Title: Quantum Extremal surfaces

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Abstract: <p>I will describe a new proposal for defining the holographic
 entanglement entropy at subleading orders in N (on the boundary) or
 hbar (in the bulk). This involves a new concept of "quantum extremal
 surfaces" defined as the surface which extremizes the sum of the area
 and the bulk entanglement entropy. This conjecture reduces to
 previous conjectures in suitable limits, and satisfies some nontrivial
 consistency checks. Based on arXiv:1408.3203</p>

Quantum Extremal Surfaces

Aron Wall

mostly based on arXiv:1408.3203 with Netta Engelhardt

We are grateful to Don Marolf, Ahmed Almheiri, Gary Horowitz, Raphael Bousso, Zachary Fisher, William Kelly, Dalit Engelhardt, Mark Van Raamsdonk, and Mudassir Moosa for conversations,

and the IAS, the Simons Foundation, NSF grants PHY12-05500 & DGE-1144085 for funding

ENTROPY = AREA

+ quantum corrections!

$$S_{BH} = \frac{A}{4G\hbar}$$

Bekenstein-Hawking entropy has an entropic interpretation for:

1. Black hole thermodynamics (S enters “First” & Second Laws)
(area increase theorem closely related to Penrose singularity theorem)

2. Holographic Entanglement Entropy (RT, HRT, LM...)

3. Covariant entropy bound? (Bousso, FMW, BCFM...)

4. General codimension 2 surfaces???
(Jacobson, Bianchi-Myers...)
motivated by area law for entanglement, cut off at Planck scale

In each case, the area is the leading order classical piece, but we can also consider quantum corrections. Can “quantize” these ideas by including entanglement entropy contributions.

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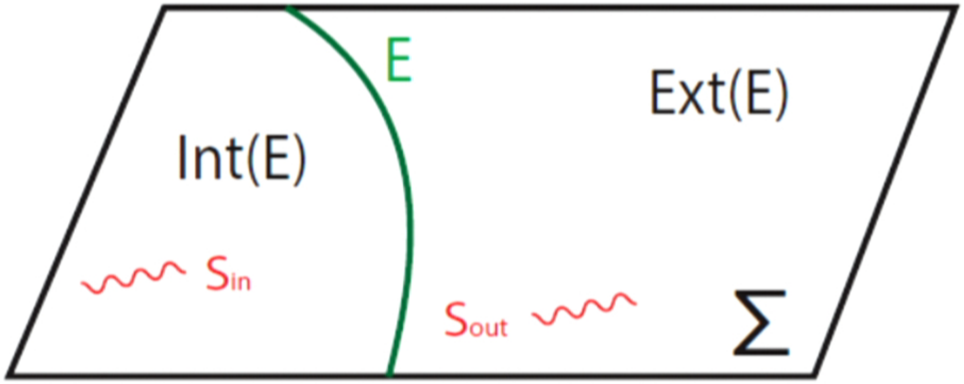

Given any Cauchy surface Σ, and a surface E which divides it into two regions Int(E) and Ext(E), can define entanglement entropy:

$$S_{\rm ent} = -{\rm tr}(\rho \ln \rho)$$

where ρ is the density matrix restricted to one side or the other. for a pure total state, doesn't matter which side ($\rho_{\rm out}$ or $\rho_{\rm in}$), since $S_{\rm in} = S_{\rm out}$.

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$S_{\rm ent}$ is UV divergent, but divergences are local.

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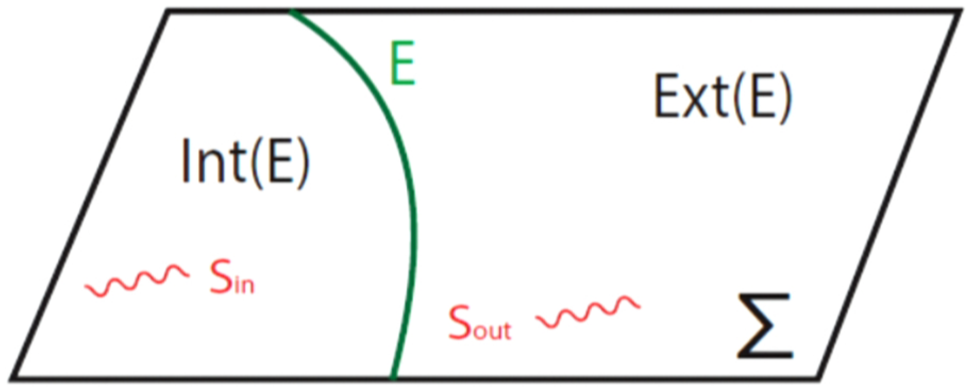

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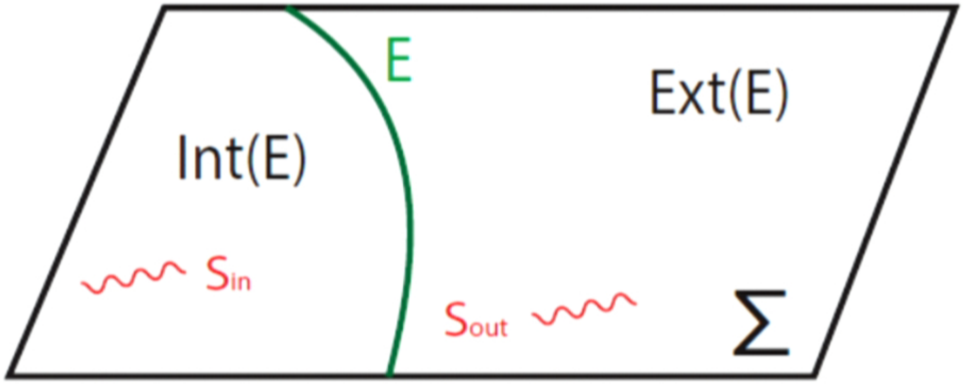

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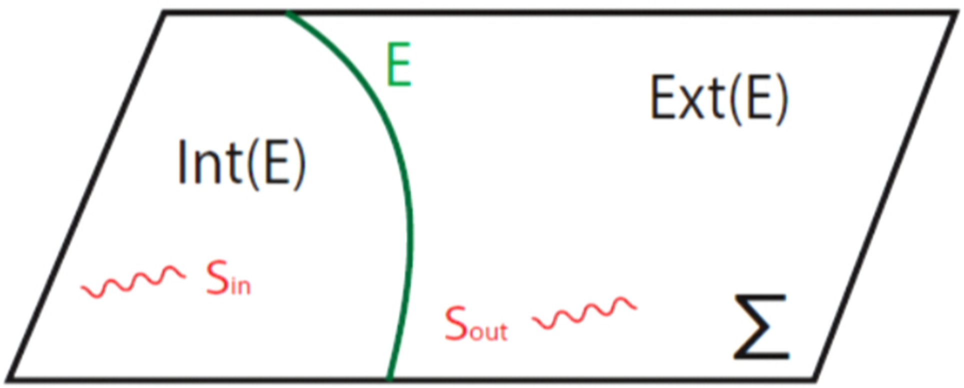
The Generalized Entropy

If the theory is GRAVITATIONAL, then we can also define a finite "generalized entropy" of E:

$$S_{\rm gen} = \frac{\langle A \rangle}{4G\hbar} + S_{\rm out} + \text{counterterms}$$

or we can use $S_{\rm in}$.

counterterms are local geometrical quantities used to absorb EE divergences, (e.g. leading order area law divergence corrects 1/G)



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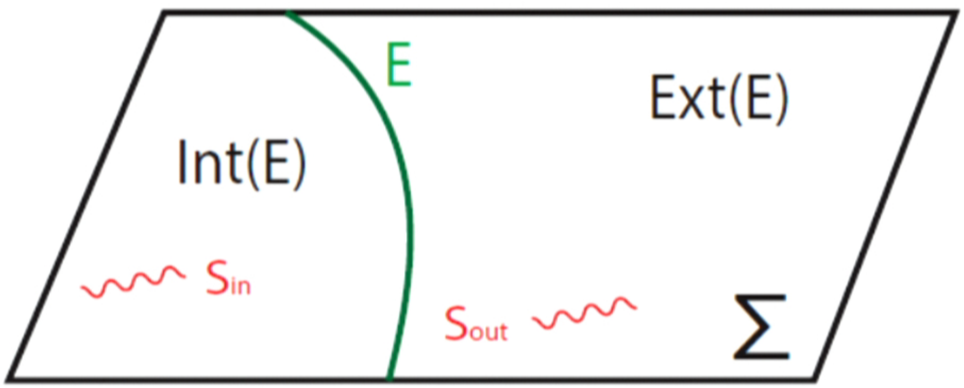

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HBAR EXPANSION

Can expand contributions to metric wrt $\hbar$:

$$g_{ab} = g_{ab}^{(0)} + g_{ab}^{(1/2)} + g_{ab}^{(1)} + g_{ab}^{(3/2)} + \dots$$

- $g_{ab}^{(0)}$: classical background
- $g_{ab}^{(1/2)}$: quantized linearized gravitons
- $g_{ab}^{(1)}$: gravitational fields sourced by matter & gravitons
- $g_{ab}^{(3/2)} + \dots$: higher order garbage

Associated corrections to S_{gen} one power of $\hbar$ lower.

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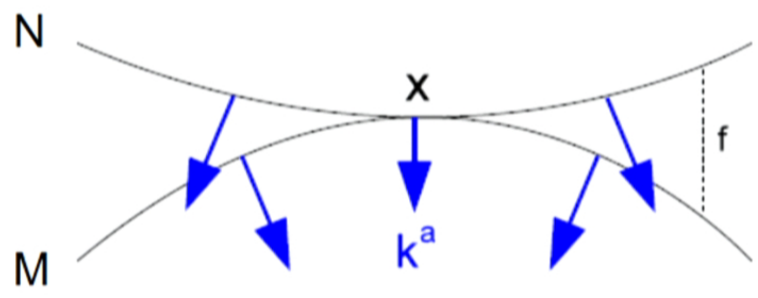
A Useful Montonicity Result



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and let M lie "outside" or on N (i.e. towards the direction of motion)

Then for variations of the surfaces near x along k^a:

$$\left.\frac{\delta S_{\rm gen}}{\delta N^a}k^a\right|_x \geq \left.\frac{\delta S_{\rm gen}}{\delta M^a}k^a\right|_x$$

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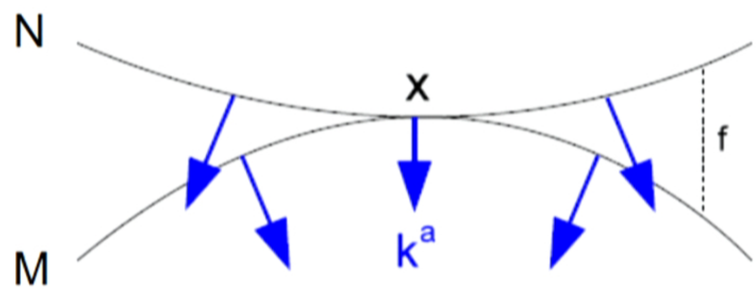

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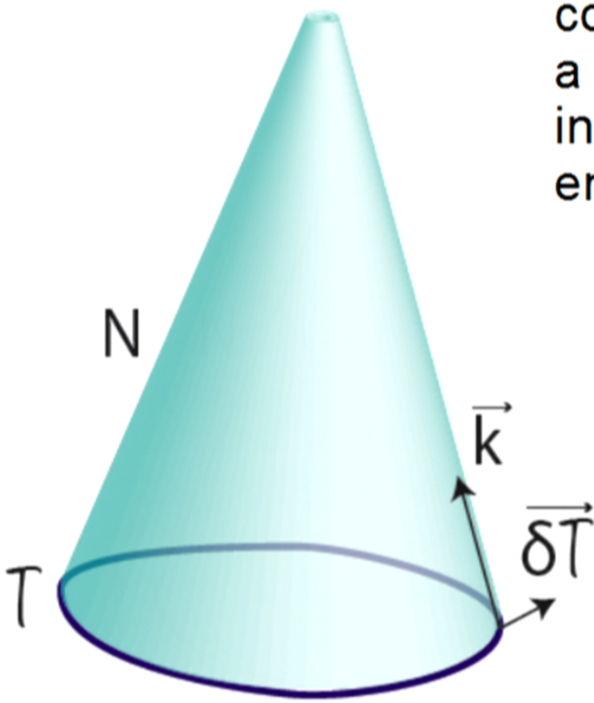
Example 2: QUANTUM SINGULARITY THEOREM



A quantum trapped surface $\mathcal{T}$ is a codimension 2 surface such that a null surface shot out has initially decreasing generalized entropy:

$$\frac{\delta S_{\mathrm{gen}}}{\delta \mathcal{T}^a} k^a < 0$$

Assuming that the GSL is true, these can be used to prove a semiclassical analogue of the Penrose singularity theorem, without using the null energy condition (Wall, arXiv:1010.5513)

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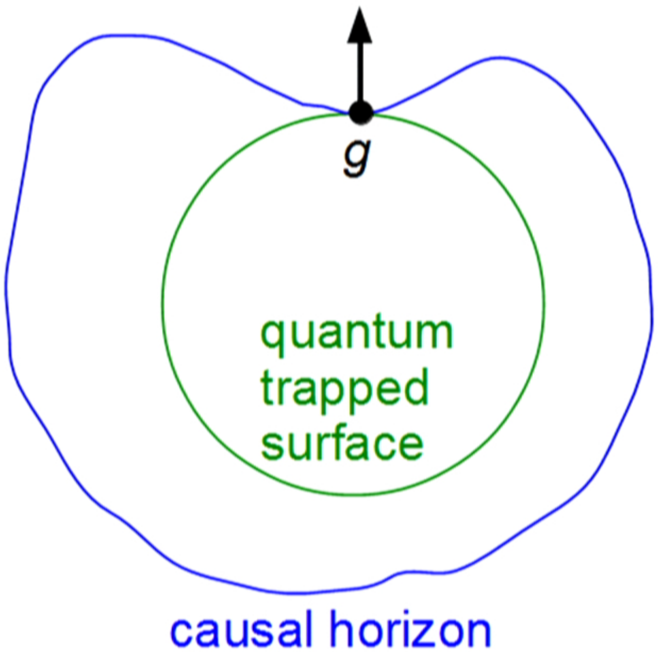
null geodesic incompleteness from GSL

diagram shows space at one time



Let the *g* represent the horizon generator which extends infinitely far to the future.

The boundary of the past of *g* is a causal horizon which touches the trapped surface at *g*, and is required by causality to be on or outside of it everywhere.

Monotonicity theorem from earlier shows that generalized entropy increases near *g* faster for the surface on the inside than the outside. *Hence GSL violated.*

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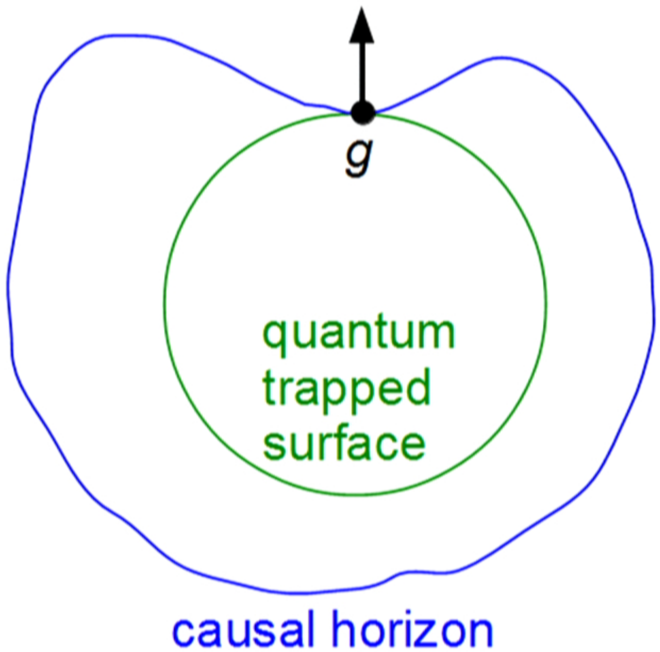

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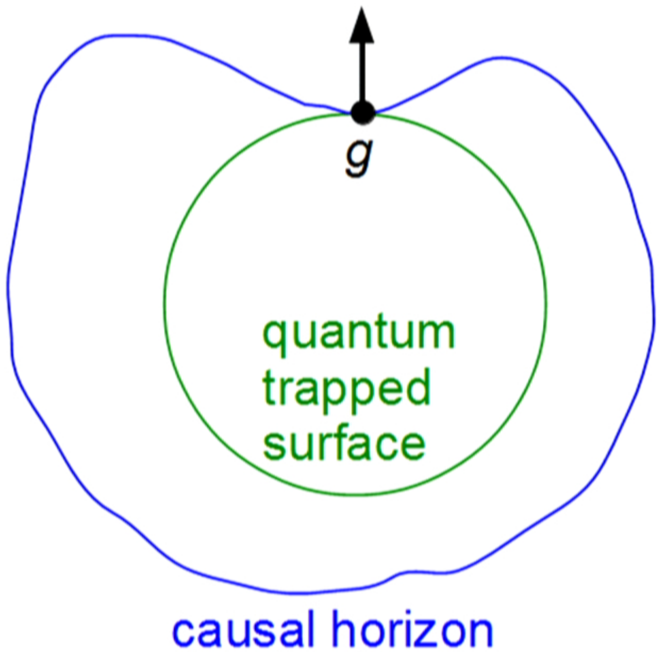

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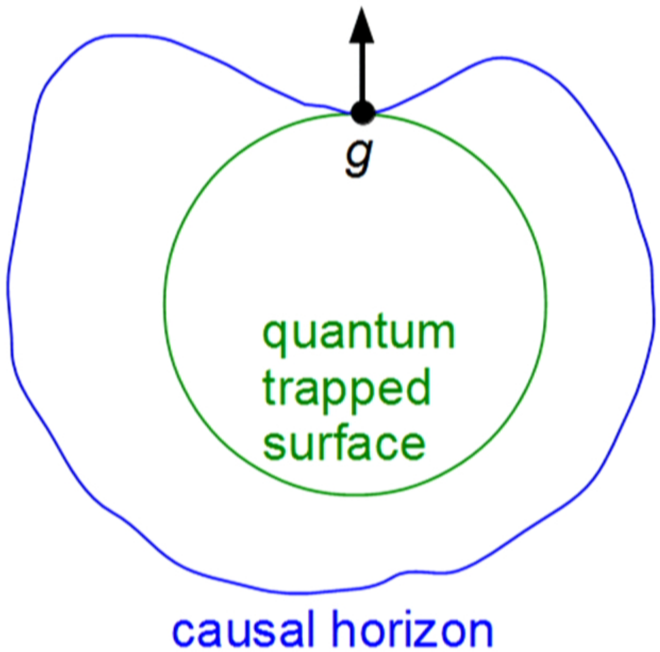

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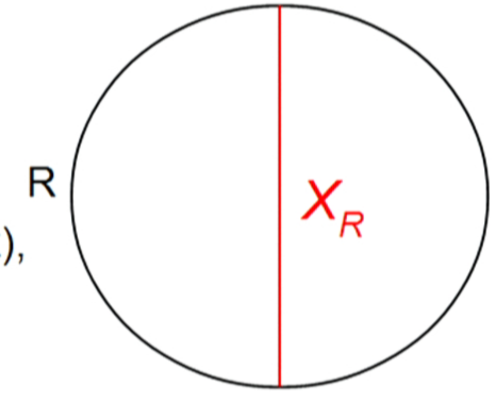
Want to calculate the entropy of a region in a boundary CFT using a codimension 2 surface in the bulk gravity theory.

HRT: Find surface X_R which extremizes area: $\frac{\delta A}{\delta X^a} = 0$

$$S_A = A(X_R)$$

at leading order in N (classical bulk)



FLM: To calculate leading order quantum corrections (N^0 on boundary, hbar^0 to S_bulk), use

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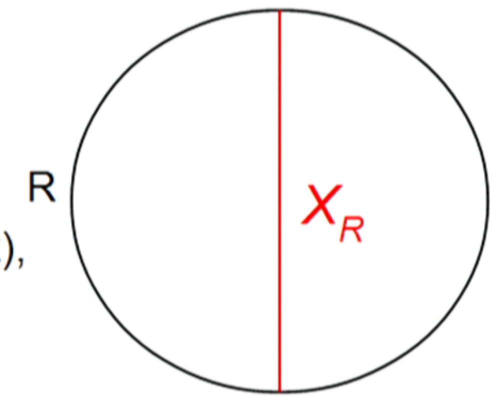

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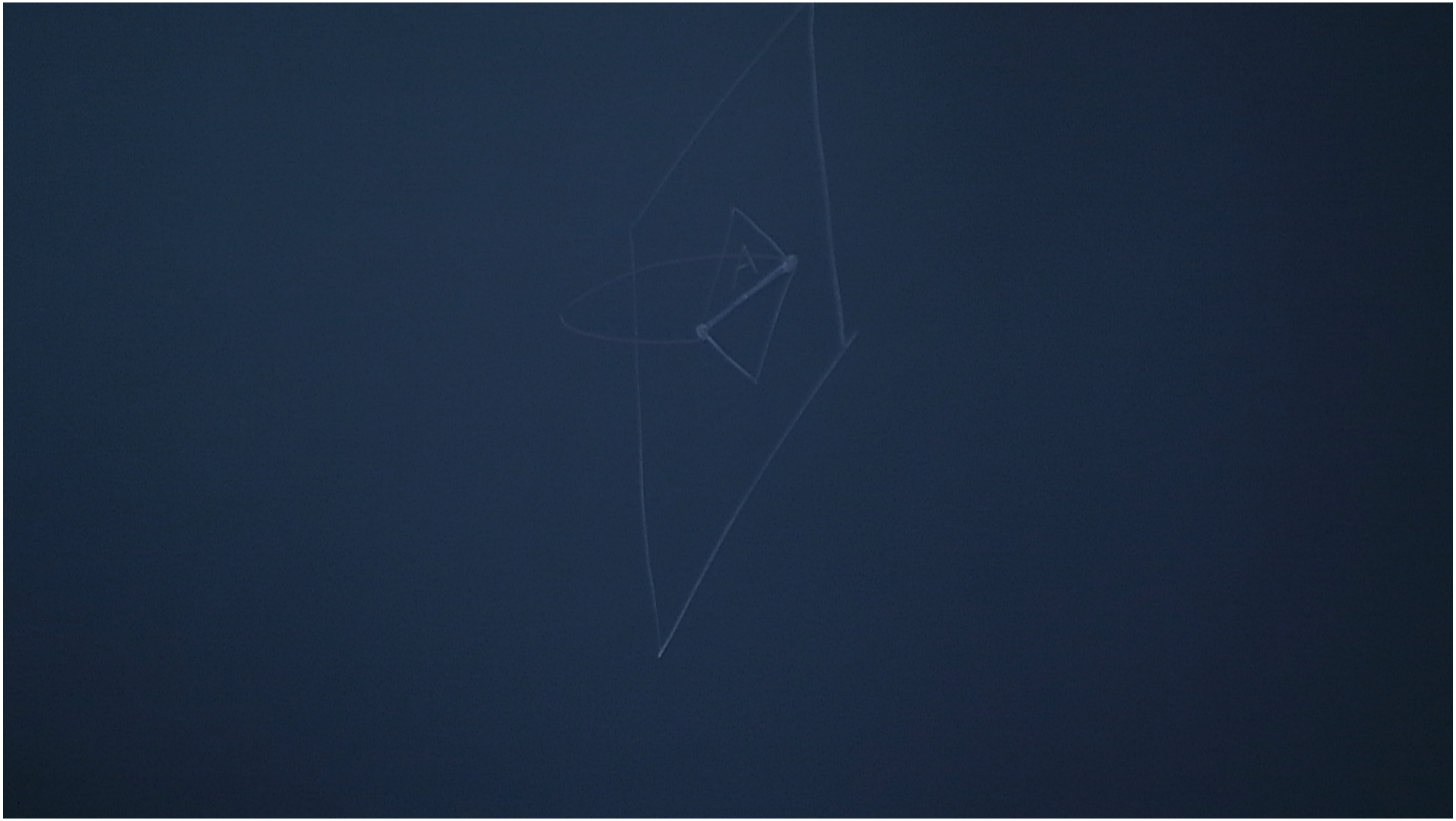

Comparison to FLM proposal

* FLM *derived* their formula from a path integral argument, for the classical (LM) and leading order quantum corrections (FLM).
(They only claim it works for static RT, but no obvious problem for ext surfaces!)

* We *assume* our formula works at all orders in hbar, and show that it has nice properties one might expect of the entangling surface.

* At leading order in the quantum corrections, the 2 proposals identify different surfaces, but they have the same entropy at leading order. At higher orders in hbar, the 2 proposals need not agree!
(though FLM never claimed their result should be used at higher orders…)

* Our proposal is much easier to prove theorems about. For example, quantum extremal surfaces always lie outside the causal wedge, but this is not true of "classical extremal surfaces".

* Our proposal consistent with idea that one should extremize the higher curvature corrections
(e.g. Hung-Myers-Smolkin 11, de Boer-Kulaxizi-Parnachev 11, Dong 13...).

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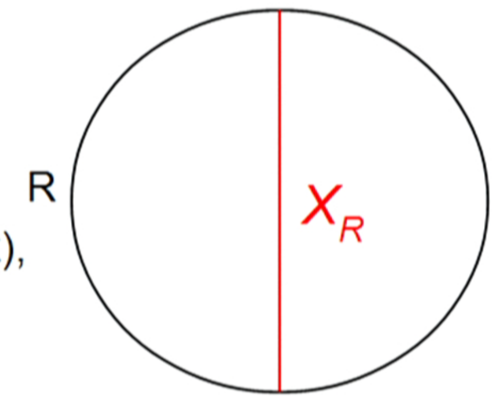

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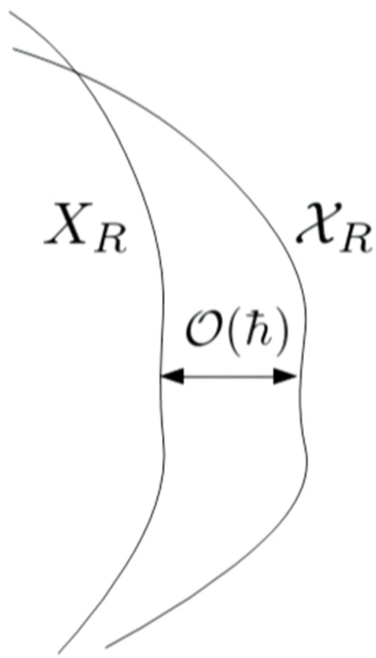
PROPOSALS AGREE TO LEADING QUANTUM ORDER



if you ignore $\sqrt{\hbar}$ graviton effects then surfaces are separated by $\mathcal{O}(\hbar)$

we are interested in the order unity part of $S_{\rm gen}$. Mostly these are suppressed by powers of $\hbar$ since surfaces are close, but

$$S_{BH} = \frac{A}{4G\hbar}$$

has an $\hbar$ in the denominator.

but that is OK, since first order variations of the area away from X_R vanish.

If you don't ignore gravitons, not clear they still agree at this order
what happens to FLM argument in this case?

?

Quantum Extremal Surface is deeper than Causal Surface

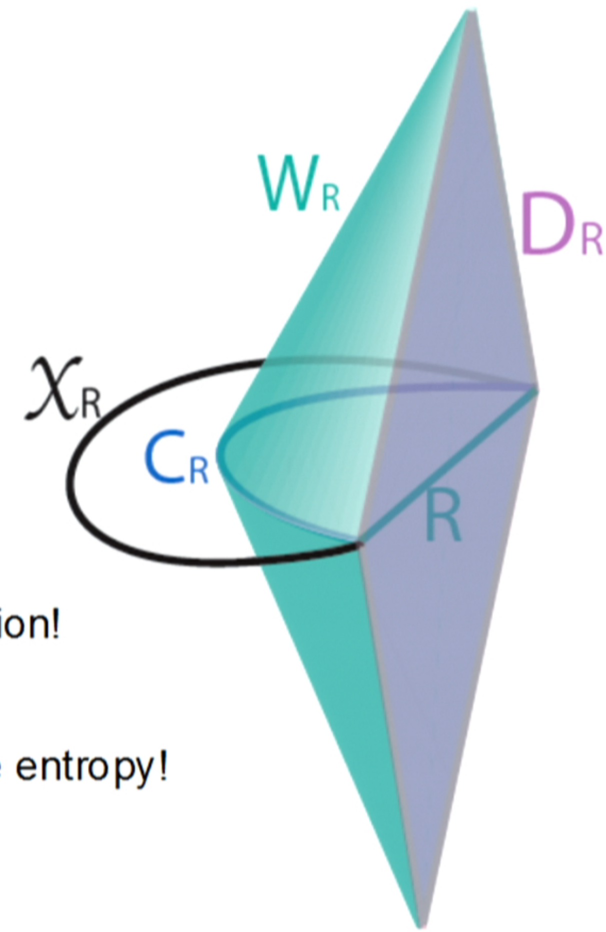
C_R is intersection of past & future horizons

assume the future horizon obeys the GSL
(and the past horizon the time-reversed GSL)

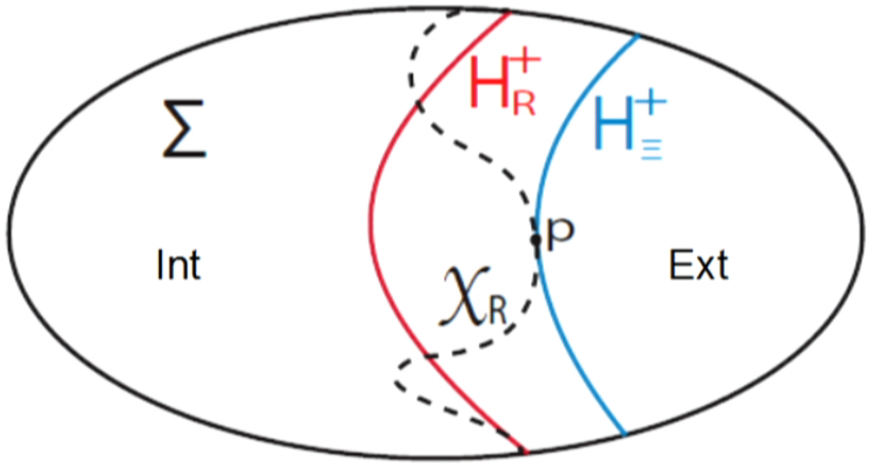
then quantum extremal surface $\mathcal{X}_R$
is spacelike further into the interior

not true for X_R on spacetimes with
quantum fields violating the null energy condition!

This means $S_{\text{gen}}(X_R)$ could in principle be
affected by unitary operators, so it can't be the entropy!



PROOF BY CONTRADICTION:



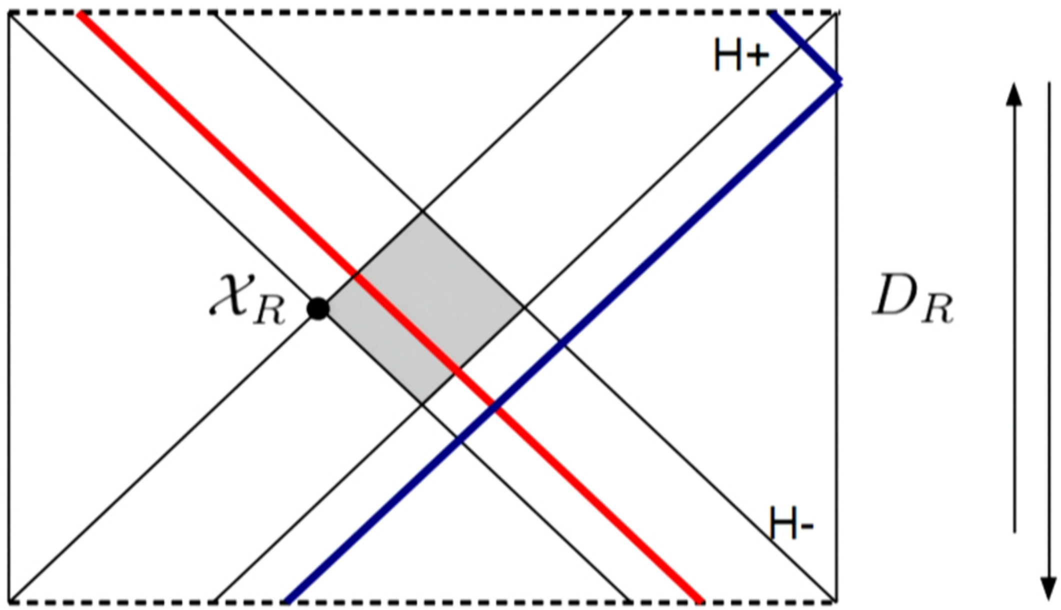
You can continuously deform the horizons H+ and H- until one of them touches $\mathcal{X}_R$ at a point p. Let it be the future horizon H+.

An H+ has increasing $S_{\rm gen}$ by the GSL.

and $\mathcal{X}_R$ has stationary $S_{\rm gen}$ by definition.

But monotonicity theorem says $\mathcal{X}_R$'s is increasing faster!

Shenker-Stanford Construction



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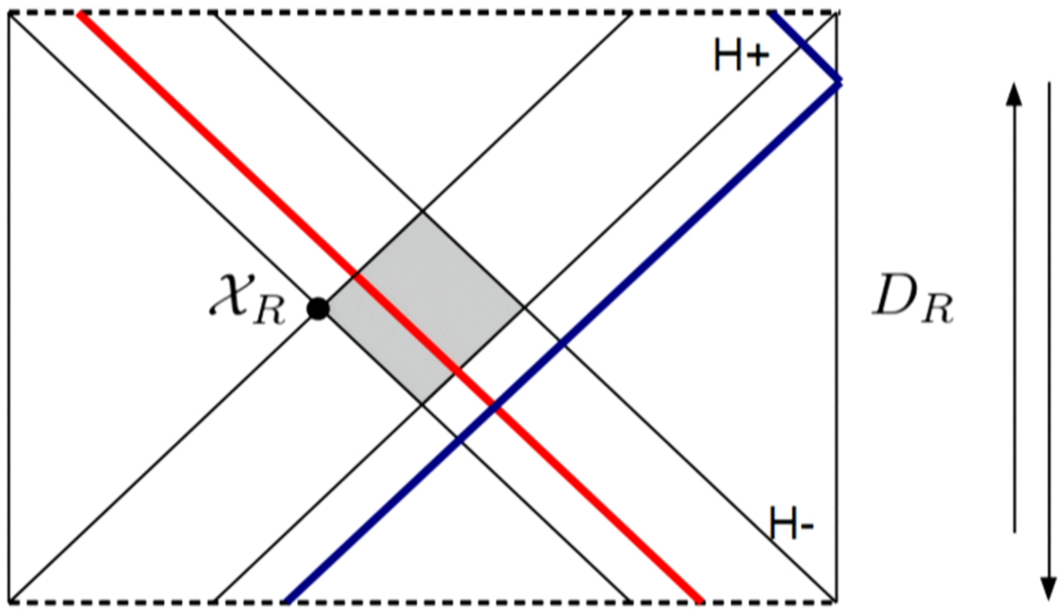

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Shenker-Stanford Conclusions

* The SS construction can be used to measure and/or influence the region behind the causal wedge.

* but C_R can never get past the closest $\mathcal{X}_R$, assuming the GSL (or null energy condition classically)

* so as long as each step involves *causal* bulk signaling, these things remain invariant:

# of quantum extremal surfaces
their geometry
any spacetime regions behind the extremal surfaces

A limit on bulk reconstruction, or can we do better with nonlocal effects?

Is it always possible to get the causal surface arbitrarily close to the nearest extremal surface?

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Barriers

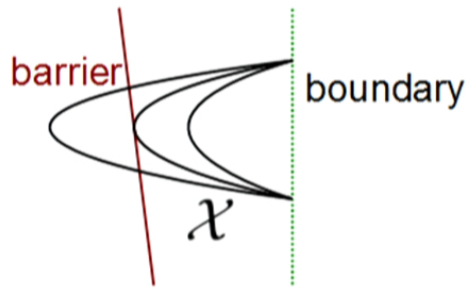
In arXiv:1312.3699 we (EW) had identified "barriers" to extremal surfaces
A barrier is a codimension 1 surface that can't be crossed by a continuous deformation of extremal surfaces anchored to one side.

Relevant to questions about which spacetime regions reconstructable from HRT, but proofs don't require asymptotically AdS.

Some of these results continue to hold for quantum extremal surfaces, but now it only makes sense to think about codimension 2 surfaces.

Main result: A null surface whose generalized entropy is nonincreasing for all slicings acts as a barrier to quantum extremal surfaces.

Proof from monotonicity theorem, as usual.



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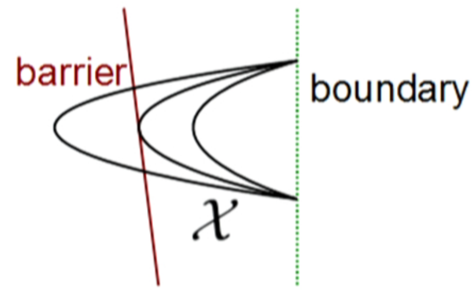

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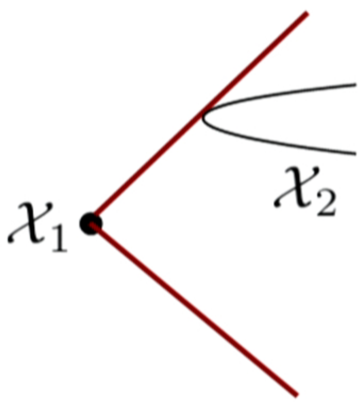
Quantum extremal surfaces are themselves barriers

Proof assumes that if $S_{\rm gen}$ starts to decrease, it continues to decrease

this is strongly suggested by quantum singularity result, but stronger statement than GSL. Can be proven semiclassically for free fields (Wall, forthcoming.)

Can be viewed as “quantum” version of Generalized Covariant Entropy Bound (Strominger-Thomson 04, Bousso-Fisher-Wall (forthcoming))



barrier constructed by shooting out null surfaces from $\mathcal{X}_1$ to past and future, towards boundary

whichever null surface $\mathcal{X}_2$ tries to cross first, there's a contradiction.

Summary

When the bulk experiences quantum corrections, the natural generalization of HRT is a surface which extremizes $S_{\rm gen}$.

Agrees with FLM entropy to leading quantum order, but not the same surface as what they proposed.

Many important classical results can be generalized to $\mathcal{X}$, using the GSL but not the null energy condition, e.g.

1. spacelike deeper than causal surface, so SS can't get past it
2. barrier theorems

Natural home for our conjecture is perturbative quantum gravity, but...

WORRYING ABOUT QUANTUM SPACETIMES

1. Since the quantum corrections are *operators*, how should we deal with quantum superpositions of different geometries?

choose particular coordinate gauge?
or require surface to be extremal as eigenvalue Eq'n

$$\frac{\delta A}{\delta X^a}|\Psi\rangle = 0, \text{ not just } \left\langle \frac{\delta A}{\delta X^a} \right\rangle = 0$$

?

requires $|\Psi\rangle$ to include d.o.f. associated with surface location:

$$\mathcal{H}_{\rm bulk} \otimes \mathcal{H}_{\rm surface}$$

since location of surface isn't really a physical field, need operator

$$\Omega : (\mathcal{H}_{\rm surface} \to \mathbb{C})$$

to reduce to “correct” state of matter fields alone.

also need to linearize $S_{\rm gen}$ to make it into an operator.

need for classical relations to be replaced with operator equations and for the surfaces named in proofs to be simultaneously localizable

WORRYING ABOUT QUANTUM SPACETIMES

1. Since the quantum corrections are *operators*, how should we deal with quantum superpositions of different geometries?

choose particular coordinate gauge?
or require surface to be extremal as eigenvalue Eq'n

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