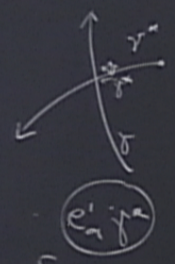
Title: Quantum Gravity Review-10

Date: Mar 02, 2015 10:15 AM

URL: http://pirsa.org/15030007

Abstract:

# The length operator

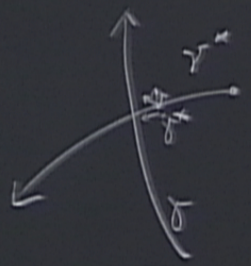



$\delta_{ij} e^i_a e^j_b = q_{ab}$

$\delta_{ij} e^i_a e^j_b \dot{\gamma}^a \dot{\gamma}^b = |\dot{\gamma}^a|^2$

$$\hat{\ell}^2_{c^*} |j, m, n\rangle := \left(\sum_k \hat{E}^k_e \hat{E}^k_e\right) |j, m, n\rangle = -\hbar^2 \sum_{p,q}$$

# The length operator




$$\hat{\ell}^2_{e^*}\,|j,m,n\rangle := \left(\sum_R \hat{E}^k_e \hat{E}^k_e\right)$$

$$E^i = \frac{1}{dt} R_{e^* T_0}$$

$$\int_{\cdot} e^i_a e^j_b = q_{ab}$$

$$\int_{\cdot} e^i_a e^j_b \dot{\gamma}^{*a} \dot{\gamma}^{*b} = |\dot{\gamma}^*_{e^*}|^2$$

operator

$$\hat{L}^2_{e^*} |j,m,n\rangle := \left(\sum_k \hat{E}^k_e \hat{E}^k_e\right) |j,m,n\rangle = -\hbar^2 \sum_{p,q} |$$

$$\vec{E}^j = \frac{d}{dt} R_{e^{tT_j}}$$

$$|m,n\rangle = -t^2 \sum_{p,q} |j,m,q\rangle \sum_{k} D^{j}_{qp}(T^k)\, D^{i}_{pn}(T^k)$$

operator

$$\hat{\ell}_e^2 |j,m,n\rangle := \left(\sum_k \hat{E}_e^k \hat{E}_e^k\right) |j,m,n\rangle = -\hbar^2 \sum_{p,q} |j,m,q\rangle \sum_R D^j_{qp}(R$$

$$\hat{E}_e^k |j,m,n\rangle = i\hbar L_e^k |j,m,n\rangle = i\hbar \frac{d}{dt}\Big|_{t=0} R_{\exp(tT^k)} |j,m,n\rangle$$

$$= i\hbar \sum_p |j,m,p\rangle \underbrace{\frac{d}{dt}\Big|_{t=0} D^j_{pn}(\exp(tT^k))}_{= D^j_{pn}(T^k)}$$

operator

$$\hat{\ell}^2_{c^*} |j,m,n\rangle := \left(\sum_k \hat{E}^k_e \hat{E}^k_e\right) |j,m,n\rangle = -\hbar^2 \sum_{p,q} |j,m,q\rangle \sum_k D^j_{qp}(T^k) D^i_{pn}$$

$$\sum_k D^j_{qp}(T^k)\, D^j_{pn}(T^k) \qquad [C^j, D^j(T^e)] = 0$$

## length operator

$$\hat{\ell}^2_{e^*} \, |j, m, n\rangle := \left(\sum_k \hat{E}^k_e \hat{E}^k_e\right) |j, m, n\rangle = -\hbar^2 \sum_{p,q} |$$

$$C^j_{qn} = \sum_k D^j_{qp}(T^k)^{\cdot} \, D^j_{pn}(T^k)$$

on $V_j$

$\uparrow \epsilon \quad + \quad \uparrow \epsilon$

## length operator

$$\hat{\ell}^2_{e^*} |j, m, n\rangle := \left(\sum_k \hat{E}^k_e \hat{E}^k_e\right) |j, m, n\rangle = -\hbar^2 \sum_{p,q}$$

$$C^j_{qn} = \sum_k D^j_{qp}(T^k)\, D^j_{pn}(T^k)$$

on $V_d$

$\epsilon$ + $\epsilon$

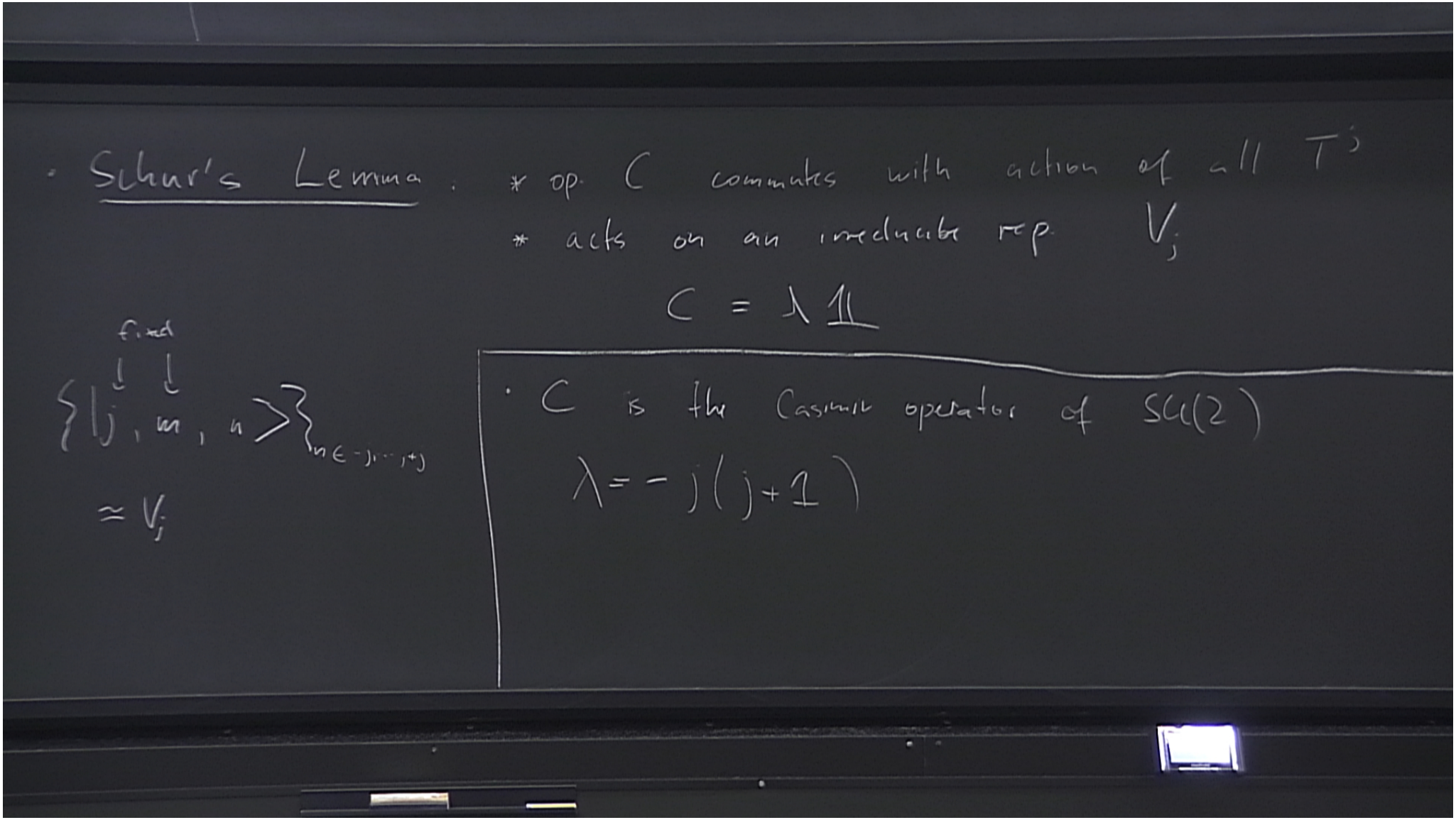
Operator is a Casimir operator

$$|j,m,n\rangle = -t^2 \sum_{p,q} |j,m,q\rangle \sum_{k} D^{j}_{qp}(T^k)\, D^{j}_{pn}(T^k)$$

$$(T^k)$$

$$[C^j, D^j(T^\ell)] = 0$$

$\epsilon$


· Schur's Lemma : * op. C commutes with action of all T^j
* acts on an irreducible rep. V_j
C = λ 𝟙
fixed
{|j, m, n⟩}_{n∈{-j,...,+j}}
≃ V_j
· C is the Casimir operator of SU(2)
λ = -j(j+1)

· Schur's Lemma : * op. $C$ commutes with action of all $T^j$

* acts on an irreducible rep. $V_j$

$$C = \lambda \mathbb{1}$$

fixed

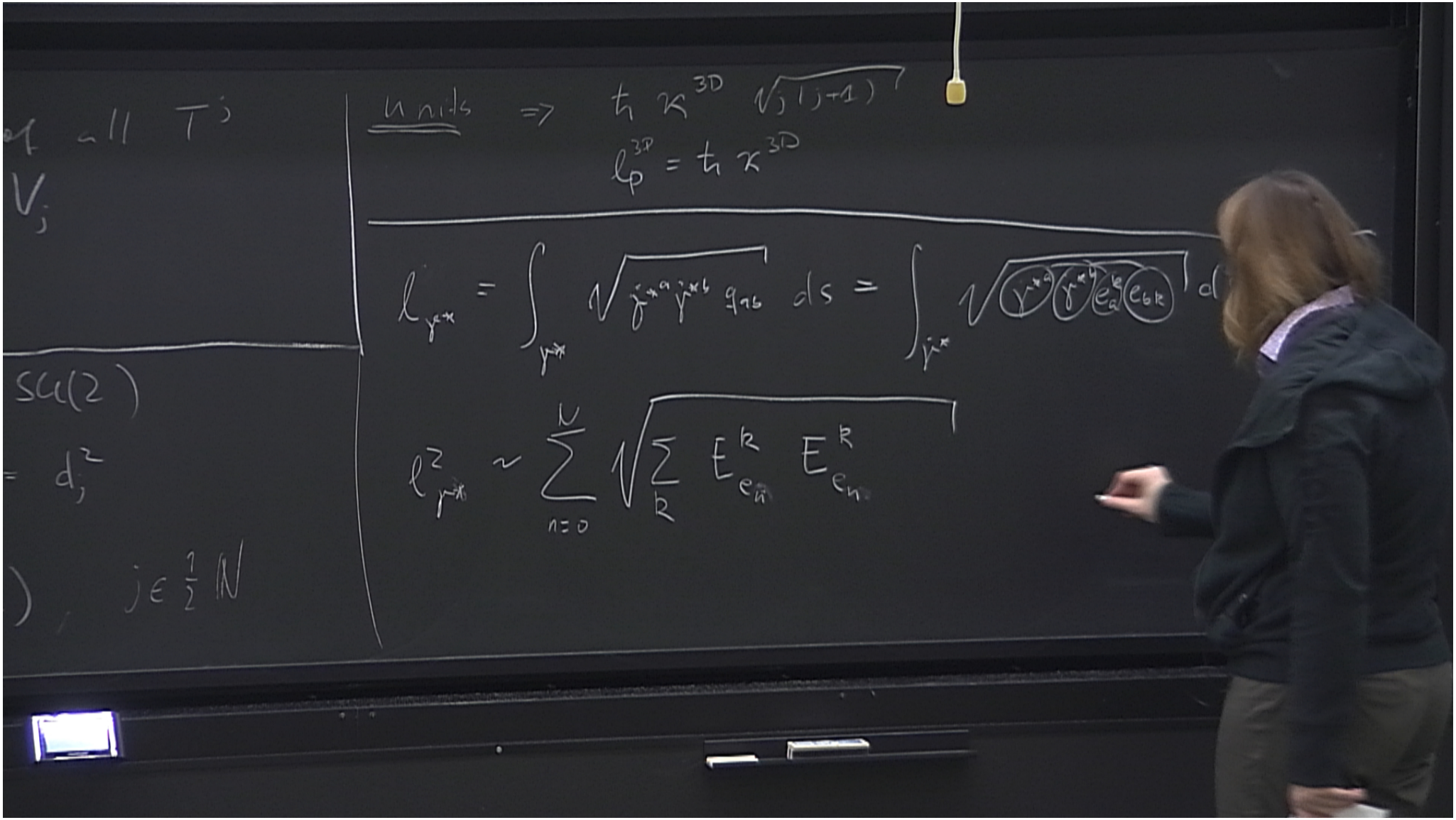
$\{|j, m, n\rangle\}_{m \in -j, \dots, +j}$

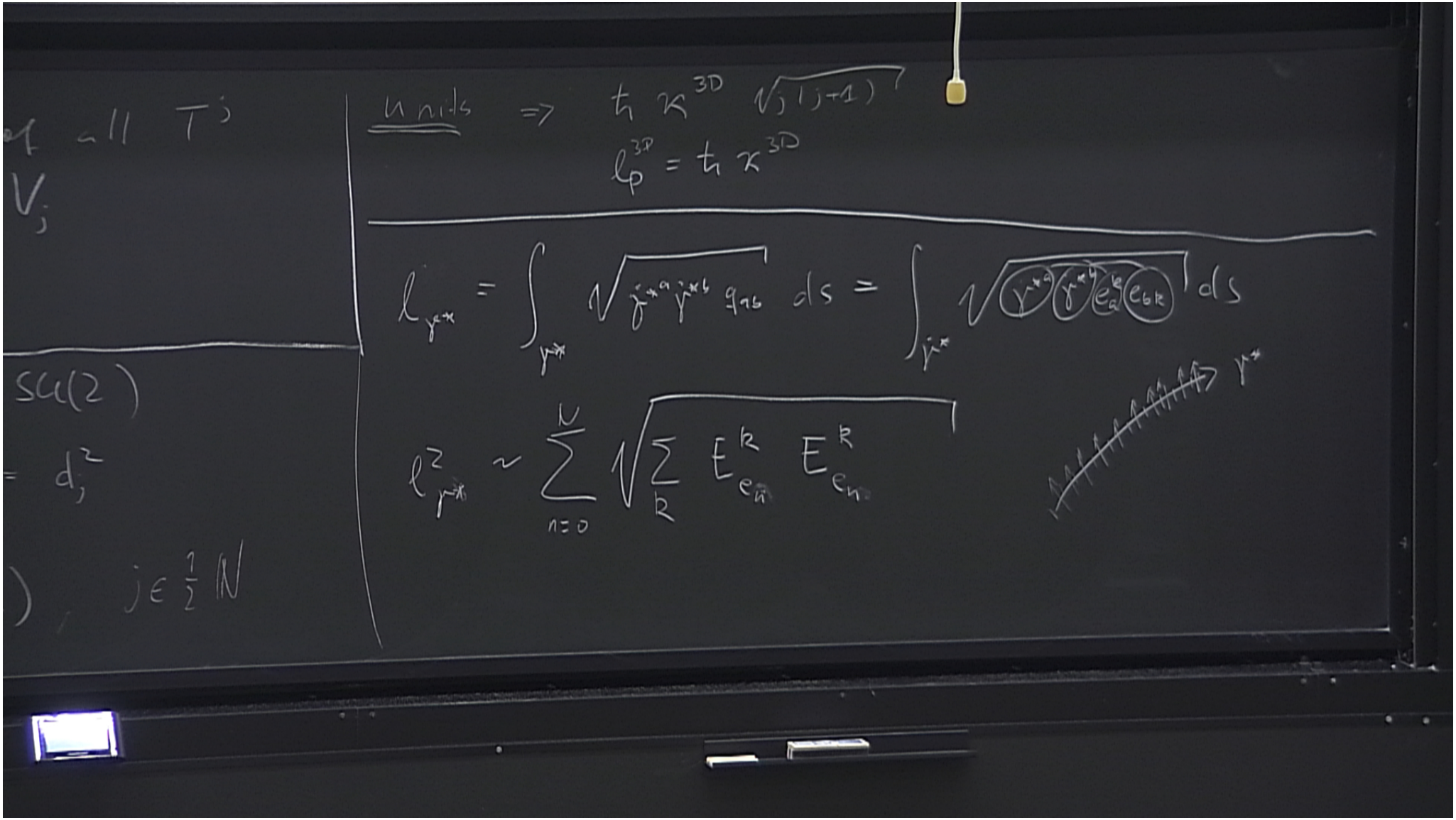
$\simeq V_j$

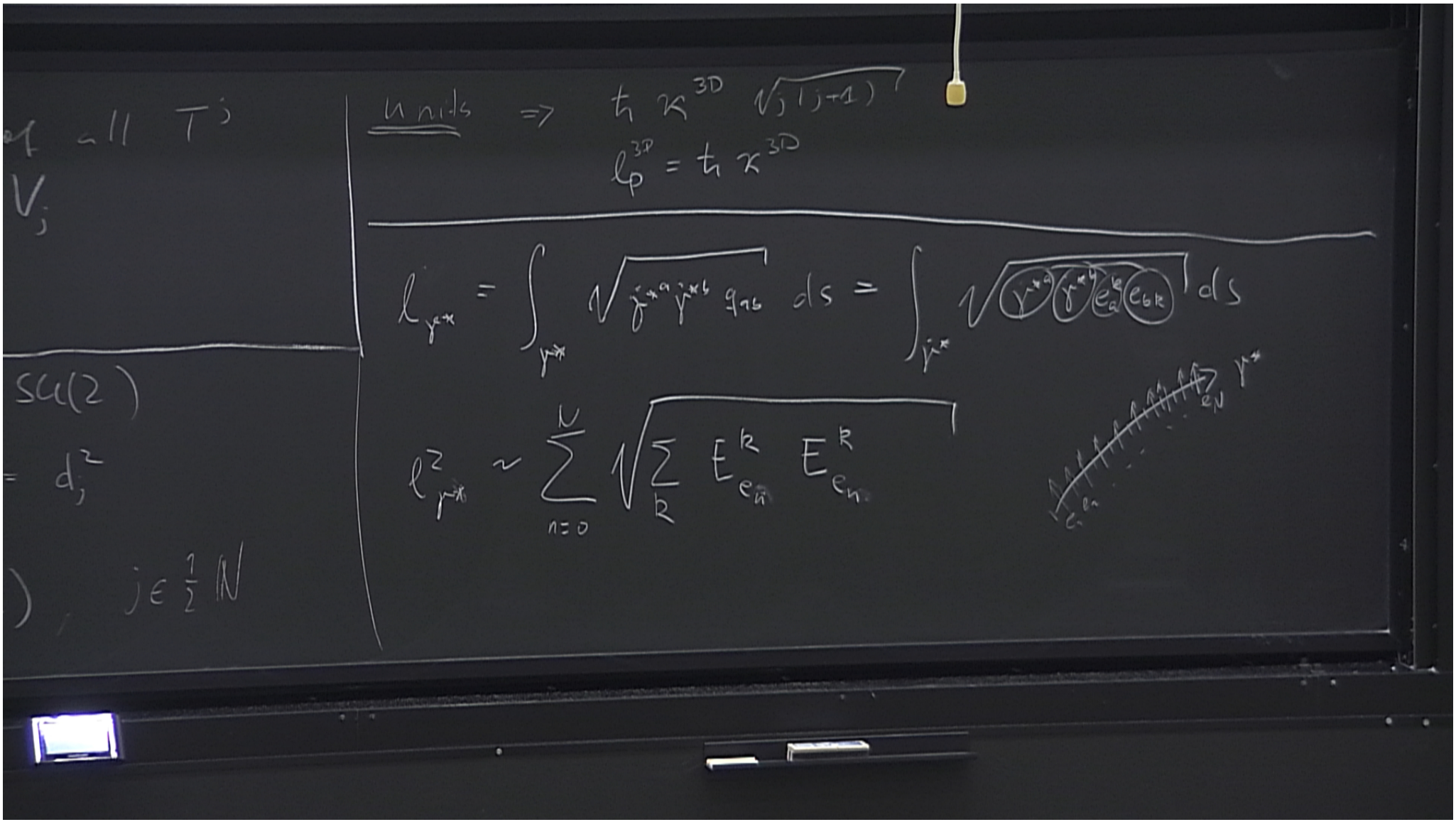
· $C$ is the Casimir operator of $SU(2)$

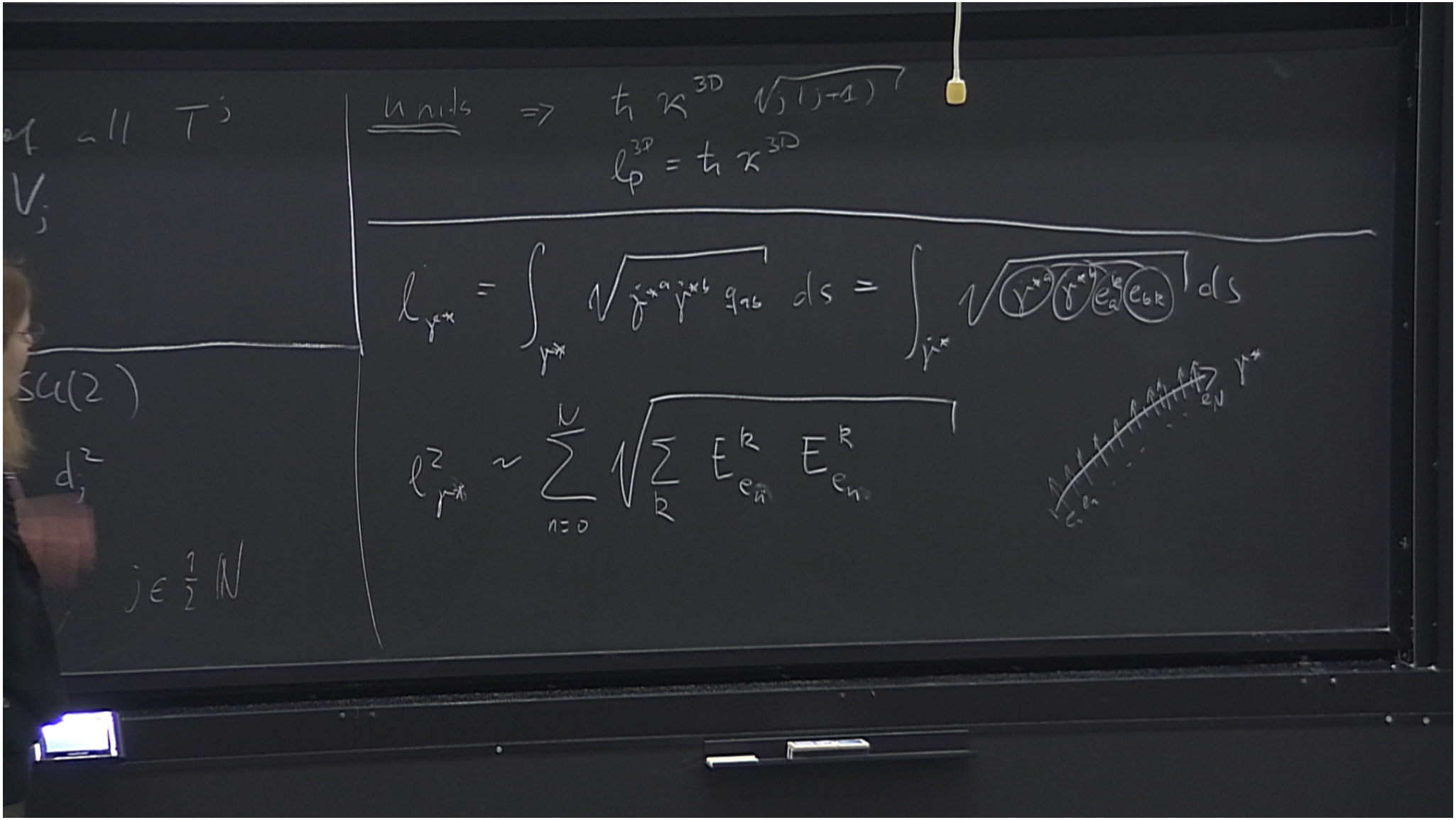
$\lambda = -j(j+1)$, degeneracy $= d_j^2$

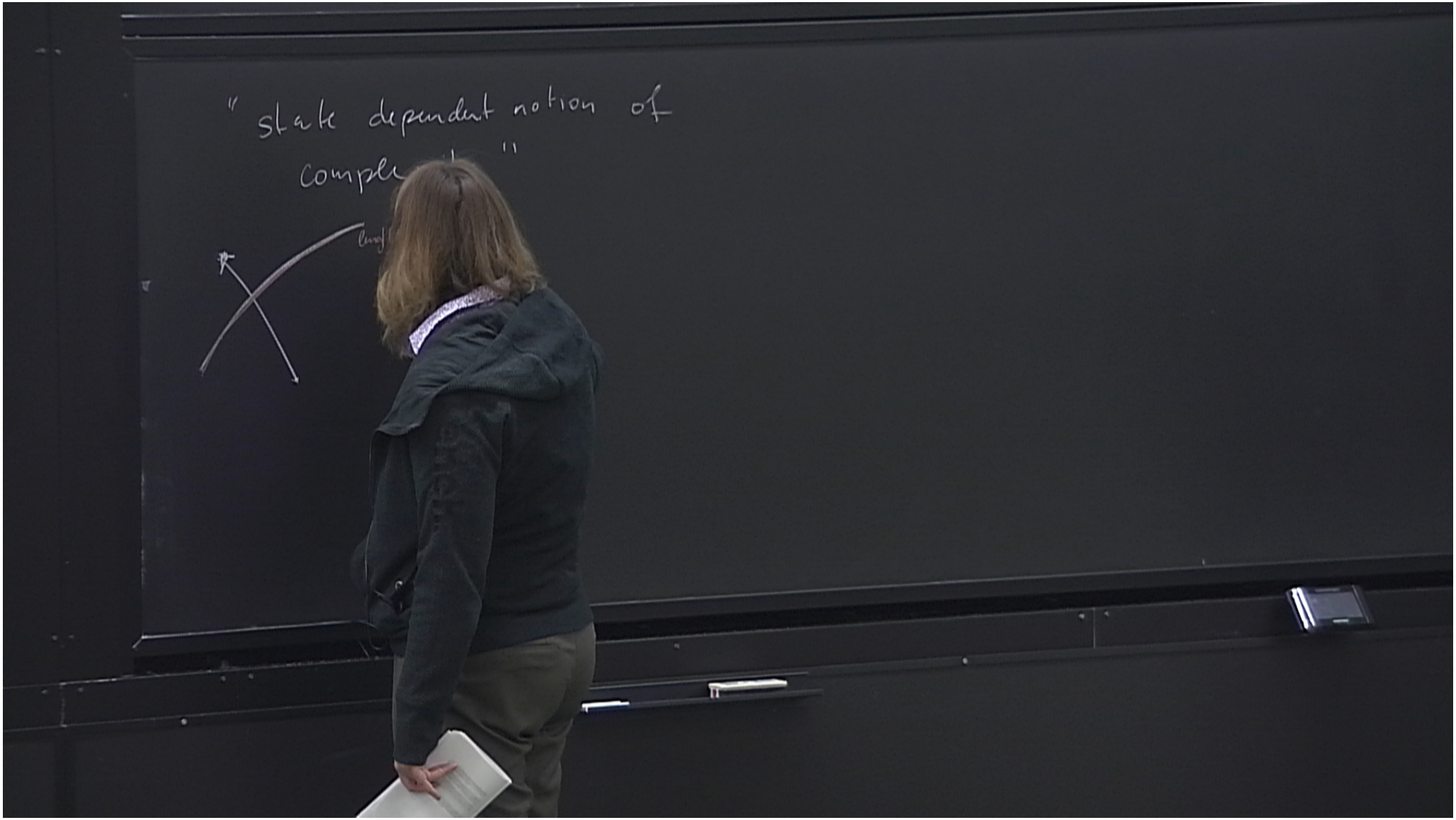
$\ell_{ext}^2$ has a spectrum $\hbar^2 j(j+1)$, $j \in \frac{1}{2}\mathbb{N}$

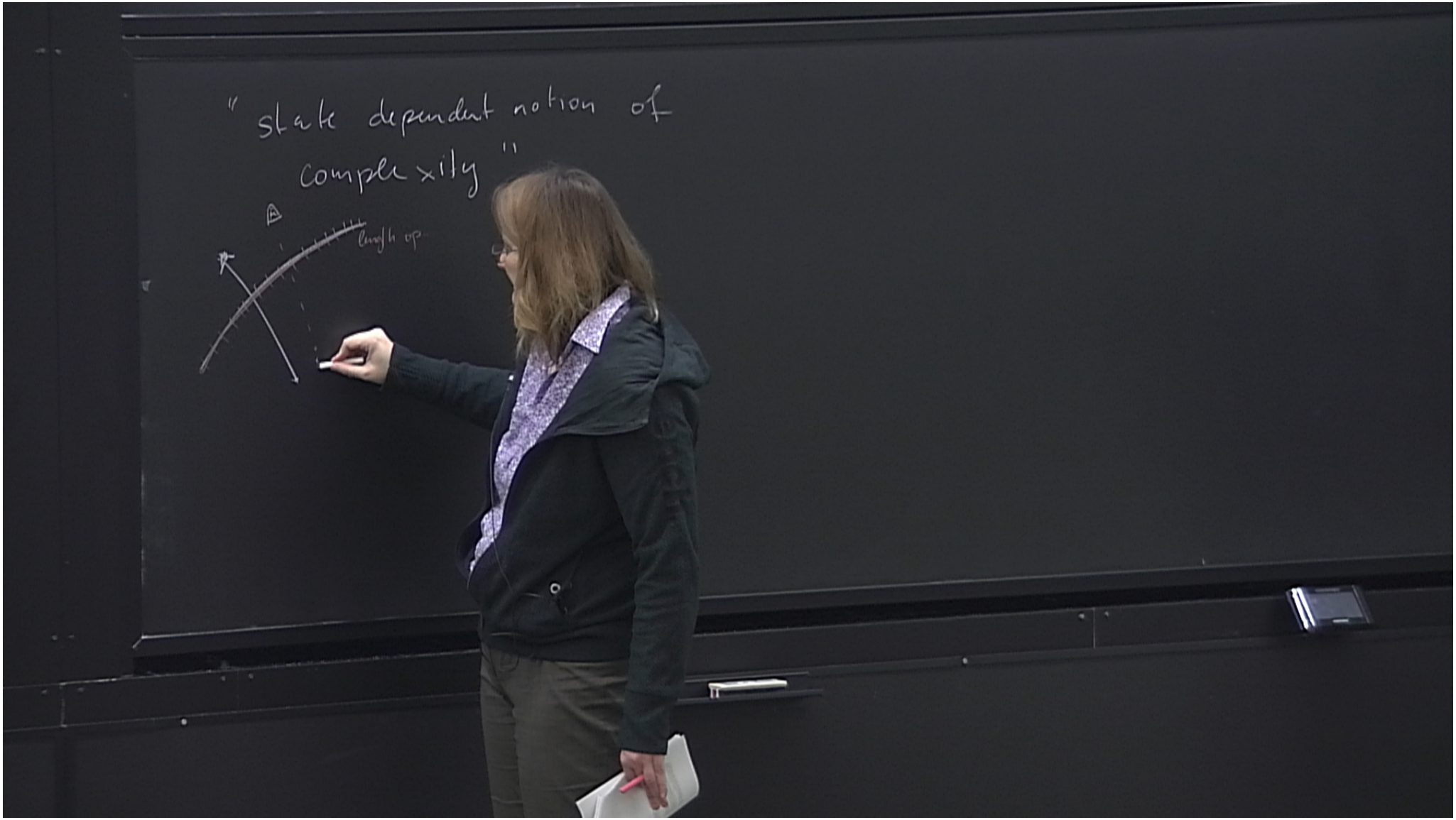

units
SU(2)
$j \in \frac{1}{2}\mathbb{N}$

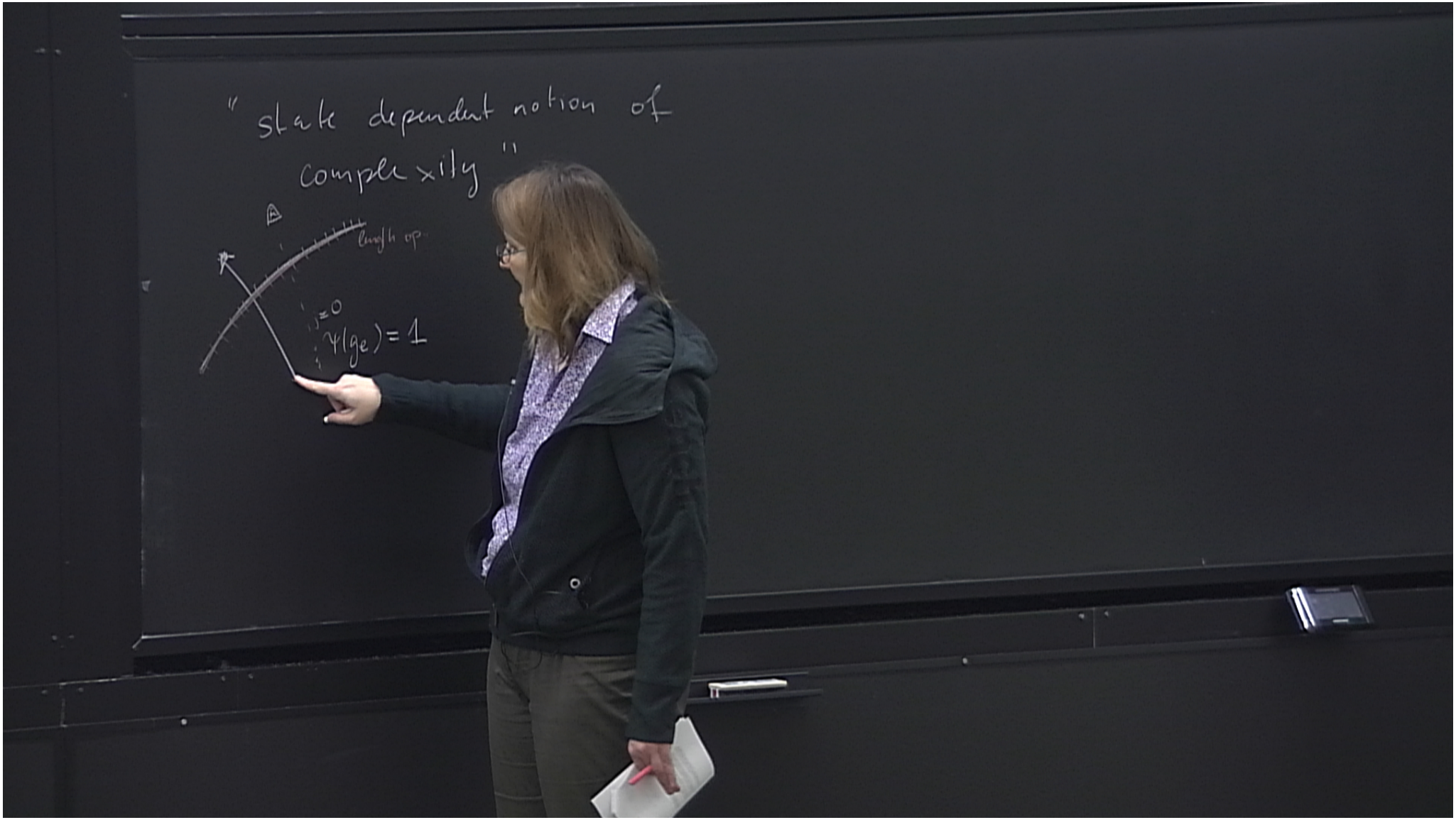

of all T^j
V_j
SU(2)
= d_j^2
j ∈ 1/2 N
units ⇒ t κ^3D √(j(j+1))
ℓ_P^3D = t κ^3D
ℓ_γ* = ∫_γ* √(ẋ^a ẋ^b q_ab) ds = ∫_γ* √(γ^ab e^a_k e^b_k) ds
ℓ²_γ* ~ Σ_{n=0}^{N} √(Σ_R E^R_{e_n} E^R_{e_n})
γ*

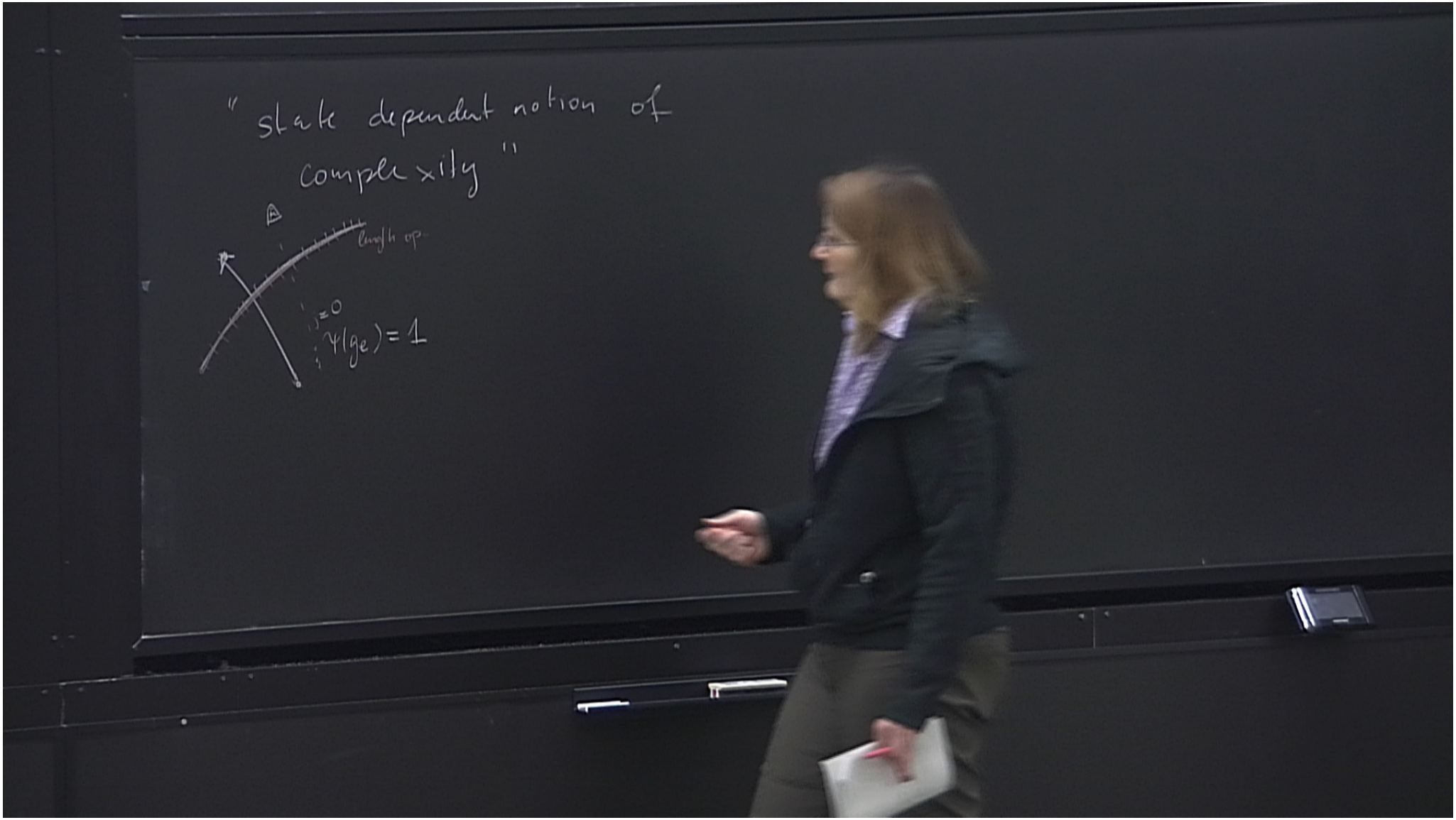

all $T^j$
$V_j$
$SU(2)$
$= d_j^2$
$j \in \frac{1}{2}\mathbb{N}$
units $\Rightarrow$ $t_1 \kappa^{3D}$ $\sqrt{j(j+1)}$
$\ell_p^{3D} = t_1 \kappa^{3D}$
$\ell_{\gamma^*} = \int_{\gamma^*} \sqrt{\dot{\gamma}^{*a} \dot{\gamma}^{*b} q_{ab}}\, ds = \int_{\gamma^*} \sqrt{\gamma^{*a} \gamma^{*b} e^k_a e_{bk}}\, ds$
$\ell^2_{\gamma^*} \sim \sum_{n=0}^{N} \sqrt{\sum_R E^k_{e_n} E^k_{e_n}}$
$\gamma^*$
$e_N$


units
SU(2)
$j \in \frac{1}{2}\mathbb{N}$

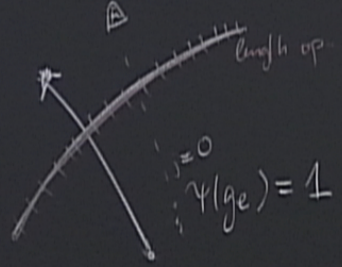

" state dependent notion of

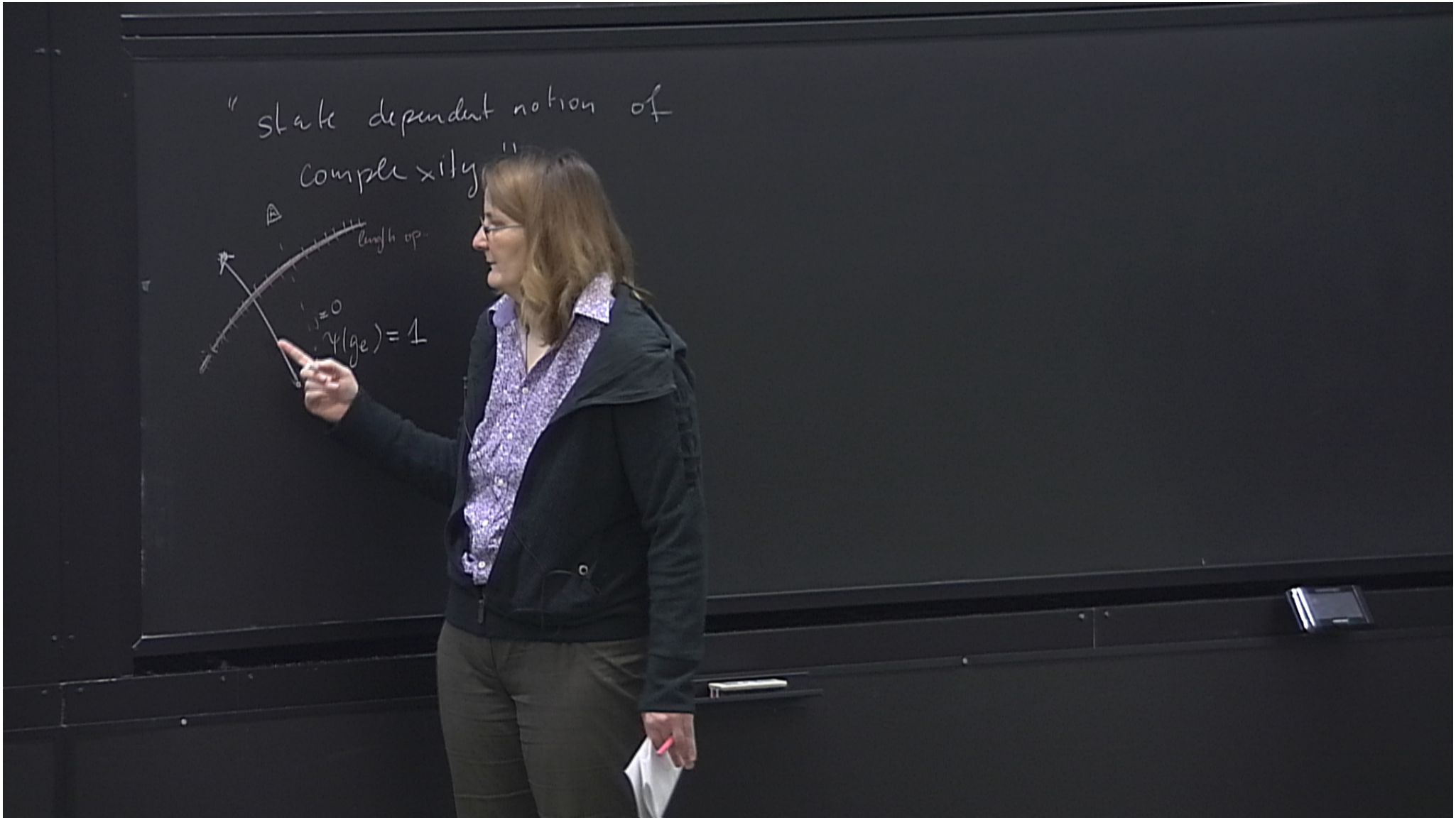

" state dependent notion of
complexity "

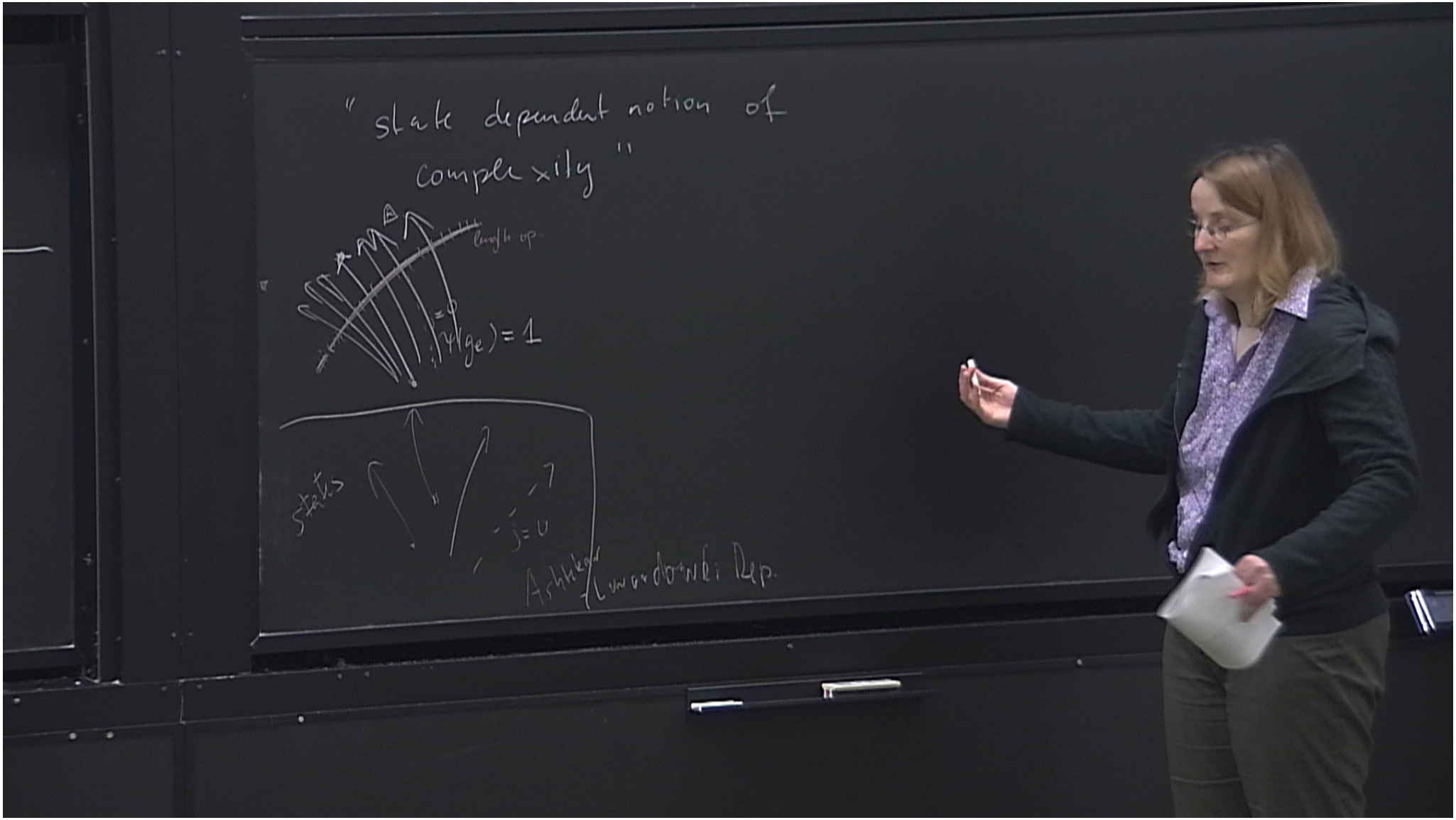

" state dependent notion of
complexity "
rough op.
j=0
Y(ge) = 1


" state dependent notion of
complexity "
length op.
j=0
Ψ(ge) = 1

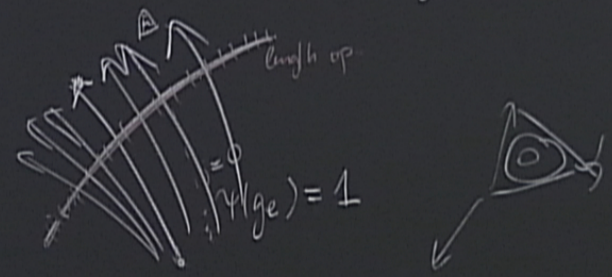
"state dependent notion of complexity"

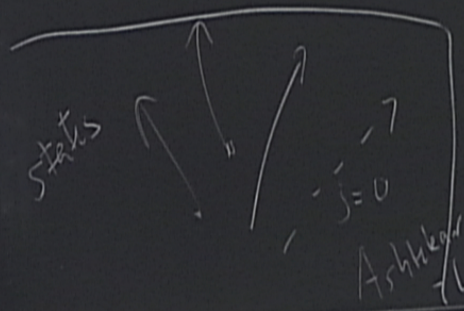


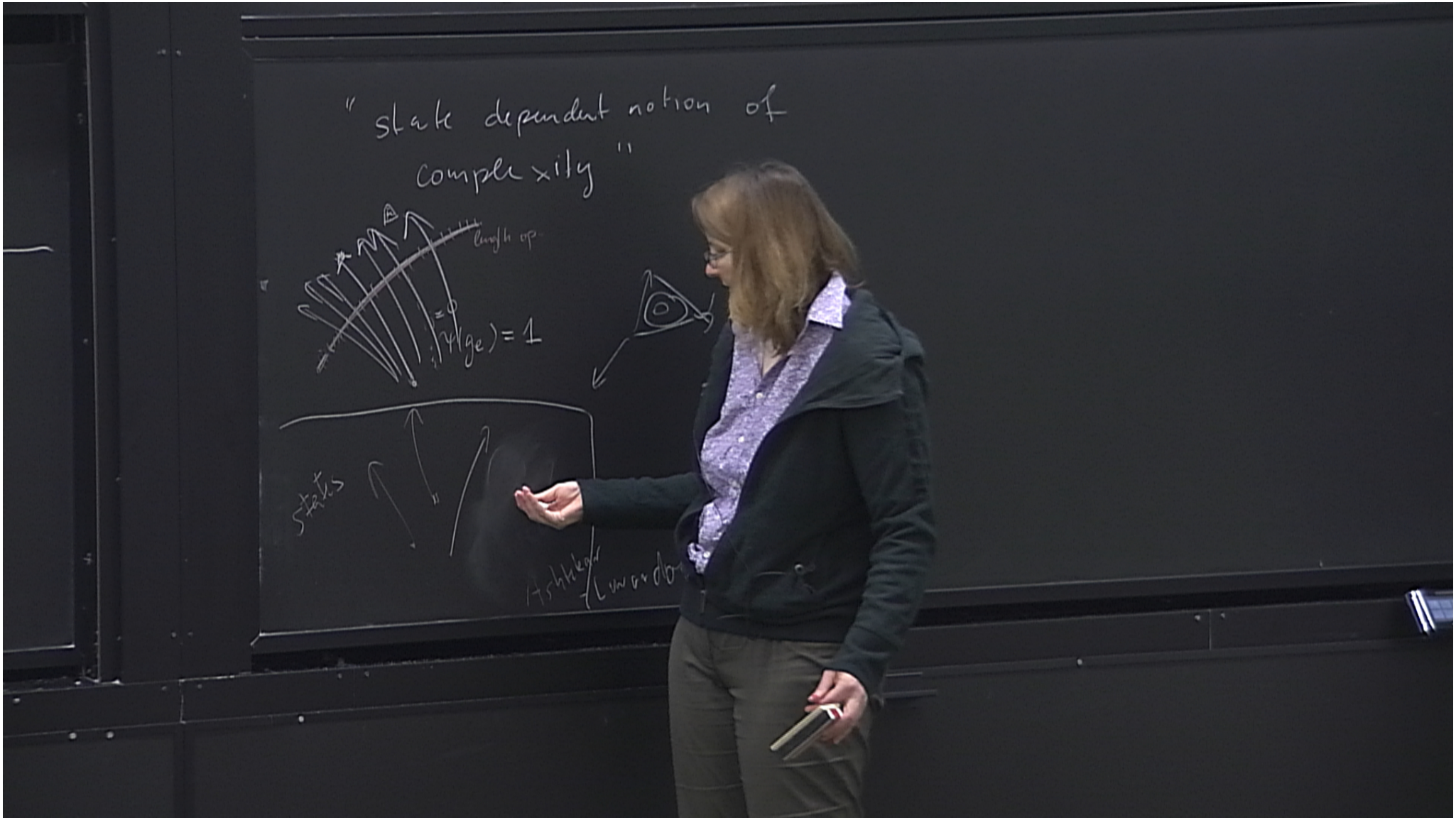

" state dependent notion of
complexity "
A
Length op.
$\psi(ge) = 1$

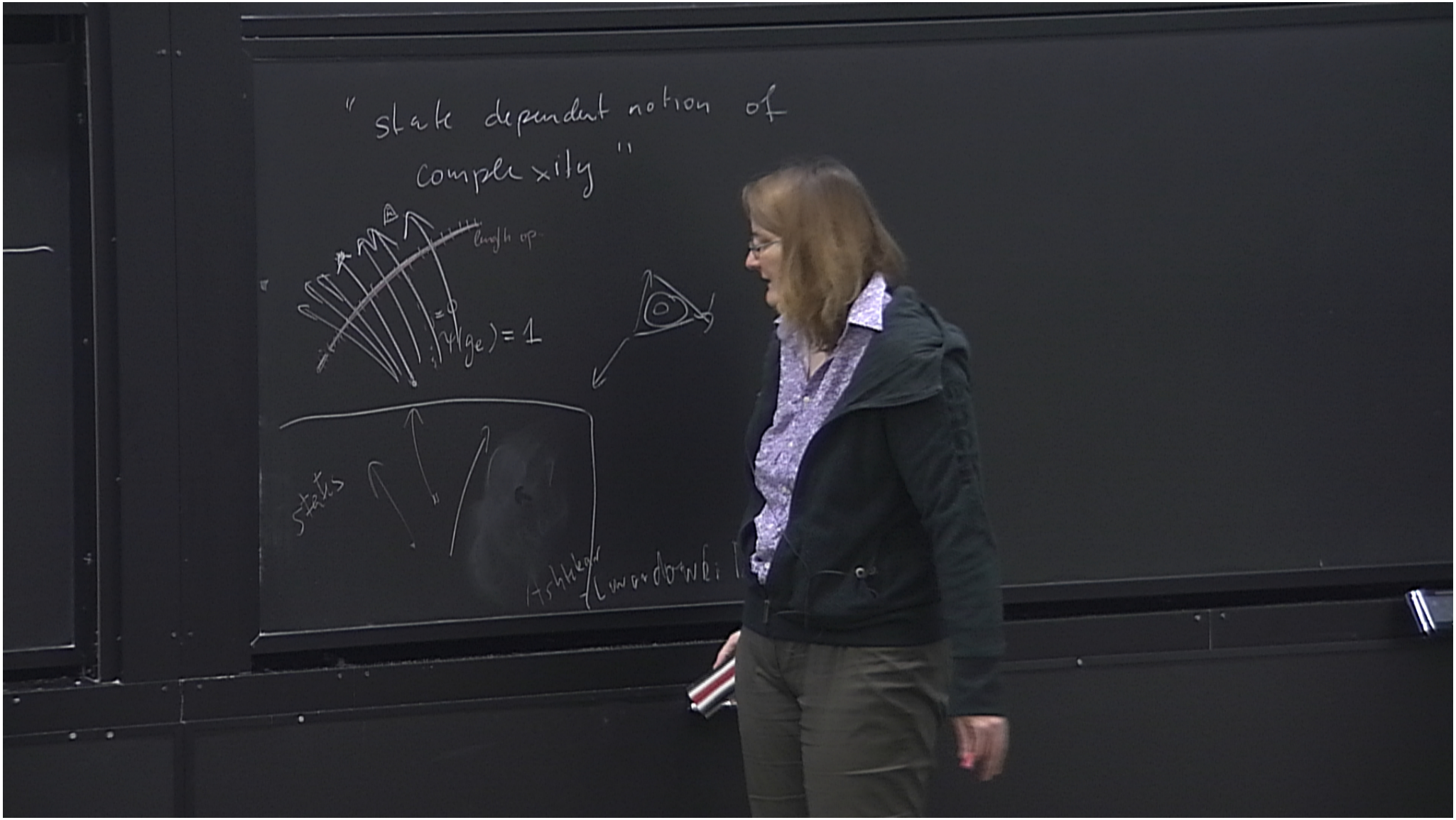

" state dependent notion of complexity "
length op.
$\chi(ge) = 1$
states
$j = 0$
Ashtekar/Lewandowski Rep.

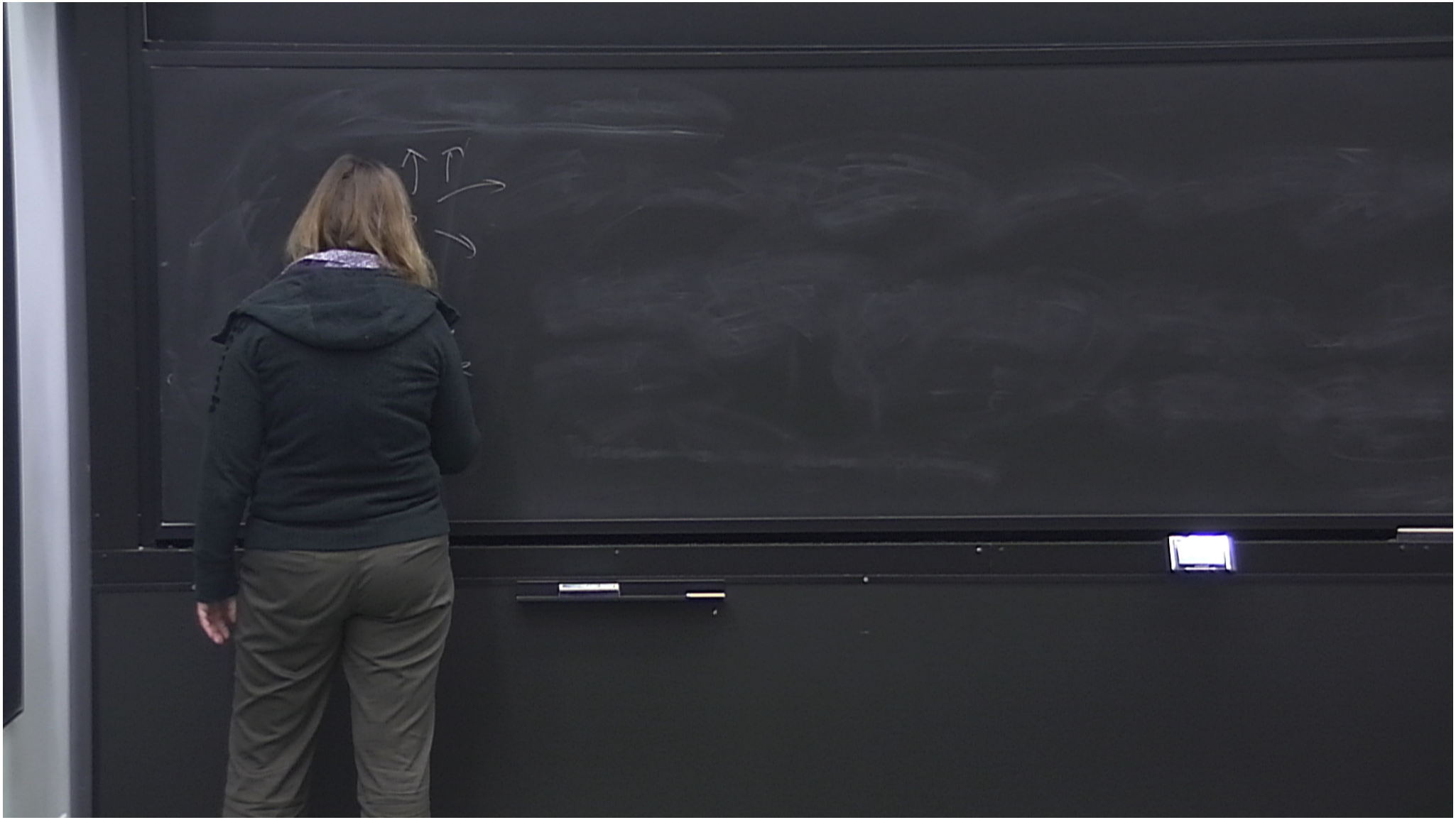
"state dependent notion of complexity"

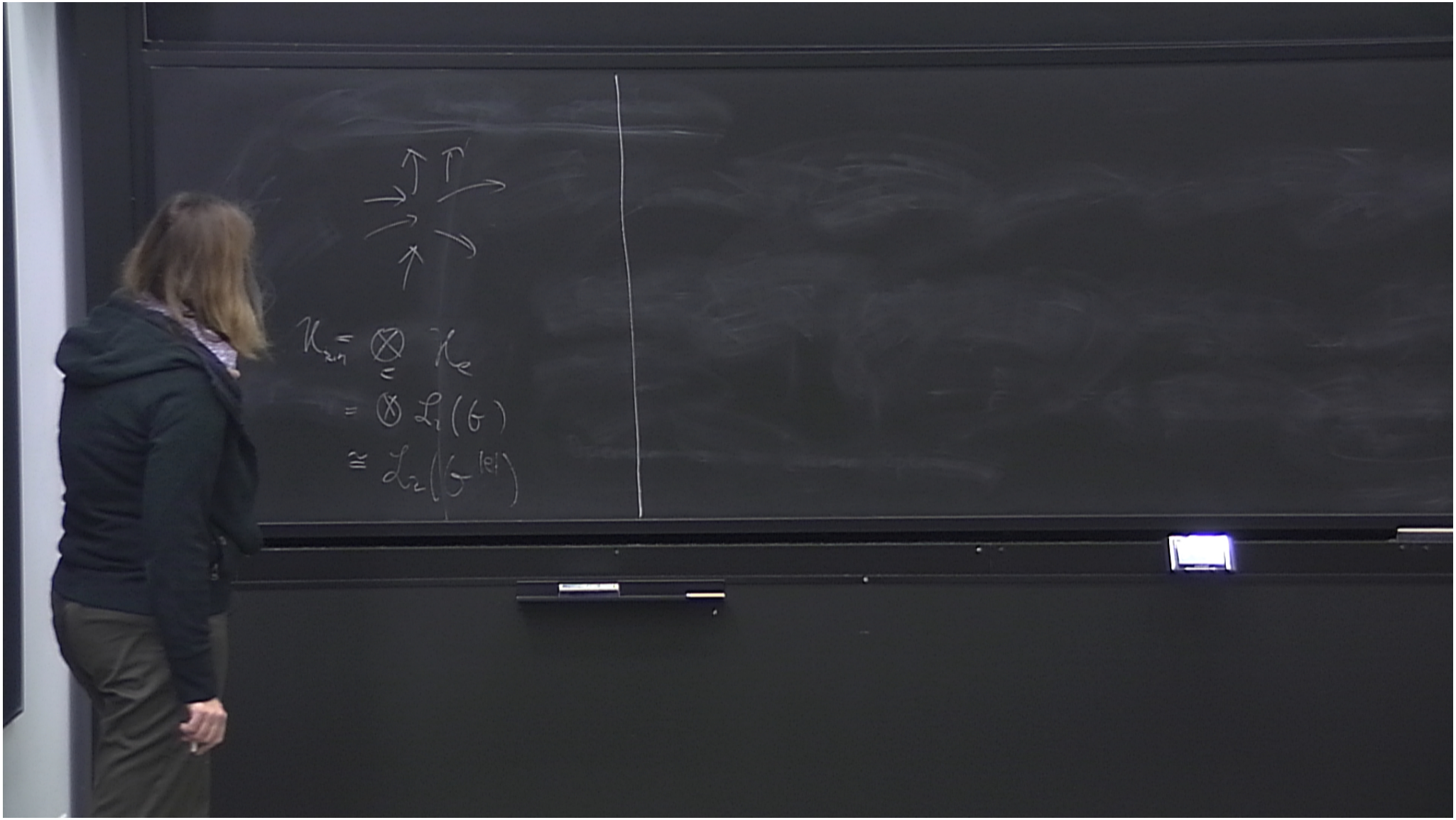


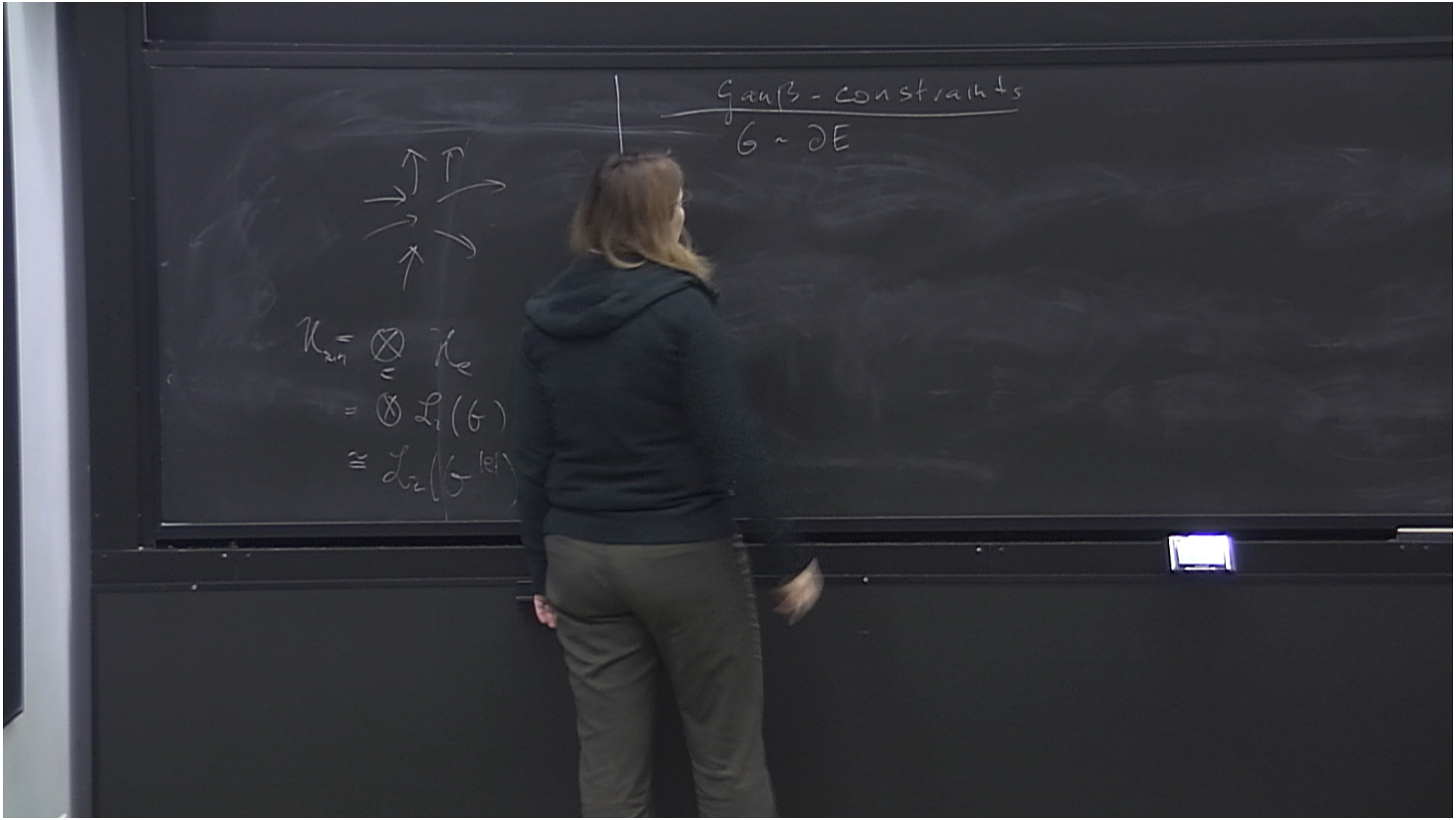



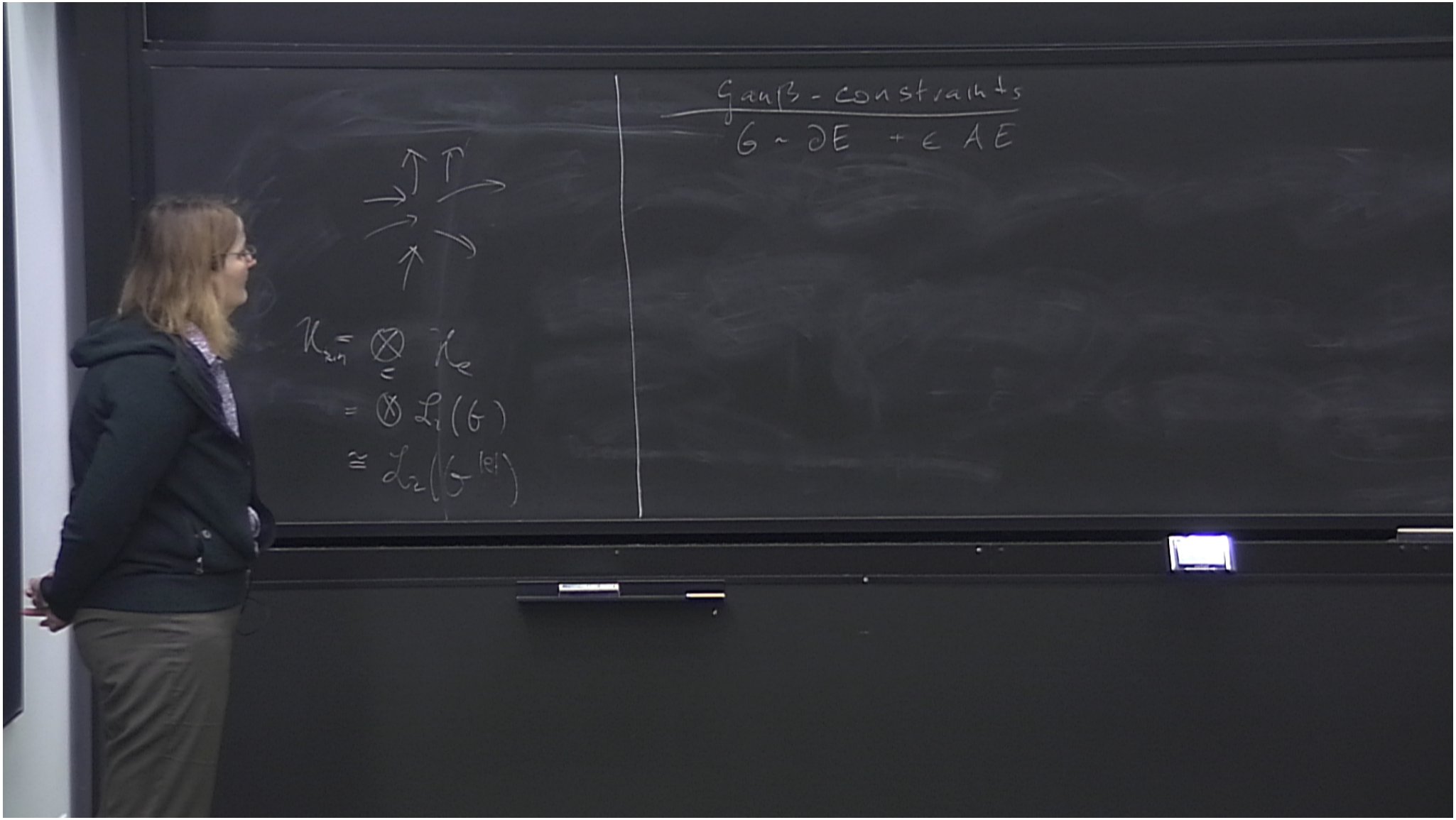

" state dependent notion of
complexity "
length op
=0
$\chi(ge) = 1$
states

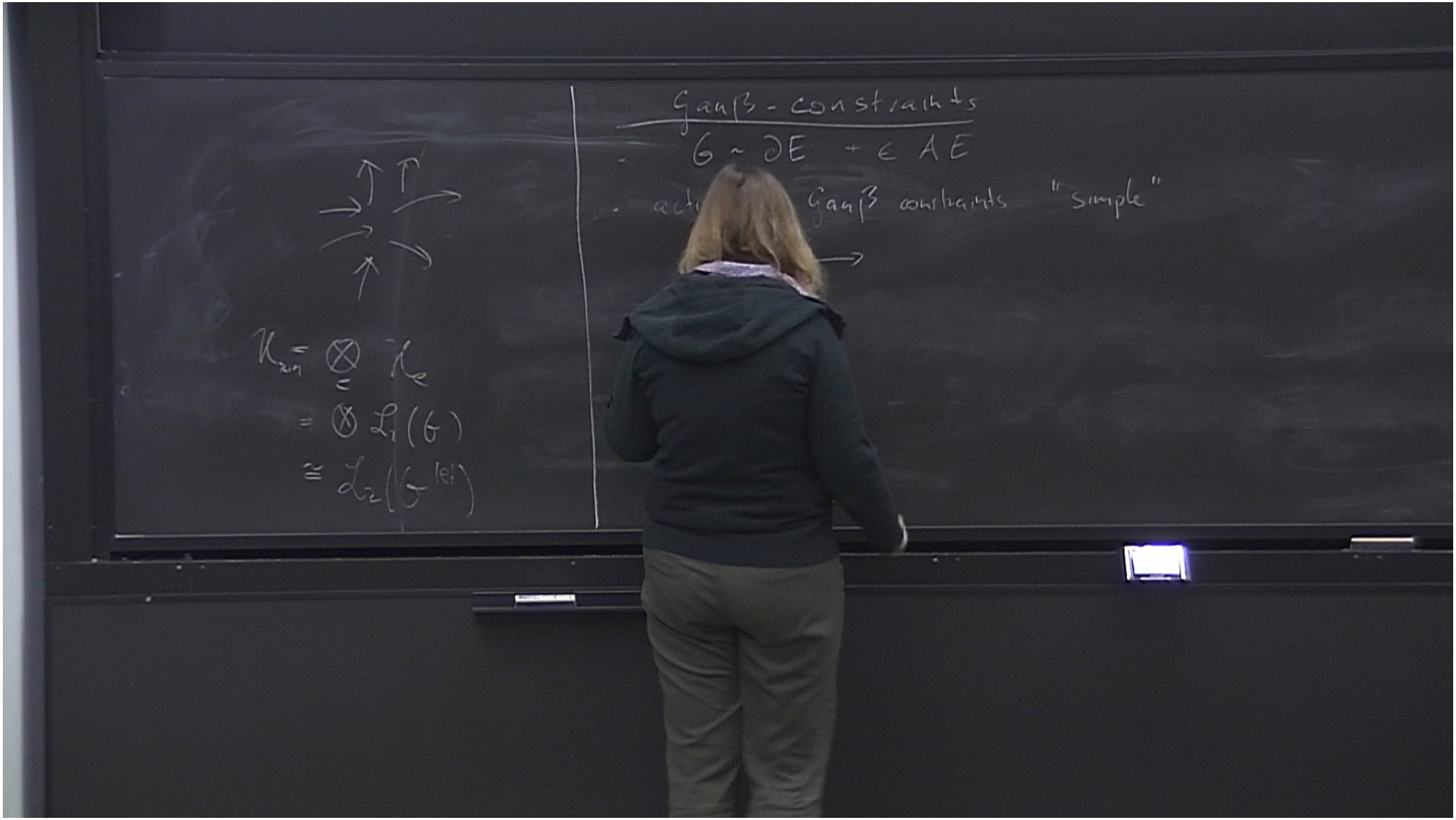

" state dependent notion of
complexity "
Length op
=0
=1

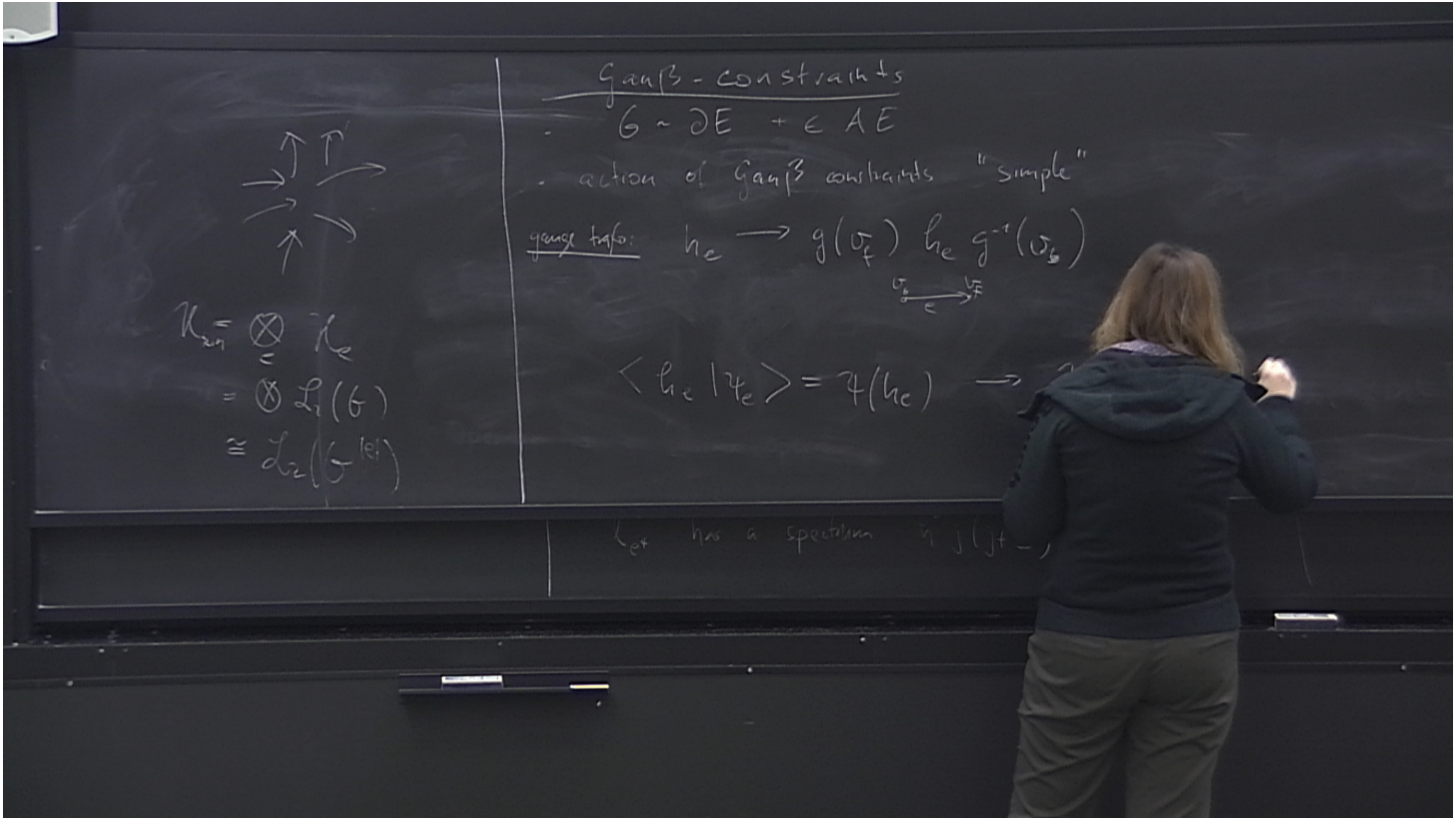


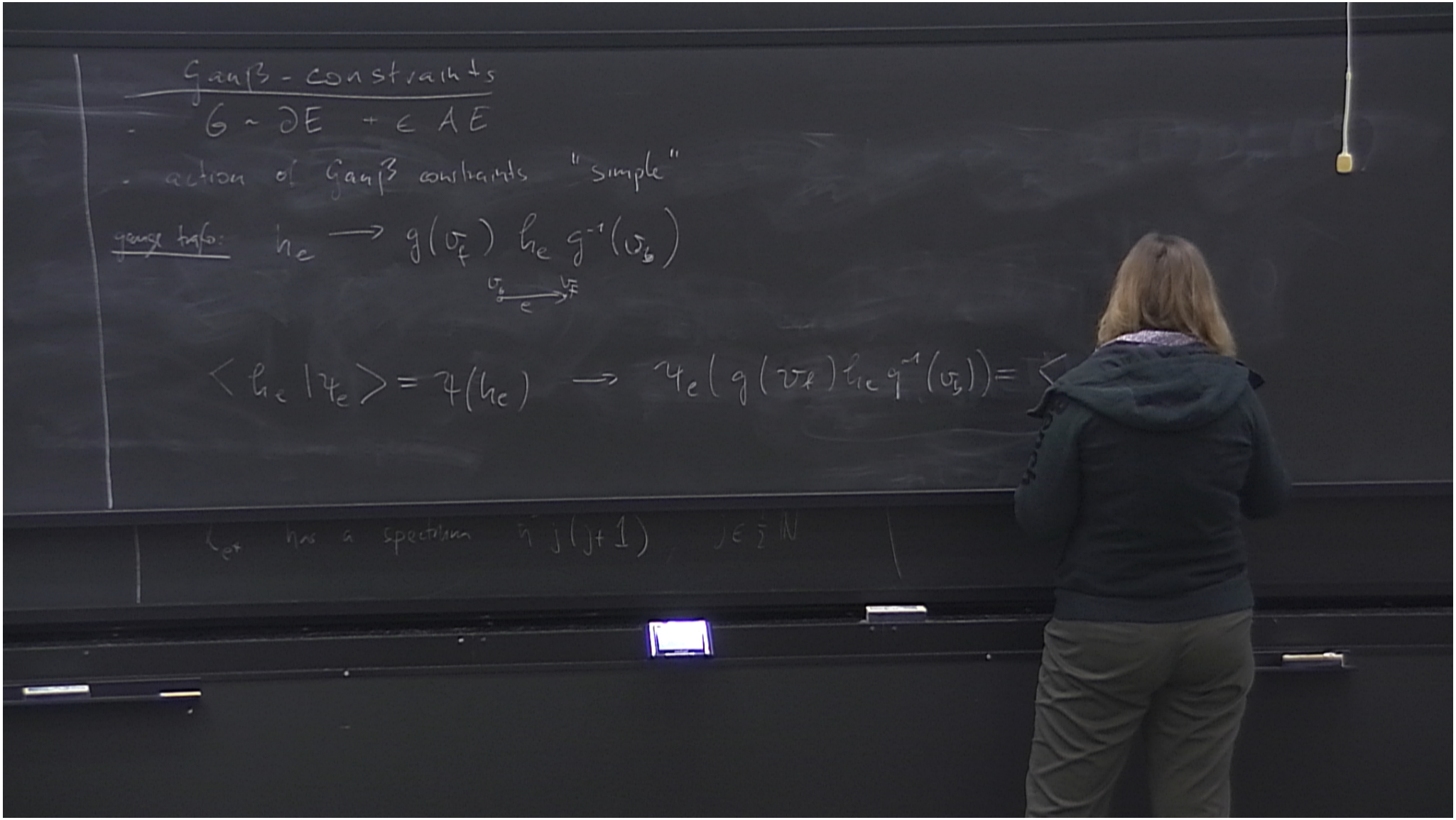

$\mathcal{H}_{sym} = \bigotimes_{e} \mathcal{H}_e$
$= \bigotimes \mathcal{L}_2(G)$
$\cong \mathcal{L}_2(G^{|E|})$

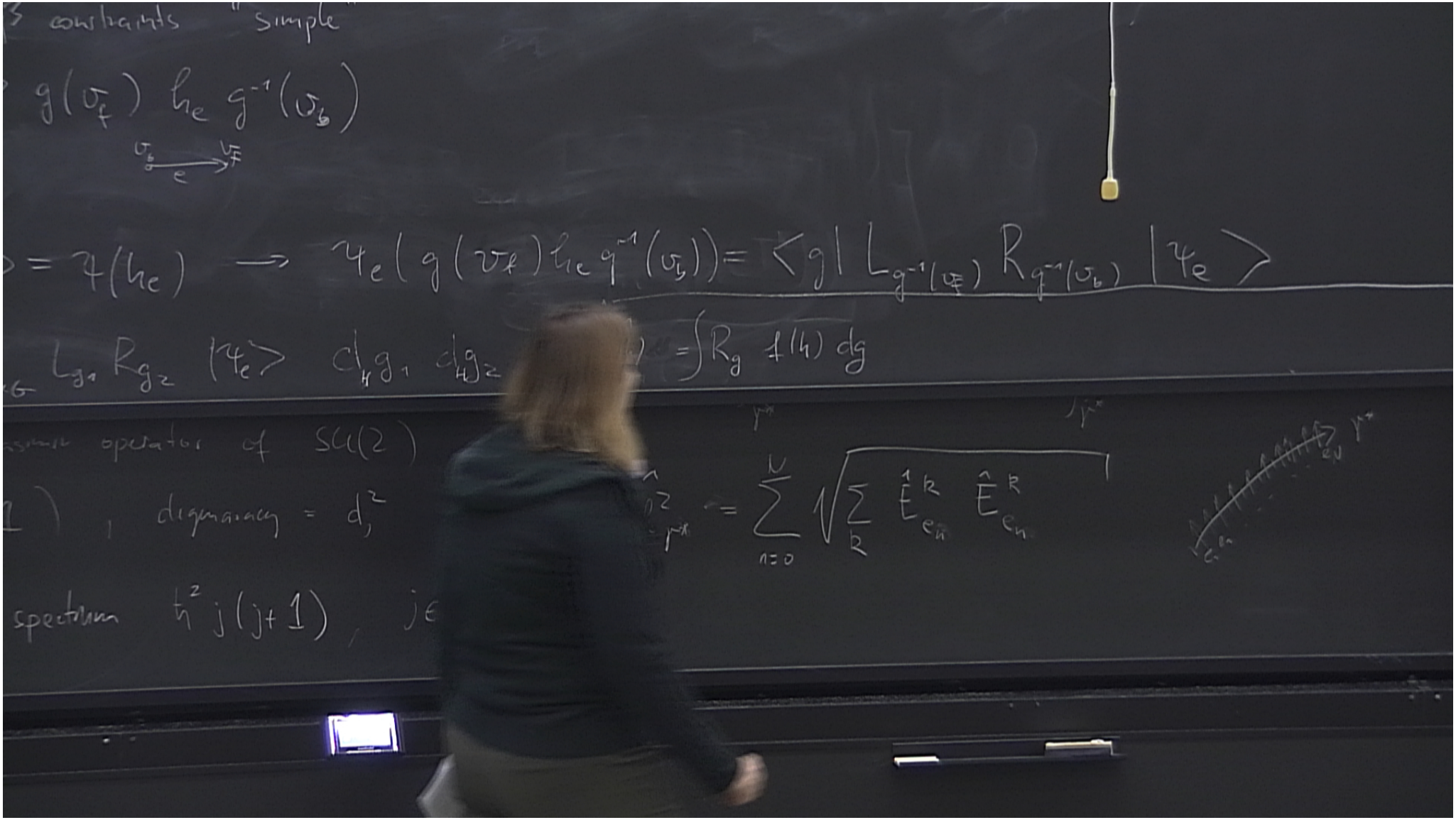

Gauß - constraints
G ~ ∂E
Γ
$\mathcal{H}_{sin} = \bigotimes_e \mathcal{H}_e$
$= \bigotimes \mathcal{L}_2(G)$
$\cong \mathcal{L}_2(G^{|e|})$

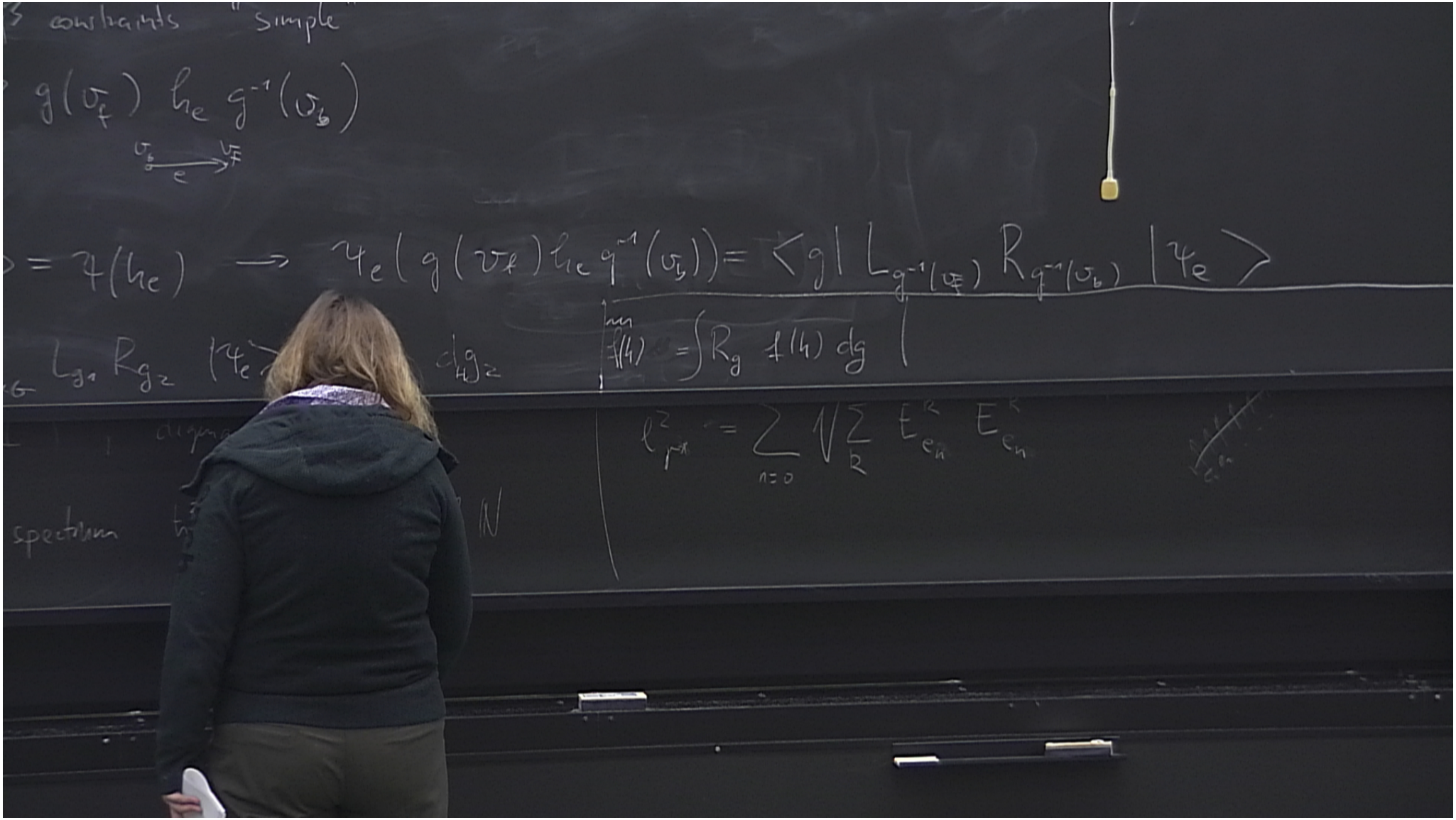

Gauß-constraints
G ~ ∂E + ∈ AE
Γ
$\mathcal{H}_{sin} = \bigotimes_e \mathcal{H}_e$
$= \bigotimes \mathcal{L}_2(G)$
$\cong \mathcal{L}_2(G^{|e|})$

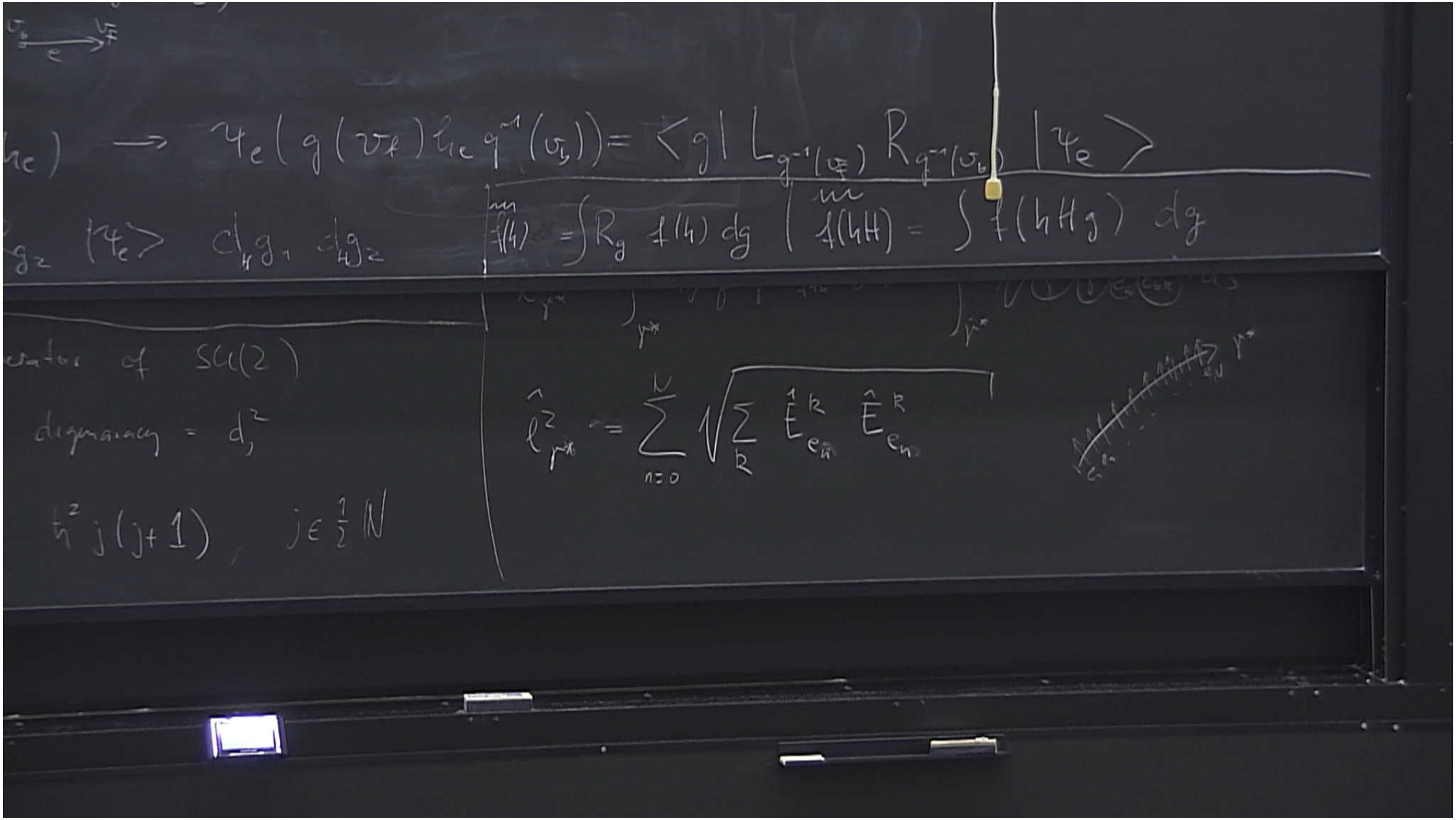

Gauß - constraints
G ~ ∂E + ε AE
Gauß constraints "simple"

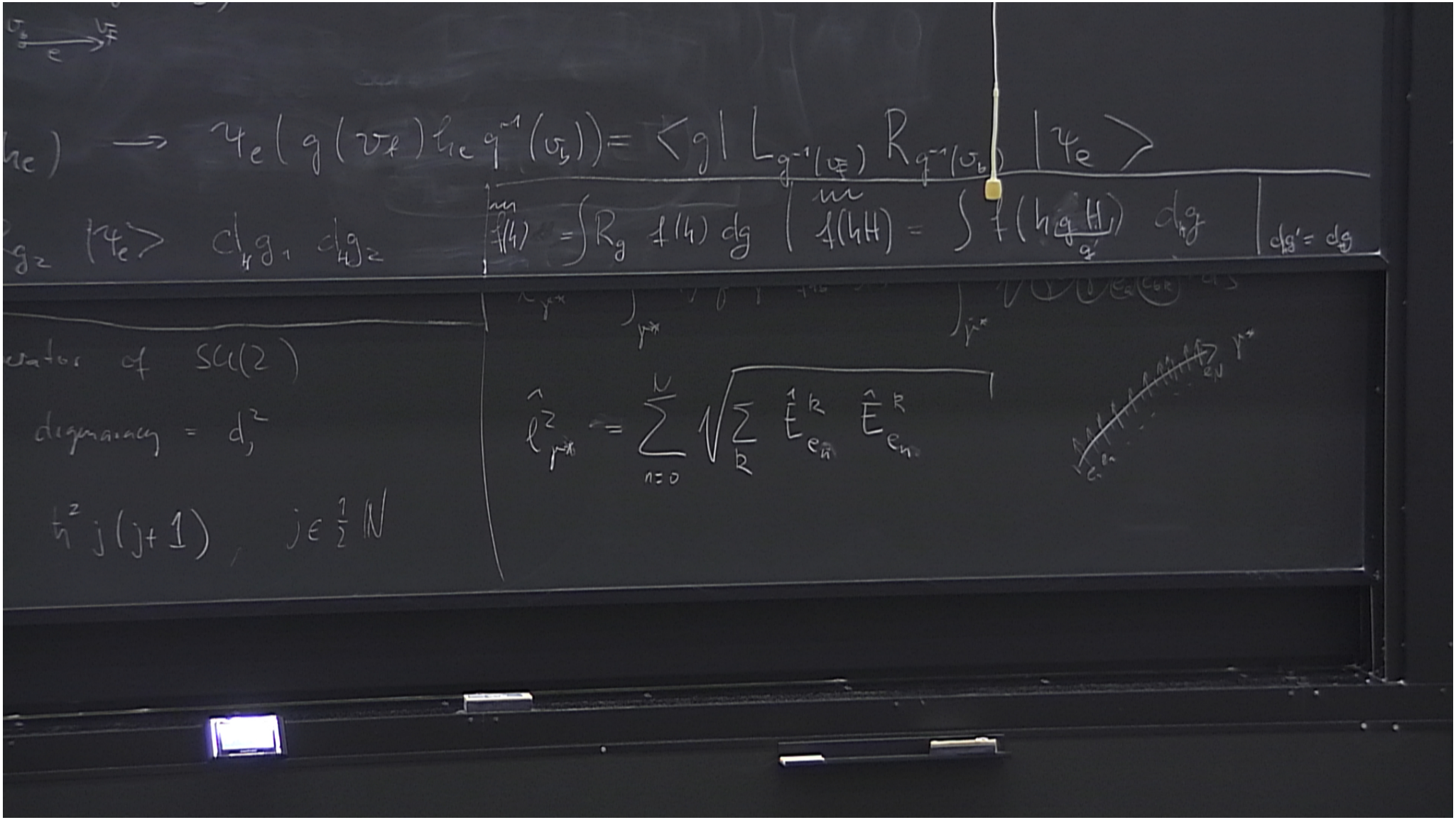

Gauß-constraints
G ~ ∂E + ε AE
- action of Gauß constraints "simple"
gauge trafo: $h_e \longrightarrow g(v_t)\, h_e\, g^{-1}(v_s)$
$\mathcal{H}_{kin} = \bigotimes_e \mathcal{H}_e$
$= \bigotimes_e L_2(G, |e|)$
$\cong L_2(G^{|E|})$
$\langle h_e | \gamma_e \rangle = \gamma(h_e) \longrightarrow$
Let has a spectrum

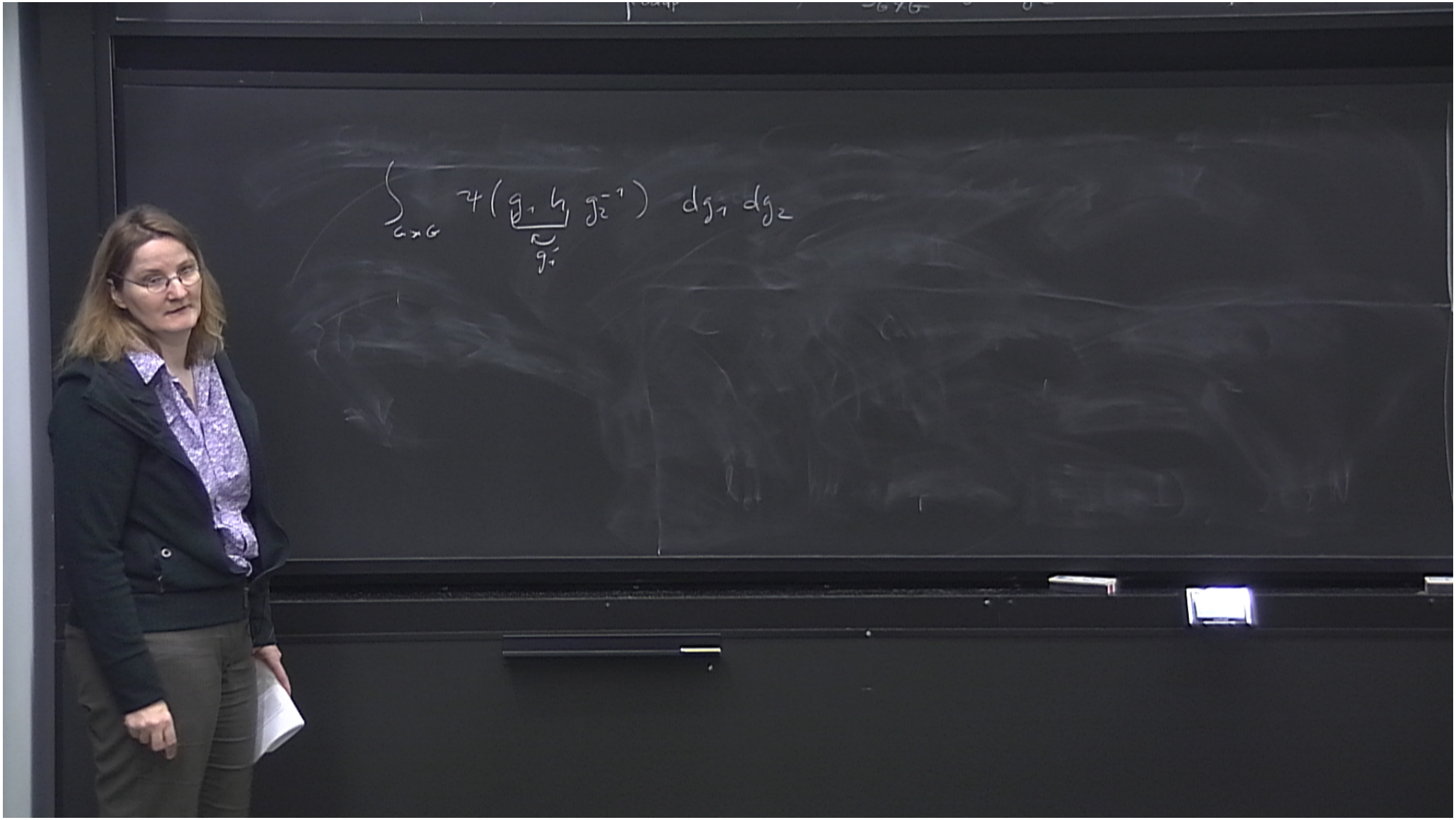

Gauß - constraints
G ~ ∂E + ε AE
- action of Gauß constraints "simple"
gauge trafo: $h_e \longrightarrow g(v_t)\, h_e\, g^{-1}(v_s)$
$v_s \xrightarrow{e} v_t$
$\langle h_e | \psi_e \rangle = \psi(h_e) \longrightarrow \psi_e(g(v_t)\, h_e\, g^{-1}(v_s)) =$
$\hat{G}_{ef}$ has a spectrum $\hbar\, j(j+1)$ $j \in \frac{1}{2}\mathbb{N}$

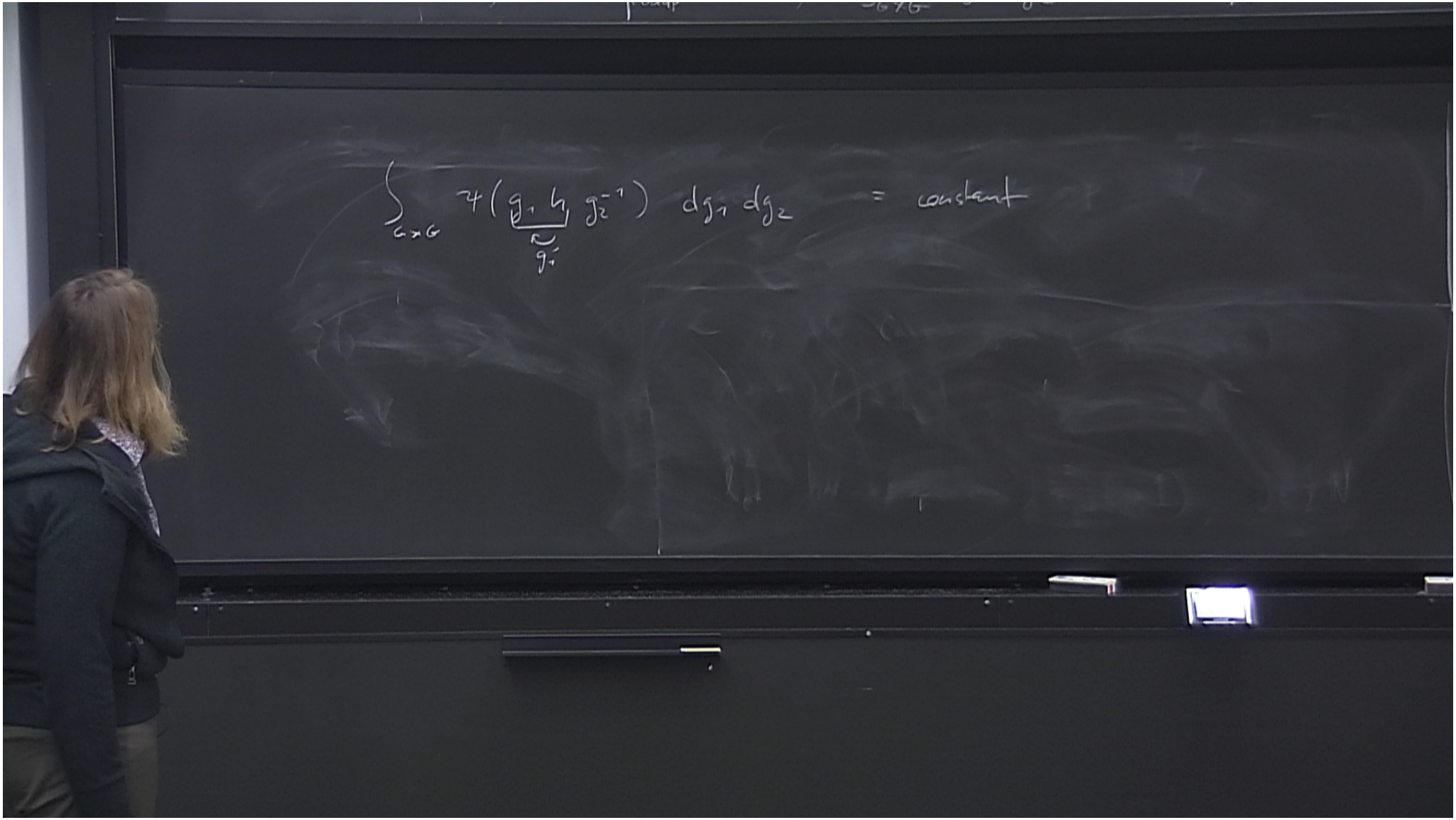

constraints "simple"
$g(v_f)\, h_e\, g^{-1}(v_b)$
$v_b$
$e$
$v_f$
$= \mathcal{T}(h_e) \longrightarrow \psi_e(g(v_f)\, h_e\, g^{-1}(v_b)) = \langle g | L_{g^{-1}(v_f)}\, R_{g^{-1}(v_b)} | \psi_e \rangle$
$L_{g_1} R_{g_2} |\psi_e\rangle$
$= \int R_g\, f(h)\, dg$
operator of $SU(2)$
, degeneracy $= d_j^2$
spectrum $\hbar^2 j(j+1)$
$= \sum_{n=0}^{N} \sqrt{\sum_R \hat{E}^R_{e_n} \hat{E}^R_{e_n}}$

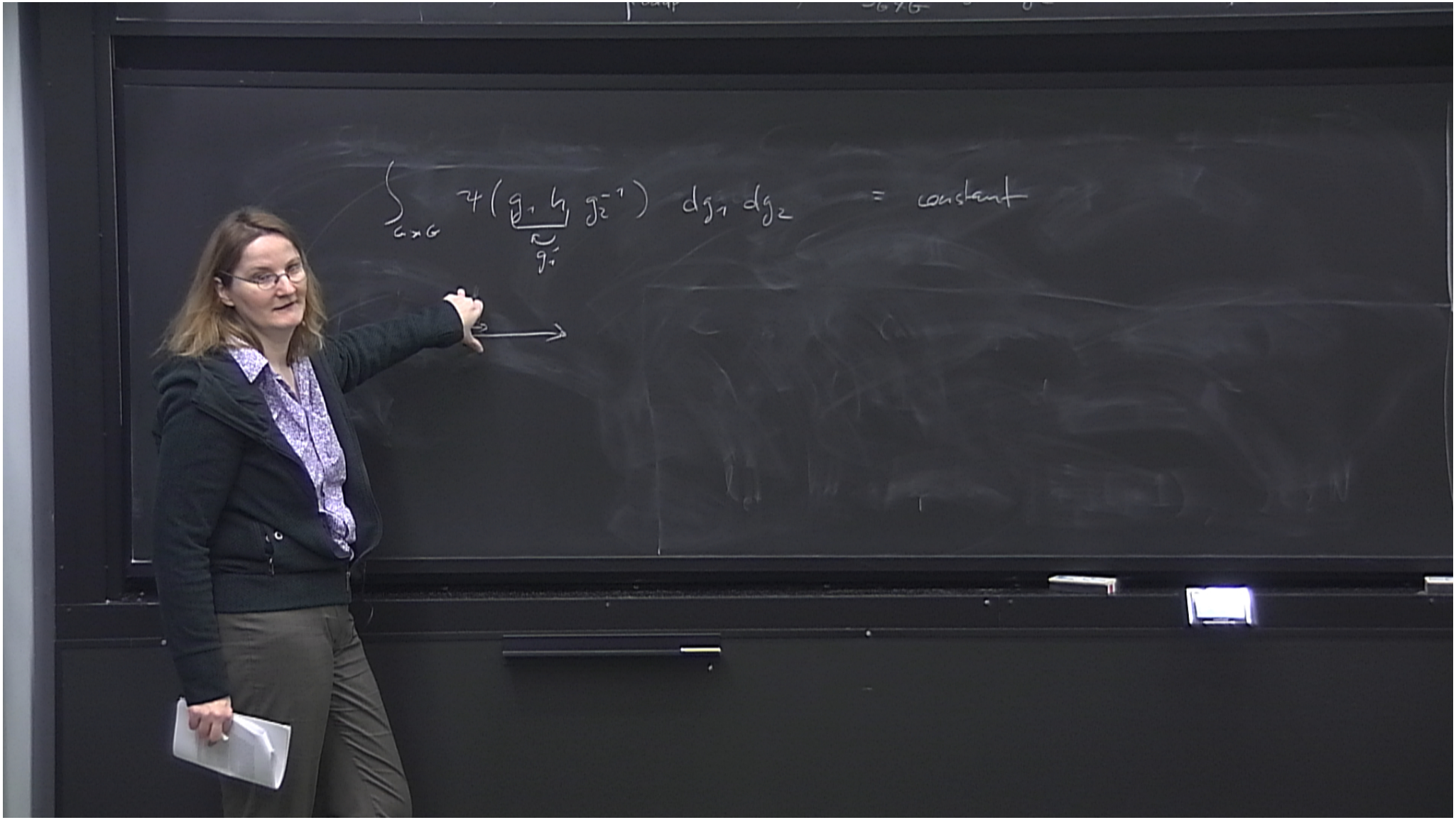

constraints "simple"
$g(v_f)\, h_e\, g^{-1}(v_b)$
$v_b$ e $v_f$
$= \pi(h_e) \longrightarrow \psi_e(g(v_f) h_e g^{-1}(v_b)) = \langle g | L_{g^{-1}(v_f)} R_{g^{-1}(v_b)} | \psi_e \rangle$
$L_{g_1} R_{g_2} |\psi_e\rangle$
$dg_2$
$f(h) = \int R_g f(h)\, dg$
spectrum
N
$\ell^2_{\mu^*} = \sum_{n=0} \sqrt{\sum_R} E^k_{e_n} E^k_{e_n}$


$\psi_e(g(v_f)\, h_e\, g^{-1}(v_i)) = \langle g | L_{g^{-1}(v_f)} R_{g^{-1}(v_i)} | \psi_e \rangle$
$f(h) = \int R_g f(h)\, dg$
$f(hH) = \int f(hHg)\, dg$
$\hat{\ell}^2_{\mu^*} = \sum_{n=0}^{N} \sqrt{\sum_R \hat{E}^{1\,R}_{e_n} \hat{E}^{R}_{e_n}}$
$t^2 j(j+1), \quad j \in \frac{1}{2}\mathbb{N}$


$\psi_e(g(v_f)\, h_e\, g^{-1}(v_i)) = \langle g | L_{g^{-1}(v_f)} R_{g^{-1}(v_i)} | \psi_e \rangle$
$f(h) = \int R_g f(h)\, dg$
$f(hH) = \int f(hgH)\, dg$
$dg' = dg$
$\hat{\ell}^2_{\mu^*} = \sum_{n=0}^{N} \sqrt{\sum_R \hat{E}^{1\,k}_{e_n} \hat{E}^{R}_{e_n}}$
degeneracy $= d_j^2$
$\hbar^2 j(j+1)$, $j \in \frac{1}{2}\mathbb{N}$


$\int_{G\times G} \varphi(g_1 h g_2^{-1})\, dg_1\, dg_2$
$g_1$


$\int_{G\times G} \psi(g_1 h g_2^{-1})\, dg_1\, dg_2 = \text{constant}$
$g_1$


$\int_{G\times G} \psi(g_1 h g_2^{-1})\, dg_1\, dg_2 = \text{constant}$
$g_1$

$$\int_{G\times G} \psi(g_1 h g_2^{-1})\, dg_1\, dg_2 = \text{constant}$$

$g_1' = g_1 h$

$\psi(g_1, g_2) \Rightarrow \psi(g_2 \cdot g_1')$ where $g_1' = g_1 h$, $h = g_1^{-1} g_1'$

$g_1 \quad g_1^{-1} g_1' \quad g_2$

