⟩+|111⟩)(|000⟩+|111⟩)
|1⟩ → (|000⟩−|111⟩)(|000⟩−|111⟩)(|000⟩−|111⟩)
R2θ = (1 0

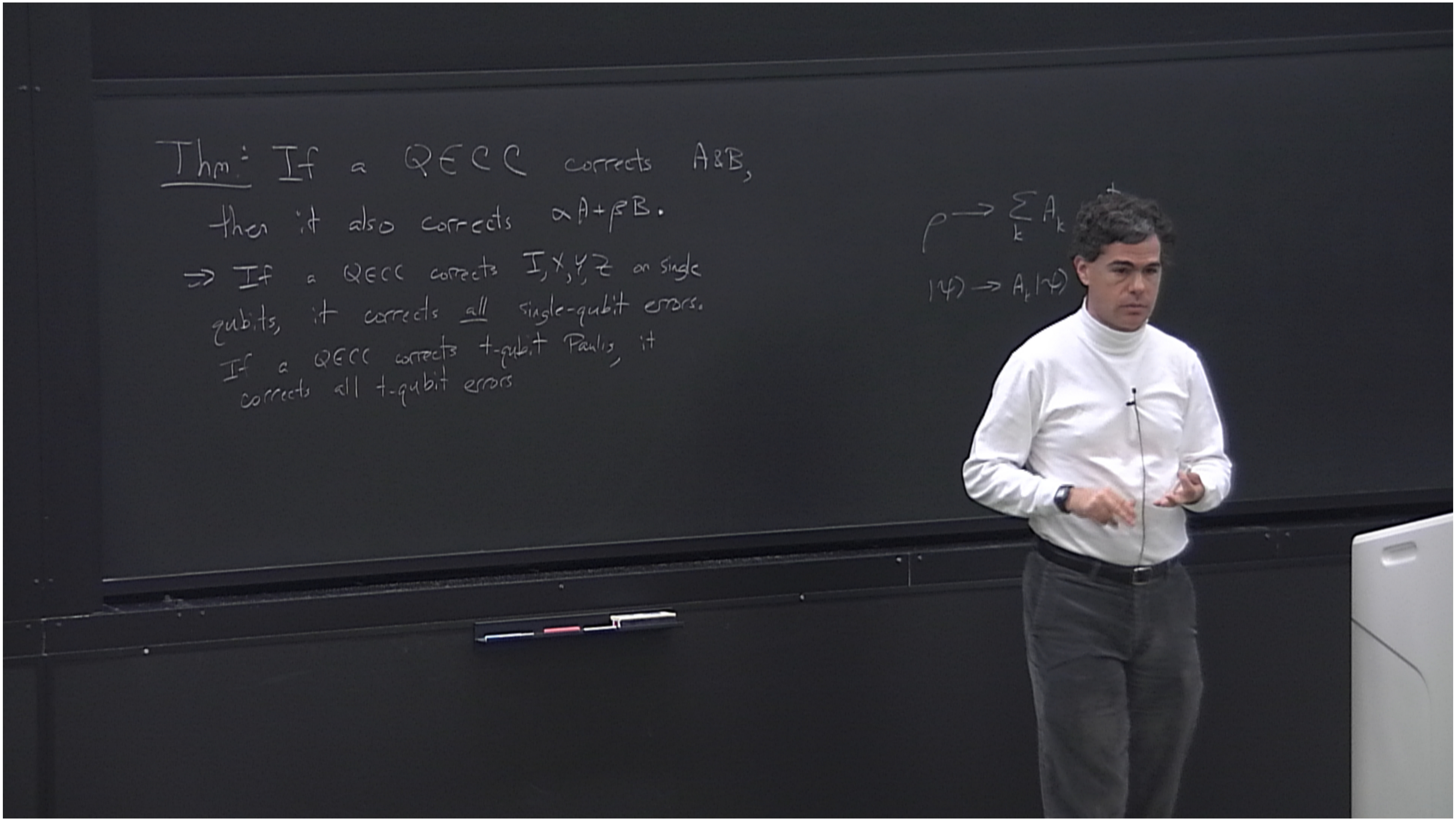

R_{zθ} = (1 0 / 0 e^{i2θ}) = e^{iθ} (e^{-iθ} 0 / 0 e^{iθ}) ∝ cos θ I − i sin θ Z
R_{zθ}^{(j)} |ψ⟩ |0⟩ = (cos θ |ψ⟩ − i sin θ Z^{(j)} |ψ⟩)

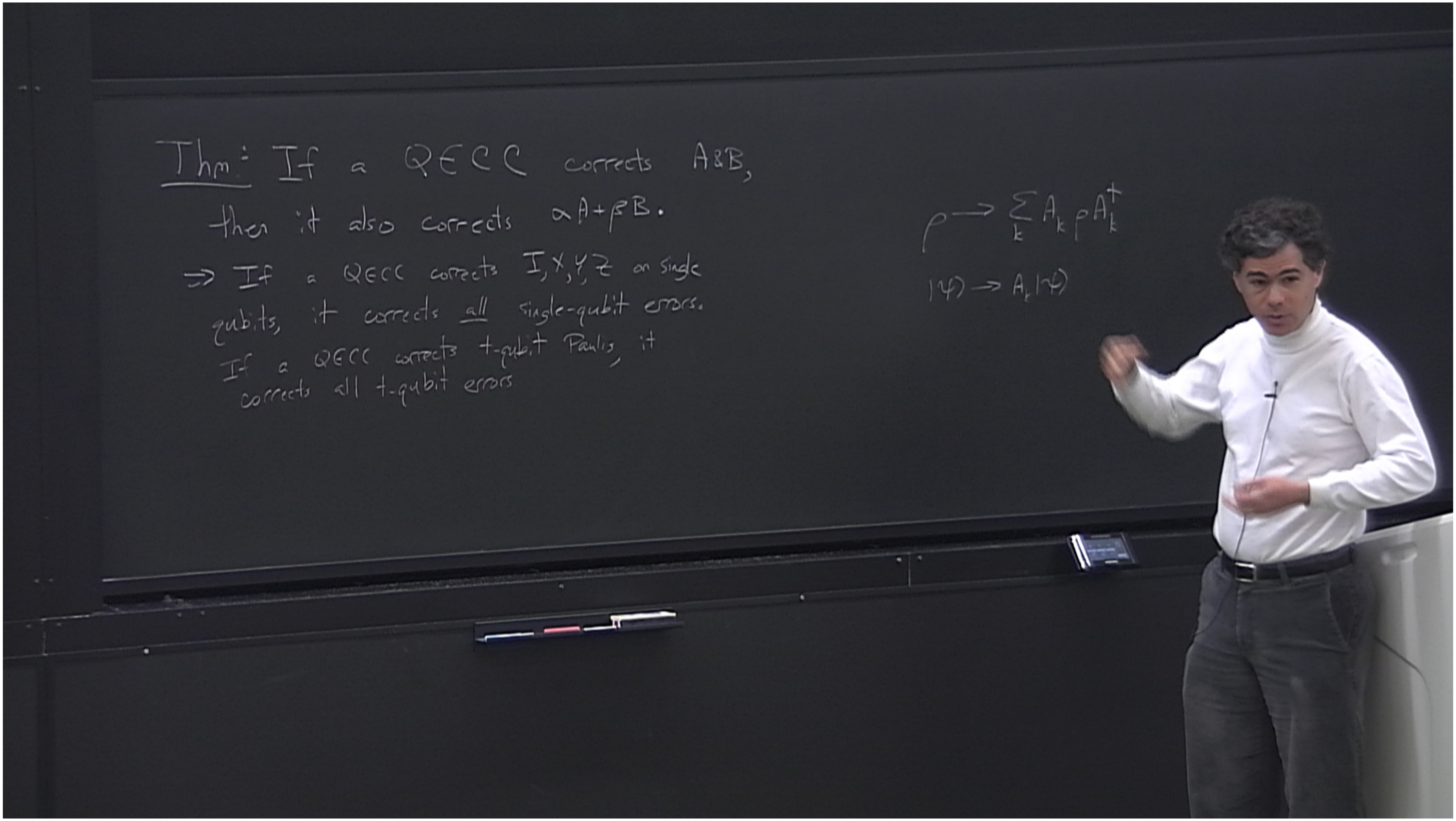

$R_{z\theta} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i2\theta} \end{pmatrix} = e^{i\theta} \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix} \propto \cos\theta\, I - i\sin\theta\, Z$
$R_{z\theta}^{(j)} |\psi\rangle|0\rangle = (\cos\theta|\psi\rangle - i\sin\theta\, Z^{(j)}|\psi\rangle)|0\rangle$
$\rightarrow \cos\theta\, |\psi\rangle|\text{"no error"}\rangle - i\sin\theta\, Z^{(j)}|\psi\rangle|\text{"}Z^{(j)}\text{"}\rangle$
Measure ancilla : Prob. $\cos^2\theta$: $|\psi\rangle|\text{no error}\rangle$
Prob. $\sin^2\theta$: $Z^{(j)}|\psi\rangle|Z^{(j)}\rangle$

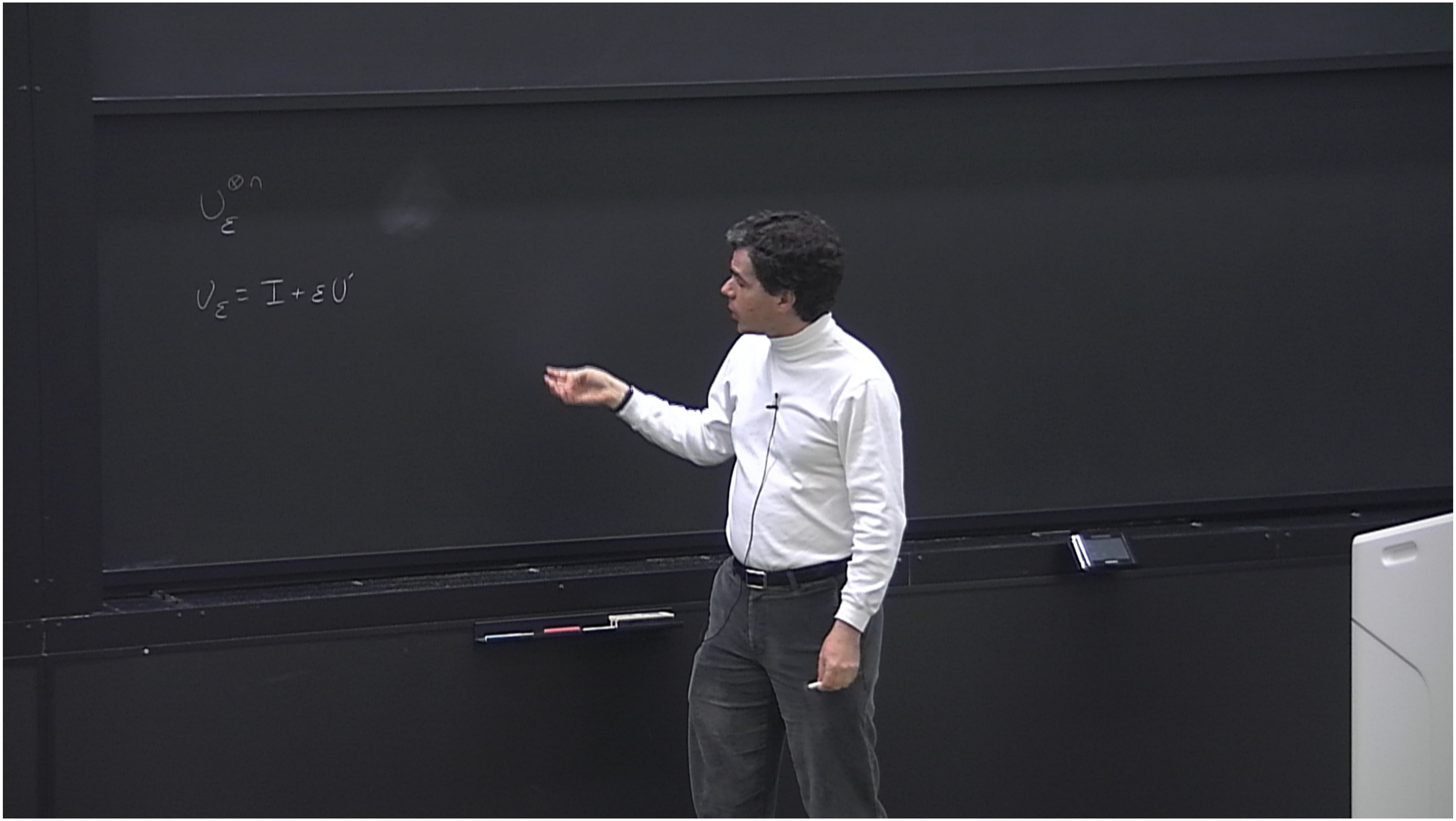

Thm: If a QECC corrects A&B,
then it also corrects
B.
⇒ If a QECC corrects
on single
qubits, it

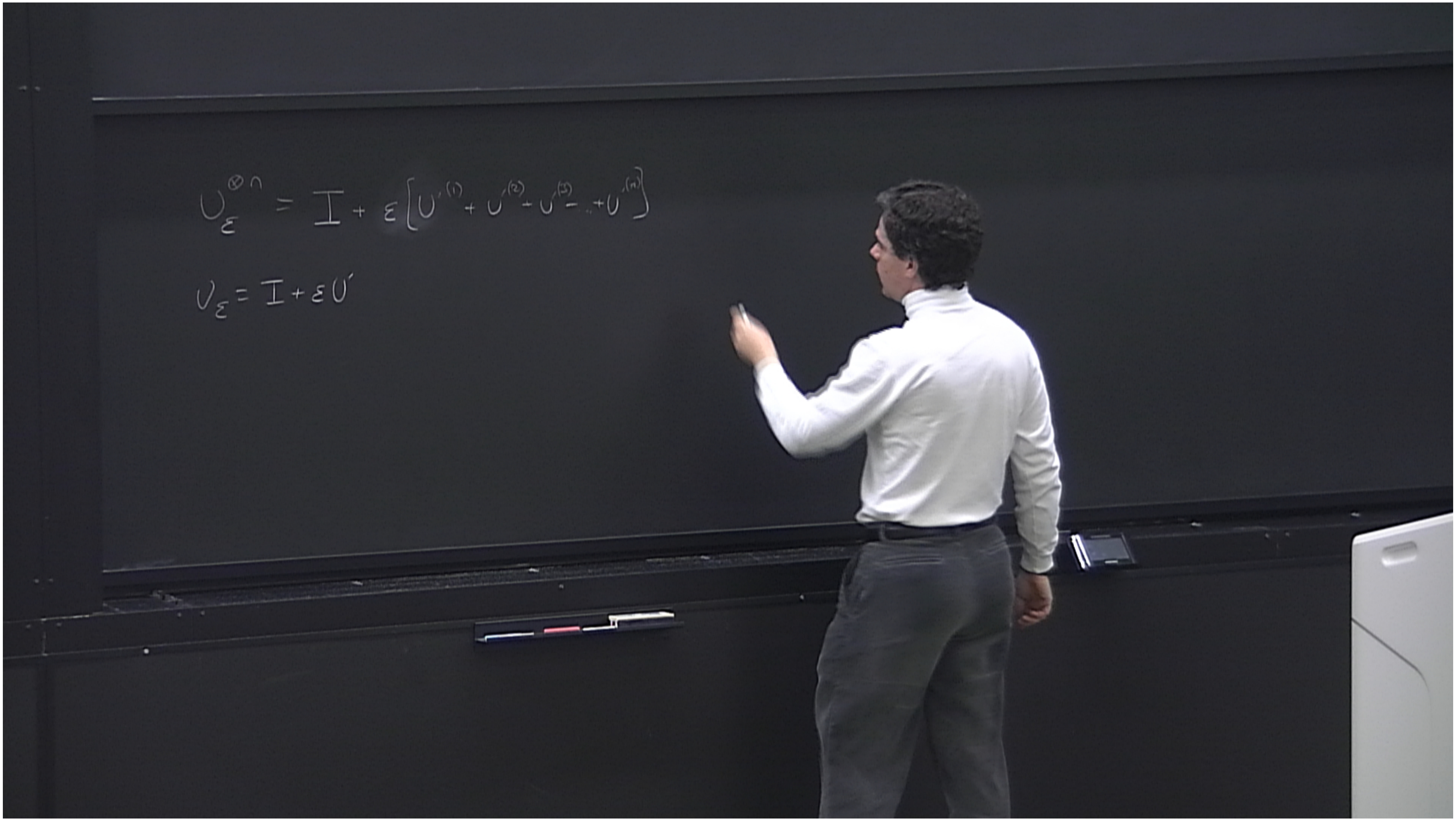

Thm: If a QECC corrects A&B,
then it also corrects αA+βB.
⇒ If a QECC corrects I, X, Y, Z on single
qubits, it corrects all single-qubit errors

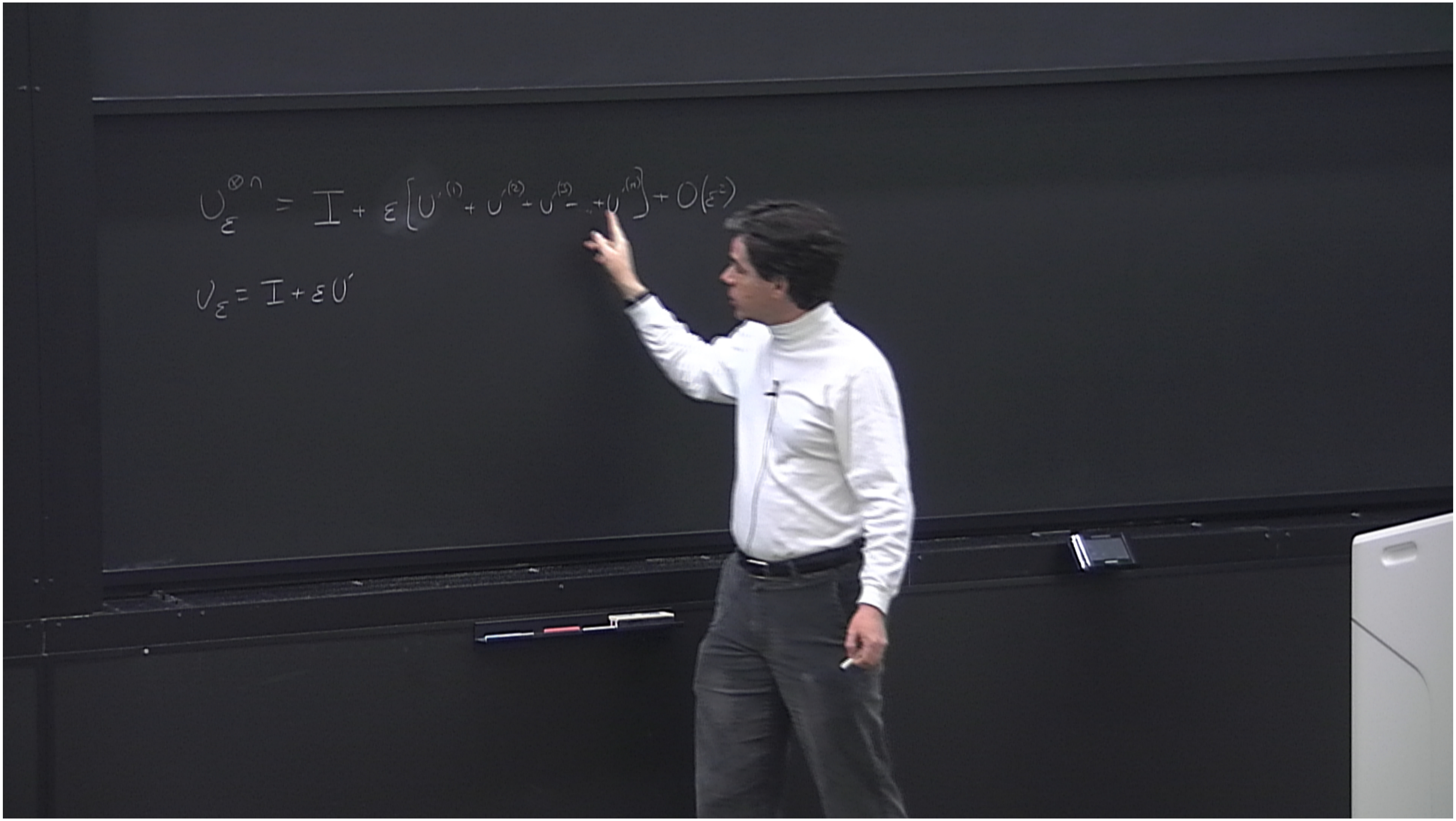

X = (0 1 / 1 0)
Z = (1 0 / 0 −1)
Y = (0 −i / i 0) = iXZ
I = (1 0 / 0 1)
|+⟩
|+⟩
Thm: If a QEC
then it also corrects
⇒ If a QECC corrects
qubits, it corrects all

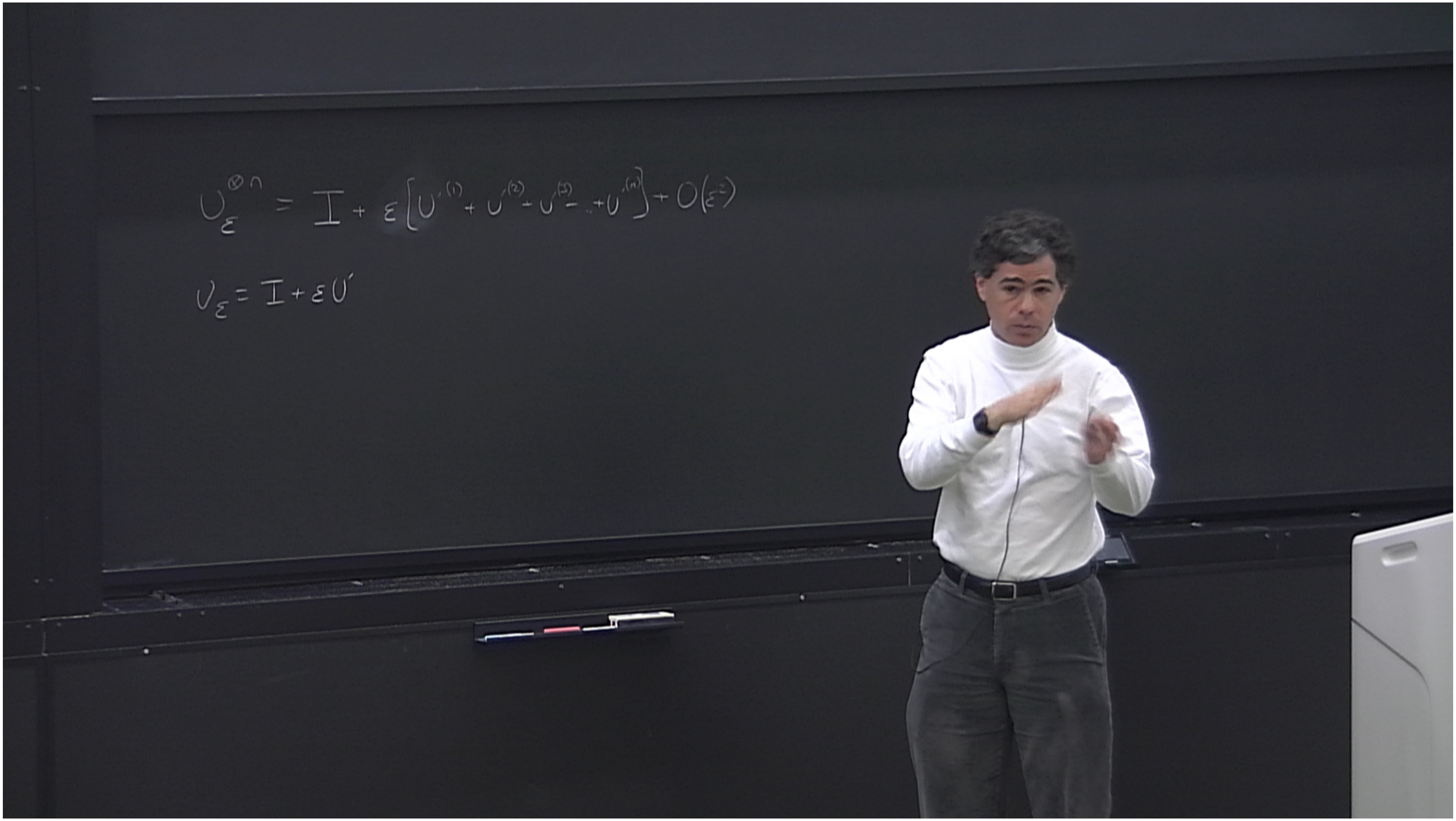

Thm: If a QECC corrects A&B,
then it also corrects αA+βB.
⇒ If a QECC corrects I, X, Y, Z on single
qubits, it corrects all single-qubit errors.
If a QECC corrects t-qubit Paulis, it
corrects all t-qubit errors
|ψ⟩ → A_k|ψ⟩

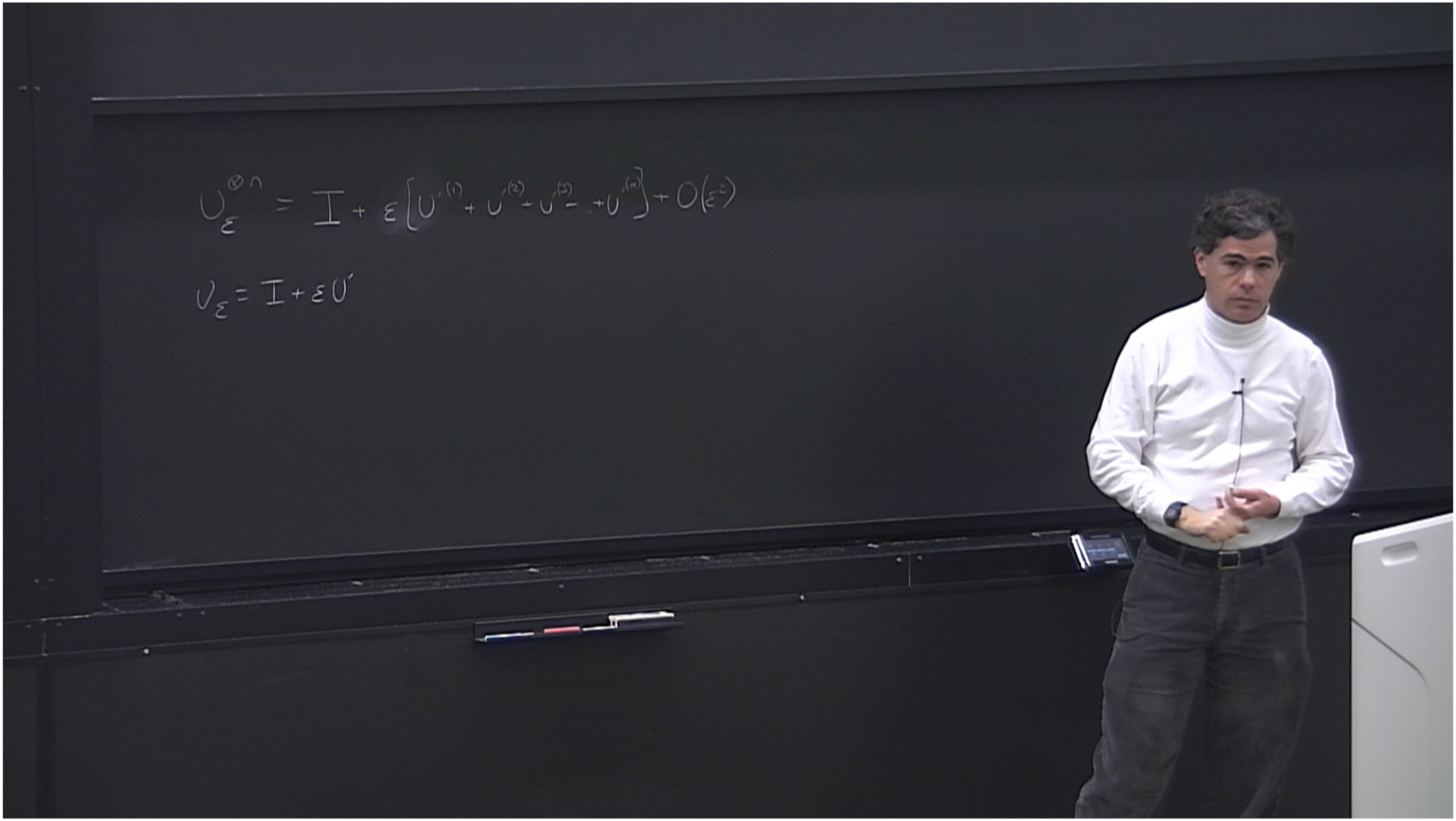

Thm: If a QECC corrects A&B,
then it also corrects αA+βB.
⇒ If a QECC corrects I, X, Y, Z on single
qubits, it corrects all single-qubit errors.
If a QECC corrects t-qubit Paulis, it
corrects all t-qubit errors
ρ → Σk Ak ρ Ak†
|ψ⟩ → Ak|ψ⟩

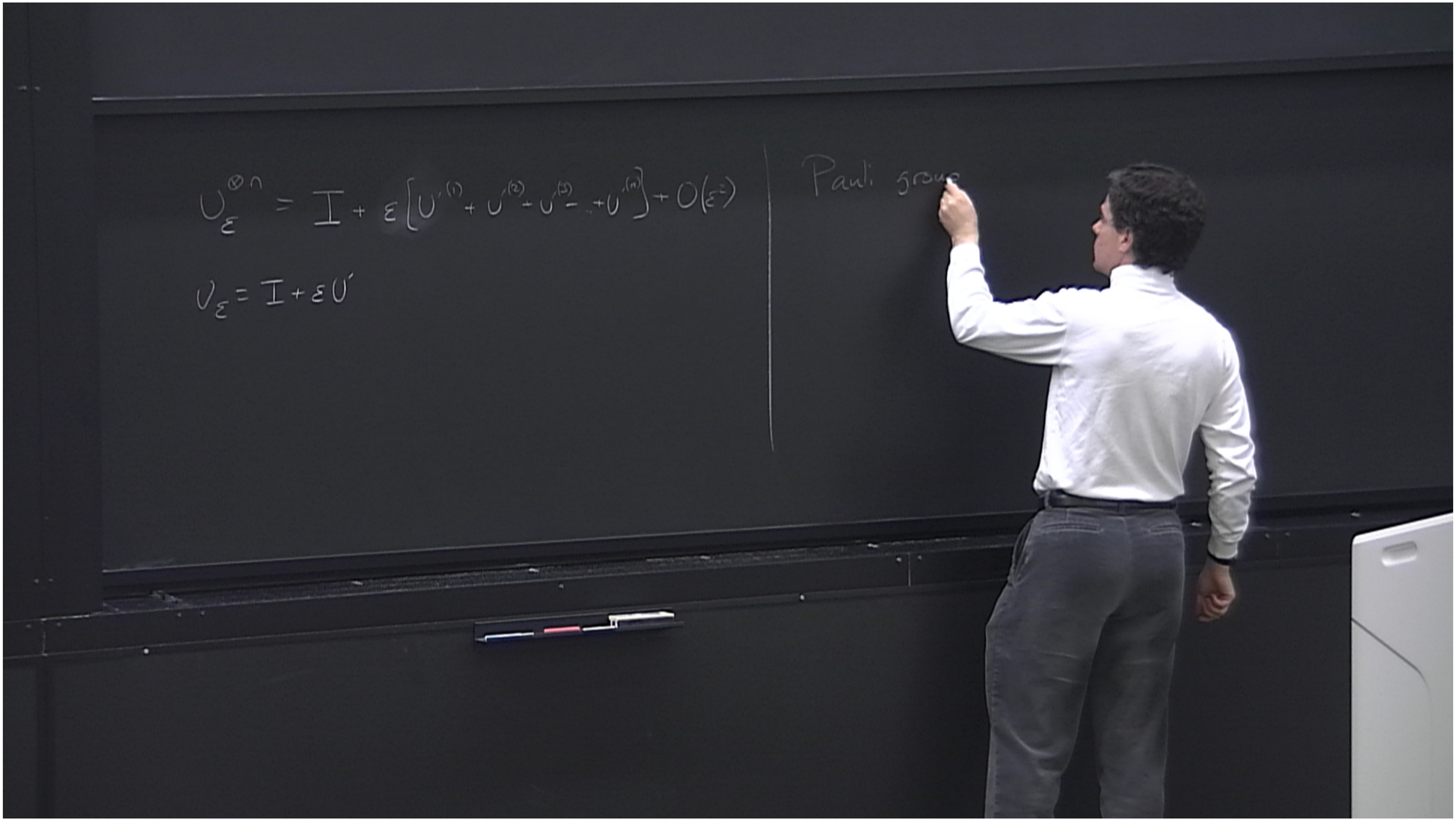

$U_\varepsilon^{\otimes n}$
$U_\varepsilon = I + \varepsilon U'$

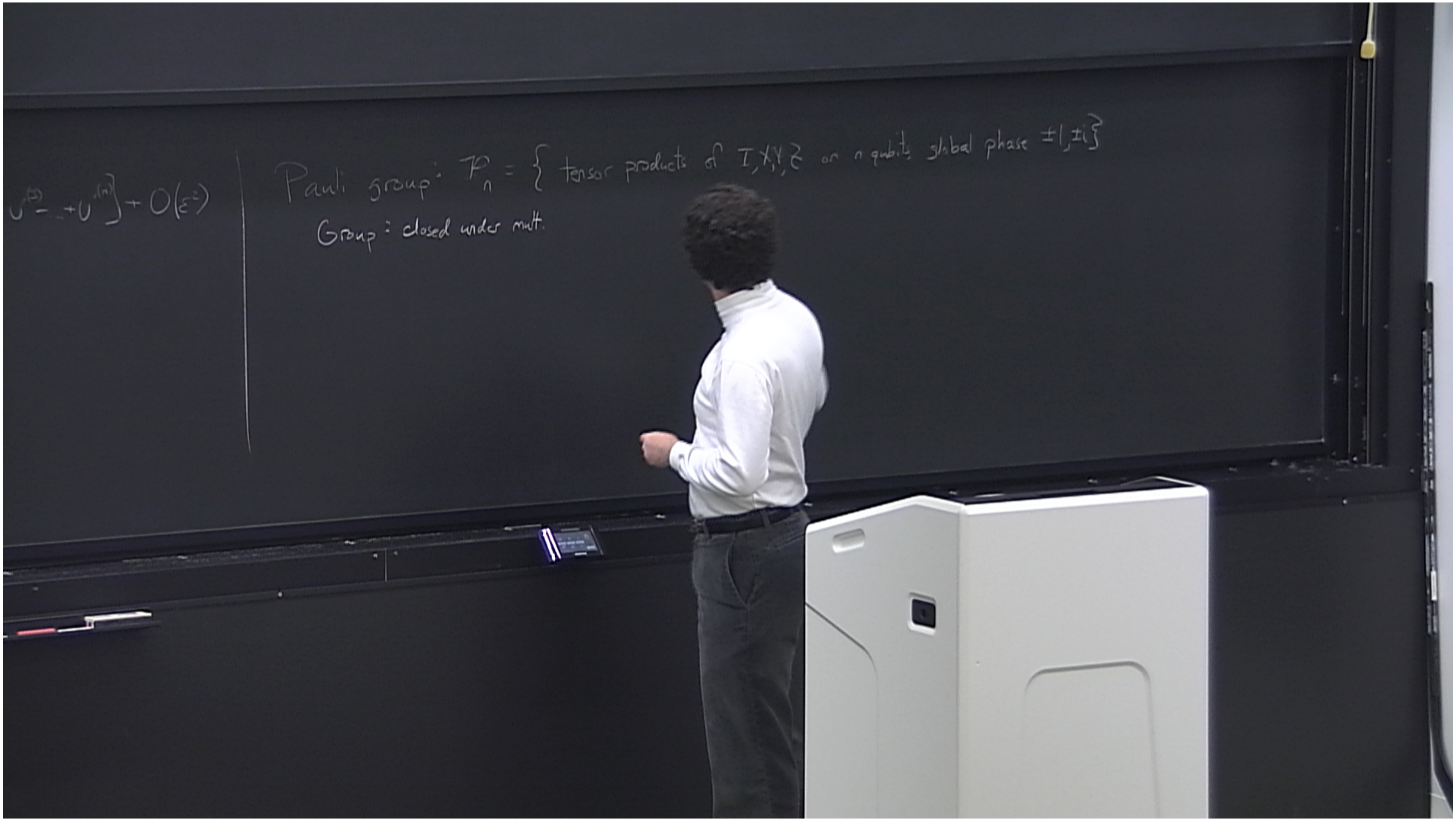

$U_\varepsilon^{\otimes n} = I + \varepsilon\left[U'^{(1)} + U'^{(2)} + U'^{(3)} + \dots + U'^{(n)}\right]$
$U_\varepsilon = I + \varepsilon U'$

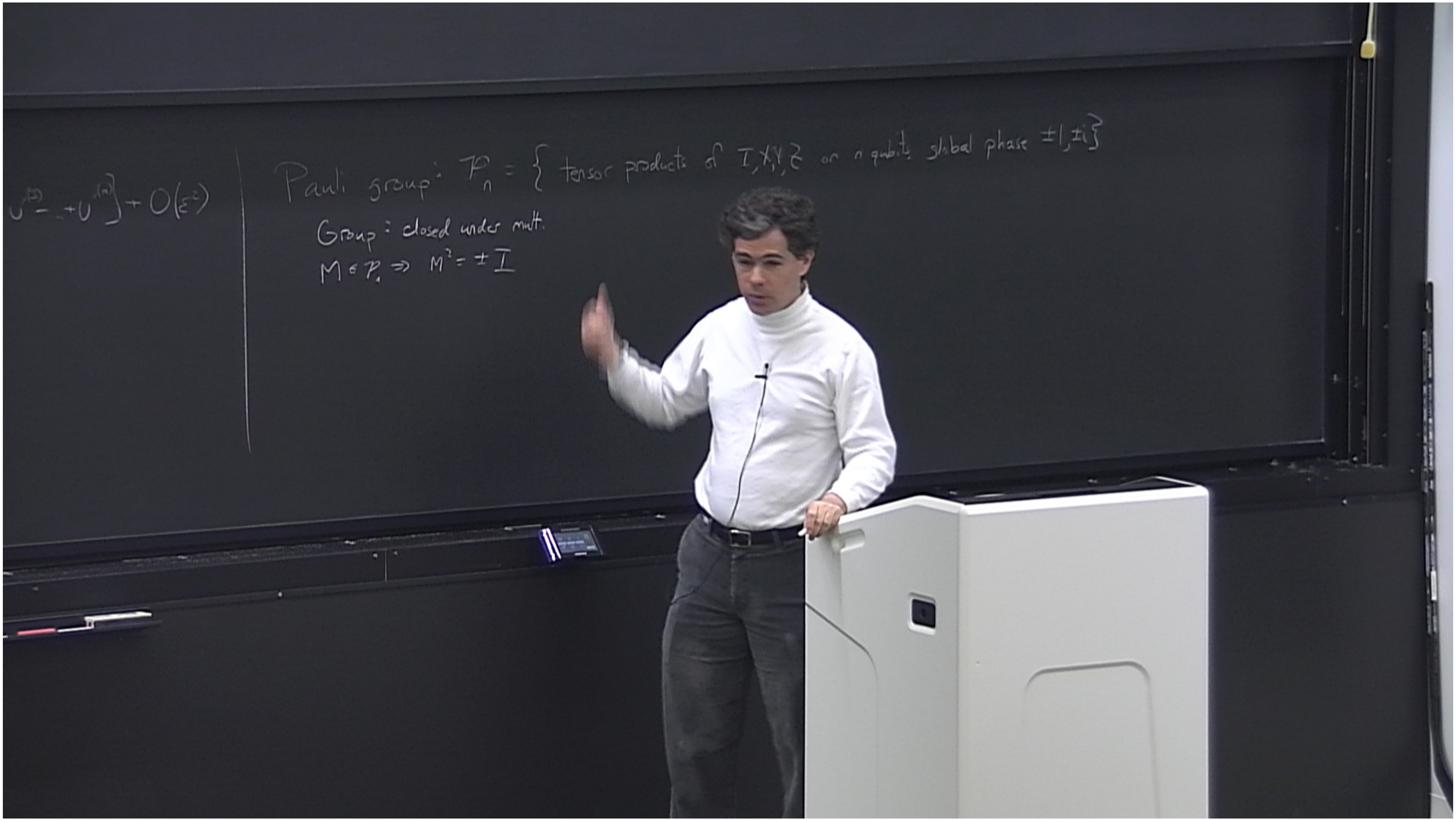

$U_\varepsilon^{\otimes n} = I + \varepsilon\left[U'^{(1)} + U'^{(2)} + U'^{(3)} + \dots + U'^{(n)}\right] + O(\varepsilon^2)$
$U_\varepsilon = I + \varepsilon U'$

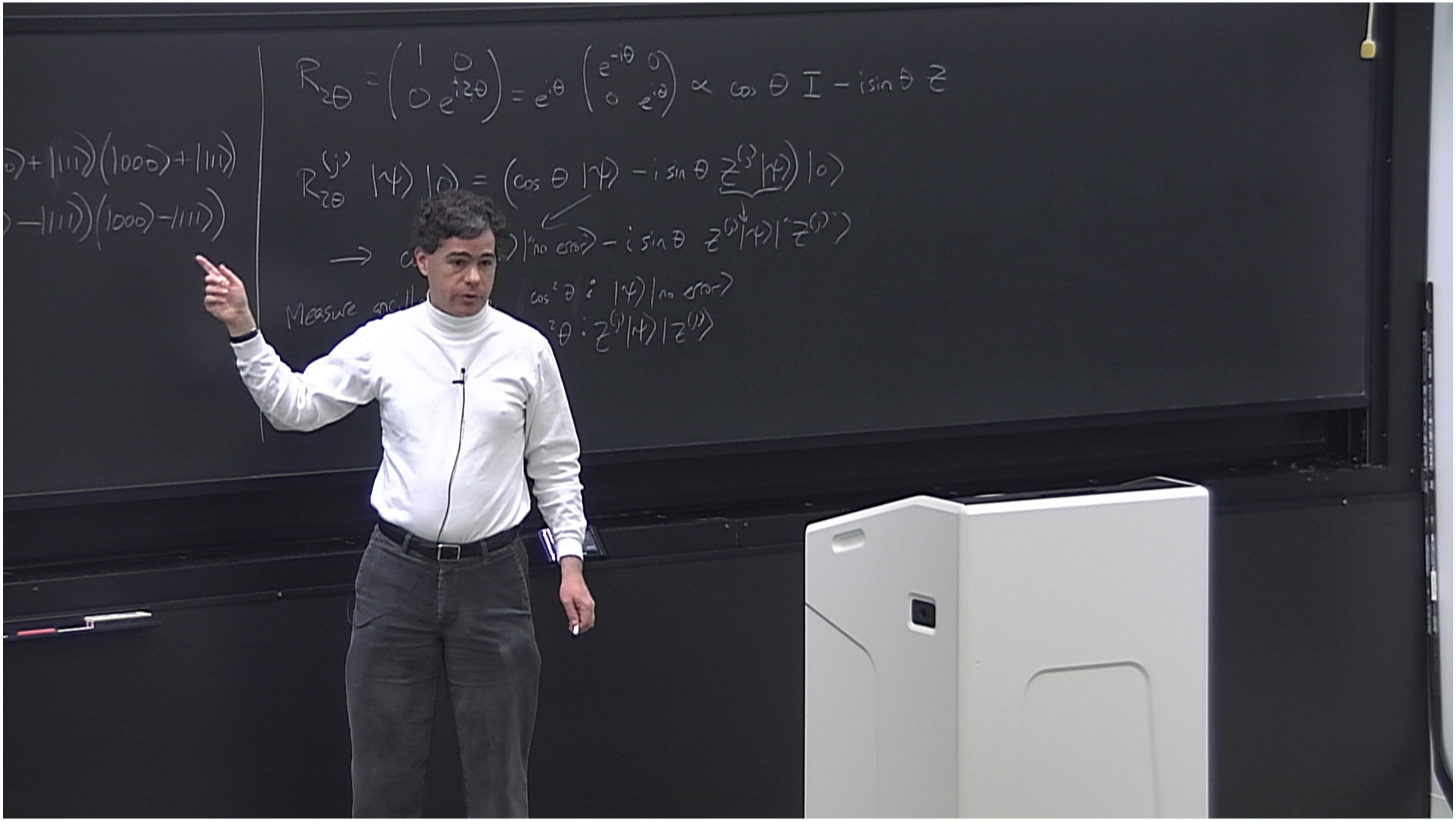

$U_\varepsilon^{\otimes n} = I + \varepsilon\left[U'^{(1)} + U'^{(2)} + U'^{(3)} + \dots + U'^{(n)}\right] + O(\varepsilon^2)$
$U_\varepsilon = I + \varepsilon U'$

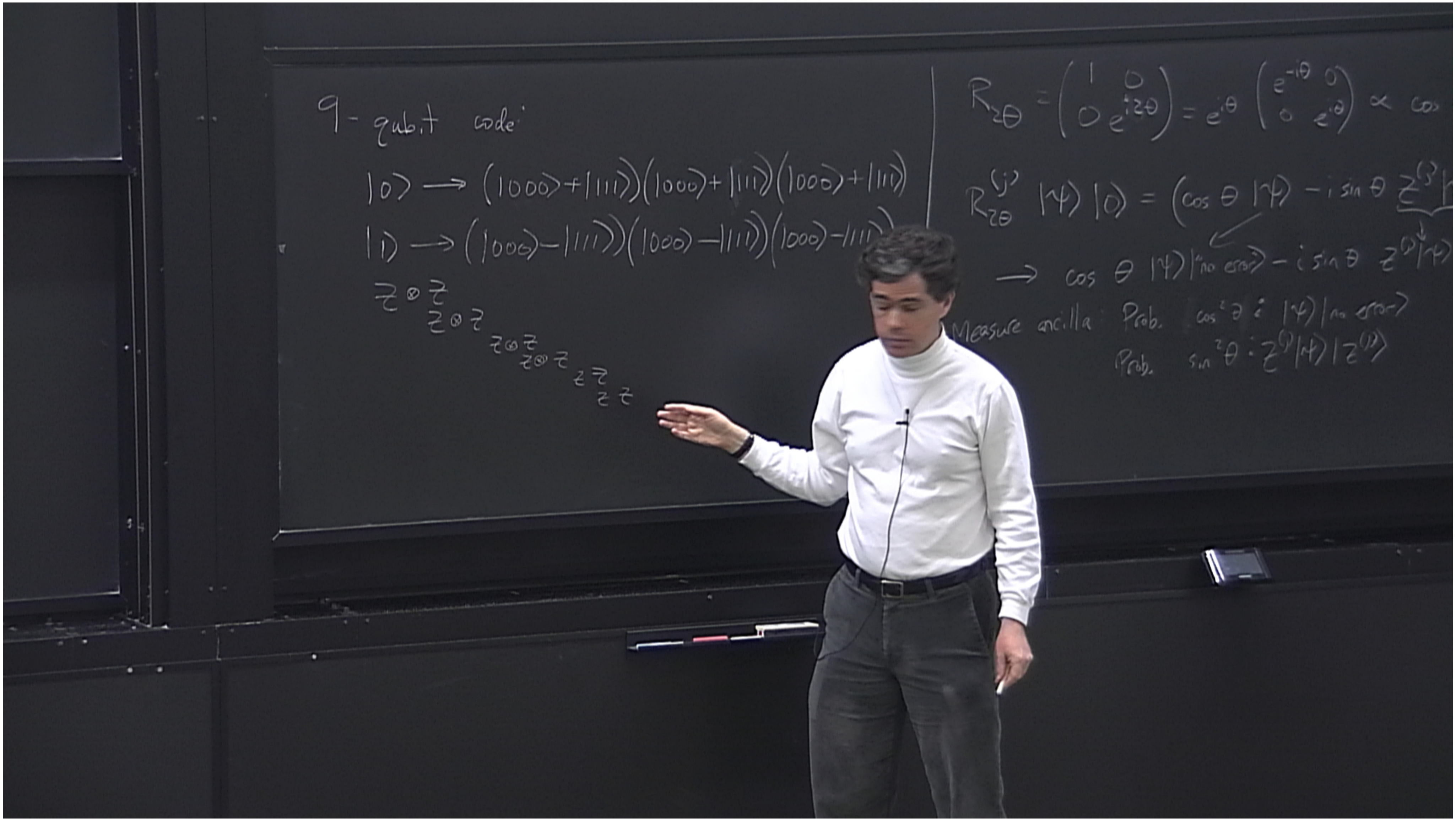

$U_\varepsilon^{\otimes n} = I + \varepsilon\left[U'^{(1)} + U'^{(2)} + U'^{(3)} + \dots + U'^{(n)}\right] + O(\varepsilon^2)$
$U_\varepsilon = I + \varepsilon U'$

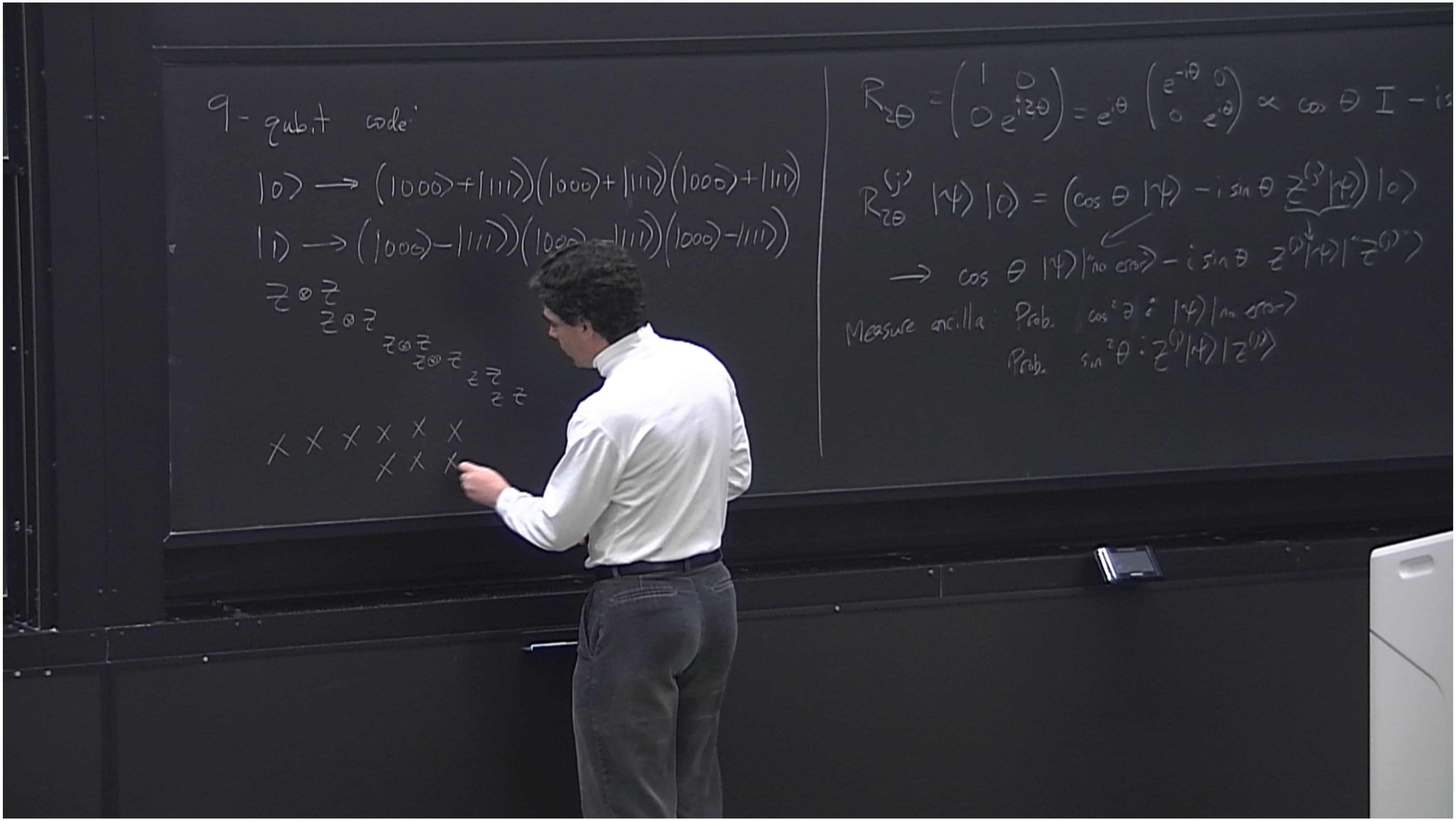

$U_\varepsilon^{\otimes n} = I + \varepsilon[U'^{(1)} + U'^{(2)} + U'^{(3)} + \dots + U'^{(n)}] + O(\varepsilon^2)$
$U_\varepsilon = I + \varepsilon U'$
Pauli grou

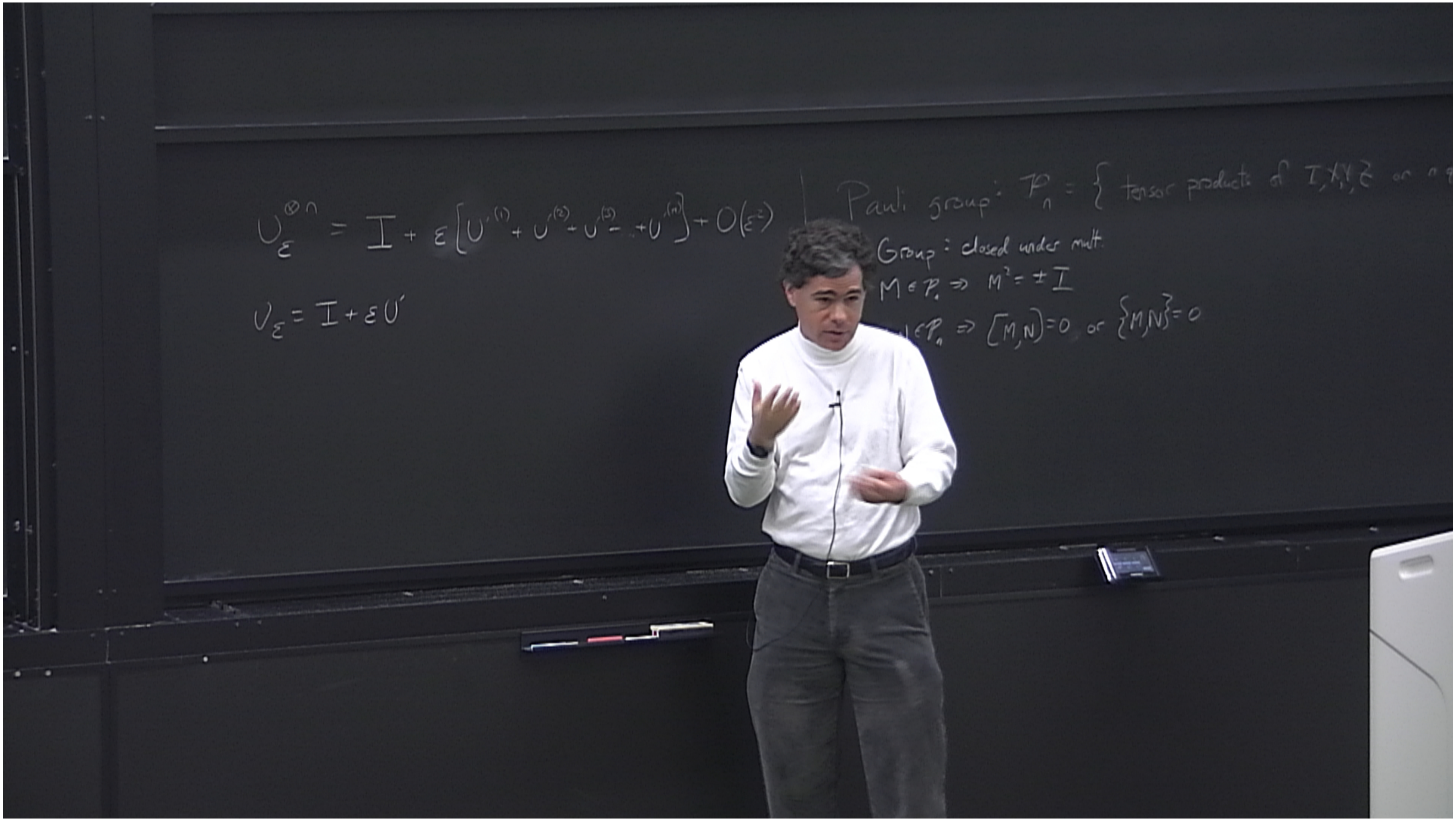

Pauli group: P_n = { tensor products of I, X, Y, Z on n qubits global phase ±1, ±i }
Group: closed under mult.
+ O(ε²)

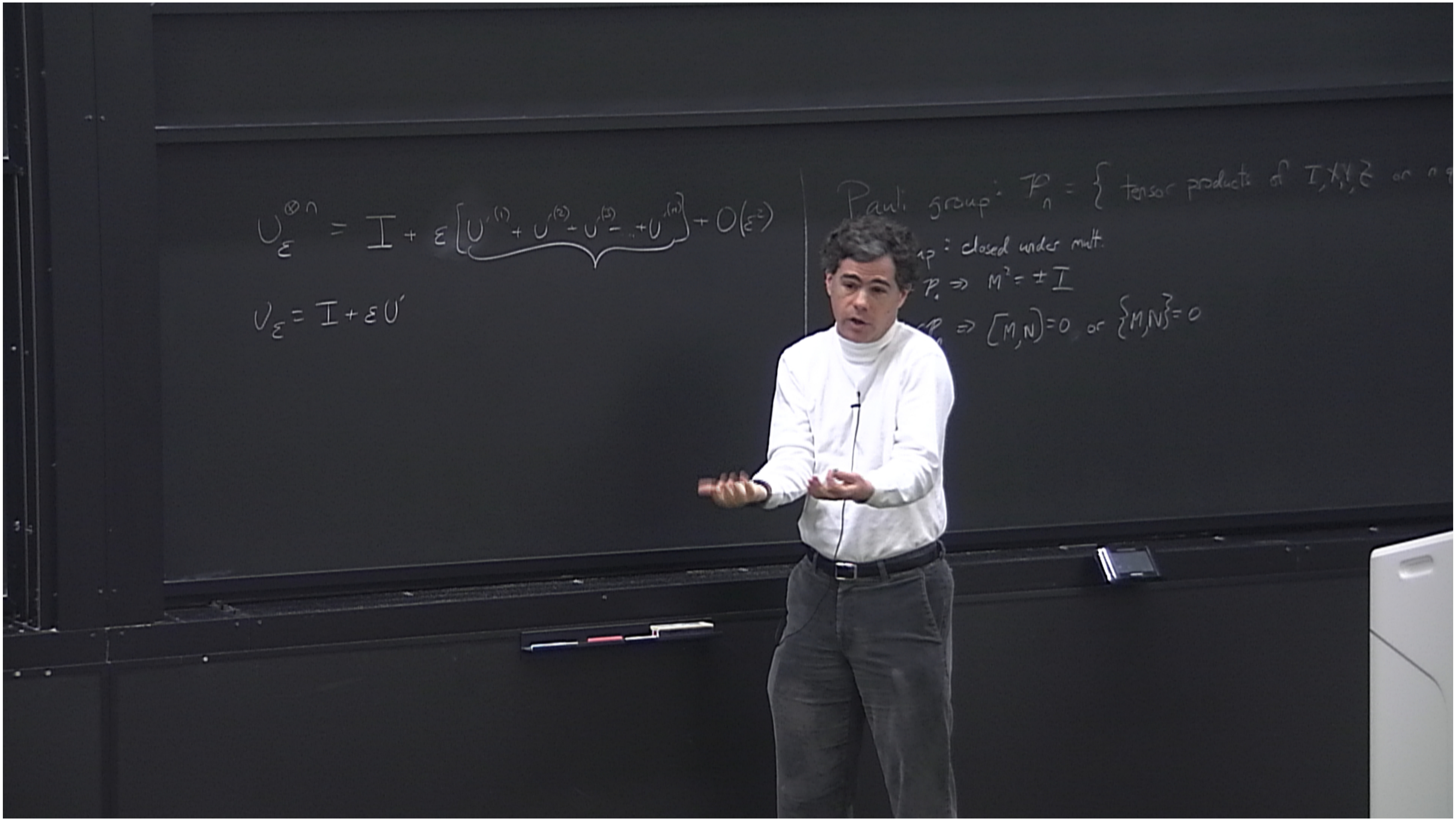

+ O(ε²)
Pauli group: P_n = { tensor products of I, X, Y, Z on n qubits global phase ±1, ±i }
Group: closed under mult.
M ∈ P_n ⇒ M² = ± I


$R_{z\theta} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i2\theta} \end{pmatrix} = e^{i\theta} \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix} \propto \cos\theta \, I - i \sin\theta \, Z$
$R_{z\theta}^{(j)} |\psi\rangle |0\rangle = (\cos\theta |\psi\rangle - i \sin\theta \, Z^{(j)} |\psi\rangle) |0\rangle$
Measure


9-qubit code:
|0⟩ → (|000⟩+|111⟩)(|000⟩+|111⟩)(|000⟩+|111⟩)
|1⟩ → (|000⟩−|111⟩)(|000⟩−|111⟩)(|000⟩−|111⟩)
Z⊗Z
Measure ancilla
Prob.


9-qubit code
|0⟩ → (|000⟩+|111⟩)(|000⟩+|111⟩)(|000⟩+|111⟩)
|1⟩ → (|000⟩−|111⟩)(|000⟩−|111⟩)(|000⟩−|111⟩)
Measure ancilla: Prob. cos²θ : |ψ⟩|no error⟩
Prob. sin²θ : Z^(j)|ψ⟩|Z^(j)⟩


$U_\varepsilon^{\otimes n} = I + \varepsilon[U'^{(1)} + U'^{(2)} + U'^{(3)} + \dots + U'^{(n)}] + O(\varepsilon^2)$
$U_\varepsilon = I + \varepsilon U'$
Pauli group: $\mathcal{P}_n = \{$ tensor products of I, X, Y, Z on n
Group: closed under mult.
$M \in \mathcal{P}_n \Rightarrow M^2 = \pm I$
$\in \mathcal{P}_n \Rightarrow [M,N]=0$ or $\{M,N\}=0$


$U_\varepsilon^{\otimes n} = I + \varepsilon[U'^{(1)} + U'^{(2)} + U'^{(3)} + \dots + U'^{(n)}] + O(\varepsilon^2)$
$U_\varepsilon = I + \varepsilon U'$
Pauli group: $\mathcal{P}_n$ = { tensor products of I, X, Y, Z on n q
: closed under mult.
$\Rightarrow M^2 = \pm I$
$\Rightarrow [M,N]=0$ or $\{M,N\}=0$