Title: Doing Physics with Shape Dynamics

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Abstract: <p>Shape Dynamics is a theory of gravity which replaces relativity of simultaneity for spatial conformal invariance, maintaining the same degree of symmetry of General Relativity while avoiding some of its shortcomings.</p>

<p>In SD several kinds of singularities of GR become unphysical gauge artefacts, and the presence of a preferred notion of simultaneity fits better into the structure of quantum theory. In this talk I will outline the present status of research in SD on black holes and gravitational collapse, on the emergence of spacetime and on the first-order formulation of the theory.</p>

DOING PHYSICS WITH SHAPE DYNAMICS

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Templeton Frontiers Colloquium

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[*'A Shape Dynamics Tutorial'* (upcoming book) arXiv:1409.0105]

[several upcoming papers (stay tuned on the arXiv)]

GR's classical problems

General Relativity 'predicts its own demise' (J. Wheeler '70s)

Schwarzschild spacetime:

$$ds^2 = -\left(1-\frac{2m}{r}\right)dt^2 + \left(1-\frac{2m}{r}\right)^{-1}dr^2 + r^2 d\Omega^2$$

Kretschmann scalar: $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{48m^2}{r^6} \xrightarrow[r\to 0]{} \infty$

Big-bang singularity:

$$ds^2 = -dt^2 + a^2(t)\, d\Sigma^2\,, \qquad R = 6\frac{(\dot{a})^2 + a\,\ddot{a} + k}{a^2}$$

common expectation: only quantum gravity will resolve singularities

GR's quantum problems

Spacetime diffeomorphism invariance problematic for quantum gravity,

in particular: freedom to slice spacetime into hypersurfaces of simultaneity.

Hamiltonian constraint $\mathcal{H} = \frac{1}{\sqrt{g}}\left(p^{ij}p_{ij} - \frac{1}{2}(\mathrm{tr}\, p)^2\right) - \sqrt{g}\, R$: quadratic in momenta $p^{ij} \rightarrow$ does not correspond to a vector field on configuration space, unlike gauge symmetries like Yang-Mills or 3D diffeomorphisms.

Refoliations locally indistinguishable from dynamical evolution of local fields, so $\mathcal{H}$ intertwines gauge symmetry and dynamics. A preferred time coordinate fits better the structure of quantum theory.

Dropping $\mathcal{H}$ altogether is not a good idea: it is responsible for GR having 2 propagating degrees of freedom. Dropping it introduces a new scalar dof.

SD trades refoliation invariance for another local gauge invariance which does correspond to a vector field in a configuration space. The resulting theory has 2 dofs and is locally equivalent to GR.

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A message from the 60-70's

Arnowitt–Deser–Misner (60's)

Hamiltonian formulation of GR: $^{(4)}g_{\mu\nu} = \begin{pmatrix} -N^2 + g_{ij}\,\xi^i\,\xi^j & g_{ik}\,\xi^k \\ g_{jk}\,\xi^k & g_{ij} \end{pmatrix},$

Einstein action: $\int d^4x \sqrt{^{(4)}g}\; {}^{(4)}R = \int dt d^3x \left(\dot{g}_{ij} p^{ij} + N\,\mathcal{H}[g,p] + \xi^i \mathcal{H}_i[g,p] \right),$

3D-Diffeo constraint: $\boxed{\mathcal{H}_i = -2\,\nabla_j p^j{}_i \approx 0}$ (p^{ij} must be *transverse*)

Hamiltonian constraint:

$$\boxed{\mathcal{H} = \frac{1}{\sqrt{g}} \left(p^{ij} - \frac{1}{3} g^{ij} \mathrm{tr}\, p \right) \left(p_{ij} - \frac{1}{3} g_{ij} \mathrm{tr}\, p \right) - \frac{1}{6} \frac{(\mathrm{tr}\, p)^2}{\sqrt{g}} - \sqrt{g}\, R}$$

Lichnerowicz, York, Choquet–Bruhat (70's)

Conformal method: in *CMC slicing* $\quad \mathrm{tr}\, p = g_{ij} p^{ij} = \frac{3}{2} \tau \sqrt{g}$

$\mathcal{H} \approx 0$ and $\mathcal{H}_i \approx 0$ **decouple** and turn into **elliptic equations**.

Each solution of GR is specified by a *3d conformal geometry* and a symmetric *transverse-traceless (TT) tensor*

$$\left.\begin{array}{ll} \boxed{-2\nabla_j p^j{}_i} & = \text{diffeomorphisms} \\ & \delta g_{ij} = \nabla_i \xi_j + \nabla_j \xi_i \\ \boxed{\mathrm{tr}\, p = g_{ij}\, p^{ij}} & = \text{conformal transformations} \\ & \delta g_{ij} = \omega^4\, g_{ij} \end{array}\right\} p^{ij}_{\mathrm{TT}} \quad \begin{array}{l} \text{generates} \\ \text{changes} \\ \text{of } \textit{shape} \end{array}$$

conformal geometries represent the physical configuration space of GR,
TT-tensors are their conjugate momenta.

The exceptionality of CMC/maximal slicing

Marsden–Tipler [Phys. Rep. **3** 1980] argued that all physically relevant singularities are avoided by maximal slicing.

In the CMC case the same holds except for crushing singularities (big-bang-like).

ON OUR TODO LIST: Study singularity theorems in SD.

What is Shape Dynamics?

A different Hamiltonian theory of gravity
equivalent to GR in ADM formulation in a particular gauge

Linking Theory:

- Variables: 3-metric & momenta (g_{ab}, p^{ab}) + scalar field (ϕ, π_ϕ)
- Gauge redundancies:
  - 3-diffeomorphisms
  - refoliation invariance
  - conformal invariance $g_{ab} \to \omega^4 g_{ab}, \quad \phi \to \phi - \log \omega$

↙ ↘

GR: gauge $\phi = 0$
- refoliation invariance (simultaneity)
- 3-diffeomorphism invariance

SD: gauge $\pi_\phi = 0$
- conformal invariance (rel. of scale)
- 3-diffeomorphism invariance

↘ ↙

In a common gauge, they're equivalent (ADM in 'CMC' gauge)

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Local volume dofs: $\sqrt{g}$, conjugate dilatational momenta: $\operatorname{tr} p$.

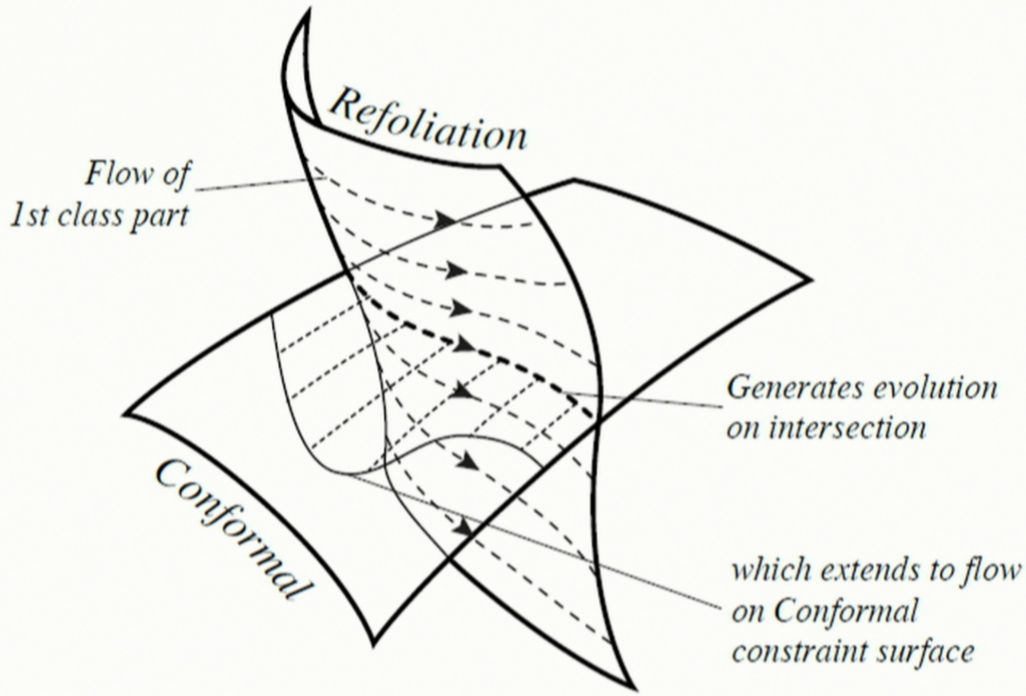
SD repackages those in the following way:

- local volume form $\sqrt{g}/V$: gauge,
- fluctuations of dilatational momentum $\operatorname{tr} p - \sqrt{g}\langle \operatorname{tr} p\rangle = 0$,
- global volume V: York Hamiltonian,
- global dilatational momentum $\langle \operatorname{tr} p\rangle$: CMC time.

York Hamiltonian: $H_{\text{York}}(\tau) = \int d^3x \sqrt{g}\, \Omega^6[g, p; x, \tau)$
Ω solution of Lichnerowicz–York equation:

$$8\,\Omega\,\Delta\,\Omega + \frac{\left\| p - \frac{1}{3} g \operatorname{tr} p \right\|^2}{g\Omega^6} - \frac{3}{8}\tau^2\,\Omega^6 - \Omega^2\,R + \textit{(matter)} = 0\,.$$

The 'iconic diagram' of SD



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Are GR & SD different only at the quantum level?

NO!

Different already at the classical level - different requirements behind them:

In GR one *postulates* a 4D spacetime and looks for solutions which represent smooth 4D manifolds (and sometimes doesn't find them: singularities).

In SD one postulates a smooth 3D conformal geometry evolving in time.

Then the same assumptions (e.g. spherical symmetry, a certain distribution of matter & gravity waves, etc.) give rise to inequivalent solutions in GR & SD.

Example:

The wormhole solution in Shape Dynamics

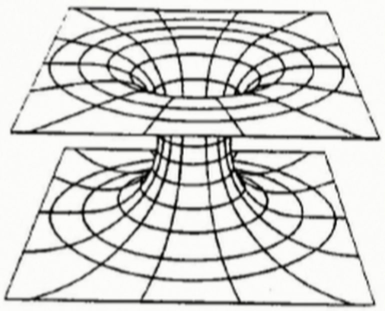
SD wormhole (inferred) 4-metric

$$ds^2 = -\left(\frac{1 - r_h/r}{1 + r_h/r}\right)^2 dt^2 + \left(1 + \frac{r_h}{r}\right)^4 \left[dr^2 + r^2 d\Omega^2\right]$$

SD wormhole has *no singularities*:

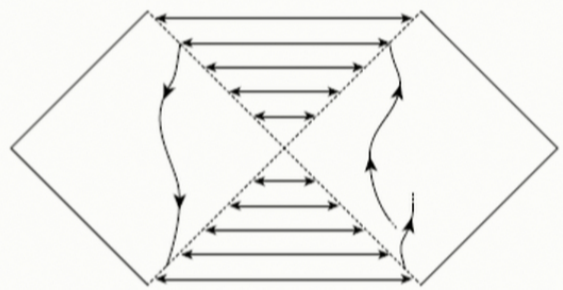
- at horizon $r = r_h$ 4-metric degenerate $\det g_{\mu\nu} = 0$, but 3-metric regular $\det g_{ab} \neq 0$, and all curvature invariants are finite,
- *inversion symmetry* between horizon interior and exterior: $r \to r_h^2/r$,
- interior is another asymptotically flat region: it takes *infinite proper time* to reach the origin $r = 0$ - like a point at infinity.

Properties & open questions on SD wormholes



- Not diffeomorphic to Schwarzschild: two Schwarzschild exteriors glued together,
- at horizon Raychaudhuri equation singular: discontinuity in expansion scalar,
- Einstein equations break down at the horizon: equivalent to singular $T^{\mu\nu} \propto \delta(r - r_h)$.

- Closed timelike curves?
- Penrose diagram of the wormhole?
- What happens to infalling matter?



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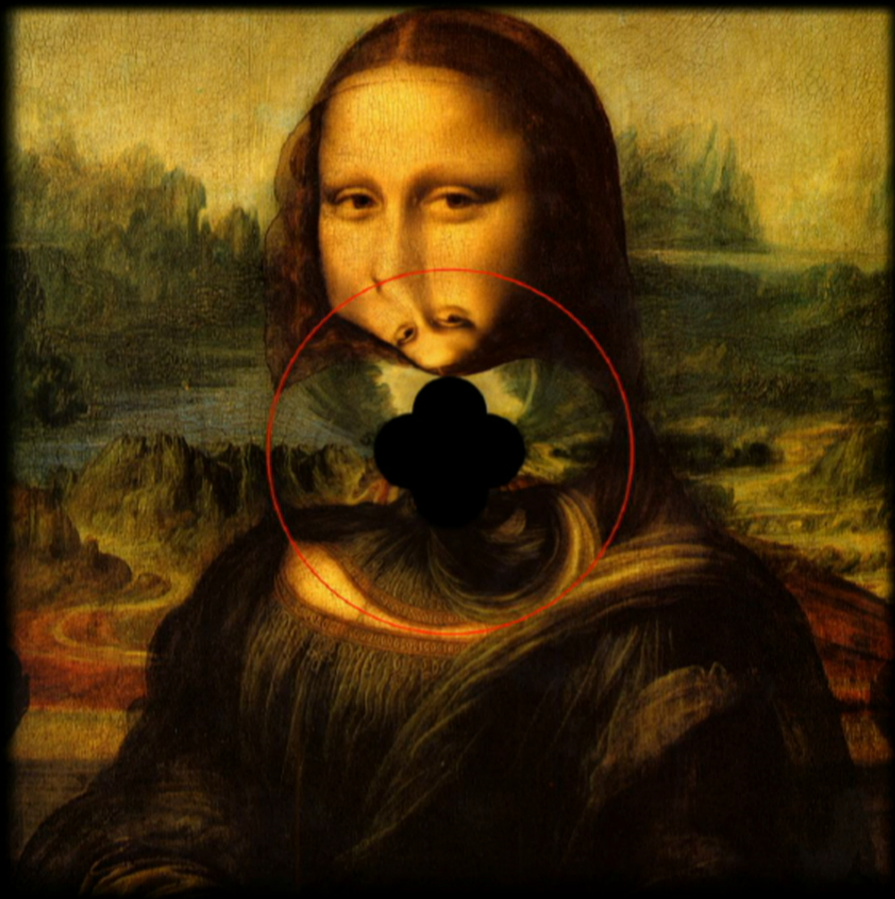

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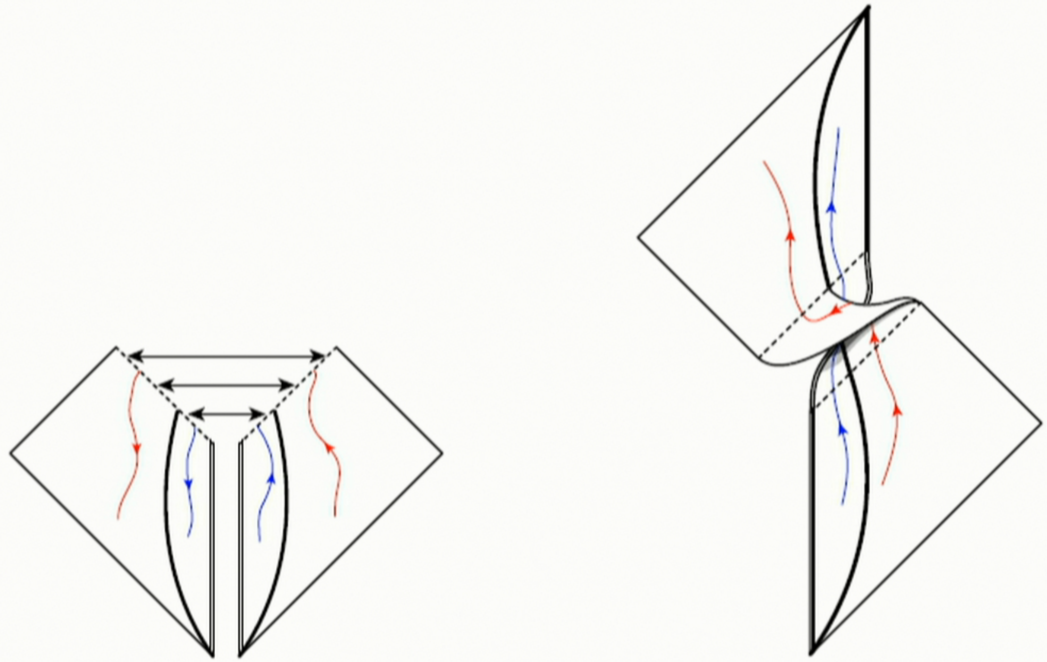
Thin shell collapse (work in progress)

Tentative Penrose diagram: no closed timelike curves and no singularities



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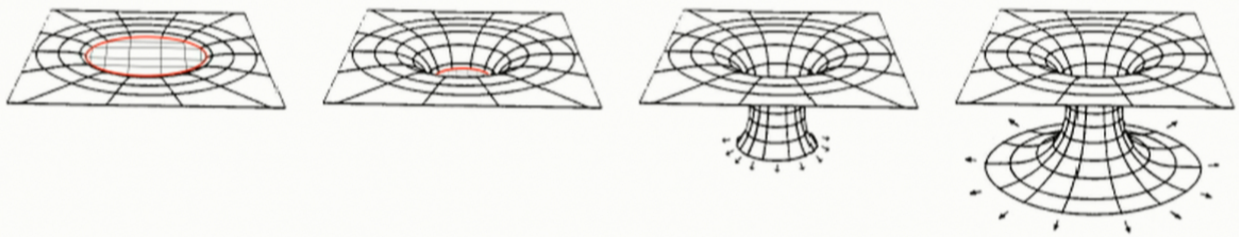
Thin shell collapse (work in progress)



Expanding compact flat region inside the shell.

In spherically symmetric case full exact calculation including backreaction essentially finished. Can reduce the dynamics to two global degrees of freedom (shell area and radial momentum). Remains to simulate solutions.

Big-bang singularities in SD

2+1 dim. Bianchi I model with torus topology:
homogeneous metric $g_{ij} = V^{\frac{1}{3}}\tilde{g}_{ij}(\tau_a)$, Teichmüller parameters τ_a

$$H_{\text{York}} = \sqrt{p^{ij}\, p_{ij}}$$

H_{York} generates curves in Teichmüller space that can be continued indefinitely.

But in the gauge in which we have equivalence with GR:

$$N_{\text{CMC}} = \frac{1}{\sqrt{\frac{3}{8}\tau^2 - 2\Lambda}}, \quad \Omega = \left(\frac{p^{ij}\, p_{ij}}{\frac{3}{8}\tau^2 - 2\Lambda}\right)^{\frac{1}{12}}$$

these curves can translate into spacetimes which can have crushing singularities ($\Omega \to 0$) anywhere on the curve.

Still admits 'shape' singularities: curves can hit the boundary of Teicmüller space (pancake or cigar-shaped universe).

How to construct spacetime from an SD solution

Take a solution of SD + matter : it is an evolving conformal geometry, parametrized by York time.

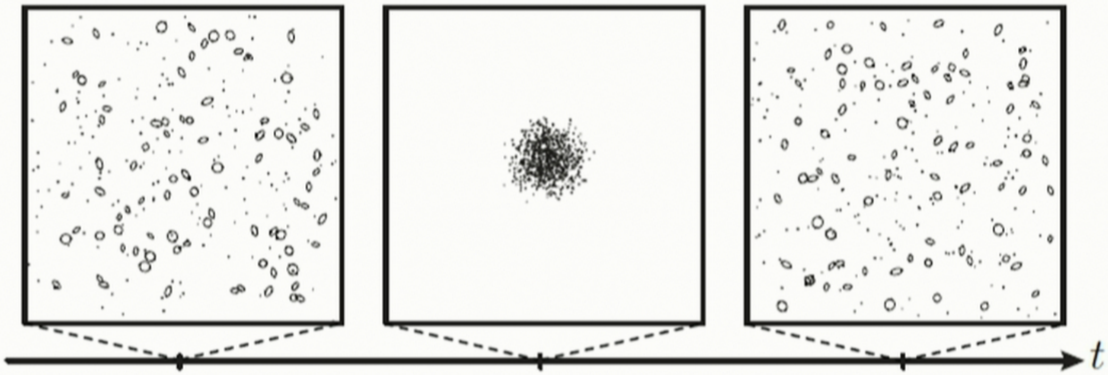
You can construct a (piecewise smooth) spacetime manifold from it, by solving the LY equation and the lapse-fixing equation.

What is the interpretation of the constructed 4-metric? The spacelike and timelike distances it measures are those read by *idealized* measuring rods and clocks, *i.e.* ignoring their backreaction on the geometry.

It is striking that the theory [...] introduces two kinds of physical things, i.e. (1) measuring rods and clocks, (2) all other things, e.g., the electromagnetic field, material point, etc. This, in a certain sense, is inconsistent; strictly speaking measuring rods and clocks would have to be represented as solutions of the basic equations... not, as it were, as theoretically self-sufficient entities.

(Einstein, 1949)

We studied an example of a precise realization of Einstein's idea in a toy model of a closed universe: the N-body problem with $\boldsymbol{J}_{\text{tot}} = E_{\text{tot}} = 0$:



at the central 'big bang-like' state the motions are chaotic, no stable subsystems that can be used to measure lengths and time intervals.

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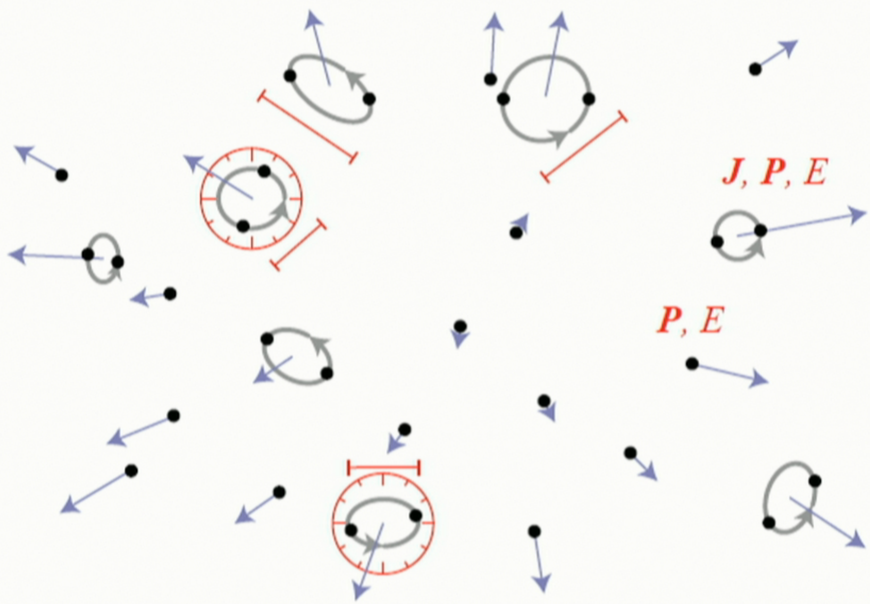
A late stages the system breaks up into clusters, subclusters and planetary systems, which stabilize as the universe expands. Only the total energy E_{tot}, angular $\boldsymbol{J}_{\text{tot}}$ and linear momentum $\boldsymbol{P}_{\text{tot}}$ are conserved, but asymptotically also E, $\boldsymbol{J}$ and $\boldsymbol{P}$ of isolated subsystems become conserved.



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Tim's 'experienced spacetime'

T. Koslowski made a first step towards a generalization of this to full SD [arXiv:1501.03007]: construct spacetime *experienced* by matter fluctuations.

1. Start with a background solution of SD: a conformal geometry - represented in some gauge as a 3-metric $g_{ij}(x,\tau)$ and momenta $p^{ij}(x,\tau)$,

2. add $\mathcal{O}(\sqrt{\varepsilon})$ matter fluctuations, *e.g.* free scalar field (quadratic Hamiltonian $H_{\mathsf{m}}[g_{ij},\varphi,\pi] := \frac{1}{2}\sqrt{g}(\pi^2 + \partial_i\varphi\partial^i\varphi + m^2\varphi^2)$, and solve LY equation:

$$8\,\Omega\,\Delta\,\Omega + \frac{\left\| p - \frac{1}{3} g\,\mathrm{tr}\,p \right\|^2}{g\Omega^6} - \frac{3}{8}\tau^2\,\Omega^6 - \Omega^2\,R + \varepsilon H_{\mathsf{m}}[g_{ij}\Omega^4,\varphi,\pi] = 0\,,$$

   at first order in ε, setting $\Omega = \Omega_0 + \varepsilon\,\Omega_1 + \mathcal{O}(\varepsilon^2)$.

3. Calculate the York Hamiltonian $H_{\mathsf{York}} = \int d^3x \sqrt{g}\,\Omega^6$ up to first order in ε,

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5. the $\mathcal{O}(\varepsilon)$ term is $\int d^3x \sqrt{g}\, N_{\mathsf{cmc}}\, H_{\mathsf{m}}[g_{ij}, \varphi, \pi]$, where N_{cmc} is the solution to the zeroth-order lapse-fixing equation:

$$8\,\Delta(N_{\mathsf{cmc}}\Omega_0) + 8\, N_{\mathsf{cmc}}\,\Delta\,\Omega_0 - \left(\frac{6 \left\| p - \frac{1}{3} g \,\mathrm{tr}\, p \right\|^2}{g \Omega_0^7} \frac{9}{4} \tau^2\, \Omega_0^5 + 2\,\Omega_0\, R \right) N_{\mathsf{cmc}} = \text{const.}$$

6. Then the matter fluctuations evolve like a scalar field on the background metric $\Omega_0^4\, g_{ab}$ with lapse N_{cmc}:

$$H_{\mathsf{effective}} = \int d^3x\, \Omega_0^6\, \sqrt{g}\; N_{\mathsf{cmc}}(x, \tau)\; H_{\mathsf{m}}[\Omega_0^4\, g_{ij}, \varphi, \pi] + \mathcal{O}(\varepsilon^2)$$

and the equations of motion for φ are

$$\Box\varphi - m^2\, \varphi = 0\,,$$

where $\Box$ is the D'Alembertian associated to the 4-metric

$$ds^2 = -N_0^2\, d\tau^2 + \Omega_0^4\, g_{ij} dx^i dx^j\,.$$

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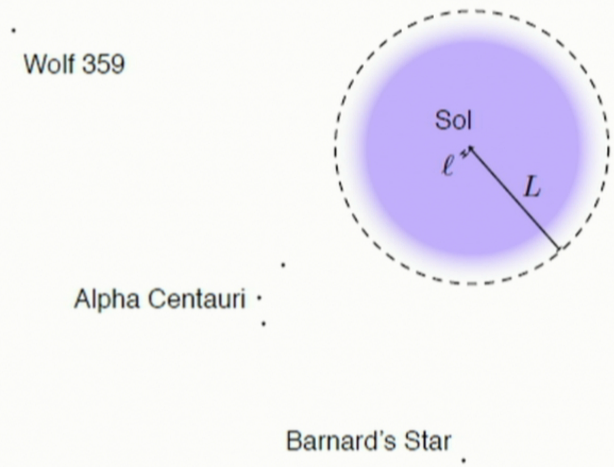
Isolated subsystems & emergence of Minkowski

Boundaries (*e.g.* asymptotically flat) against SD's relational principles.

But AF models well matter evolving in a large empty region of 'finite infinity':

If $L \gg \ell$ and the typical time scales within ℓ are smaller than the time light takes to propagate through L we can approximate the region $r < R$ with an asymptotically Euclidean space, $g_{ij} \sim \delta_{ij}$, $p^{ij} \sim 0$.



Then matter perturbations in the small region $r < \ell$ evolve according to an effective Hamiltonian which is invariant under the conformal isometries of 3D Euclidean space:

$$p_a = \frac{\partial}{\partial x^a}, \qquad \omega_a = \epsilon_{abc} x^b \frac{\partial}{\partial x^c},$$
$$\mathcal{D} = x^a \frac{\partial}{\partial x^a}, \qquad \xi^a = 2\, x^a x^b \frac{\partial}{\partial x^b} - x^b x_b \frac{\partial}{\partial x^a},$$

the conserved charges are:

$$\mathcal{C}[\chi] = \mathcal{H}_i[\chi^i] - \frac{2}{3}\mathcal{Q}[\nabla_i \chi^i],$$

and they close a SO(4,1) Poisson algebra. The generator associated to $\mathcal{D}$ turns out to be proportional to the Hamiltonian.

Quantum fields on a SD background are representations of this algebra.

Tim checked out the canonical quantization of a scalar representation. Taking the Inönü–Wigner contraction of $SO(4,1)$ into the Poincaré group $ISO(3,1)$ one reproduces the scalar field Hamiltonian on Minkowski.

First order formulation of SD

- To couple fermions to SD we need frame fields e^{I}_{a}, or *vielbeins*.

- We want them to transform conformally: Lee Smolin has a proposal for defining Ashtekar variables in SD, check out [arXiv:1407.2909].

- I think they should transform also under special conformal transformations: vielbeins are internal vectors and those, unlike the metric g_{ij}, distinguish between Weyl and special conformal transformations.

- Tim's canonical quantization on a SD background further motivates considering full $SO(4,1)$ in place of dilatations alone.

- Promising framework: Cartan geometry (non-flat model spaces) using $G = SO(4,1)$ and the *maximal parabolic subgroup* H as vertical subgroup. Model space G/H is compact!

Conclusions

- Learning how to construct familiar space-time notions from SD solutions,
- we are getting the first solutions of SD+matter,
- studying the result of grav. collapse: frozen star or pocket universe?
- TBD: what happens to matter crossing the horizon?
- TBD: what of singularity theorems in SD?
- Study QFT on a SD background.
  Does it differ physically from Poincaré-invariant QFT?
- Quantization of SD itself?
- First-order formalism? Can couple fermions? New physics?