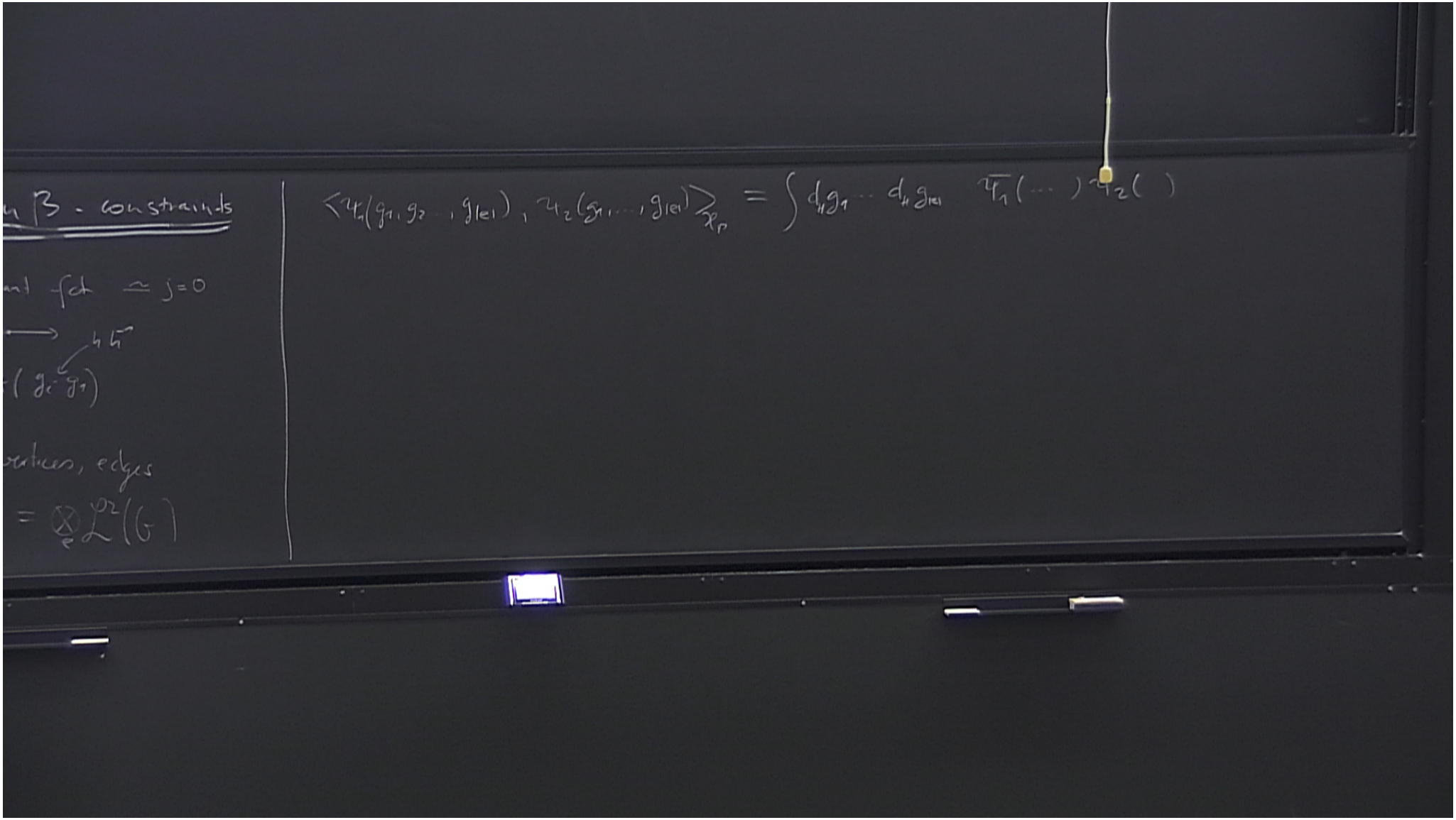
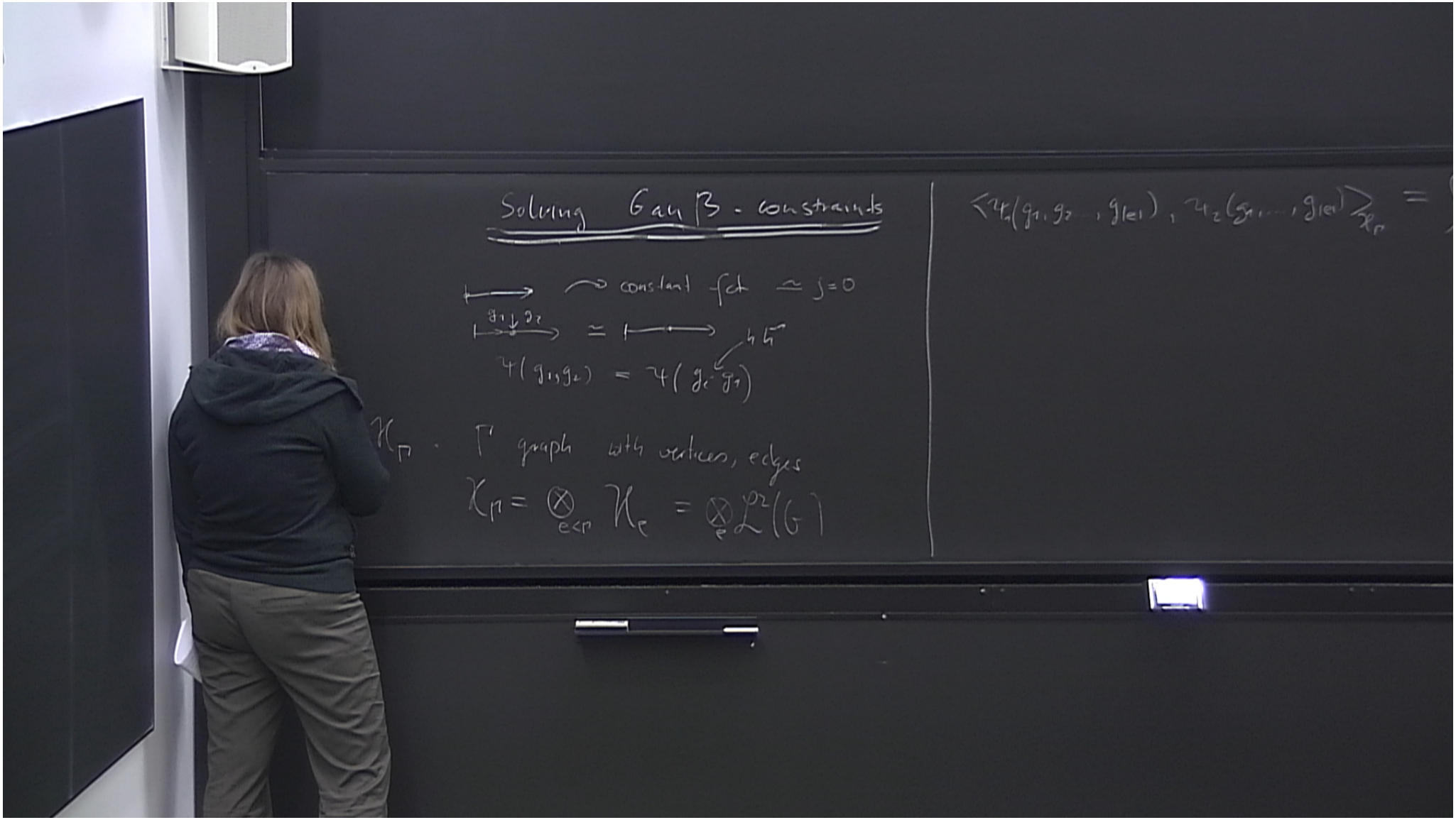
Title: Quantum Gravity Review-11

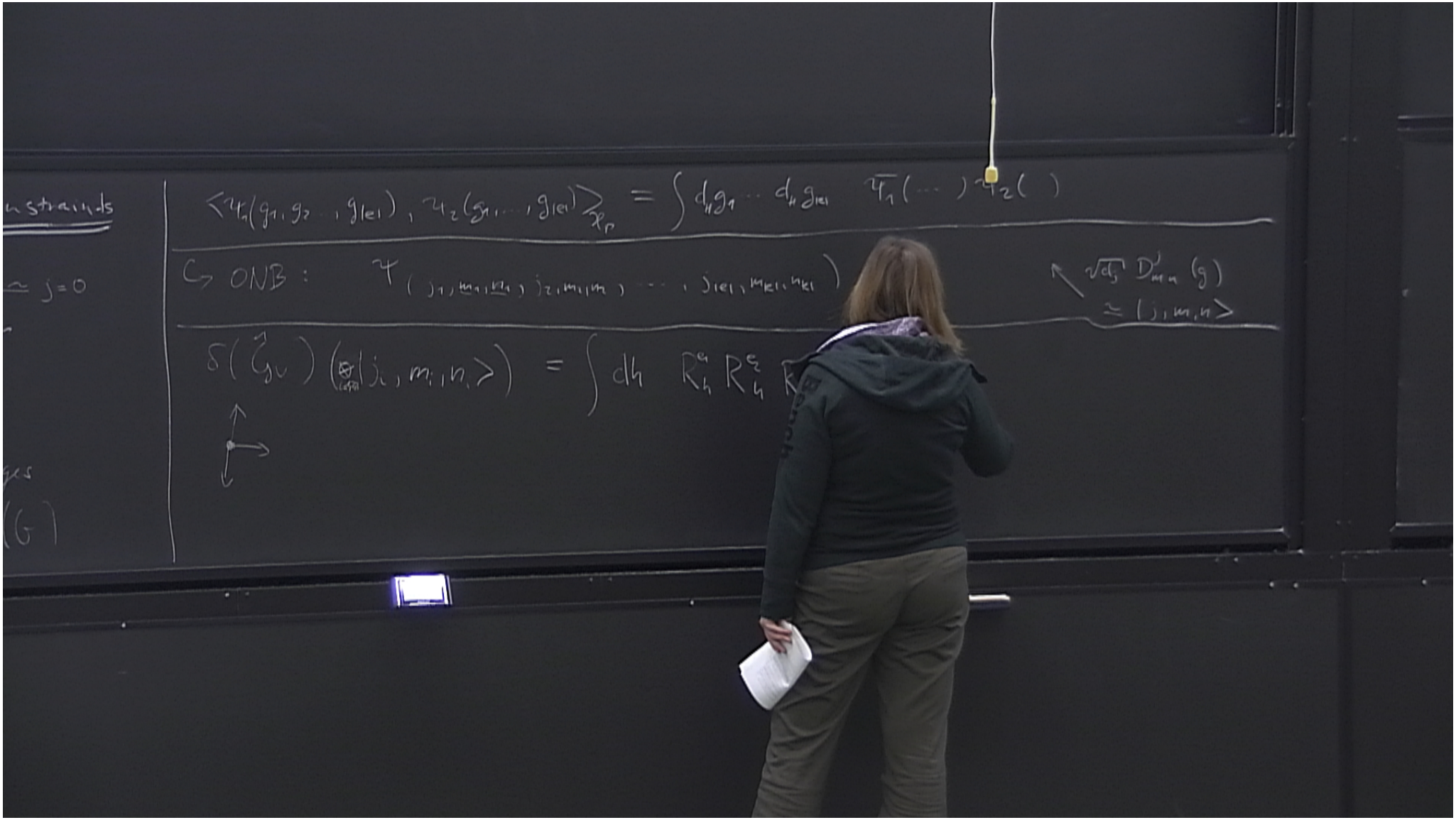
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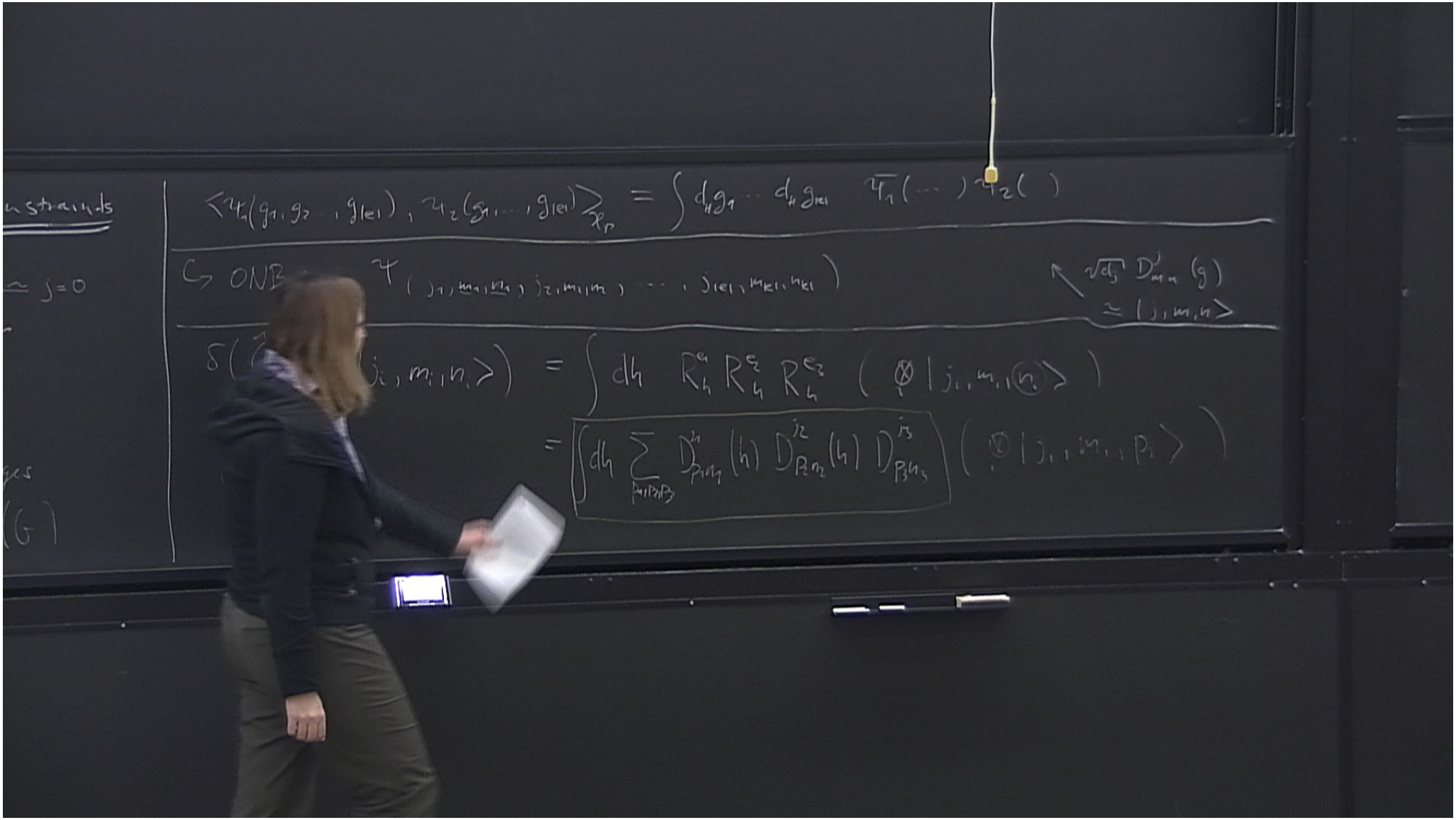
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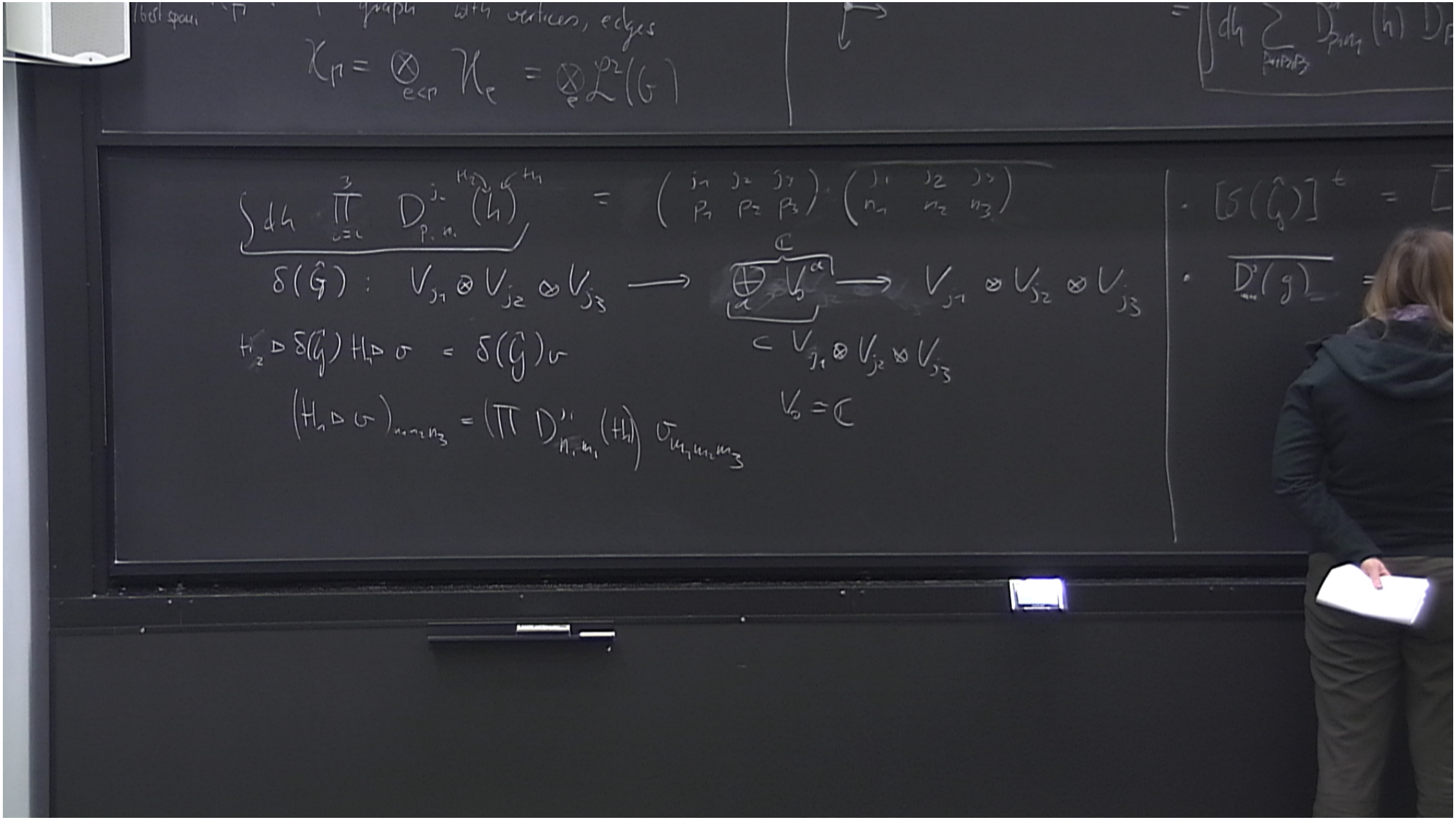
Abstract:

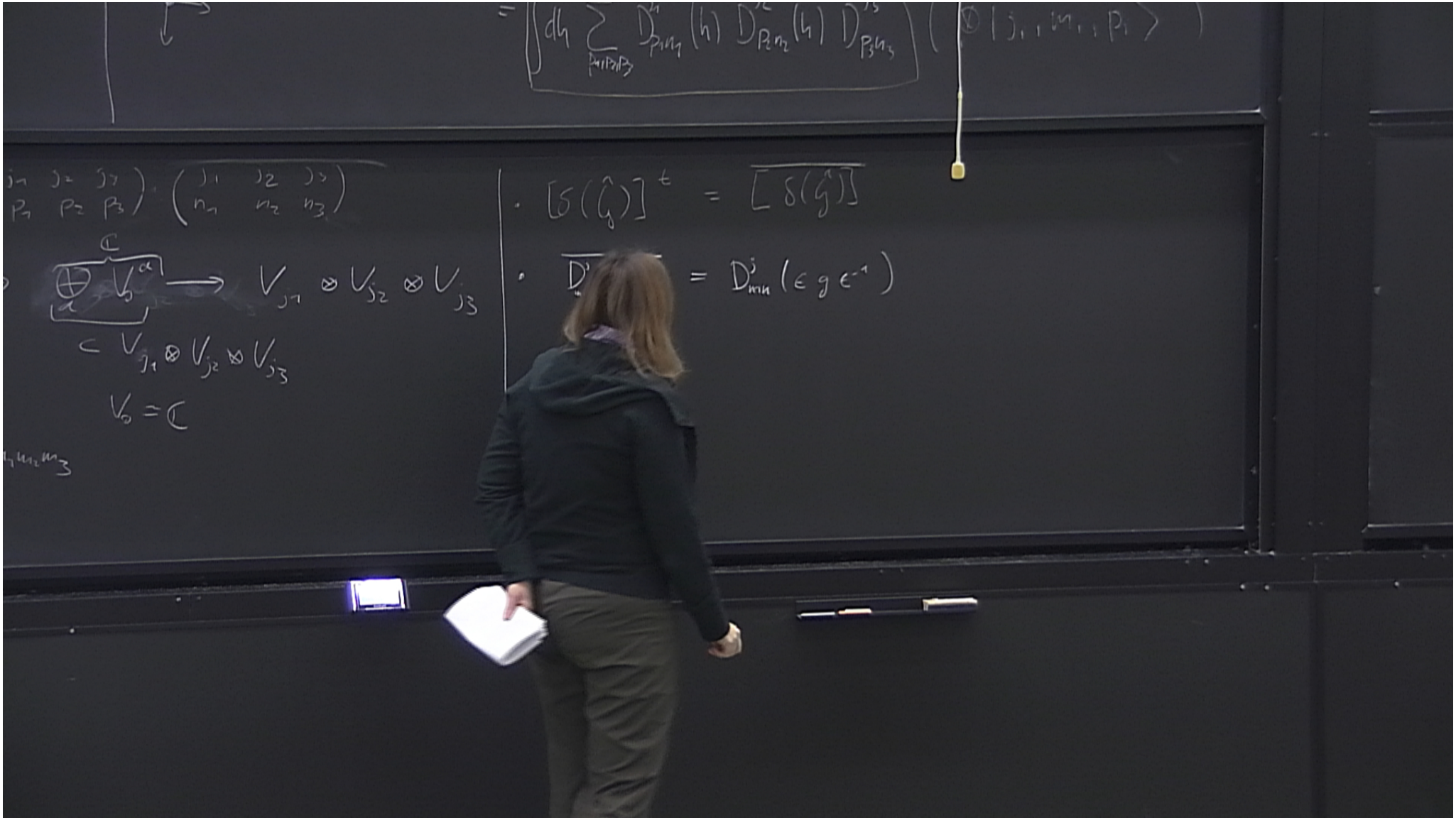

B - constraints
$\langle \psi_1(g_1, g_2, \ldots, g_{|e|}), \psi_2(g_1, \ldots, g_{|e|}) \rangle_{\chi_p} = \int dg_1 \cdots dg_{|e|} \; \psi_1(\cdots)\psi_2(\cdots)$
$= \bigotimes_e L^2(G)$

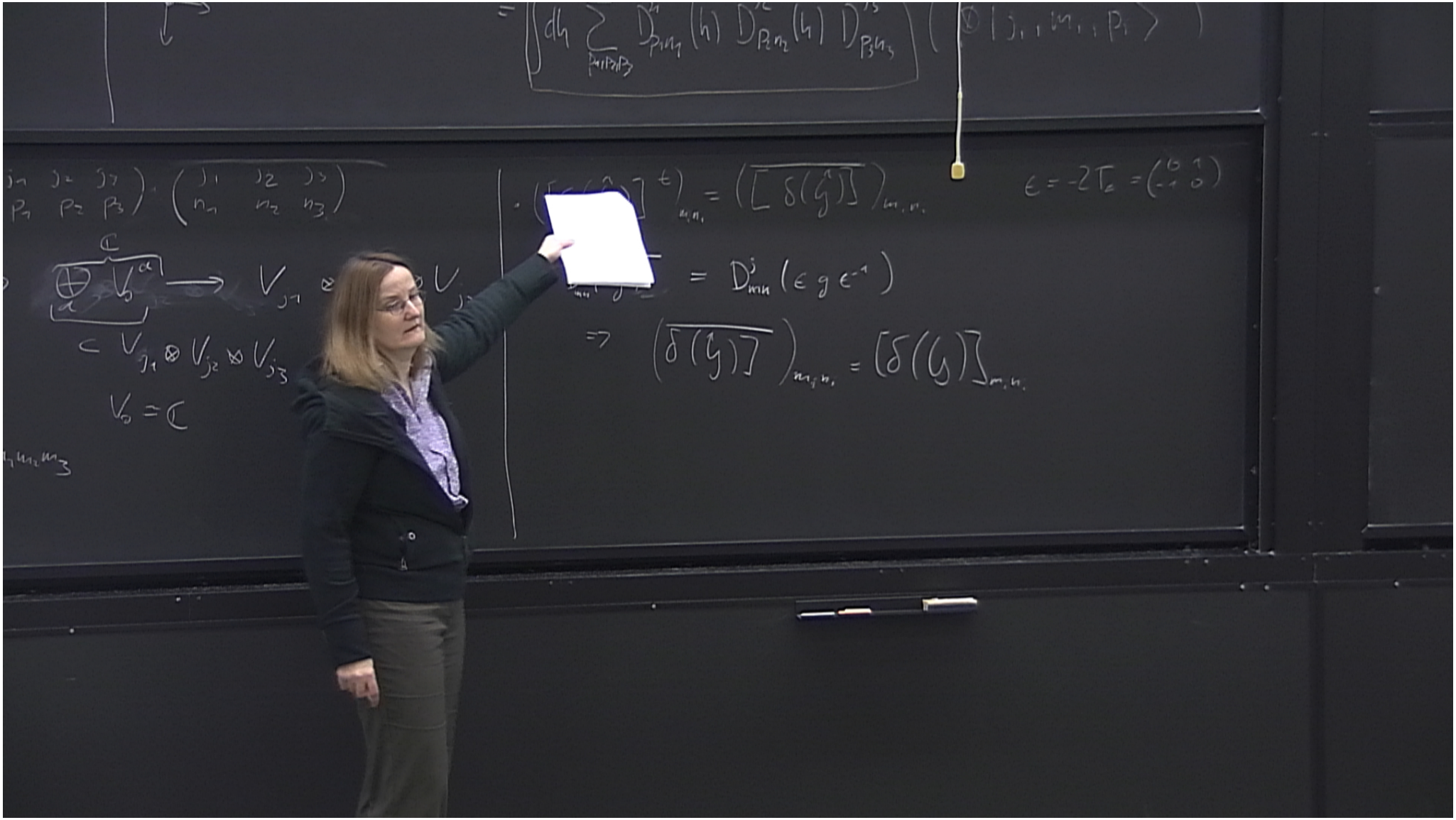

Solving Gauß - constraints
constant fct ≃ j=0
$\mathcal{H}_\Gamma$ · Γ graph with vertices, edges
$\mathcal{H}_\Gamma = \bigotimes_{e<\Gamma} \mathcal{H}_e = \bigotimes_e L^2(G)$

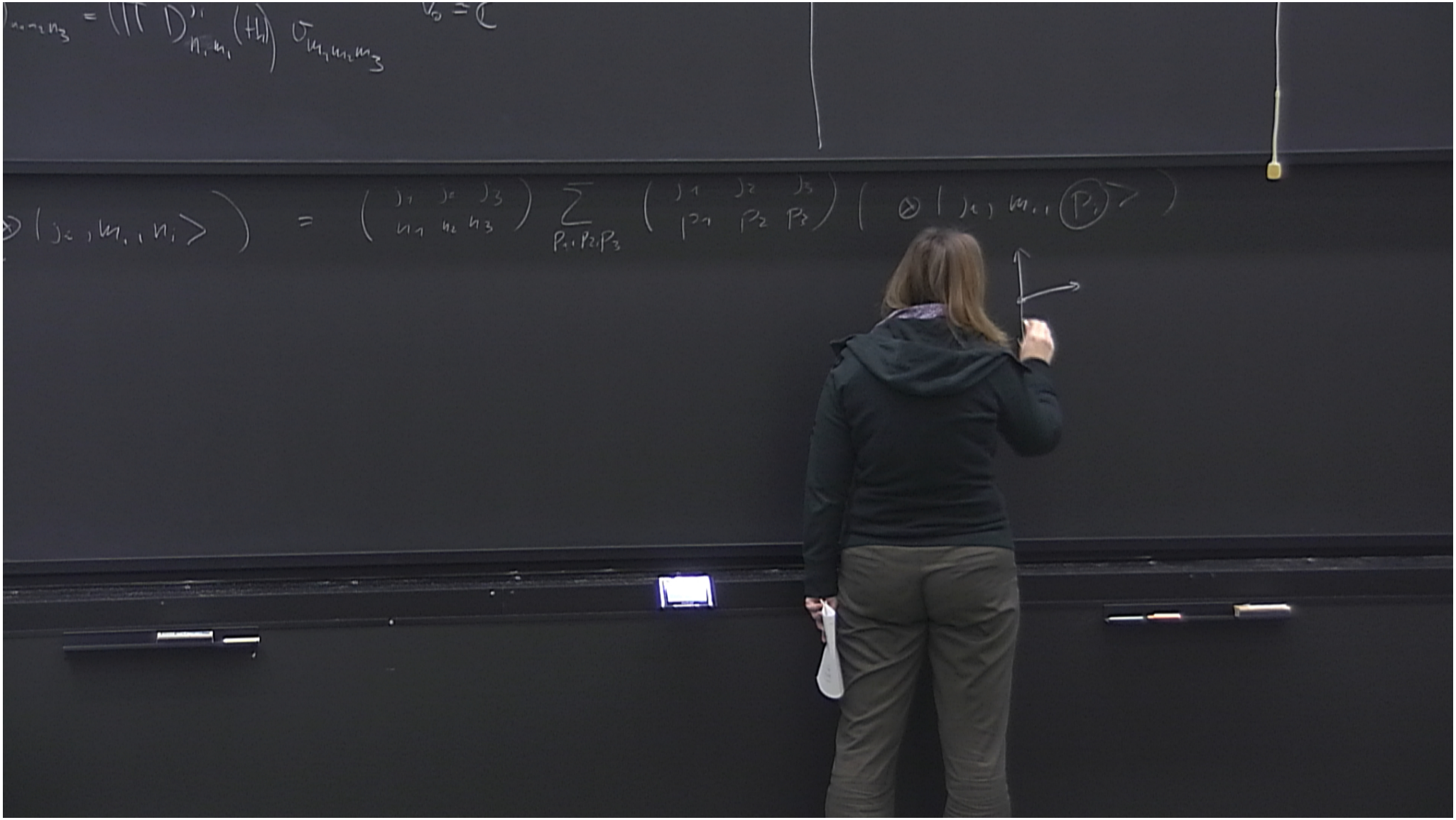

constraints
$\simeq j=0$
$\langle \psi_1(g_1, g_2, \dots, g_{|G|}), \psi_2(g_1, \dots, g_{|G|}) \rangle_{\mathcal{H}_n} = \int dg_1 \cdots dg_{|G|}\ \overline{\psi_1}(\dots)\,\psi_2(\dots)$
$\hookrightarrow$ ONB: $\psi_{(j_1,m_1,\kappa_1),(j_2,m_2,\kappa_2),\dots,(j_{|G|},m_{|G|},\kappa_{|G|})}$
$\sqrt{d_j}\, D^j_{m,n}(g) \simeq |j,m,n\rangle$
$\delta(\vec{g}_u)(|j,m,n\rangle) = \int dh\ R^u_h R^v_h$

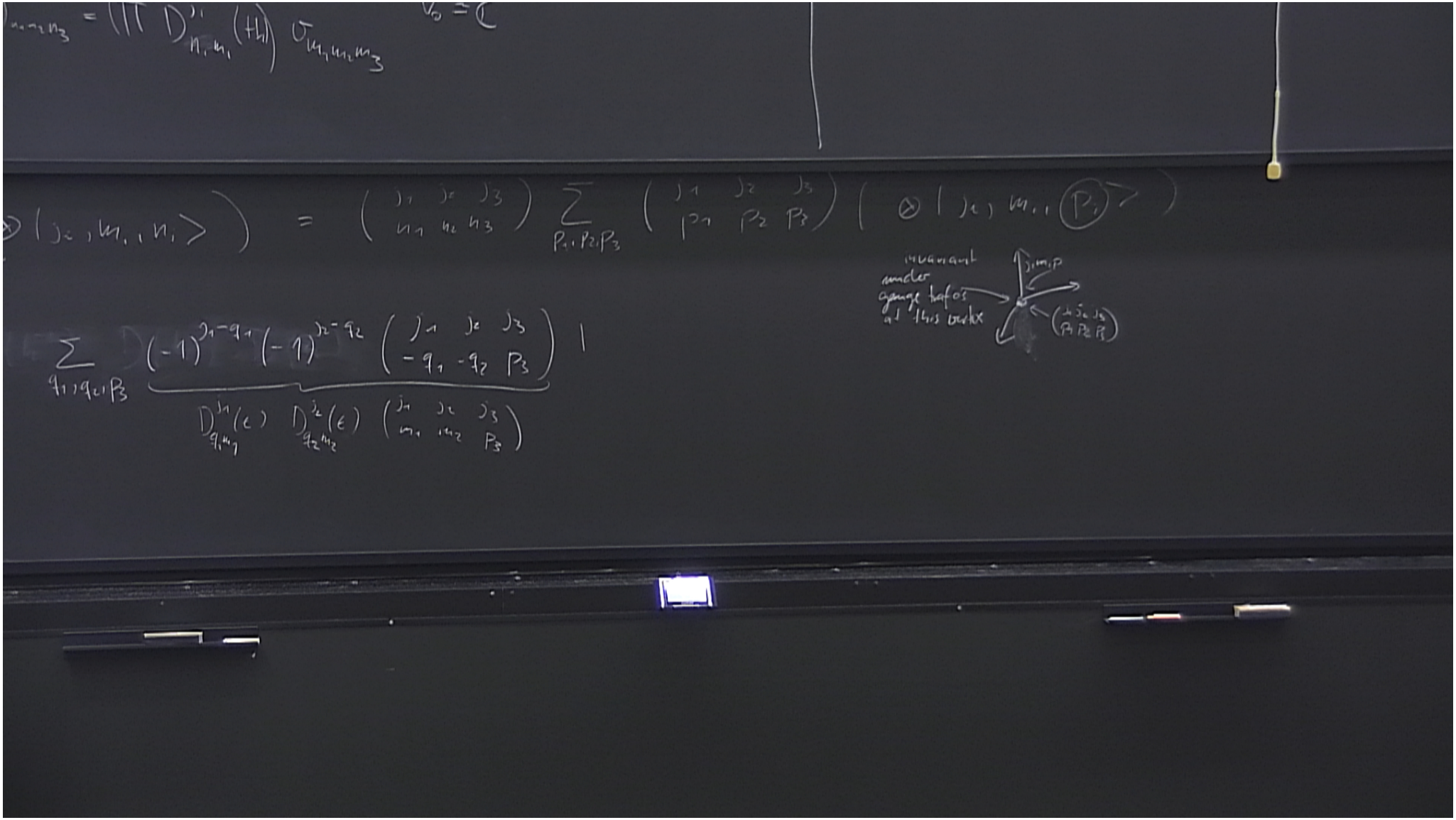

constraints
$\simeq j=0$
$\langle \psi_1(g_1, g_2, \dots, g_{|e|}), \psi_2(g_1, \dots, g_{|e|}) \rangle_{\chi_n} = \int dg_1 \cdots dg_{|e|}\ \overline{\psi_1}(\dots)\,\psi_2(\dots)$
$\hookrightarrow$ ONB
$\sqrt{d_j}\, D^j_{m\,n}(g) \simeq |j, m, n\rangle$
$\delta(\dots |j_i, m_i, n_i\rangle) = \int dh\ R^{e_1}_h R^{e_2}_h R^{e_3}_h \left(\bigotimes_i |j_i, m_i, n_i\rangle \right)$
$= \int dh \sum_{p_1 p_2 p_3} D^{j_1}_{p_1 m_1}(h)\, D^{j_2}_{p_2 m_2}(h)\, D^{j_3}_{p_3 m_3} \left(\bigotimes_i |j_i, m_i, p_i\rangle \right)$

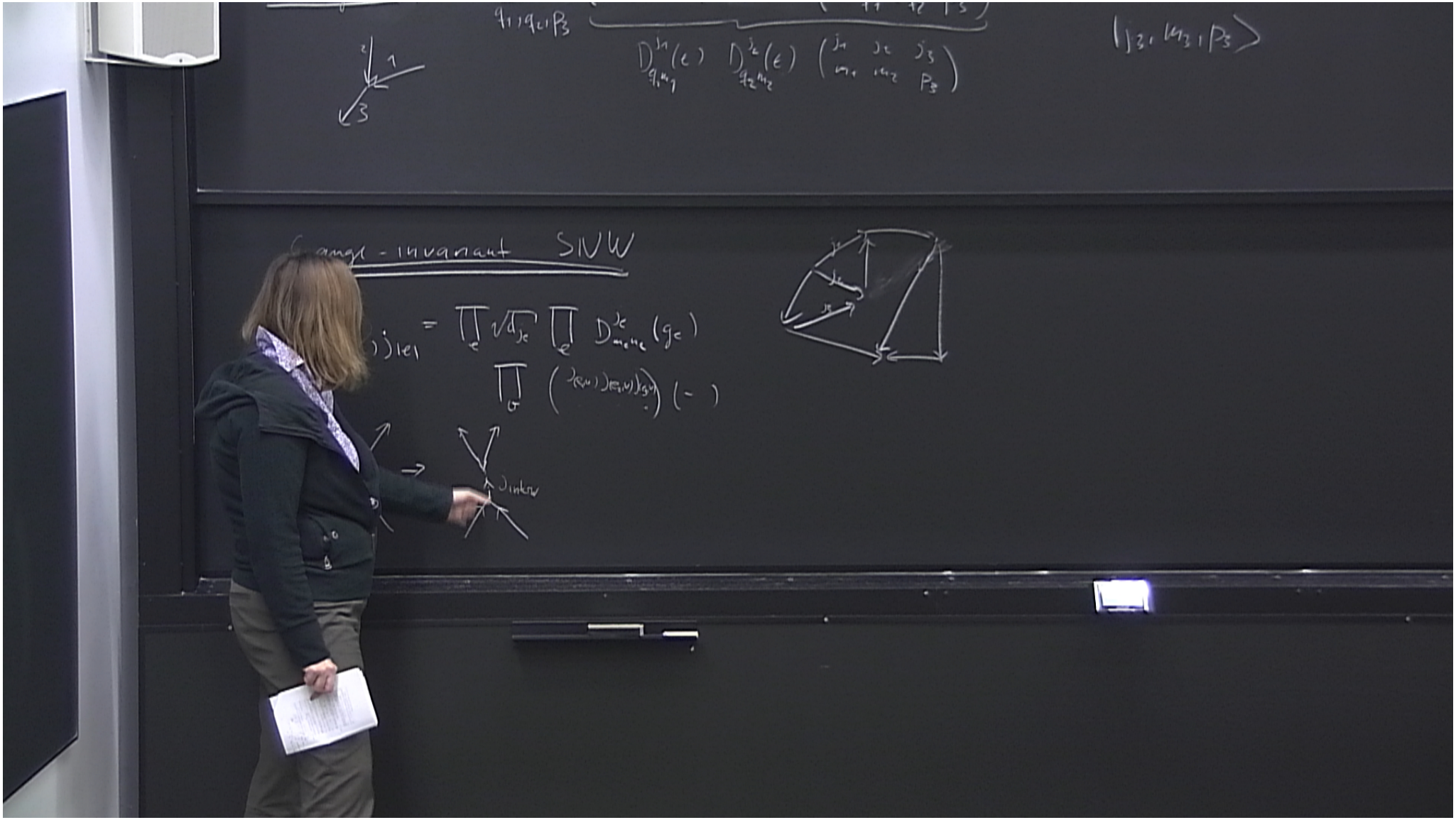

graphs with vertices, edges
$\mathcal{H}_\Gamma = \bigotimes_{e\in\Gamma} \mathcal{H}_e = \bigotimes_e L^2(G)$
$\int dh \prod_{v=1}^{3} D^{j_v}_{p_v m_v}(h)$
$= \begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & p_2 & p_3 \end{pmatrix} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$
$\delta(\hat{G}) : V_{j_1} \otimes V_{j_2} \otimes V_{j_3} \longrightarrow \bigoplus_\alpha V_0^\alpha \longrightarrow V_{j_1} \otimes V_{j_2} \otimes V_{j_3}$
$\subset V_{j_1} \otimes V_{j_2} \otimes V_{j_3}$
$V_0 = \mathbb{C}$
$\mathbb{C}$
$\delta(\hat{G})\, H_1 \triangleright \sigma = \delta(\hat{G})\, \sigma$
$(H_1 \triangleright \sigma)_{m_1 m_2 m_3} = \left(\prod D^{j_v}_{m_v m_v'}(h_1)\right) \sigma_{m_1' m_2' m_3'}$
$[\delta(\hat{G})]^\dagger =$
$\overline{D^j(g)} =$

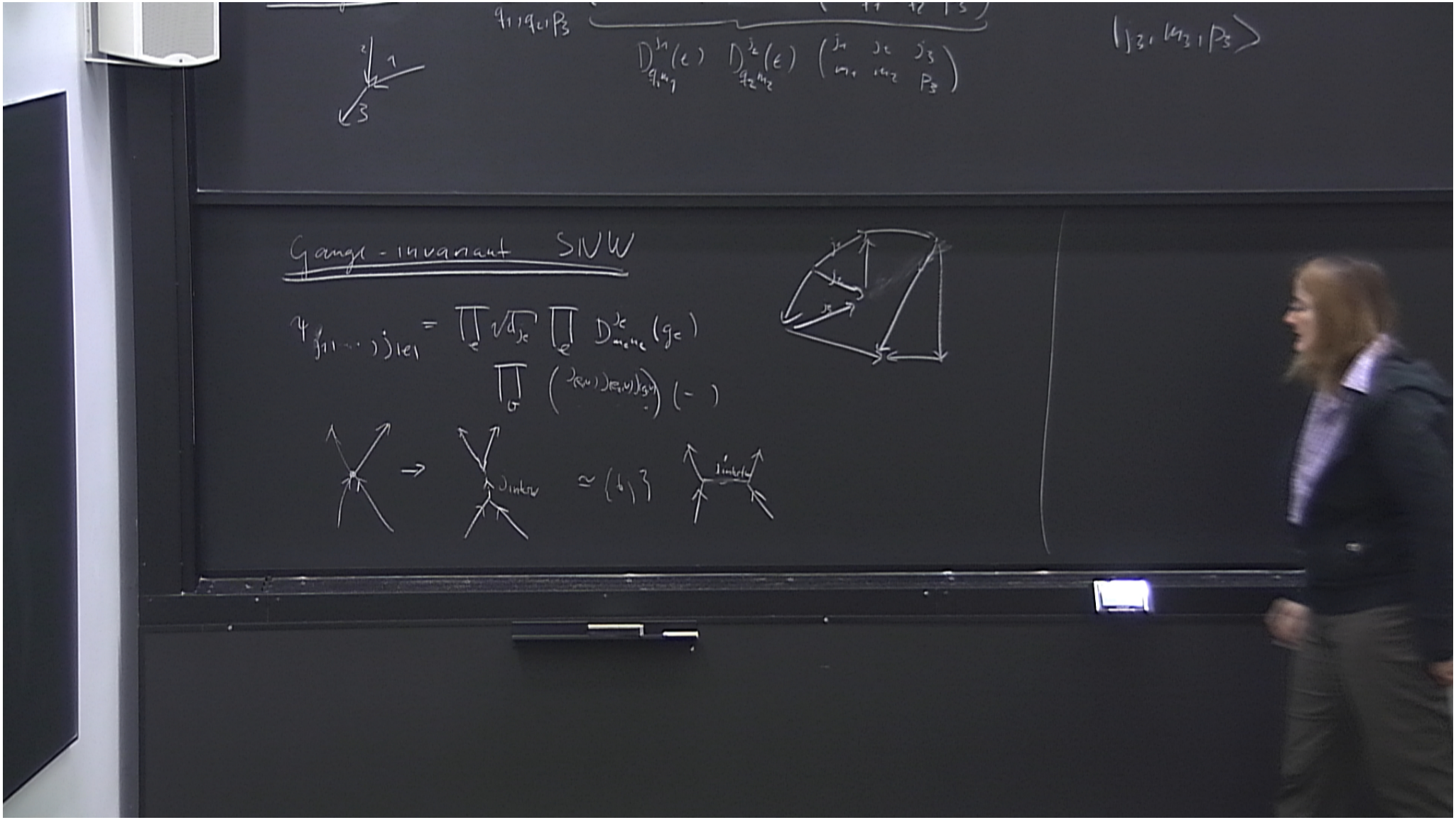

$\int dh \sum_{\beta_1\beta_2\beta_3} D^{j_1}_{\beta_1 m_1}(h)\, D^{j_2}_{\beta_2 m_2}(h)\, D^{j_3}_{\beta_3 m_3}(h)$
$[\delta(\hat{g})]^t = \overline{[\delta(\hat{g})]}$
$\overline{D^j} = D^j_{mn}(\epsilon g \epsilon^{-1})$
$V_{j_1} \otimes V_{j_2} \otimes V_{j_3}$
$V_0 = \mathbb{C}$

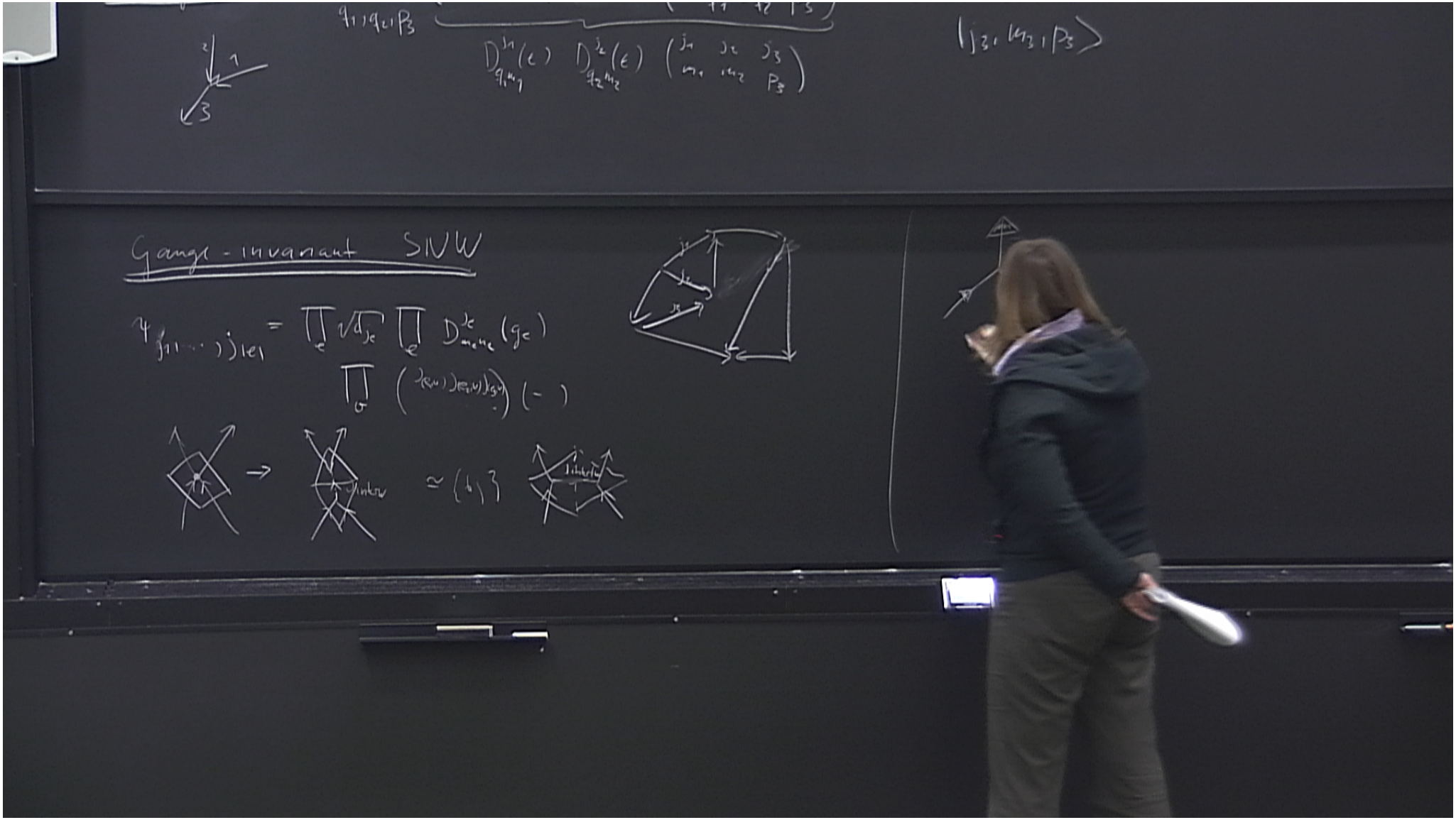

$\epsilon = -2\Gamma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
$= D^j_{mn}(\epsilon g \epsilon^{-1})$
$\Rightarrow \overline{(\delta(g)]}_{m_1 m_2} = [\delta(g)]_{m_1 m_2}$
$V_0 = \mathbb{C}$

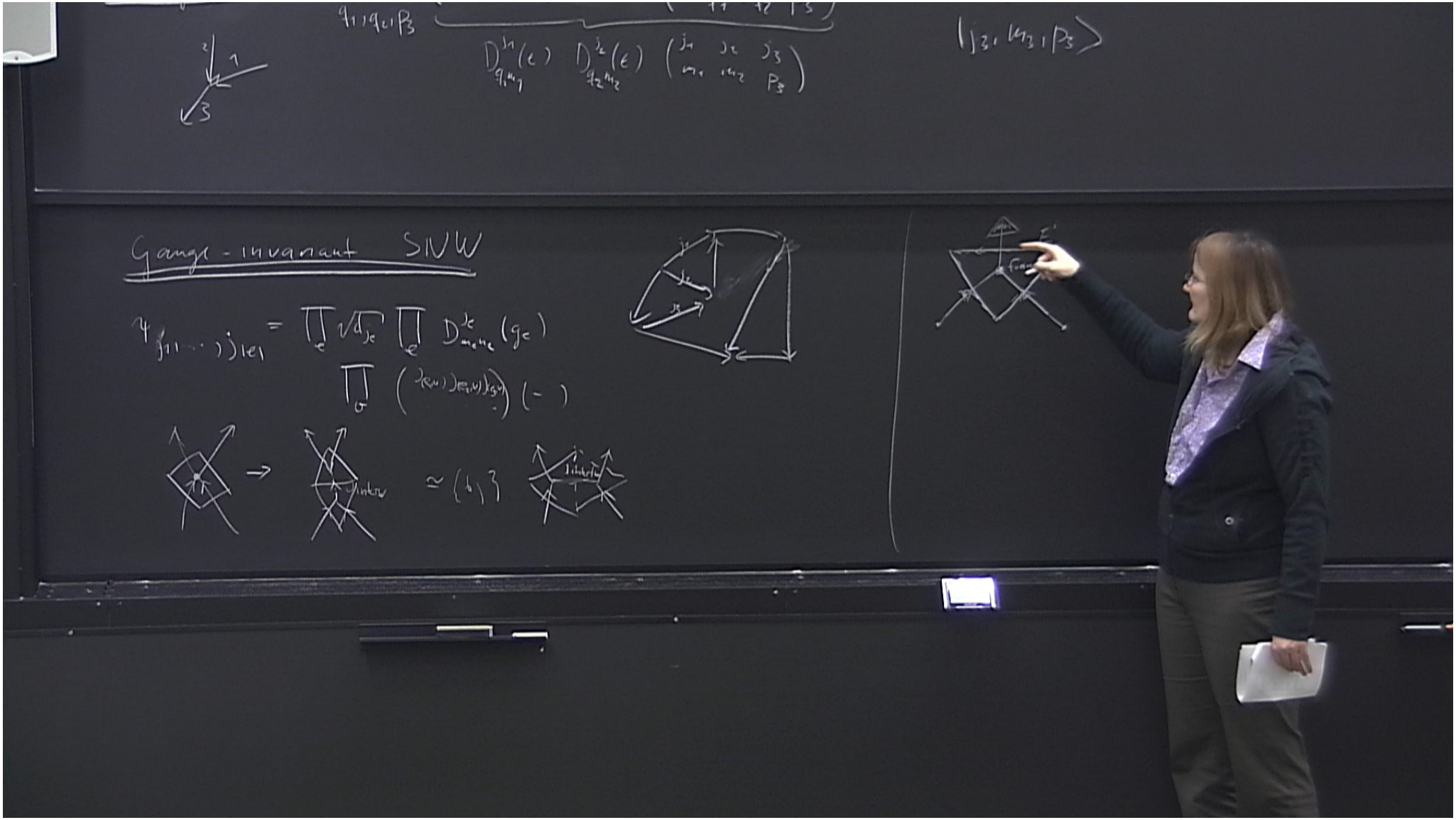


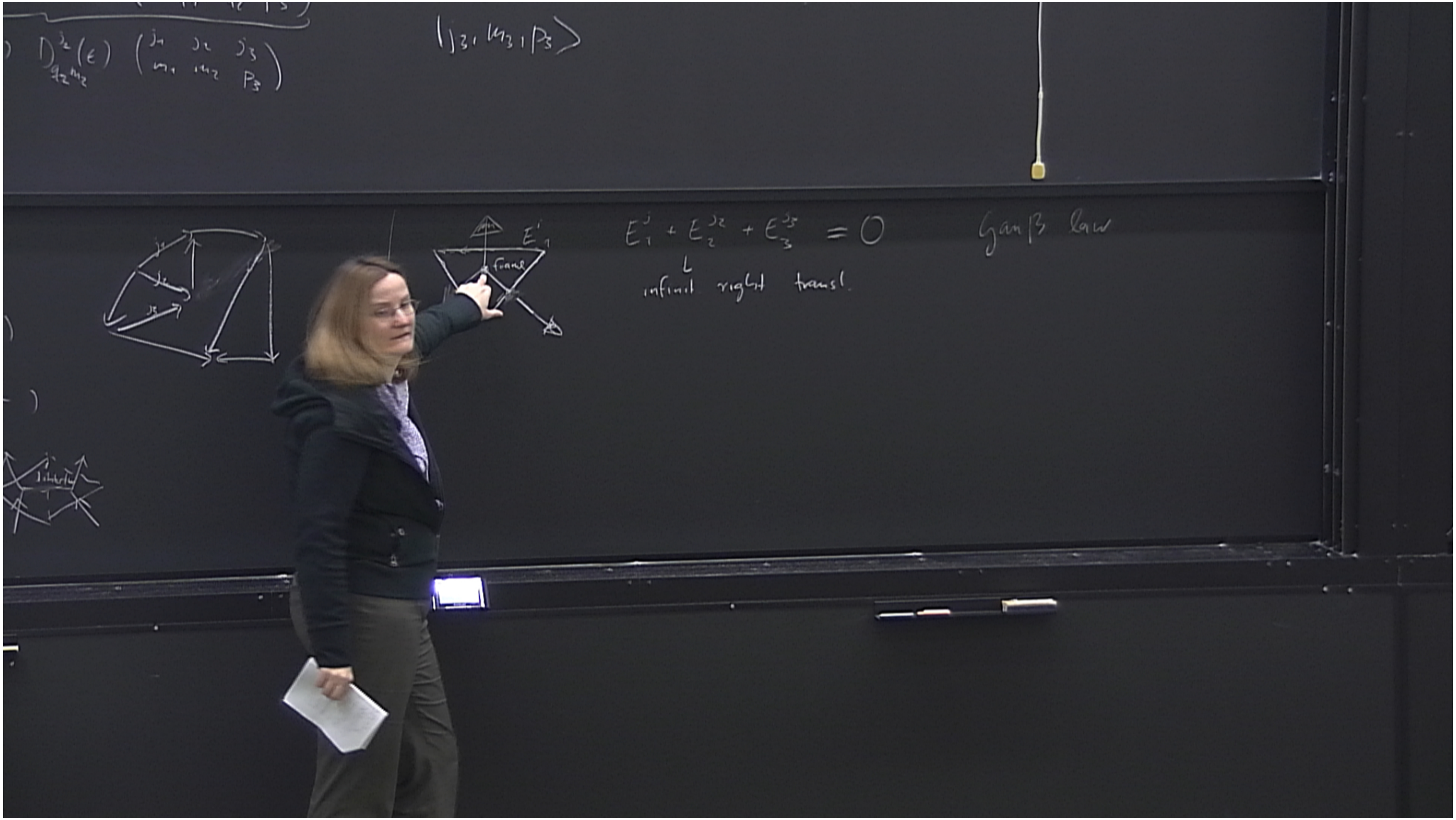

invariant under gauge trafos at this vertex

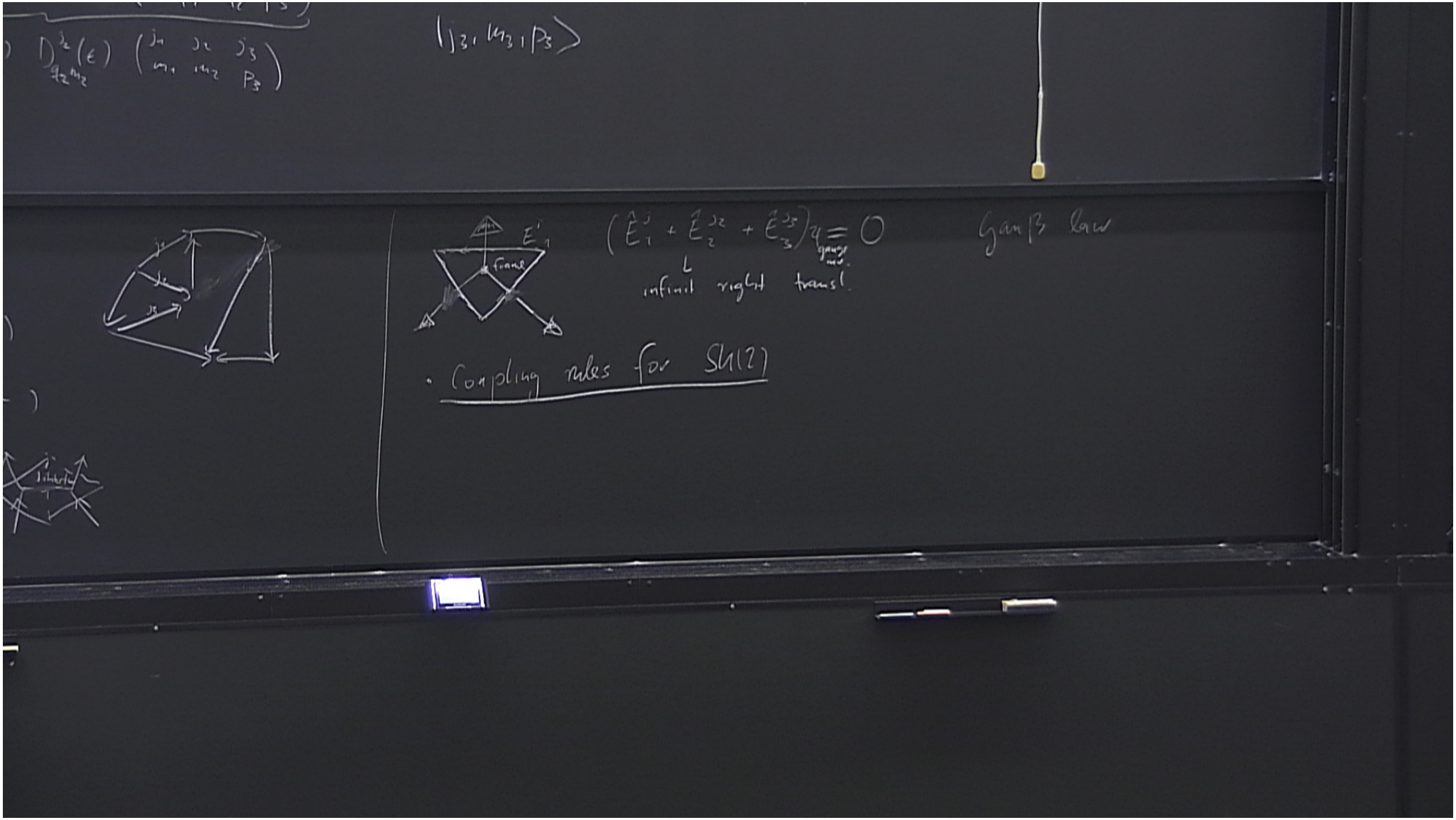

Group - invariant SNW

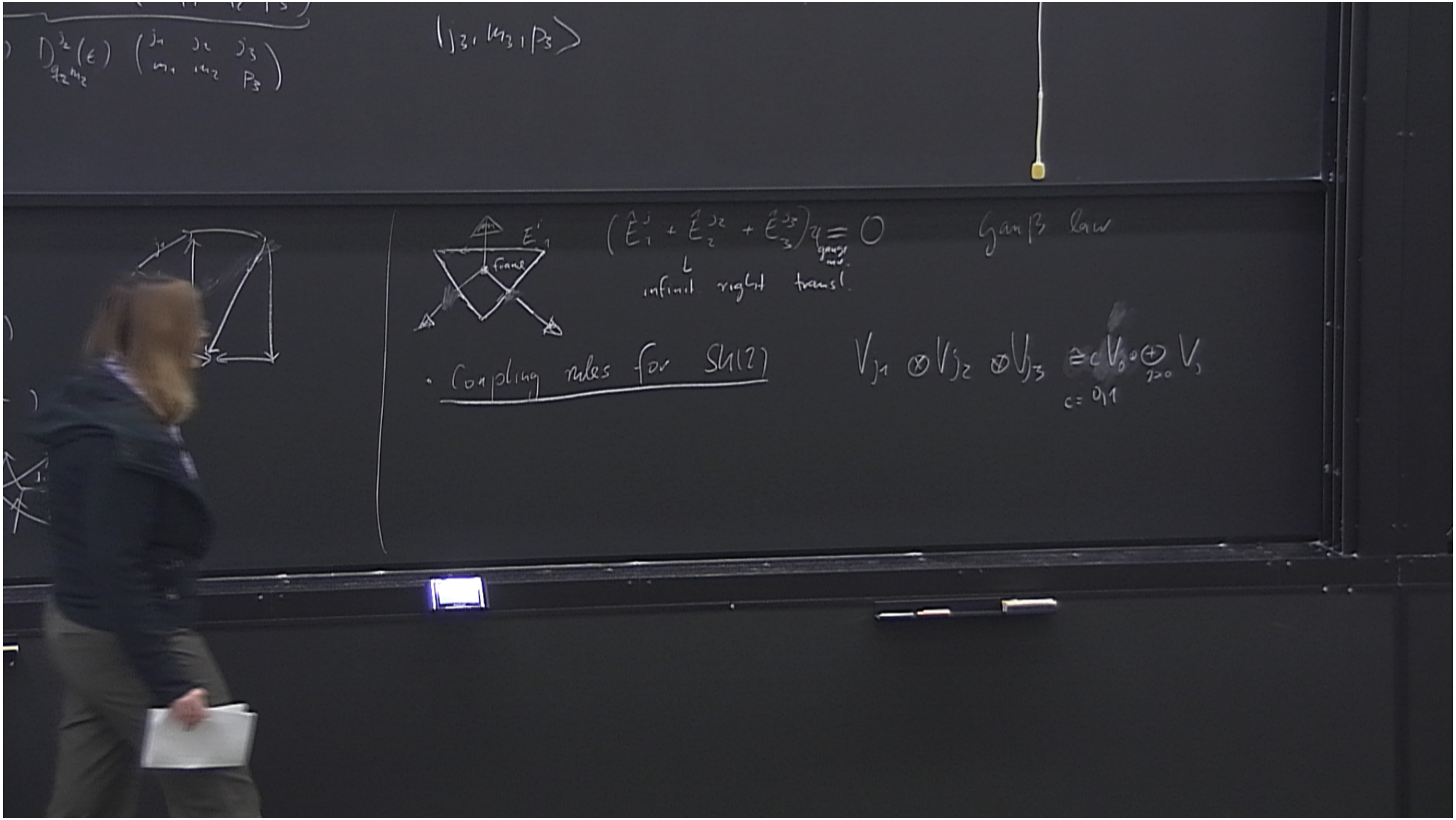

Gauge - invariant SNW

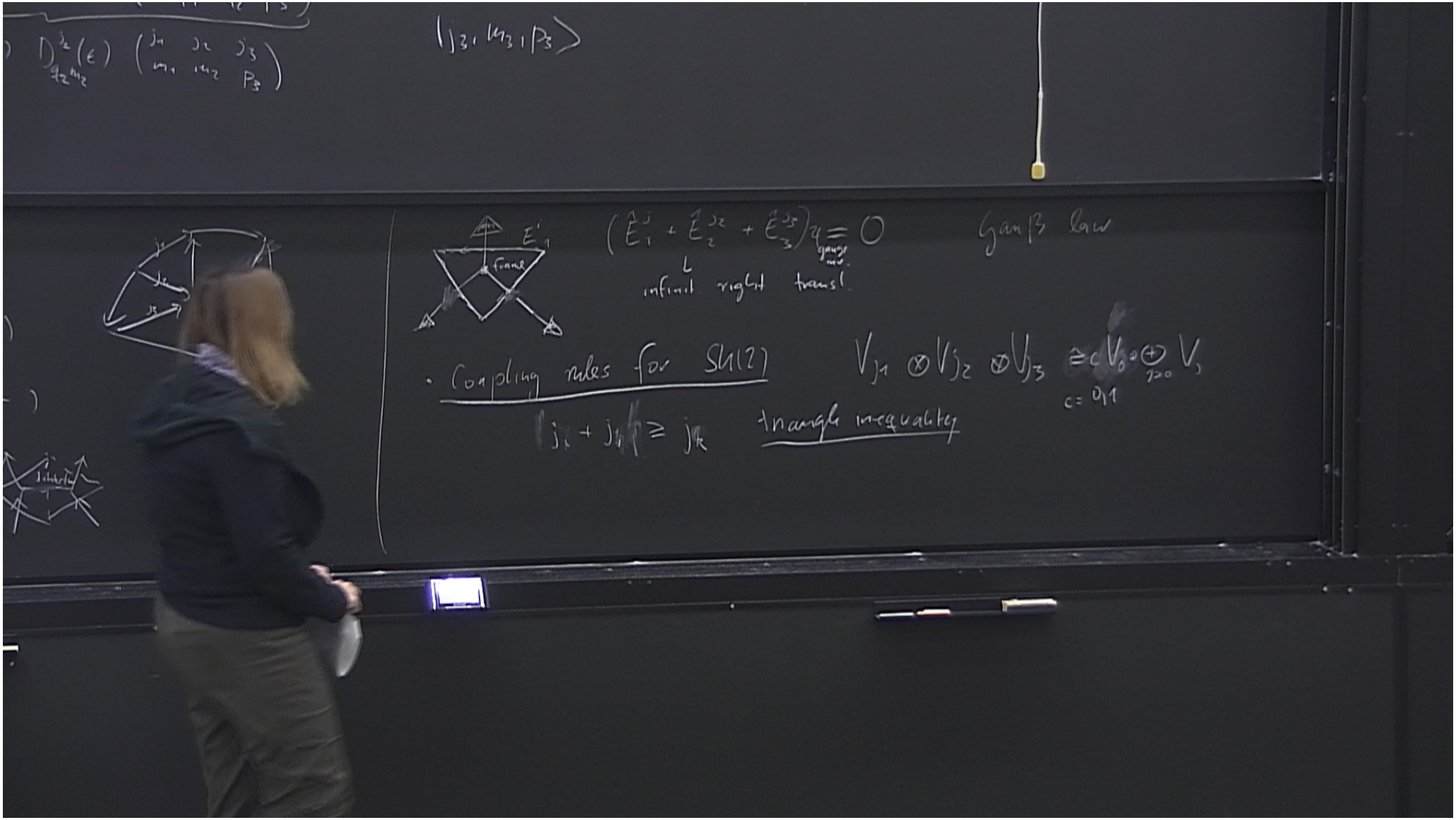

Gauge-invariant SNW

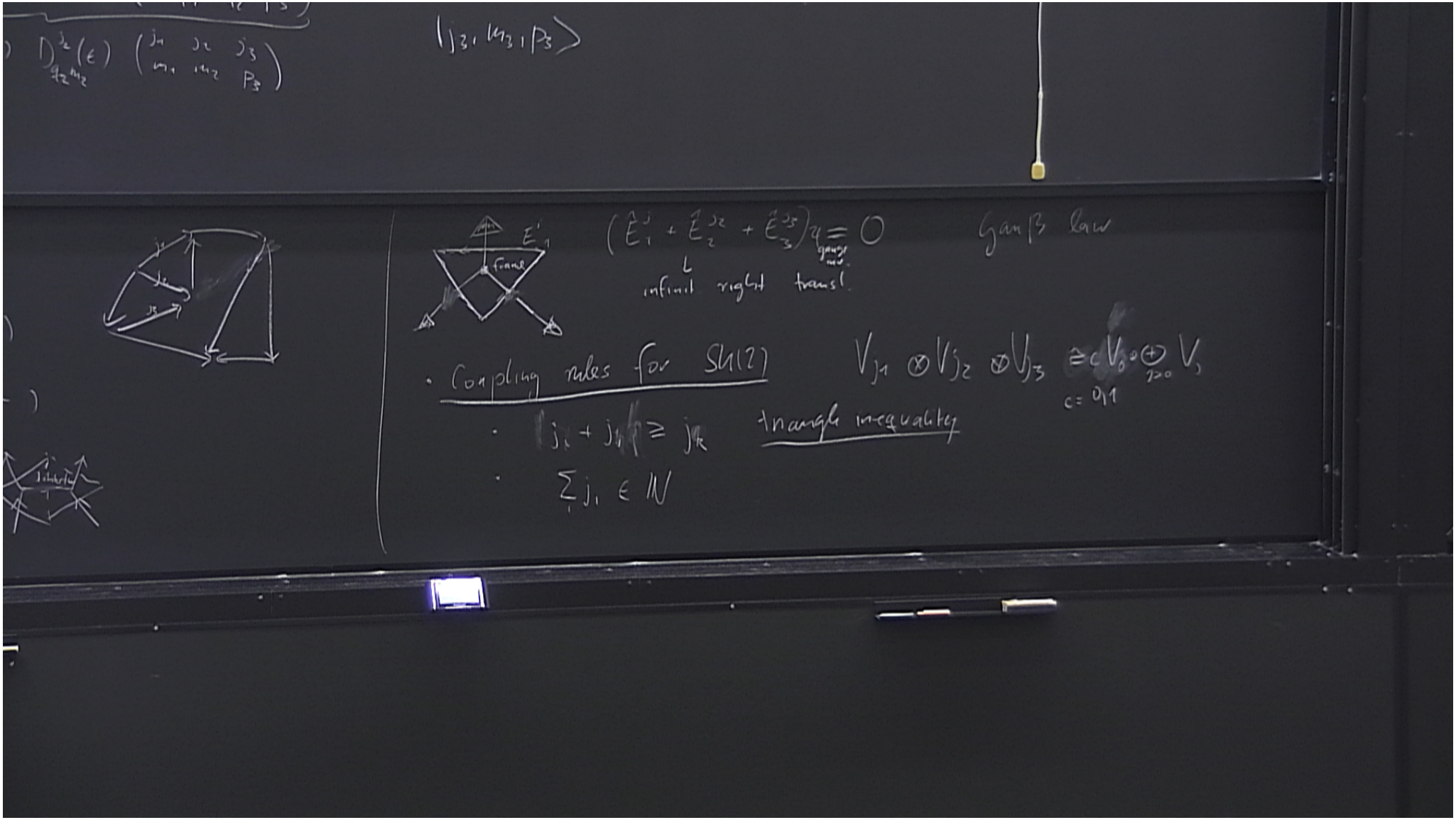

Gauge-invariant SNW

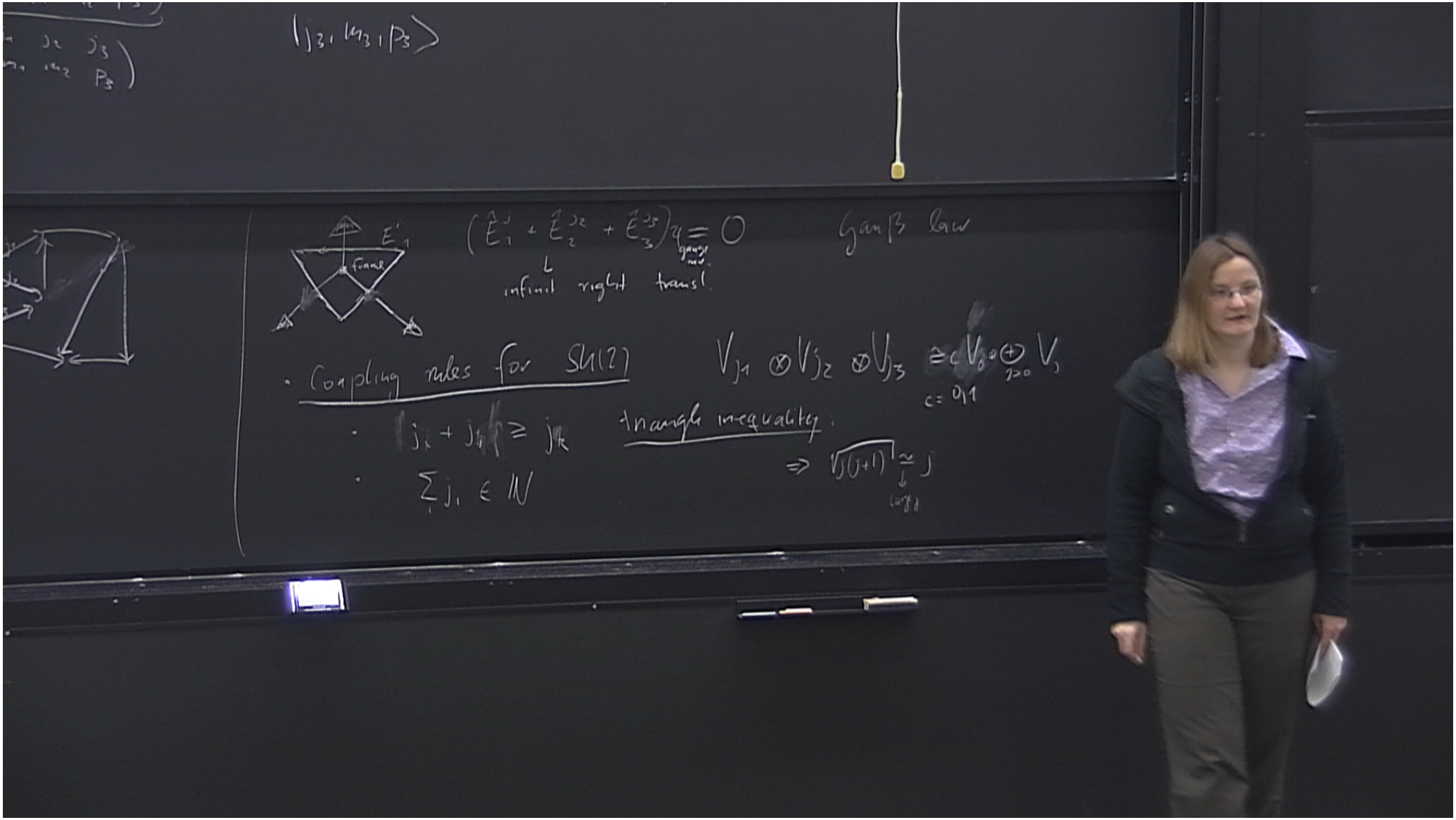

$E_1^{j_1} + E_2^{j_2} + E_3^{j_3} = 0$
Gamb law
infinit right transl.
frame


$|j_3, m_3, p_3\rangle$
$D^{j_2}_{m_2 m_2}(\epsilon)$ $\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & p_3 \end{pmatrix}$
E^i_1
frame
$(\hat{E}^{j_1}_1 + \hat{E}^{j_2}_2 + \hat{E}^{j_3}_3)\psi = 0$
gauge inv.
infinit right transl.
Gauß law
• Coupling rules for SU(2)

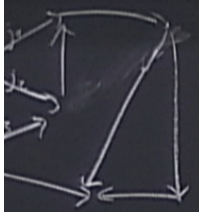

frame
infinit right transl.
Gauß law
Coupling rules for SU(2)

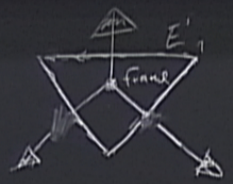

Gauss law
infinit right transl.
Coupling rules for SU(2)
triangle inequality


frame
Gauß law
infinit right transl.
Coupling rules for SU(2)
triangle inequality


$|j_3, k_3, p_3\rangle$
frame
$(\hat{E}^{j_1}_1 + \hat{E}^{j_2}_2 + \hat{E}^{j_3}_3)\,q = 0$
gauge inv.
infinit right transl
Gauß law
• Coupling rules for SU(2)
$V_{j_1} \otimes V_{j_2} \otimes V_{j_3} \cong \bigoplus_{j \geq 0} V_j^{\oplus c}$
$c = 0,1$
• $|j_1 + j_2| \geq j_3$ triangle inequality.
• $\sum_i j_i \in \mathbb{N}$
$\Rightarrow \sqrt{j(j+1)} \simeq j$
large j

$|j_3, k_3, p_3\rangle$





$(\vec{E}^{j_1}_1 + \vec{E}^{j_2}_2 + \vec{E}^{j_3}_3)\,\psi = 0$ gauge inv.

Gauß law

↳ infinit right transl

- Coupling rules for $SU(2)$ $\quad V_{j_1} \otimes V_{j_2} \otimes V_{j_3} \cong \bigoplus_{j \geq 0} V_j^{\oplus c}$, $c = 0,1$

  - $j_1 + j_2 \geq j_3$ triangle inequality.

  - $\sum_i j_i \in \mathbb{N}$

$\Rightarrow$ $\sqrt{j(j+1)} \overset{\text{large } j}{\simeq} j$ length of edge