Title: Exact holographic mapping, tensor networks and space-time geometry

Date: Feb 27, 2015 11:00 AM

URL: http://pirsa.org/15020134

Abstract: <p>Holographic duality is a duality between gravitational systems and non-gravitational systems. In this talk, I will propose a different approach for understanding holographic duality named as the exact holographic mapping. The key idea of this approach can be summarized by two points: 1) The bulk theory and boundary theory are related by a unitary mapping in the Hilbert space. 2) Space-time geometry is determined by the structure of correlations and quantum entanglement in a quantum state. When applied to lattice systems, the holographic mapping is defined by a unitary tensor network. For free fermion boundary theories, I will discuss how different bulk geometries are obtained as dual theories of different boundary states. A particularly interesting case is the AdS black hole geometry and the interpretation of the interior of a black hole. We will also discuss dual geometries of topological states of matter.</p>

Outline

- Background and motivation
- The definition of exact holographic mapping (EHM) and the free fermion example
- Thermodynamics and black hole geometry
- Optimal disentanglers and the interior of blackhole
- Holographic dual of topological states
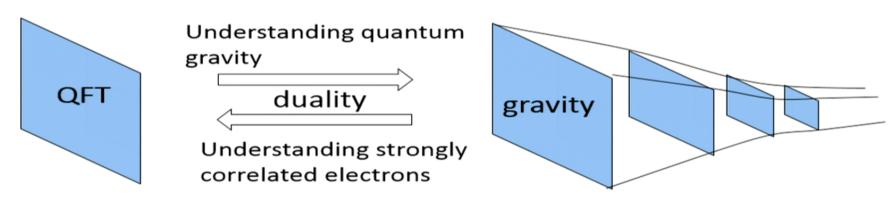- Discussion and summary

Ref: XLQ, **arXiv:1309.6282** (2013)
Unpublished works
1) Michael Zaletel, XLQ
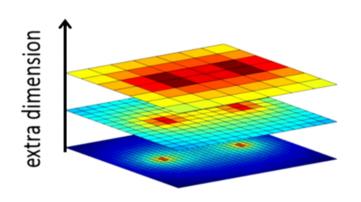2) Yingfei Gu, Chinghua Lee, Gil Y. Cho, Xueda Wen, Shinsei Ryu, XLQ

Holographic duality

QFT

Understanding quantum gravity

duality

Understanding strongly correlated electrons

gravity

- Holographic duality, or AdS/CFT (Maldacena '97, Witten '98, Gubser, Klebanov & Polyakov '98)

- The extra dimension can be interpreted as energy scale. Bulk equation of motion ↔ renormalization group flow. (E. Akhmedov, '98, Heemskerk & Polchinski '10)

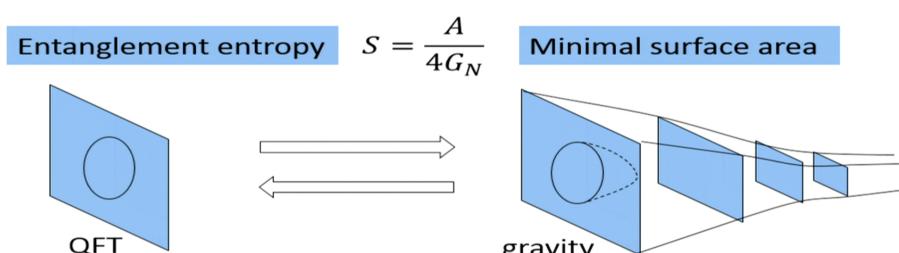- Application to condensed matter (For a review, see S. Sachdev, Annual Review of Condensed Matter Physics 3, 9 (2012))

extra dimension

Holographic duality

- Holographic duality also relates **space-time geometry** with **quantum entanglement**.

- Bekenstein–Hawking formula

Black hole entropy $\quad S = \dfrac{A}{4G_N} \quad$ Black hole area

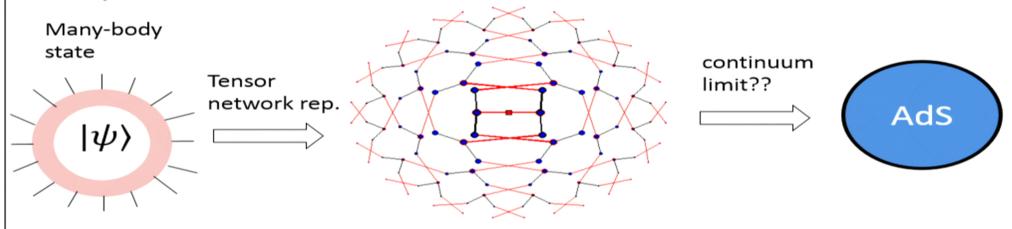- Ryu-Takayanagi formula (S. Ryu & T. Takayanagi PRL '06)

Entanglement entropy $\quad S = \dfrac{A}{4G_N} \quad$ Minimal surface area

QFT $\qquad\qquad\qquad\qquad\qquad$ gravity

Tensor networks

- Multiscale Entanglement Renormalization Ansatz (MERA). A unitary tensor network describing critical states (Vidal '07)

- MERA has been proposed to be related to AdS/CFT (Swingle '10, Evenbly&Vidal '11, Haegeman et al '11, Nozaki et al '12, Hartman&Maldacena '13)

- How this relation exactly works remain an open question

Many-body state

$|\psi\rangle$

Tensor network rep.

continuum limit??

AdS

Step back and think about two questions about quantum space-time geometry

- Classical space-time: Riemann manifold consisting of points, and the distance between points given by geodesics.

- Quantum space-time geometry: How to define points in the geometry, and the distance between points?

- **Points:** A set of points corresponds to a direct product decomposition of the Hilbert space $\mathbb{H} = \prod_{\mathbf{x}}^{\otimes} \mathbb{H}_{\mathbf{x}}$.

- **Distance:** Distance between points shall be measured by physical correlation functions. <u>More correlated sites are closer to each other.</u>

How to define distance between points?

- A choice of points is only meaningful if physical correlations are local.

- Example: Massive free fermion
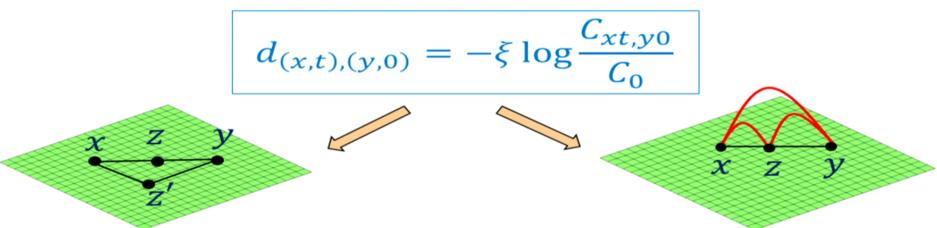
- Real space basis x. Imaginary time Correlation function $C_{xt,y0} = \langle G | c_{xt}^+ c_{y0} | G \rangle \propto C_0 e^{-d_{(x,t),(y,0)}/\xi}$. Natural distance definition

$$d_{(x,t),(y,0)} = -\xi \log \frac{C_{xt,y0}}{C_0}$$

- For all massive state, this definition works and recovers the classical space-time geometry.

- Mutual information $I_{xy} = S_x + S_y - S_{xy}$ can be used as an unbiased measure of equal-time correlation. (Wolf et al '08)

From gapped states to critical states

$$d_{(x,t),(y,0)} = -\xi \log \frac{C_{xt,y0}}{C_0}$$

Gapped states

Geodesic distance restored

Triangle inequality holds

$$d_{xz'} + d_{z'y} \geq d_{xy}$$

Critical states

$$C_{xy} \propto |x - y|^{-2\Delta}$$

$$d_{xy} \propto \log |x - y|$$
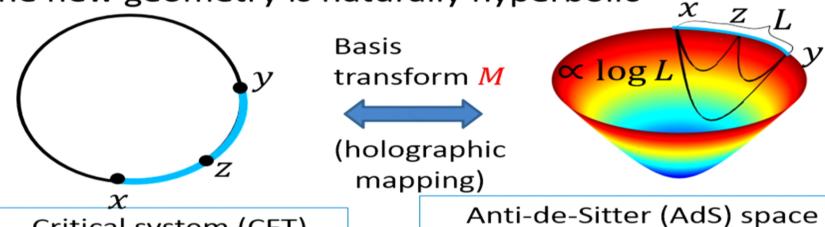
$$d_{xz'} + d_{z'y} > d_{xy}$$

- A new basis and new geometry is required for critical system

Relation to holographic duality

- Lesson from the discussion so far:

- A new basis may be defined for critical systems, which leads to a new geometry.

- The new geometry is naturally hyperbolic

Basis transform M

(holographic mapping)

$\propto \log L$
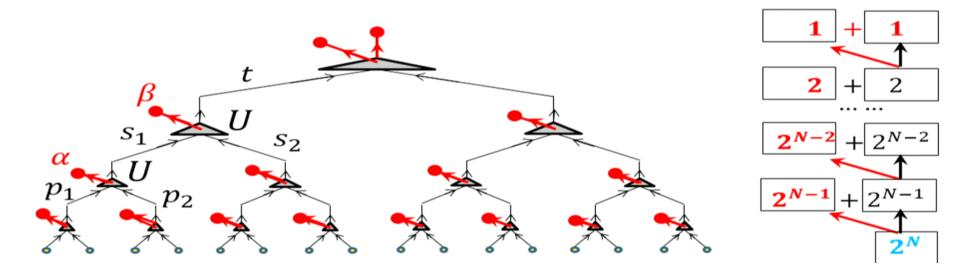
Critical system (CFT)

$$C_{xy} \propto |x - y|^{-2\Delta}$$

Scaling dimensions

Anti-de-Sitter (AdS) space

$$C_{xy} \propto e^{-\frac{d_{xy}}{\xi}}$$
$$d_{xy} \propto \log |x - y|$$

Mass of bulk fields
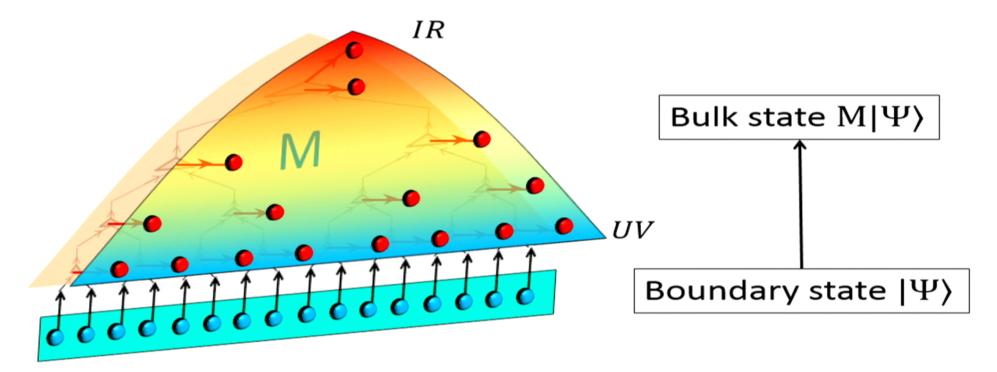
Definition of the exact holographic mapping (EHM)

- 1. Starting from 2^N sites, U maps each two neighboring sites to a "low energy" site and a **"high energy"** site
  2. Repeat step 1 the 2^{N-1} low energy sites.

- Unitary mapping $M = \prod_{\text{network}} U$ maps **boundary** (2^N sites) to **bulk** ($1 + 1 + 2 + \cdots + 2^{N-1} = 2^N$ sites)

- A modification of MERA

Definition of the exact holographic mapping
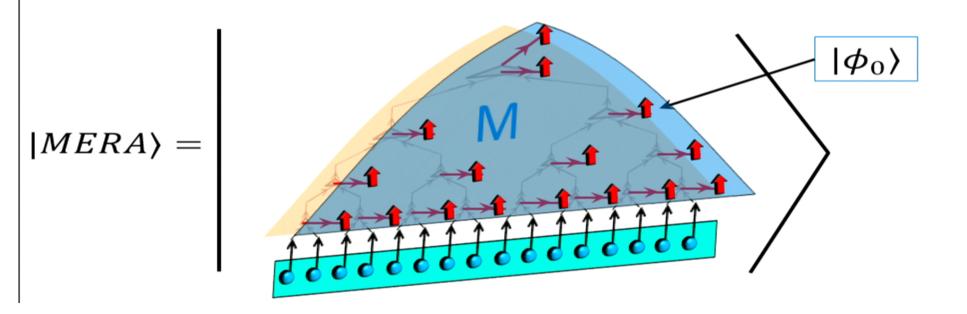
- A "lossless" generalization of real space RG.
- Degrees of freedom at different energy scales are all kept, and they can entangle with each other.

Relation of EHM and MERA
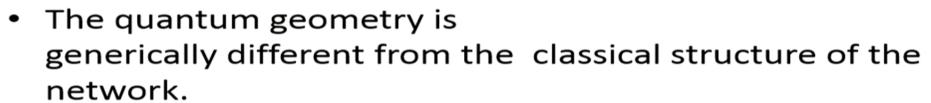
- A MERA state corresponds to acting the reverse EHM of a direct product bulk state $|MERA\rangle = M^{-1} \prod^{\otimes} |\phi_0\rangle$

- The goal of EHM is to *allow bulk states to entangle* and use that to probe the geometry.

$$|MERA\rangle =$$

$|\phi_0\rangle$
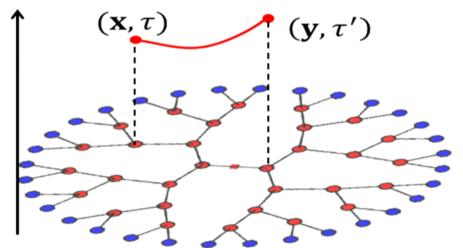
Exact holographic mapping: bulk geometry

- Bulk space-time geometry determined by bulk correlation functions $d_{(x,\tau),(y,\tau')} = -\xi \log \dfrac{\langle O_{(x,\tau)} O_{(y,\tau')} \rangle}{c_0}$.
  (τ is the imaginary time)

- Mutual information can be used for equal time distance. (A realization of **ER=EPR** (Maldacena&Susskind))

- Different choices of mapping U correspond to different choices of "spatial slices".

- The quantum geometry is generically different from the classical structure of the network.

Exact holographic mapping: free fermion
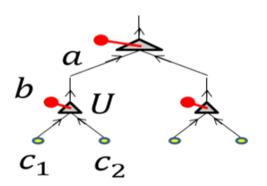
- 1+1d lattice Dirac fermion

$$H = \sum_k c_k^+ \left(\sin k\, \sigma_x + (m + 1 - \cos k)\sigma_y\right)c_k,$$

- Unitary mapping

- $$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \to \begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} c_1 + c_2 \\ c_1 - c_2 \end{pmatrix}$$

- In many-body Hilbert space
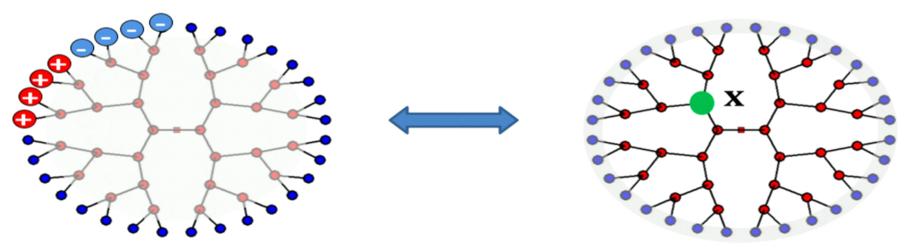
$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$|00\rangle\ |10\rangle\ |01\rangle\ |11\rangle$

Exact holographic mapping: free fermion

- Basis transformation between boundary and bulk
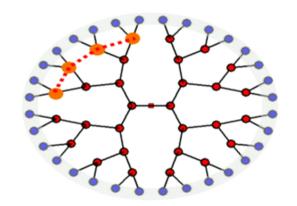
"Haar wavelet" wavefunctions at the boundary $\phi_{\mathbf{x}}(i)$

Local basis in the bulk

$$b_{\mathbf{x}} = \sum_i \phi_{\mathbf{x}}^*(i) c_i$$

Ground state of critical system

- $T = m = 0$

- **Spatial direction:** compare the distance obtained from $d_{xy} = -\xi \log \dfrac{I_{xy}}{I_0}$ to the AdS space distance

- Embedding of the bulk sites in AdS is determined by scale invariance (special for the critical state)

$$(j, n) \rightarrow (\rho, \theta), \; j = 1, 2, \ldots, 2^{N-n}$$

$$\rho = 2^{N-n} \frac{1}{2\pi}, \; \theta = \left(j - \frac{1}{2} + \frac{1}{2^{n+1}} \right) \frac{1}{\rho},$$

- Angle direction $d_{(x,n)(y,n)}/\xi \simeq \dfrac{2R}{\xi} \log \dfrac{|x-y|}{R}$

Ground state of critical system

- $T = m = 0$

- **Spatial direction:** compare the distance obtained from $d_{xy} = -\xi \log \frac{I_{xy}}{I_0}$ to the AdS space distance

- Embedding of the bulk sites in AdS is determined by scale invariance (special for the critical state)
$$(j, n) \rightarrow (\rho, \theta), \, j = 1, 2, \ldots, 2^{N-n}$$
$$\rho = 2^{N-n} \frac{1}{2\pi}, \, \theta = \left(j - \frac{1}{2} + \frac{1}{2^{n+1}}\right) \frac{1}{\rho},$$

- Angle direction $d_{(x,n)(y,n)}/\xi \simeq \frac{2R}{\xi} \log \frac{|x-y|}{R}$

Ground state of critical system

- Fitting of the angle direction distance
  ➡ AdS radius
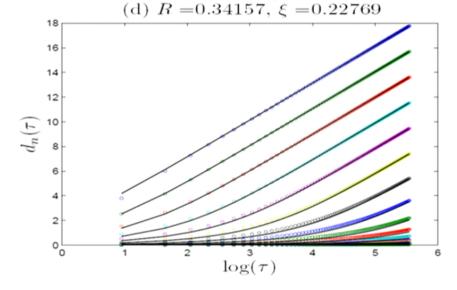
$$R = \left(64\pi^2 \log 4\right)^{-\frac{1}{6}} \simeq 0.33, \xi = R/3,$$

(Ching Hua Lee &XLQ)
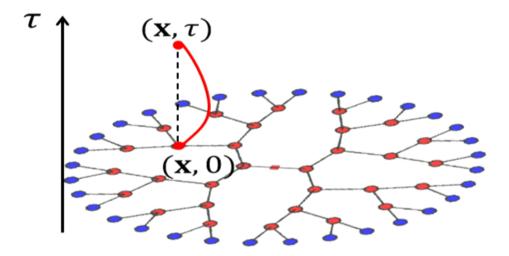
- Radial direction $d_{(x,n),(x,1)}$, qualitatively agree

(a) $R = 0.32842, \xi = 0.1086, n = 8$

(b) $R = 0.32842, \xi = 0.32388$

Ground state of critical system

- **Time-direction**: Imaginary time direction distance defined by $d_{(x,\tau)(x,0)} = -\xi \log \langle T b_{x\sigma}(\tau) b_{x\sigma}^+(0) \rangle$

- Compared with AdS_3 formula $d_{(x,\tau),(x,0)} =$

$$R \operatorname{acosh}\left[\left(\frac{\rho^2}{R^2} + 1\right)\cosh\frac{\tau}{R} - \frac{\rho^2}{R^2}\right]$$

- R can be independently fitted $R \simeq 0.34$

(d) $R = 0.34157$, $\xi = 0.22769$
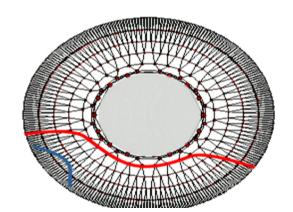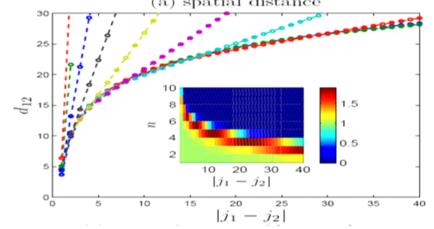
Finite temperature and black hole

- $T > 0, m = 0$. Geometry is modified for non-critical systems, even if the same mapping is chosen.

- **Spatial direction:** $d_{(x,n),(y,n)}$ Cross-over from AdS ($\propto \log|x - y|$) to Euclidean ($\propto |x - y|$).

- IR limit $n \to N$, *Stretched horizon* region.

- angle-direction distance $d_{0n,xn} \simeq (1 - 2\pi T)^{2^{n+1}} x \Rightarrow$ Area saturates to a finite value $2\pi\rho \simeq 4\pi T \cdot 2^N$ (Ching Hua Lee &XLQ)
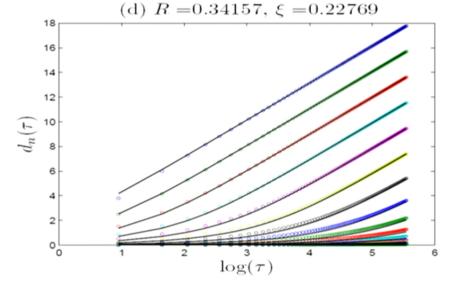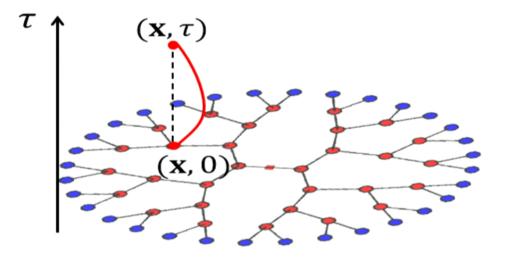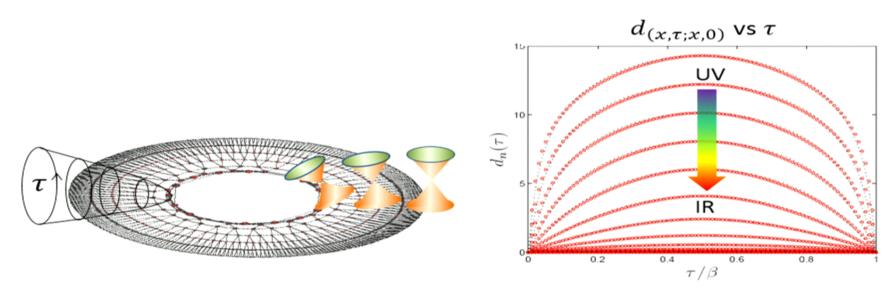
(a) spatial distance

Ground state of critical system

- **Time-direction**: Imaginary time direction distance defined by $d_{(x,\tau)(x,0)} = -\xi \log \langle T b_{x\sigma}(\tau) b^+_{x\sigma}(0) \rangle$

- Compared with AdS_3 formula $d_{(x,\tau),(x,0)} =$
$$R \operatorname{acosh}\left[\left(\frac{\rho^2}{R^2} + 1\right) \cosh\frac{\tau}{R} - \frac{\rho^2}{R^2}\right]$$

- R can be independently fitted $R \simeq 0.34$

(d) $R = 0.34157$, $\xi = 0.22769$

Finite temperature and black hole

- **Time direction**: In IR region, fermion bandwidth $\ll T$

- The time dependence of correlation function exponentially slows down. $d_{(x,\tau),(x,0)} \to 0$ in IR

- General reason: reduced density matrix of the stretched horizon region $\rho_{IR} \propto I$ ➡ Trivial time evolution.

$d_{(x,\tau;x,0)}$ vs τ
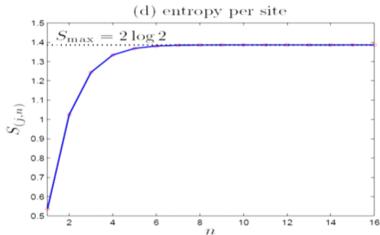
Finite temperature and black hole
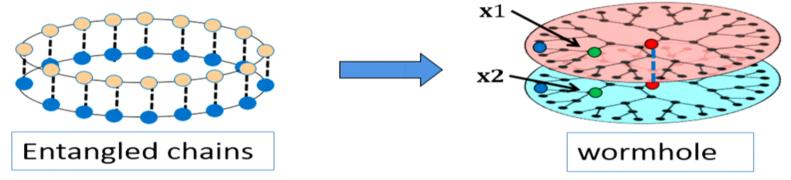
- The observed behavior agrees with BTZ blackhole metric

- $$ds^2 = (\rho^2 - b^2)d\tau^2 + R^2 \frac{d\rho^2}{\rho^2 - b^2} + \rho^2 d\theta^2$$
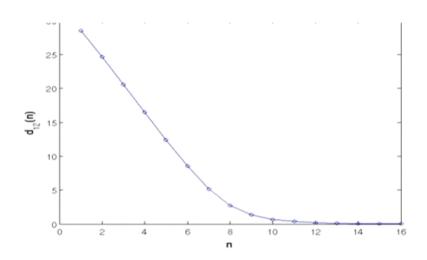
- Near the horizon $\rho \to b$, the time direction becomes infinitely short. We observe this behavior in the correlation functions, because the states in IR are in the infinite temperature limit.

- Each bulk-site in the IR carries maximal entanglement entropy.

- $\log D = \log 4$.

- The entropy in this region shall be considered as the black-hole entropy

(d) entropy per site

$S_{\max} = 2\log 2$

Einstein-Rosen bridge

Entangled chains

wormhole
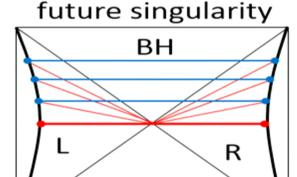
- There are different black hole geometries.

- A pure state of a two-leg ladder system in a thermal double state $|\Psi\rangle = \sum_n e^{-\beta E_n/2} |\bar{n}\rangle_L |n\rangle_R$ is dual to a worm hole (Maldacena '01)

- Time evolution of two-sided blackhole (Hartman&Maldacena '13)
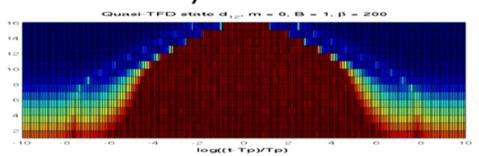
Quantum quench on the wormhole geometry
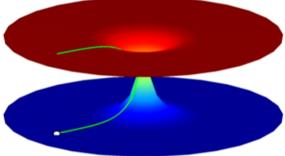
- The blackhole interior can be indirectly probed by the distance between outside points (Hartman & Maldacena JHEP '13)

- Time evolution by $H_R - H_L$ gives

$$|\Psi(t)\rangle = \sum_n e^{-\left(2it+\frac{\beta}{2}\right)E_n} |\bar{n}\rangle_L |n\rangle_R,$$

and changes the entanglement structure between the two sides.
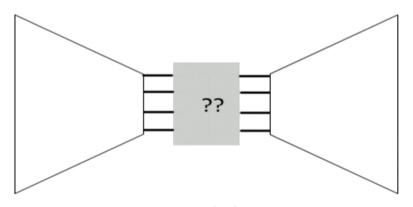
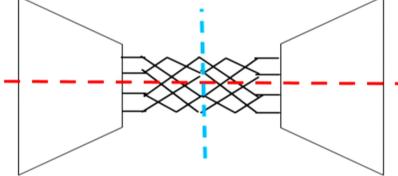- In free theory, the worm hole stretches and returns in time $t \sim L/2$

future singularity

BH

L   R

WH

Quasi-TFD state d_{12}, m = 0, B = 1, β = 200

log((t-Tp)/Tp)
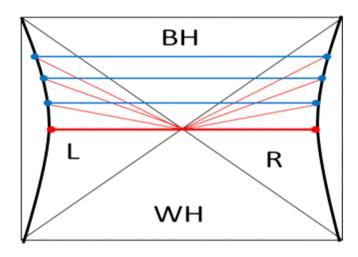
Tensor network point of view

- Outside frame (red)

BH

L

R

WH

- Hartman-Maldacena tensor network. Does it correspond to the minimal surface (blue)?
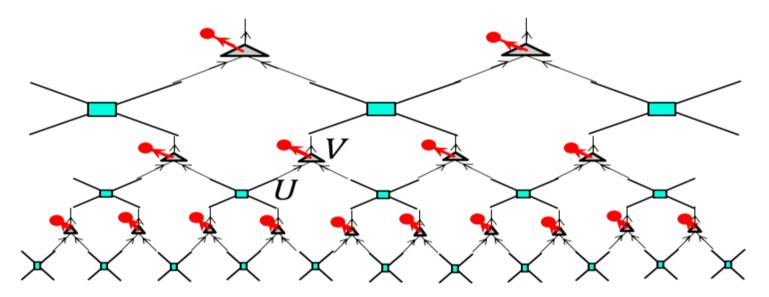
- It looks more like the green slice. The scrambling of time evolution is removed. Throat size never shrinks.
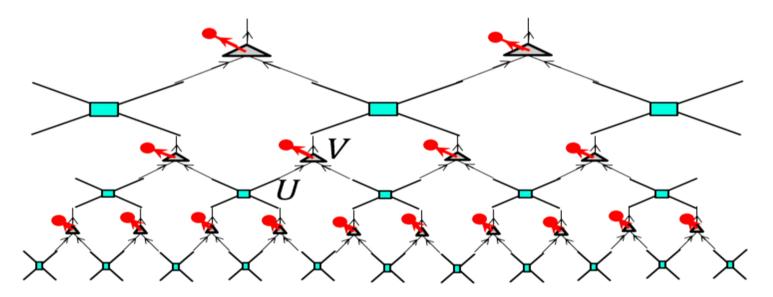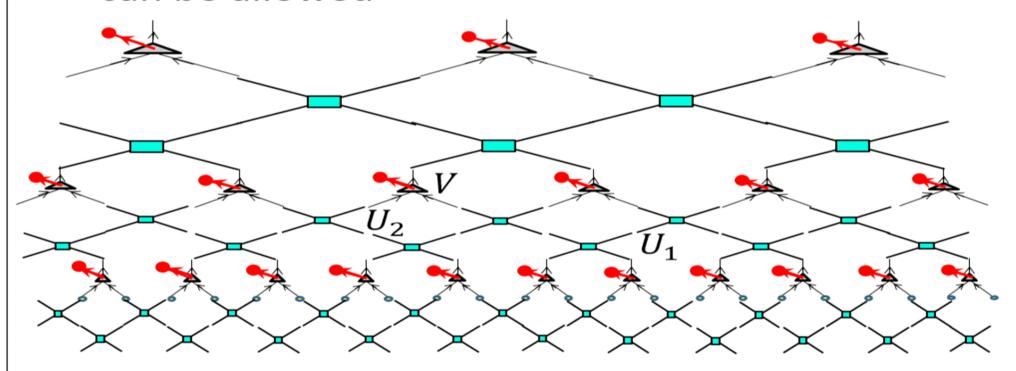
??

More general EHM tensor networks

- In general, the suitable EHM should depend on the physical state.

- In the same way as MERA, we can modify EHM by introducing disentanglers

More general EHM tensor networks

- In general, the suitable EHM should depend on the physical state.

- In the same way as MERA, we can modify EHM by introducing disentanglers

More general EHM tensor networks

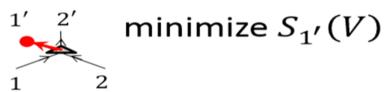- More generally, multilayers of disentanglers can be allowed

Criteria of optimal EHM tensor networks

- Although there is probably no unique choice of EHM which is the "best" choice, there shall be one family of tensor networks that are "optimal", which means the remaining entanglement of the bulk states is most short-ranged compared to other EHM with the same structure.
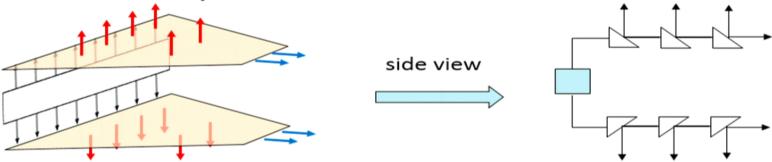
- Criteria:

- For disentanglers

$$1' \quad 2' \quad 3' \quad 4'$$
$$U$$
$$1 \quad 2 \quad 3 \quad 4$$

minimize $S_{2'3'}(U)$

- For coarse-graining

$$1' \quad 2'$$
$$1 \quad 2$$

minimize $S_{1'}(V)$

- "Distill" the entanglement to fewer IR qubits

Possibly related criteria in Evenbly&Vidal PRB '10
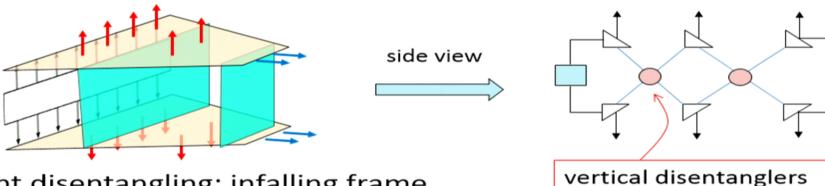
Application of the optimal EHM to the thermal double state

- Two different disentangling scheme can be applied to the doubled system.
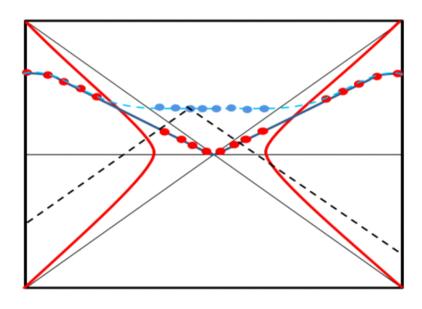

Separate disentangling: outside frame

side view

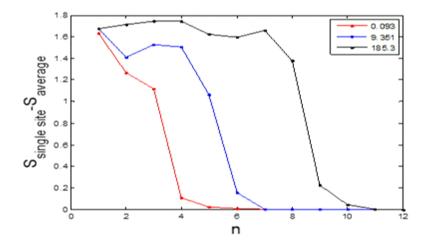Joint disentangling: infalling frame

side view

vertical disentanglers

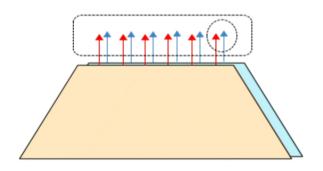# Application of the optimal EHM to the thermal double state

- For the quenched thermal double state $|\Psi(t)\rangle = \sum_n e^{-\frac{\beta E_n}{2} - 2iE_n t}|n_L\rangle|n_R\rangle$ the separate disentangling stops working at energy scale $E \sim T = 1/\beta$.

- **Outside frame** (red dots): separate left and right bulk regions. Left and right are not smoothly connected

- **Infalling frame** (blue dots): states in IR depends on both sides of the boundary.

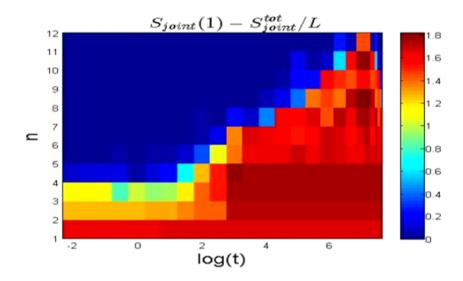- Blue states are further disentangled compared to red, to define a smooth geometry.

Numerical results

- Carrying the disentangling procedure until the IR states are direct products $\rho_{IR} \simeq \prod_i \rho_i$

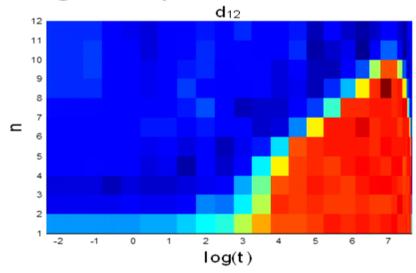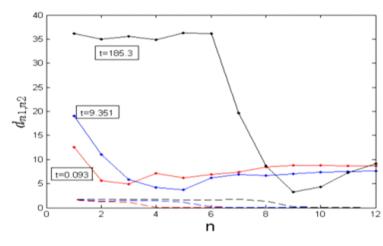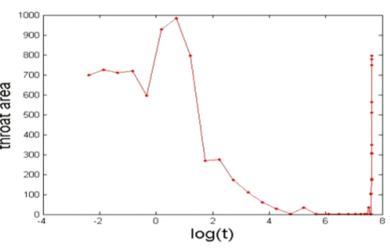- This factorization can be judged by the value of $\sum S(\rho_i) - S(\rho_{IR})$

$$S_{joint}(1) - S_{joint}^{tot}/L$$

Numerical results

- Distance between bulk sites
- The throat remains smooth, with a shrinking area (and expand again at $t \sim L/2$)
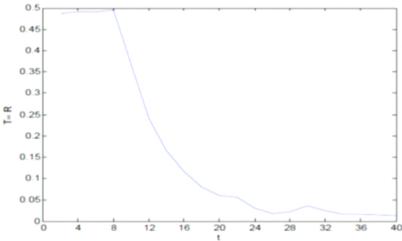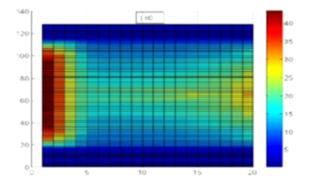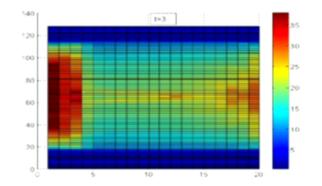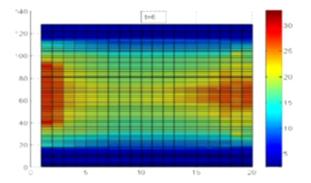- Question: Can we see the singularity?

Topological edge states and black hole evaporation

- Temperature vs time
- Difference from semiclassical blackhole: specific heat is still positive

Summary and open questions

Quantum many body system → **Exact holographic mapping** → Points (A new set of local basis) → **Correlation defines distance** → Emergent space-time geometry

- Relation between quantum entanglement and space-time geometry.

- A quantum model for blackholes.

- A possible new approach to strongly correlated condensed matter systems.

- Open questions:
  ◊ Application to physical strongly correlated problems.
  ◊ The black-hole information paradox.
  ◊ How to describe a "quantum" geometry, beyond the concepts of points and distance?

Topological edge states and black hole evaporation

- Temperature vs time

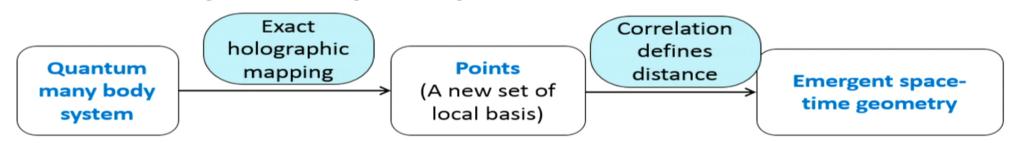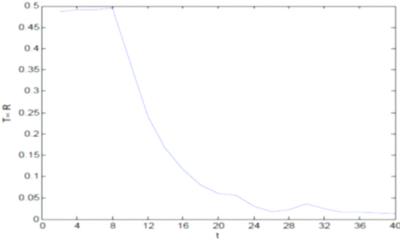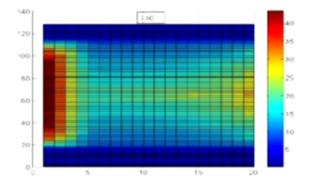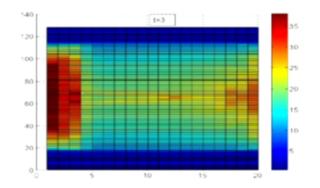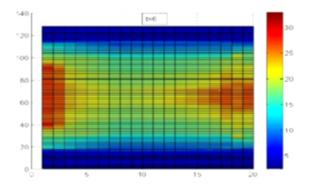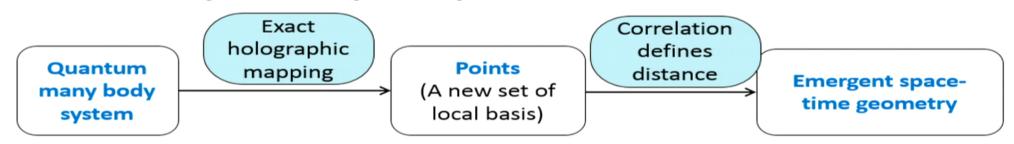- Difference from semiclassical blackhole: specific heat is still positive

Summary and open questions

```
┌──────────┐   ( Exact    )   ┌──────────┐   ( Correlation )   ┌──────────┐
│ Quantum  │   ( holographic)─▶│  Points  │   ( defines     )─▶│ Emergent │
│many body │──▶( mapping   )   │(A new set│   ( distance    )   │ space-   │
│ system   │                   │of local  │                    │ time     │
└──────────┘                   │ basis)   │                    │ geometry │
                               └──────────┘                    └──────────┘
```

- Relation between quantum entanglement and space-time geometry.

- A quantum model for blackholes.

- A possible new approach to strongly correlated condensed matter systems.

- Open questions:
  ◊ Application to physical strongly correlated problems.
  ◊ The black-hole information paradox.
  ◊ How to describe a "quantum" geometry, beyond the concepts of points and distance?