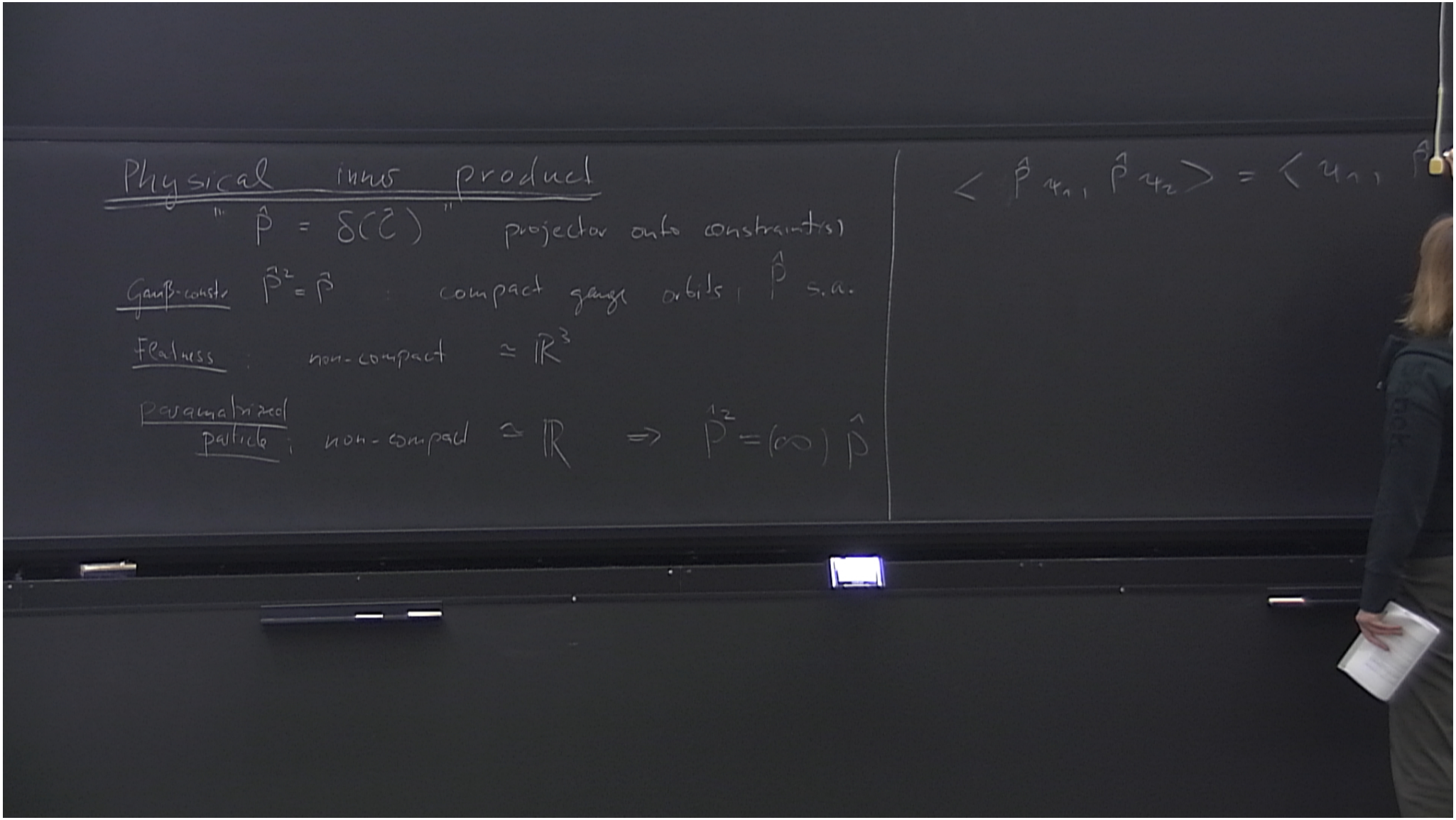
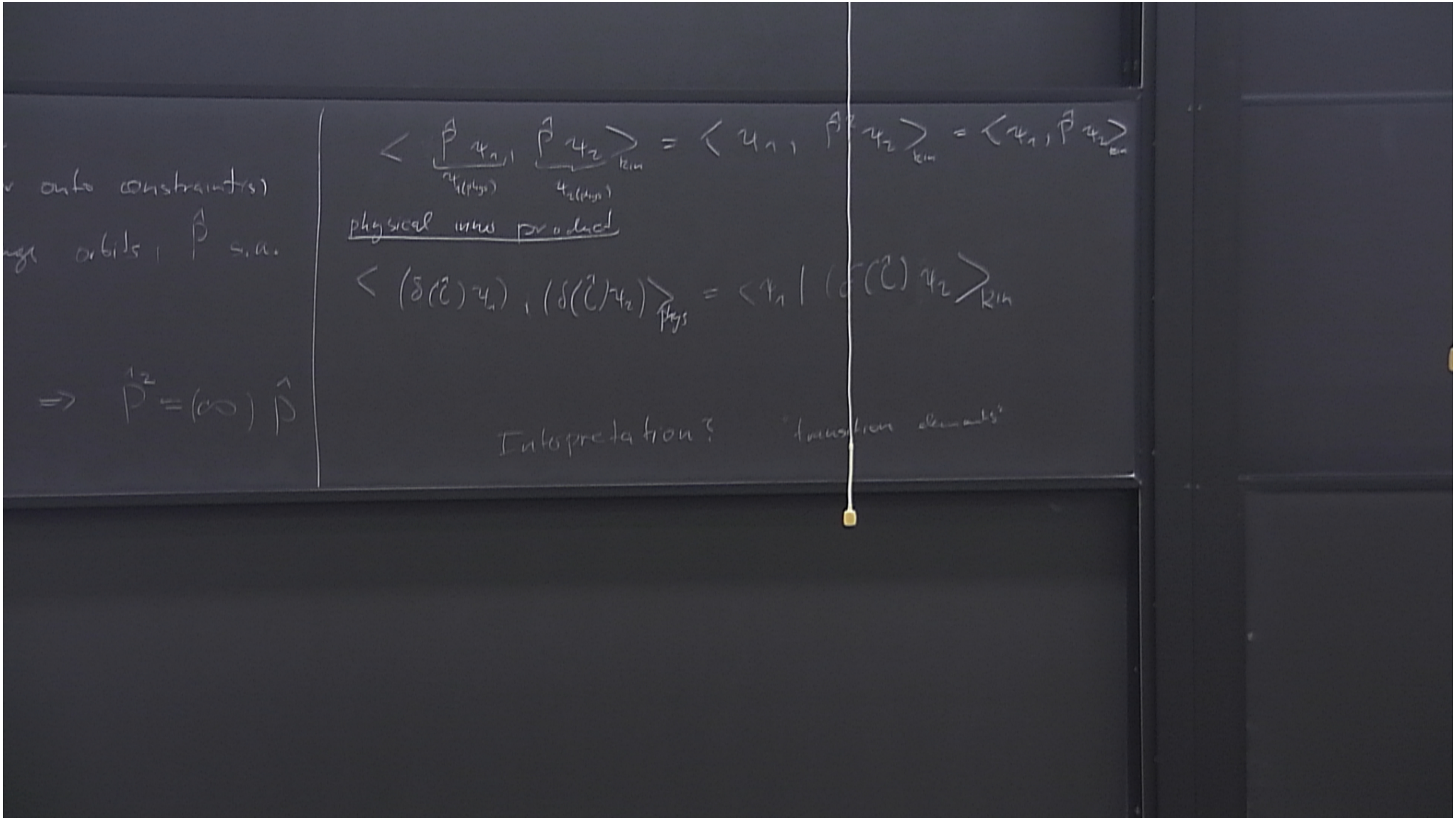
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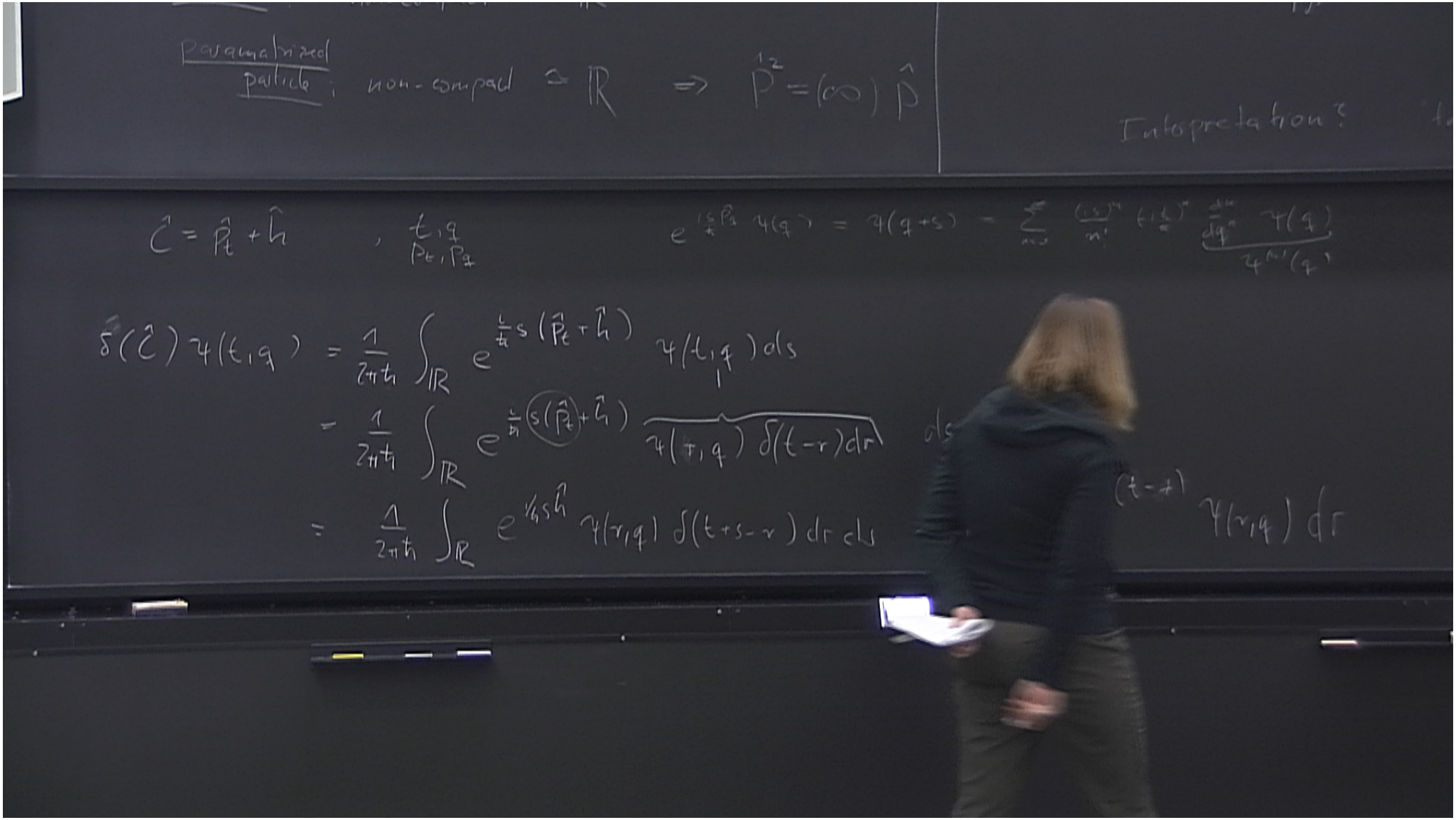
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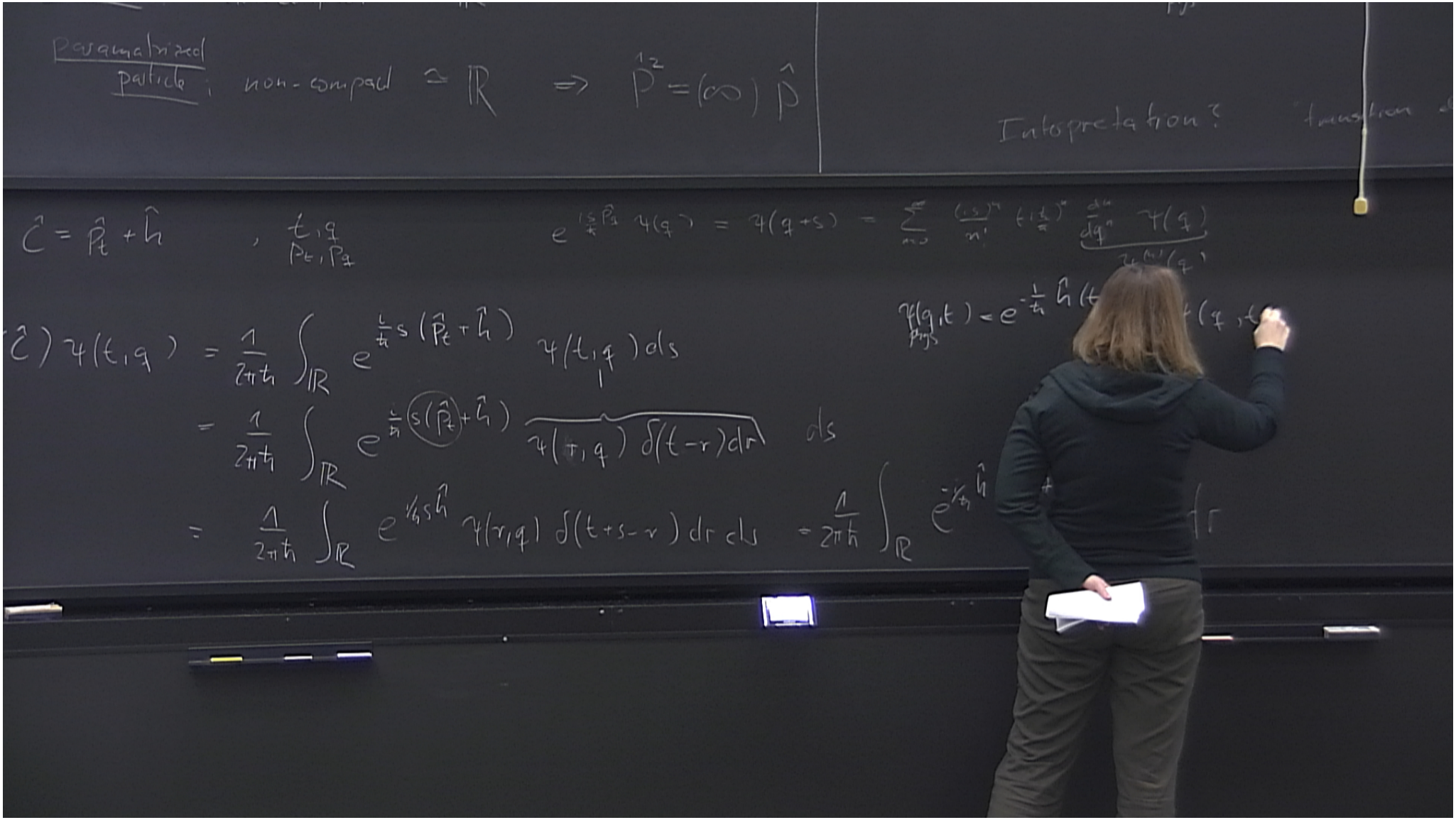
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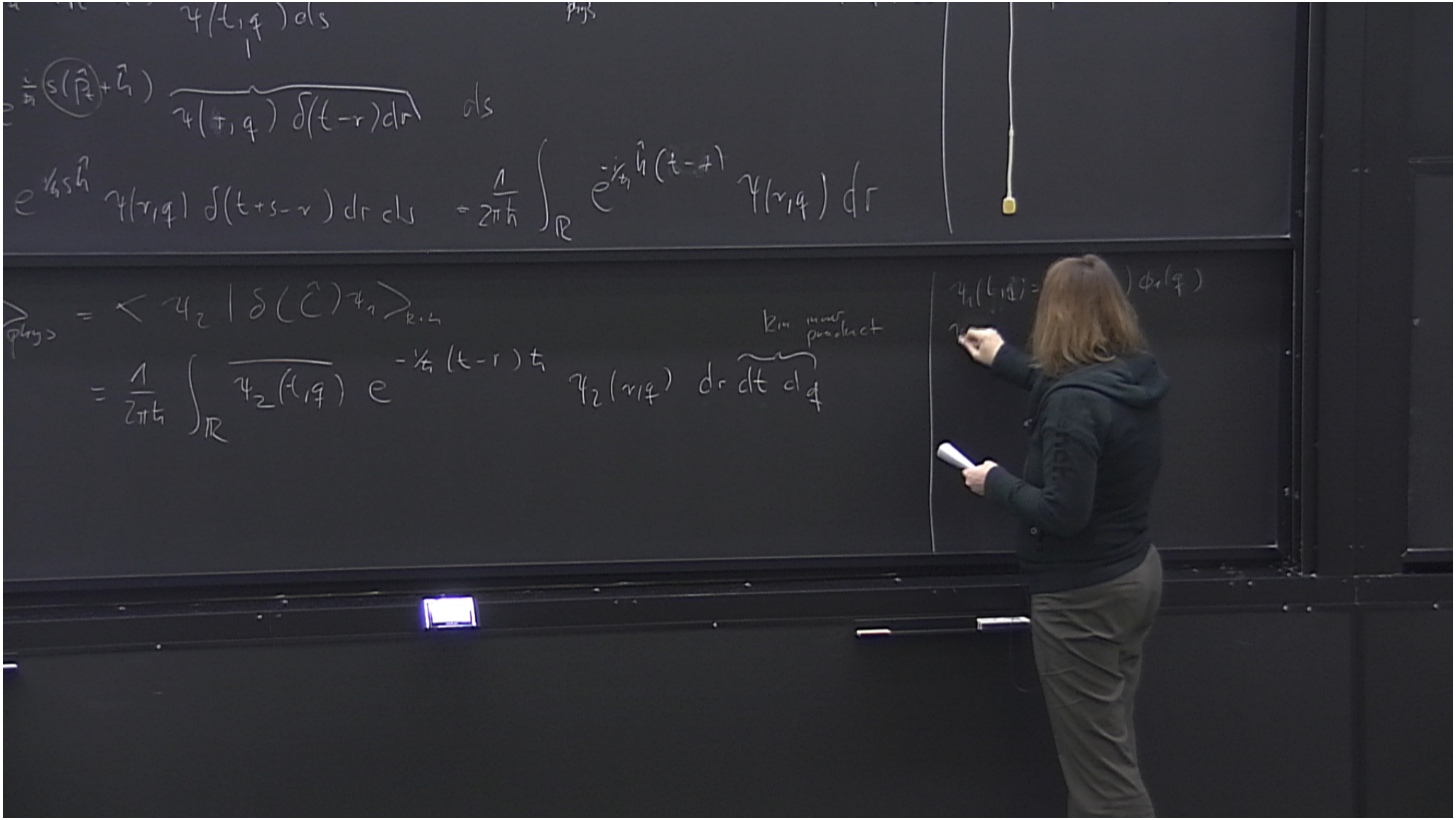
Abstract:

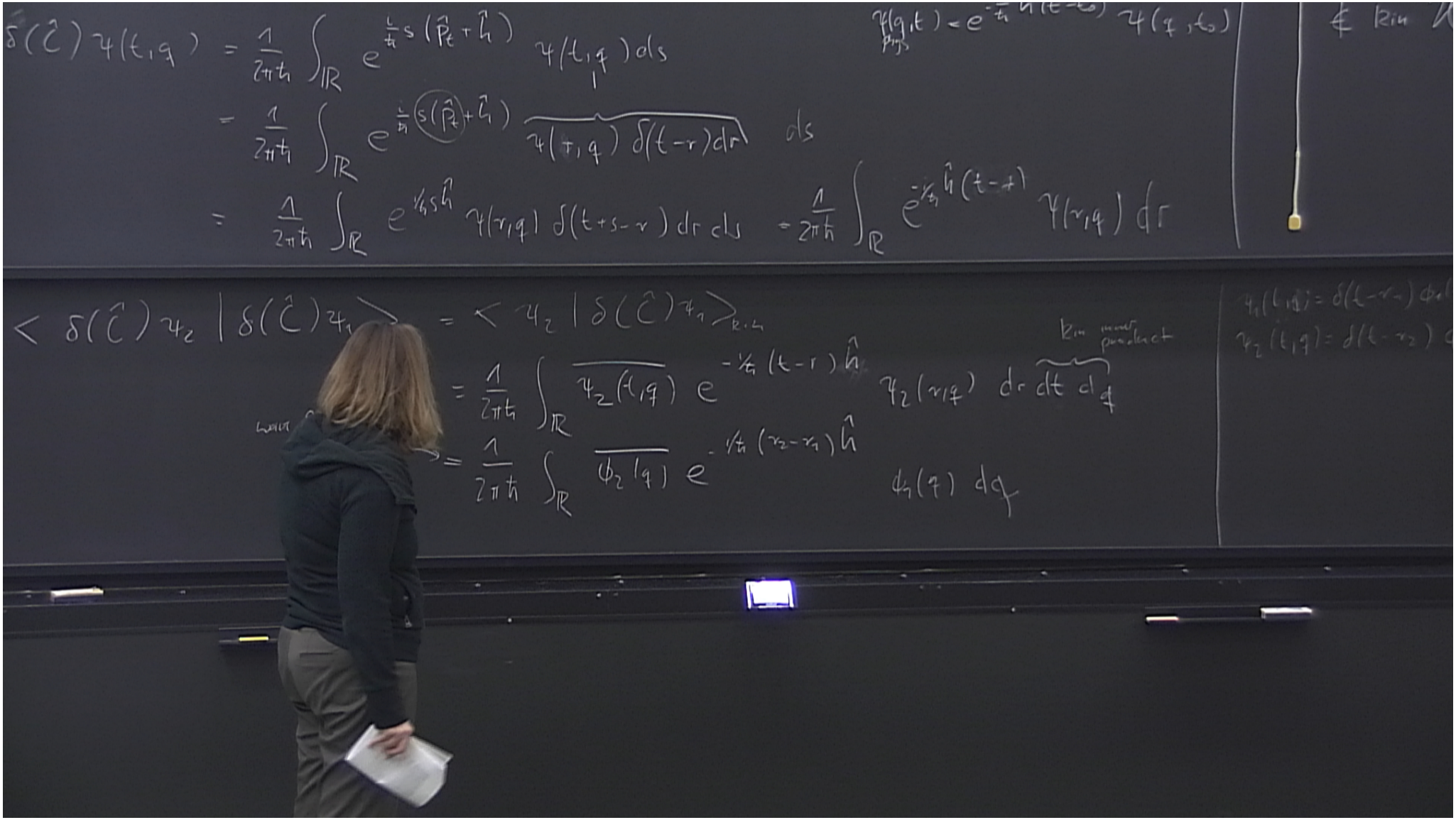

Physical inner product
"$\hat{P} = \delta(\hat{C})$" projector onto constraint(s)
Gauß-constr $\hat{P}^2 = \hat{P}$: compact gauge orbits, $\hat{P}$ s.a.
Flatness : non-compact $\simeq \mathbb{R}^3$
Reparametrized particle : non-compact $\simeq \mathbb{R}$ $\Rightarrow$ $\hat{P}^2 = (\infty)\,\hat{P}$
$\langle \hat{P}\psi_1, \hat{P}\psi_2 \rangle = \langle \psi_1,$

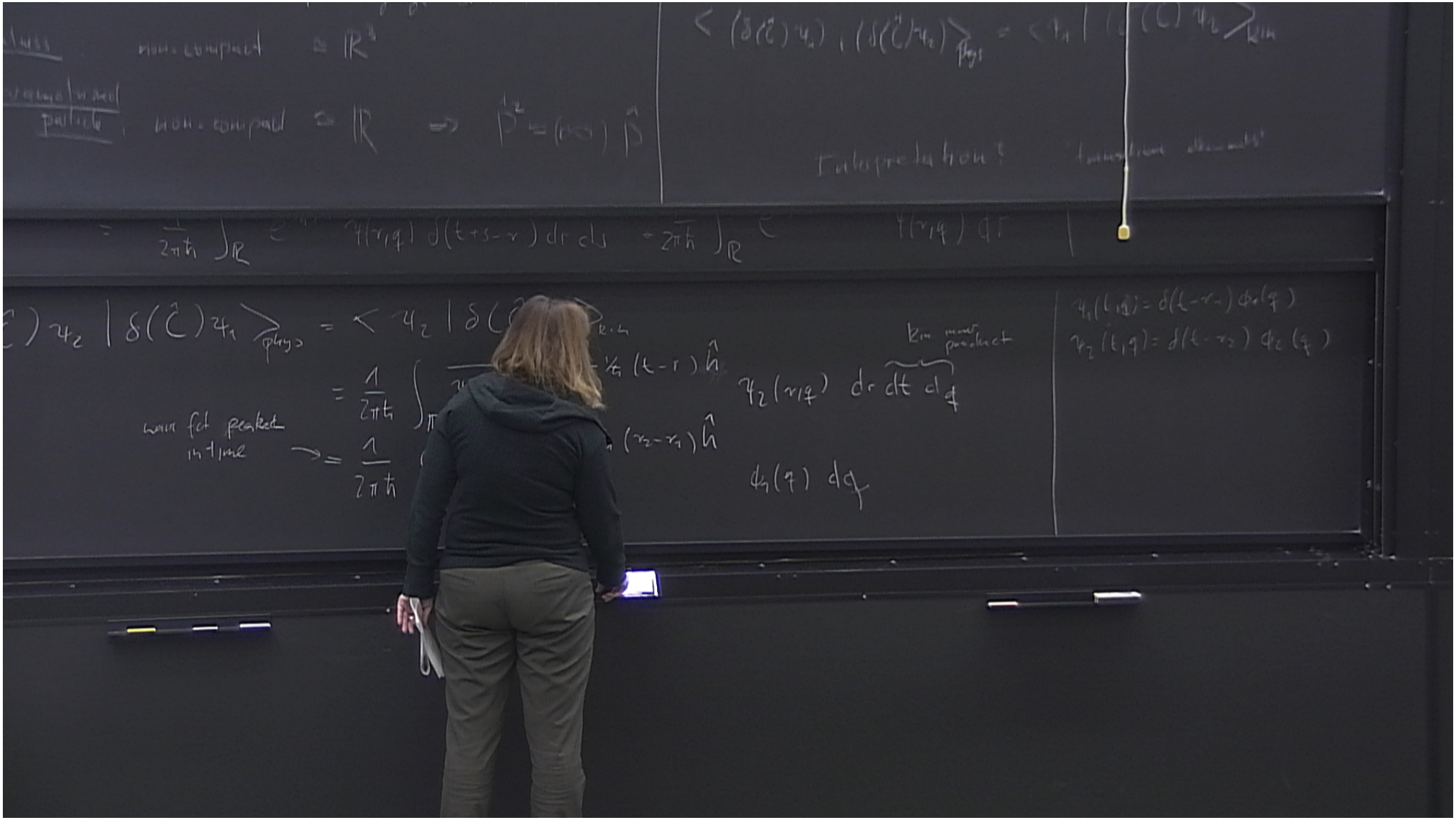

onto constraint(s)
orbits, $\hat{P}$ s.a.
$\Rightarrow \hat{P}^2 = (\infty)\,\hat{P}$
$\langle \hat{P}\psi_1, \hat{P}\psi_2 \rangle_{kin} = \langle \psi_1, \hat{P}^2\psi_2 \rangle_{kin} = \langle \psi_1, \hat{P}\psi_2 \rangle_{kin}$
$\psi_{1(phys)}$
$\psi_{2(phys)}$
physical inner product
$\langle (\delta(\vec{C})\psi_1), (\delta(\vec{C})\psi_2) \rangle_{phys} = \langle \psi_1 | (\delta(\vec{C})\psi_2 \rangle_{kin}$
Interpretation?
'transition elements'

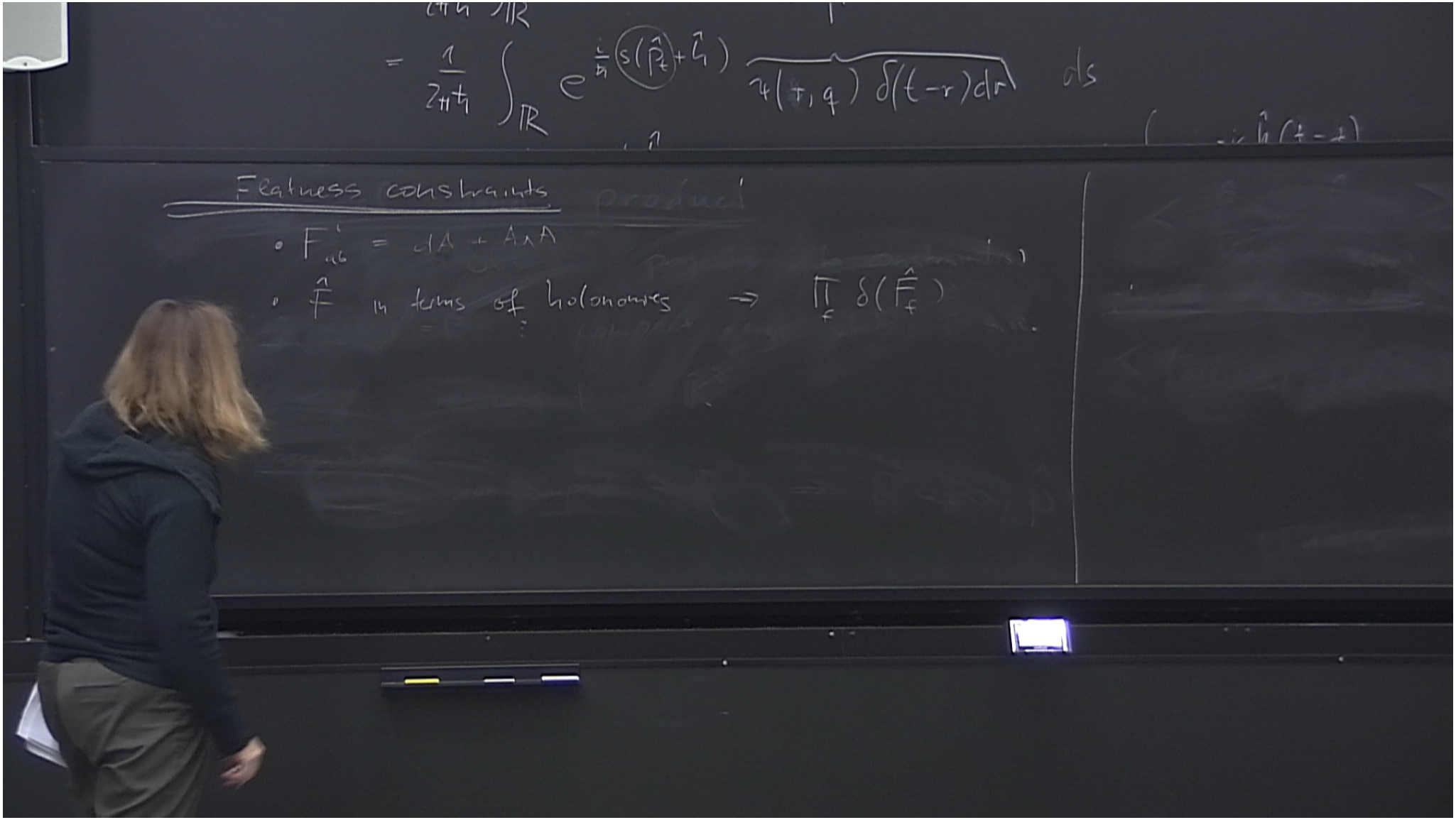

Parametrized particle: non-compact $\simeq \mathbb{R} \Rightarrow \hat{P}^2 = (\infty)\,\hat{P}$
Interpretation?
$\hat{C} = \hat{P}_t + \hat{H}$, t, q / P_t, P_q
$e^{\frac{i}{\hbar} s \hat{P}_t} \psi(q) = \psi(q+s) = \sum_{n=0}^{\infty} \frac{(is)^n}{n!} \left(\frac{t}{\hbar}\right)^n \frac{d^n}{dq^n} \psi(q)$
$\psi^{(n)}(q)$
$\delta(\hat{C})\,\psi(t,q) = \frac{1}{2\pi\hbar} \int_{\mathbb{R}} e^{\frac{i}{\hbar} s (\hat{P}_t + \hat{H})}\, \psi(t,q)\, ds$
$= \frac{1}{2\pi\hbar} \int_{\mathbb{R}} e^{\frac{i}{\hbar} s (\hat{P}_t + \hat{H})}\, \psi(\tau,q)\, \delta(t-\tau)\, d\tau$
$= \frac{1}{2\pi\hbar} \int_{\mathbb{R}} e^{\frac{i}{\hbar} s \hat{H}}\, \psi(\tau,q)\, \delta(t+s-\tau)\, d\tau\, ds$
$(t-\tau)$
$\psi(\tau,q)\, d\tau$


Parametrized particle: non-compact $\simeq \mathbb{R} \Rightarrow \hat{p}^2 = (\infty)\,\hat{p}$
Interpretation?
$\hat{C} = \hat{p}_t + \hat{h}$, t, q / p_t, p_q
$e^{\frac{i}{\hbar} s \hat{p}_q} \psi(q) = \psi(q+s) = \sum_{n=0}^{\infty} \frac{(is)^n}{n!} \frac{(i\hbar)^n}{\hbar^n} \frac{d^n}{dq^n} \psi(q)$
$\hat{C})\psi(t,q) = \frac{1}{2\pi\hbar} \int_{\mathbb{R}} e^{\frac{i}{\hbar} s (\hat{p}_t + \hat{h})} \psi(t,q)\, ds$
$= \frac{1}{2\pi\hbar} \int_{\mathbb{R}} e^{\frac{i}{\hbar} s (\hat{p}_t + \hat{h})} \psi(\tau,q)\, \delta(t-\tau)\, d\tau\, ds$
$= \frac{1}{2\pi\hbar} \int_{\mathbb{R}} e^{\frac{i}{\hbar} s \hat{h}} \psi(\tau,q)\, \delta(t+s-\tau)\, d\tau\, ds = \frac{1}{2\pi\hbar} \int_{\mathbb{R}}$


kin inner product


$\delta(\hat{C})\psi(t,q) = \frac{1}{2\pi\hbar}\int_{\mathbb{R}} e^{\frac{i}{\hbar}s(\hat{P}_t+\hat{h})}\,\psi(t,q)\,ds$
$= \frac{1}{2\pi\hbar}\int_{\mathbb{R}} e^{\frac{i}{\hbar}s(\hat{P}_t+\hat{h})}\,\psi(\tau,q)\,\delta(t-\tau)\,d\tau\,ds$
$= \frac{1}{2\pi\hbar}\int_{\mathbb{R}} e^{i/\hbar s\hat{h}}\,\psi(\tau,q)\,\delta(t+s-\tau)\,d\tau\,ds = \frac{1}{2\pi\hbar}\int_{\mathbb{R}} e^{-i/\hbar\hat{h}(t-\tau)}\,\psi(\tau,q)\,d\tau$
$\psi(q,t) = e^{-\frac{i}{\hbar}\hat{h}(t-t_0)}\,\psi(q,t_0)$
$\in$ kin
$\langle \delta(\hat{C})\psi_2 \mid \delta(\hat{C})\psi_1\rangle = \langle \psi_2 \mid \delta(\hat{C})\psi_1\rangle_{kin}$
$= \frac{1}{2\pi\hbar}\int_{\mathbb{R}} \overline{\psi_2(t,q)}\, e^{-i/\hbar(t-\tau)\hat{h}}\,\psi_2(\tau,q)\,d\tau\,\overbrace{dt\,dq}^{\text{kin inner product}}$
$= \frac{1}{2\pi\hbar}\int_{\mathbb{R}} \overline{\phi_2(q)}\, e^{-i/\hbar(\tau_2-\tau_1)\hat{h}}\,\phi_1(q)\,dq$
$\psi_1(t,q) = \delta(t-\tau_1)\,\phi_1$
$\psi_2(t,q) = \delta(t-\tau_2)\,\phi_2$


non-compact
particle
non-compact
Interpretation
kin. inner product
wave fct peaked in time

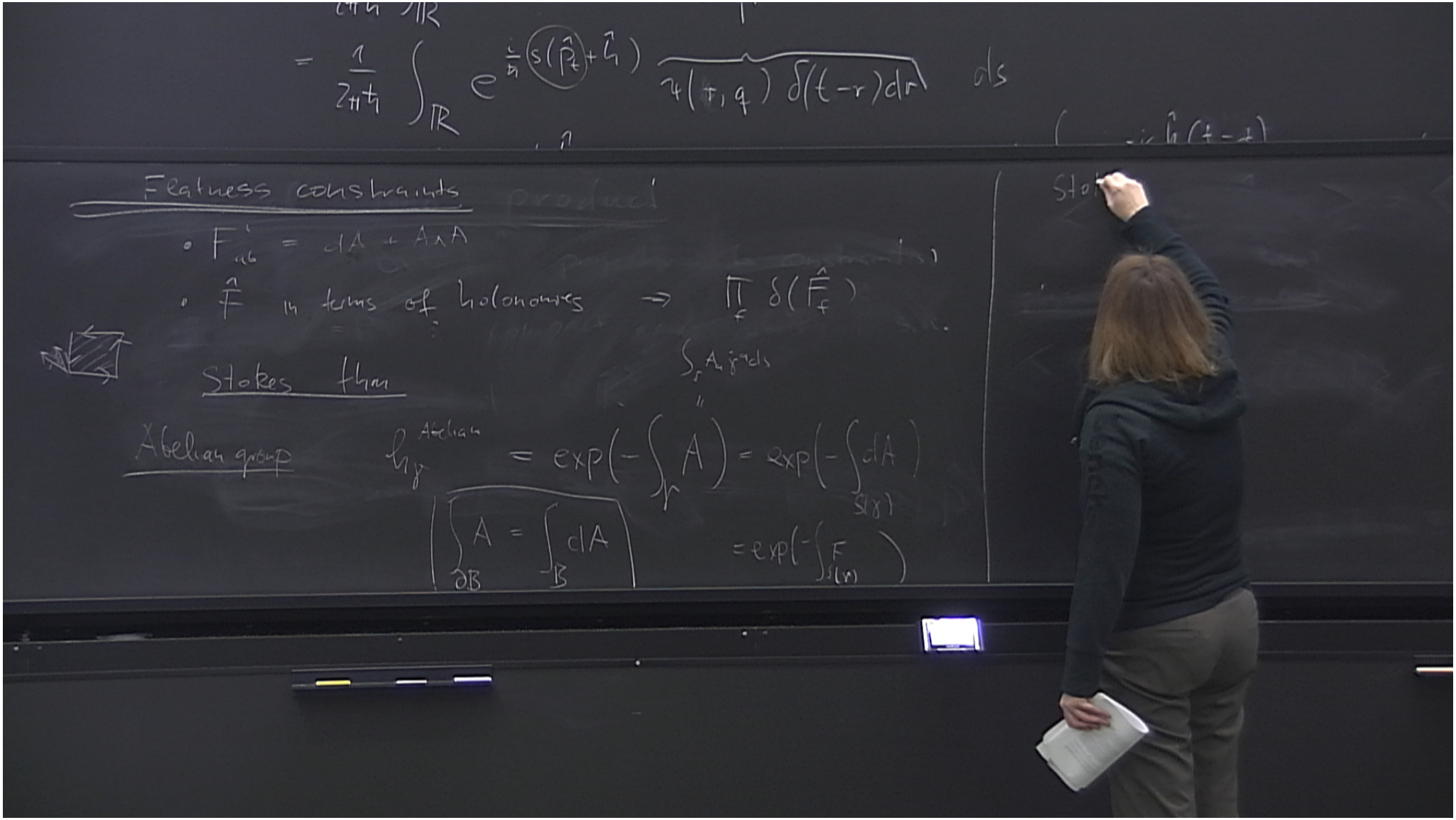

$= \frac{1}{Z_{\Gamma} t_{\gamma}} \int_{\mathbb{R}} e^{\frac{i}{\hbar} S(\hat{p}_t + \zeta_t)} \overbrace{\eta(\tau, q)\, \delta(t-\tau) d\tau}\, ds$
Flatness constraints
• $F^i_{ab} = dA + A \wedge A$
• $\hat{F}$ in terms of holonomies $\Rightarrow$ $\prod_f \delta(\hat{F}_f)$

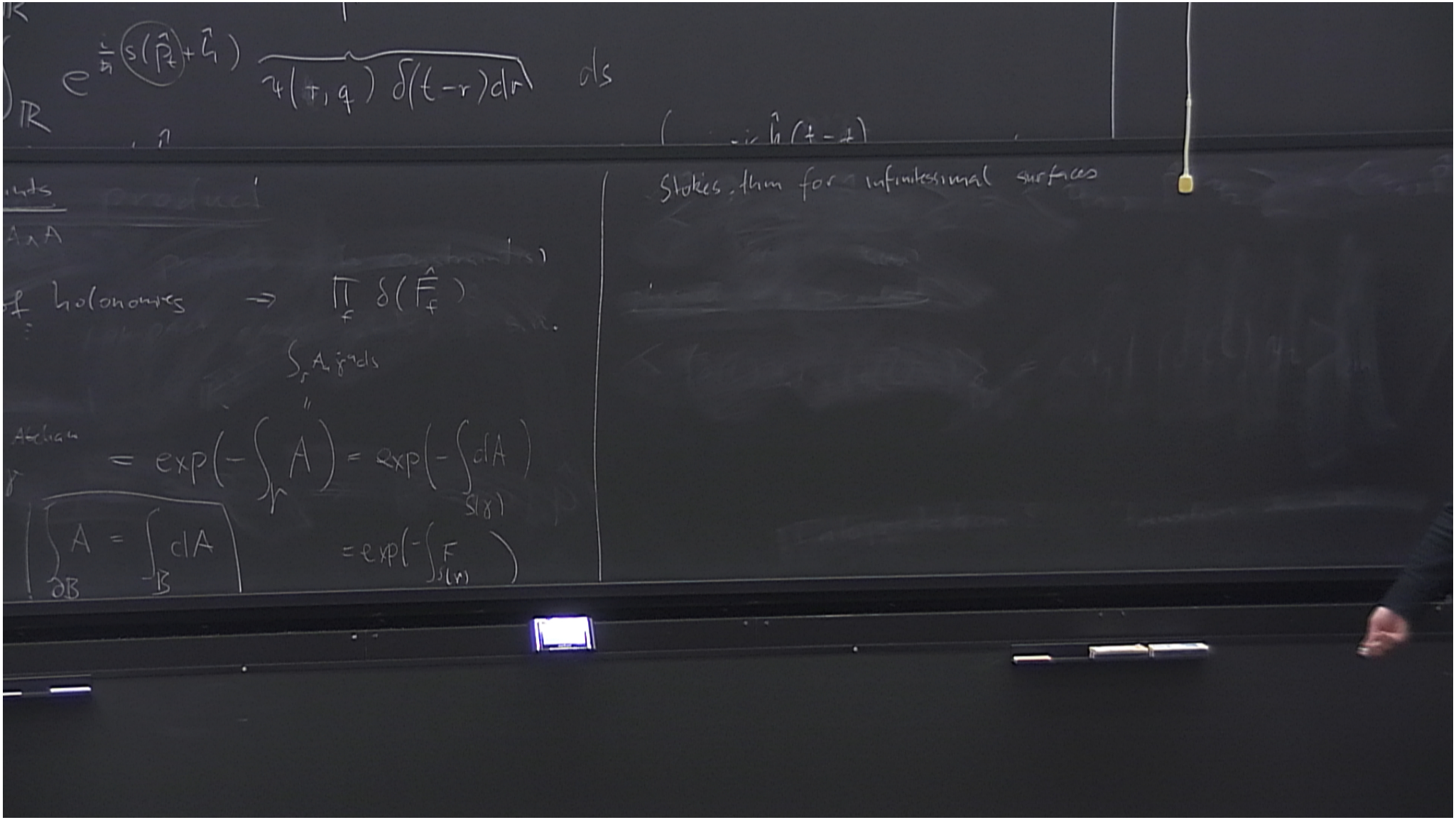
$$= \frac{1}{Z_H Z_q} \int_{\mathbb{R}} e^{\frac{i}{\hbar} S(\hat{p}_t + \hat{z}_t)} \overbrace{\eta(\tau, q)\, \delta(t-\tau) d\tau}\, ds$$

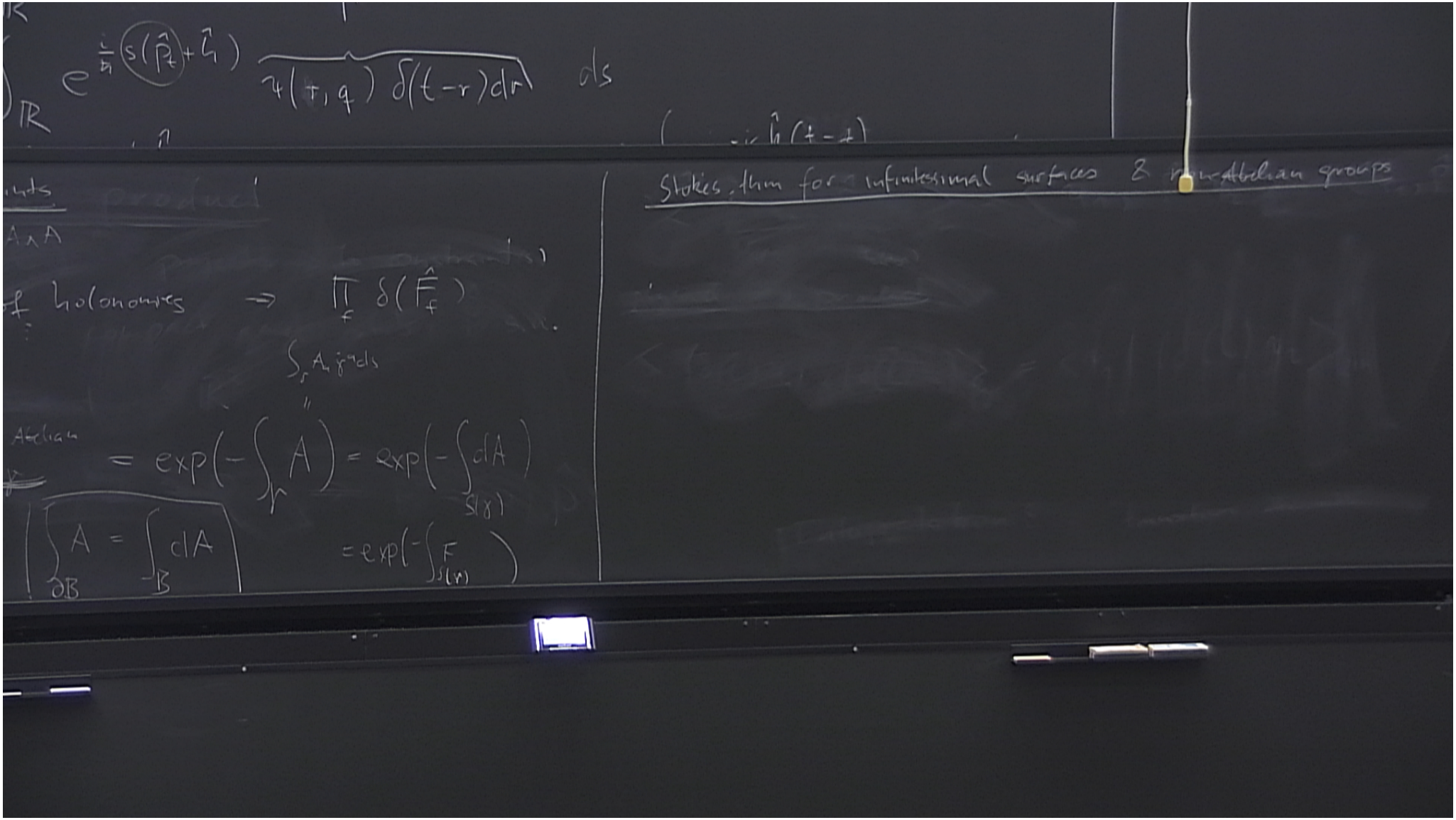
Flatness constraints

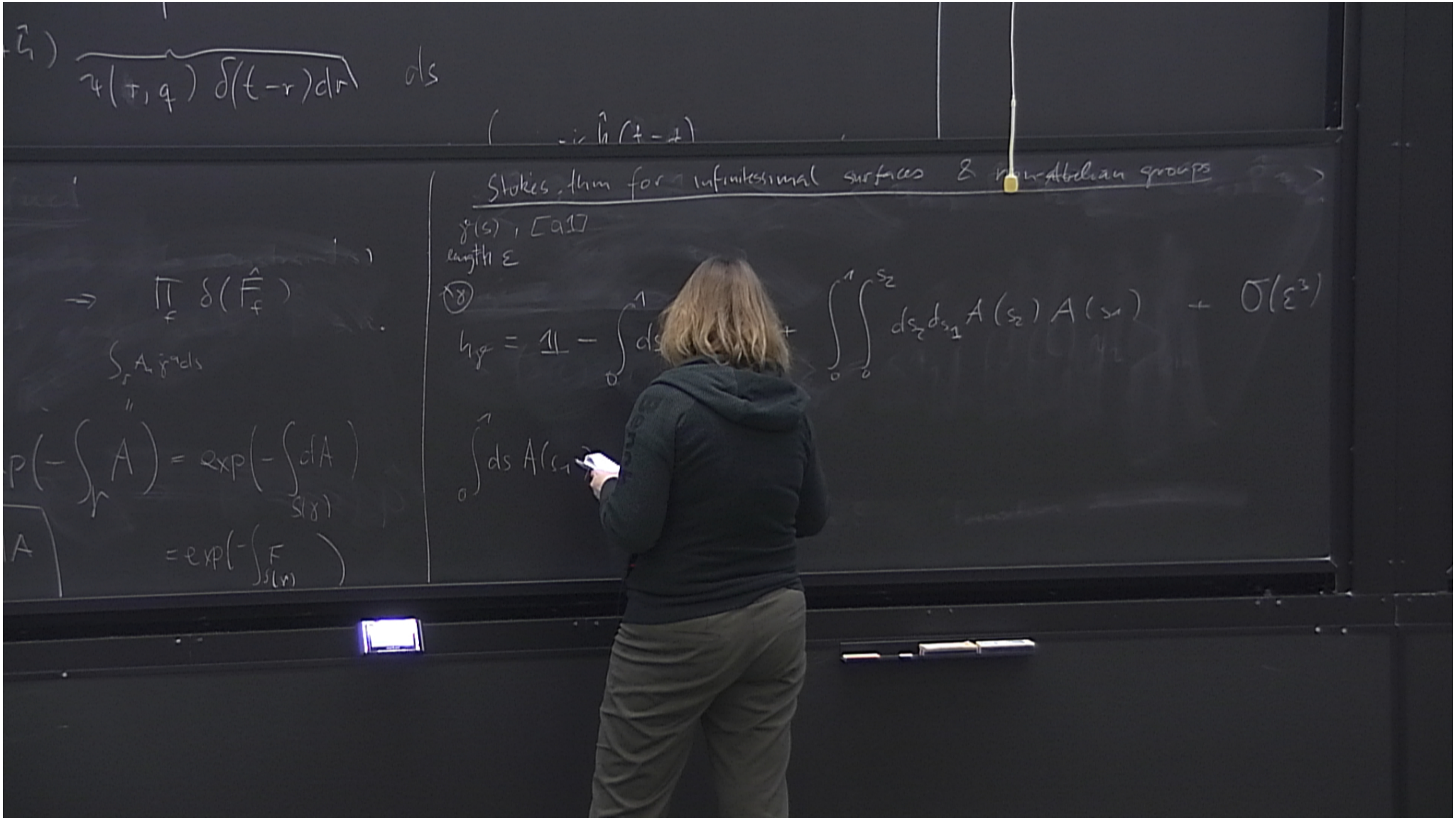
- $F^i_{ab} = dA + A \wedge A$
- $\hat{F}$ in terms of holonomies $\Rightarrow \prod_f \delta(\hat{F}_f)$

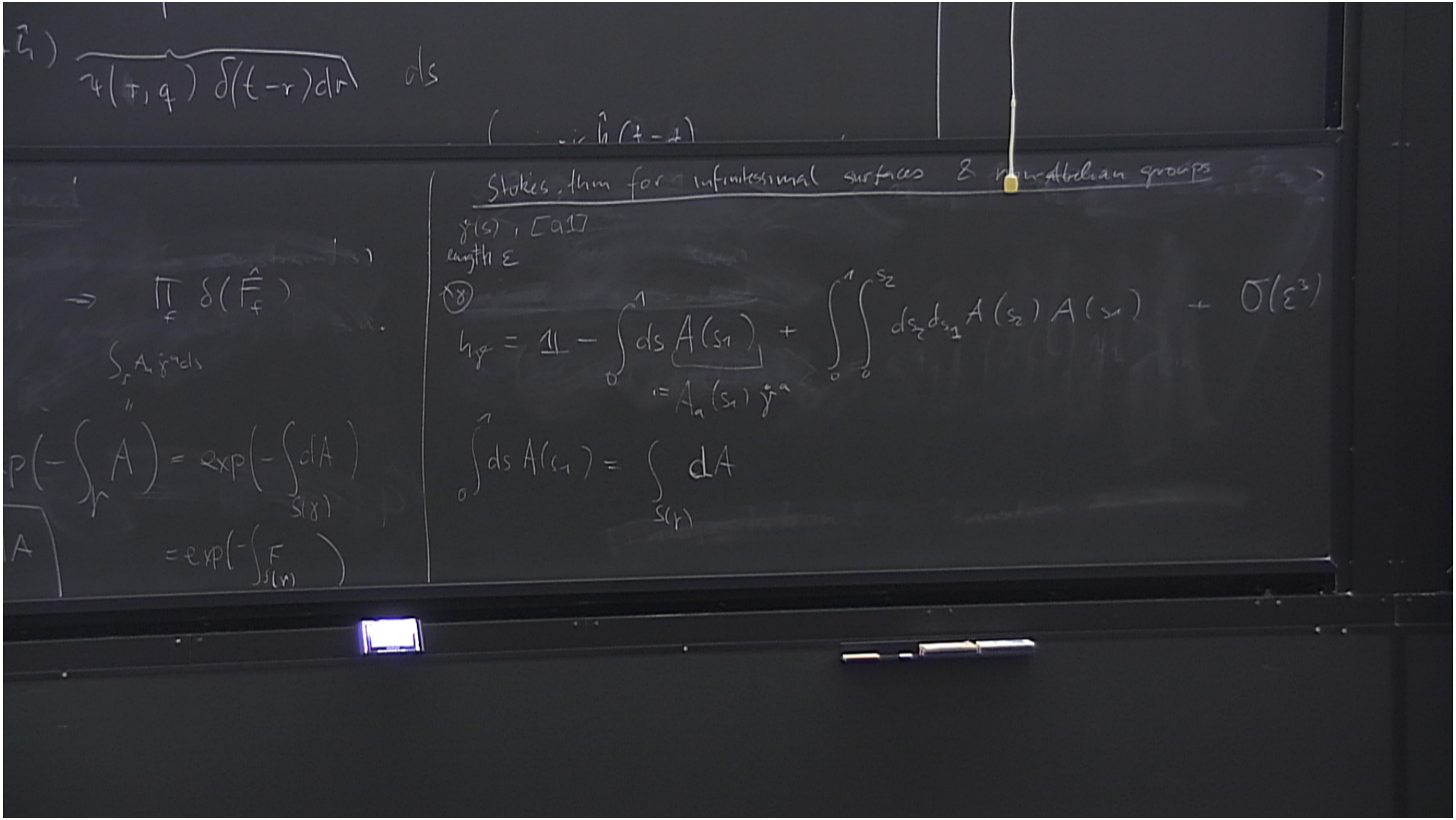
Stokes thm

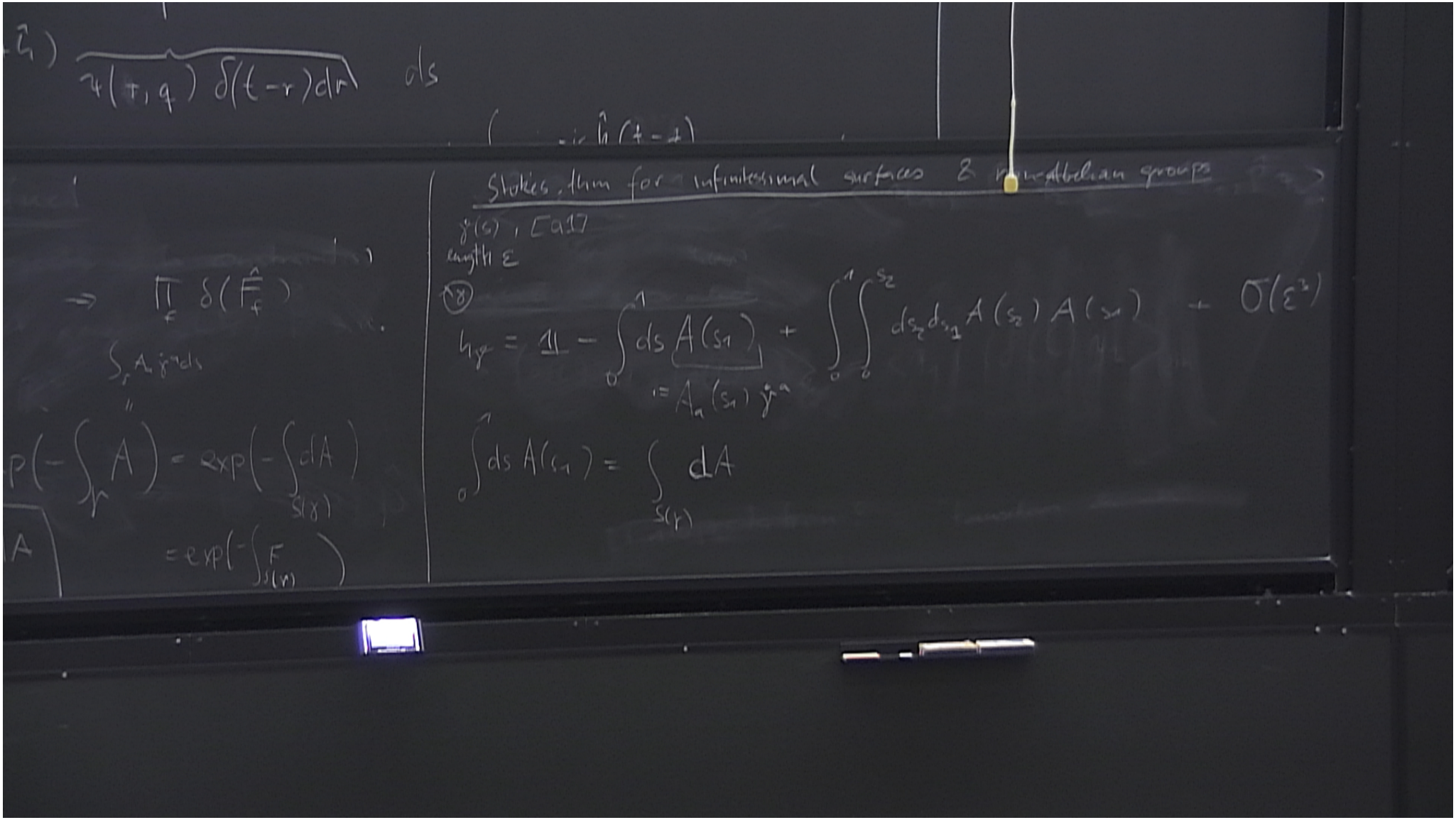

Flatness constraints
$F^i_{ab} = dA + A \wedge A$
$\hat{F}$ in terms of holonomies $\Rightarrow \prod_f \delta(\hat{F}_f)$
Stokes thm
Abelian group
$h_\gamma^{\text{Abelian}} = \exp\left(-\int_\gamma A\right) = \exp\left(-\int_{S(\gamma)} dA\right)$
$\int_{\partial B} A = \int_B dA$
$= \exp\left(-\int_{S(\gamma)} F\right)$


Stokes theorem for infinitesimal surfaces
holonomies
Action


Stokes thm for infinitesimal surfaces & non-Abelian groups
holonomies
Abelian


Stokes thm for infinitesimal surfaces & non-Abelian groups
length ε


Stokes thm for infinitesimal surfaces & non-Abelian groups
length ε

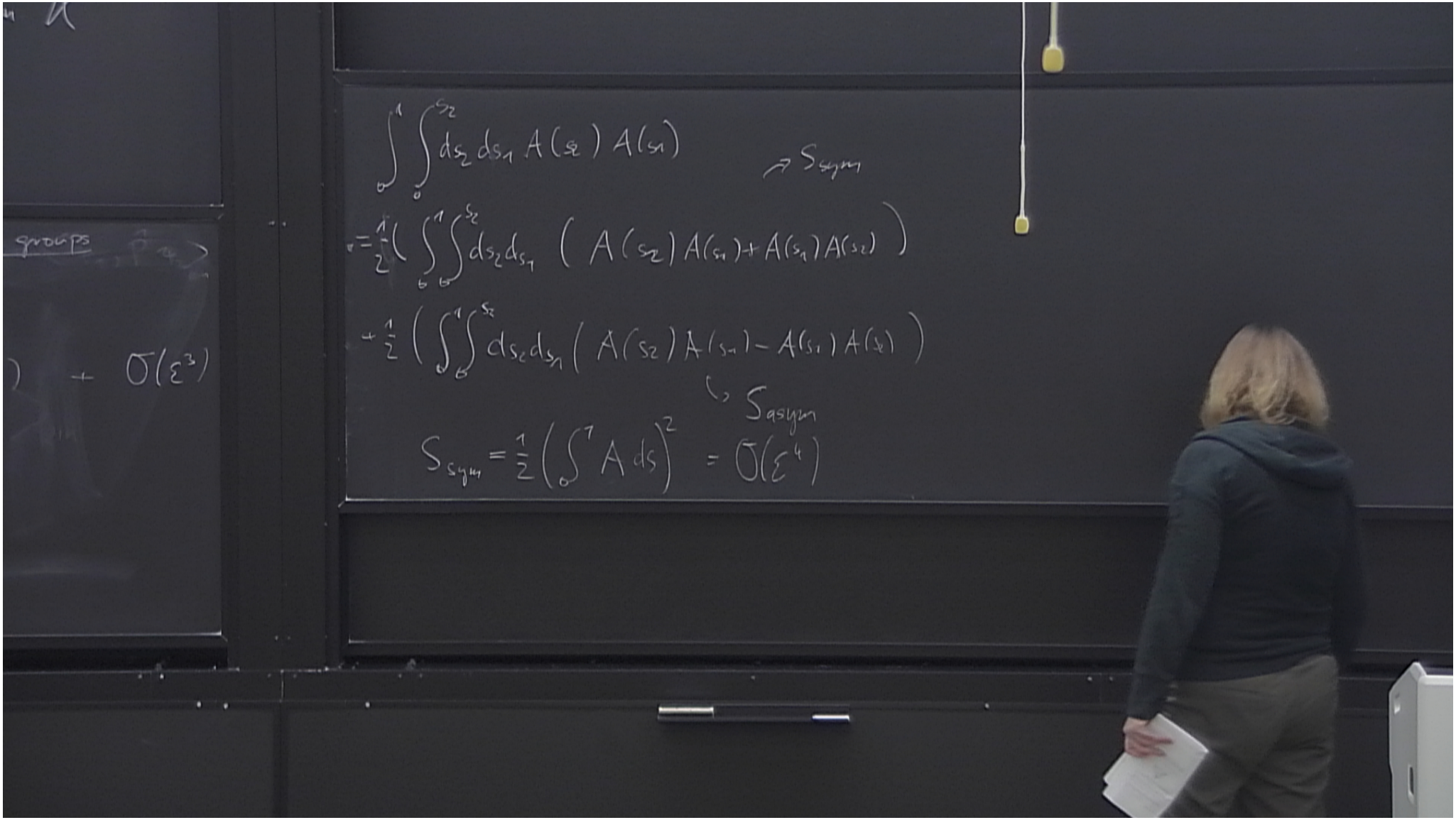

Stokes theorem for infinitesimal surfaces & non-abelian groups
length ε
$\gamma(s)$: [0,1]
$\Rightarrow \prod_f \delta(\hat{F}_f)$
$\int_\gamma A_i \dot{\gamma}^i ds$
$\exp(-\int_\gamma A) = \exp(-\int_{S(\gamma)} dA)$
$= \exp(-\int_{S(\gamma)} F)$
$U_\gamma = \mathbb{1} - \int_0^1 ds\, A(s_1) + \int_0^1 \int_0^{s_2} ds_2 ds_1 A(s_2) A(s_1) + \mathcal{O}(\varepsilon^3)$
$:= A_a(s_1)\dot{\gamma}^a$
$\int_0^1 ds\, A(s_1) = \int_{S(\gamma)} dA$

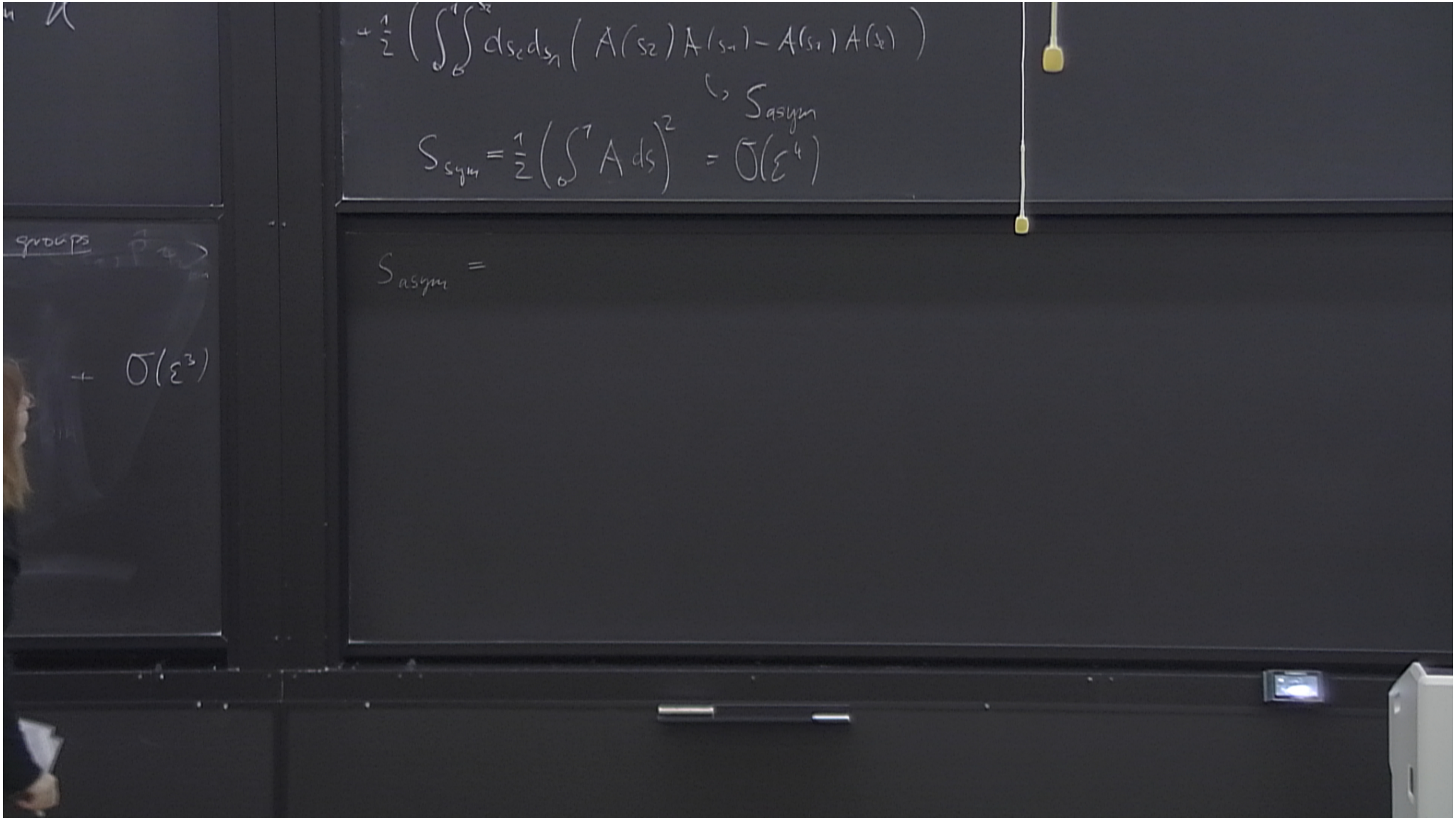
$$\int_0^1 \int_0^{s_2} ds_2\, ds_1\, A(s_2)\, A(s_1) \qquad \rightarrow S_{sym}$$

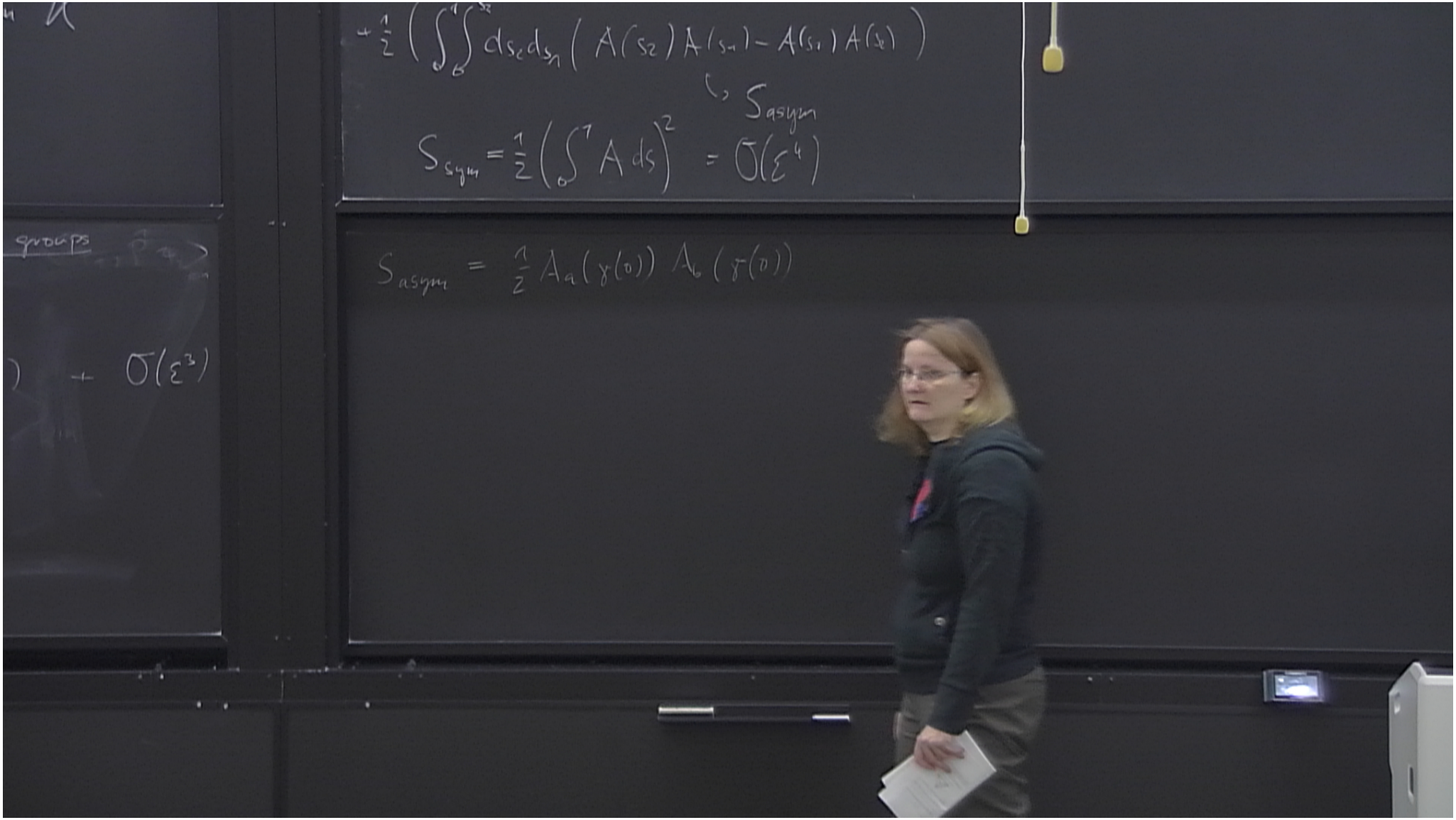
$$= \frac{1}{2}\left(\int_0^1 \int_0^{s_2} ds_2\, ds_1 \left(A(s_2) A(s_1) + A(s_1) A(s_2) \right) \right.$$

$$\left. + \frac{1}{2} \left(\int_0^1 \int_0^{s_2} ds_2\, ds_1 \left(A(s_2) A(s_1) - A(s_1) A(s_2) \right) \right. \right.$$

$$\hookrightarrow S_{asym}$$


groups
$\int_0^1 \int_0^{s_2} ds_2 ds_1 A(s_2) A(s_1)$
$\rightarrow S_{sym}$
$= \frac{1}{2}\left(\int_0^1 \int_0^{s_2} ds_2 ds_1 \left(A(s_2)A(s_1) + A(s_1)A(s_2) \right) \right.$
$+ \frac{1}{2}\left(\int_0^1 \int_0^{s_2} ds_2 ds_1 \left(A(s_2)A(s_1) - A(s_1)A(s_2) \right) \right.$
S_{asym}
$S_{sym} = \frac{1}{2}\left(\int_0^1 A\, ds \right)^2 = \mathcal{O}(\varepsilon^4)$
$+ \mathcal{O}(\varepsilon^3)$


groups
$+ \frac{1}{2}\left(\int_0^1\int_0^{s_2} ds_2 ds_1 \left(A(s_2)A(s_1) - A(s_1)A(s_2)\right)\right)$
$\hookrightarrow S_{asym}$
$S_{sym} = \frac{1}{2}\left(\int_0^1 A\, ds\right)^2 = \mathcal{O}(\varepsilon^4)$
$S_{asym} =$
$+ \mathcal{O}(\varepsilon^3)$


groups
$+ \frac{1}{2}\left(\int_0^1\int_0^{s_2} ds_2 ds_1 \left(A(s_2)A(s_1) - A(s_1)A(s_2)\right)\right.$
$\hookrightarrow S_{asym}$
$S_{sym} = \frac{1}{2}\left(\int_0^1 A\, ds\right)^2 = \mathcal{O}(\varepsilon^4)$
$S_{asym} = \frac{1}{2} A_a(\gamma(0))\, A_b(\gamma(0))$
$+ \mathcal{O}(\varepsilon^3)$

$$+\frac{1}{2}\left(\int_0^1\int_0^{s_2} ds_2 ds_1 \left(A(s_2)A(s_1) - A(s_1)A(s_2)\right)\right.$$

$\hookrightarrow S_{asym}$

$$S_{sym} = \frac{1}{2}\left(\int_0^1 A\,ds\right)^2 = \mathcal{O}(\epsilon^4)$$

$$S_{asym} = \frac{1}{2} A_a(\gamma(0))\, A_b(\gamma(0)) \int_0^1 ds_2 \int_0^{s_2} ds_1 \left(\dot{\gamma}^a(s_2)\dot{\gamma}^b(s_1) - \dot{\gamma}^a(s_1)\dot{\gamma}^b(s_2)\right) + \mathcal{O}(\epsilon^3)$$

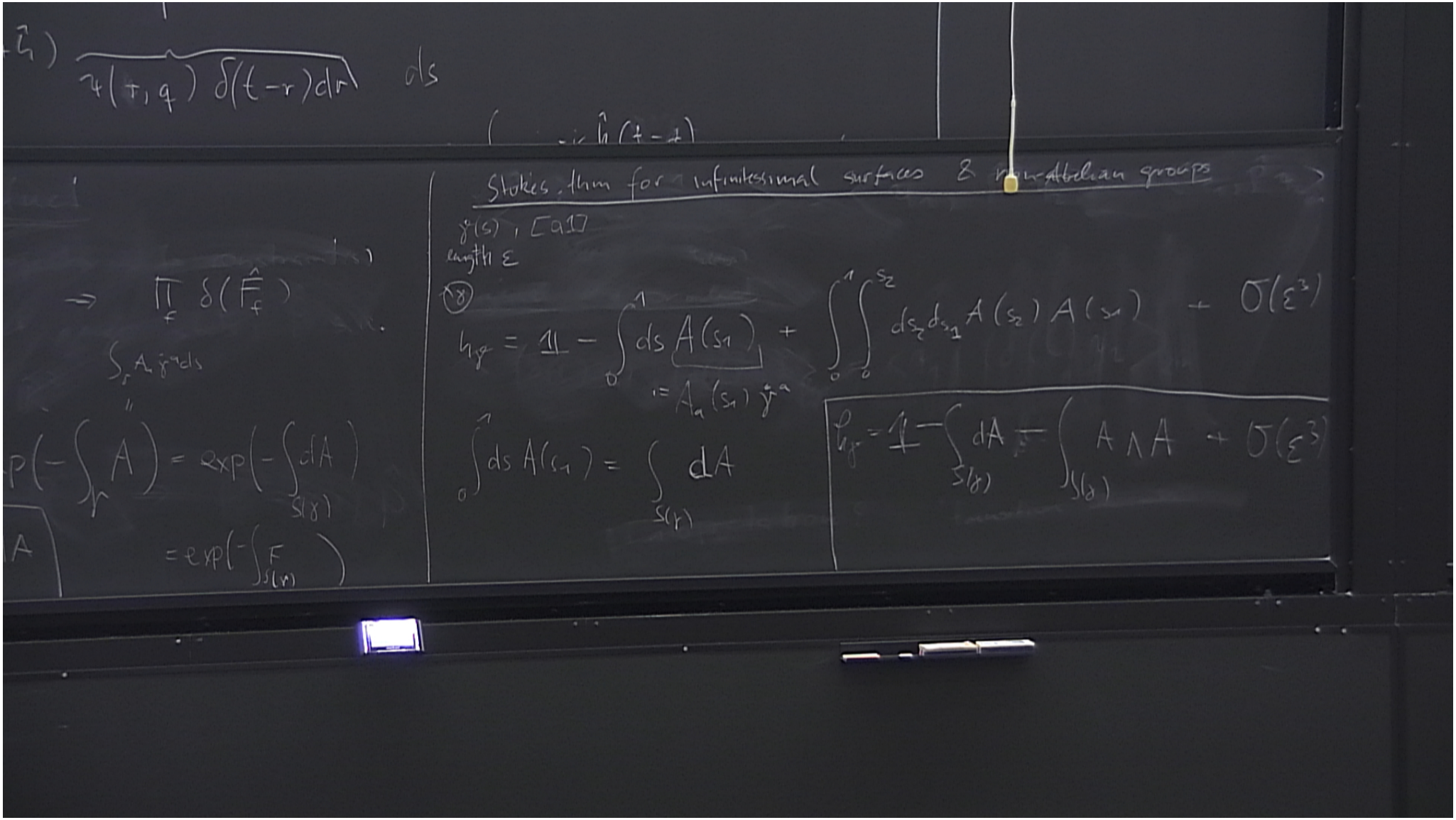
$$S_{asym} = -\int_{S(\gamma)} A \wedge A + \mathcal{O}(\epsilon^3)$$

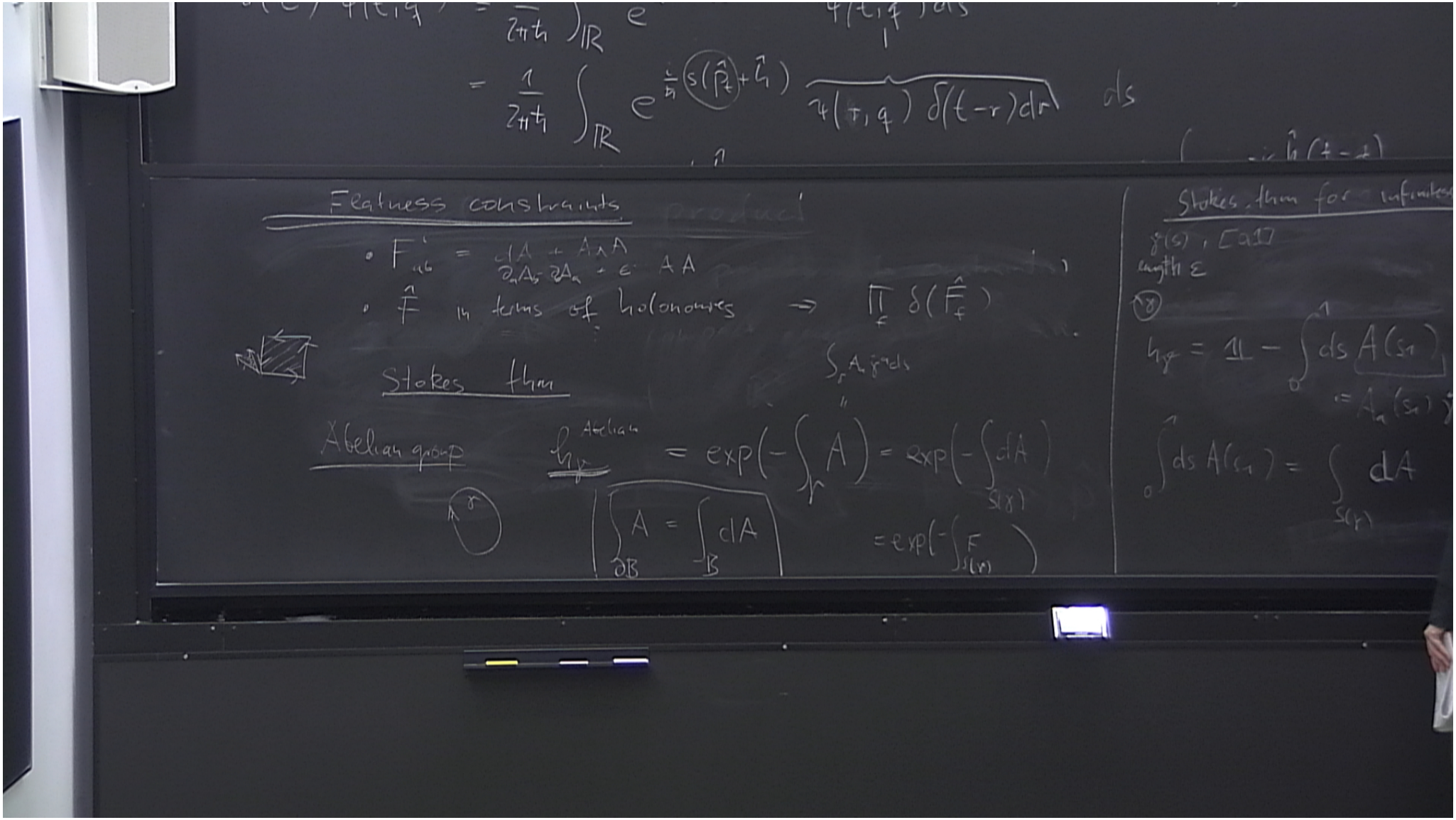
$$\int_0^1 ds \left(\frac{d\gamma^a}{ds}\gamma^b(s) - \frac{d\gamma^b}{ds}\gamma^a(s)\right)$$

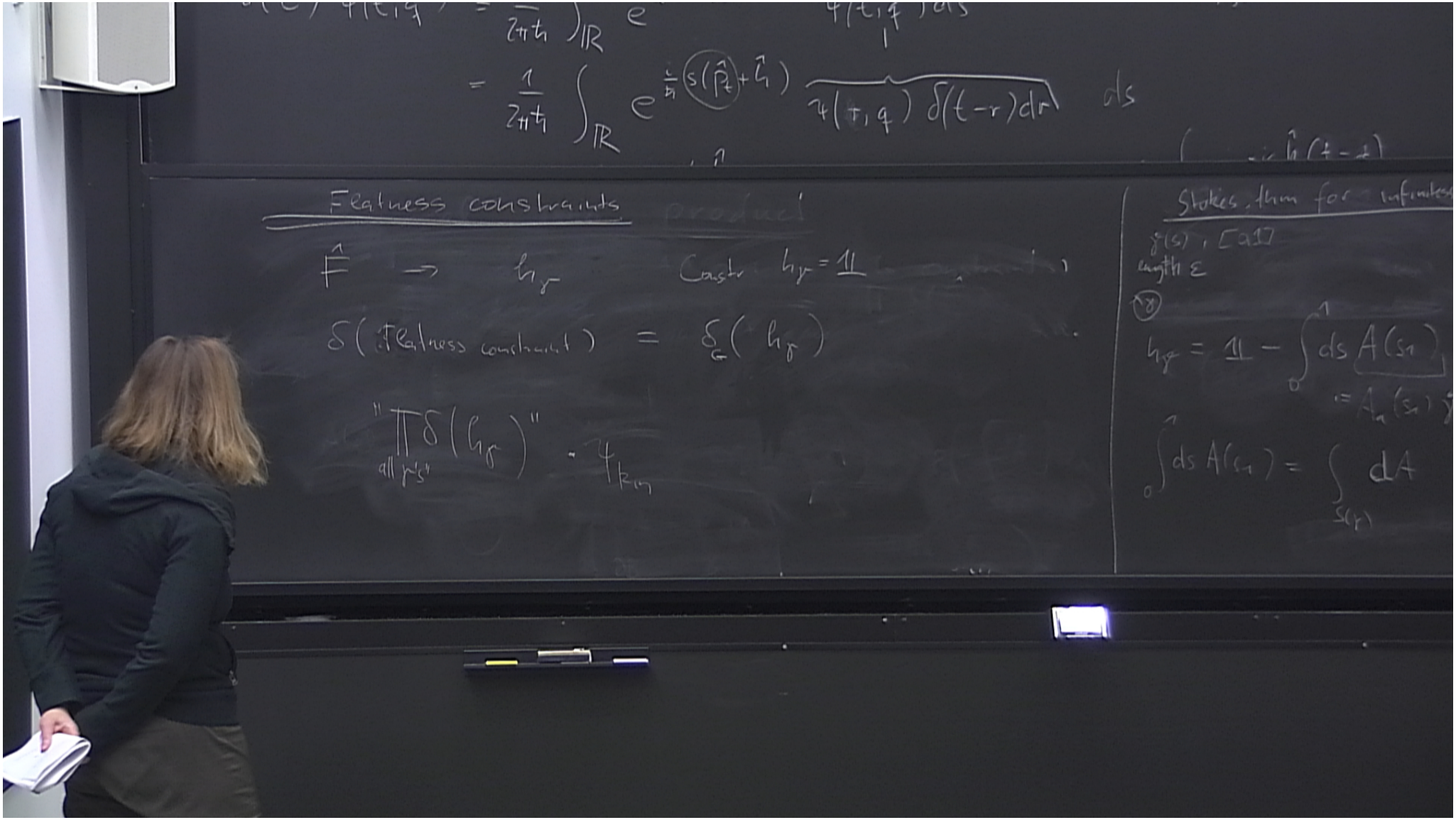
$$= \int_\gamma x^b dx^a - x^a dx^b$$

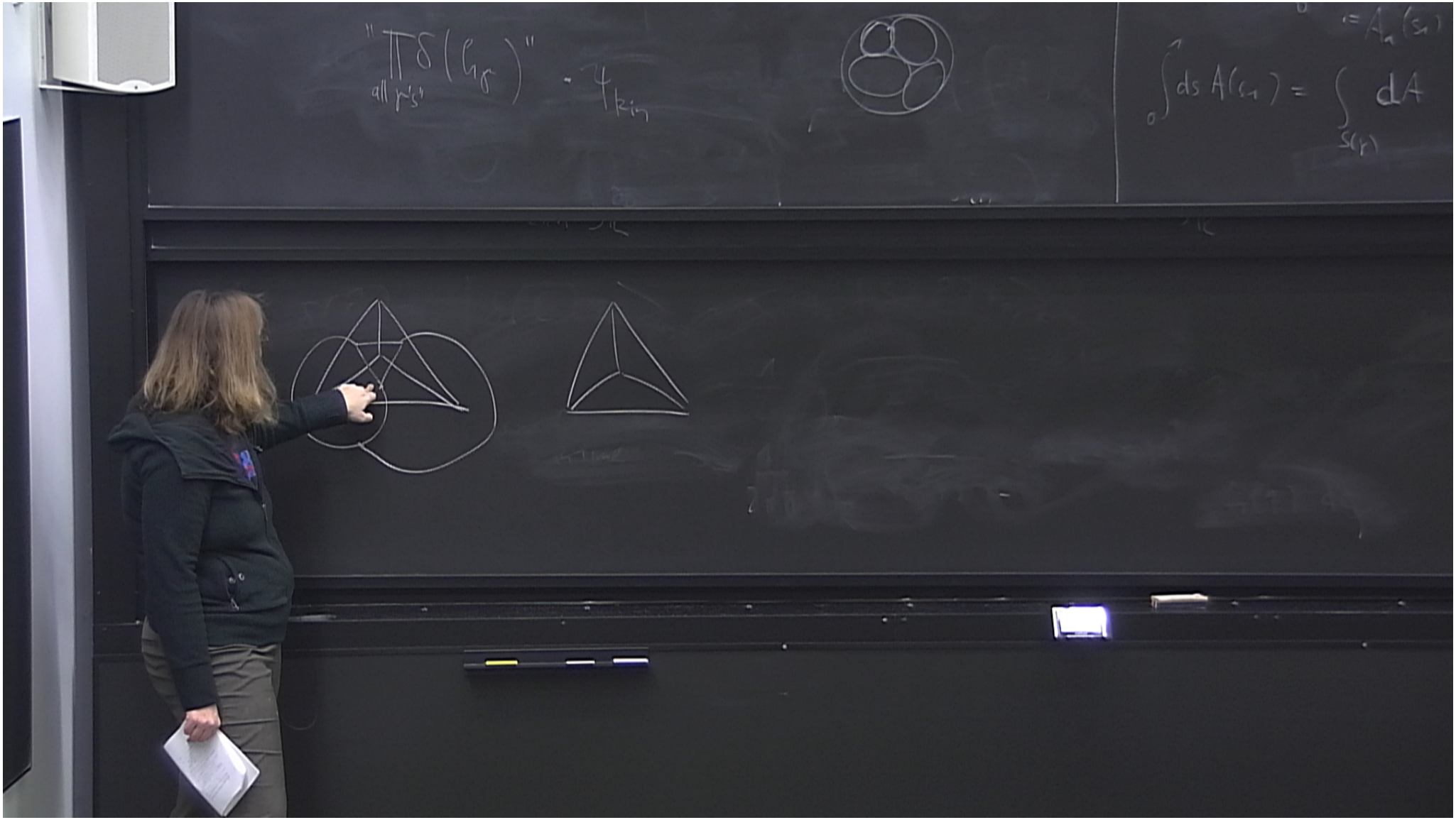
$$= \int_{S(\gamma)} d\left(x^b dx^a - x^a dx^b\right)$$

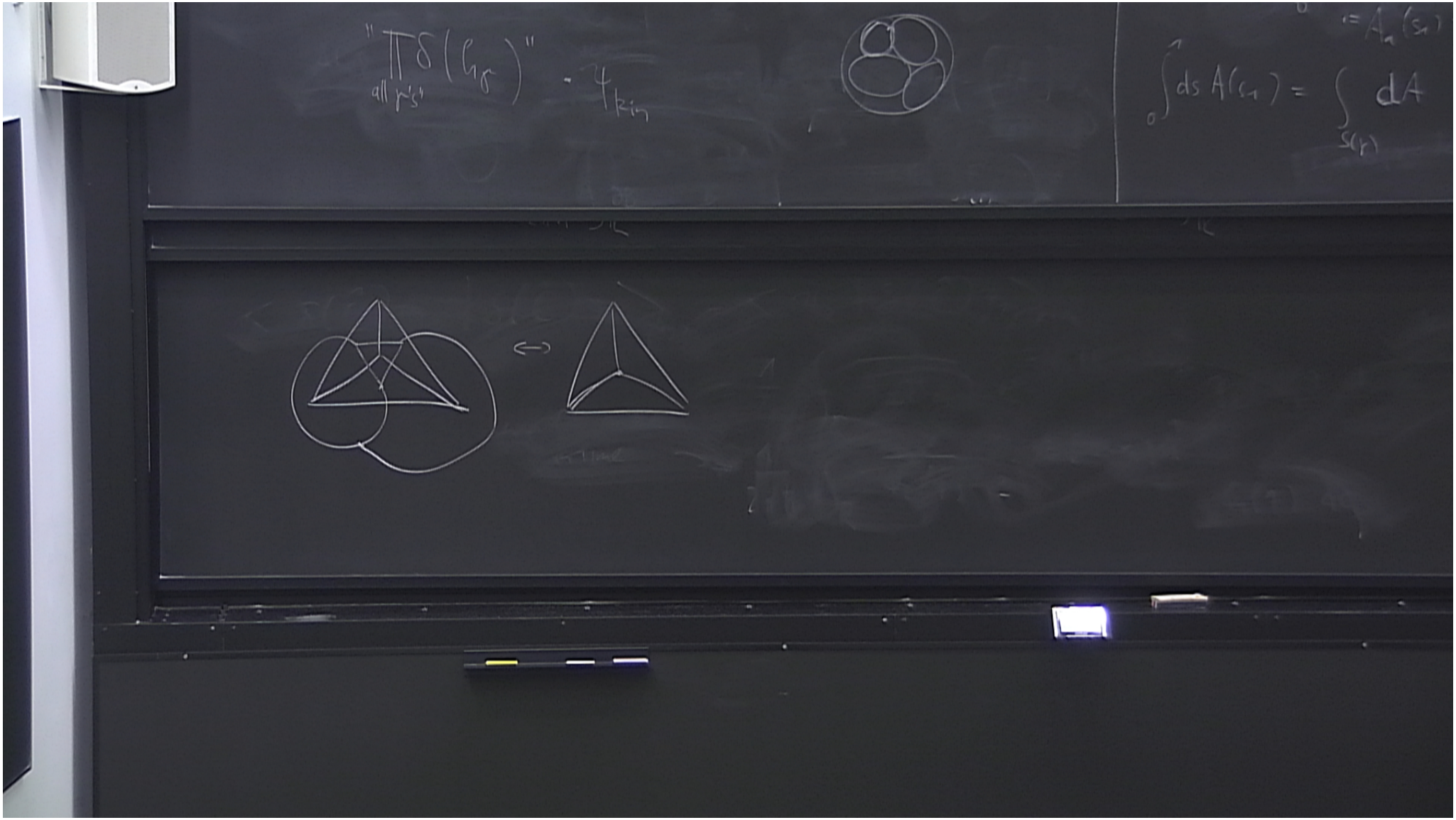
$$= -2\int_{S(\gamma)} dx^a \wedge dx^b$$


Stokes thm for infinitesimal surfaces & non-Abelian groups
length ε


Flatness constraints
Stokes thm
Abelian group
Stokes thm for infinitesimal


Flatness constraints
Stokes' thm for infinitesimal


"∏ δ(ρ_σ)"
all σ's
· 4_kin
∫ ds A(s_1) = ∫ dA
S(s_1)


"∏ δ(C_σ)"
all σ's
· 4_kin
= A_n(s_1)
∫ ds A(s_1) = ∫ dA
S(s_1)