Title: Local tests of global entanglement and a counterexample to the generalized area law

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Abstract: <p>We introduce a technique for applying quantum expanders in a distributed fashion, and use it to solve two basic questions: testing whether a bipartite quantum state shared by two parties is the maximally entangled state and disproving a generalized area law. In the process these two questions which appear completely unrelated turn out to be two sides of the same coin. Strikingly in both cases a constant amount of resources are used to verify a global property.</p>

local tests of global entanglement

and a counterexample to the generalized area law

D. Aharonov, A. Harow, Z. Landau, **D. Nagaj**, M. Szegedy, U. Vazirani

universität wien

1 q. expanders

maximally entangled states

2 entanglement

testing and communication

1 **q. expanders**

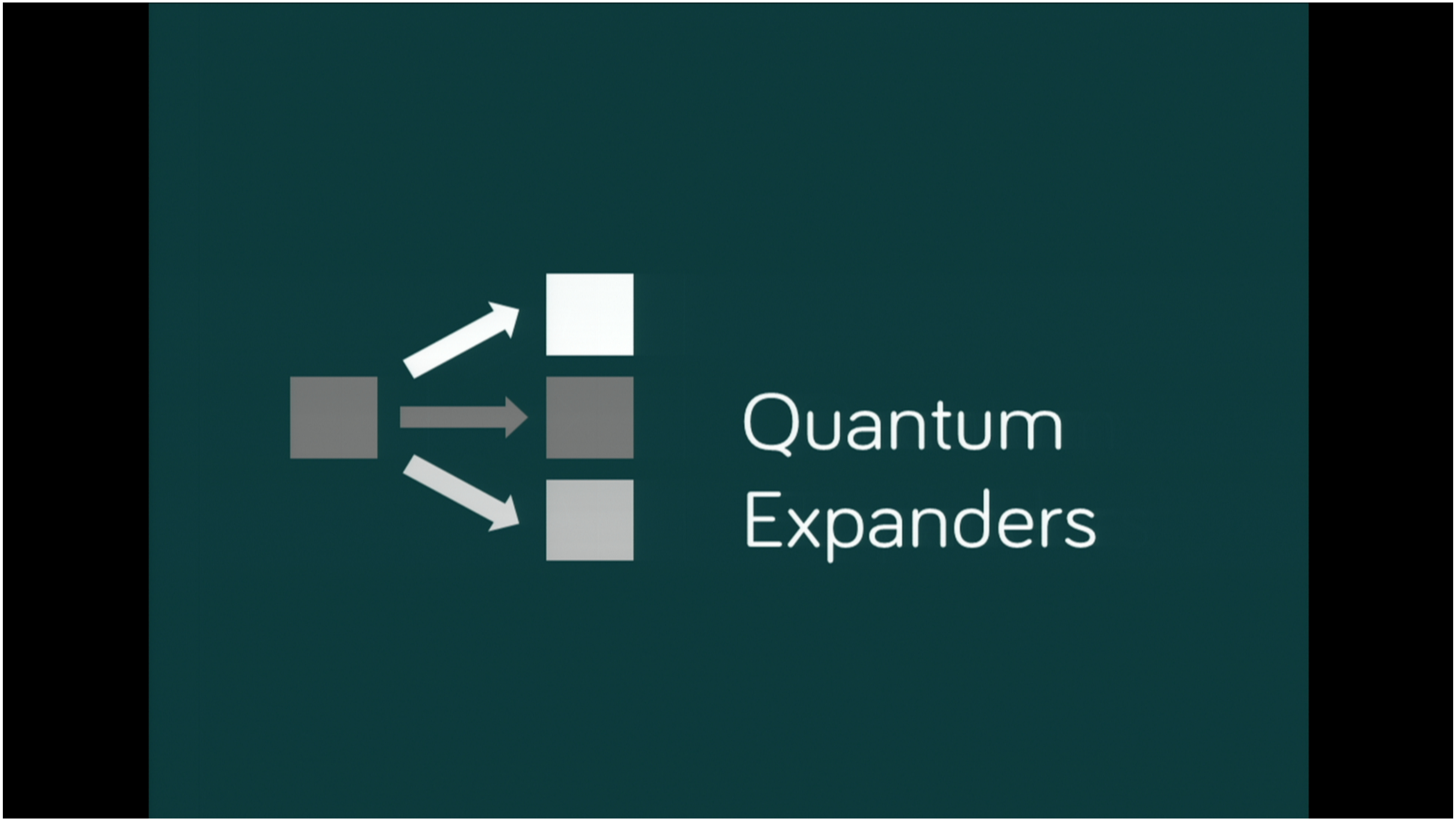
maximally entangled states

2 **entanglement**

testing and communication

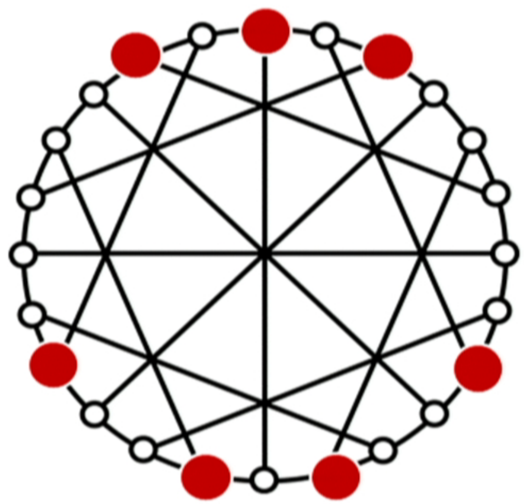
3 **area law**

gaps, connections, correlations


Quantum
Expanders

1 Classical expanders

- walk on these graphs? mix fast!



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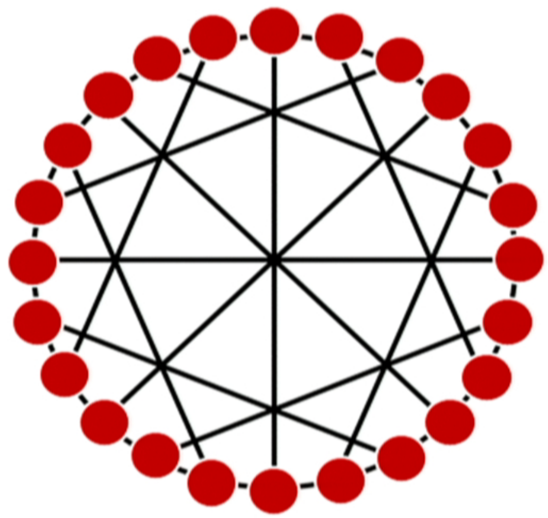

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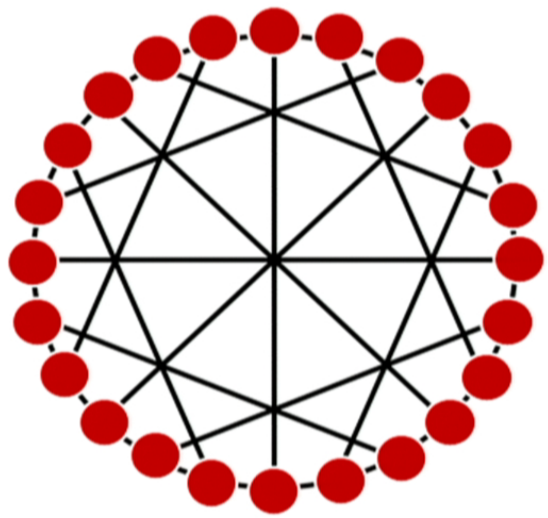

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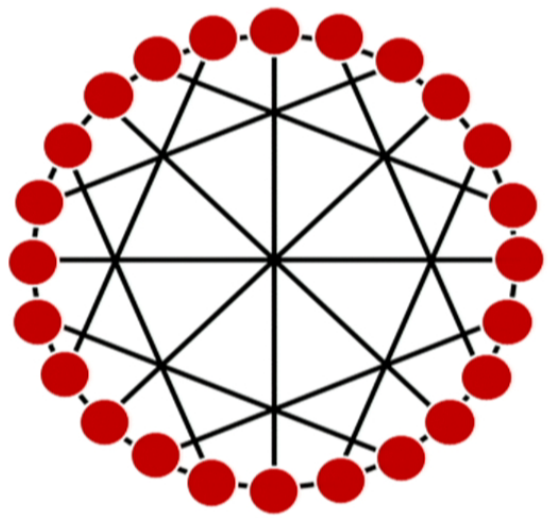

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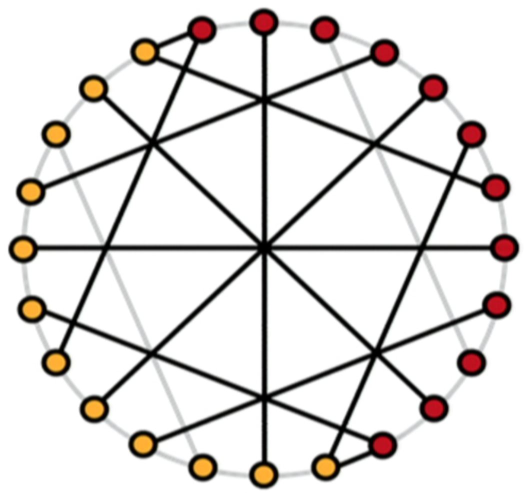

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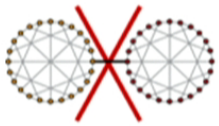
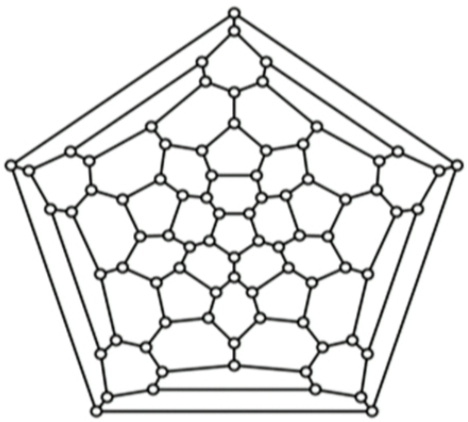
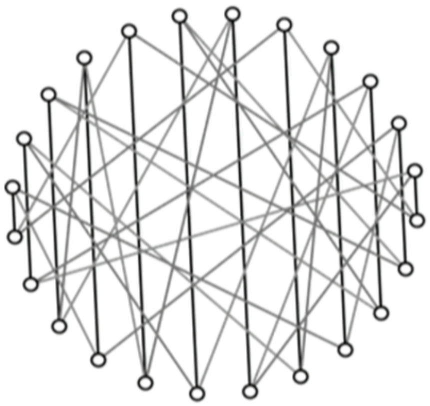
  examples: Ramanujan, Cayley







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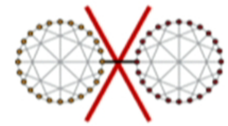
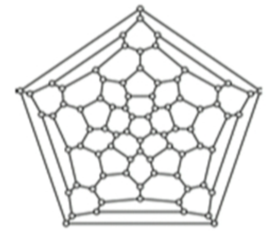
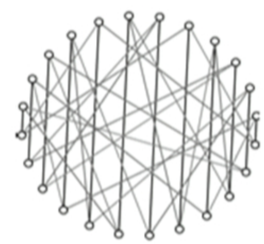




- explicit, constant-degree
  approximations to the full graph

  normalized adjacency matrix
  second largest eigenvalue $1-\lambda$



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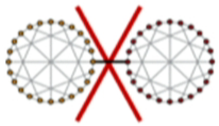

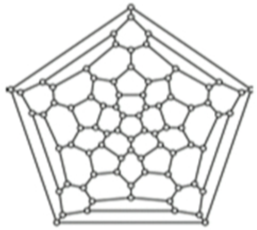

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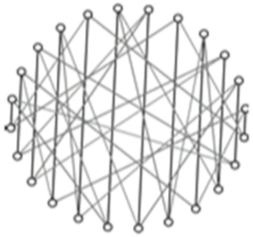

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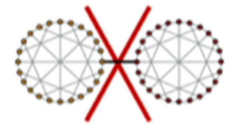

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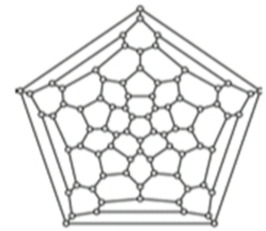

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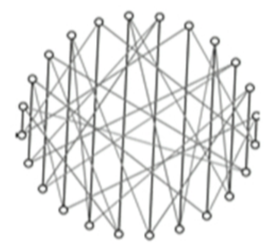
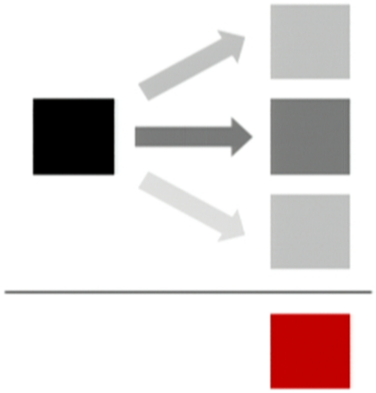


- a review [Hoory Linial Wigderson]
  a talk [Harrow quantum expanders youtube]

1 Mixing up something quantum

- applying random unitaries

$$\mathcal{E}(X) = \frac{1}{k}\sum_{i=1}^{k} U_i X U_i^\dagger$$



- classical expanders: explicit, constant-degree approximations to the full graph fast-mixing

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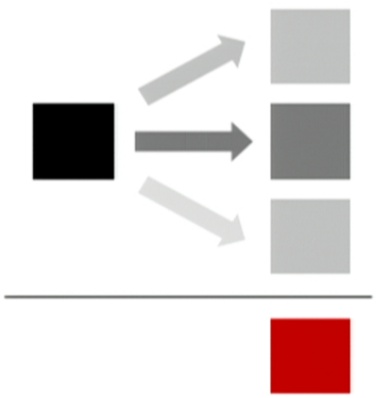

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- transform $N \times N$ matrices

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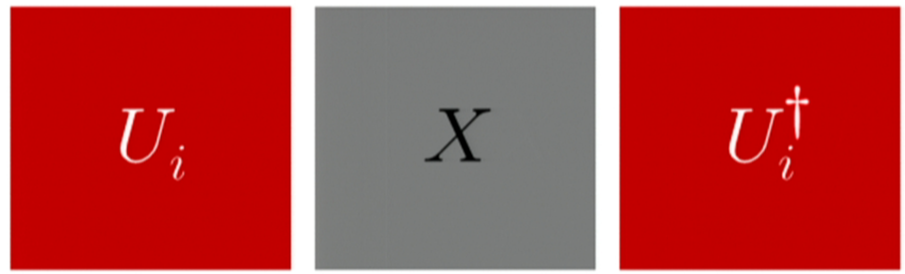

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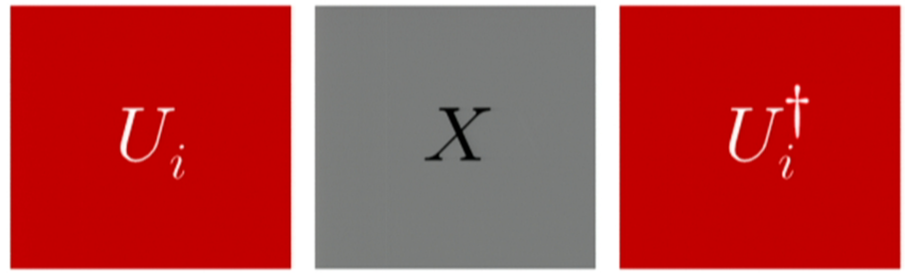

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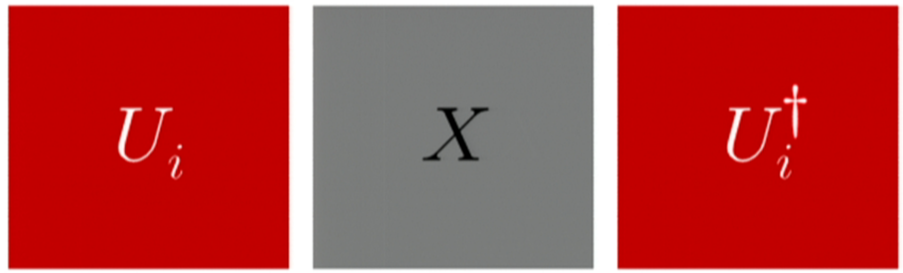

$$X = \mathbb{I}$$

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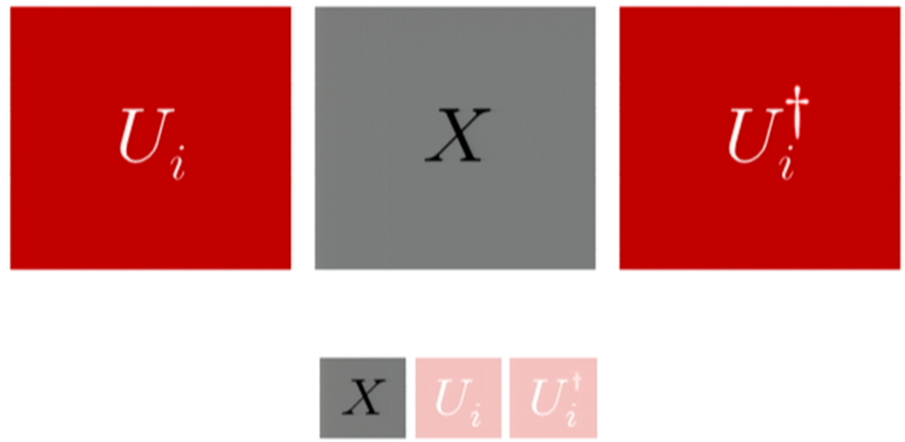

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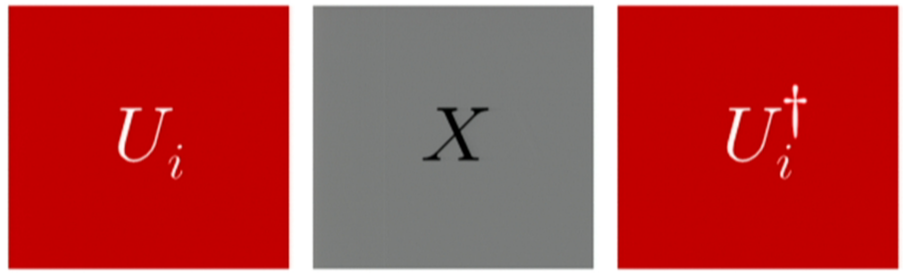

- interpreting matrices as 2-register states

$$\underset{\text{density matrix}}{X} = \sum_{a,b} X_{ab} |a\rangle\langle b|$$

1 Quantum expanders

- transform *N*×*N* matrices

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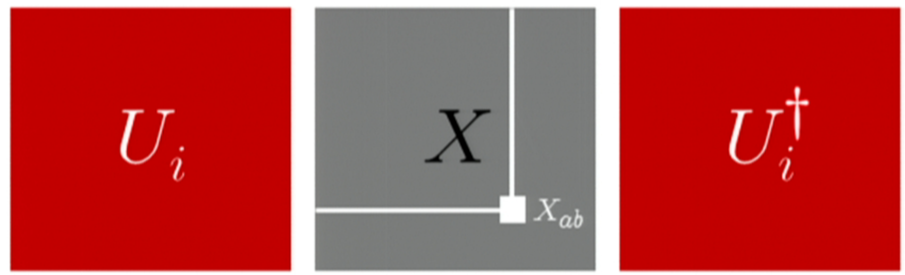

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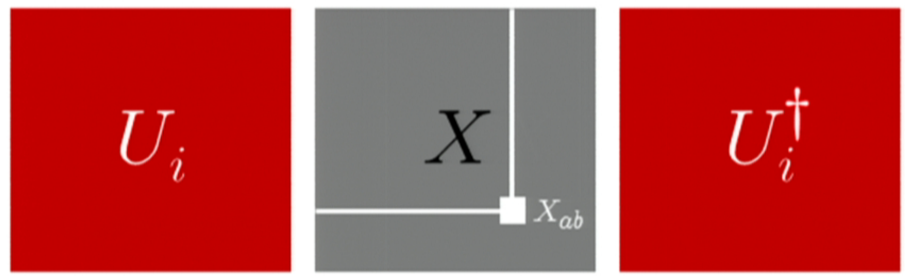

- interpreting matrices as 2-register states

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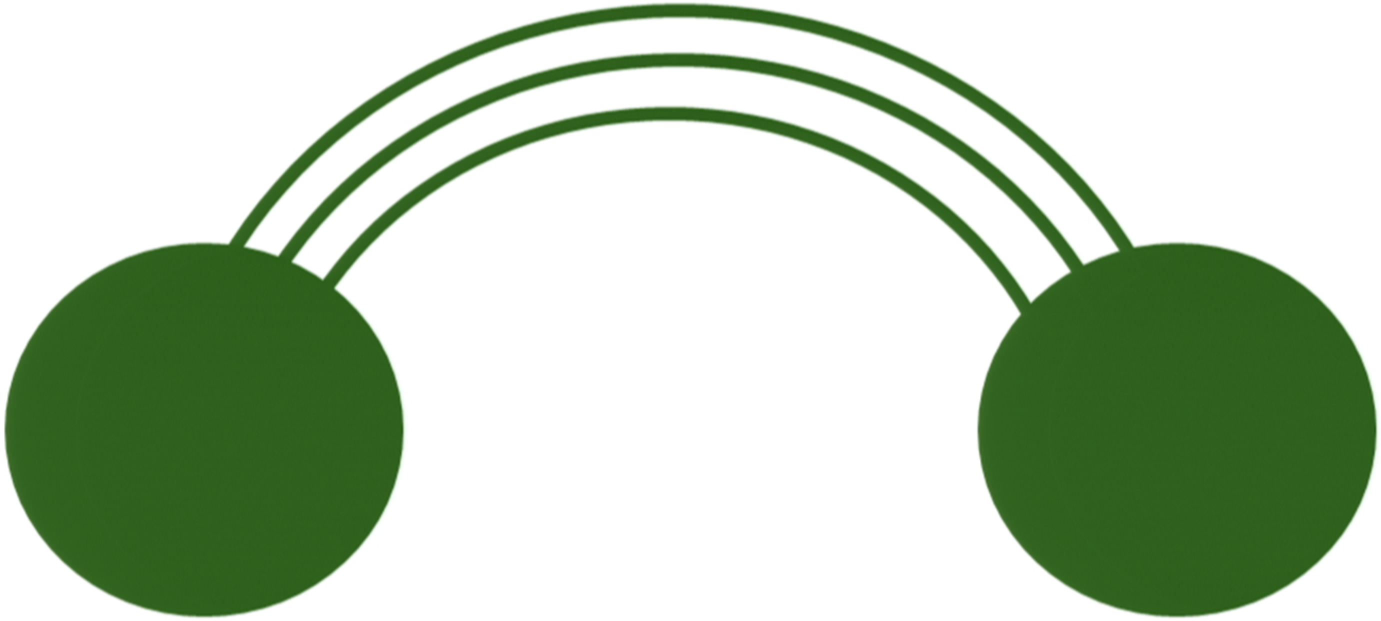
density matrix

2 EPR testing

- how costly is it to certify that we share a maximally entangled state?

$$\frac{1}{\sqrt{N}}\sum_{x=1}^{N}|x\rangle|x\rangle$$

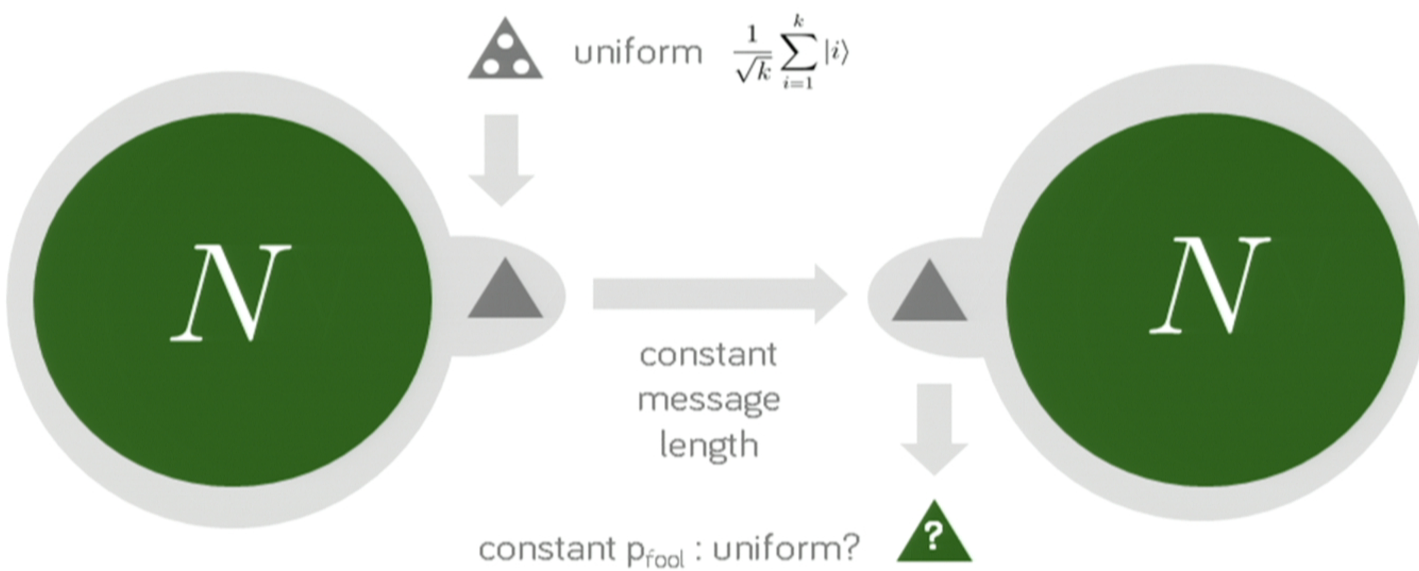


2 EPR testing

- when does the qutrit remain uniform?
  for max. entangled X

$$\frac{1}{\sqrt{3}}\sum_{i=1}^{3}|i\rangle\,(U_i\otimes U_i^*)\,\frac{1}{\sqrt{N}}\sum_{x=1}^{N}|x\rangle|x\rangle$$

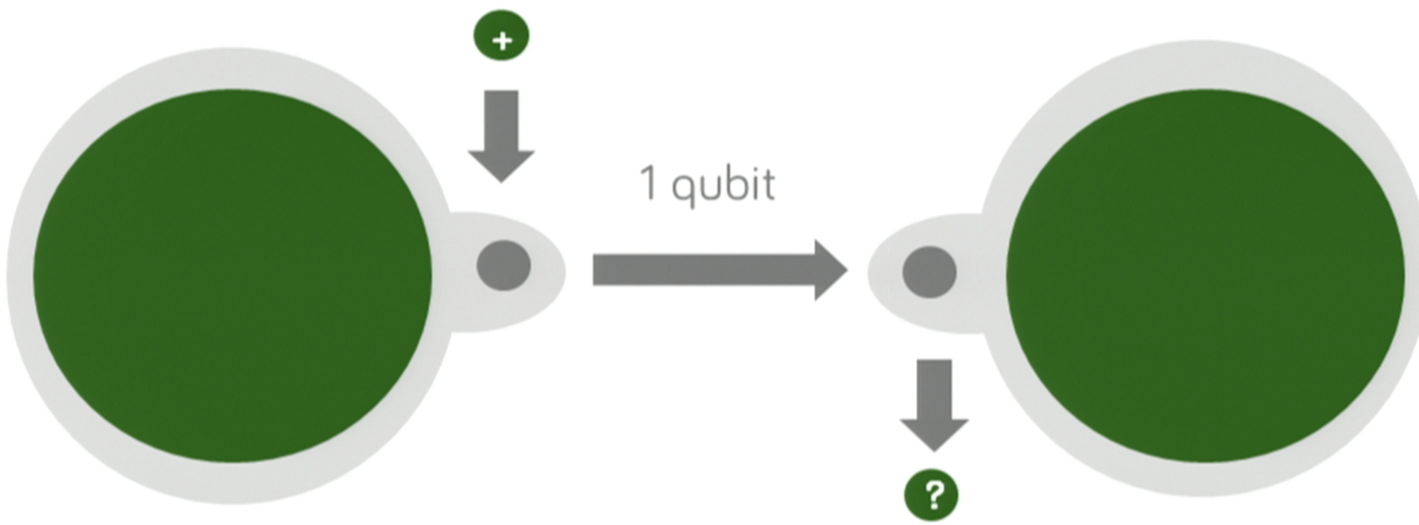
- quantum expander property … soundness



2 EPR testing: slimming down the protocol

- 2 shared bits of randomness, 1 qubit of communication.
- Pick a random $i \in \{1, 2, 3\}$. Use a qubit $|+\rangle$ to control the application of U_i.

  Use the qubit to control the application of U_i^*. Test if the qubit is $|+\rangle$.



few connections

local interactions

constant gap

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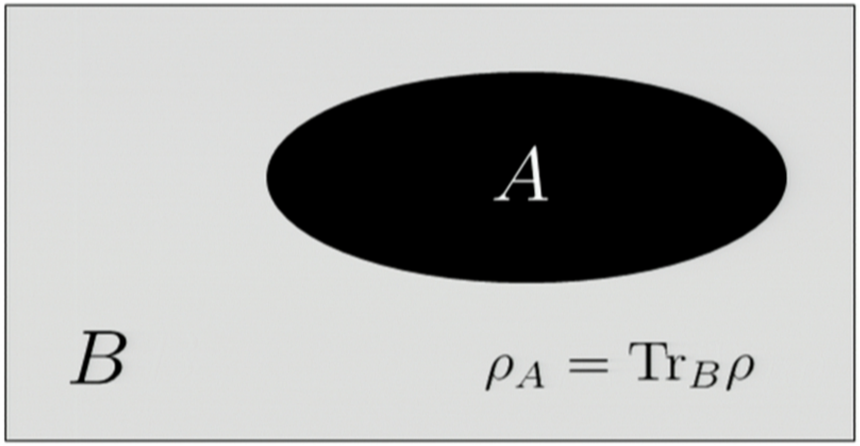
global correlations

3 Ground states of gapped quantum spin systems

- entanglement entropy

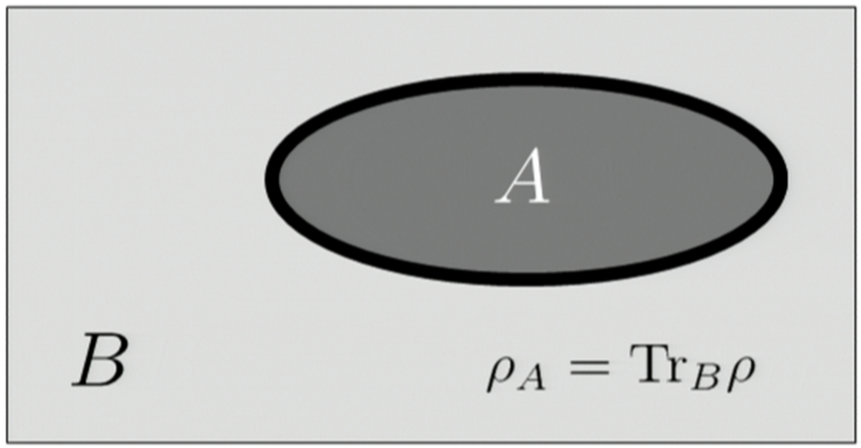
$$S = -\mathrm{Tr}(\rho_A \ln \rho_A)$$



3 Ground states of gapped quantum spin systems

- entanglement entropy

$$S = -\mathrm{Tr}(\rho_A \ln \rho_A) \sim$$ ~~volume~~ surface area



area law ➡ a simple ground state?

3 Gapped 1D Hamiltonians

- Nothing closer than Δ to the ground state.

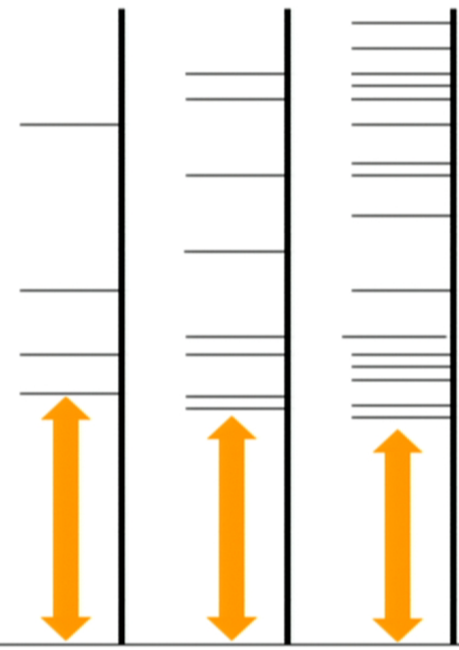
the AKLT (spin-1) chain

$$\sum_{j=1}^{N-1} \vec{S}_j \cdot \vec{S}_{j+1} + \frac{1}{3}\left(\vec{S}_j \cdot \vec{S}_{j+1}\right)^2$$

a biased walk in 1D

$$\sum_{j=1}^{N-1} \left(|j\rangle - B|j+1\rangle\right)\left(\langle j| - B\langle j+1|\right)$$

$N \to \infty$



Without a gap, the entropy can be large. [Verstraete, Latorre+]

3 Ground states of gapped H's & the area law

- entanglement entropy

$$S = -\mathrm{Tr}(\rho_A \ln \rho_A) \sim$$ ~~volume~~ surface area

1D ... theorems [Hastings 07, Arad+ 13]

algorithms [White 92, Vidal 03, Landau+ 13]

2D ... we're close

- generalized area conjecture

entropy ~ cut size

3 Generalized area conjecture: the counterexample

- an $N \times 3 \times 3 \times N$ dimensional system
- a gapped, frustration-free Hamiltonian
  an $O(1)$ interaction between the qutrits
- a unique, very entangled ground state
  $O(N)$ entanglement entropy across the cut

3 The 4-particle ($N \times 3 \times 3 \times N$) Hamiltonian

- a projector P_L with ground states $\frac{1}{\sqrt{3}}\left(|1\rangle|x\rangle + |2\rangle A|x\rangle + |3\rangle B|x\rangle\right)$

- as a vector

x
Ax
Bx

$\otimes|j\rangle \otimes |y\rangle$

as a matrix

X_1	X_2	X_3
AX_1	AX_2	AX_3
BX_1	BX_2	BX_3

P_L

3 The 4-particle ($N \times 3 \times 3 \times N$) Hamiltonian

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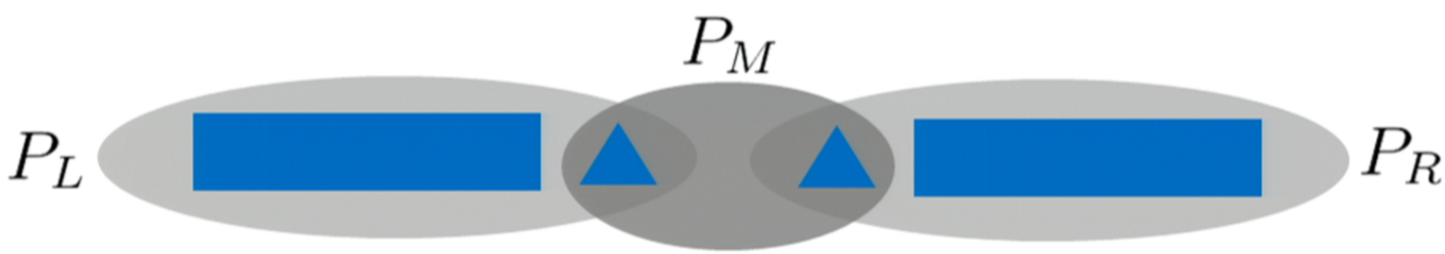
  a projector P_R — $(|1\rangle|y\rangle + |2\rangle A|y\rangle + |3\rangle B|y\rangle)\,/\sqrt{3}$
  a projector P_M — enforce symmetry: 12 & 21
  13 & 31

X	XA	XB
AX	AXA	AXB
BX	BXA	BXB



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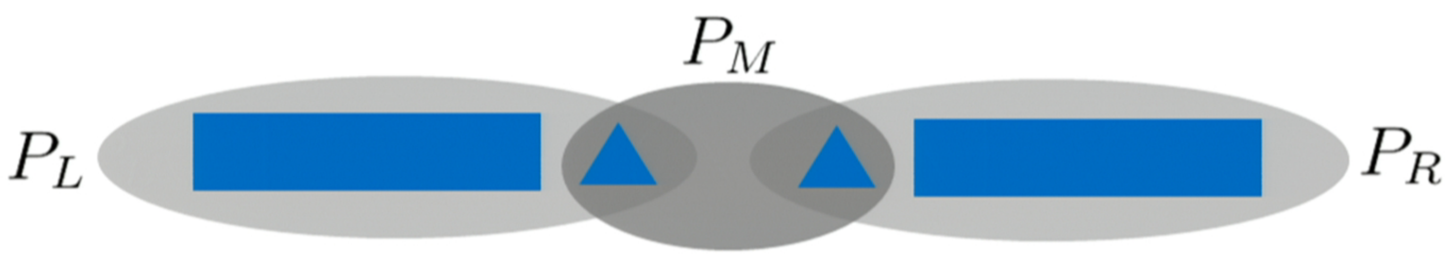
- who commutes with A and B?

  only the identity, as $[I, A, B]$ is a q. expander

I	A	B
A	AA	AB
B	BA	BB

$$\frac{1}{3\sqrt{N}}$$

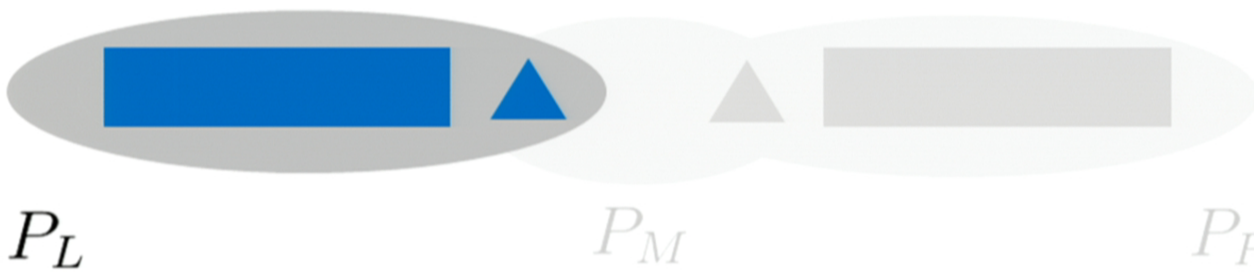


3 Making the counterexample local

- a quantum circuit, history state

prepare $$\frac{1}{\sqrt{3}}\left(|1\rangle|x\rangle + |2\rangle A|x\rangle + |3\rangle B|x\rangle\right)$$

from $$\frac{1}{\sqrt{3}}\left(|1\rangle + |2\rangle + |3\rangle\right)|x\rangle$$



3 The history state: a ground state of a qudit chain

2-local

c-o-n-d-i-t-i-o-n-s

clock encoding

state progression

initialization

idling

$$|\cdots\rangle|0\rangle \otimes |0\rangle$$

$$|\varphi_t\rangle \otimes |t\rangle$$

$$|\varphi_{t+1}\rangle \otimes |t+1\rangle$$

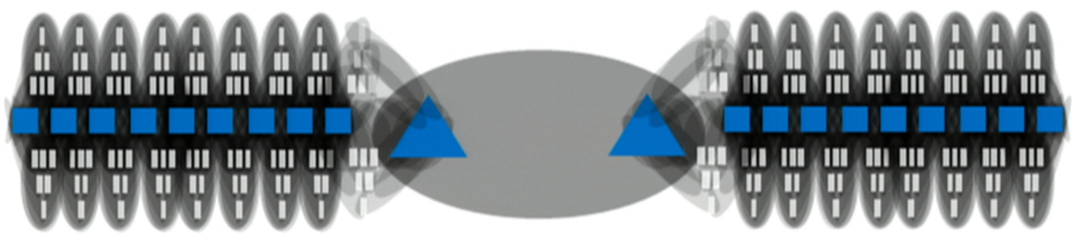
$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} |\varphi_t\rangle \otimes |t\rangle$$

most of the state has the result

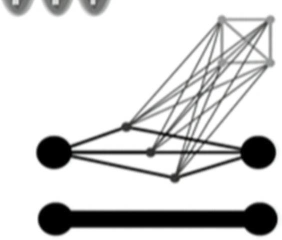
3 Making the counterexample local

- a quantum circuit, history state
  Kitaev's LH, 1D, qudits
  an approx. groundstate, a small 1/poly(n) gap

- rescale P_L, P_R (not the middle!)
  a constant gap, huge couplings



- decompose using gadgets [Cao, N.]



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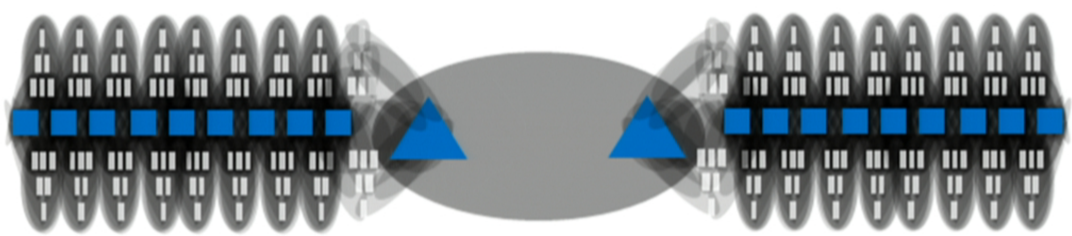
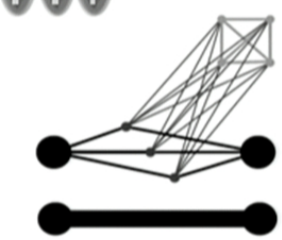
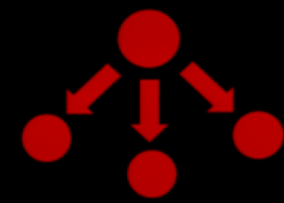


- decompose using gadgets [Cao, N.]
  ~~huge couplings~~ many spins, high degree



1 **q. expanders**

maximally entangled states



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