Title: Black hole chemistry: thermodynamics with Lambda

Date: Feb 19, 2015 01:00 PM

URL: http://pirsa.org/15020092

Abstract: <p>The mass of a black hole has traditionally been identified with its energy. We describe a new perspective on black hole thermodynamics, one that identifies the mass of a black hole with chemical enthalpy, and the cosmological constant as thermodynamic pressure. This leads to an understanding of black holes from the viewpoint of chemistry, in terms of concepts such as Van derWaals fluids, reentrant phase transitions, and triple points. Both charged and rotating
black holes exhibit novel chemical-type phase behaviour, hitherto unseen.</p>


SHK Reebok

Black holes as thermodynamic objects

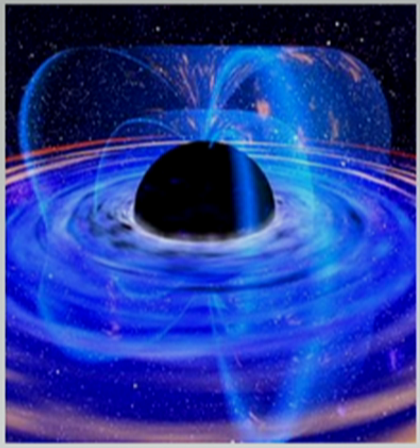
If someone points out to you that your pet theory of the universe is in disagreement with Maxwell's equations-then so much the worse for Maxwell's equations. If it is found to be contradicted by observation-well these experimentalists do bungle things sometimes. But if your theory is found to be against the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation.

Sir Arthur Stanley Eddington

Gifford Lectures (1927), *The Nature of the Physical World* (1928), 74.

Schwarzschild black hole:

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega^2$$



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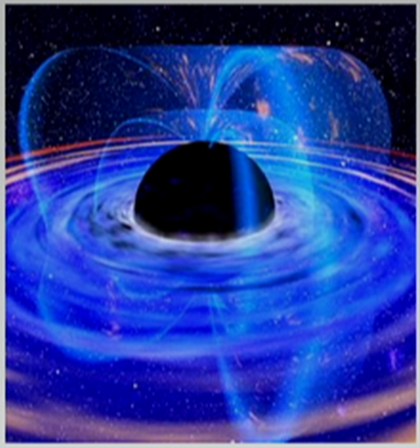
- asymptotic mass (total energy)

$$M = -\frac{1}{8\pi}\int_{S_\infty} *dk\,, \quad k^a = (\partial_t)^a$$



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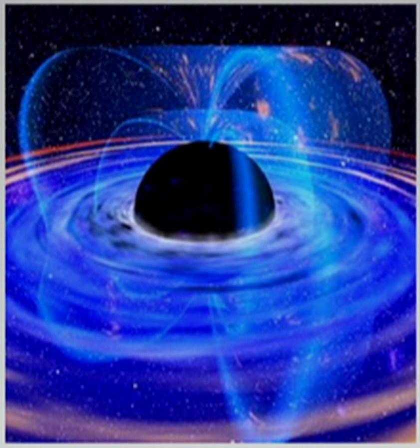

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surface gravity $(k^b \nabla_b k^a)_{|H} = \kappa k^a_{|H}$ ⇒ $\kappa = \dfrac{1}{4M}$

surface area $A = 4\pi r_h^2$ never decreases

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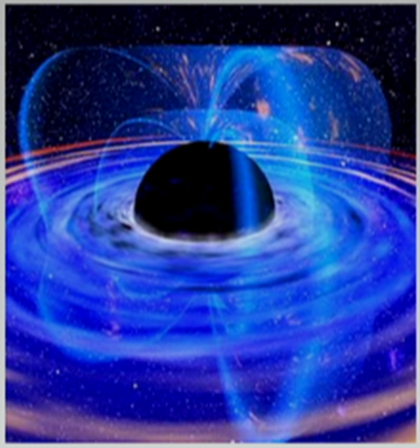

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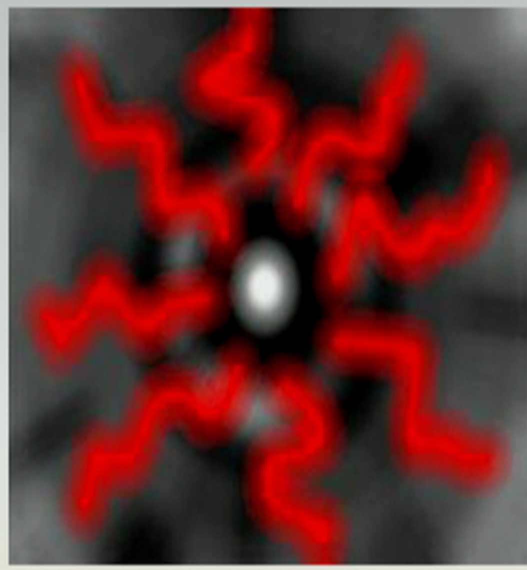
- black hole mechanics:

$dM = \kappa dA$ ⟺ (Bekenstein?) $dE = TdS$

Hawking (1974):

When quantum effects are taken into account, black holes radiate away particles as black body.



$$T = \frac{\kappa}{2\pi}, \quad S = \frac{A}{4}$$

derivation used QFT in curved spacetime

Other approaches: Euclidean path integral approach, Tunneling approach, LQG, String theory, ...

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Black hole thermodynamics

- First law of black hole thermodynamics:

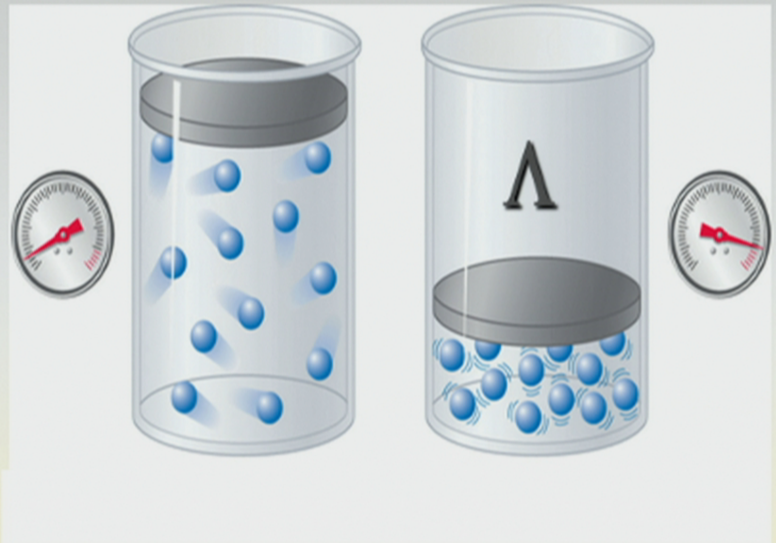
$$\delta M = T\delta S + \sum_i \Omega_i \delta J_i + \Phi \delta Q$$

- Smarr-Gibbs-Duhem relation:

$$\frac{d-3}{d-2}M = TS + \sum_i \Omega_i J_i + \frac{d-3}{d-2}\Phi Q$$

Where is the PdV term?

Λ as thermodynamic pressure & thermodynamic volume



Proposal

- Consider an asymptotically AdS black hole spacetime

- Identify the cosmological constant with a thermodynamic pressure

$$P = -\frac{1}{8\pi}\Lambda = \frac{3}{8\pi}\frac{1}{l^2}$$

- Allow this to be a “dynamical” quantity

Thermodynamic volume

1st Law of black hole thermodynamics:

D.Kastor, S.Ray, and J.Traschen, *Enthalpy and the Mechanics of AdS Black Holes*, Class. Quant. Grav. 26 (2009) 195011, [arXiv:0904.2765].

$$\delta M = T\delta S + \Theta\,\delta P + \dots$$

- Introduces PdV term into black hole thermodynamics
- Mass M interpreted as **enthalpy** rather than energy

$$U = M + \epsilon V = M - PV$$

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Thermodynamic volume of black holes:

$$V = \left(\frac{\partial M}{\partial P}\right)_{S,\dots}$$

Schwarzschild(-AdS):

$$V = \frac{4}{3}\pi r_+^3$$

Other definitions of volume

"Naive" geometric volume:

$$\mathrm{Vol} = \int_{r_0}^{r_+} dr \int d\Omega \sqrt{-g}$$

- M.K. Parikh, Volume of black holes, Phys. Rev. D73, 124021 (2006).
- W. Ballik and K. Lake, The volume of stationary black holes and the meaning of the surface gravity, arXiv:1005.1116 [gr-qc].

Killing co-potentials volume:

$$\nabla \cdot \xi = 0 \;\Rightarrow\; \nabla \cdot \omega = \xi\,.$$

D. Kastor, S. Ray and J. Traschen, Enthalpy and the mechanics of AdS black holes, Class. Quantum Grav. 26, 195011 (2009), arXiv:0904.2765 [hep-th].

$$V = -\int_H *\omega_\xi\,.$$

(Komar integration in AdS)

Kerr BH: $$V = \frac{r_+ A}{D-1} = \frac{4\pi}{3} r_+ (r_+^2 + a^2)\,.$$

(unique-constructed from KY, r_0=0)

Vector volume: W.Ballik and K.Lake, Vector volume and black holes, Phys.Rev.D88, 04038 (2013).

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Good definition of volume: isoperimetric inequality

Isoperimetric Inequalities (analogue of Penrose inequalities)

M. Cvetic, G.W Gibbons, DK, C.N. Pope, *Black hole enthalpy and an entropy inequality for the thermodynamic volume,* Phys. Rev. D84 (2011) 024037, [arXiv:1012.2888].

$$\mathcal{R} = \left(\frac{(d-1)\mathcal{V}}{\omega_{d-2}}\right)^{\frac{1}{d-1}} \left(\frac{\omega_{d-2}}{\mathcal{A}}\right)^{\frac{1}{d-2}}$$

$$\omega_d = \frac{2\pi^{\frac{d+1}{2}}}{\Gamma\left(\frac{d+1}{2}\right)}$$

Conjecture: for any AdS black hole $\mathcal{R} \geq 1$

"For a black hole of given **thermodynamic volume** V, the entropy is maximised for Schwarzschild-AdS"

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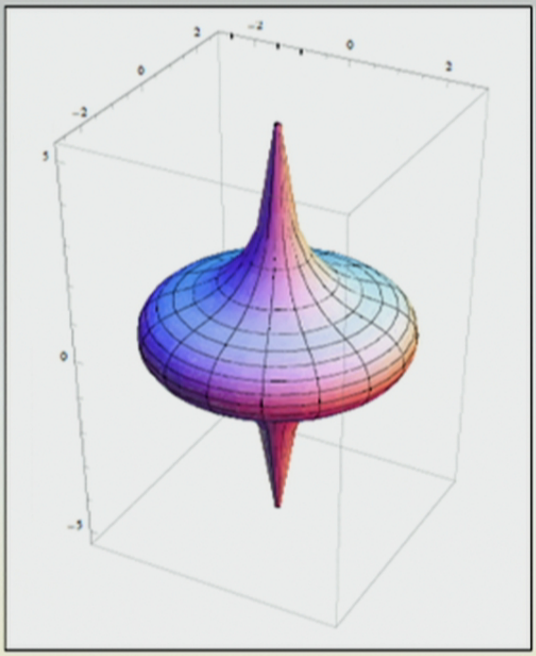
Super-entropic black holes

...recent work with R.Hennigar, R.Mann, N.Musoke, arXiv:1411.4309

D.Klemm, *Four-dimensional black holes with unusual horizons*, arXiv:1401.3107.



- Non-compact horizon with finite horizon area
- Thermodynamic volume

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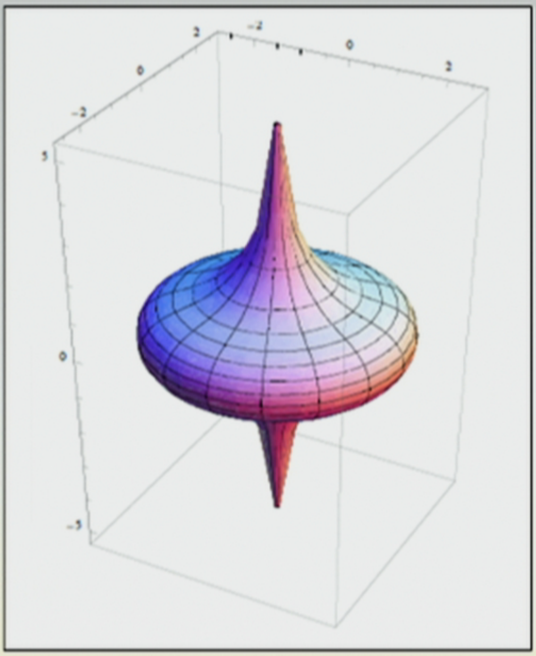

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Allows one to derive the valid Smarr relation (scaling argument)

Euler's theorem:

$$f(\alpha^p x, \alpha^q y) = \alpha^r f(x,y) \Rightarrow r f(x,y) = p\left(\frac{\partial f}{\partial x}\right) x + q\left(\frac{\partial f}{\partial y}\right) y.$$

Mass of black hole: $M = M(A, P)$

since $[P] = L^{-2}, \quad [A] = L^2, \quad [M] = L \Rightarrow$

$$M = 2A\left(\frac{\partial M}{\partial A}\right) - 2P\left(\frac{\partial M}{\partial \Lambda}\right) + dM = \kappa dA + V dP$$

Smarr relation: $M = 2(TS - VP)$

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Thermodynamic machinery

- **Study**: charged and rotating AdS black holes in a canonical (fixed Q or J) ensemble. Relate to **fluid thermodynamics**, by comparing the "same physical quantities"

- The corresponding thermodynamic potential is **Gibbs free energy**

$$G = M - TS = G(P, T, J_1, \ldots, J_N, Q)\,.$$

equilibrium state corresponds to the **global minimum** of G.

- **Local thermodynamic stability**: positivity of the specific heat

$$C_P \equiv C_{P,J_1,\ldots,J_N,Q} = T\left(\frac{\partial S}{\partial T}\right)_{P,J_1,\ldots,J_N,Q}$$

- **Phase diagrams**: P-T diagrams

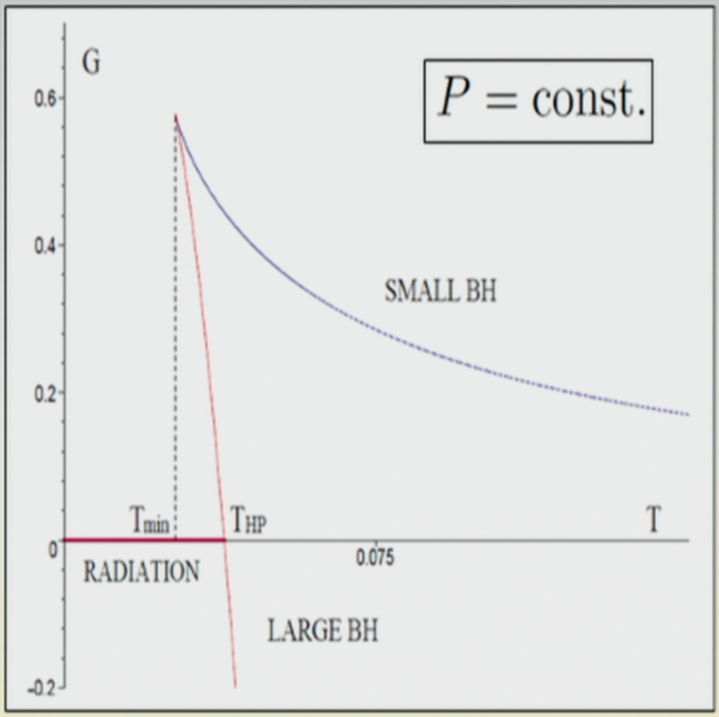
- **Critical points:** calculate critical exponents,….

a) Schwarzschild-AdS black hole

Hawking-Page transition:

$$f = k - \frac{2M}{r} + \frac{r^2}{l^2}\,.$$

S.W. Hawking & D.N. Page, *Thermodynamics of black holes in anti-de-Sitter space*, Commun. Math. Phys. 87, 577 (1983).



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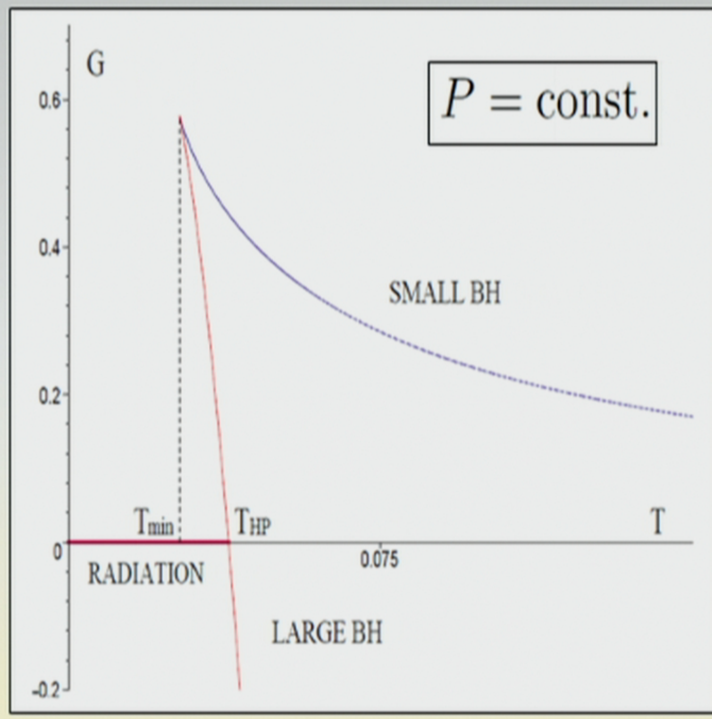

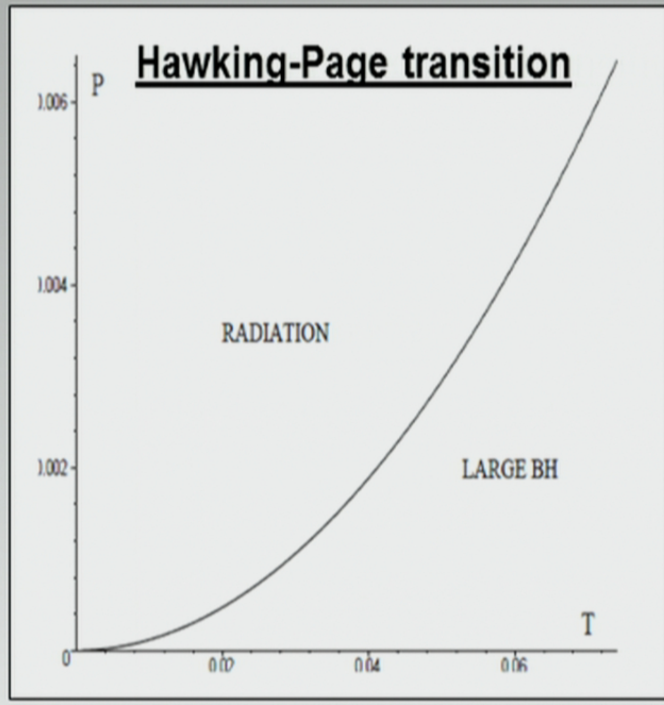

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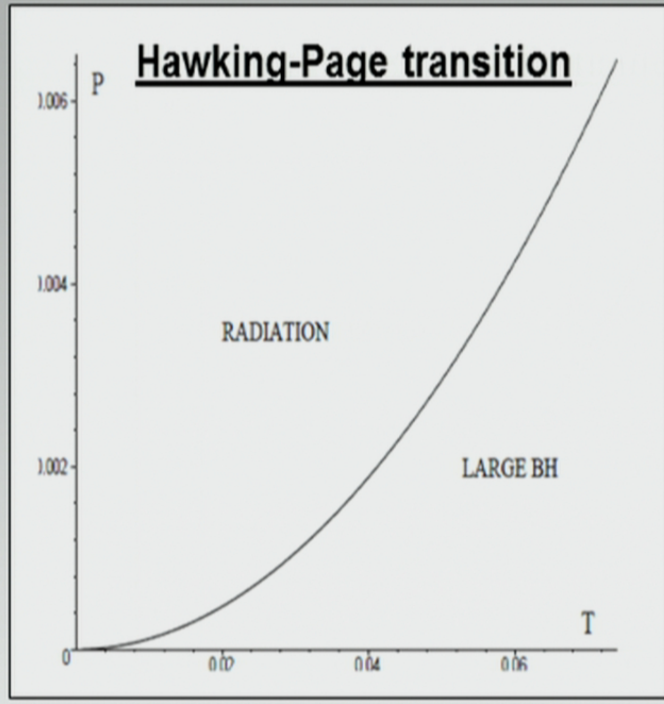
"<u>fluid interpretation</u>": solid/liquid PT (infinite coexistence line)



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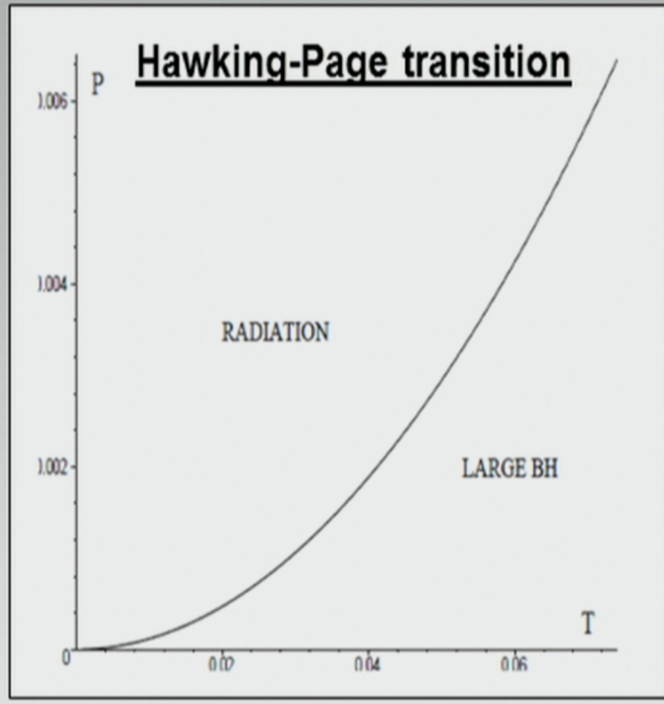
Equation of state: depends on the horizon topology

$$Pv = T - \frac{k}{2\pi}\frac{1}{v}$$

$$v = 2r_+ l_P^2 = 2\left(\frac{3V}{4\pi}\right)^{\frac{1}{3}} = 6\frac{V}{N}, \quad N = \frac{A}{l_P^2}$$

Planar black holes correspond to ideal gas! Can we go beyond?



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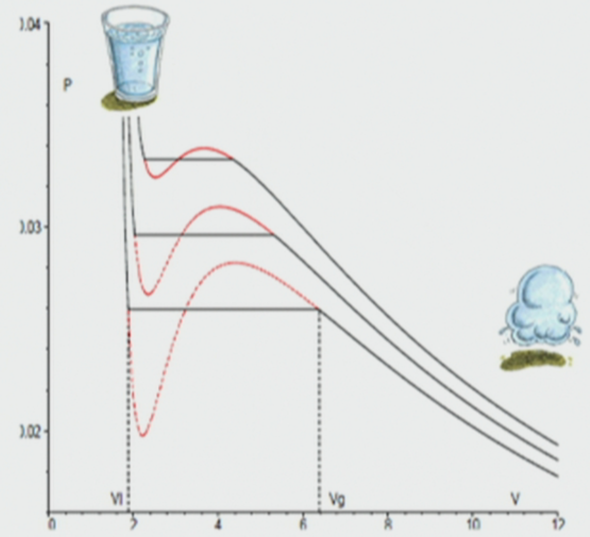

FIG. 2. **Maxwell's equal area law.** The 'oscillating' (dashed) part of the isotherm $T < T_c$ is replaced by an isobar, such that the areas above and below the isobar are equal one another.

$$\left(P + \frac{a}{v^2}\right)(v - b) = T$$

Parameter a measures the **attraction** between particles (a>0) and b corresponds to "**volume of fluid particles**".

Critical point:

$$\rho_c = \frac{P_c v_c}{T_c} = \frac{3}{8}$$

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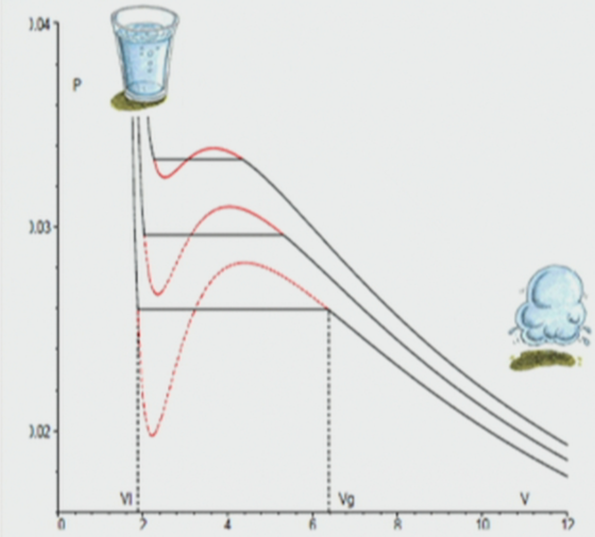

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Analogy complete?

Equation of state:

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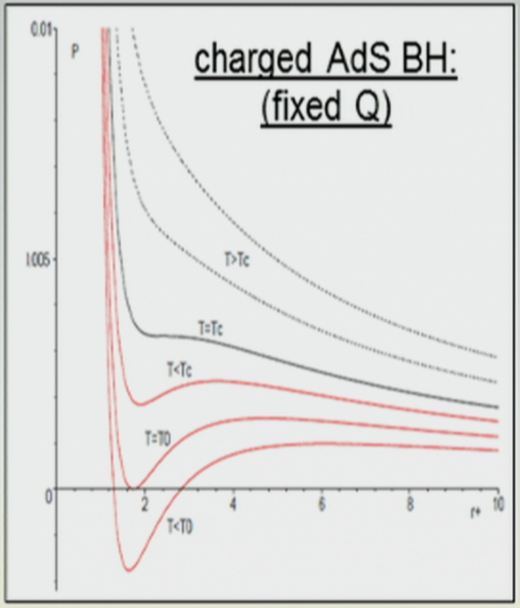

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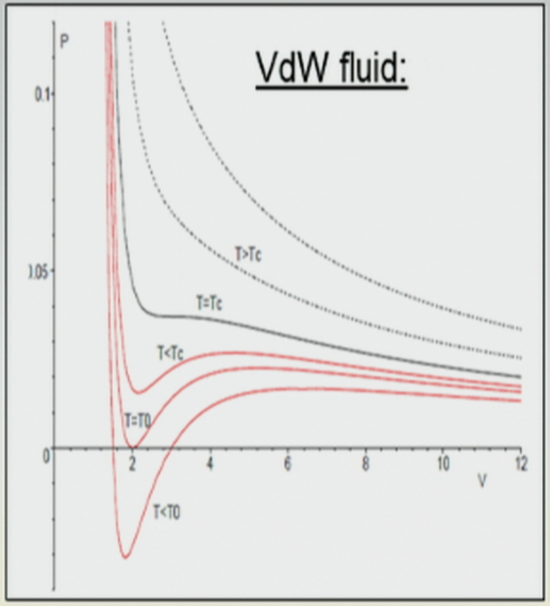
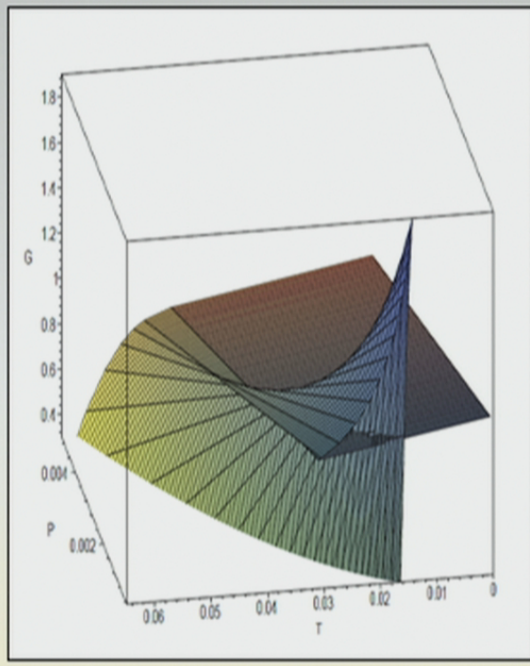


Gibbs free energy: demonstrates standard swallow tail behavior

$$G = G(T,P) = \frac{1}{4}\left(r_+ - \frac{8\pi}{3}Pr_+^3 + \frac{3Q^2}{r_+}\right)$$



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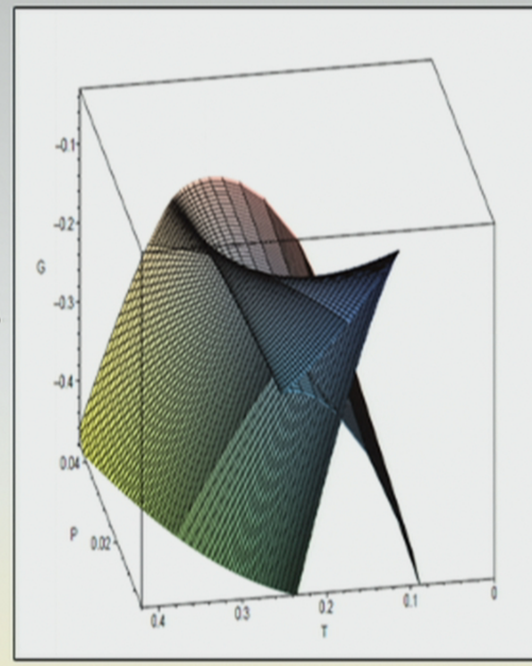
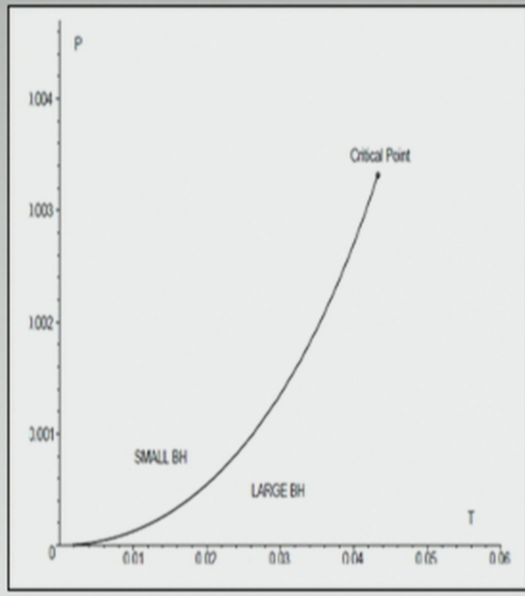


Coexistence line



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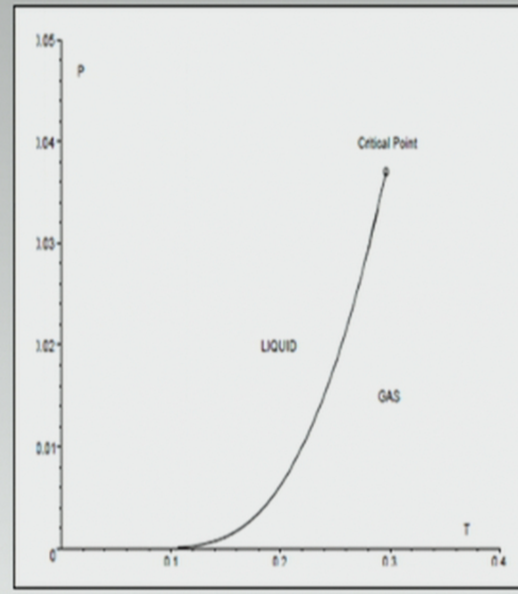

- **Clausius-Clapeyron** and **Ehrenfest** equations are satisfied

- **MFT critical exponents** $\alpha = 0\,, \quad \beta = \frac{1}{2}\,, \quad \gamma = 1\,, \quad \delta = 3$

  govern specific heat, volume, compressibility and pressure at the vicinity of critical point.

Van der Waals black hole

A. Rajagopal, DK, R.Mann, *Van der Waals black hole*, arXiv:1408.1105.

search for:

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

where

$$T = \frac{f'(r_+)}{4\pi} = \left(P + \frac{a}{v^2}\right)(v - b)$$

and

$$V = \left(\frac{\partial M}{\partial P}\right)_{S,\dots} \qquad v = k\frac{V}{N}, \quad N = \frac{A}{L_{pl}^2}$$

solution:

$$f = 2\pi a - \frac{2M}{r} + \frac{r^2}{l^2}\left(1 + \frac{3}{2}\frac{b}{r}\right) - \frac{3\pi a b^2}{r(2r + 3b)} - \frac{4\pi a b}{r}\log\left(\frac{r}{b} + \frac{3}{2}\right)$$

c) Reentrant phase transition

A system undergoes an RPT if a **monotonic** variation of any thermodynamic quantity results in two (or more) phase transitions such that the **final state is macroscopically similar** to the initial state.

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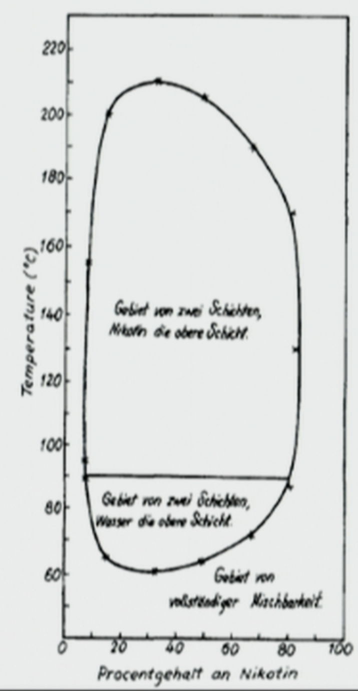
First observed by Hudson (1904) in a nicotine/water mixture

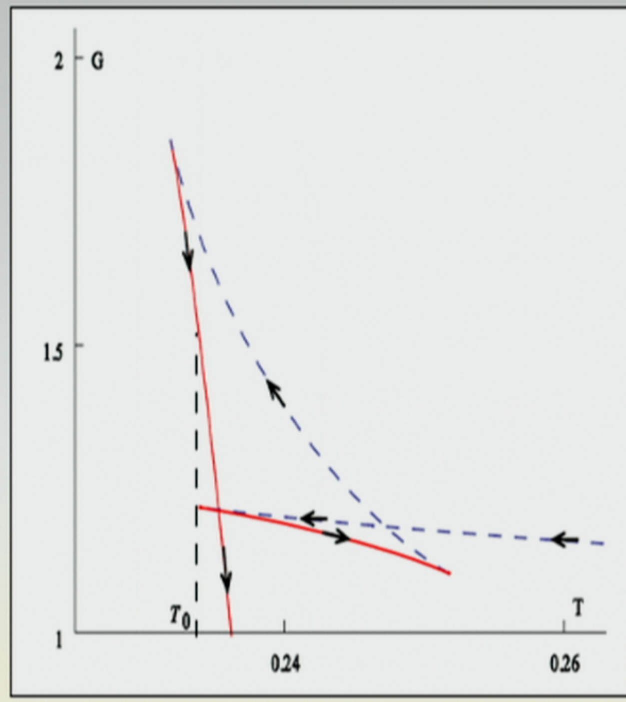
Z. Phys. Chem. 47 (1904) 113.

T. Narayanan, A. Kumar / Physics Reports 249 (1994) 135–218



AdS analogue: large/small/large black hole phase transition in singly spinning Kerr-AdS BH in 6 dimensions

N.Altamirano, DK, R.B. Mann, *Reentrant phase transitions in rotating AdS black holes*, arXiv:1306.5756 (2013).



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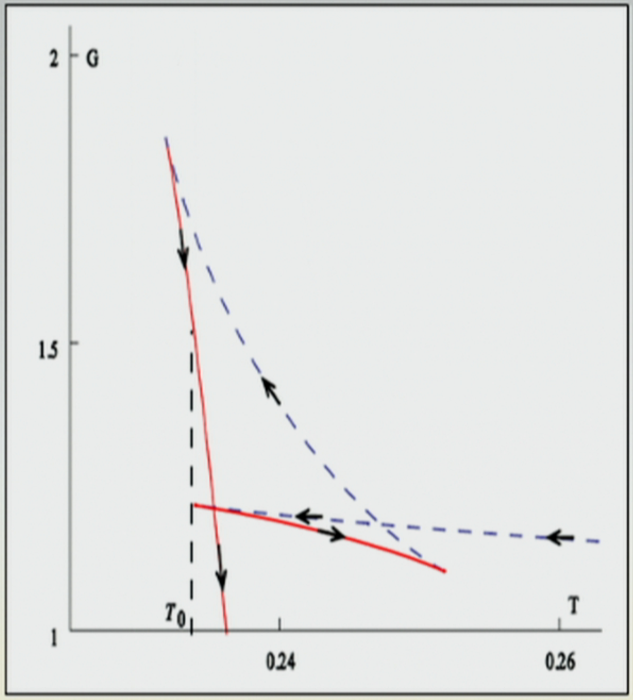
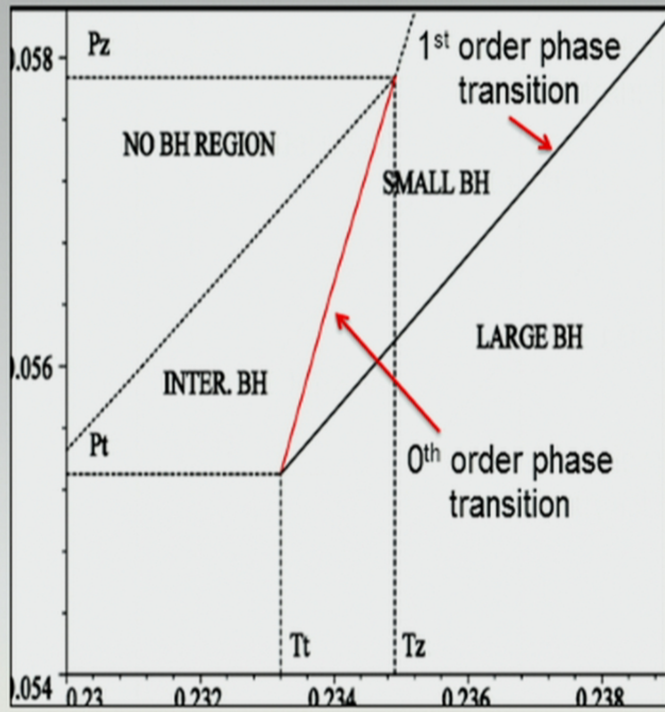


accompanied by a peculiar zeroth-order phase transition

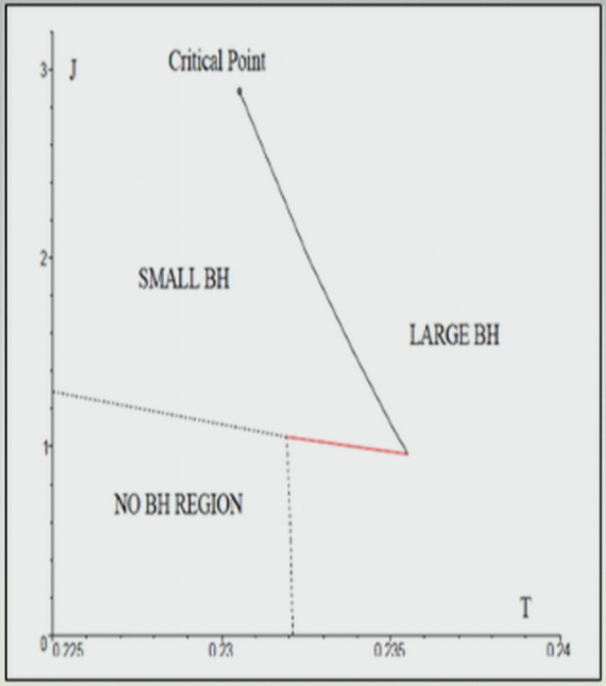
P-T phase diagram



Low T	Medium T	High T
mixed	water/nicotine	mixed
intermediate BH	small BH	large BH

J-T phase diagram

The discovered RPT does not require variable Λ!

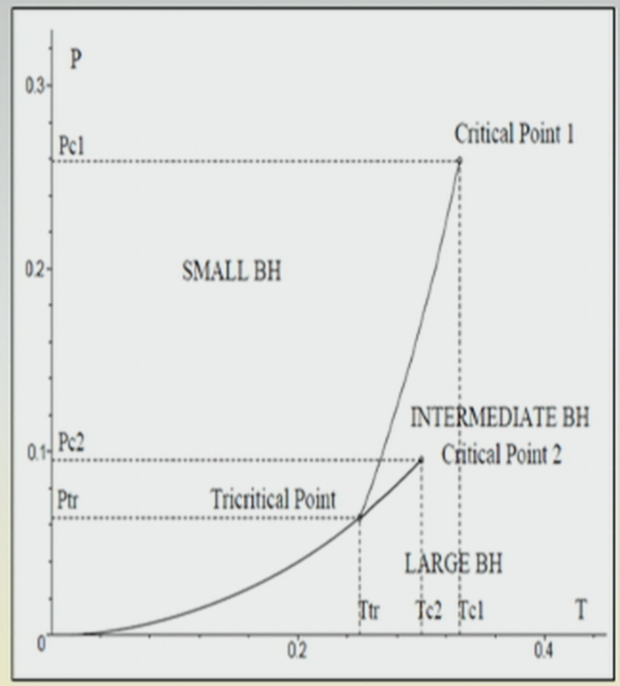


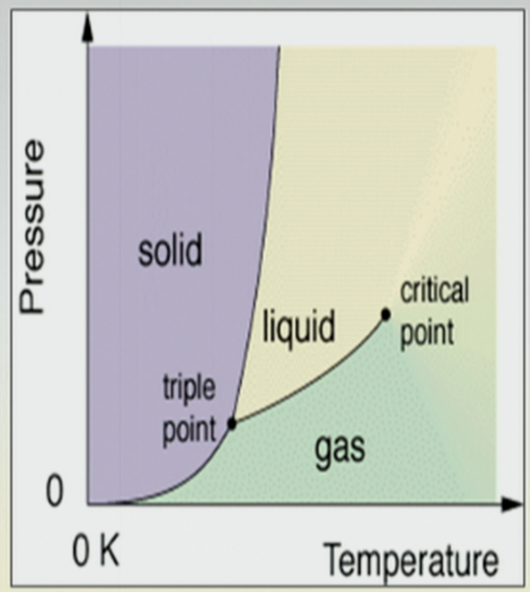
Occurs in any d>6: “two components”: BH vs. Black brane?

d) Triple point and solid/liquid/gas analogue

large/small/large black hole phase transition and a triple point in multiply spinning Kerr-AdS BH in 6 dimensions with certain ratio q of the two angular momenta.

N.Altamirano, DK, R.B. Mann, Z. Sherkatghanad, *Kerr-Ads analogue of tricritical point and solid/liquid/gas phase transition,* arXiv:1308.2672 (2013).



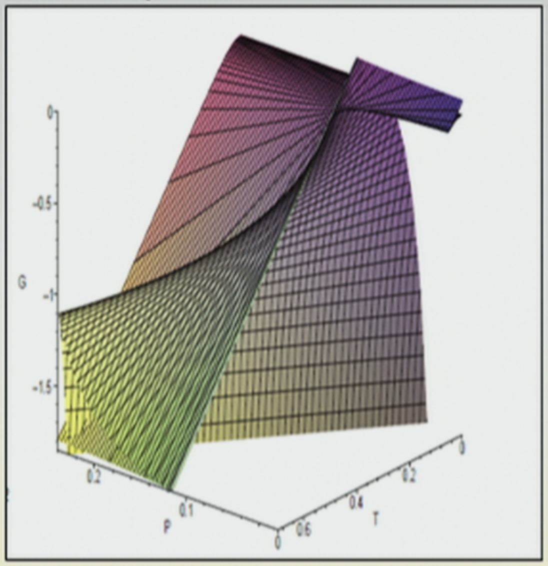


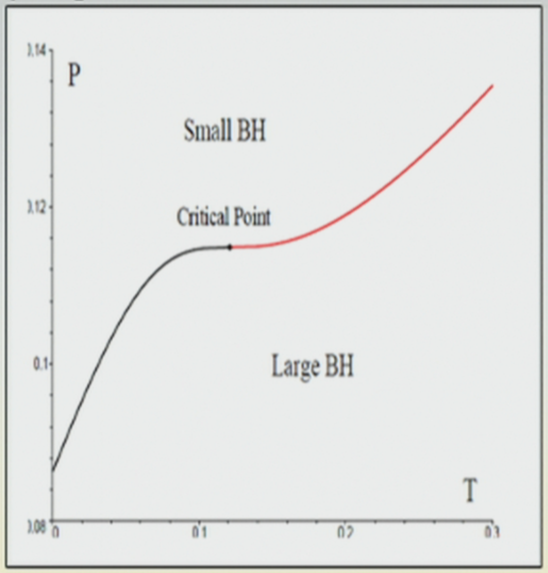
e) Isolated critical point from Lovelock gravity

- Lovelock higher curvature gravity

$$\mathcal{L} = \frac{1}{16\pi G_N} \sum_{k=0}^{K} \hat{\alpha}_{(k)} \mathcal{L}^{(k)}$$

- Special tuned Lovelock couplings, odd-order K





B. Dolan, A. Kostouki, DK, R. Mann, arXiv:1407.4783.

Critical exponents:

$$\alpha = 0, \quad \beta = 1, \quad \gamma = K - 1, \quad \delta = K .$$

Comments:

- cf. mean field theory critical exponents

$$\alpha = 0, \quad \beta = \frac{1}{2}, \quad \gamma = 1, \quad \delta = 3$$

- satisfies Widom relation and Rushbrooke inequality

$$\gamma = \beta(\delta - 1) \qquad \alpha + 2\beta + \gamma \geq 2$$

- Prigogine-Defay ratio

$$\Pi = 1/K$$

...indicates more than one order parameter?

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Conclusions

1) Thermodynamics is a **governing principle**, black holes are not an exception!
2) Recently people have been playing with the idea of identifying the cosmological constant with the dynamical **pressure**. This gives a way of defining the **volume of black holes**.
3) Gain some **useful properties:** Isoperimetric inequalities, consistent Smarr relation, compressibility (B. Dolan),....?
4) One can also search for analogues with "**every day TDs of simple substances**": solid/liquid, Van der Waals, reentrant phase transitions, triple points, solid/liquid/gas phase transitions, heat engines (C. Johnson), isolated critical point...
5) Can also be extended to **dS black hole spacetimes** (arXiv:1301.5926).
6) Is there an interpretation in **AdS/CFT correspondence**? (C. Johnson-arXiv:1404.5982, B. Dolan-arXiv:1406.7267)