Title: Geometry from Compression

Date: Feb 03, 2015 02:00 PM

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Abstract: <p>Recent research has suggested deep connections between geometry and entropy. This connection was first seen in black hole thermodynamics, but has been more fully realized in the Ryu-Takayanagi proposal for calculating entanglement entropies in AdS/CFT. We suggest that this connection is even broader: entropy, and in particular compression, are the fundamental building blocks of emergent geometry. We demonstrate how spatial geometry can be derived from the properties of a recursive compression algorithm for the boundary CFT. We propose a general algorithm for constructing MERA-like tensor networks and elucidate connections to the mathematical field of integral geometry.</p>

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Gravity is Entropic:

From this perspective, gravity is best understood as a thermodynamic theory. Entropy generates spacetime, but seems *agnostic* about what it is purified by.

- Black hole thermodynamics *[Bekenstein and Hawking]*
- Einstein equations as equation of state *[Jacobson]*
- Gravity as entropic force *[Verlinde]*
- Extremal surfaces from entanglement entropy *[Ryu-Takayanagi; Hubeny, Rangamani, Takayanagi]*
- AdS geometry as a MERA entanglement network *[Swingle]*
- *AdS Rindler horizons and ER =EPR [Van Raamsdonk; Maldacena, Susskind]*
- *Linearized Einstein equations from EE [Lashkari, McDermott, Faulkner, Hartman; Myers, Van Raamsdonk]*

Gravity is Entropic:

From this perspective, gravity is thermodynamic and entropy generates spacetime, but seems *agnostic* about what it is purified by.

Gravity is Pure:

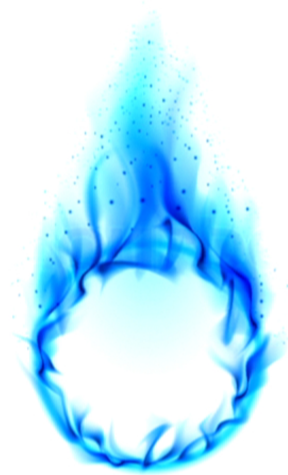
From the other perspective, the microscopic structure of entanglement purification is important.

- Through AdS/CFT we have confirmed that gravity can be described by a microscopic unitary theory
- EFT in curved space-time: vacuum state is a particular entangled state
- Eternal AdS black hole described by particular TFD state

Which of these perspectives is correct? Is there a middle ground between the two perspectives?

Firewalls

[Almheiri, Marolf, Polchinski, Sully; Braunstein]

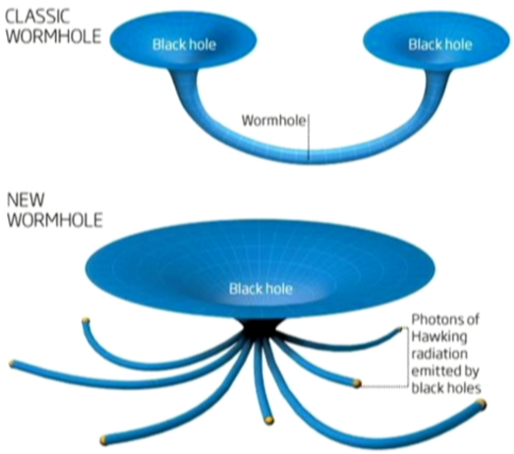


Resolving the tension between these two perspectives isn't simply a question about quantum gravity at the Planck scale.

The black hole information paradox and the question of firewalls hinges on which of these two perspectives we believe:

- The reliance on EFT and the belief that the vacuum has a fixed structure seems to lead inevitably to firewalls
  - A smooth vacuum state at the horizon requires fields to be in the local Rindler state at the horizon, with entanglement between the outgoing and ingoing Hawking partners.

- The incompatibility of mutual entanglements that leads to the firewall can be resolved if one tracks entanglement, but not its purification.
- This entropic approach builds a smooth geometry by constructing the interior Hawking modes from whatever the exterior Hawking mode happens to be entangled with.
  - This leads to constructions like the proposal of Papadodimas and Raju for building non-linear (state dependent) interior operators and the EPR=ER proposal of Maldacena and Susskind.



Black holes are an invaluable pressure test for our ideas about quantum gravity, but they also add to the confusion about what we are doing.

- Life is confusing enough without immediately confronting, for example, whether quantum gravity can accommodate violations of quantum mechanics.
- While this may drive straight to the heart of the issue, perhaps something can be learned by less invasive surgery of what we think we know.

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It seems valuable to explore the tension between entropy and purity in the absence of black holes.

Questions we would like to answer:

1. **What states have good geometric duals?**
   To what extent is this geometry determined by entropic CFT quantities? What information about the state is necessary?

2. **Is there an entropic interpretation of the holographic RG and the emergence of the radial direction?**

3. **What is the entropic meaning of areas in spacetime?**
   What is this entropy actually counting?

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   What is this entropy actually counting?

In this talk I will describe what I think is substantive progress in answering all three of these questions.

Our Starting Point

The classic results of black hole thermodynamics suggest we should associate an entropy to black hole horizons (or any killing horizons):

$$S_{BH} = \frac{A}{4G_N}$$

Natural to then ask: can we associate a notion of entropy to any choice of bulk area?

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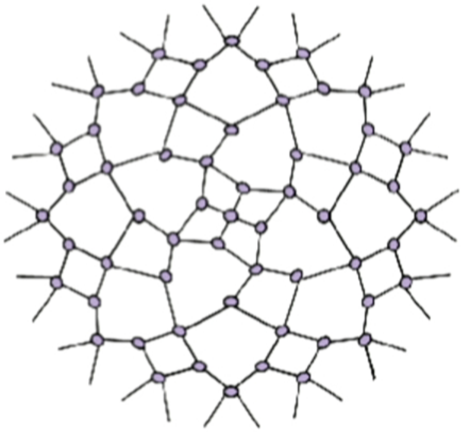
Spacetime Engtanglement Conjecture [Bianchi, Myers]:

In a theory of quantum gravity, for large regions of smooth spacetime, the entanglement entropy between the degrees of freedom describing the given region with those describing its complement will be given by the BH formula at leading order.

Hints from MERA:

For a CFT, one can efficiently represent certain low-energy states by a network of unitary operators. Swingle has suggested that the structure of this lattice for the vacuum state of a CFT mimics the coarse structure of AdS.
(Q1)

[Vidal; Swingle]



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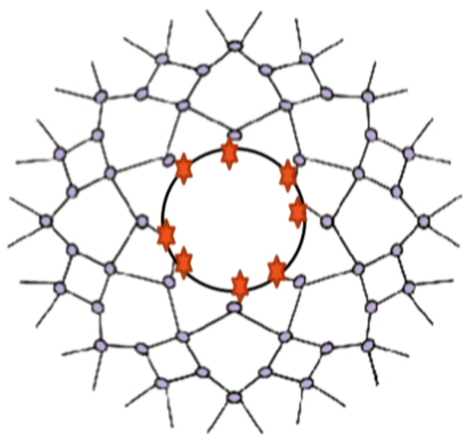
[Vidal; Swingle]

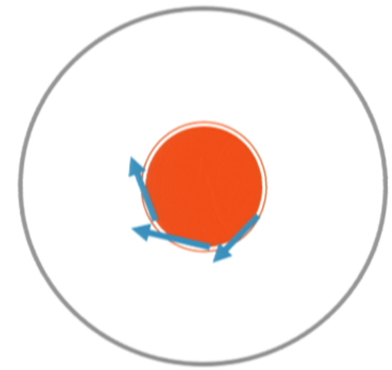
- Lattice points deeper in the bulk encode IR entanglement in the CFT **(Q2)**



- When we cut out a region of the MERA lattice, we remove a number of unitary operators proportional to the area of the cut.
- The number of possible states that fills in the lattice is proportional to the area of the cut. **(Q3)**
- However, the subspace spanned by these states does not necessarily form a tensor factor of the Hilbert space.

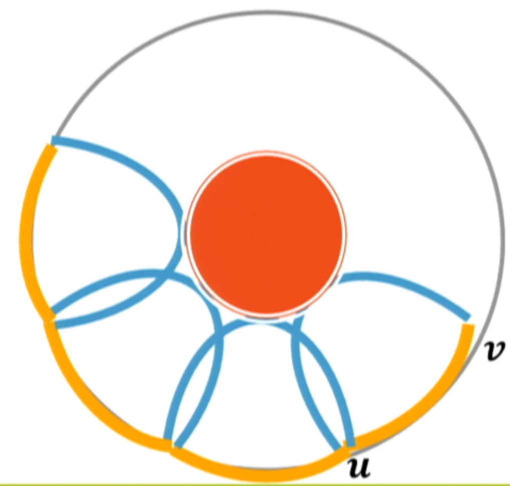
Differential entropy is the continuum analog of our procedure of assigning an entropy to a length in the MERA.

[BALASUBRAMANIAN, CHOWDHURY, CZECH, DE BOER, HELLER]



- Differential entropy says that we can rewrite the integral for the length of the boundary of a region in terms of it's tangent vectors.
- We can do a (non-local) transformation from the tangent space to the boundary-anchored geodesics that the tangents lie on.
- For geometries that are '**boundary-rigid,'** these geodesics are always minimal. RT tell us their length is actually the Entanglement entropy of the region.
- The integral takes a miraculously simple form:

$$S_{\text{diff}} = \int \mathrm{d}\lambda \frac{du(\lambda)}{d\lambda} \frac{\partial S[v(\lambda), u(\lambda)]}{\partial u}$$



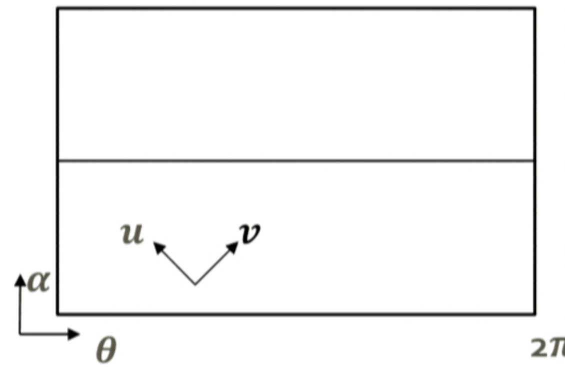
The Radon transform and the Crofton formula can be naturally extended to hyperbolic space (uniquely determined by symmetry):

- Let Γ be the space of planes in $\mathbb{H}^2$, with the unique invariant measure. Then:

$$L(C) = \frac{1}{4}\int_\Gamma \#(\gamma \cap C)\, d^2\gamma$$

- The invariant Crofton measure on the space of planes is given by

$$\mathrm{d}^2\gamma = \frac{1}{\sin^2(u-v)}\mathrm{d}u\mathrm{d}v$$



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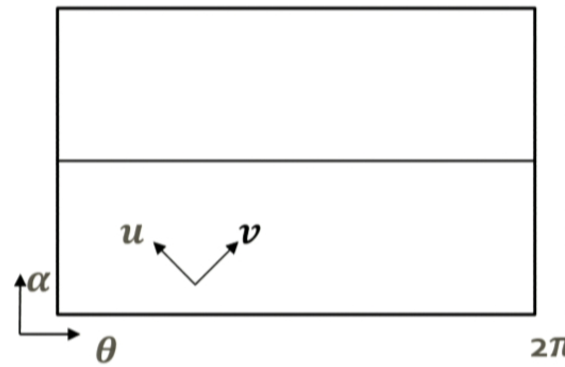

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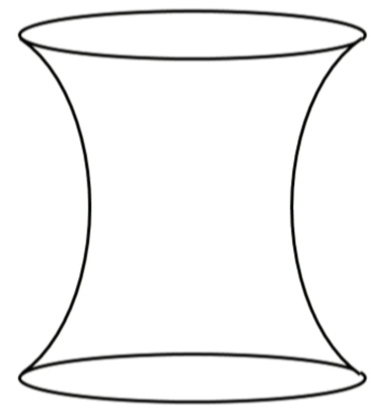
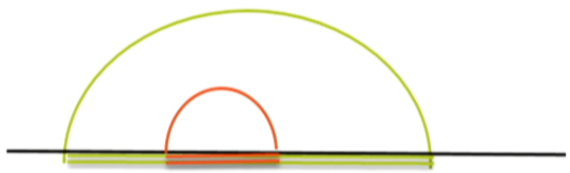
- Taking the Crofton form to be the **curvature** density, we find Kinematic space is **Lorentzian de Sitter space**.

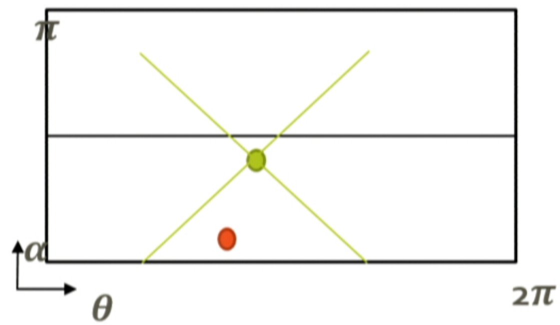


Causal Structure

- What is the meaning of the causal structure in Kinematic space?

- Time-like:





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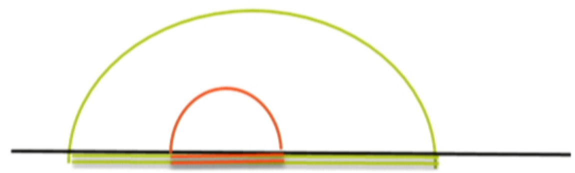
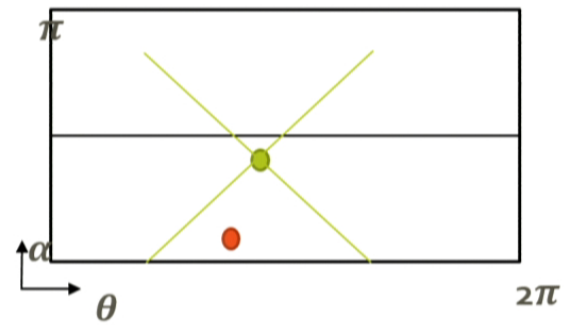
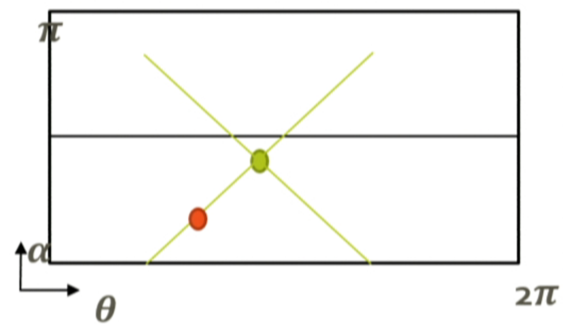




- Light-like:



General Geometries

- Is there anything that can be done in more general geometries?
  - The derivation of the correct measure in integral geometry relied on symmetry…

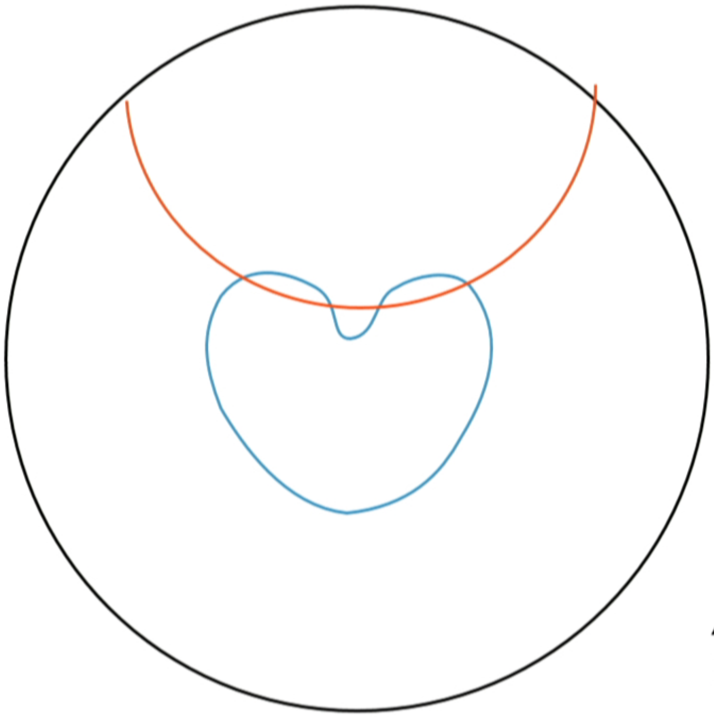
In fact, YES! We can use the meaning of this causal structure as a guide to choose an appropriate curvature form on the space.

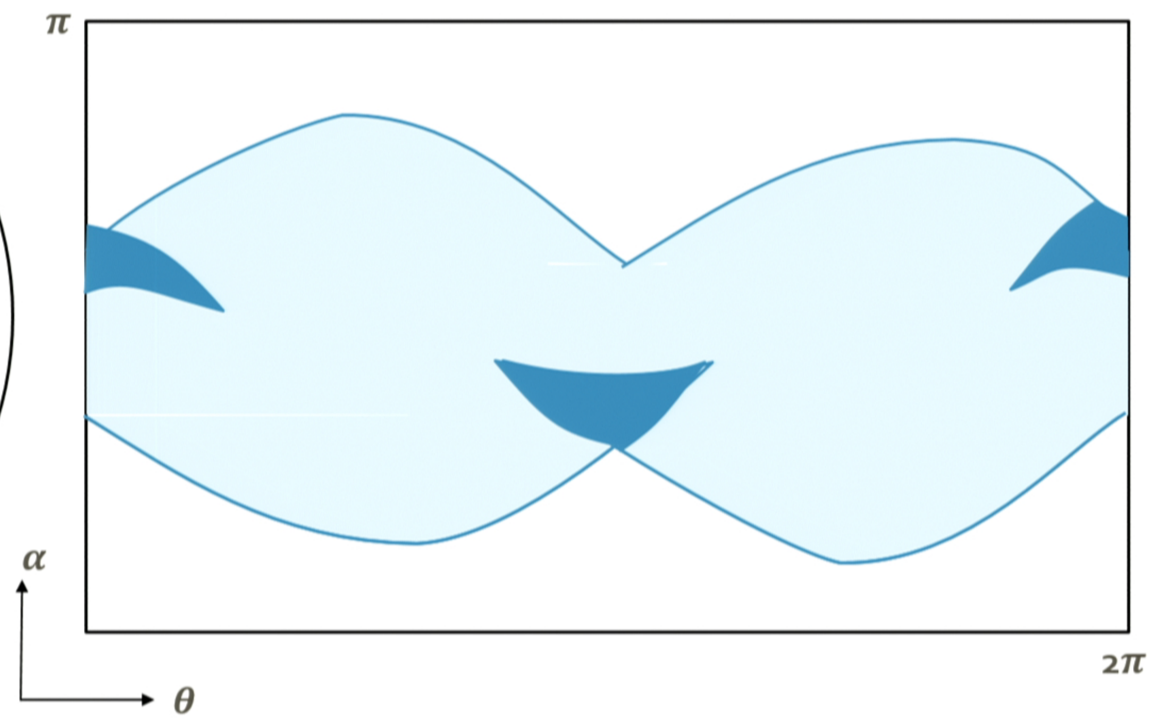
- There is a natural Crofton Form that reproduces the lengths of any curve in any geometry whose tangent space is covered by boundary-anchored geodesics.
- The Crofton measure is given by

$$\mathrm{d}^2\gamma = \partial_u \partial_v L(u,v) \mathrm{d}u \mathrm{d}v$$

- It will be natural again to interpret this density as the curvature density $\mathrm{d}^2\gamma = \mathrm{d}V R$. This is the curvature form of some asymptotically de Sitter, Lorentzian metric on the cylinder

- Cusps from non-convex curves are jut regions with higher intersection number because geodesics can enter and leave a region multiple times



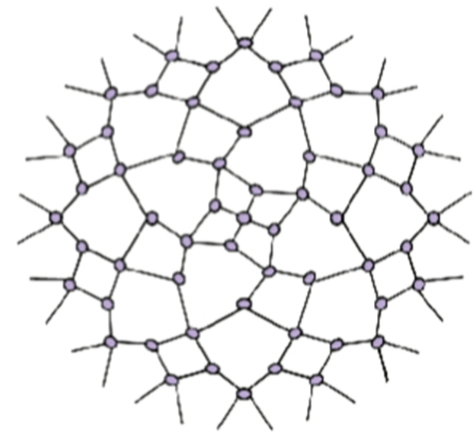


LESSONS FROM MERA

What is MERA?

[Vidal]

- MERA is a variational ansatz for solving the ground state of a CFT on a spatial lattice

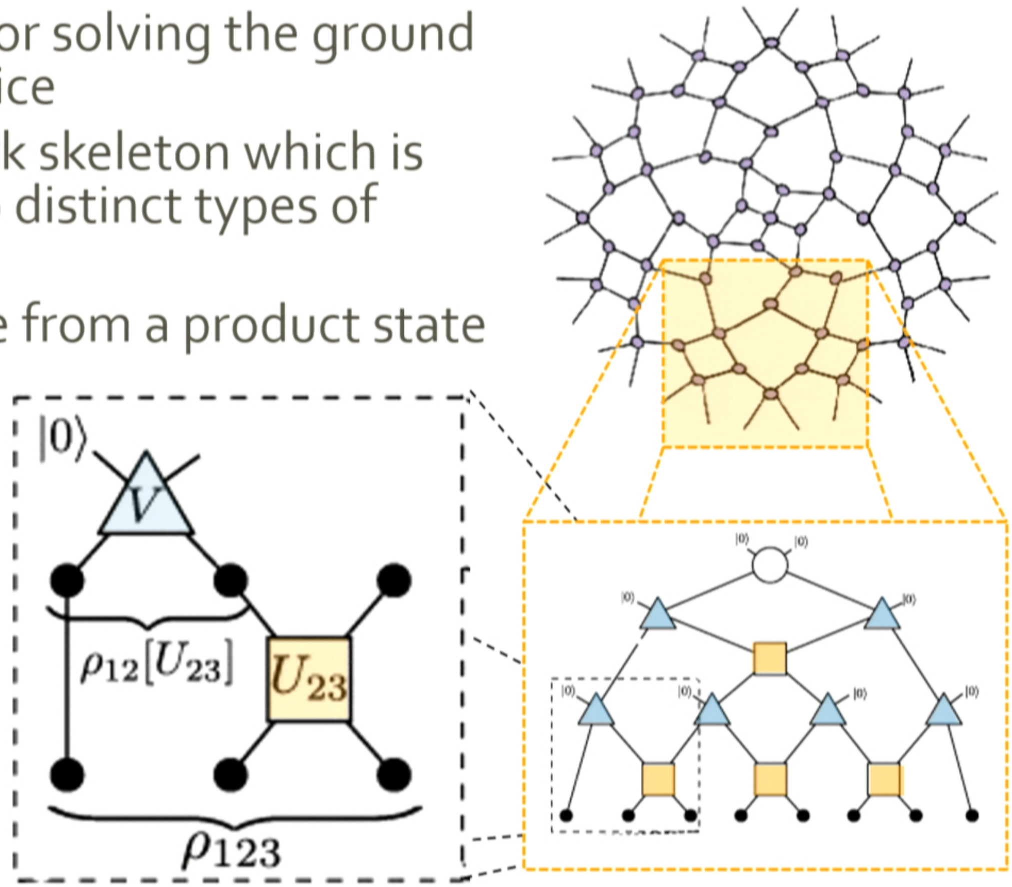


[Img: Evenbly, Vidal]

What is MERA?

[Vidal]

- MERA is a variational ansatz for solving the ground state of a CFT on a spatial lattice
- It is a particular tensor network skeleton which is sequentially composed of two distinct types of tensor nodes.
- These nodes prepare the state from a product state by:
  1. The removal of local entanglement by **disentanglers**
  2. Coarse-graining by **isometries**



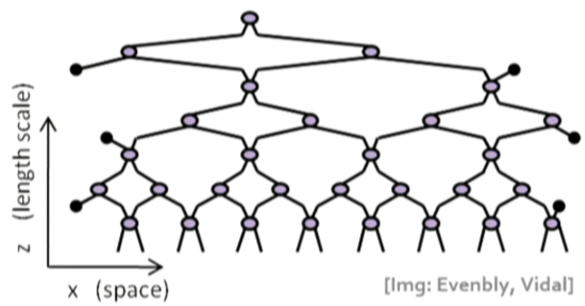
[Img: Evenbly, Vidal]

MERA and Holography

[Swingle; Evenbly, Vidal]

- The MERA network has an additional spatial dimension to the boundary state.
  - It is related to the length-scale of entanglement coarse-graining.
- It is then natural to view the MERA as related to a holographic geometry.

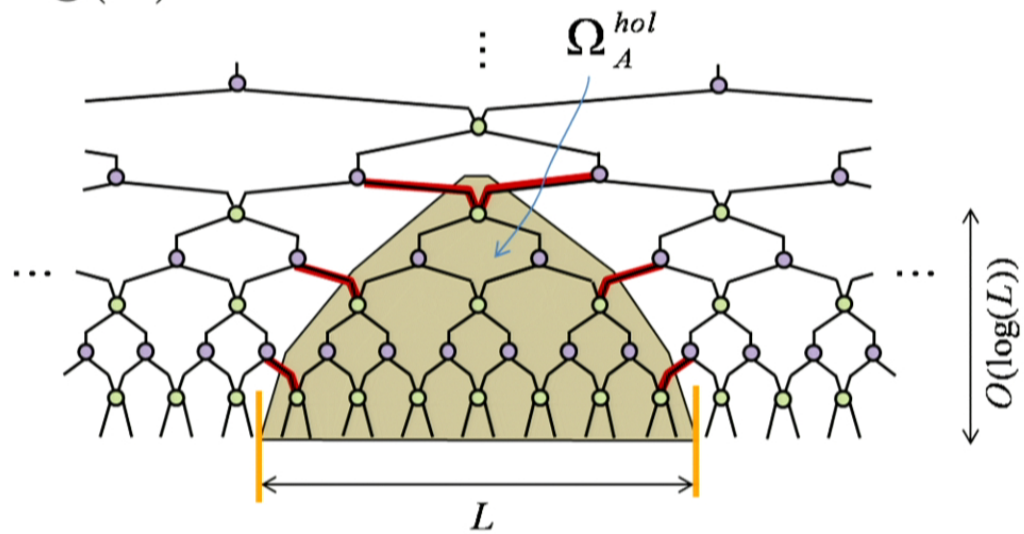
- This can be made precise:
  - Associate a fixed distance to crossing each line of the lattice.
  - In coordinates z = log(L), the corresponding metric is that of a spatial slice of AdS_2:



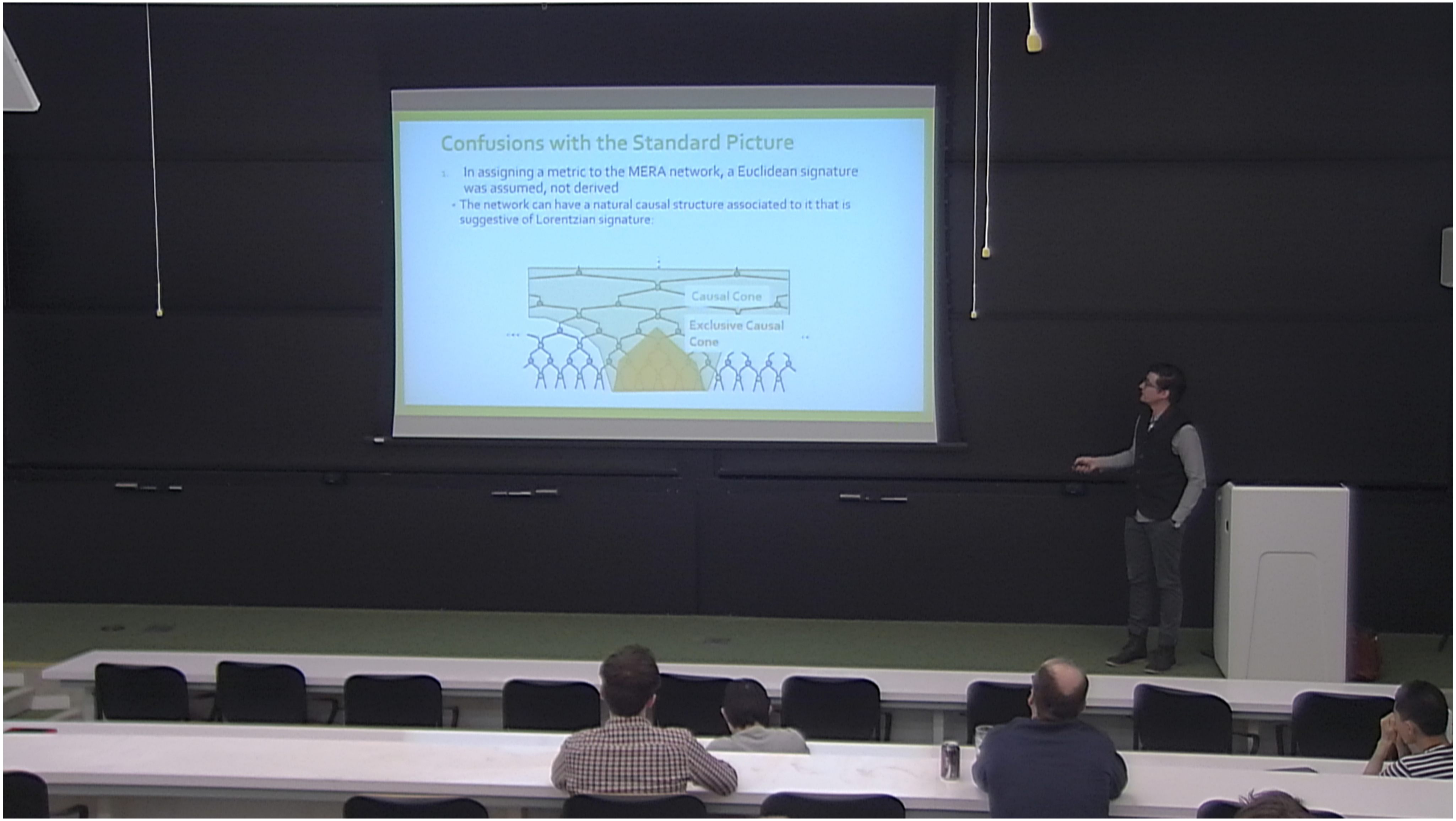
$$\mathrm{d}s^2 = \frac{1}{z^2}\left(\mathrm{d}z^2 + \mathrm{d}x^2\right)$$

- MERA viewed as a discretized AdS2 geometry also realizes the Ryu-Takayanagi procedure for calculating entanglement entropies:
  - Consider a minimal cut in the MERA network that is homologous to some boundary region of length L (measured in lattice sites)
  - The length of the cut is then given by:

$$d \sim \log(L)$$



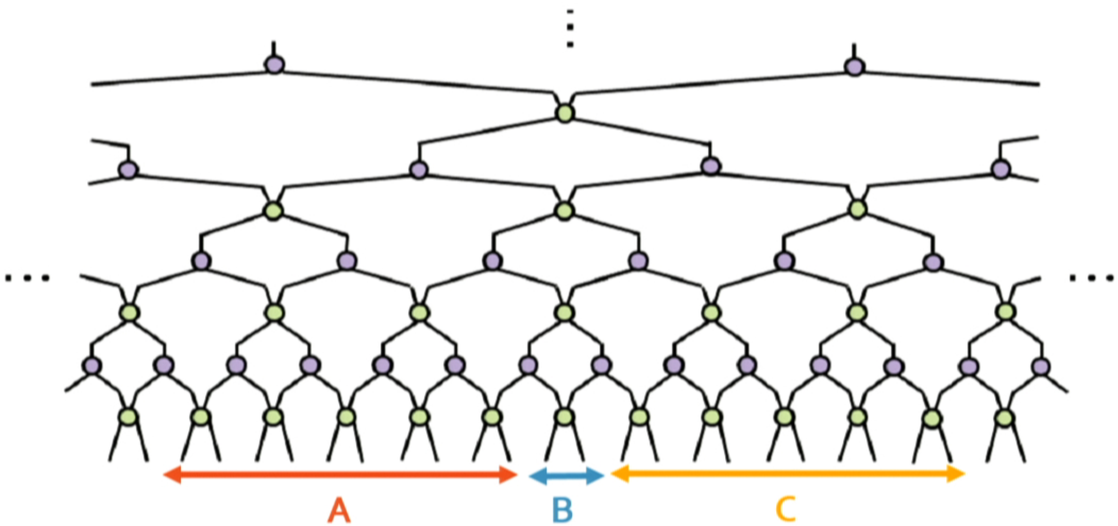
[Img: Evenbly, Vidal]


Confusions with the Standard Picture
1. In assigning a metric to the MERA network, a Euclidean signature was assumed, not derived
• The network can have a natural causal structure associated to it that is suggestive of Lorentzian signature:
Causal Cone
Exclusive Causal Cone

Lessons from MERA for general TN

Principles of Tensor Networks for Gravitational States:

1. The TN should encode compressed states along the 'RT' surfaces.



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Principles of Tensor Networks for Gravitational States:

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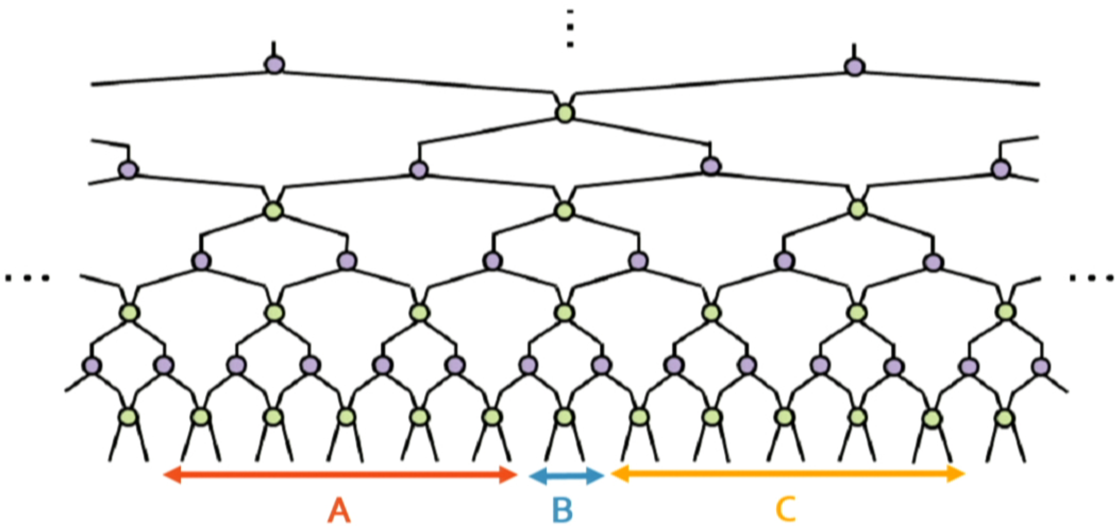
- If the boundaries of the RT region are uncorrelated, then **information and correlations are determined by local properties** of the network:



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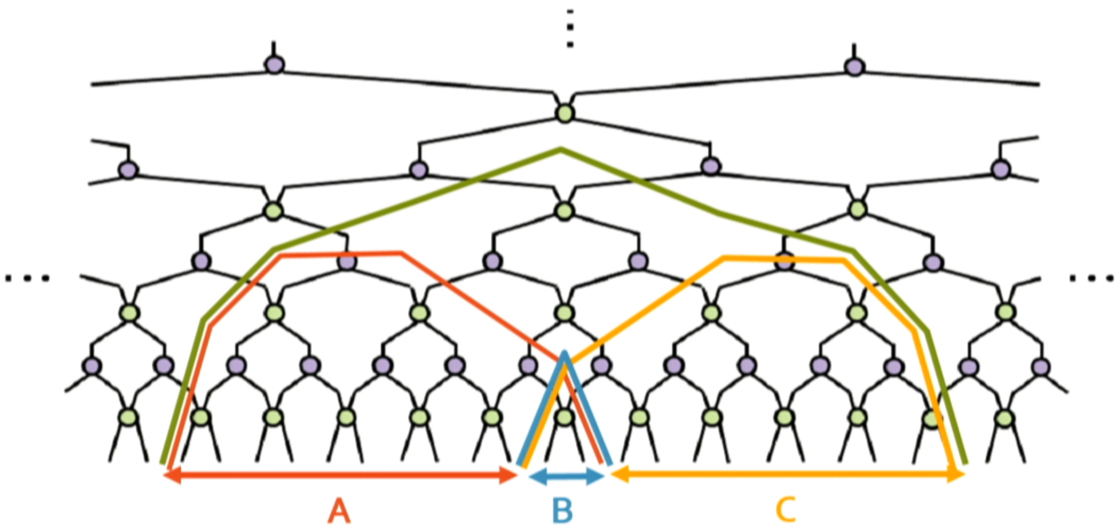

$$S_{AB} + S_{BC} - S_{ABC} - S_{B}$$

Lessons from MERA for general TN

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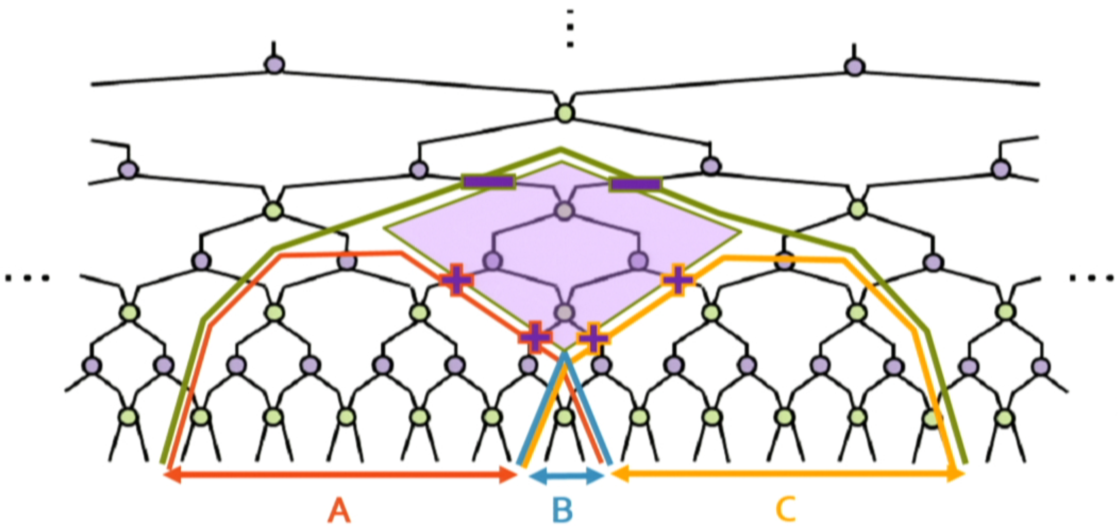


$$S_{AB} + S_{BC} - S_{ABC} - S_{B}$$

$$=$$

$$I(A, C|B)$$

Conditional Mutual Information

BASKET WEAVING 101

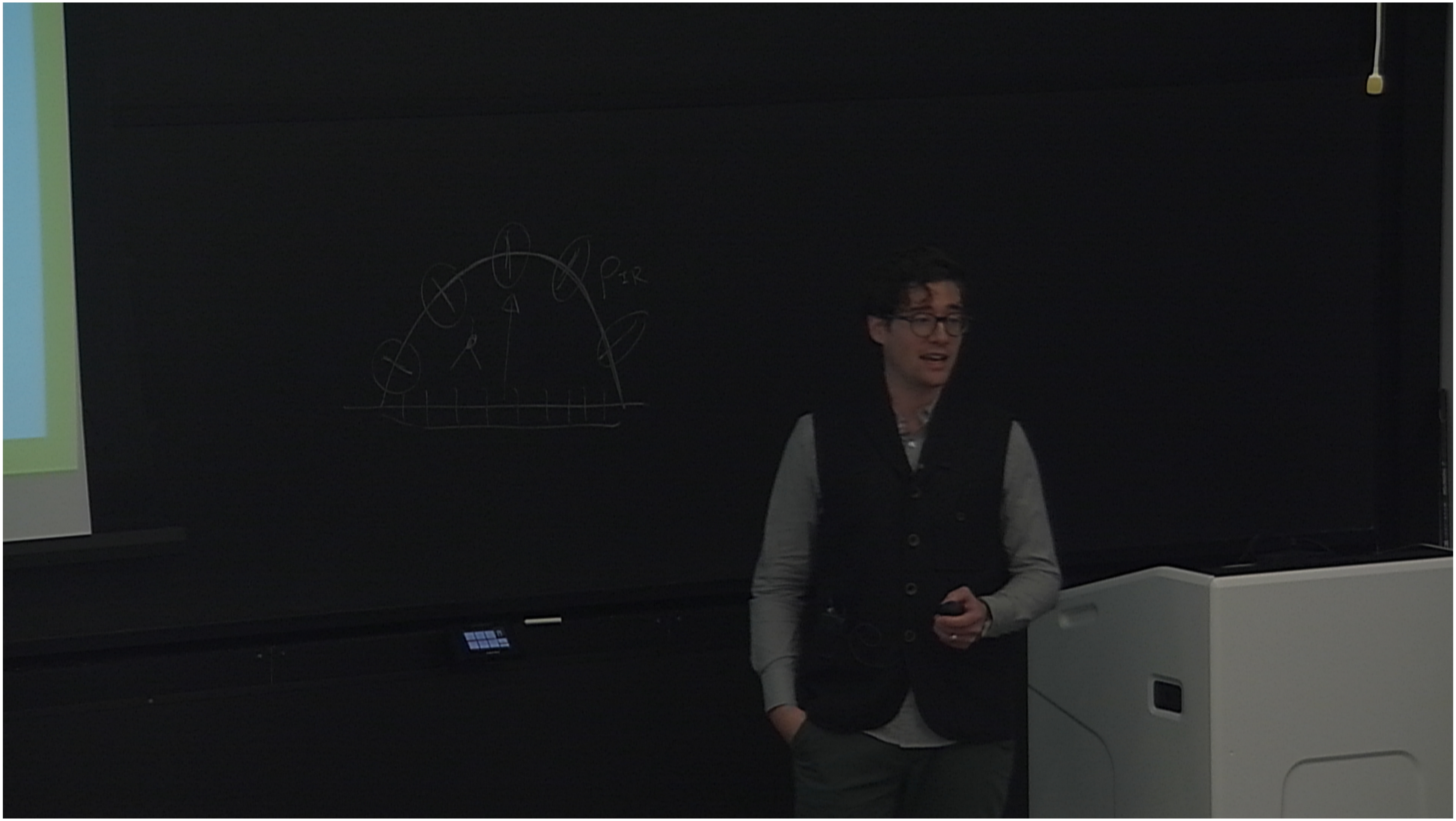
In which we assemble the qbits and pieces of a tensor network

Lessons from MERA for general TN

- Picture from MERA is not perfect. But, I would like to translate the workings of MERA into general principles for constructing a tensor network. What is MERA ***really*** doing that's useful?
- MERA only realizes the RT procedure because the links cut are assumed to be uncorrelated, so distances and cuts are both additive and locally measurable. This is the heart of the connection between tensor networks and geometry.
  - This is equivalent to saying that the RT surface in the TN encodes the **compressed state** of the interval.
- We argue this should be elevated to a principle:

Principles of Tensor Networks for Gravitational States:

1. **The TN should encode compressed states along the 'RT' surfaces.**

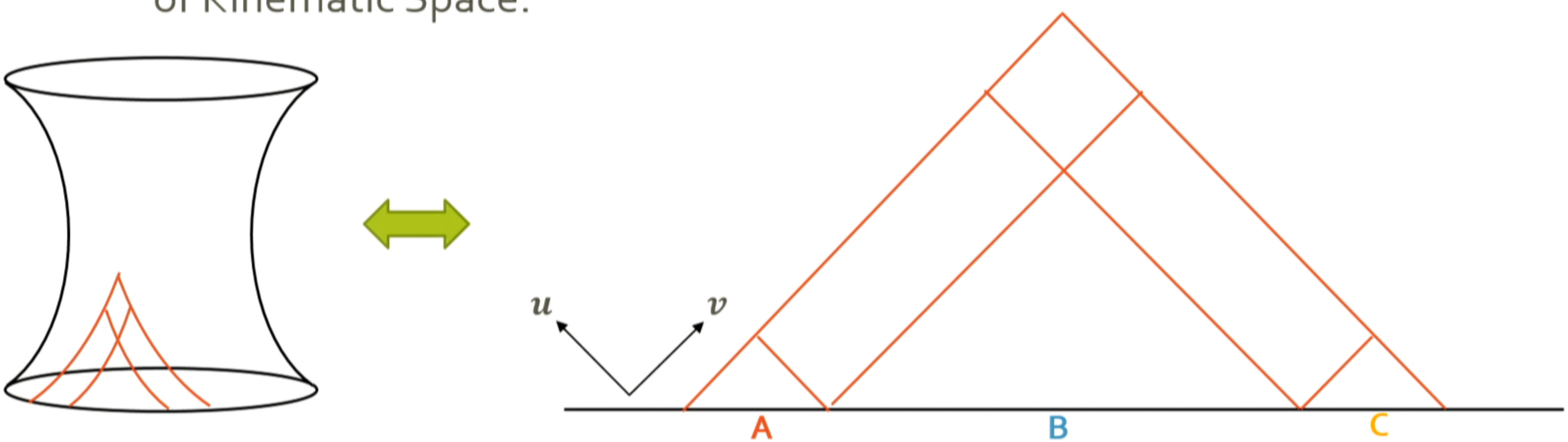


Interpreting Kinematic Geometry

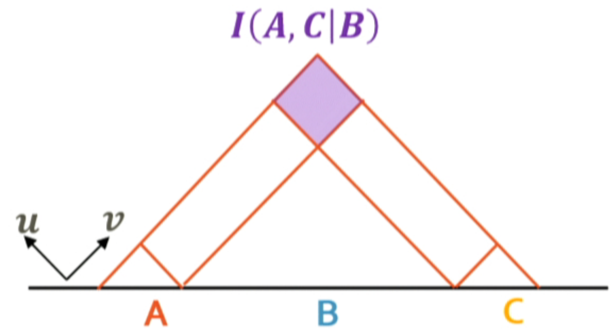
- Recall that local geometric invariants of Kinematic Space can be written in terms of entanglement entropy.
  - Notably, the curvature density was given by

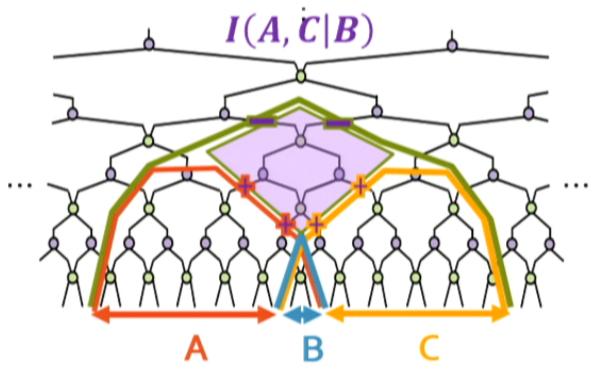
$$\mathrm{d}V\ R = \partial_u \partial_v S(u,v)\mathrm{d}u\mathrm{d}v$$

  - Let's see what this density gives us integrated over specific causal regions of Kinematic Space:



- Notice that the marked region in Kinematic space describes the exact same quantity as does the marked region in MERA for the ground state of a CFT
- It is just a small leap now to suggest that the MERA network is actualy describing not hyperbolic space, but its dual de Sitter space!
  - The 'RT' surfaces should actually be understood as the causal domain of dependence of the boundary region of dS2
  - The tensor nodes aren't describing a point of discretized H2, but are points of discretized dS2 associated to a particular geodesic/boundary interval





- But, we didn't just find a Kinematic space for the AdS vacuum.
- Using our principled approach to understand what MERA is doing, and our knowledge of Kinematic geometry, can we find efficient tensor networks for more general states?
- In hindsight, let's modify our principles:

Principles of Tensor Networks for General States:

1. **The TN should encode compressed states along the boundary of the domain of dependence of a boundary interval.**
   The purpose of the domain of dependence is then to extract all the mutual information a region has with its sub-sytems.

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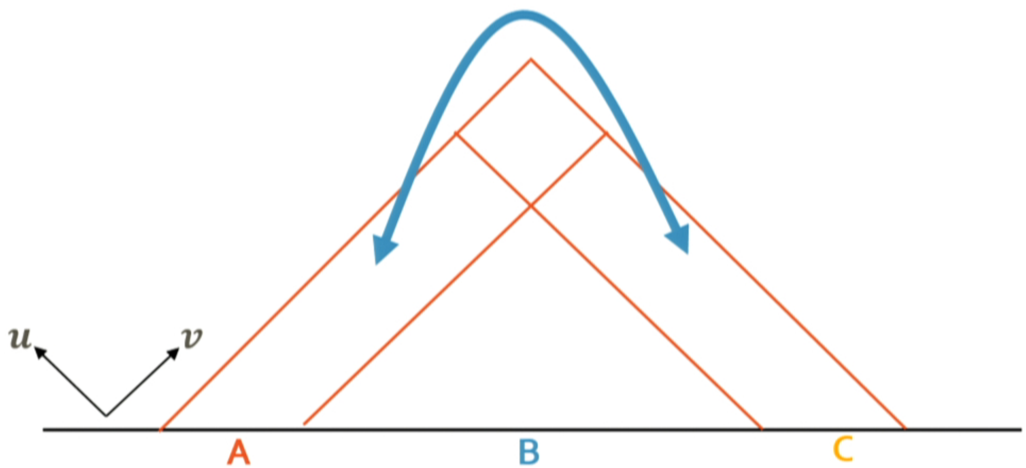
2. **Conditional Mutual Information is encoded locally by the curvature of Kinematic space.**

- The requirement that our network performs compression along the future boundary is extremely powerful:
  - Correlations between subsystems A and C cannot be created outside the future boundary of ABC because this is a compression surface,



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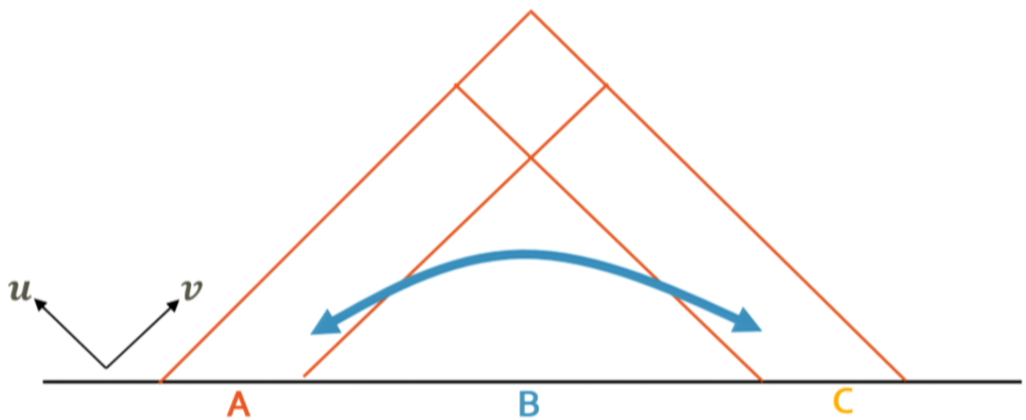
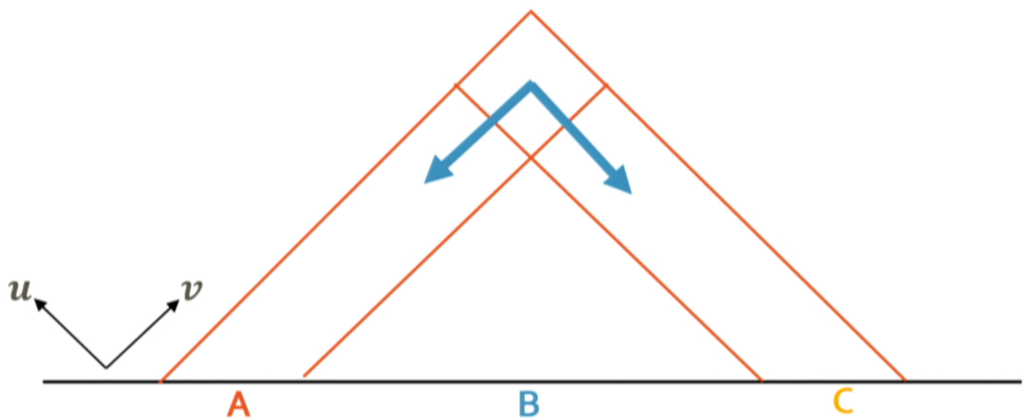
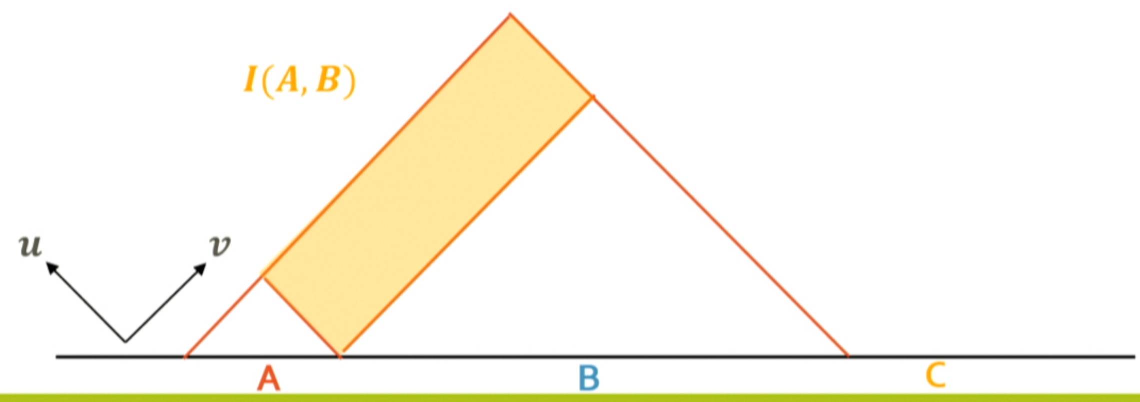


- The requirement that our network performs compression along the future boundary is extremely powerful:
  - Correlations between subsystems A and C cannot be created outside the future boundary of ABC because this is a compression surface,
    - Nor can it propagate outwards from the future boundary of B
- So all correlations must arise from the part of the network in the top diamond
- Information flows in a compression network along light rays



- Consider the colored region of Kinematic space below. It's easy to check that this encodes the Mutual Information I(A,B).
  - Since the red borders encode compressed states for the corresponding boundary region, mutual information between regions A and B can only be created by tensor nodes in the orange region.



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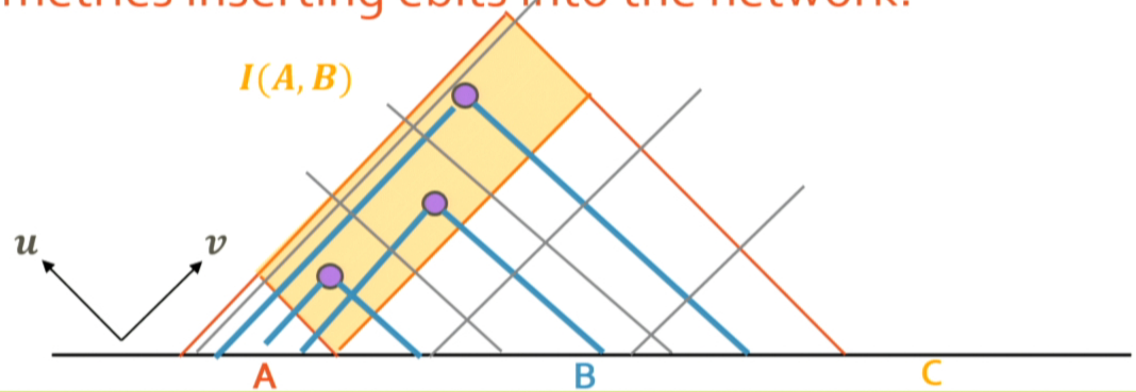
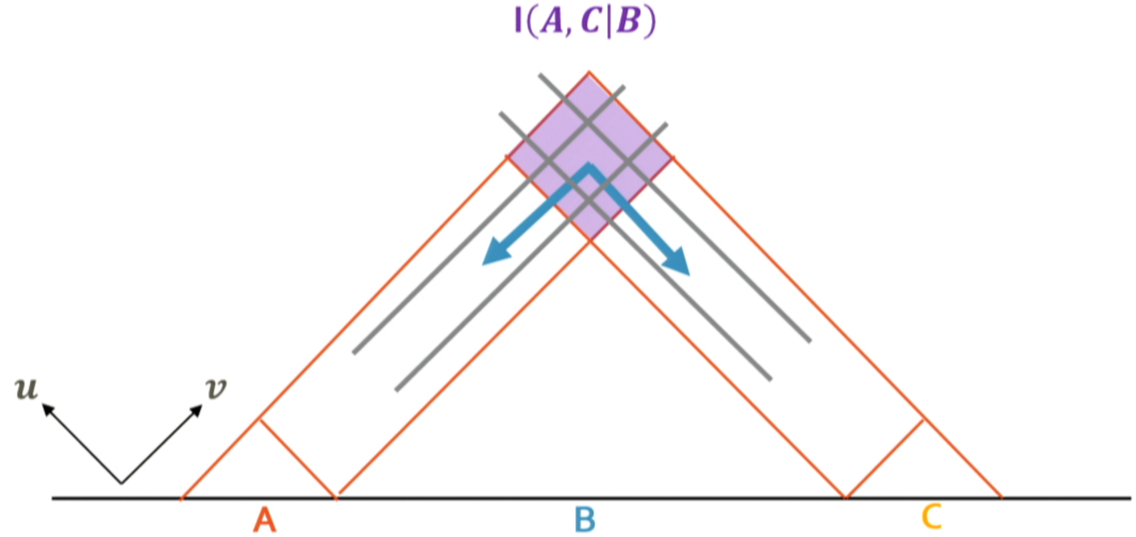
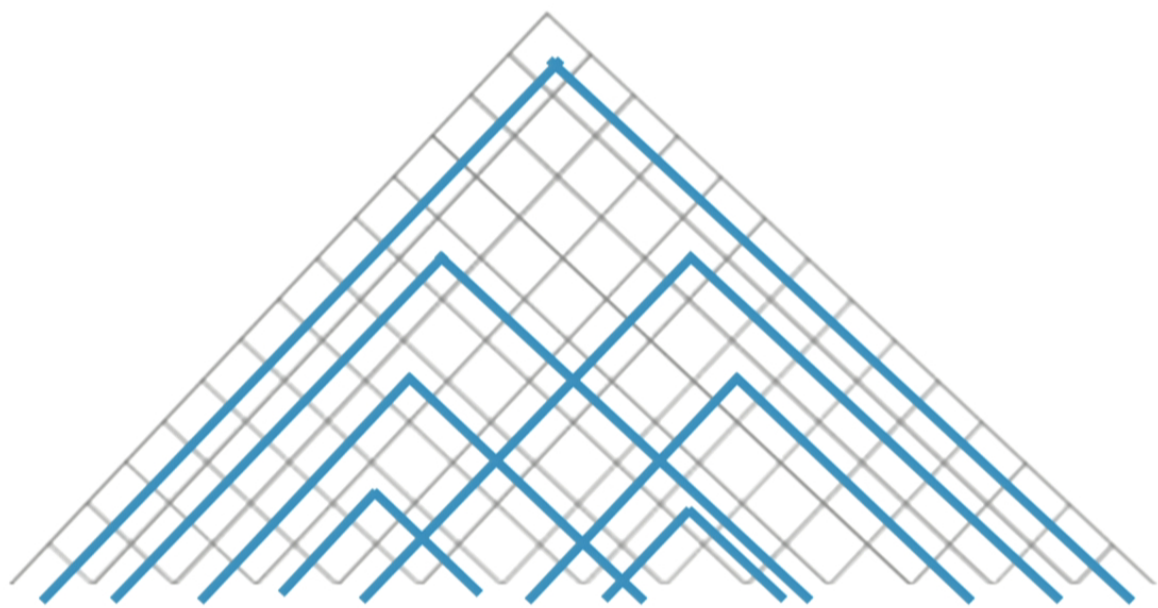
  - Since the red borders encode compressed states for the corresponding boundary region, mutual information between regions A and B can only be created by tensor nodes in the orange region.
  - As in MERA, the density integral must be equal to a boundary term measuring the difference in the number of network lines crossing the top and bottom edges of the region.
    - The cuts are uncorrelated ⇒ additive contributions to entropy.
- This is perfectly consistent if we interpret the curvature integral as measuring the density of isometries inserting ebits into the network.



- Likewise, consider the Conditional Mutual Information I(A,C|B):
  - If all boundaries are uncorrelated, then CMI is just the difference in lines cutting the top and bottom and the colored region
  - The density integral that reproduces this is exactly the density of isometries inserting ebits into the network

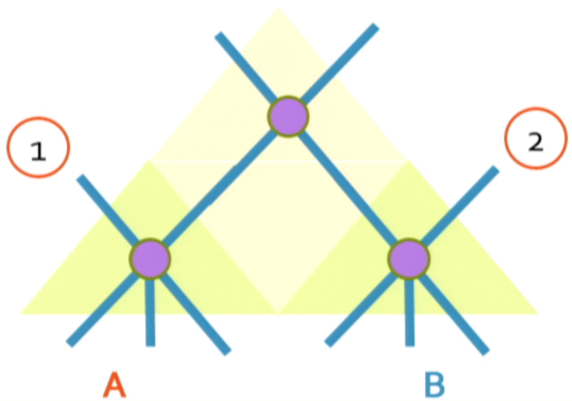


- So for an arbitrary choice of entanglement entropies $S(u, v)$, we can use the corresponding curvature density to add lines to the network that prepare a state with exactly these entropies



The Role of Disentanglers

1. First, we can compress two side-by-side regions A and B independently.
2. But we need to be able to iterate the compression process to the joint system AB.
3. So all mutual information between A and B must be directed to the inward pointing legs.
   The outward pointing legs at 1 and 2 must have zero mutual information because they are unaffected by the next step.



Tripartite Information

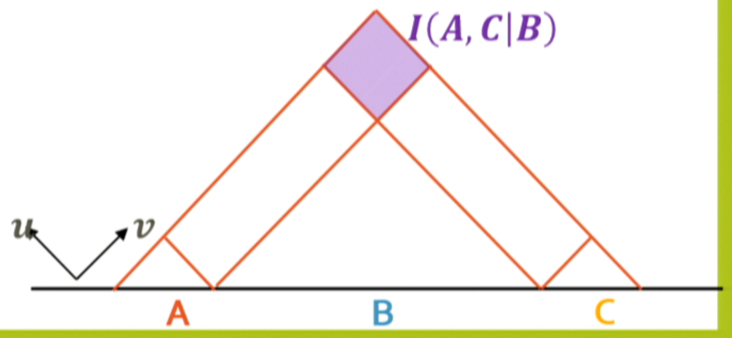
- This is an appropriate time to make an important aside. Tripartite information can also be written

$$I_3(A,B,C) = I(A,C) - I(A,C|B)$$

- Since all mutual information between A and C must arise from the purple region of our network, we conclude that

$$I_3(A,B,C) \leq 0$$

- This is NOT generic, but rather a restriction on the states we can build



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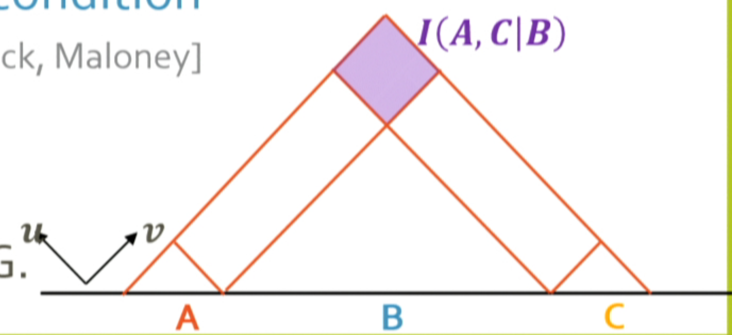
- Negative tripartite information is a necessary condition to have geometric bulk dual. [Hayden, Headrick, Maloney]
- We conclude that our tensor networks can construct **Quasi-Gravitational States.**
  - Should be a wide enough class for interesting QG.



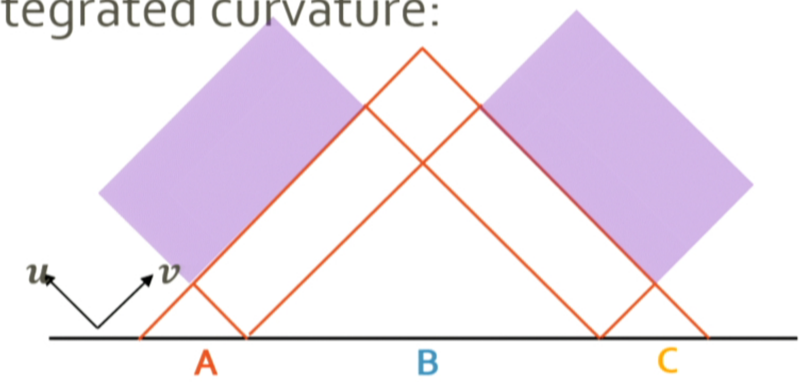
- To understand the relationship between disentanglers and tripartite information, it will be useful to examine their behavior in a **state with a geometric dual.**
  - Let's consider three contiguous regions again, but let's work in the regime where A and C have non-zero mutual information so that

$$S(A \cup C) = S(B) + S(ABC)$$

  - Then some basic algebra gives

$$I_3(A, B, C) = -S(B|A) - S(B|C)$$

- This corresponds to a simple region of integrated curvature:



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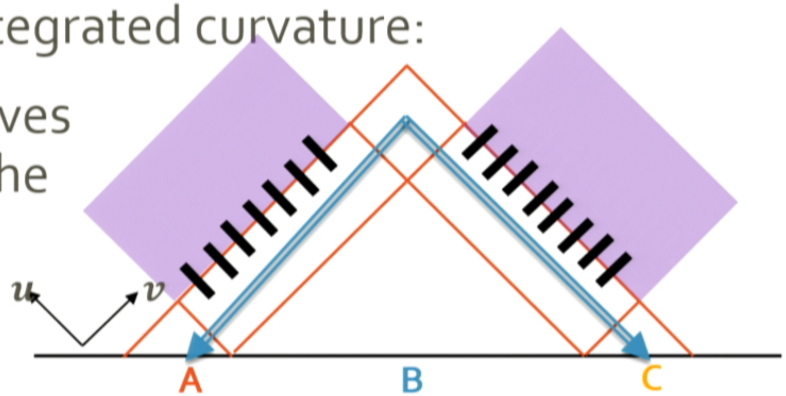
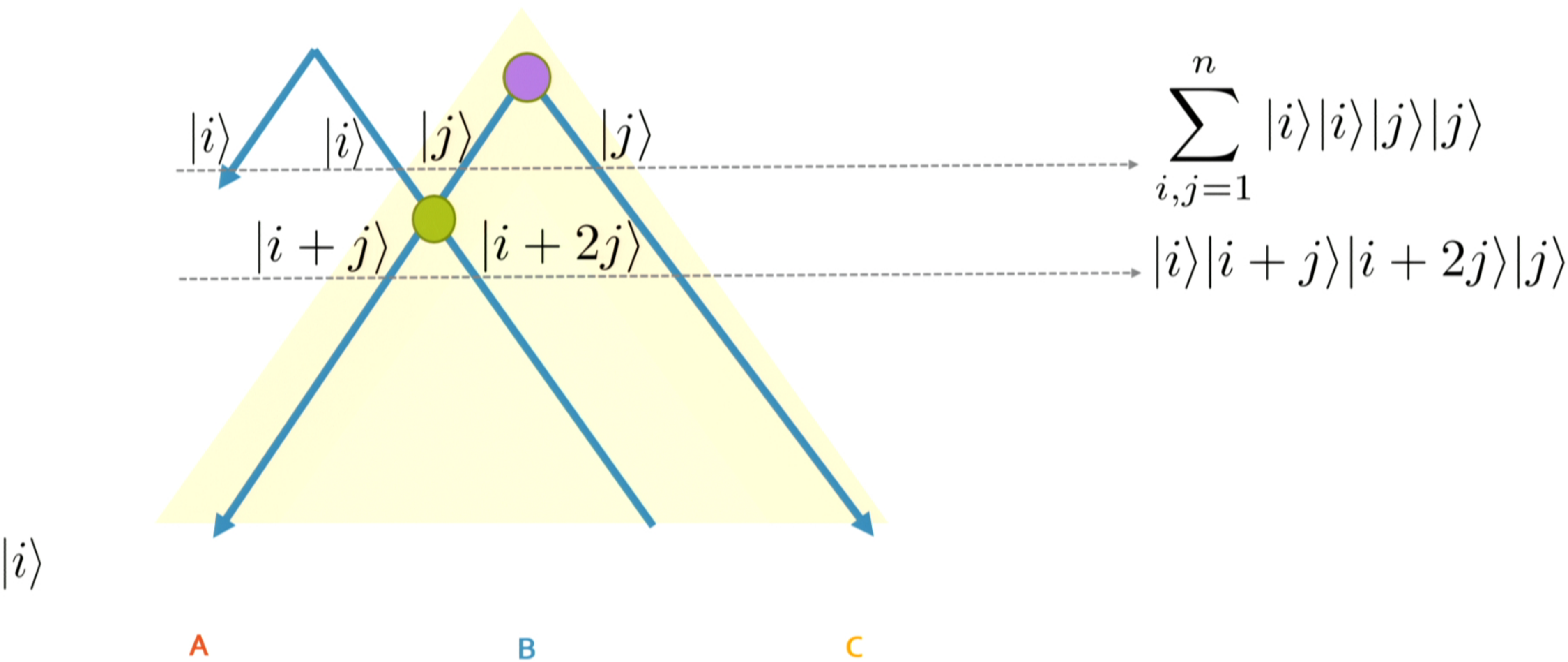
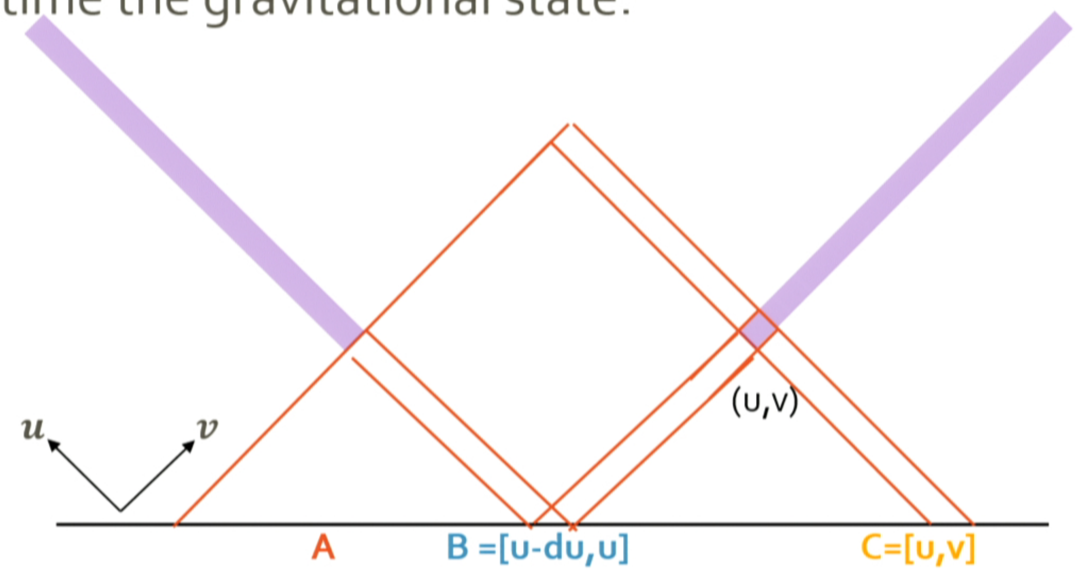
- This corresponds to a simple region of integrated curvature:
- The integrated curvature in this region gives twice the total number of lines crossing the lower surface. So each line destroys one ebit of mutual information.



- We can easily model this mixing that destroys the potential mutual information between two regions:



- Gravitational states then can be seen to be 'maximally mixing' in that every pair of crossing lines must distribute the information amongst themselves as does our toy model, and in doing so destroys the maximal amount of information.
- We want to quantify how far away our disentanglers are away from a gravitational state where they are maximally mixing.
- To do this, consider one last time the gravitational state:



- Thus in a general state, we define ***two different*** metric/curvature structures:

$$\mathrm{d}V^{(3)}R^{(3)} = -\partial_v I_3(A, [u - du, u], [u, v])dv$$

$$\mathrm{d}V^{(2)}R^{(2)} = \partial_u \partial_v S(u, v)dudv$$

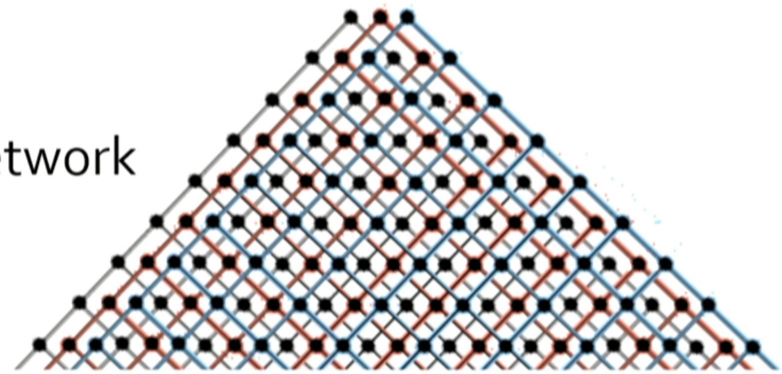
- The condition that our state has a good geometric dual is that these two densities are identical.
  - In that case, we can count the insertion of lines into our network either by mutual information created, or dually by mutual information destroyed.

IMPLICATIONS AND OPEN QUESTIONS

Orbifolds

- Our tripartite metric is sufficient to distinguish between N copies of a CFT and the symmetric orbifold:

- For N copeies, the different copies of the network don't mix with each other.



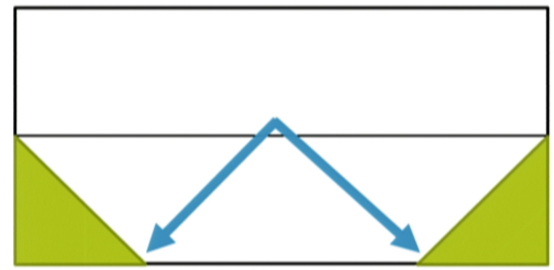
$$\mathrm{d}V^{(2)}R^{(2)} = N\mathrm{d}V^{(3)}R^{(3)}$$

- In the symmetric orbifold theory, the lines becomes sufficiently mixed between the different copies so that

$$\mathrm{d}V^{(2)}R^{(2)} = \mathrm{d}V^{(3)}R^{(3)}$$

Smooth IR Geometry vs Singularities

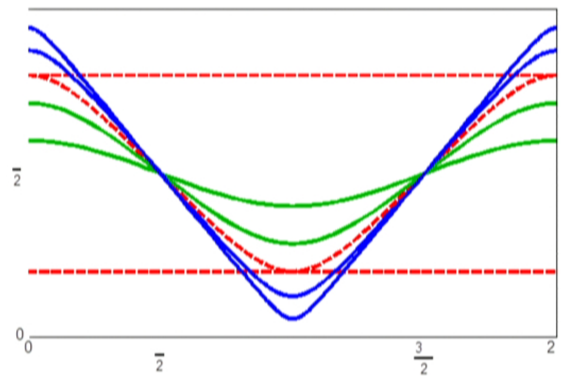
- Our construction of the tensor network depended on our ability to cleanly direct information to the L or R. What happens at large distances when this breaks down?
  - Our L/R distinction fails precisely when $\alpha > \pi/2$. Do we need to be worried about constructing the network in this region?

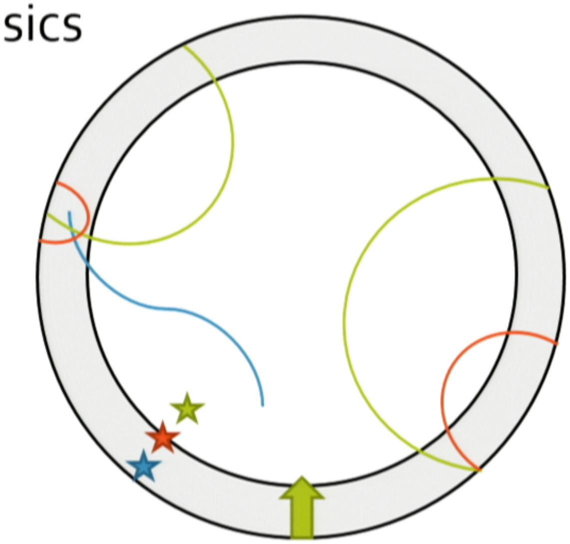


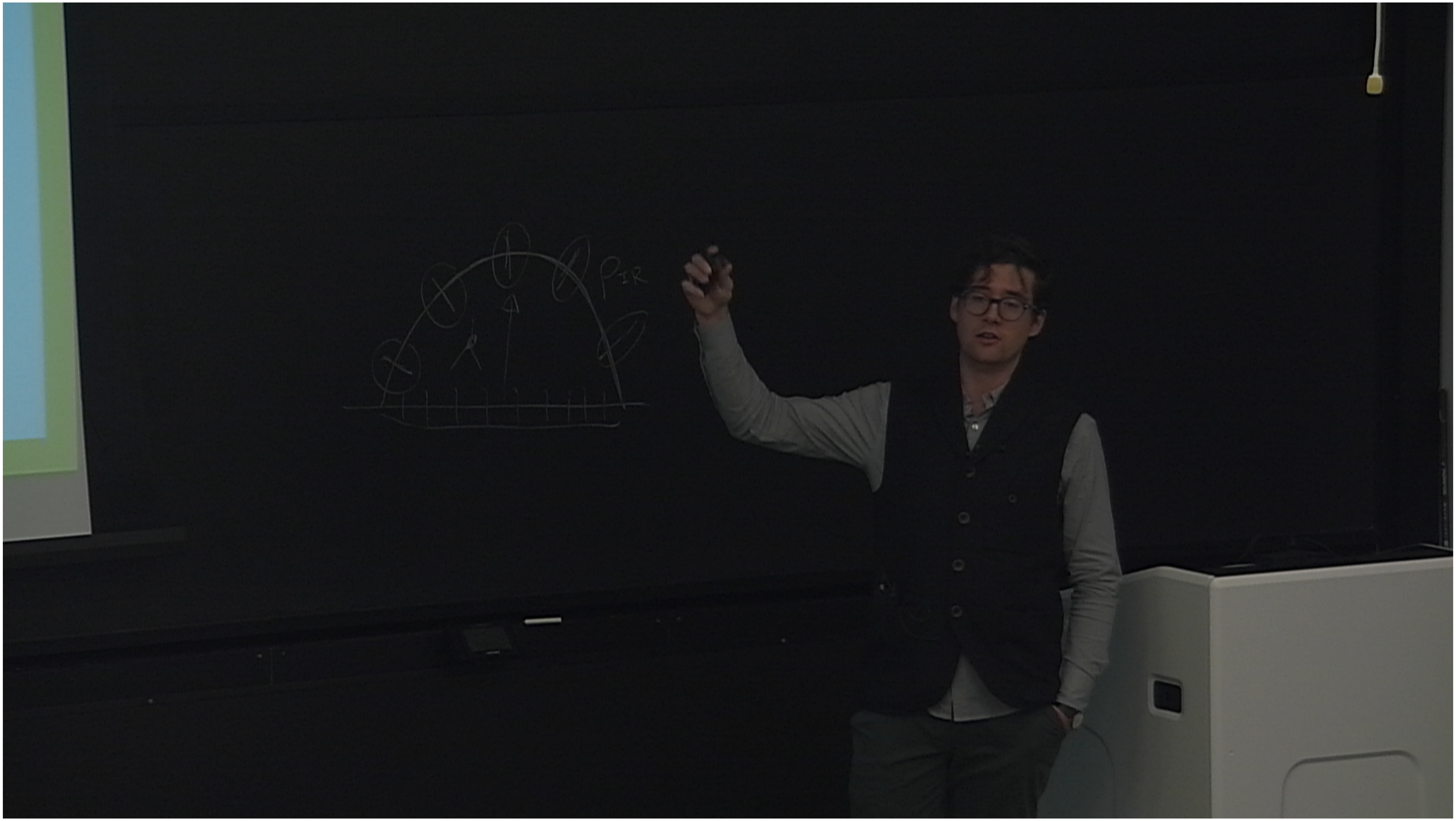
- Surprisingly, no! We know from differential entropy that the length of a curve that circles the origin is 0.
  - So it must follow that there are no lines that cross the corresponding surface in Kinematic space as we approach a point.

Kinematic Space and Holographic RG

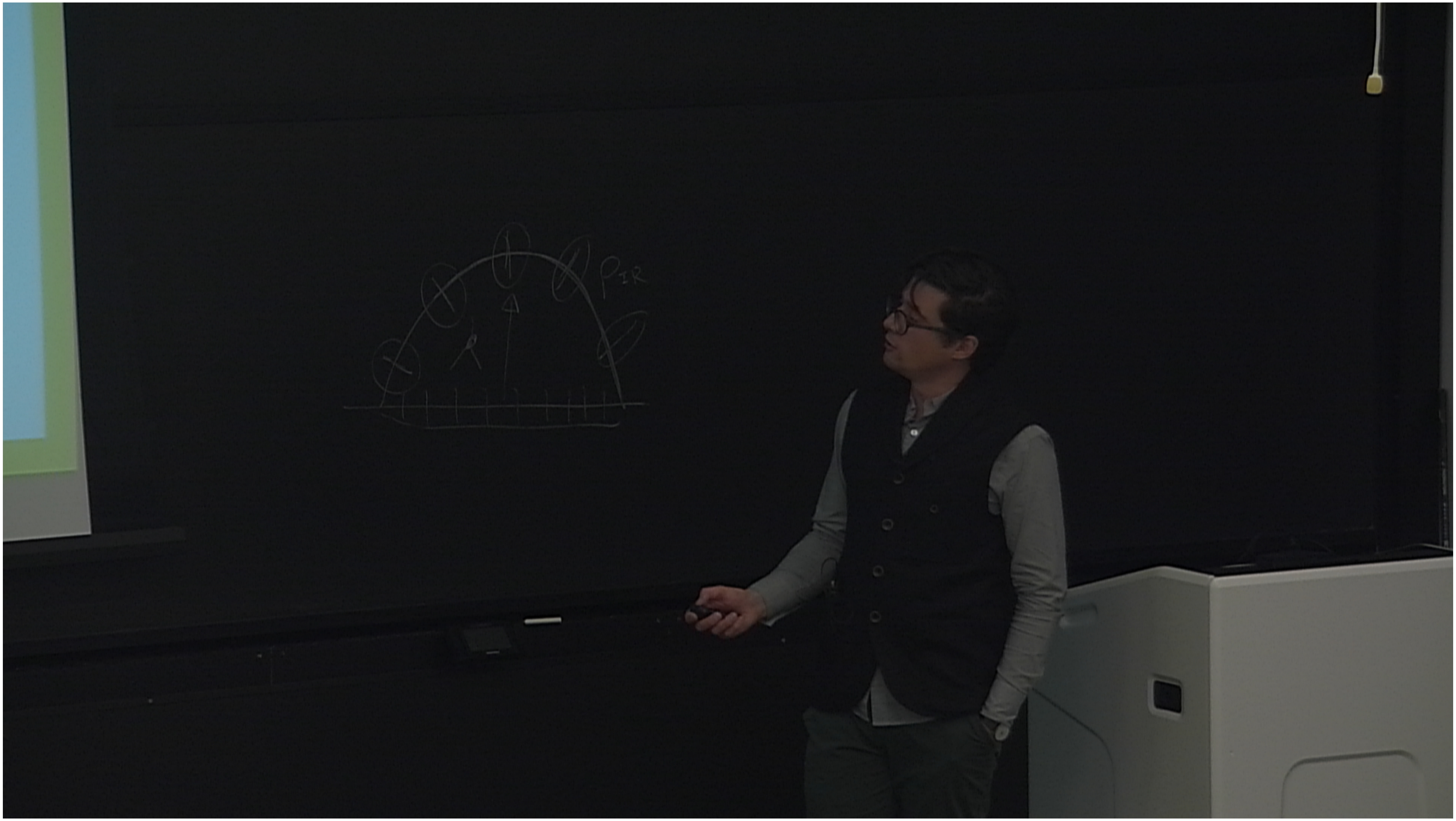
- Natural to ask what the real space holographic RG looks like in Kinematic Space.
  - It is not just as simple as removing the top and bottom or undoing part of the tensor network.
- Easy to see what's happening by examining the geodesics themselves under RG.
  - See that that the bulk RG can be understood as a transformation on the causal structure of the dual space.



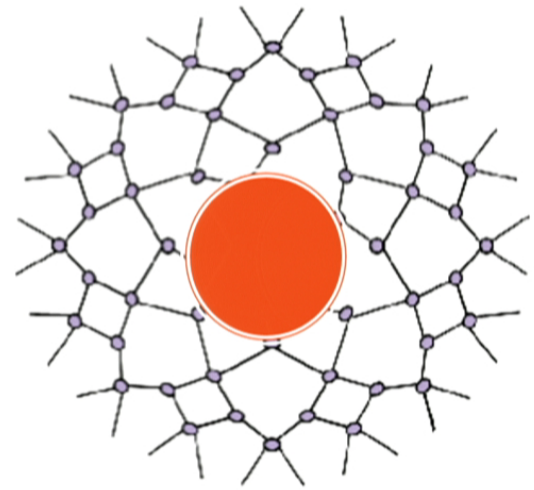


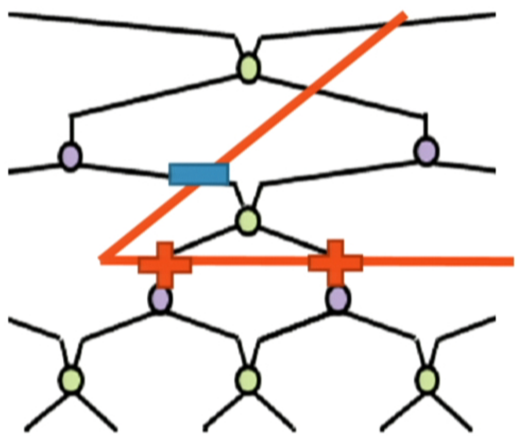


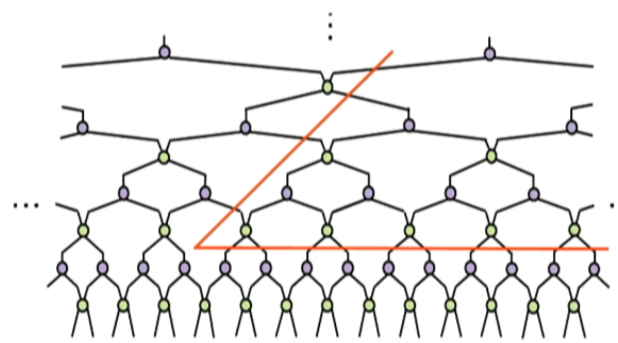
ρ_{IR}


ρIR

- But the non-perturbative perspective given to us by MERA sees the counting very differently.
- We are counting the different states that can be prepared by different choices of the IR network.
- What does this extra spike do to the entropy?
- **Not all cuts can count positively towards the entropy.**
  - Some must be interpreted as **negative constraints**.







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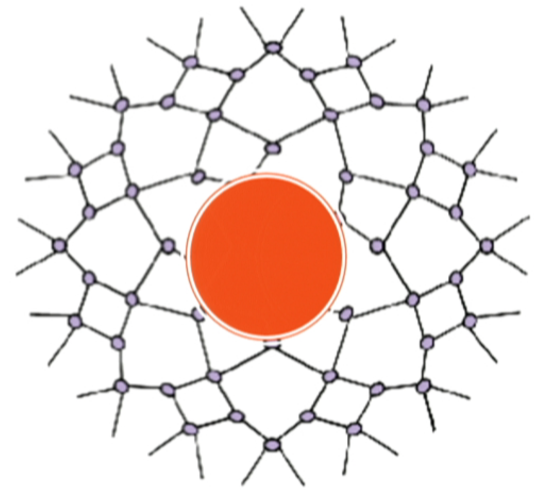
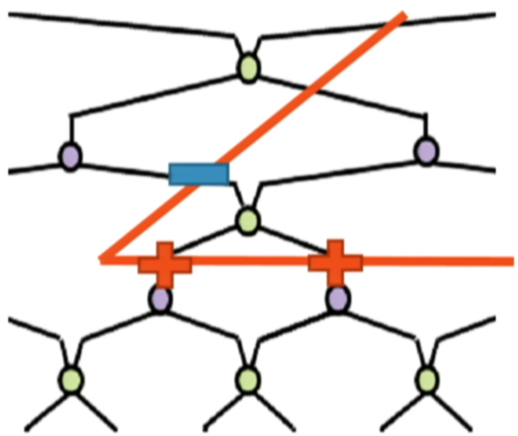
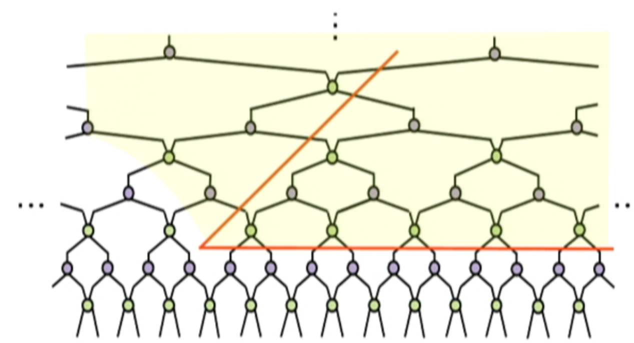
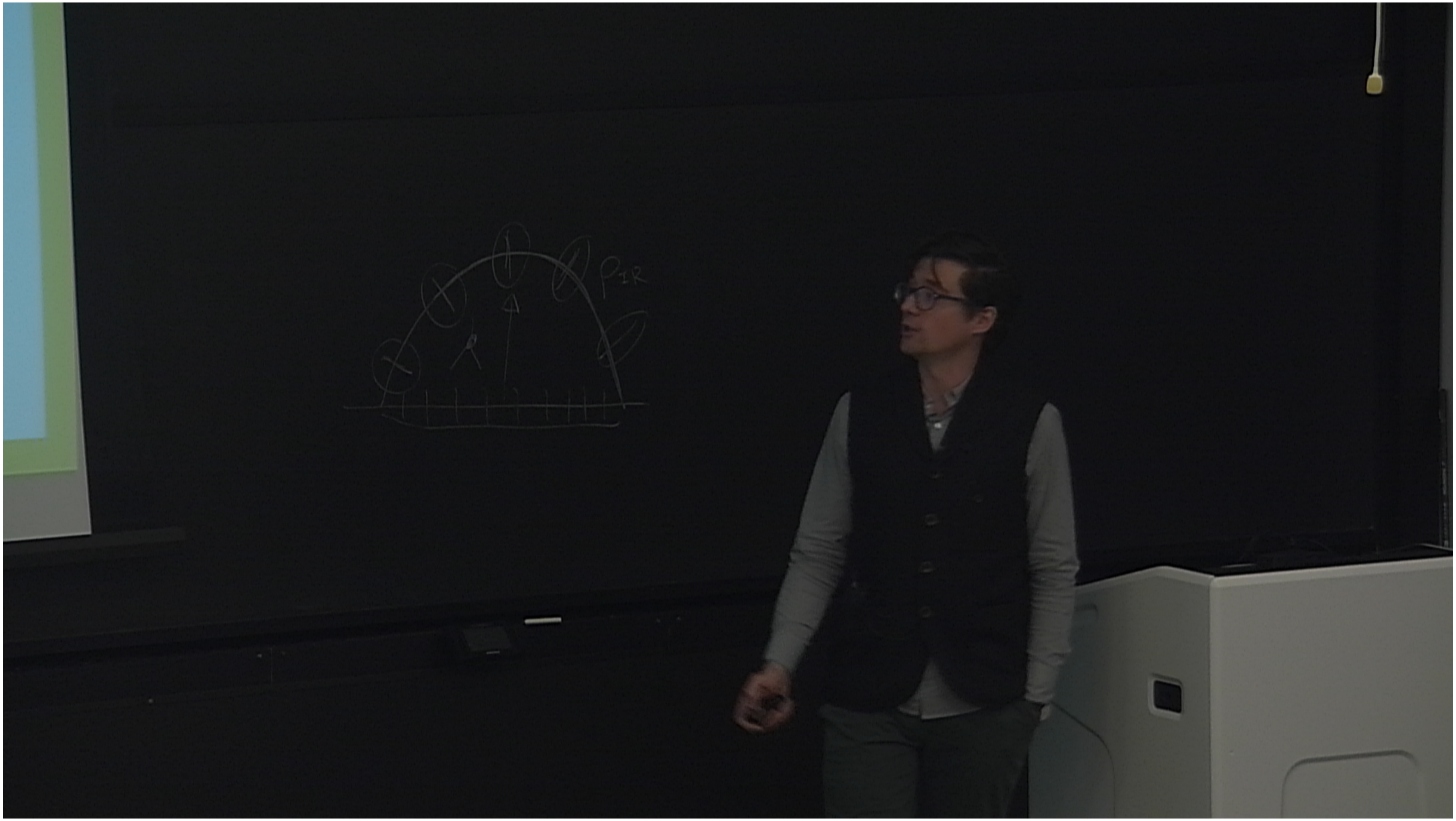






Crosses the causal cone of sites already cut by surface.



ρ_{IR}