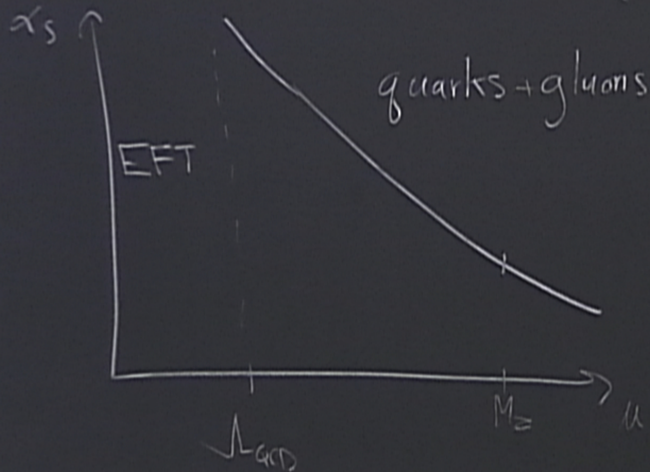
Title: Beyond Standard Model-10

Date: Mar 02, 2015 09:00 AM

URL: http://pirsa.org/15030001

Abstract:

$$\text{QCD:} \quad \mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} + \sum_I \bar{q}_I \left(i\gamma^\mu D_\mu - m_I\right) q_I \,, \quad I = u, d$$



$$\Lambda_{QCD} \sim 200 \text{ MeV}$$

$$\left.\begin{aligned} m_u &= 2.5 \text{ MeV} \\ m_d &= 5.3 \text{ MeV} \end{aligned}\right\} \ll \Lambda_{QCD}$$

$$\langle \bar{q}_{R I}\, q_{L J} \rangle = \delta_{IJ} \Lambda_{QCD}^3$$

u, d

$$\mathcal{L} = -\frac{1}{4} G_{\mu\nu}^{a} G^{\mu\nu a} + \bar{Q}_L i\gamma^\mu D_\mu Q_L + \bar{Q}_R i\gamma^\mu D_\mu Q_R$$

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

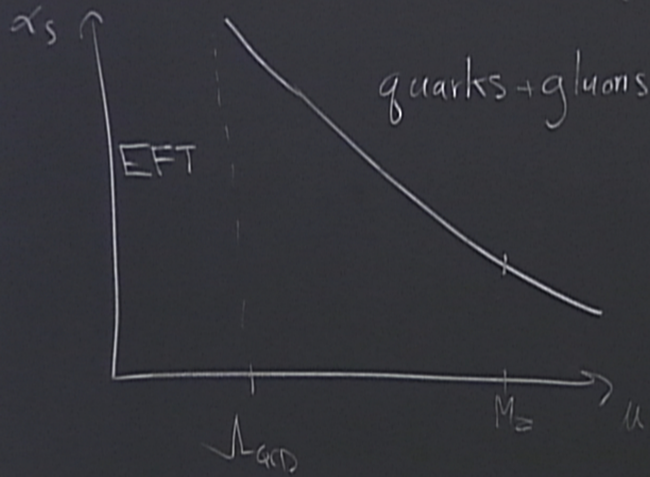
Invariant under: $\begin{cases} Q_L \to L Q_L \\ Q_R \to Q_R \end{cases}, \quad L \in SU(2)_L$

$\begin{cases} Q_L \to Q_L \\ Q_R \to R Q_R \end{cases}, \quad R \in SU(2)_R$

$U(1)_A \begin{cases} Q_L \to e^{i\alpha_A} Q_L \\ Q_R \to e^{-i\alpha_A} Q_R \end{cases}$

$\begin{cases} Q_L \to e^{i\alpha_V} Q_L \\ Q_R \to e^{i\alpha_V} Q_R \end{cases}, \quad U(1)_V$

$$\text{QCD:} \quad \mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} + \sum_I \bar{q}_I \left(i\gamma^\mu D_\mu - m_I \right) q_I \, , \quad I = u, d$$



$$\Lambda_{QCD} \sim 200 \text{ MeV}$$

$$\left. \begin{aligned} m_u &= 2.5 \text{ MeV} \\ m_d &= 5.3 \text{ MeV} \end{aligned} \right\} \ll \Lambda_{QCD}$$

$$\langle \bar{q}_{R I} \, q_{L J} \rangle = \delta_{IJ} \Lambda^3_{QCD}$$

$$\rightarrow L R^\dagger (\;)$$

$SU(2)_V \subset SU(2)_L \times SU(2)_R$

$\quad 3 \qquad\qquad 3 \qquad\qquad 3$

$L = R$

$$\begin{cases} L = e^{i\alpha^a_L t^a}, \quad t^a = \sigma^a/2 \\ R = e^{i\alpha^a_R t^a} \end{cases}$$

$\alpha^a_L = \alpha^a_R$

$\langle \ldots \rangle = \Lambda^3_{QCD}\, \delta_{IJ}$ breaks 3 generators $\Rightarrow$ 3 NGB = Nambu Goldstone Boson

$$\Sigma(x) = \exp[2i\pi^a(x)t^a/f]$$

↳ fields

$$t^a = \sigma^a/2$$

f = dim.-1 quantity

$$\Sigma(x) \xrightarrow{G_{Flav}} L\Sigma(x)R^\dagger$$

$$\begin{cases} L = e^{ic_A^a t^a} e^{ic_V^a t^a} \\ R = e^{-ic_A^a t^a} e^{ic_V^a t^a} \end{cases}$$

$$V: \Sigma(x) \to V\Sigma V^\dagger = \exp[2i(V\pi^a t^a V^\dagger)/f]$$

$$\exp[2i(\underbrace{V\pi^a t^a V^\dagger})/f]$$

$$V \pi^a t^a V^\dagger \to \pi^a t^a + \pi^a c_V^b [t^b, t^a] + \dots$$

$$\Rightarrow \pi^a \to \pi'^a = \left(\delta^{ac} - \epsilon^{abc} c_V^b\right) \pi^c$$

$$A: \quad \pi \to \pi' = \pi + \underbrace{f c_A^a}_{\text{shift, non-linear}} t^a + \Theta\left(c_A^{a\,2}\right)$$

$$\Phi = (v + h/\sqrt{2})\, e^{i a(x)/f}$$

U(1), spontareously broken by $\langle \Phi \rangle = v$

$$a \to a + f\,, \quad f \sim v$$

$$\mathrm{tr}(\underbrace{\Sigma^\dagger \Sigma}_{\mathbb{1}}) \to \mathrm{tr}[(R \Sigma^\dagger L^\dagger)(L$$

$$\mathrm{tr}(\underbrace{\Sigma^\dagger\Sigma}_{\mathbb{1}}) \to \mathrm{tr}[(R\Sigma^\dagger \underbrace{L^\dagger)(L}_{\mathbb{1}}\Sigma R)] = \mathrm{tr}(R\Sigma^\dagger\Sigma R^\dagger)_{\mathbb{1}} = \mathrm{tr}(\Sigma^\dagger\Sigma)$$

$$\mathrm{tr}(t^a t^b) = \frac{1}{2}\delta^{ab}$$

$$(AB)^\dagger = B^\dagger A^\dagger$$

$$\mathcal{L} \supset N \cdot \mathrm{tr}(\partial_\mu \Sigma^\dagger \cdot \partial^\mu \Sigma) = N\,\mathrm{tr}\left[\partial_\mu\left(1 + 2i\pi^a t^a/f + \ldots\right)^\dagger \cdot \partial^\mu\left(1 + 2i\pi^a t^a/f + \ldots\right)\right]$$

$$= N \cdot \frac{4}{f^2} \cdot \frac{1}{2} \sum_a (\partial_\mu \pi^a)^2 + \mathcal{O}\left(\frac{1}{f^3}\right) + \ldots$$

$$\mathrm{tr}(\underbrace{\Sigma^\dagger\Sigma}_{\mathbb{1}}) \to \mathrm{tr}[(R\Sigma^\dagger \underbrace{L^\dagger)(L}_{\mathbb{1}}\Sigma R)] = \mathrm{tr}(R\Sigma^\dagger\Sigma R^\dagger) = \mathrm{tr}(\ldots$$

$\mathbb{1}$

$\mathrm{tr}(t^a \ldots$

$$(AB)^\dagger = B^\dagger A^\dagger$$

$$\to N = \frac{f^2}{4}$$

$$\mathcal{L} \supset N \cdot \mathrm{tr}(\partial_\mu \Sigma^\dagger \cdot \partial^\mu \Sigma) = N\,\mathrm{tr}\left[\partial_\mu (1 + 2i\pi^a t^a/f + \ldots)^\dagger \cdot \partial^\mu (1 + 2i \ldots\right.$$

$$= N \cdot \frac{4}{f^2} \cdot \frac{1}{2} \sum_a (\partial_\mu \pi^a)^2 + \mathcal{O}\left(\frac{1}{f^3}\right) + \ldots$$

$$\ldots R)] = \mathrm{tr}(R\,\Sigma^\dagger \Sigma R^\dagger) = \mathrm{tr}(\Sigma^\dagger \Sigma)$$

$$\mathbb{1}$$

$$\mathrm{tr}(t^a t^b) = \frac{1}{2}\delta^{ab}$$

$$N = \frac{f^2}{4}$$

$$\mathrm{tr}\left[\partial_\mu\left(1 + 2i\,\pi^a t^a/f + \ldots\right)^\dagger \cdot \partial^\mu\left(1 + 2i\,\pi^a t^a/f + \ldots\right)\right]$$

$$\cdot \frac{4}{f^2}\cdot\frac{1}{2}\sum_a (\partial_\mu \pi^a)^2 + O\left(\frac{1}{f^3}\right) + \ldots \qquad \left(\frac{f}{f}\right)$$

$$\mathcal{L} \supset N \; \mathrm{tr}\left(\partial_\mu \Sigma \cdot \partial \Sigma\right)$$

p^2 $=$

$$\mathcal{L}_{p^4} = L_1 \left[\mathrm{tr}\left(\partial \Sigma^\dagger \cdot \partial \Sigma\right)\right]^2$$
$$+ L_2 \, \mathrm{tr}\left(\partial_\mu \Sigma^\dagger \partial_\nu\right) \mathrm{tr}\left(\partial^\mu \Sigma^\dagger \partial^\nu \Sigma\right)$$
$$+ L_3 \, \mathrm{tr}\left(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma \cdot \partial_\nu \Sigma^\dagger \partial^\nu \Sigma\right)$$
$$+ \ldots$$

$$= N \cdot \frac{4}{F^2} \cdot \frac{1}{2} \sum_a (\partial_\mu \pi^a)^2 + \mathcal{O}\left(\frac{1}{F^3}\right) + \cdots$$

$$Q = t^3_L + t^3_R + \frac{1}{6} \underset{U(1)_V}{\mathbb{1}}$$

$u_L : \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

$d_L : -\frac{1}{2} + \frac{1}{6} = \frac{1}{3}$

$U(1)_{em} : e^{i\alpha t^3_L} e^{i\alpha t^3_R} e^{i\alpha/6}$

$\pi^0 = \pi^3$

$\pi^\pm = \sqrt{2}\left(\pi^1 \mp i\pi^2\right)$

$f = ?$

$m_\pi \simeq 135\ \mathrm{MeV}$

j^{μ}_{V}, $j^{\mu a}_{L}$, $j^{\mu a}_{R}$

$$j^{\mu a}_{L} = \bar{q}\gamma^{\mu}P_L t^a q$$

$$\to j^{1}_{\mu L} + j^{2}_{\mu L} = \frac{i}{\sqrt{2}} f \partial_\mu \pi^- + \mathcal{O}\left(\frac{1}{f}\right)$$

$$j^{\mu a}_{L} W^a_\mu \quad \frac{g^2}{2m_W^2}\left(\bar{u}\gamma^\mu P_L d\right)\left(\bar{\mu}\gamma_\mu P_L \nu_\mu\right)$$

$$\Rightarrow f = f_\pi = 93\,\mathrm{MeV} \sim \Lambda_{QCD}$$

$$\langle \mu^- \bar{\nu}_\mu | H_{int} | \pi^-(p) \rangle \Rightarrow \text{rate for } \Gamma_{\pi^-}$$

$$-\mathcal{L} \supset \bar{Q}_L M Q_R + h.c.$$

$$M = \mathrm{diag}(m_u, m_d)$$

$$= \bar{u}_L m_u u_R + \bar{d}_L m_d d_R + h.c.$$

$\rightarrow -$

$$\rightarrow \bar{Q}_L L^\dagger M R Q_R$$

Pretend $M \rightarrow L M R^\dagger$

$$\rightarrow \ -\mathcal{L} \supset \frac{1}{2}\tilde{\Lambda}^3 \, \mathrm{tr}(M^\dagger \Sigma)$$

$$\sim \Lambda^3_{QCD}$$

$$m_\pi^2 f_\pi^2 = \tilde{\Lambda}^3 (m_u + m_d)$$

$m_\pi \simeq 135$ MeV

pNGB