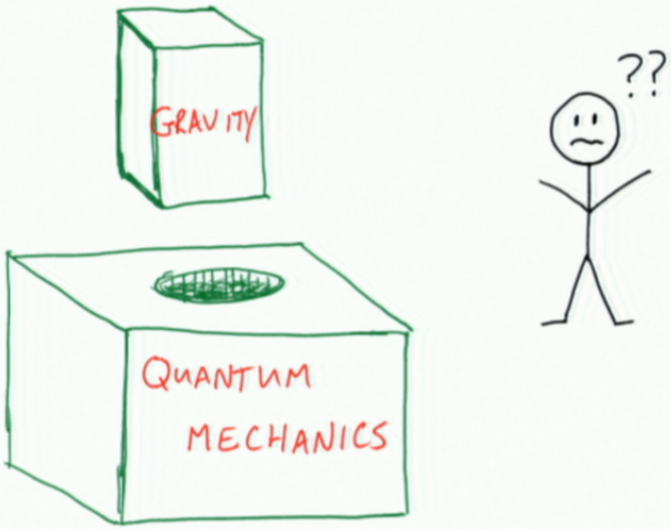
Title: Gravity and Entanglement

Date: Feb 18, 2015 02:00 PM

URL: http://pirsa.org/15020086

Abstract: The AdS/CFT correspondence from string theory provides a quantum theory of gravity in which spacetime and gravitational physics emerge from an ordinary non-gravitational system with many degrees of freedom. In this talk, I will explain how quantum entanglement between these degrees of freedom is crucial for the emergence of a classical spacetime, and describe progress in understanding how spacetime dynamics (gravitation) arises from the physics of quantum entanglement."

One of the greatest challenges for theoretical physics:



1997: Remarkable progress via AdS/CFT correspondence in string theory (Maldacena)

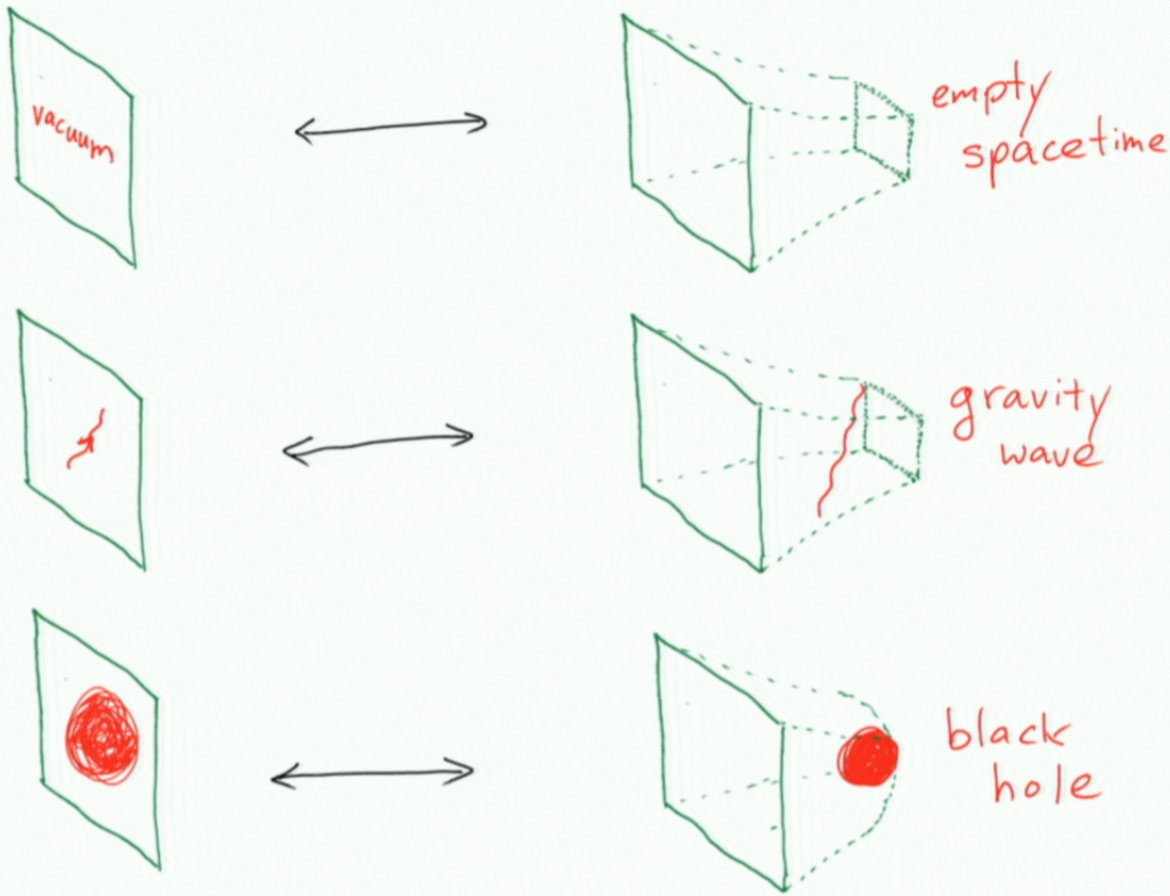
QUANTUM GRAVITY (certain examples)

=

exactly equivalent

ORDINARY QUANTUM SYSTEM (eg. QFT on fixed spacetime)

Different QFT states ⟷ Different spacetimes



BIG QUESTION:

How/why do spacetime/gravity emerge from QFT physics?

Recent work: physics of entanglement + quantum information is crucial!

Start with a single spin:

Possible states form a vector space:

basis: $|\uparrow\rangle, \; |\downarrow\rangle$

general state: $c_\uparrow |\uparrow\rangle + c_\downarrow |\downarrow\rangle$

QUANTUM SUPERPOSITION

Crucial point:

$$a|\uparrow\rangle|\uparrow\rangle + b|\uparrow\rangle|\downarrow\rangle + c|\downarrow\rangle|\uparrow\rangle + d|\downarrow\rangle|\downarrow\rangle$$

cannot generally be written as

$$\left(c^1_\uparrow|\uparrow\rangle + c^1_\downarrow|\downarrow\rangle\right)\left(c^2_\uparrow|\uparrow\rangle + c^2_\downarrow|\downarrow\rangle\right)$$

If not, spin 1 is **ENTANGLED** with spin 2.

e.g. NOT ENTANGLED:

$$|\uparrow\rangle|\downarrow\rangle$$

$$|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\downarrow\rangle = (|\uparrow\rangle + |\downarrow\rangle)|\downarrow\rangle$$

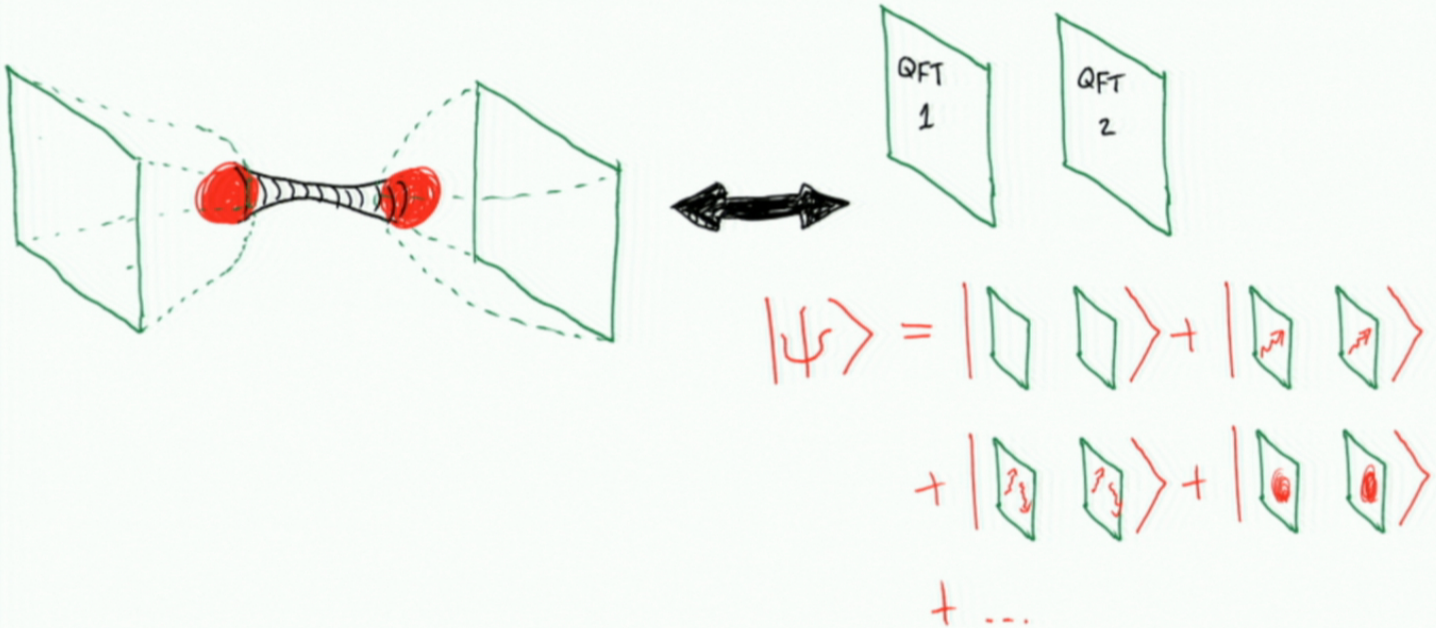
ENTANGLED:

$$|\uparrow\rangle|\uparrow\rangle + |\downarrow\rangle|\downarrow\rangle$$

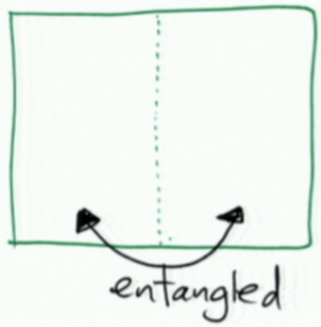
$$|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle$$

Maldacena 2001:

2 separate spacetimes connected by wormhole ⟷ 2 separate QFTs entangled with one another

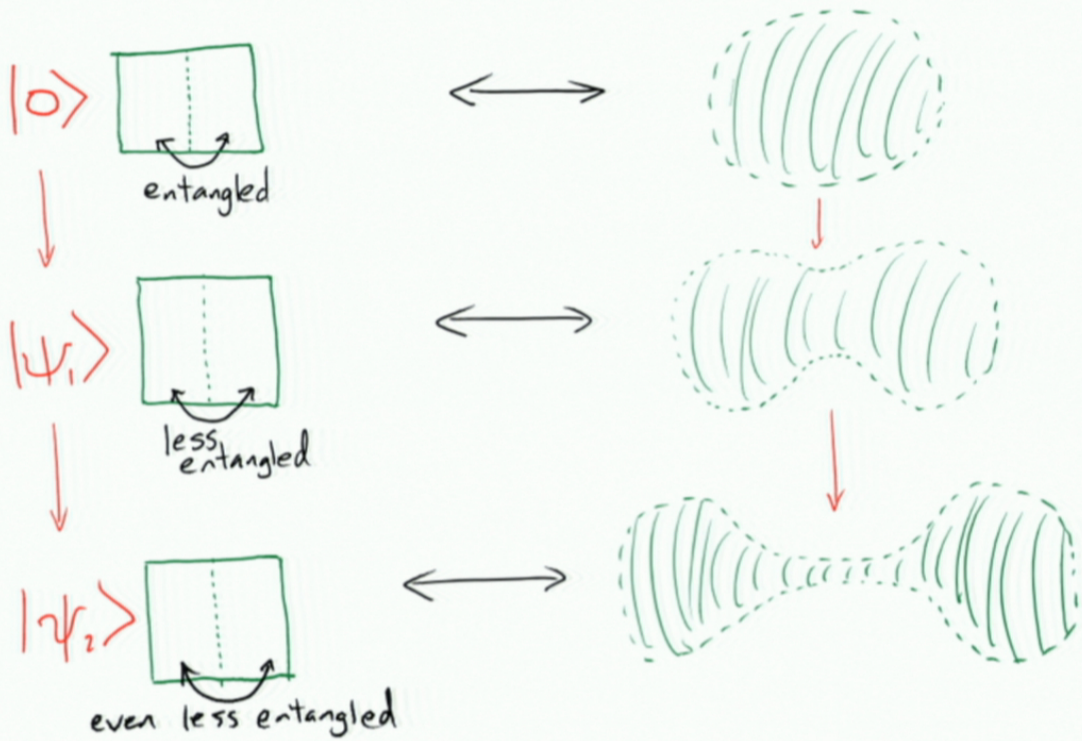


BUT: lots of entanglement already in QFT ground state dual to empty spacetime.



QFT fields in any region entangled with fields outside

What happens if we remove this entanglement?



2 regions of space pinch off from each other!

MIR; MVR, Czech, Nogueira, Karczmarek

Suggests that classical spacetime geometry emerges via entanglement of degrees of freedom in dual QFT!

MVR, Swingle 2009

No classical spacetime without quantum entanglement.

Can we be more precise?

Next step: better understand description of quantum subsystems & quantify entanglement

$$a|\uparrow\rangle|\uparrow\rangle + b|\uparrow\rangle|\downarrow\rangle + c|\downarrow\rangle|\uparrow\rangle + d|\downarrow\rangle|\downarrow\rangle$$

What is the state of the first spin?

Generally: NO state $c_\uparrow|\uparrow\rangle + c_\downarrow|\downarrow\rangle$ gives the same results for measurements on the first spin

General answer: **ENSEMBLE** of states

Measurements on single spin can always be reproduced by:

$$\boxed{\begin{array}{ll} \text{Probability } p_1 : & c_\uparrow^1|\uparrow\rangle + c_\downarrow^1|\downarrow\rangle \\ \text{Probability } p_2 : & c_\uparrow^2|\uparrow\rangle + c_\downarrow^2|\downarrow\rangle \end{array}}$$

orthogonal states $|\psi_i\rangle$

add to 1

= Ensemble

= Mixed state

= Density matrix $\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$

For entangled states:

p_1 and p_2 both nonzero: there is classical uncertainty about the state of spin 1

e.g. NOT ENTANGLED:

state of spin 1:

$$|\uparrow\rangle|\downarrow\rangle \longrightarrow |\uparrow\rangle \quad p = 1$$

$$|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\downarrow\rangle \longrightarrow |\uparrow\rangle + |\downarrow\rangle \quad p = 1$$

ENTANGLED:

$$|\uparrow\rangle|\uparrow\rangle + |\downarrow\rangle|\downarrow\rangle \longrightarrow \boxed{\begin{array}{ll} |\uparrow\rangle & p = \frac{1}{2} \\ |\downarrow\rangle & p = \frac{1}{2} \end{array}}$$

ENTANGLED:

$$\frac{1}{\sqrt{2}}|\uparrow\rangle|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle|\downarrow\rangle \longrightarrow \begin{array}{ll} |\uparrow\rangle & p = \frac{1}{2} \\ |\downarrow\rangle & p = \frac{1}{2} \end{array}$$

LESS ENTANGLED:

$$\sqrt{\frac{99}{100}}|\uparrow\rangle|\uparrow\rangle + \sqrt{\frac{1}{100}}|\downarrow\rangle|\downarrow\rangle \longrightarrow \begin{array}{ll} |\uparrow\rangle & p = \frac{99}{100} \\ |\downarrow\rangle & p = \frac{1}{100} \end{array}$$

= less classical uncertainty

Measurements on single spin can always be reproduced by:

$$\boxed{\begin{array}{ll} \text{Probability } p_1 : & c^1_\uparrow|\uparrow\rangle + c^1_\downarrow|\downarrow\rangle \\ \text{Probability } p_2 : & c^2_\uparrow|\uparrow\rangle + c^2_\downarrow|\downarrow\rangle \end{array}}$$

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For entangled states:

p_1 and p_2 both nonzero: there is classical uncertainty about the state of spin 1

Quantify entanglement via ENTROPY

Ensembles familiar from stat mech:

MICROCANONICAL ENSEMBLE:

$$|E_i\rangle \quad p_i = \frac{1}{N}$$

in $[E, E+\delta E]$

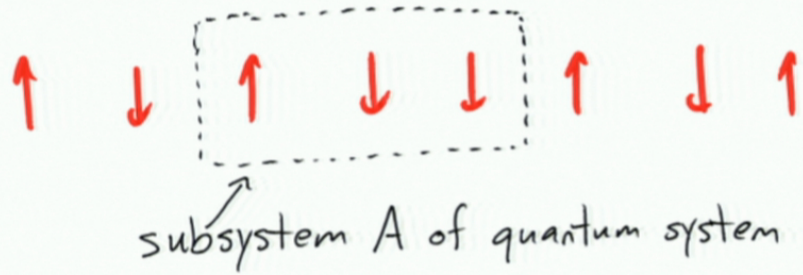
CANONICAL ENSEMBLE:

$$|E_i\rangle \quad p_i = \frac{1}{Z} \cdot e^{-E_i/kT}$$

Define entropy as: $S = -\sum p_i \log(p_i)$

Von Neumann

Entanglement Entropy

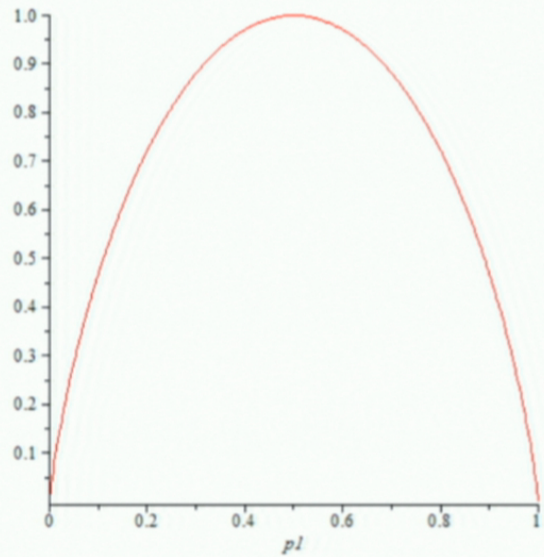


in state $\boxed{|\psi_i\rangle \text{ prob } p_i}$

Entanglement w. rest of system quantified by the entropy of this ensemble

$$S(A) = -\sum p_i \ln(p_i)$$

Example: 2 spins $\alpha|\uparrow\rangle|\uparrow\rangle + \beta|\downarrow\rangle|\downarrow\rangle$

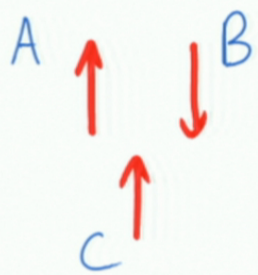


$p_1 = |\alpha|^2$

$p_2 = |\beta|^2 = 1 - p_1$

$S = -p_1 \log_2 p_1 - (1-p_1)\log_2(1-p_1)$

example: 3 spins



3 entanglement entropies

$$0 \leq S_A, S_B, S_C \leq 1$$

Interesting aside: these must satisfy triangle inequalities

← geometrical way to represent information about entanglement.

Example: 2 oscillators

Ground state:

Uncoupled:

$$|0\rangle|0\rangle$$

Coupled:

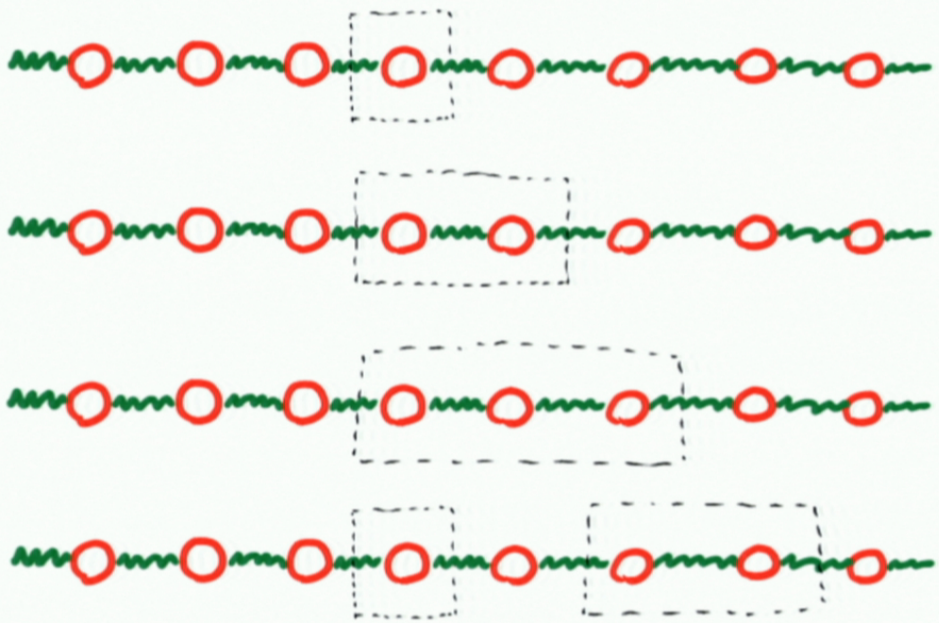
Ground state:

$$\sum c_{nm}|n\rangle|m\rangle$$

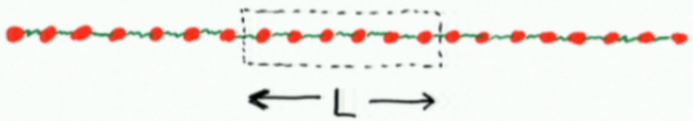
$|\psi_n\rangle, p_n$: eigenvectors & eigenvalues of matrix $c^t c$

Many oscillators:

Can look at $\{p_i\}$ or entanglement entropy for any subsystem:



In limit of $\varepsilon \to 0$, system defines a quantum field theory with exact scaling symmetry.



Ground state entanglement entropy for interval

$$S = \frac{1}{3}\log\left(\frac{L}{\varepsilon}\right)$$

divergent for $\varepsilon \to 0$

but $\frac{dS}{dL}$ or

$S_A(|\psi\rangle) - S_A(|0\rangle)$ sensible

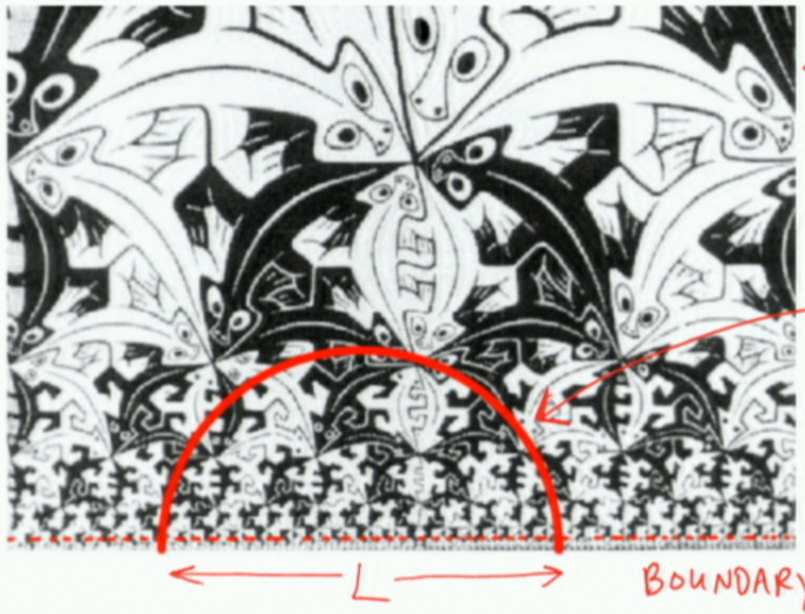
Can prove:

"central charge" = measure of number of d.o.f.

$$S = \frac{c}{3} \log\left(\frac{L}{\varepsilon}\right)$$

for ANY 1D conformal field theory.

Ryu + Takayanagi: this result can be represented geometrically.



anti-de Sitter space

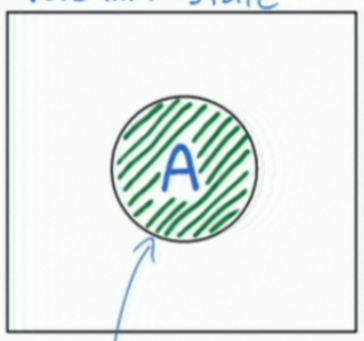
shortest length curve that has same boundary as interval L

$$\text{length} = \frac{c}{3} \ln\left(\frac{L}{\varepsilon}\right)$$

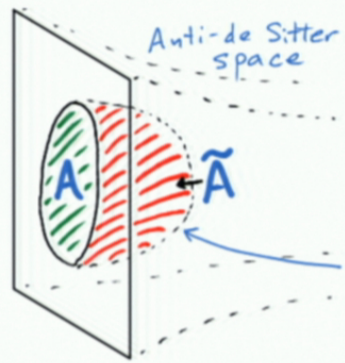
Higher dimensions:

for any CFT:

vacuum state



ball shaped region



Anti-de Sitter space

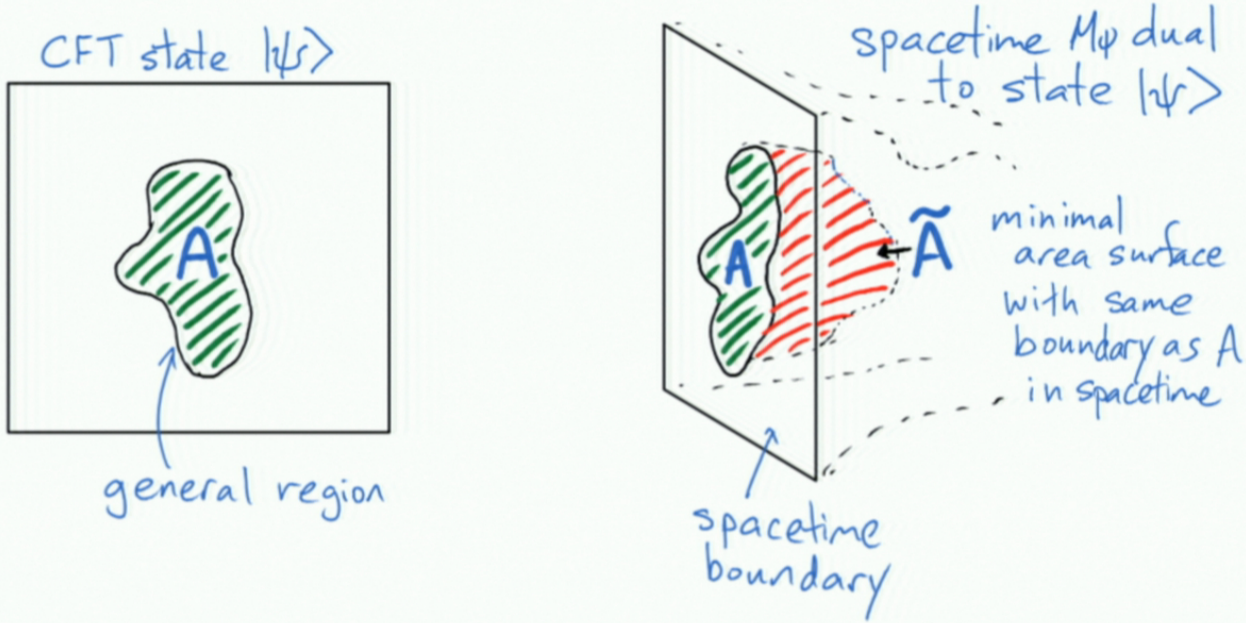
minimal area surface with same boundary as A

$$S(A) \propto \text{Area}(\tilde{A})$$

c.f. $S = \frac{\text{Area}}{4G_N}$ for black holes!

Ryu-Takayanagi: conjectured this to hold for general regions A + general states $|\psi\rangle$ in "holographic" CFTs

Geometry From Entanglement



$|\psi\rangle \rightarrow$ calculate S_A for many A $\rightarrow$ find M_ψ s.t. $\mathrm{Area}(\tilde{A}) = S_A$

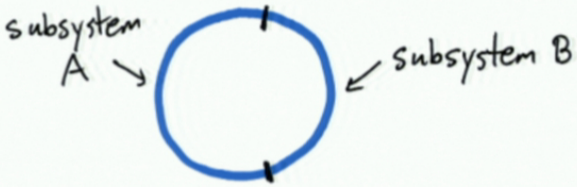
can (plausibly) reconstruct geometry from entanglement!

Summary so far:

Spacetime geometry encoded in entanglement structure of quantum state.

- RT conjecture passes many consistency checks
- Proof by Maldacena & Lewkowycz

ENTANGLEMENT STRUCTURE ⟷ SPACETIME STRUCTURE

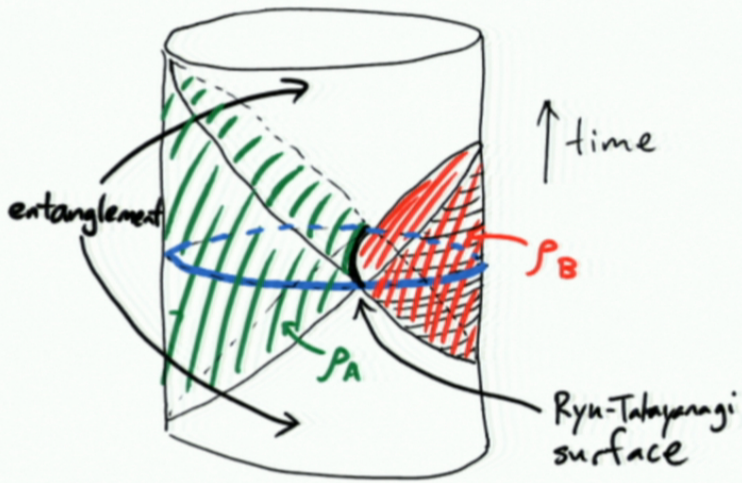


$$|\Psi_{AB}\rangle$$

ρ_A state of A

ρ_B state of B

+ entanglement info



MVR, Czech, Karczmarek, Nogueira

Same density matrix from different states

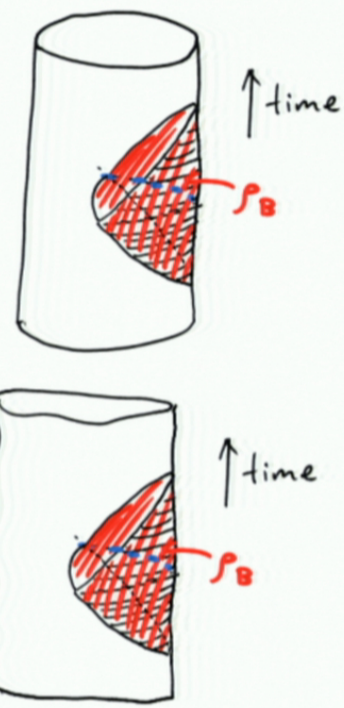
$\updownarrow$

identical spacetime region patched into different geometries.

$|\Psi_{AB}\rangle \searrow$

ρ_B state of B

$|\Psi_{CB}\rangle \nearrow$



Recently : gravitational dynamics from entanglement

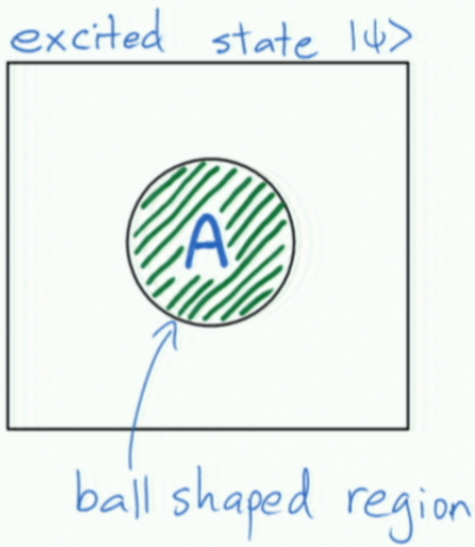
N. Lashkari, M. McDermott, MVR
T. Faulkner, M. Guica, T. Hartman, R. Myers, MVR
B. Swingle, MVR.

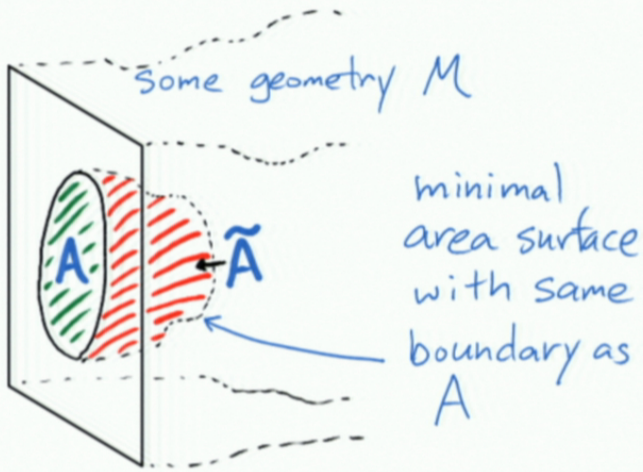
If spacetime geometry is a manifestation of QFT entanglement, can we understand spacetime dynamics from entanglement physics?

Geometries must satisfy Einstein's Equations

Will assume:

Ryu-Takayanagi holds for ball-shaped regions, for family of states $|\psi\rangle$.





Which geometries are possible?

Entanglement entropies obey constraints
(e.g. triangle inequality for spins)

subadditivity, strong subadditivity, positivity & monotonicity of relative entropy, ...

$\Downarrow$ Entropy-area connection

Constraints on geometries

A Quantum First Law of Thermodynamics:

Starting in thermal state ($p_i = \frac{1}{Z} e^{-\beta E_i}$) with temp. T:

For ANY variation of the state

$$\delta S_A = \frac{1}{T} \delta \langle E \rangle$$

entanglement entropy

energy (Hamiltonian)

Similar "First Law" for perturbations to ground state of a CFT

Blanco, Casini, Hung, Myers

$$|0\rangle \to |0\rangle + \delta|\phi\rangle$$

For any ball B:

$$\delta S_B = \delta \langle E_B \rangle$$

$$E_B = \int_B dx \; f(x) \cdot \langle \rho(x) \rangle$$

specific function on B

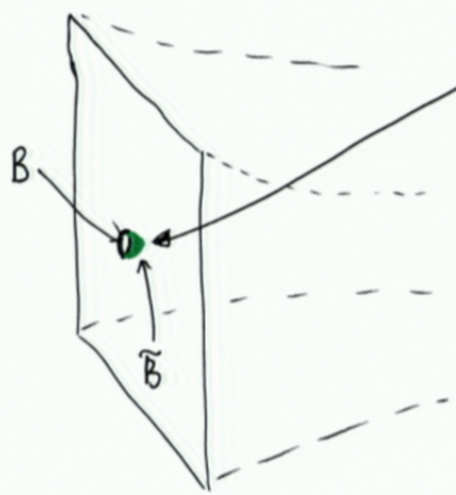
Energy density

$$\int_B dx\, f(x) \cdot \delta \langle \rho(x) \rangle = \delta S_B$$

small B

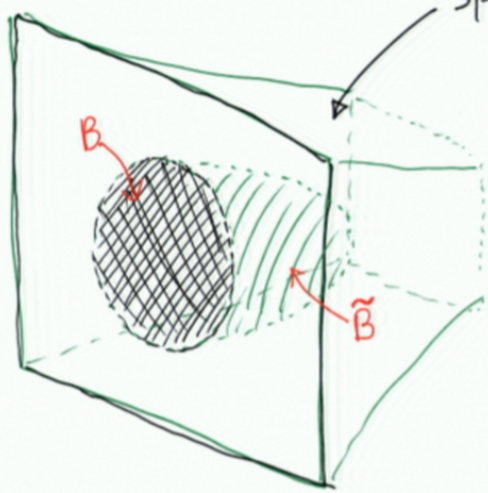
$$\langle \rho(x) \rangle \propto \lim_{R \to 0} \delta S_B$$

area of this



CFT energy density tells us geometry near boundary.

$\delta S_B = \delta E_B$ for general balls



spacetime dual to $|0\rangle + \delta|\psi\rangle$

$\|$

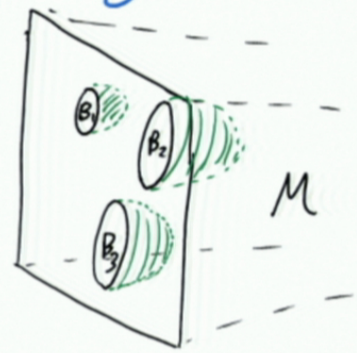
some perturbation to empty AdS space

$\delta S_B = \delta E_B$ $\longrightarrow$ relates perturbed geometry at $\tilde{B}$ to perturbed geometry at B (use Ryu + Takayanagi)

CONSTRAINT ON DUAL SPACETIME

Set of constraints for all ball-shaped regions B

Some math



GEOMETRY M MUST SATISFY EINSTEIN'S EQUATIONS!!

(linearized about Anti-de-Sitter)

More recently:

w Swingle: Refined Ryu-Takayanagi formula: $S_{CFT} = \frac{\text{Area}}{4G_N} + S_\Sigma$

$\Downarrow \delta S = \delta E$

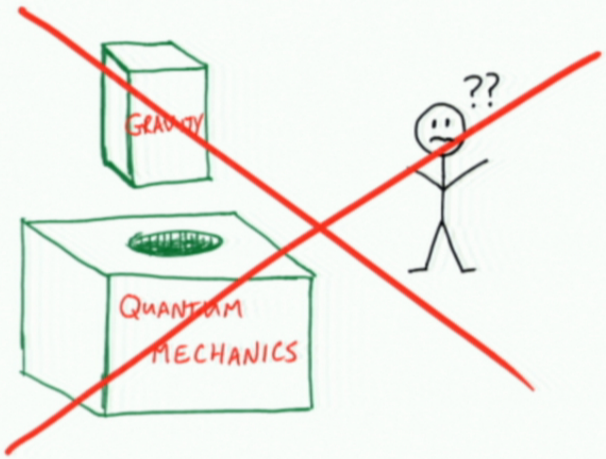
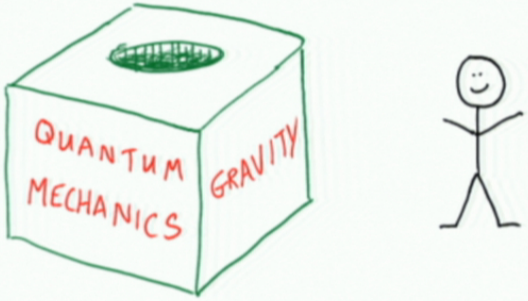
$\langle T_{\mu\nu} \rangle$ is source for linearized Einst. Eqns.

w. Lashkari
Rabideau
Sabella-Garnier

Constraints at non-linear level
(strong subadditivity + positivity/monotonicity of relative entropy)
give rise to energy conditions!

also: Rangamani, Takayanagi
Hubeny, Bhattacharya
Ogawa, Lin, Marolf, Stoica

SUMMARY

structure of entanglement in QFT ⟷ spacetime geometry

Constraints on QFT entanglement ⟷ Einstein Equations (so far: linearized)