Title: UV Properties of Galileons

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Abstract: <p>It is well known that S-matrix Analyticity, Lorentz invariance and Unitarity place strong constraints on whether Effective Field Theories can be UV completed. A large class of gravitational field theories such as Massive Gravity and DGP inspired braneworld models contain as limits Galileon theories which in the past have been argued to violate the conditions necessary for a UV completion. I will present arguments that imply quite the opposite, although Galileons do not admit a standard Wilsonian UV completion, I will use the recently discovered Galileon duality to argue that these theories can exhibit a non-Wilsonian completion consistent with the `Classicalizationâ€™ proposal. These theories have quintessentially gravitational properties such as non-polynomially bounded scattering amplitudes, absence of local off-shell observables and an exponentially soft 2-2 scattering amplitude. These properties show up in a violation of the standard Wightman axioms and are consistent with many of the known properties of UV completions of gravity. I will argue that the superluminal solutions found in the low energy EFT are absent in the UV completion.</p>

GALILEONS AS EFT

- Galileon Lagrangian arises as Decoupling/Scaling Limits of whole host of Infrared Modified theories of gravity, such as DGP/ Massive Gravity
- We may define Galileon in Minkowski via EFT of a single scalar field with the following nonlinearly realized symmetry

$$\pi \to \pi + v_\mu x^\mu + c$$

GALILEONS AS EFT

$$\pi \to \pi + v_\mu x^\mu + c$$

Follow the EFT recipe

$$\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi / \Lambda^3$$

$$S_{\text{Galileon}} = S_{\text{A}} + S_{\text{B}}$$

Finite number of `Wess-Zumino' terms

$$S_{\text{A}} = \int \mathrm{d}^d x \Lambda^\sigma \pi \sum_{n=0}^{d} \alpha_n \epsilon\, \epsilon\, \Pi^n \eta^{d-n}$$

Nicolis et al 2006

Infinite number of irrelevant operators

$$S_{\text{B}} = \int \mathrm{d}^d x \Lambda^{\sigma+1} \sum_{n,m} \beta_{n,m} \frac{\partial^{2n}}{\Lambda^{2n}} \Pi^m$$

GALILEONS AS EFT

$$\mathcal{L}_2(\pi) = \epsilon^{abcd}\epsilon^{ABCD}(\partial_a\partial_A\pi)\pi\eta_{bB}\eta_{cC}\eta_{dD}$$

$$\mathcal{L}_3(\pi) = \frac{1}{\Lambda^3}\epsilon^{abcd}\epsilon^{ABCD}(\partial_a\partial_A\pi)(\partial_b\partial_B\pi)\pi\eta_{cC}\eta_{dD}$$

$$\mathcal{L}_4(\pi) = \frac{1}{\Lambda^6}\epsilon^{abcd}\epsilon^{ABCD}(\partial_a\partial_A\pi)(\partial_b\partial_B\pi)(\partial_c\partial_C\pi)\pi\eta_{dD}$$

$$\mathcal{L}_5(\pi) = \frac{1}{\Lambda^9}\epsilon^{abcd}\epsilon^{ABCD}(\partial_a\partial_A\pi)(\partial_b\partial_B\pi)(\partial_c\partial_C\pi)(\partial_d\partial_D\pi)\pi$$

GALILEONS AS EFT

No problem computing trees and loops perturbatively

Finite number of `Wess-Zumino' terms

$$S_{\rm A} = \int {\rm d}^d x \Lambda^\sigma \pi \sum_{n=0}^{d} \alpha_n \epsilon\, \epsilon\, \Pi^n \eta^{d-n}$$

Exhibit Non-renormalization theorem

Infinite number of terms

$$S_{\rm B} = \int {\rm d}^d x \Lambda^{\sigma+1} \sum_{n,m} \beta_{n,m} \frac{\partial^{2n}}{\Lambda^{2n}} \Pi^m$$

are renormalized

THE PROBLEM

Thus at level of EFT, for momenta $k < \Lambda$

everything looks great!

perturbation theory breaks down at $k = \Lambda$

The problem is
there is no LOCAL, LORENTZ INVARIANT, UNITARY
UV COMPLETION FOR WHICH THESE ARE AN EFT OF

WHY?

In a local field theory, operators commute outside the lightcone

$$[\pi(x), \pi(y)] = 0 \qquad (x-y)^2 > 0$$

From this, and the assumption of stability (all states have positive energy and mass) we derive the Jost-Lehmann-Dyson representation

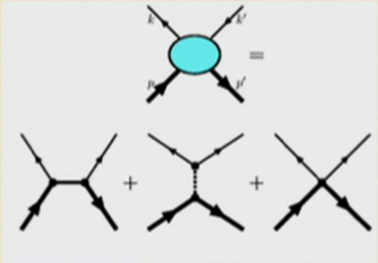
$$\langle P_f | [\pi(x/2), \pi(-x/2)] | P_i \rangle = \int_0^\infty d\mu D(\mu, P_i, P_f, x) \Delta_\mu(x)$$

$$\Delta_\mu(x) = 0, \quad x^2 > 0$$

DISPERSION RELATIONS

$$\langle P_f | [\pi(x/2), \pi(-x/2)] | P_i \rangle = \int_0^\infty d\mu D(\mu, P_i, P_f, x) \Delta_\mu(x)$$

In a generic field theory, D grows as a polynomial in μ



From this one can prove that the
forward scattering amplitude A(s,0)
is analytic in the complex s plane
(modulo branch cuts and poles on real axis)
with a **finite** number of subtractions

Hepp 1964

POLYNOMIAL BOUNDEDNESS

In a local field theory:
Wightman functions are tempered distributions

$$W(\{k_i\}) = \prod_i \int d^d k_i \langle 0|\pi(x_1)\pi(x_2)\ldots\pi(x_N)e^{-\sum_i ik_i.x_i}|\rangle$$

Momentum space growth is bounded by a polynomial in k

$$W(\{k_i\}) < C|\sum_i |k_i||^N$$

Scattering amplitudes are bounded by a polynomial in k for complex k

$$A(\{k_i\}) < C|\sum_i |k_i||^N$$

FROISSART-MARTIN BOUND

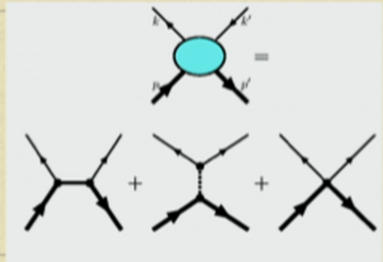
Assuming only Polynomial Boundedness, and (proven) analyticity in the Martin-Lehmann ellipse

Number of subtractions in dispersion relation is never more than 2!

$$A(s) \leq \text{constant} \;\; s(\ln s)^2$$

$$\sigma(s) \sim \frac{Im(A(s))}{s} < \frac{c}{m^2}(\ln s)^2$$

FORWARD SCATTERING DISPERSION



Unsubtracted Dispersion Relation

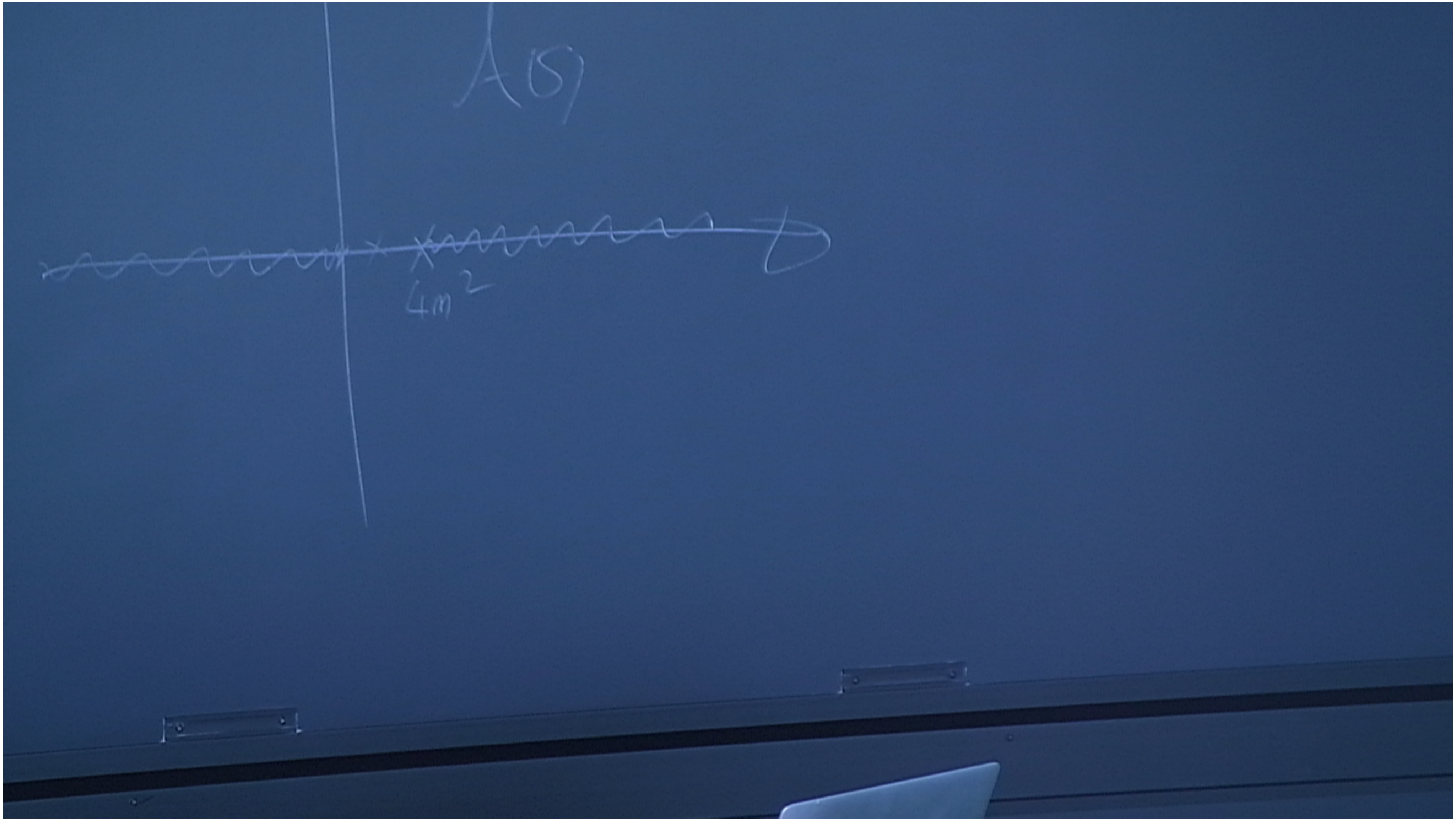
$$A(s) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{\mathrm{Im}(A(s)}{s' - s} ds' + \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{\mathrm{Im}(A(s)}{s' + 4m^2 + s} ds' + \text{pole terms}$$

after subtractions

$$A(s) = A_0 + sA_1 + \frac{1}{\pi} s^2 \int_{4m^2}^{\infty} \frac{\mathrm{Im}[A(s')]}{s'^2(s' - s)} + \frac{1}{\pi}(4m^2 - s)^2 \int_{4m^2}^{\infty} \frac{\mathrm{Im}[A(s')]}{s'^2(s' - 4m^2 + s)} + \text{pole terms}$$

subtractions

crossing symmetry


A(s)
4m²

FORWARD SCATTERING DISPERSION

$$A(s) = A_0 + sA_1 + \frac{1}{\pi}s^2 \int_{4m^2}^{\infty} \frac{\mathrm{Im}[A(s')]}{s'^2(s'-s)} + \frac{1}{\pi}(4m^2-s)^2 \int_{4m^2}^{\infty} \frac{\mathrm{Im}[A(s')]}{s'^2(s'-4m^2+s)} + \text{pole terms}$$

subtractions

crossing symmetry

Differentiating

$$A''(0) = \frac{4}{\pi} \int_0^{\infty} \frac{\mathrm{Im}[A(s')]}{s'^3} + \dots$$

FORWARD SCATTERING DISPERSION

$$A''(0) = \frac{4}{\pi}\int_0^\infty \frac{\mathrm{Im}[A(s')]}{s'^3} + \dots$$

optical theorem $\mathrm{Im}[A(s)] = s\sigma(s)$

$$A''(s = s_0) = \frac{4}{\pi}\int \mathrm{d}s \frac{s\sigma(s)}{s^3} + \cdots > 0$$

BUT FOR ALL GALILEON MODELS!

at best lowest order contribution

$$A(s,t,u) \sim \frac{1}{\Lambda^6}(s^3+t^3+u^3)+\ldots$$

$$A''(0)=0$$ in limit $m \to 0$

Adams et al 2006

which violates

$$A''(s=s_0)=\frac{4}{\pi}\int \mathrm{d}s \frac{s\sigma(s)}{s^3}+\cdots>0$$

CONCLUSIONS

Galileons do not admit a local, Lorentz invariant, UV completion

Adams et al 2006

ASSUMPTIONS

Locality/causality implies analyticity through JLD representation

Locality implies polynomial boundedness (temperedness assumption)

Polynomial boundedness plus analyticity implies Froissart-Martin bound

Together with unitarity imply $A''(s) > 0$

All of these statements are
Rigorously proven in Axiomatic Field Theory

FORWARD SCATTERING DISPERSION

$$A''(0) = \frac{4}{\pi}\int_0^\infty \frac{\mathrm{Im}[A(s')]}{s'^3} + \dots$$

optical theorem $\mathrm{Im}[A(s)] = s\sigma(s)$

$$A''(s = s_0) = \frac{4}{\pi}\int \mathrm{d}s \frac{s\sigma(s)}{s^3} + \dots > 0$$

END OF THE ROAD …. OR?

We should never give up Unitarity!

Would rather not give up Lorentz invariance since then lose motivation for considering Galileons e.g. Lorentz invariant massive gravity

What about locality?

LOCALITY IN GRAVITY

We already know of one theory in which polynomial boundedness, aka locality is violated, at least for fixed angle scattering

General Relativity

This is transparent already in eikonel limit

$$A(s,t) \sim e^{i(s(-t)^{(D-4)/2})^{1/(D-3)}}$$

and is expected in Black-Hole regime

$$A(s,t) \sim e^{R_S(\sqrt{s})\sqrt{t}}, \quad t > 0$$

Giddings-Porto 2009

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Giddings-Porto 2009

ANALYTICITY WITHOUT POLYNOMIAL BOUNDEDNESS

However if we give up analyticity, then we lose any notion of causality (even macro-causality)

So the best possible solution (and hence our conjecture) is:

Galileons admit a UV completion with a Lorentz invariant, Unitarty, Analytic, crossing symmetric S-matrix, but without polynomial boundedness

ANALYTICITY WITHOUT POLYNOMIAL BOUNDEDNESS

This transforms the previous arguments: without P.B. we lose Froissart-Martin

Even if the forward scattering amplitude is P.B. ……

Now, Number of subtractions in forward scattering dispersion relation can be more than 2!

ANALYTICITY WITHOUT POLYNOMIAL BOUNDEDNESS

$$A(s) = A_0 + A_1 s + A_2 s^2 + A_3 s^3 + \qquad (\\ \frac{1}{\pi} s^4 \int_{4m^2}^{\infty} \frac{\mathrm{Im}[A(s')]}{s'^4(s'-s)} + \frac{1}{\pi}(4m^2 - s)^4 \int_{4m^2}^{\infty} \frac{\mathrm{Im}[A(u')]}{u'^4(u' - 4m^2 + s)} + \text{pole terms}$$

$$A''''(0) = 4! \frac{2}{\pi} \int_0^{\infty} \frac{\mathrm{Im}[A(s')]}{s'^5} > 0 \qquad A''''(0) \;\sim\; s^4/\Lambda^8$$

can be achieved with Galileon operator

$$\mathcal{L}_{12} \sim \frac{1}{\Lambda^8}(\partial_\mu \partial_\nu \pi \partial^\mu \partial^\nu \pi)^2$$

Galileon symmetry is not in conflict with Unitarity and Analyticity!

SPECTRAL REPRESENTATION

Start with simplest case:

$$W(x,y) = \langle 0|\hat{O}^\dagger(x)\hat{O}(y)|0\rangle = \int_0^\infty \mathrm{d}\mu\, \rho_O(\mu)\, W_\mu(x,y)$$

Unitarity $\rho_O(\mu) \geq 0$ Stability $\mu \geq 0$

$$W_\mu(x,y) \approx \frac{(2\sqrt{\mu})^{1/2}}{(4\pi|x-y|)^{3/2}} e^{-\sqrt{\mu}|x-y|}\,, \quad (x-y)^2 > 0\,, \quad \mu|x-y|^2 \gg 1\,,$$

$$W_\mu(x,y) \approx -ie^{-i\pi/4} \frac{(2\sqrt{\mu})^{1/2}}{(4\pi|x-y|)^{3/2}} e^{-i\sqrt{\mu}|x-y|}\,, \quad (x-y)^2 < 0\,, \quad \mu|x-y|^2 \gg 1$$

LOCALIZABLE, QUASI-LOCAL, NON-LOCALIZABLE

$$W(x,y) = \langle 0|\hat{O}^\dagger(x)\hat{O}(y)|0\rangle = \int_0^\infty \mathrm{d}\mu\, \rho_O(\mu)\, W_\mu(x,y)$$

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Define order of growth

$$\rho_O(\mu) \sim e^{\sigma\mu^\alpha} \times \text{subdominant terms}$$

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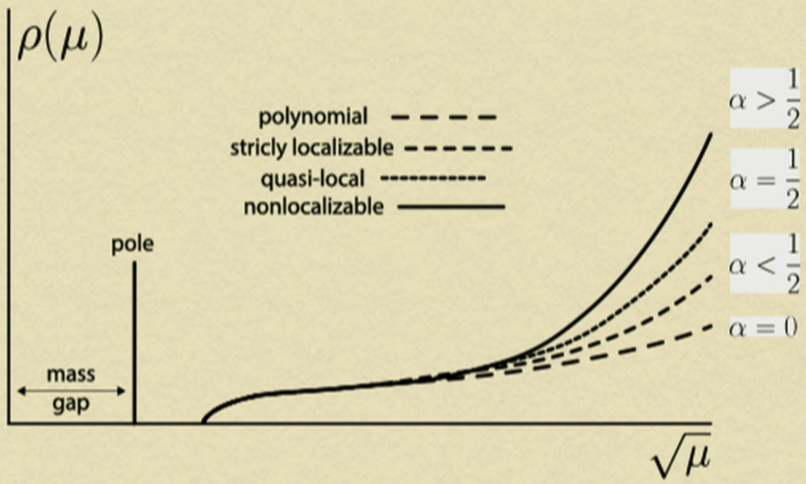

NON-LOCALIZABLE FIELDS

$$\rho_O(\mu) \sim e^{\sigma \mu^\alpha} \times \text{subdominant terms}$$

If $\alpha > 1/2$

the position space Wightman function does not exist!

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NON-LOCALIZABLE FIELDS

what does exist

$$W(f,g) = \int_0^\infty \mathrm{d}\mu \int \frac{\mathrm{d}^d k}{(2\pi)^d} f^*(k) g(k) 2\pi \rho_O(-k^2)$$

$$f(k), g(k) < C e^{-\frac{1}{2}\sigma|k|^{2\alpha}} \text{ as } |k| \to \infty$$

such functions are called Gelfand-Shilov distributions

Idea promote Wightman axioms from tempered distributions to Gelfand-Shilov distributions

IDEA IS NOT NEW!

Historical work: Jaffe, Taylor, Efimov, Guttinger, Fradkin, Fainberg, Iofa …

In particular Steinmann 1970 shows non-localizable fields have LSZ formalism, cluster decomposition, CPT, asymptotic states

More recently has been proposed for Little String Theories and M-theory ….. and is implicit in Classicalization proposal of Dvali et al

LITTLE STRING THEORIES

LSTs describe M-theory compactified on a T^5

Decoupling limit of N coincident five-branes in String theory

$M_{\text{Planck}} \to \infty,\ g_s \to 0 \quad M_{\text{string}}$ kept fixed

Infrared Limit are 6 dimensional superconformal field theories

No UV fixed point

T-duality and stringy behavior survive

Exponential growth of spectral density associated with Hagedorn

$$\rho(\mu) \sim e^{\frac{cM_{\text{string}}\sqrt{\mu}}{\sqrt{N}}}$$

LSTs are QUASI-LOCAL FIELD THEORIES Kaputsin '99

QUANTUM GRAVITY/M-THEORY

Argument due to Aharony and Banks '98

In field theory with UV fixed point at finite volume, density of states grows as

$$\rho(E) \sim e^{\mathrm{S}_{\mathrm{entropy}}} \sim e^{c' V^{1/d} E^{(d-1)/d}}$$

Certain special operators grow at most as power

Expect KL spectral density to scale with density of states (unless operators have small overlap with high energy states)

Generic operators either tempered or strictly local fields

QUANTUM GRAVITY/M-THEORY

Argument due to Aharony and Banks '98

In Quantum Gravity we expect high energy properties to be dominated by production of black holes

$$\rho(E) \sim e^{S_{\text{BH entropy}}} = e^{c(E/M_{\text{Pl}})^{\frac{d-2}{d-3}}}$$

If operators spectral densities scale with density of states: Observables in quantum gravity are Non-localizable fields

Fits perfectly with the rule of thumb that there are no local gauge invariant observables in quantum gravity

LOCALITY BOUND IN GR

Phrased differently, density of states scales as

$$\rho(E) \sim e^{E r_*(E)}$$

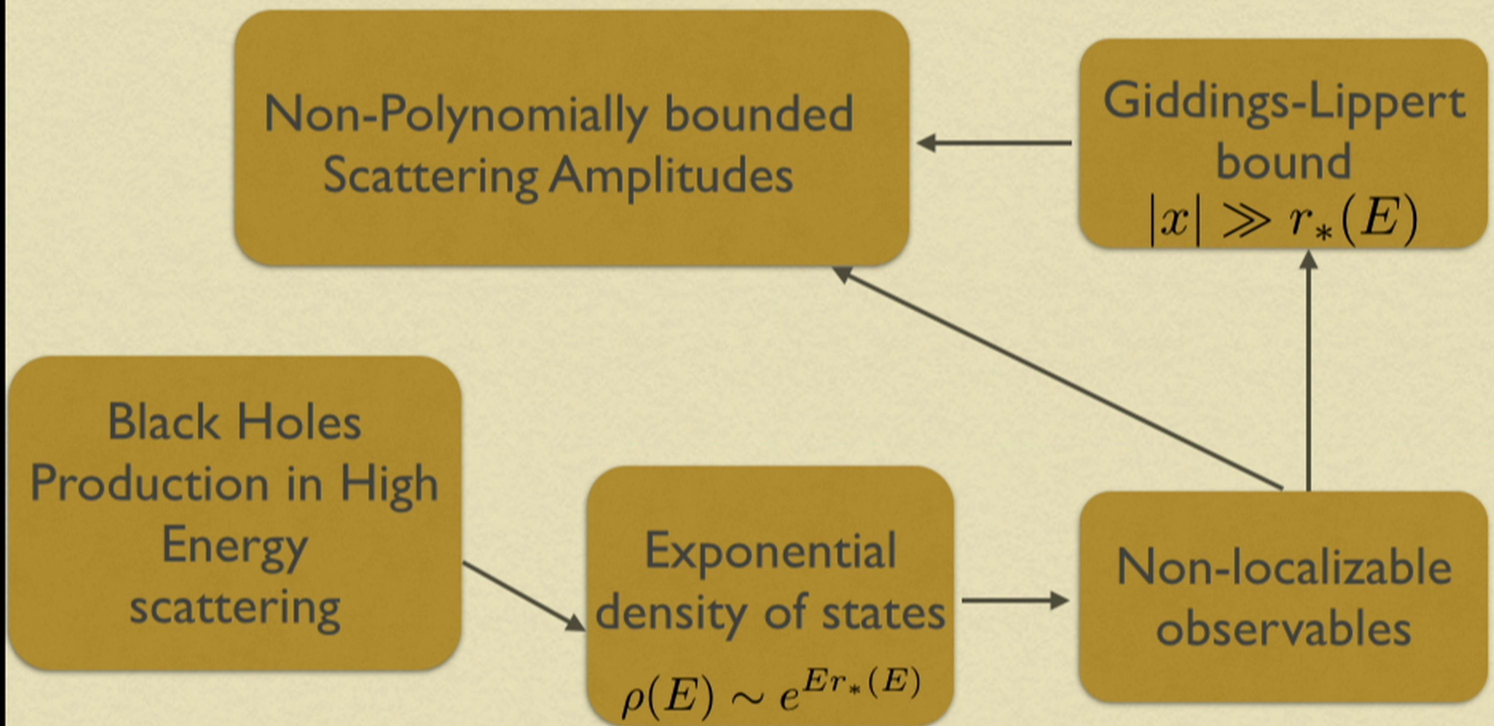
where here $r_*(E)$ is Schwarzschild radius

This implies Giddings-Lippert 2001 locality bound

We can only talk about locality in gravity at distances

$$|x| > r_*(E)$$

LOCALITY IN GRAVITY



WHAT HAS THIS TO DO WITH GALILEONS?

Aren't Galileons just scalar fields theories? What has this gravitational non-locality to do with them?

VAINSHTEIN MECHANISM!

Just like GR, Galileons have a built in length scale at which they become strongly coupled

Vainshtein effect

When curvature is large $R \gg m^2$ recover GR

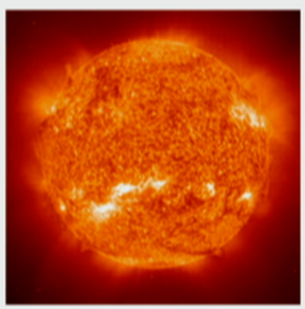
When curvature is small $R \ll m^2$ fifth force propagates

Determines characteristic Vainshtein radius $\frac{M}{M_P^2 r_V^3} \sim m^2$

Screened region $r \ll r_V$ $r_V = (r_s m^{-1})^{1/3}$

Weak coupling region $r \gg r_V$

For Sun



$m^{-1} \sim 4000Mpc$

$r_s \sim 3km$

$r_V \sim 250pc$

VAINSHTEIN MECHANISM

Normal phrasing: Couple Galileon to a source of matter of mass M

In 4D, Galileons strongly coupled inside Vainshtein radius

$$r_V = \Lambda^{-1}\left(\frac{M}{M_{\rm Pl}}\right)^{1/3}$$

However Galileon profile has FINITE SELF-ENERGY !?!

$$E \sim r_V^{-1}\left(\frac{M}{M_{\rm Pl}}\right)^2$$

$$r_V(E) \sim \Lambda^{-1}\left(\frac{E}{\Lambda}\right)^{1/5}$$ and so we have $$r_*(E) = r_V(E)$$

VAINSHTEIN MECHANISM

When two high energy particles collide they produce a Black Hole

In GR, the size is the Schwarschild radius, in Massive Gravity its the Vainshtein radius

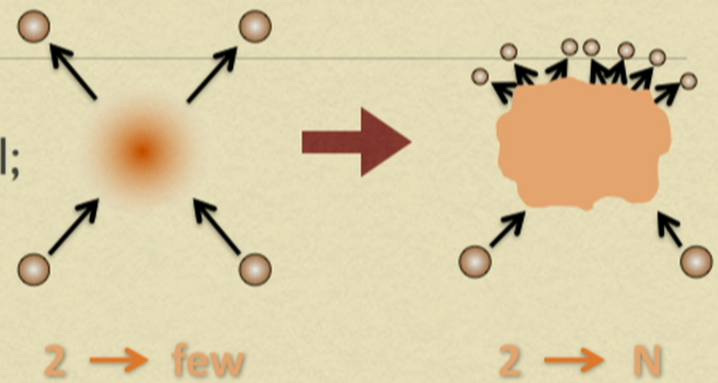
$$r_*(E) \sim \Lambda^{-1} \left(\frac{E}{\Lambda}\right)^{1/5}$$

Natural to suppose same Gravitational non-locality spreads out to Vainshtein radius

Gives us a locality bound: $|x| > r_*(E)$

CLASSICALIZATION

Vainshtein radius is the
Classicalization radius of Dvali et al;



Proposed picture:
High Energy scattering of two hard quanta, produce metastable bound state (classicalon) of N quanta which decays into N soft quanta with

$$N \sim E r_*(E)$$

Typical energy of final quanta $k \sim \frac{E}{N} \sim \frac{1}{r_*(E)}$

Final density of states $\rho(E) \sim e^N \sim e^{E r_*(E)}$

CLASSICALIZATION

Final density of states $\rho(E) \sim e^N \sim e^{Er_*(E)}$

In all Vainshtein/classicalizing theories,
$r_*(E)$ grows with energy

e.g. for Galileons in d dimensions $r_*(E) = \frac{1}{\Lambda}\left(\frac{E}{\Lambda}\right)^{\frac{1}{d+1}}$

Thus all Vainshtein/Classicalization theories are expected to be non-localizable field theories

CLASSICALIZATION

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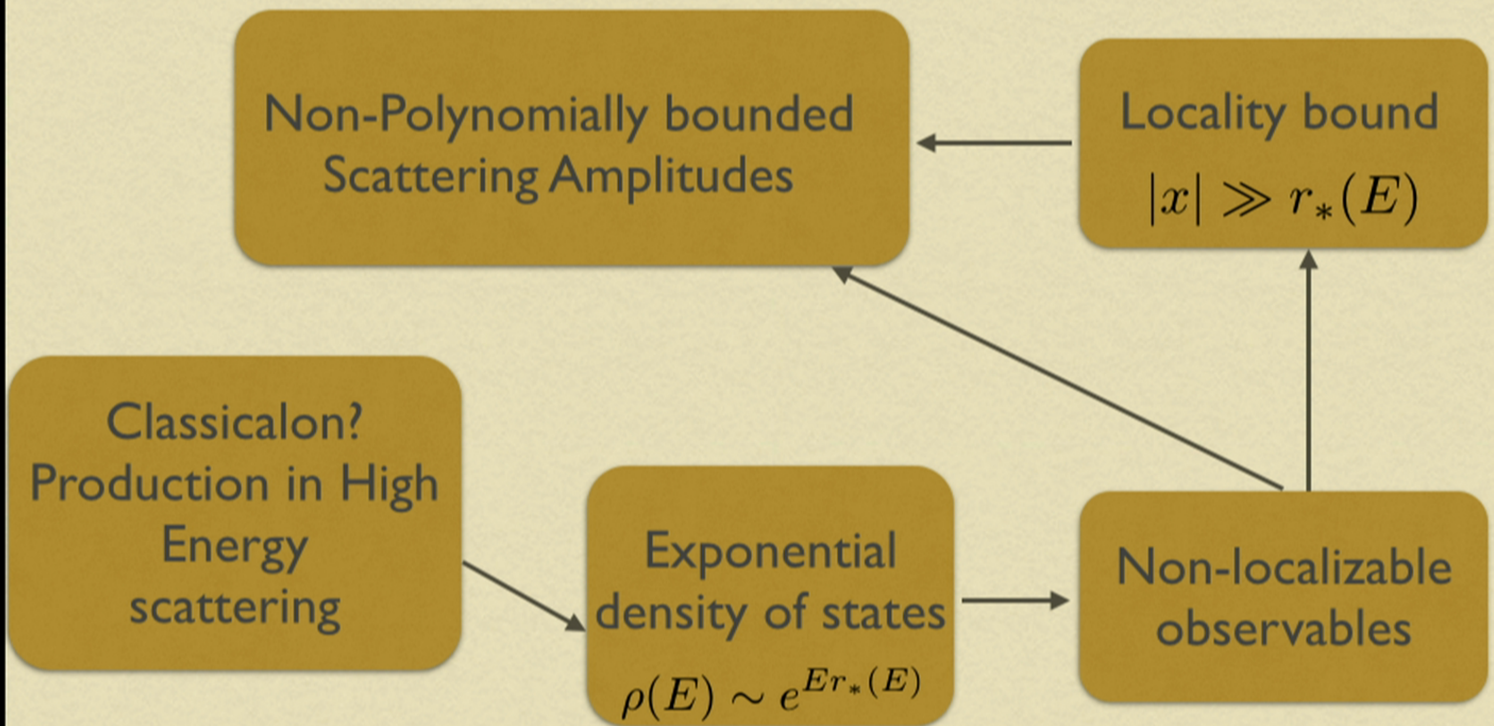
CLASSICALIZATION



UV PROPERTIES OF GALILEONS

THE CONJECTURE:

Galileons are non-localizable quantum fields,
whose spectral densities grow as

$$\rho(E) \sim e^N \sim e^{E r_*(E)} \sim e^{(E/\Lambda)^{(d+2)/(d+1)}}$$

Remarkable we can PROVE this for one special Galileon model

GALILEON DUALITY

To prove the conjecture, we will make use of the existence of the Galileon Duality

de Rham, Fasiello, Tolley 2013
Curtright Fairlie 2012

Under this transformation:
Galileon → Galileon with distinct coefficients

GALILEON DUALITY

$$\tilde{x}^\mu = x^\mu + \frac{1}{\Lambda^3}\partial^\mu \pi(x)$$

$$x^\mu = \tilde{x}^\mu - \frac{1}{\Lambda^3}\tilde{\partial}^\mu \tilde{\pi}(\tilde{x})$$

GALILEON DUALITY

$$\tilde{x}^\mu = x^\mu + \frac{1}{\Lambda^3}\partial^\mu \pi(x)$$

Galileon

$$x^\mu = \tilde{x}^\mu - \frac{1}{\Lambda^3}\tilde{\partial}^\mu \tilde{\pi}(\tilde{x})$$

Dual Galileon

$$\partial^\mu \pi(x) = \tilde{\partial}^\mu \tilde{\pi}(\tilde{x})$$

$$\tilde{\pi}(\tilde{x}) = \pi(x) + \frac{1}{2\Lambda^3}(\partial\pi(x))^2$$

$$\pi(x) = \tilde{\pi}(\tilde{x}) - \frac{1}{2\Lambda^3}(\tilde{\partial}\tilde{\pi}(\tilde{x}))^2$$

Legendre Transform

GALILEON DUALITY

$$\int d^4x \; c_2\mathcal{L}_2(\pi) + c_3\mathcal{L}_3(\pi) + c_4\mathcal{L}_4(\pi) + c_5\mathcal{L}_5(\pi)$$

$$=$$

$$\int d^4x \; p_2\mathcal{L}_2(\tilde{\pi}) + p_3\mathcal{L}_3(\tilde{\pi}) + p_4\mathcal{L}_4(\tilde{\pi}) + p_5\mathcal{L}_5(\tilde{\pi})$$

$$p_n = \frac{1}{n}\sum_{k=2}^{5}(-1)^k \frac{k(5-k)}{(n-k)!(5-n)!} c_k$$

One Galileon is dual to a distinct Galileon

GALILEON DUALITY

In any dimension, there is one Galileon model which is dual to a free theory

$$\tilde{x}^\mu = x^\mu + \frac{1}{\Lambda^\sigma}\partial^\mu \pi(x)$$

$$x^\mu = \tilde{x}^\mu - \frac{1}{\Lambda^\sigma}\partial^\mu \rho(\tilde{x})$$

$$\chi(x) = \tilde{\chi}(\tilde{x})$$

$$S_{\text{Galileon}}[\pi] = \int \mathrm{d}^d x \left[-\frac{1}{2}\det(1+\Pi(x))(\partial\pi(x))^2\right] = \int \mathrm{d}^d x \left[-\frac{1}{2}(\partial\rho(x))^2\right] = S_{\text{free}}[\rho]$$

EXPLICIT FORM OF DUALITY

$$\pi(x) = \int \mathrm{d}^d y \int \frac{\mathrm{d}^d k}{(2\pi)^d} U(\rho(y)) \; e^{ik.(x-y)+\frac{ik.\partial\rho(y)}{\Lambda^\sigma}}$$

Prompts following definition of quantum field:

$$\hat{\pi}(x) = \int \mathrm{d}^d y \int \frac{\mathrm{d}^d k}{(2\pi)^d} e^{ik.(x-y)} : U(\hat{\rho}(y)) \; e^{\frac{ik.\partial\hat{\rho}(y)}{\Lambda^\sigma}} :$$

$$\hat{\rho}(x) = \int \mathrm{d}\tilde{k} \left[\hat{a}_k e^{ik.x} + \hat{a}_k^\dagger e^{-ik.x}\right]$$

Normal ordering removes infinite number of divergences, all remaining correlation functions are finite (in momentum space)

EXPLICIT FORM OF DUALITY

Prompts following definition of quantum field:

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$$\hat{\rho}(x) = \int \mathrm{d}\tilde{k}\, \left[\hat{a}_k e^{ik.x} + \hat{a}_k^\dagger e^{-ik.x}\right]$$

This definition seems well justified since in a coherent state

$$\rho_c(x) = \langle\beta|\hat{\rho}(x)|\alpha\rangle = \int \mathrm{d}\tilde{k}\, \left[\alpha_k e^{ik.x} + \beta_k^* e^{-ik.x}\right] \qquad \pi_c(x) = \int \mathrm{d}^d y \int \frac{\mathrm{d}^d k}{(2\pi)^d} U(\rho_c(y))\; e^{ik.(x-y)+\frac{ik.\partial\rho_c(y)}{\Lambda^\sigma}}$$

$$\langle\beta|\hat{\pi}(x)|\alpha\rangle = \pi_c(x)$$

This will automatically be a solution of Galileon equation of motion

same would not be true if we had not normal ordered

EVALUATION OF SPECTRAL DENSITY

Formal definition:

$$\langle 0|\hat{\pi}(x)\hat{\pi}(x')|0\rangle = \int \mathrm{d}^d y \int \frac{\mathrm{d}^d k}{(2\pi)^d} e^{ik.(x-x'-y)} \langle 0| : U(\hat{\rho}(y))\, e^{\frac{ik.\partial\hat{\rho}(y)}{\Lambda^\sigma}} :: U(\hat{\rho}(0))\, e^{\frac{-ik.\partial\hat{\rho}(0)}{\Lambda^\sigma}} : |0\rangle$$

$$= \int \mathrm{d}^d y \int \frac{\mathrm{d}^d k}{(2\pi)^d} e^{ik.(x-x'-y)} F(y,k) \langle 0| : e^{\frac{ik.\partial\hat{\rho}(y)}{\Lambda^\sigma}} :: e^{\frac{-ik.\partial\hat{\rho}(0)}{\Lambda^\sigma}} : |0\rangle$$

Wicks theorem:

$$\langle 0| : e^{\frac{ik.\partial\hat{\rho}(y)}{\Lambda^\sigma}} :: e^{\frac{-ik.\partial\hat{\rho}(0)}{\Lambda^\sigma}} : |0\rangle = e^{-\frac{1}{2}k^\mu k^\nu \partial_\mu \partial_\nu \langle 0|\rho(y)\rho(0)|0\rangle/\Lambda^{2\sigma}}$$

$$= \int \mathrm{d}^d y \int \frac{\mathrm{d}^d k}{(2\pi)^d} e^{ik.(x-x'-y)} F(y,k) \exp\left[\frac{a_d}{\Lambda^{2\sigma}}\left(\frac{k^2}{y_-^d} - d\frac{(k\cdot y)^2}{y_-^{d+2}}\right)\right] \qquad \frac{1}{y_-^d} = \frac{1}{(|\vec{y}|^2 - (y^0 - i\epsilon)^2)^{d/2}}$$

Wightman function

EVALUATION OF SPECTRAL DENSITY

$$\langle 0|\hat{\pi}(x)\hat{\pi}(y)|0\rangle = \int_0^\infty \mathrm{d}\mu \int \frac{\mathrm{d}^d k}{(2\pi)^d} e^{ik.(x-y)} \rho(-k^2) 2\pi\delta(k^2+\mu)\theta(k^0)$$
$$= \int_0^\infty \mathrm{d}\mu \int \frac{\mathrm{d}^{d-1} k}{(2\pi)^{d-1} 2\omega_k(\mu)} e^{ik.(x-y)} \rho(\mu)\,.$$

Spectral Density

$$\theta(k^0)\theta(-k^2)2\pi\rho_{\pi,0}(-k^2) = \int \mathrm{d}^4 y\, e^{-ik.y} \frac{1}{4\pi^2 y_-^2} \sum_{n=0}^{\infty} \frac{1}{n!} \left[\frac{1}{2\pi^2 \Lambda^6} \left(\frac{k^2}{y_-^4} - 4\frac{(k\cdot y)^2}{y_-^6} \right) \right]^n$$

$$2\pi\rho_{\pi,0}(\mu) = \frac{1}{4\pi^2} \sum_{n=0}^{\infty} \sum_{r=0}^{n} \frac{\mu^{3n-1}}{\Lambda^{6n}} \frac{(-1)^r 4^{n-r}}{r!(n-r)!(2\pi^2)^n} \frac{(4\pi^2)^{3n-r+1}(6n-2r-2)!}{(16\pi^2)^{3n-r}(4n-2)!(3n-r-1)!(3n-r)!}$$

Sum over states

$$2\pi\rho_{\pi,0}(\mu) = 2\pi\delta(\mu) + \frac{a}{3\pi\mu}\sqrt{\frac{2}{15}}\, e^{\left(\frac{5\cdot 3^{1/5}}{2^{9/5}\pi^{2/5}}\right)\left(\frac{\mu^3}{\Lambda^6}\right)^{1/5}}$$

Exponential growth

EVALUATION OF SPECTRAL DENSITY

Repeating in D dimensions, we find (in any duality frame) there exists at least one field whose spectral density grows as

$$\rho(E) \sim e^{N} \sim e^{E r_*(E)} \sim e^{(E/\Lambda)^{(d+2)/(d+1)}}$$

Dominant contribution in sum comes from N-particle states with

$$N \sim \left(\frac{E}{\Lambda}\right)^{(d+1)/(2(d+2)} \sim E\, r_*(E)$$

consistent with classicalization proposal

Thus this-Galileon (at least) is a Non-localizable field as conjectured

INTERACTING EXAMPLE - EFT OF INFINITELY LONG STRINGS

Cooper et al 2014

This was for a free theory, however a (non-Galileon) interacting example with similar properties exists

LEEFT superluminal propagation

Nambu-Goto with wrong sign

$$S = l_s^2 \int \mathrm{d}^2\sigma \sqrt{-Det[\eta_{ab} + l_s^2 \partial_a X^i \partial_b X^i]}$$

$$E \sim \int \mathrm{d}\sigma \, l_s^2 \sqrt{-Det[\eta_{ab} - l_s^2 \partial_a X^i \partial_b X^i]} + \dots \qquad E \sim l_s^2 r_*(E) \qquad r_*(E) \sim l_s^{-2} E$$

$$A^*(s) = A(s^*) \qquad e^{2i\delta(s)} = e^{-isl_s^2/4}, \quad \mathrm{Im}(s) > 0,$$

$$A(-s) = A(s) \qquad e^{2i\delta(s)} = e^{isl_s^2/4}, \quad \mathrm{Im}(s) < 0.$$

Exact, unitary, crossing symmetric, Analytic, Lorentz invariant S-matrix

Non-polynomially bounded

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EXPECTED FORM FOR GALILEONS

Appears to be no obstruction to having a *Lorentz Invariant, Crossing Symmetric, Analytic S-matrix*

Violation of P.B. implies no Micro-causality

Analyticity of S-matrix ensure Macro-causality

$$\hat{\phi}(f_{x_0}) = \int \mathrm{d}^d x f_{x_0}(x)\hat{\phi}(x) \qquad \left[\hat{\phi}(f_{x_0}), \hat{\phi}(g_{y_0})\right] \to 0\,, \quad (x_0 - y_0)^2 \to +\infty$$

Macro-causality implies absence of superluminal propagation

Resolves conflict from apparent S.L. propagation in LEEFT

SUMMARY

One can prove that the LEEFT Galileons cannot be UV completed by a Local, Lorentz invariant field theory

But there appears to be no obstruction with UV completing with a Non-localizable Lorentz invariant field theory

Non-localizablity has been argued to be an essential feature of gravitational theories - exponential density of states associated with black holes

We have been able to show, by an explicit UV quantization of a specific Galileon that Galileons are also non-localizable field theories

Explicit S-matrix work in progress, but appears to be no obstruction, provided we accept non-P.B.