Title: Quantum Gravity Review-13

Date: Mar 05, 2015 10:15 AM

URL: http://www.pirsa.org/15030010

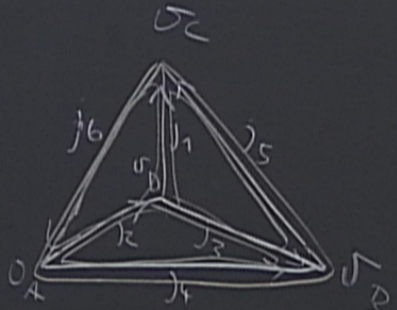
Abstract:

Solution of flatness constraints

$\delta(F) \longrightarrow \delta_G(h_\gamma)$

dual graph of the tetrahedron

Σ = sphere



$\longrightarrow \prod_f \delta(H_f)$



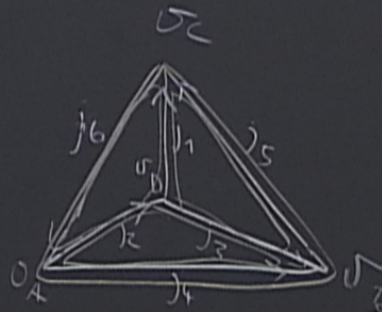
$H_4 = g_2 \, h_4 \, h_3 \, h_2 \, g_2^{-1}$

integral version of the Bianchi identity $dF = 0$

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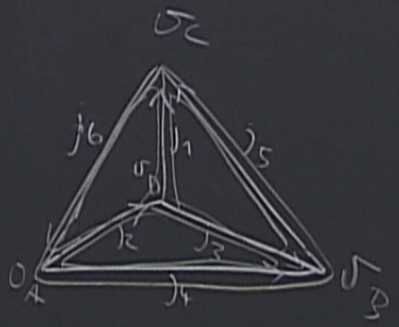
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$\longrightarrow \prod_f \delta(H_f) = P_{(F=0)}$



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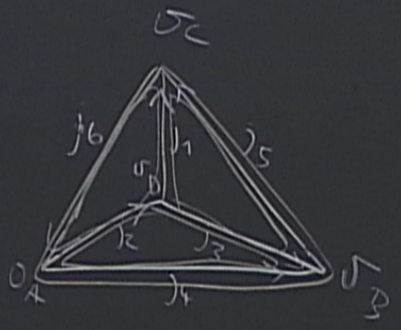
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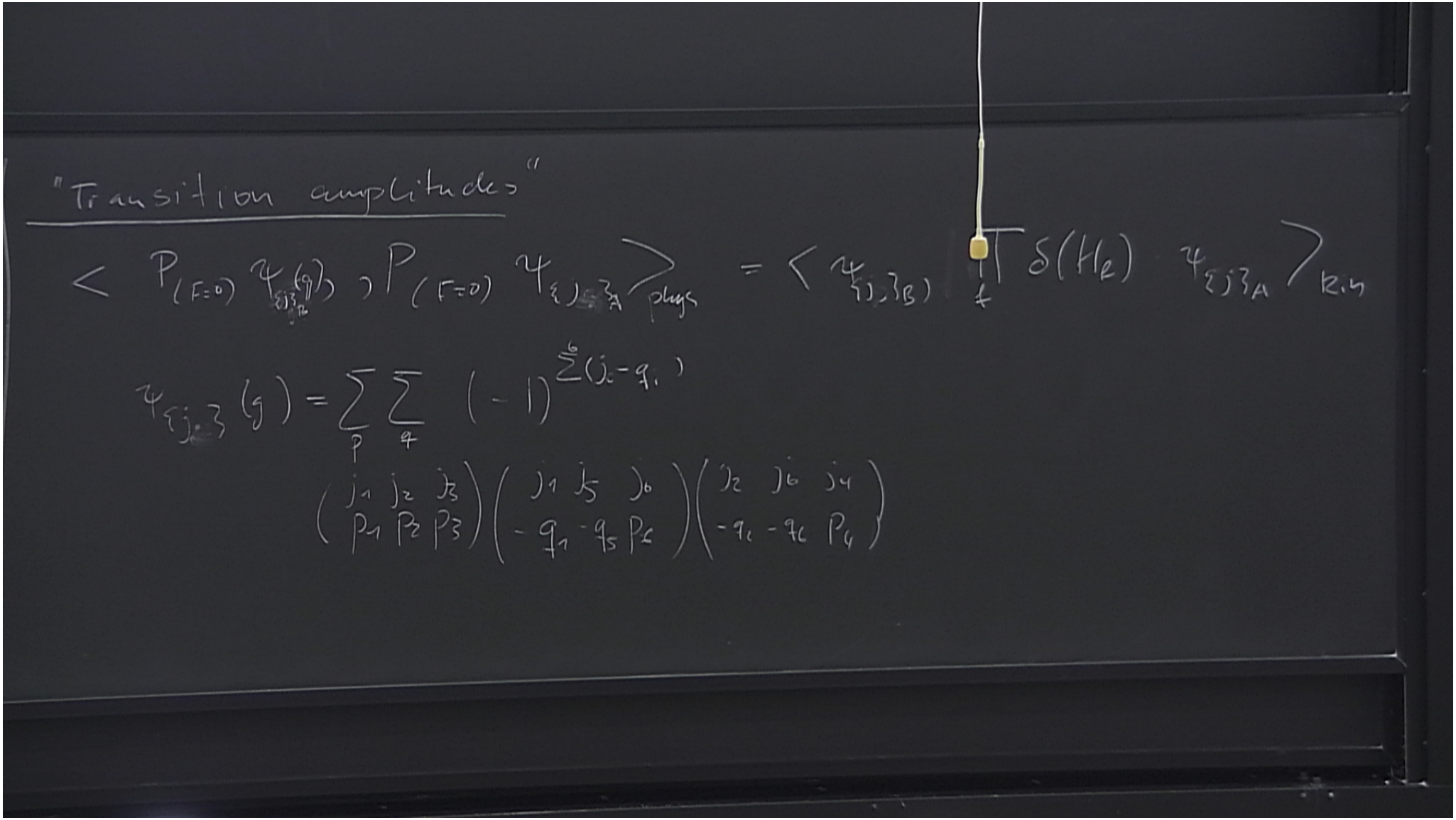


$$\longrightarrow \prod_{(\text{thread } f)} \delta(H_f) = P_{(F=0)}$$



$$H_4 = g_2 \, t_{l_1} t_{l_3} t_{l_2} g_2^{-1}$$

integral version of the Bianchi identity $dF = 0 \Leftrightarrow \boxed{\partial\delta S = 0}$


"Transition amplitudes"
$\langle P_{(F=0)}\, \psi_{\{j_i\}}(q) \rangle P_{(F=0)}\, \psi_{\{j_i\}_A} \rangle_{\text{plugs}} = \langle \psi_{\{j_i\}_B} \; T \; \delta(H_E) \; \psi_{\{j_i\}_A} \rangle_{K.H}$
$\psi_{\{j_i\}}(q) = \sum_P \sum_q (-1)^{\sum (j_i - q_i)}$
$\begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & p_2 & p_3 \end{pmatrix} \begin{pmatrix} j_1 & j_5 & j_6 \\ -q_1 & -q_5 & p_5 \end{pmatrix} \begin{pmatrix} j_2 & j_6 & j_4 \\ -q_2 & -q_6 & p_4 \end{pmatrix}$

"Transition amplitudes"

$$\langle P_{(F=0)}\, \psi_{\{j_i\}_B}(g_i) \rangle P_{(F=0)}\, \psi_{\{j_i\}_A} \rangle_{\text{phys}} = \langle \psi_{\{j_i\}_B} \Big| \prod_t \delta(H_t)\ \psi_{\{j_i\}_A} \rangle_{\text{kin}}$$

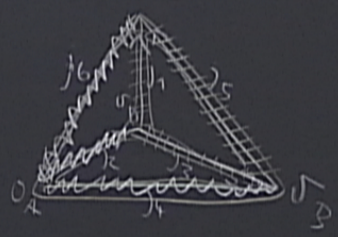
$$\psi_{\{j_i\}_A}(g_{ij}) = \sum_p \sum_q (-1)^{\sum_i (j_i - q_i)} \begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & p_2 & p_3 \end{pmatrix} \begin{pmatrix} j_1 & j_5 & j_6 \\ -q_1 & -q_5 & p_6 \end{pmatrix} \begin{pmatrix} j_2 & j_6 & j_4 \\ -q_2 & -q_6 & p_4 \end{pmatrix} \begin{pmatrix} j_3 & j_4 & j_5 \\ -q_3 & -q_4 & p_5 \end{pmatrix} \left(\prod_i \sqrt{d_{q_i}} \right) \left(\prod_i D^{j_i}_{q_i p_i}(g_i) \right)$$

Solution of flatness constraints

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dual triang of the tetrahedron

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$\longrightarrow \prod_{(\text{thread})} \delta(H_f) = P_{(F=0)}$

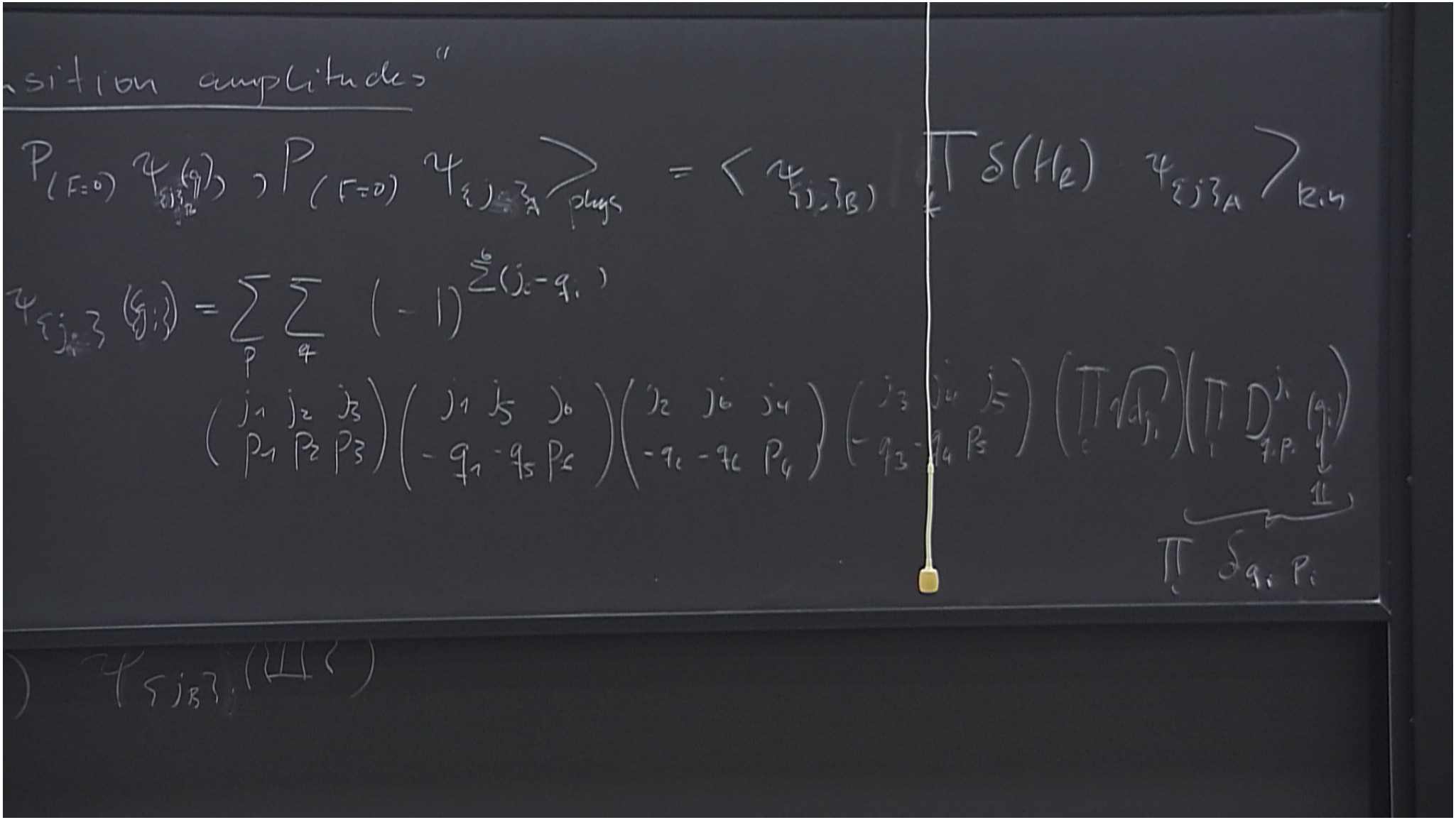


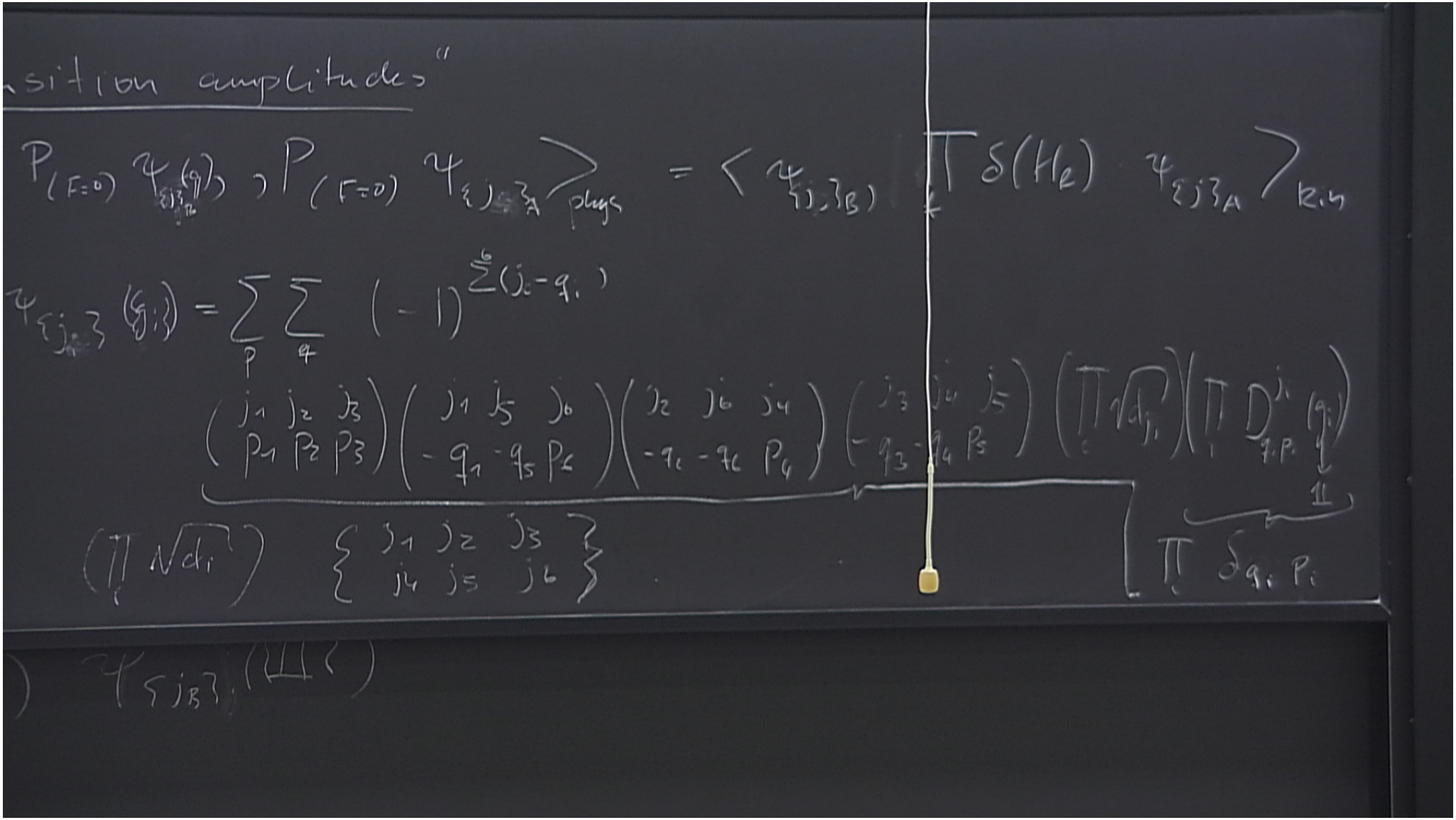
$H_4 = g_2 \, h_1 \, h_3 \, h_2 \, g_2^{-1}$

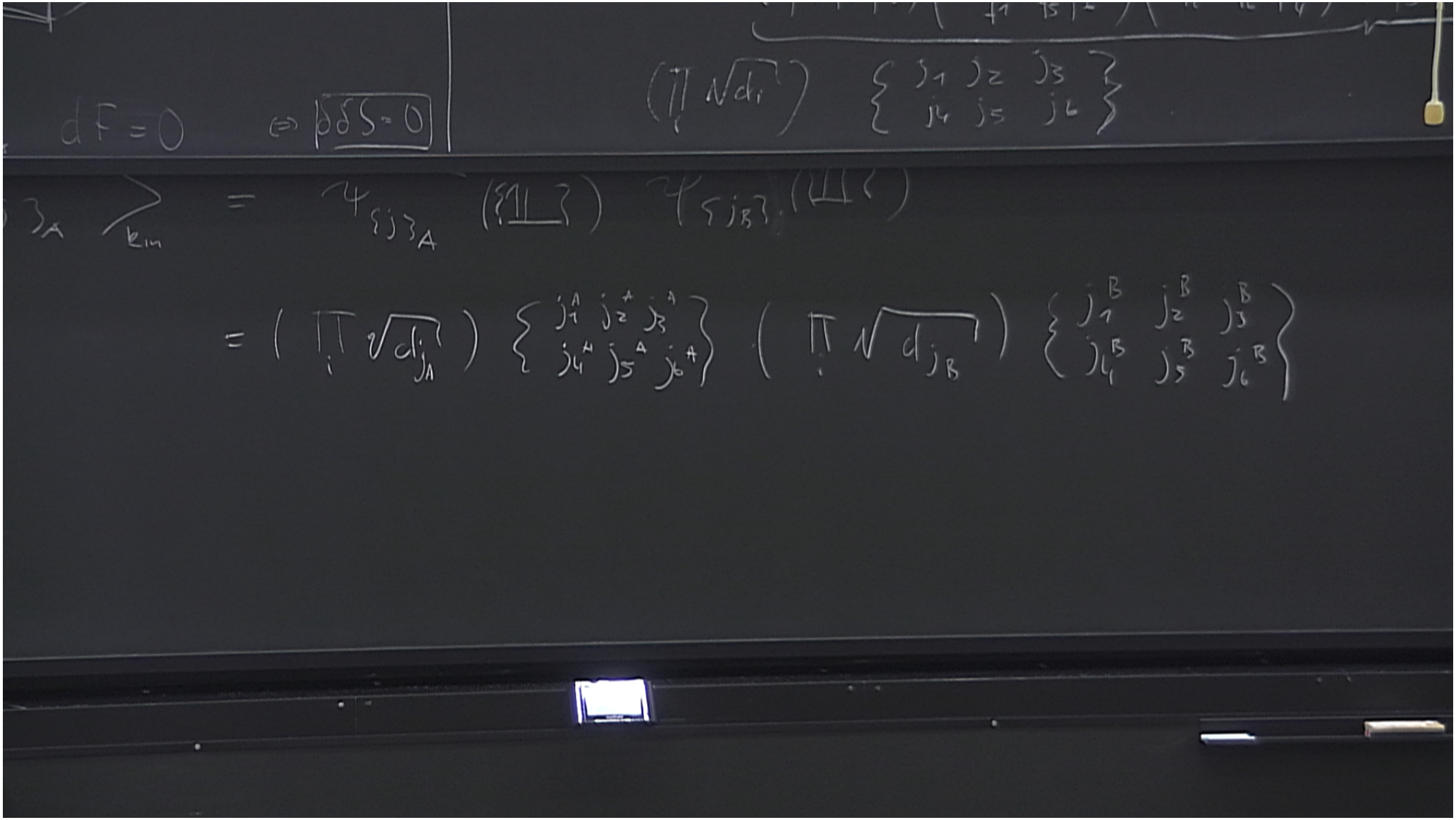
Integral version of the Bianchi identity $dF = 0 \Leftrightarrow \boxed{\partial S = 0}$

"Tra

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$P_{(F=0)}\,\psi_{\{j\}_B}(q) \rangle\, P_{(F=0)}\,\psi_{\{j\}_A}\rangle_{phys} = \langle \psi_{\{j\}_B} \prod \delta(H_k)\, \psi_{\{j\}_A}\rangle_{kin}$
$\psi_{\{j_i\}}(g_i) = \sum_p \sum_q (-1)^{\sum (j_i - q_i)}$
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$\prod \delta_{q_i p_i}$
$\psi_{\{j_B\}}(\Pi g)$


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$P_{(F=0)}\,\psi_{\{j\}_B}(q) \rangle\, P_{(F=0)}\,\psi_{\{j\}_A} \rangle_{\text{plugs}} = \langle \psi_{\{j\}_B} | \prod \delta(H_k)\, \psi_{\{j\}_A} \rangle_{\text{kin}}$
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$\left(\prod \sqrt{d_i}\right) \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$
$\prod \delta_{q_i p_i}$
$\psi_{\{j_B\}}(\mathrm{III}')$


dF = 0 ⇔ δδS = 0

$H_4 = g_2 H_1 H_3 H_2 g_2^{-1}$

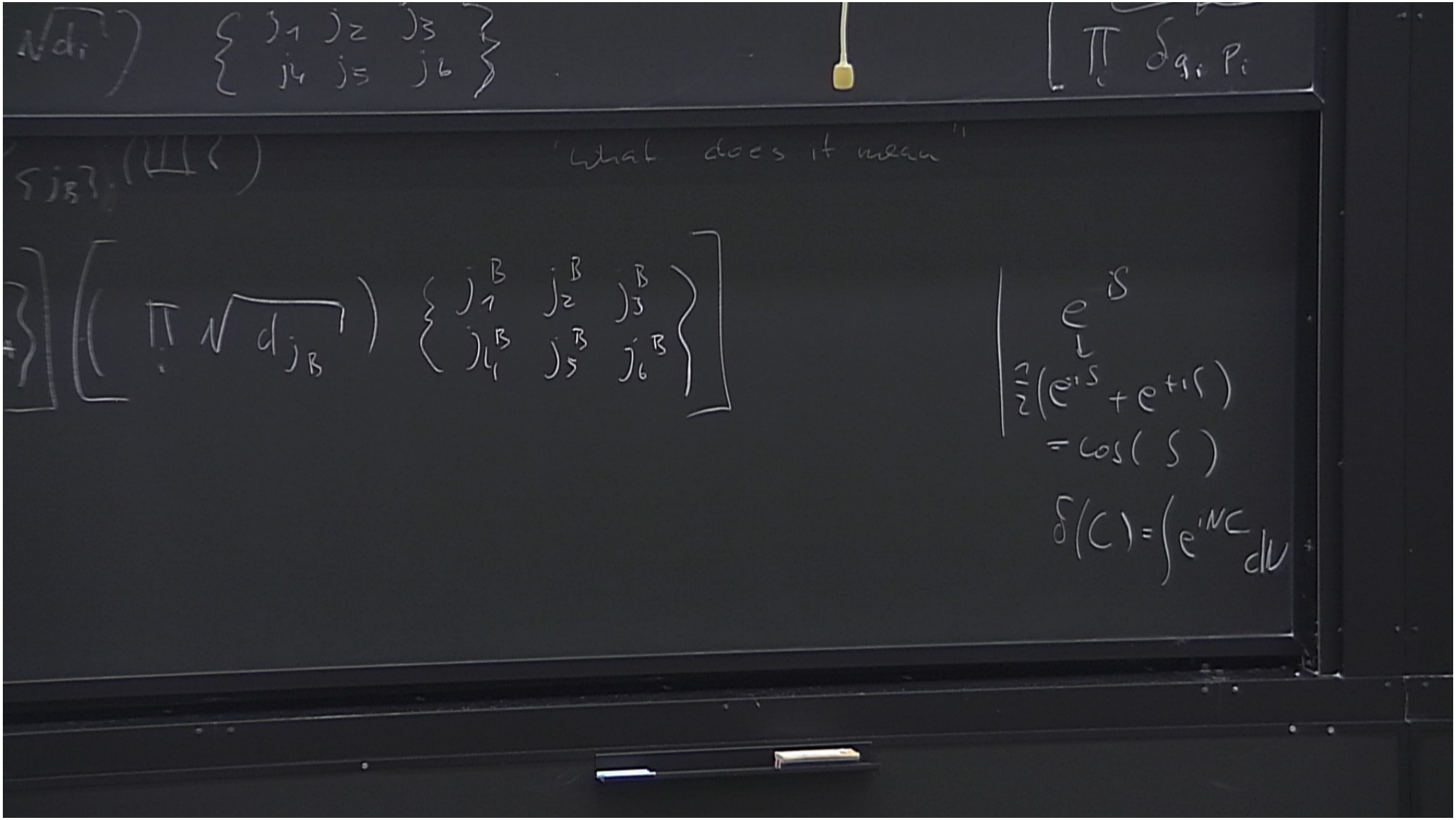
integral version of the Bianchi identity, $dF = 0 \Leftrightarrow \delta\delta S = 0$

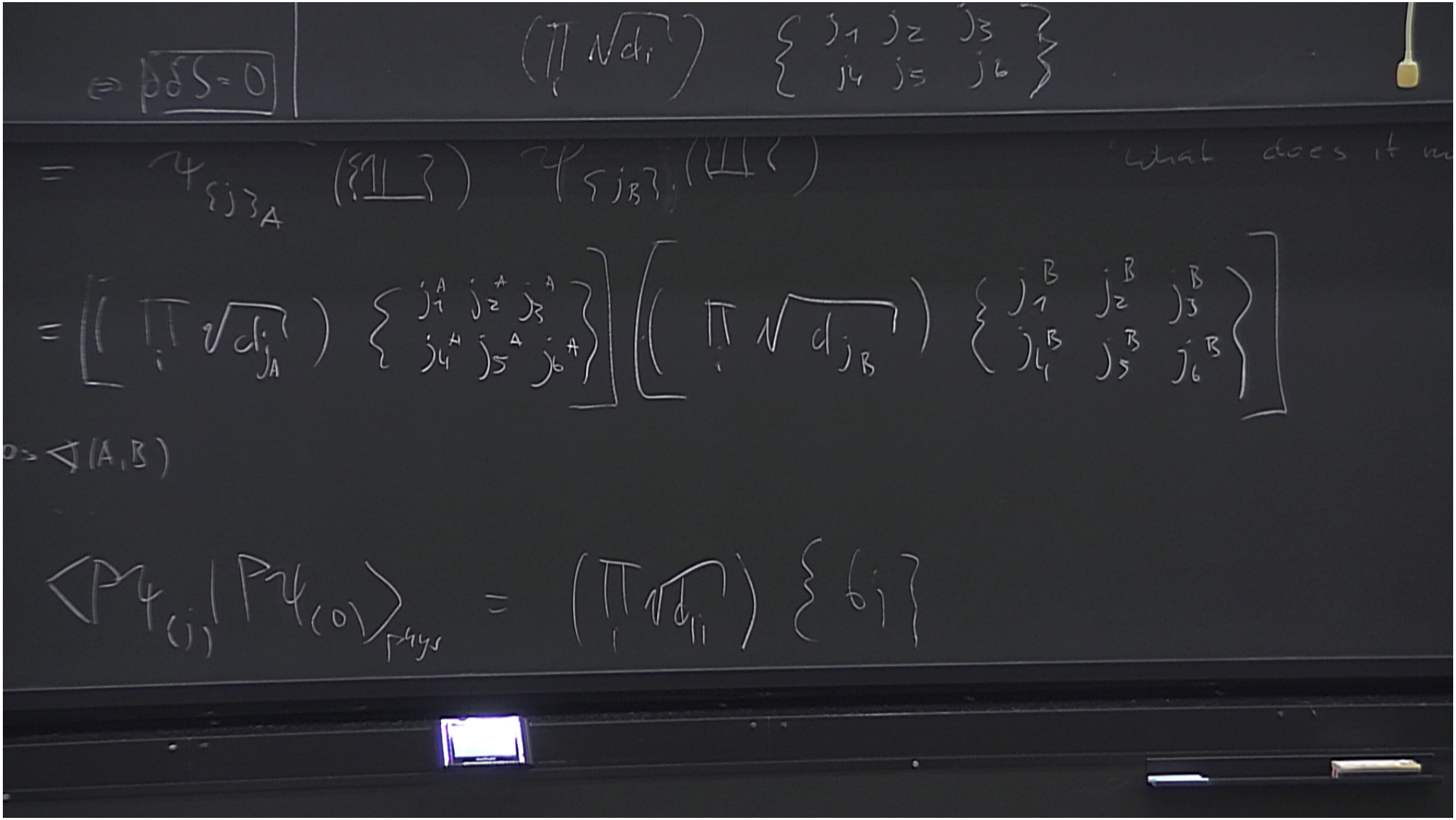
$(?! \sqrt{d_i})$

$\langle \psi_{\{j\}_B} | \prod_{3F's} \delta(H_F) | \psi_{\{j\}_A} \rangle_{Kin} = \psi_{\{j\}_A}(\{1\}) \; \psi_{\{j_B\}}(\{1\})$

$\langle \psi_{(B)} | \psi_{(A)} \rangle = \left(\prod_i \sqrt{d_{j_A^i}} \right) \begin{Bmatrix} j_1^A & j_2^A & j_3^A \\ j_4^A & j_5^A & j_6^A \end{Bmatrix} \left(\prod_i \right.$

$\langle v_A | v_B \rangle = |v_A| |v_B| \cos \sphericalangle(A,B)$


'what does it mean'
$\delta(C) = \int e^{iNC} dV$


$\Leftrightarrow \boxed{\hat{p}\hat{S}=0}$
$\left(\prod_i \sqrt{d_i}\right) \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$
$= \Psi_{\{j\}_A}(\{1\!\!1\}) \Psi_{\{j_B\}}(1\!\!1)$
what does it m
$= \left[\left(\prod_i \sqrt{d_{j_A}}\right) \begin{Bmatrix} j_1^A & j_2^A & j_3^A \\ j_4^A & j_5^A & j_6^A \end{Bmatrix}\right] \left[\left(\prod_i \sqrt{d_{j_B}}\right) \begin{Bmatrix} j_1^B & j_2^B & j_3^B \\ j_4^B & j_5^B & j_6^B \end{Bmatrix}\right]$
$\Rightarrow \nabla(A,B)$
$\langle P\Psi_{\{j\}} | P\Psi_{\{0\}} \rangle_{Phys} = \left(\prod_i \sqrt{d_{j_i}}\right) \begin{Bmatrix} j_i \end{Bmatrix}$

Path integral

$$\int e^{iS(e,A)} \mathcal{D}e\,\mathcal{D}A$$

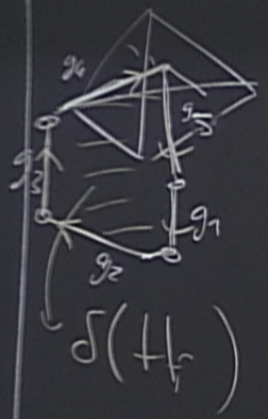
$$= \int e^{i\int_M (F\cdot e)} \mathcal{D}e\,\mathcal{D}A$$

$$\simeq \int \prod_{\text{points}} \delta(F)\, \mathcal{D}A$$

"integral over all flat connections"

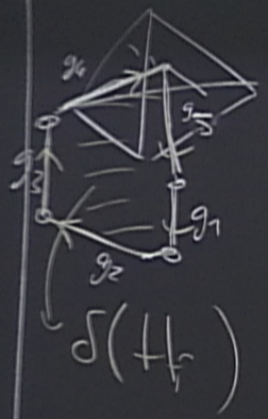
→ Dual Triangulation

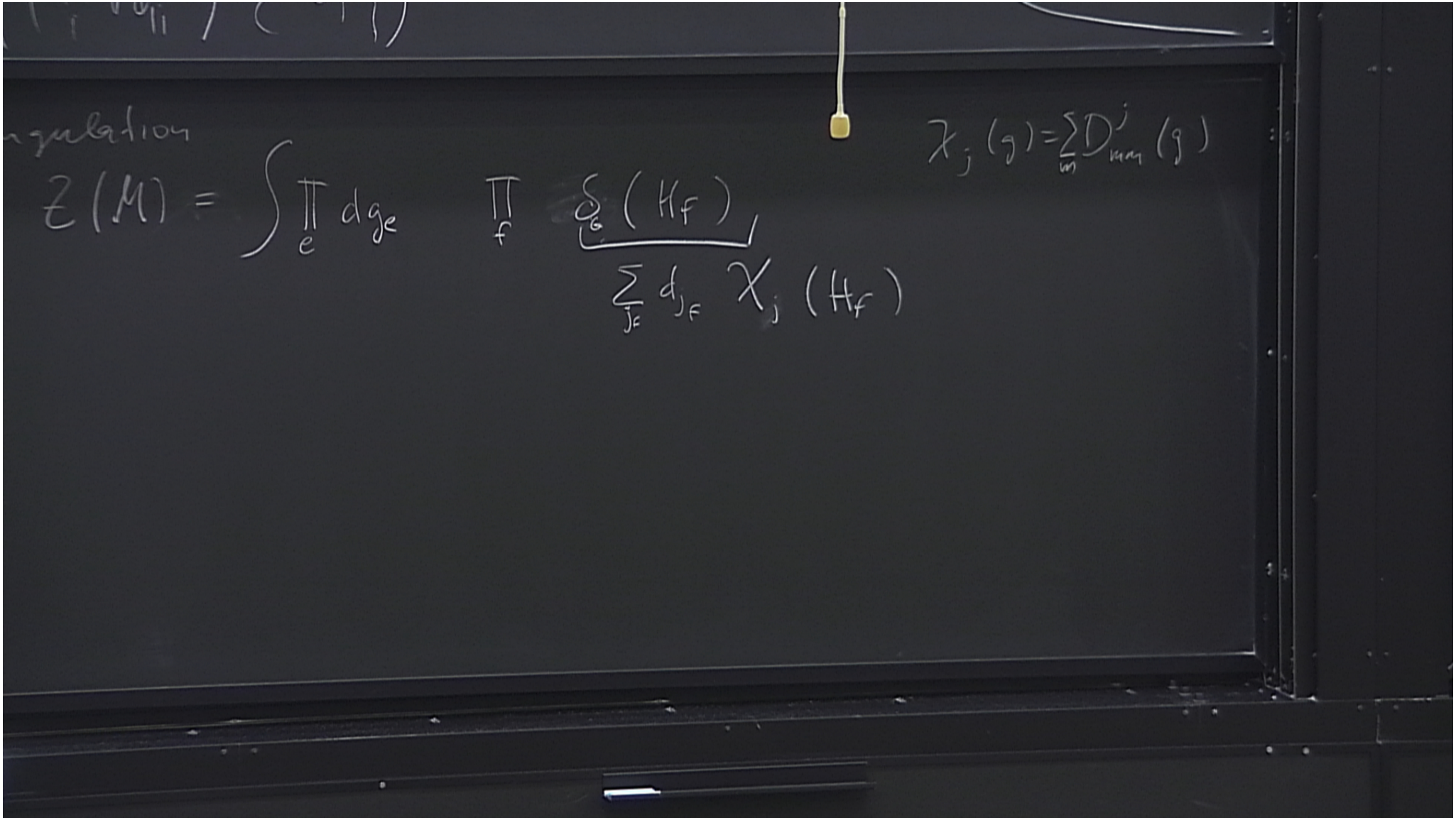
$$\Rightarrow \quad Z(M) = \int \prod_e dg_e \quad \prod_f \delta_G(H_f)$$

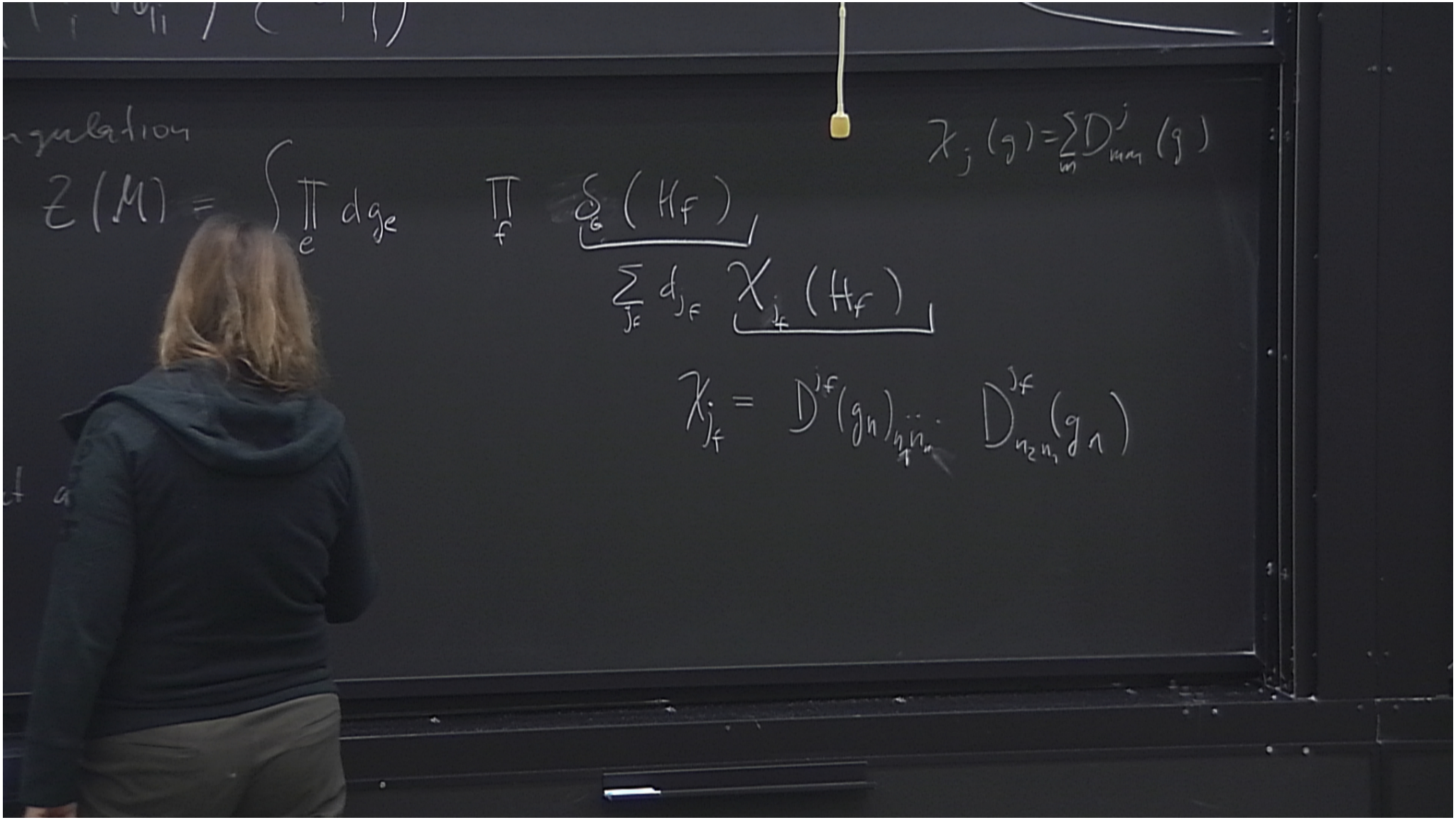


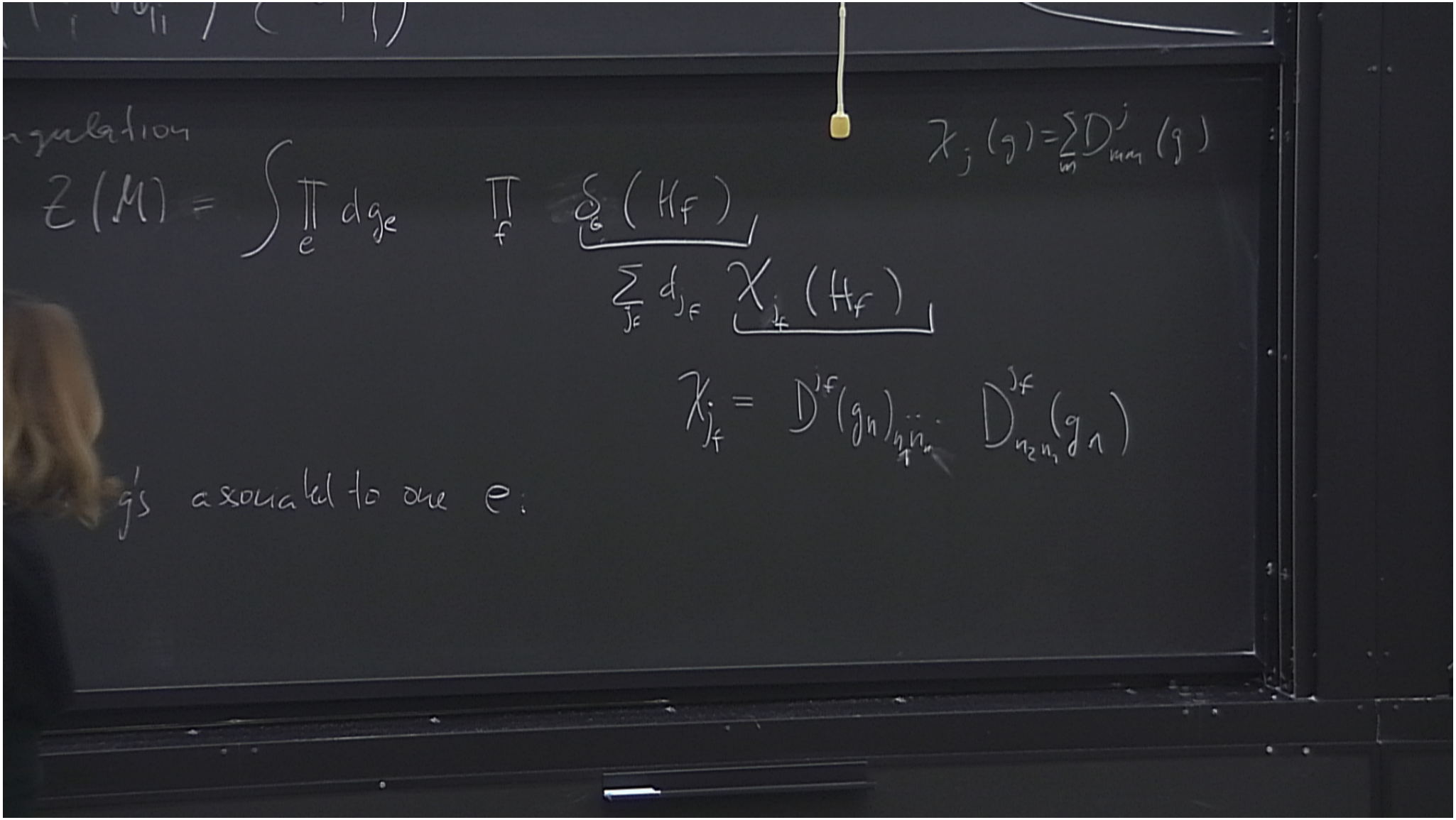
→ Dual Triangulation

$$\Rightarrow Z(M) = \int \prod_e dg_e \prod_f \frac{\delta_G(H_f)}{\sum_{j_F} d_{j_F} \chi_j(H_f)}$$




$\chi_j(g) = \sum_{mn} D^j_{mn}(g)$
$Z(M) = \int \prod_e dg_e \prod_f \frac{\delta_G(H_f)}{\sum_{j_f} d_{j_f} \chi_j(H_f)}$


$\chi_j(g) = \sum_m D^j_{mm}(g)$
$Z(M) = \int \prod_e dg_e \prod_f \delta_G(H_f) \Big/ \sum_{j_f} d_{j_f} \chi_{j_f}(H_f)$
$\chi_{j_f} = D^{j_f}(g_n)_{n_1 n_1} \cdots D^{j_f}_{n_2 n_1}(g_1)$


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$Z(M) = \int \prod_e dg_e \prod_f \delta_G(H_f)$
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Dual Triangulation

$$\Rightarrow Z(M) = \int \prod_e dg_e \prod_f \delta_G(H_f)$$

$$\chi_j(g) = \sum_{m,n} D^j_{mn}(g)$$

$$\delta_G(H_f) = \sum_{j_f} d_{j_f} \chi_{j_f}(H_f)$$

$$\chi_{j_f} = D^{j_f}(g_n)_{m_1 n_1} \cdots D^{j_f}_{m_2 n_1}(g_1)$$

- collect all g's associated to one e:

$$\int \prod_{f \supset e} D^{j_f}_{m_f n_f}(g_e)\, dg = \Big($$

Dual Triangulation

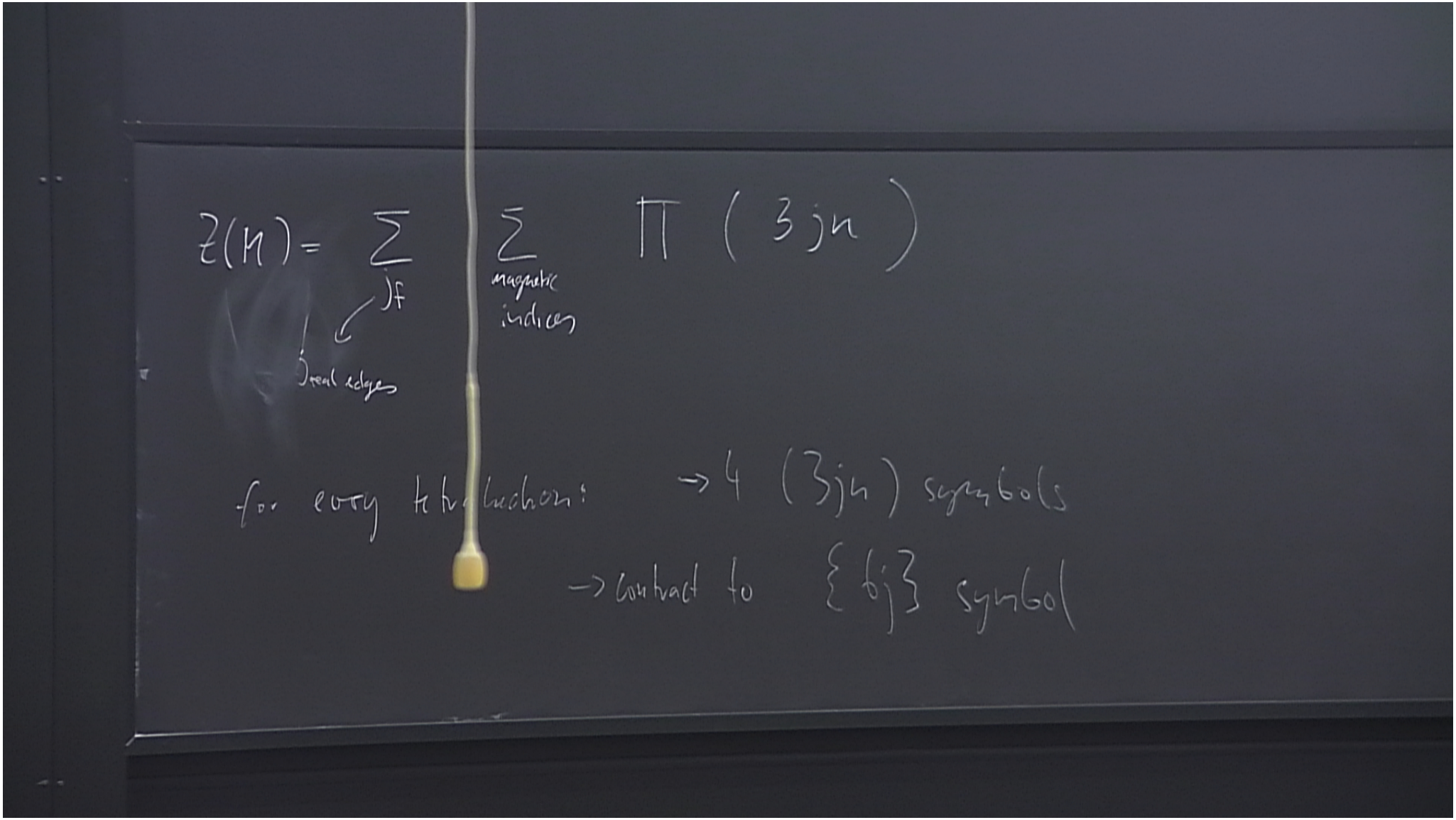
$$\Rightarrow Z(M) = \int \prod_e dg_e \prod_f \underbrace{\delta_G(H_f)}_{\sum_{j_f} d_{j_f} \underbrace{\chi_{j_f}(H_f)}}$$

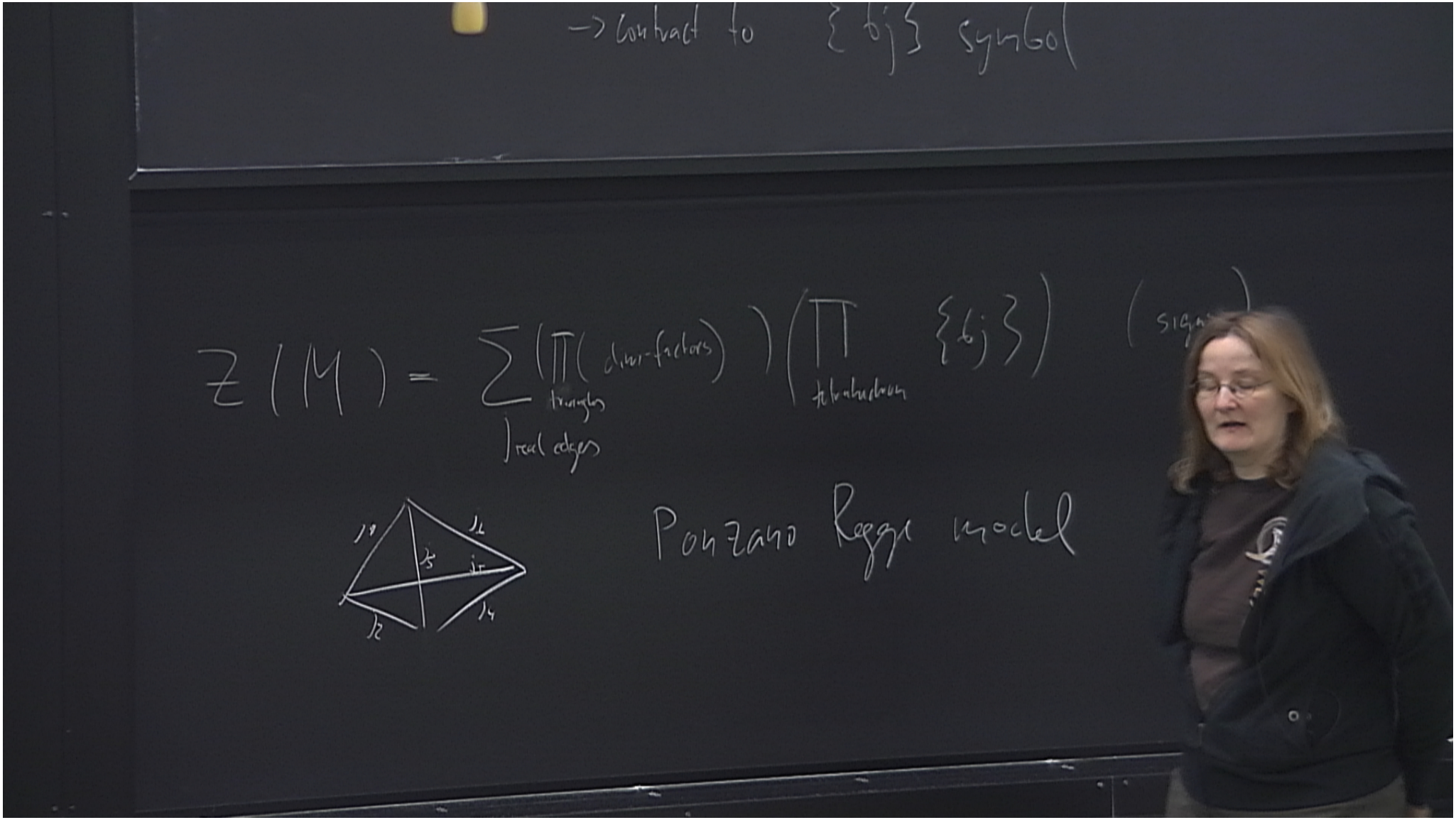
$$\chi_j(g) = \sum_{m} D^j_{mm}(g)$$

$$\chi_{j_f} = D^{j_f}(g_n)_{m_1 m_n} \cdots D^{j_f}_{m_2 m_1}(g_1)$$

- collect all g's associated to one e:

$$\int \prod_{f \supset e} D^{j_f}_{m_f n_f}(g_e) \left(dg_e^{(3)}\right) = \begin{pmatrix} j_{f_1} & j_{f_2} & j_{f_3} \\ m_1 & m_2 & m_3 \end{pmatrix} \begin{pmatrix} j_{f_1} & j_{f_2} & j_{f_3} \\ n_1 & n_2 & n_3 \end{pmatrix}$$


$Z(H) = \sum_{\text{if}} \sum_{\text{magnetic indices}} \prod (3jm)$
! 3 real edges
for every tetrahedron: → 4 (3jm) symbols
→ contract to {6j} symbol


-> contract to {6j} symbol
Z(M) = Σ (Π (diff-factors)) (Π {6j}) (sign)
triangles
real edges
tetrahedra
Ponzano Regge model